Functional Architectures for Complex Behaviors: Analysis and Modeling of Interacting Processes in a Hierarchy of Time Scales

Dionysios Perdikis[1], Raoul Huys[2], and Viktor Jirsa[2]

[1] Max-Planck-Institute for Human Development,
Lentzealle 94, 14195, Berlin, Germany
perdikis@mpib-berlin.mpg.de
[2] Theoretical Neuroscience Group, Institut de Neurosciences des Systèmes, Inserm,
UMR1106, Aix-Marseille Université, Faculté de Médecine,
27, Boulevard Jean Moulin 13005, Marseille, France
viktor.jirsa@univ-amu.fr, raoul.huys@univ-amu.fr
http://ins.univ-amu.fr/research-teams/theoretical-neurosciences-group

Abstract. Synergetics' applications in the sciences of cognition and behavior have focused on instabilities leading to phase transitions between competing behavioral or perceptual patterns. Inspired by this scientific tradition, functional architectures are proposed as a general theoretical framework aiming at modeling the nonstationary, multiscale dynamics of complex behaviors, beyond the neighborhood of instabilities. Such architectures consist of interacting dynamical processes, operating in a hierarchy of time scales and functionally differentiated according to their mutual time scale separations. Here, the mathematical formalism of functional architectures is presented and exemplified through simulations of cursive handwriting. Then, the implications for the analysis of complex behaviors are discussed.

Keywords: functional architectures, structure flows on manifolds, hierarchies of time scales

1 Introduction

The idea that human function (be it motoric, perceptual or cognitive) is composed of elementary processes acting as functional units or "primitives" is often expressed in the biological and life sciences. Identifying such functional units implies a time scale separation between such units and the resulting composite processes. Thus, there is a relationship between time scale separation and functional differentiation of processes. Moreover, interactions among these time scales are essential for the organization of multiscale behaviors that are characterized by circular causality. For instance, serial behaviors, such as speech and handwriting, constitute a class of complex behaviors where slow processes sequentially activate functional units (elements of a sequence), modifying the

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fast dynamics of the level of units, which often results in non-stationary processes (top-down influence); at the same time, bringing functional units into meaningful relationships (here serial order) is how a complex behavior emerges (bottom-up composition). The field of nonlinear dynamics has offered descriptions of functional or behavioral units [1, 2] such as excitable point attractor systems and limit cycles to model discrete and rhythmic behaviors, respectively, as well as more complex ones (e.g. multistable or chaotic). Moreover, Synergetics [3–5] has focused on the instabilities leading to phase transitions among competing, low-dimensional, cognitive or behavioral patterns of order parameters, and the related experimentally observed phenomena, such as hysteresis, critical slowing down and fluctuations, and metastability. The present work proposes functional architectures [6–8] to account for the ensemble of interactions constituting the organization of complex human function, outside the neighborhood of instabilities as well. Functional architectures deal with a dual task: to provide a general formalism of the functional units' dynamics, and to propose dynamical mechanisms responsible for temporarily establishing a functional unit, and subsequently destabilizing it, in order to transit to another one (a role played by control parameters whose dynamics is not explicitly treated in Synergetics), for complex function to emerge.

2 Structured Flows on Manifolds modeling Functional Units

In a recent advance, 'Structured Flows on Manifolds' (SFM) [8, 9] has been proposed (in the spirit of Synergetics) to link the dynamics of large-scale brain networks interacting with bodily and environmental dynamics (high dimensional systems) to low-dimensional phenomenological descriptions of functional (or behavioral) dynamics. SFM suggest that during the engagement in a specific function, the high-dimensional network dynamics collapses (via a fast adiabatic contraction) on a functionally relevant subset of the phase (state) space, the so-called manifold. The phase flow is the structured dynamics on the manifold that evolves for the duration of a specific functional process:

$$\begin{aligned} \tau\dot{u}_i &= -g_i(\{u_i\},\{s_k\})u_i + \mu f_i(\{u_i\},\{s_k\}) \\ \tau\dot{s}_k &= -s_k + h_k(\{u_i\},\{s_k\}) \\ \{u_i\} &\in \Re^N, \{s_k\} \in \Re^M, N \ll M, \mu \ll 1 \end{aligned} \tag{1}$$

where $g_i()$, $f_i()$, and $h_i(.)$ define the globally attractive manifold, the slow (due to the small value of μ) flow on it, and the fast dynamics towards the manifold, respectively. τ is the time constant of the fast contraction. The requirement for SFM is to contain an inertial manifold [10], which is a global structure used in cases of reduction of infinite dimensional dynamical systems to finite dimensional spaces. The existence and global stability of an inertial manifold has to be treated in a case-by-case manner, but it is not constrained to the neighborhood of instabilities and phase transitions. After the adiabatic elimination of

