Multi-objective Modeling of Ground Deformation and Gravity Changes of Volcanic Eruptions

Piero Conca[1(✉)], Gilda Currenti[2], Giovanni Carapezza[1], Ciro del Negro[2], Jole Costanza[3], and Giuseppe Nicosia[1]

[1] Department of Computer Science, University of Catania, Catania, Italy
pieroconca@gmail.com
[2] Istituto Nazionale di Geofisica e Vulcanologia (INGV), Catania, Italy
gilda.currenti@ingv.it
[3] Istituto Italiano di Tecnologia (IIT), Milan, Italy

Abstract. Inverse modeling of geophysical observations is becoming an important topic in volcanology. The advantage of exploiting innovative inverse methods in volcanology is twofold by providing: a robust tool for the interpretation of the observations and a quantitative model-based assessment of volcanic hazard. This paper re-interprets the data collected during the 1981 eruption of Mt Etna, which offers a good case study to explore and validate new inversion algorithms. Single-objective optimization and multi-objective optimization are here applied in order to improve the fitting of the geophysical observations and better constrain the model parameters. We explore the genetic algorithm NSGA2 and the differential evolution (DE) method. The inverse results provide a better fitting of the model to the geophysical observations with respect to previously published results. In particular, NSGA2 shows low fitting error in electro-optical distance measurements (EDM), leveling and micro-gravity measurements; while the DE algorithm provides a set of solutions that combine low leveling error with low EDM error but that are characterized by a poor capability of minimizing all measures at the same time. The sensitivity of the model to variations of its parameters are investigated by means of the Morris technique and the Sobol' indices with the aim of identifying the parameters that have higher impact on the model. In particular, the model parameters, which define the sources position, their dip and the porosity of the infiltration zones, are found to be the more sensitive. In addition, being the robustness a good indicator of the quality of a solution, a subset of solutions with good characteristics is selected and their robustness is evaluated in order to identify the more suitable model.

1 Introduction

Mt Etna is one of the best monitored and most studied active volcanoes worldwide. Since the Eighties a large number of multiparametric geophysical surveys

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have been carried out on the ground surface to gain insights into the activity of the volcano. One of the first historical dataset dates back to the 1981 eruption, which is remembered because of its intensity in terms of effusive rate and amount of lava emitted, despite the relatively short time duration of the eruptive activity. Attempts had been made to separately model the recorded dataset [2,3,15]. Among the different hypotheses formulated, Bonaccorso [2] interpreted the geodetic observations (leveling and EDM) by suggesting the activation of two magmatic intrusions oriented northward: the initial deeper one starting from the summit craters and the shallower one feeding the final effusive fractures. This hypothesis was considered later on to implement a computational model of the 1981 eruption [3], with the purpose of getting a more comprehensive picture of the intrusive mechanism related to the 1981 flank eruption of Mt Etna through a joint inversion of all the available dataset (microgravity, leveling and EDM). A multi-objective optimization was performed to search the space of the model parameters and find a solution that closely fits the geophysical measurements [6,13,18]. In order to explain the discrepancy between the intrusive volumes estimated by geodetic and gravity data, the model was modified to account for the porosity of the host rock. That model was optimised by means of the evolutionary multi-objective optimization algorithm NSGA2 [8]. This paper provides insight into the optimization of the computational model proposed in [3]. In particular, it presents further investigation of the optimization capabilities of NSGA2 and, in addition, it also applies the single-objective DE algorithm to evaluate its performance with respect to the NSGA2. The paper also presents the results of a sensitivity analysis of the model in order to identify the parameters that have higher influence on its performance. Finally, an analysis of the robustness of a set of solutions is presented.

2 Single-Objective and Multi-objective Optimization

Geophysical inversion in volcanic areas focuses on exploiting data from different monitoring techniques (geodesy, gravimetry, magnetism), physical models and numerical approaches in order to identify likely magmatic sources and gain insights about the state of the volcano. Indeed, the geophysical observations collected on a volcano are the surface expressions of processes that occur deeply within the volcanic edifice. Magma migration and accumulation generate a wide variety of geophysical signals, which can be observed before and during eruptive processes. Magma ascent to the Earth's surface forces crustal rocks apart engendering stress and displacement fields and producing variations in the gravity field due to modifications in the subsurface density distribution. Ground deformation and gravity changes are generally recognized as reliable indicators of unrest, resulting from the uprising of fresh magma toward the surface. Measurements of these geophysical signals are useful for imaging the spatio-temporal evolution of magma propagation and for providing a quantitative estimate about the magma volume rising from depth. Deformation and gravity changes are generally interpreted separately from each other using physics-based models, which provide an estimate of the expected geophysical observation produced by volcanic sources. The consistency of interpretations from different observations is qualitatively checked only a posteriori. An integrated geophysical inversion based on both data set should prove a more efficient and accurate procedure for inferring magmatic sources and minimizing interpretation ambiguities. The geophysical inversion is formulated as an optimization problem, which searches the magma source parameters (location, geometry, volume, mass, etc.) $\mathbf{m} = \{m_1, \ldots, m_p\} \in M$ in order to minimize the misfit between the values of geophysical observations and their respective values estimated by the physics-based forward model. The joint inversion of different geophysical observables implies that the misfits for each *i-th* dataset are simultaneously minimized:

$$f_i\left(\mathbf{m}\right) = \left\|g_i\left(\mathbf{m}\right) - d_i^{obs}\right\| \quad \text{for} \quad i = 1, \ldots, k. \tag{1}$$

where f_i is an objective function and denotes the difference between the value calculated through $g_i(\mathbf{m})$ (forward model) and the observed value d_i^{obs} for each *i-th* geophysical observable. Therefore, the joint inversion of a multiparametric geophysical dataset can be regarded as a multiobjective optimization problem (MOP). Solving this problem means to find the set of model parameters $\mathbf{m}^*$ that satisfies a set of constraints and optimizes the objective function vector, whose elements are the objective functions:

$$\begin{aligned} &\mathbf{m}^* = \min_{\mathbf{m} \in M} F(\mathbf{m}) \quad \text{with } m_j^{min} \leq m_j \leq m_j^{max} \text{ and } j = 1, \ldots, p, \\ &\text{where } F\left(\mathbf{m}\right) = \left[f_1\left(m\right), f_2\left(m\right), \ldots, f_k\left(m\right)\right]. \end{aligned} \tag{2}$$

Here, we set up a MOP to infer the models space parameters $\mathbf{m}$ of the magmatic sources by jointly inverting the microgravity, leveling and EDM (Electroptical Distance Measurements) data gathered spanning the 1981 Etna eruption. Gravity measurements were performed using spring-based relative gravimeters along a profile circumventing the Etna edifice. Gravity changes were computed by differencing the measurements carried out from two surveys in August/September 1980 and July/August 1981, before and after the eruption. Concurrently, levelling surveys were also performed to measure elevation changes of the ground surfaces. Moreover, discrete horizontal deformation were also measured in September 1980 and May 1982 and in October 1979 and June 1981, using the EDM networks in the SW and NE area, respectively. The pattern of these geophysical dataset support the volcanological evidence that the 1981 Etna eruption was characterized by magma intrusions through fractures into the rocks. This geophysical process is simulated mathematically using solutions devised in [11,12] by solving analytically the elasto-static and gravity equations for modeling displacement and gravity changes induced by rectangular fluid-driven fractures. Two intrusive sources and two associated surrounding zones of pre-existing microfractures, which were filled with new magma are considered following the results reported in Carbone et al. [5]. Since the forward models are nonlinear operators, it calls for using robust nonlinear inversion methods. In the frame of multi-objective optimization techniques, we investigate the NSGA2 algorithm.