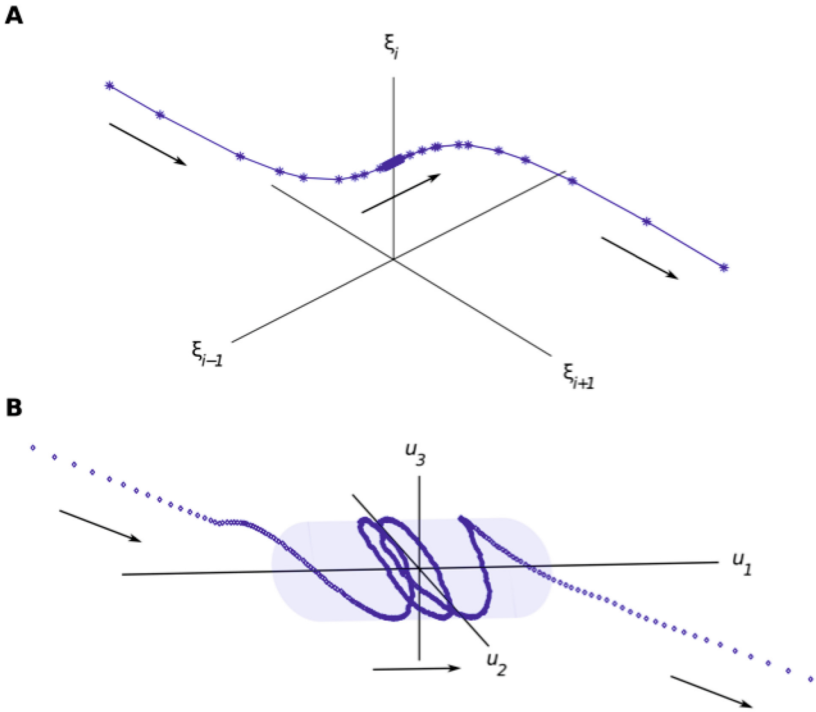

Fig. 1. Multiscale dynamics: slow operational signal and SFM emergence. Panel A: The slow operational signals $\{\xi_j\}$ converge through a fast transient to a specific ξ_j node resulting (here) in the emergence of a cylindrical manifold. Panel B: The functional dynamics $\{u_j\}$ collapses fast (also) onto the manifold where it executes a slow spiral flow. The ξ_j node's stability is sustained for the duration of the flow execution. Subsequently, the ξ_j node destabilizes, followed by the related manifold, and the dynamics moves away, again through fast transients. The density of data point is inversely proportional to the time scale of the dynamics. Figure adapted from [7].

$s_k = h(\{u_i\}, \{s_k\})$, the low-dimensional dynamics on the manifold is given by

$$\tau \dot{u}_i = F_i(\{u_i\}) = -g_i(\{u_i\})u_i + \mu f_i(\{u_i\}) \tag{2}$$

and describes (quantitatively) *functional modes*, the main building blocks of functional architectures, as autonomous, deterministic and time continuous systems. Moreover, the phase flow topology of a mode uniquely determines a system's qualitative behavior, encoding the invariant features of a dynamical process relative to quantitative variations such as robustness or stability changes due to stochastic contributions or to different contexts, thus identifying all functional possibilities within a class in a model-independent manner.

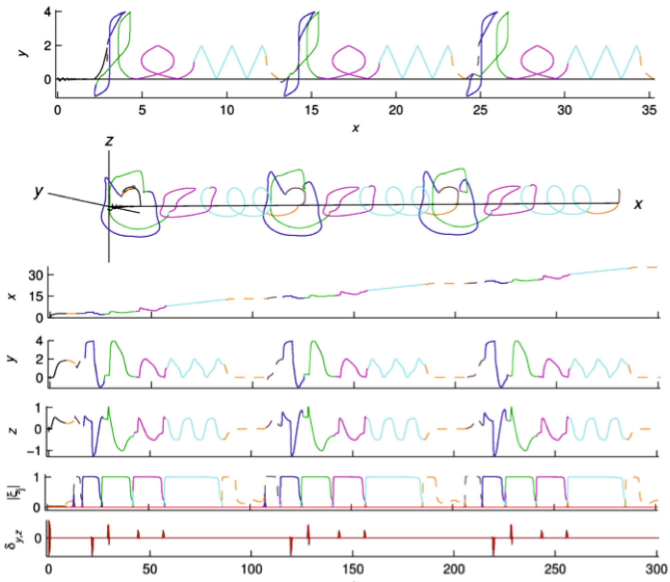

Fig. 2. The generation of the word "flow" and the operational signals involved. The word is repetitively generated after a short transient (black solid line). Four principle functional modes are used, one for each character (associated with solid blue, green, magenta and cyan lines, respectively), plus two auxiliary ones (linear point attractors) at the sequence's beginning and end (dotted dark and light brown lines, respectively). From top to bottom: three repetitions of the word in the handwriting workspace (the plane x-y), the output trajectory in the 3-dimensional functional phase space spanned by state variables x, y and z, followed by their time series, and the time series of the slow (WTA competition coefficients $|\xi_j|$) and the instantaneous ($|\delta_{x,y}|$) "kicks", light and dark red, respectively) operational signals. The mode amplitudes that do not participate in the word always have a value close to zero (red line). Figure adapted from [7].