In order to improve the search for solutions, the population of solutions and the number of generations are increased with respect to the experiments reported in [3]. In particular, the size of the population has been increased from 500 to 1,000 individuals, while the number of generations has been increased up to 10,000 from the value of 800. In addition, the single-objective optimization technique of Differential Evolution (DE) has also been used to optimize the parameters of the model. This technique evolves a population of solutions without calculating the derivatives of the objective function. The parameters that control the DE algorithm are the *scale* and the *crossover probability* that in our case have, respectively, the values 0.8 and 0.7. The population contains 1,000 individuals and is optimized for 10,000 generations, in order to perform the same number of objective function evaluations as NSGA2 and therefore provide a fair comparison. In this context the three misfits used for the multi-objective optimization (leveling, EDM and gravity) are combined into a single-objective function which is expressed by the following formula:

$$\phi(x_i) = \sqrt{\left(\frac{err_{\text{leveling}}(x_i)}{\sigma_{\text{leveling}}}\right)^2 + \left(\frac{err_{\text{EDM}}(x_i)}{\sigma_{\text{EDM}}}\right)^2 + \left(\frac{err_{\text{gravity}}(x_i)}{\sigma_{\text{gravity}}}\right)^2}; \quad (3)$$

where x_i is the i^{th} individual and σ_h are the data uncertainties. An estimate of the data uncertainty is obtained by the standard deviation of each measurements dataset, which is of 0.05 m for the leveling, 0.12 m for the EDM, and 35 μGal for the gravity data. The best solutions generated by the optimization techniques NSGA2 and DE are plotted in Figs. 1 and 2. The figures show that the NSGA2 with a population of 1,000 individuals and 10,000 generations produces better results with respect to the same algorithm using a population of size 500 and 800 generations. By contrast, the solutions generated by the DE algorithm combine lower EDM and leveling errors than NSGA2, but are not able to minimize all measures at the same time, as shown in Fig. 2. The similarity of the output

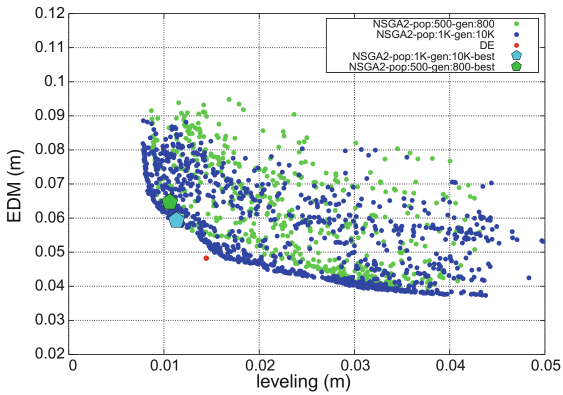

Fig. 1. Leveling error and EDM error of the solutions generated by the optimization techniques NSGA2 with two different parametric configurations and DE.

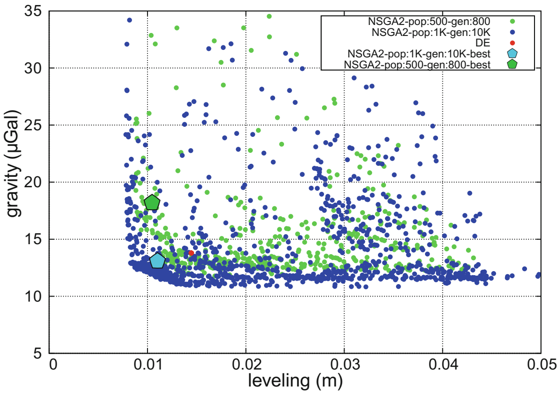

Fig. 2. Leveling error and gravity error of the solutions obtained.

values, which are concentrated in a very small region of the space of values, indicates that these solutions present little differences. This result contrasts with the large diversity of the solutions provided by NSGA2. Moreover, the values of several parameters coincide with the bounds of their respective intervals, this seems to indicate that DE is not able to search the space of parameters effectively. This could be related to the fact that this technique was natively developed for unconstrained optimization, and therefore could be more suitable to that problem rather than constrained optimization. A manual selection, performed by an expert, of the solutions found is displayed in Table 1. Moreover, a map of the Etna and the values generated by the these solutions (found by NSGA2 with a population of 1,000 individuals and 10,000 generations and with a population of 800 individuals and 500 generations) is displayed in Fig. 3.

3 Sensitivity Analysis

Sensitivity analysis (SA) is an important tool for the study of a model [14]. In fact, SA can help understand the behaviour of a model by evaluating the impact of its input parameters on the output. This information could be used, for example, to focus on a subset of parameters when optimization is performed. Moreover, SA allows to the unveil the relations between different parameters.

Concerning the model of the 1981 eruption of Mt Etna, SA is used to identify the characteristics of the magmatic intrusions whose variations affect significantly the output of the model and those which affect it marginally and are, therefore, less relevant. There are several techniques for SA, in our context the technique by Morris and the Sobol' indices were used evaluate the sensitivity of the model.

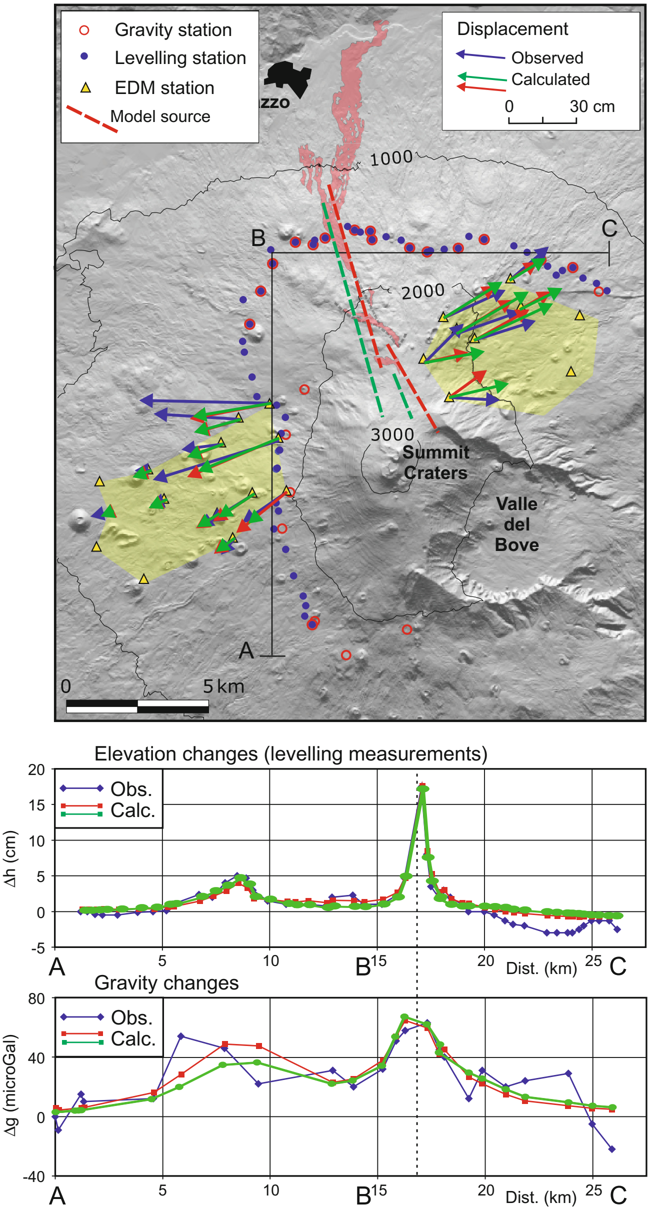

Fig. 3. Map of the Etna showing the locations of the measurement stations and the deformations measured by the EDM sensors and those calculated by the model. The plots at the bottom show the measured values (blue line) and the calculated values of, respectively, elevation and gravity changes for the NSGA2-500-800 (red line) and NSGA-1K-10K (green line) models. The details of the model parameters are reported in Table 1 (Color figure online).

Table 1. Ranges of the values of the parameters and optimal solutions selected by an expert.