3 Functional Architectures for Complex Behavioral Processes

In its most general formulation (first part of equation (3)), we can describe a functional architecture through its flow $\{F_i(.)\}$ in phase space, potentially subjected to additional dynamics $\{\sigma_i(t)\}$ called *operational signals* (for a detailed treatment see [6, 7]):

$$\tau \dot{u}_i = F_i(\{u_i\}, \{\sigma_i(t)\}) = \sum_j |\xi_j(t)|(F_{ij}(\{u_i\}) + \delta_i(t)) \tag{3}$$

where $\{F_{ij}(.)\}$ is the j-th functional mode that is available in an agent's dynamical repertoire and $\{\sigma_i(t)\}$ is a (generally) time-dependent operational signal.

The $\{\sigma_i(t)\}$ may operate on various time scales relative to the characteristic time scale of $\{F_{ij}(.)\}$. These time scale separations among modes and signals determine their functional differentiation and result in distinct functional architectures. The second part of equation (3) describes a more specific form of a functional architecture where, at each moment in time, the expressed phase flow $\{F_i(.)\}$ is given as a linear combination of all available modes, weighted by the operational signal $|\xi_j(t)|$. $|\xi_j(t)|$ modifies $\{F_i(.)\}$ on a slower time scale than the one of the functional modes $\{F_{ij}(.)\}$, except for the critical moments where abrupt transitions between modes take place. The operation of this slow signal upon the functional modes implements the basic mechanism of the functional architecture: slow dynamics drives the faster functional dynamics through an alternating sequence of fast and slow transients, each of the latter ones constituting a distinct function, described by a SFM (Fig. 1). Thus, elementary processes are sewed together to form a longer more complex one. The architecture also provides for the optional involvement of the operational signal $\{\delta_i(t)\}$ that leaves the flow $\{F_i(.)\}$ unaffected, because it acts instantaneously, just like a functionally meaningful perturbation (for instance, by moving the system beyond a threshold and thus initiating a significant change in the trajectory's evolution). The ensemble of subsystems ($|\xi_j(t)|$, $\{F_{ij}(.)\}$, $\{\delta_i(t)\}$) operating on distinct time scales ($\tau_\delta \ll \tau_F \ll \tau_\xi$, respectively) constitutes the functional architecture. In [7] we showed how the (($|\xi_j(t)|$, $\{F_{ij}(.)\}$, $\{\delta_i(t)\}$)) dynamics can be designed to organize functional modes so that serial order behavior emerges. Under the requirement that functional modes' activations do not overlap, we implemented a "winner-take-all competition" (WTA) for the dynamics, in the spirit of the Synergetic Computer [5]. Suitable dynamics for the parameters of the competition ("attentional parameters" in the terminology of the Synergetic Computer) had to be designed, so as to activate the functional modes taking part in the sequence one after the other and with the appropriate timing. As the different ingredients of the functional architecture were intricately coupled in various ways, the whole architecture was constituted as an autonomous multiscale dynamical system. We demonstrated the application of the functional architecture on cursive handwriting (Fig. 2), for which we designed a repertoire of functional modes implementing characters (or parts thereof) modeled as 3-dimensional SFM on cylindrical manifolds.

4 Implications for Analysis of Complex Behaviors

Functional architectures aim not only at providing the theoretician with hypotheses on the nature and mutual interactions of the dynamical components of complex behaviors, but also the experimenter with the means to identify such components. In [7] we analyzed data from several simulated trials of the handwriting architecture, assuming the time scale separations and the respective functional differentiation of functional modes and operational signals. We managed to identify and distinguish between all three kinds of dynamics (($|\xi_j(t)|$, $\{F_{ij}(.)\}$, $\{\delta_i(t)\}$) and thus, decompose the architecture's output into its dynamical components and segment it into time periods where different functional modes are activated. Doing so appeared to be possible only through the calculation of the output phase flow variability (as opposed to studying the variability of the output time series), i.e. by identifying the short time periods when the expressed phase flow changed because operational signals became effective. Future work could be directed towards methods of phase flow reconstruction of stochastic systems based on Fokker-Planck equation formalisms, adjusted for nonstationary, multiscale systems [11].

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