Series Parameter	Min.	Max.	NSGA2	DE	NSGA2 [3]
North source					
Z_1^N, depth of the top, m b.s.l	20	20	20	20	20
L^N, length, m	4,000	8,000	6,251	6,184.2	6,703
H^N, height, m	200	500	209.42	200	231.7
W^N, tensile opening, m	0.5	2	1.54	2	0.93
ϕ^N, azimuth (from the north)	−35	−15	−15.69	−31.18	−16
X^N, northing of top center, m	4,181,250	4,186,250	4,185,594	4,184,628	4,184,924
Y^N, easting of top center, m	496,750	499,250	497,887	498,736	497,970
δ^N, dip (from the east)	45	145	111.67	113.02	88.1
$\Delta\rho^N$, density contrast, Kg/m^3	100	500	114.8	100	116.8
North infiltration zone					
D^N, depth, m	500	2,000	1,430.76	829.43	1,325
H_I^N, height, m	100	2,000	335.68	1,999	576.5
$U{\cdot}\rho^N$, thickness·density Kg/m^2	0	50,000	18,943.01	4,118.51	13,146.76
South source					
Z_1^S, depth of the top, m b.s.l	100	1,000	505.07	841	404
L^S, length, m	1,000	5,000	2,446.51	3,024.73	3,589
H^S, height, m	500	2000	1,028.12	883.3	1,140
W^S, tensile opening, m	2	6	5.43	5.99	5.2
ϕ^S, azimuth (from the north)	−30	10	−29.05	−16	−30
X^S, northing of top center, m	4,180,000	4,181,277	4,181,277	4,180,741.2	4,181,004
Y^S, easting of top center, m	496,500	501,000	499,533.6	499,844.31	499,998.3
δ^S, dip (from the east)	45	145	118.4	111.08	131.1
$\Delta\rho^S$, density contrast, Kg/m^3	100	500	114.82	100	116.8
South infiltration zone					
D^S, depth, m	500	2,000	1934.93.79	2,000	1,589
H_I^S, height, m	100	2,000	976.65	2,000	1,409
$U{\cdot}\rho^S$, Kg/m^2	0	50,000	47,904.93	19,135.64	34,485.43
Objective function and robustness					
err_{leveling}			0.0112	0.0144	0.0106
err_{EDM}			0.0595	0.0482	0.0646
err_{gravity}			13.06	13.79	18.15
Global robustness			0.2646	0.2591	0.2618

3.1 Morris Technique

The method by Morris is one of the techniques used to analyse the sensitivity of the model to variations of its parameters [10]. This global optimization technique follows a path through the input space by modifying the value of one parameter at a time and measures the response of the model. In particular, in order to

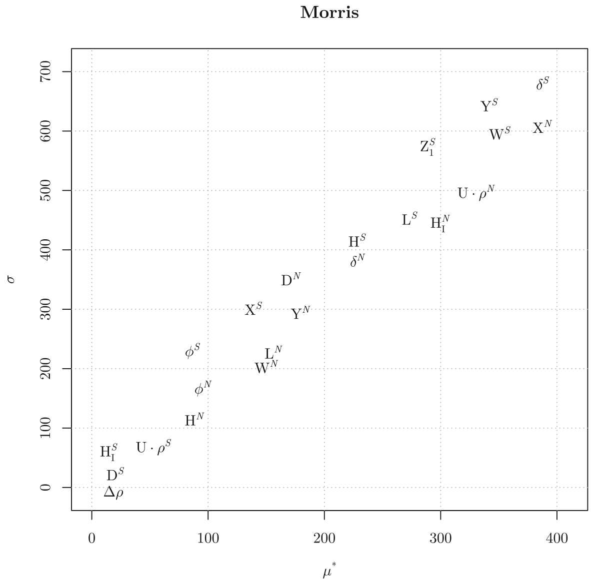

Fig. 4. Sensitivity analysis by means of the Morris method. The parameters on the upper right corner affect more largely the behaviour of the model.

quantify such response, the mean and the standard deviation of the changes to the model output are calculated for each variable. Since the mean can assume negative values, a normalization is performed. The results are shown in Fig. 4. The points of the plot near the origin of the axes have small values of mean and standard deviation and are, therefore, associated with parameters whose variations cause negligible effects to the output of the model. The other points, especially those in the top right corner, indicate large variations of the mean and are associated with parameters that strongly affect the model output when they are varied. The plot also reveals that the relationship between inputs and outputs are nonlinear since the magnitude of the effect of the variation of a parameter is related to the values of other parameters. This is suggested by the fact that large values of standard deviation are observed. In particular, these parameters control the characteristics of the deeper magmatic intrusion are its dip (δ^S), easting position (Y^S), opening (W^S), length (L^S) and depth (Z_1^S), as well as northing position (X^N), height (H_N) and thickness·, density and dip δ^N of the model of the shallower magmatic intrusion. These results are in agreement with those obtained on volcanomagnetic models performed on similar source geometries [5].

3.2 Sobol' Indices

Sobol' indices represent an effective method for estimating the sensitivity of a nonlinear model [9]. This technique, assuming that the inputs are independent, performs a decomposition of the output variance of the model in order to generate a set of indices. The higher the value of an index, the more important the effect of the parameter associated with that index in determining the output of the model [14,16]. The results, shown in Fig. 5, display the estimated value of each index along with its maximum and minimum values. They are in accordance with those obtained by the Morris technique, with the exception of the parameter δ^S, which in this case is not considered to affect the output.

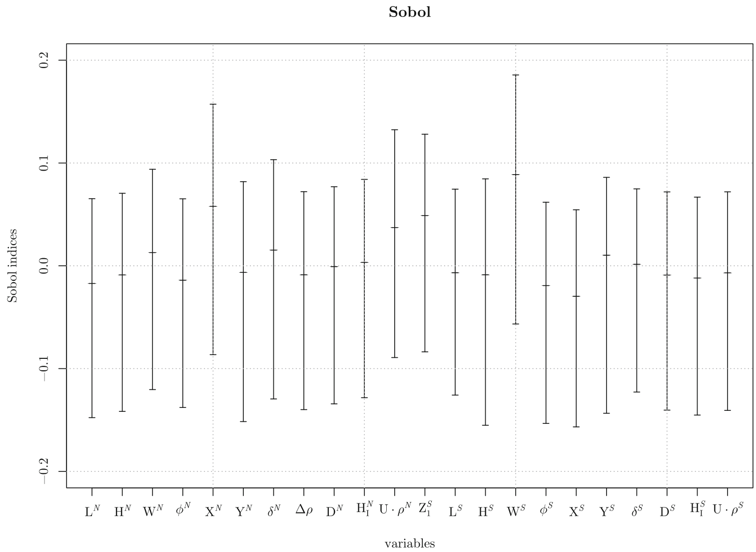

Fig. 5. Sensitivity analysis by means of the Sobol' indices.

4 Robustness Analysis

The minimization of the objective function is of primary importance for the selection of a model. However, it is not the only measure of its quality and a robustness analysis can help choose among a selection of optimal or sub-optimal solutions [1]. As a matter of fact, in many applications, if two solutions have the same objective function value, the solution which undergoes smaller variations of its objective function value when its parameters are perturbed should be preferred. For example, in the optimization of a biological model, robust solutions are preferrable as they mimic the ability of organisms to operate under different stress conditions [4,17]. In order to measure the robustness of a model, here we use the method proposed in [17]. Given a solution Ψ, a perturbation is defined as $\tau = \gamma(\Psi, \sigma_r)$, where the function γ having the form of a stochastic noise with normal distribution and standard deviation σ_r is applied to the solution Ψ. A set T consisting of several perturbations τ of Ψ is generated. A sample τ is robust to the perturbation of magnitude dictated by σ_r if the difference between the value of the objective function ϕ in correspondence of τ and the value in correspondence of the reference solution Ψ is smaller than ϵ, as expressed by the following equation:

$$\rho(\Psi, \tau, \phi, \epsilon) = \begin{cases} 1, & \text{if } |\phi(\Psi) - \phi(\tau)| \leq \epsilon. \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

An estimate of the robustness of a system Ψ is obtained by performing a set T of trials and then calculating the rate of successful trials, which is given by:

$$\Gamma(\Psi, T, \phi, \epsilon) = \frac{\sum\limits_{\tau \in T} \rho(\Psi, \tau, \phi, \epsilon)}{|T|}. \tag{5}$$

Robustness analysis is global when all the parameters are varied at the same time, while it is local if a parameter at a time is considered. Although local robustness allows to evaluate how a solution "reacts" to perturbations of specific input parameters, we believe that performing a global robustness analysis in this context is more meaningful, as it allows to observe the result of the joint perturbation of the parameters of the model (which determine the characteristics of the sources and the infiltration zones). In particular, we calculated the robustness of the solutions that we found and the robustness of the solution reported in [3], whose parameters are displayed in Table 1. The parameters of the robustness analysis have the values: $\sigma_r = 0.01$ and $\epsilon = 0.0071$, where the value of ϵ corresponds to one tenth of the minimum objective function value of NSGA2 according to the single-objective function (1), while the number of trials $|T| = 10,000$. The NSGA2 instance with a large population size and number of iterations has the highest robustness, with a value of 0.2646, while the solution reported in [3] has a slightly smaller robustness with a value of 0.2618 and the solution obtained by DE has a value of 0.2591, the lowest of the three techniques.

5 Conclusions

This paper has presented the results of the optimization of the conceptual model of the 1981 eruption at Etna volcano proposed in [3]. This model hypothesizes that the eruption was generated by two magmatic intrusions that developed in the northern flank of mount Etna. Two techniques have been used to perform the optimization of the model: the single-objective Differential Evolution technique and the multi-objective NSGA2 technique with increased population size and number of generations with respect to the original paper. The optimization performed using NSGA2 provides improved solutions with respect to those presented in [3], while DE was not able to provide good combinations of all output measures. Moreover, the solutions obtained by DE have very similar characteristics, while those found using NSGA2 feature a high diversity, this provides more meaningful information regarding the characteristics of a model. An analysis of the robustness of a selection of optimal solutions obtained was performed in order to evaluate if they were able to provide a stable output when their parameters were perturbed. Such analysis revealed that the new solution obtained by NSGA2 shows slightly higher robustness with respect to the solution previously obtained, this entails that such solutions are less susceptible to variations of their values. An analysis of the sensitivity of the model was also performed in order to identify the parameters that more significantly affect the output of the model and those which cause little effect on it. Two different methods were used: the Morris technique and the Sobol' indices. They revealed that the parameters of the model that control the characteristics of the deeper magmatic intrusion are its easting position, opening, length and depth, and the parameters that control the shallower magmatic intrusion are its position, height and thickness, density and dip. Moreover, the Morris technique highlighted that the relations between the input parameters are nonlinear.

The new optimal solution found by NSGA2 (Fig. 3, Table 1), although similar to the solution reported in [3], shows some differences. Particularly, the northern shallow source has a deeper infiltration zone and the southern source is both shorter and deeper. Since the sensitivity analysis showed that these parameters are those that may significantly affect the model outputs, the new optimal solution is preferable to the previous one. Moreover, the Morris and Sobol analyses show that the optimization problem is more sensitive to those parameters, which directly reflect their influence on the ground surface by controlling the wavelength and the extent of the geophysical variations. As expected, the sensitivity of the optimization model is also dependent on the network configurations of the measurement points. Particularly, in the 1981 Etna eruption case study no measurements were available in the more affected summit area that could have been helped in better constraining the extension of the source, especially the length and the position of the shallower intrusion.

A set of directions for the future developments of this study have been outlined. The search for solutions could be extended by the use of further optimization techniques, such as the immune-inspired algorithm opt-IA [7]. This would help shed light on the characteristics of the techniques that are effective at dealing with this optimization problem. Moreover, the information provided by the sensitivity analysis could be used to improve the optimization. For instance, the search for solutions could focus on the parameters that more largely affect the output of the model and neglect those which produce little or no variations. In addition, the information about the robustness could be used to select a set of solutions among the optimal ones found or to guide the optimization process.

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