

Coarse-Grained Order Parameter Dynamics of the Synergetic Computer and Multistable Perception in Schizophrenia

Till D. Frank and Dobromir G. Dotov

Department of Psychology, Center for the Ecological Study of Perception and Action,
University of Connecticut, 406 Babbidge Road, Storrs, CT, 06269, USA

till.frank@uconn.edu

<http://psych.uconn.edu/faculty/frank.php>

Abstract. The synergetic computer that has originally been developed as an algorithm for pattern recognition has also been used in the life sciences as a model for various self-organizing perceptual processes. Coarse-graining of the order parameter equations of the synergetic computer is discussed for sets of to-be-perceived patterns that vary in the degree to which they can be distinguished from each other. Coarse-gaining is exploited to conduct a model-based analysis on literature data of multistable perception under schizophrenia as tested in motion-induced blindness (MIB) experiments. The analysis not only supports earlier suggestions that schizophrenia reduces the occurrence frequency of the MIB effect but also suggests that the perceptual system of schizophrenia patients is characterized by a greater degree of asymmetry.

Keywords: multistable perception, schizophrenia, synergetic computer, motion-induced blindness

1 Introduction

The synergetic computer is an algorithm for pattern recognition [1]. The algorithm is based on self-organization principles and has been developed within the framework of synergetics [2]. Although the algorithm has been developed to solve pattern recognition problems [1, 3–7], it has been generalized and applied in various related, interdisciplinary fields. In particular, the algorithm has been generalized to allow for hierarchical pattern recognition processes [8]. Economic and industrial applications in the field of settlement dynamics [9, 10], job assignment problems and robotics [11–17], and signal transmission via message buffer [18] have been addressed. Although the synergetic computer describes an artificial associative memory or decision-making system, due to its roots in synergetics and the theory of self-organization, the synergetic computer has also been regarded as a benchmark model for self-organizing psychological processes and self-organizing motor control system. In this context, oscillatory phenomenon induced by certain perceptual [19, 20], and auditory [21, 22] stimuli have been

discussed and application to priming [23, 24], grasping [25, 26], and motor development during infancy [27, 28] can be found in the literature.

The pattern recognition algorithm is a winner-takes-all system that for a given initial stimulus pattern converges to a fixed point solution indicating the perception of a stored prototype pattern. The algorithm can be discussed from the perspective of the to-be-perceived and stored patterns. Alternatively, the algorithm can be studied from the perspective of the pattern amplitudes. In line with the fact that the synergetic computer is considered as a computational or artificial self-organizing system mimicking natural self-organizing systems, the amplitudes have typically been considered as order parameters [1, 29, 30].

Let ξ_k denote the order parameters of $k = 1, \dots, N$ patterns. We consider the order-parameter dynamics of the synergetic computer in the following form [1]

$$\frac{d}{dt}\xi_k = \xi_k \left(\lambda - B \sum_{m \neq k, m=1}^N \xi_m^2 - C \sum_{m=1}^N \xi_m^2 \right) \tag{1}$$

with $\lambda, B, C > 0$. Equation (1) can be cast into a form that is convenient for conducting a stability analysis of fixed points in the generalized case that will be considered in Section 3 when the attention parameter λ depends on the pattern index [7, 23–27]. Accordingly, Eq. (1) can equivalently be expressed by $d\xi_k/dt = \xi_k(\lambda - g C \sum_{m \neq k, m=1}^N \xi_m^2 - C\xi_k^2)$, where we have introduced the coupling parameter $g = 1 + B/C > 1$. The parameter C can be put to $C = 1$ without loss of generality such that

$$\frac{d}{dt}\xi_k = \xi_k \left(\lambda - g \sum_{m \neq k, m=1}^N \xi_m^2 - \xi_k^2 \right) . \tag{2}$$

Alternatively, the parameter λ and the order parameters ξ_k may be rescaled by \sqrt{C} and the rescaled equations are considered [26]. Solutions of Eq. (2) under initial conditions $\xi_k(0) \geq 0$ will be considered, which implies that all order parameters remain semi-positive definite for all times (i.e., $\xi_k(t) > 0 \forall t \geq 0$).

In what follows, we will derive order parameter equations on several levels of coarse-graining. The ideas that will be developed below are closely related to the ideas developed in earlier studies on hierarchical generalizations of the order parameter equations of the synergetic computer [8].

2 Approximative coarse-grained order parameter dynamics

In Section 2.1, we will consider first a special case that will be used in Section 3 in the application for multistable perception of schizophrenia. Subsequently, in section 2.2, the general case will be discussed.

2.1 Special case

In this section, it is assumed that all patterns $k = 2, \dots, N$ possess a common feature that is not present in the 'default' pattern $k = 1$. In this special case, we consider the course-grained order parameter U defined by

$$U = \sqrt{\sum_{s \in I_U} \xi_s^2} \tag{3}$$

with the index set $I_U = \{2, \dots, N\}$. Due to the 'winner-takes-all' property of the synergetic computer (see Section 1) it follows that if one of the order parameters ξ_k^* with $k^* \in I_U$ becomes finite in the stationary case, then $U = \xi_{k^*}^* > 0$. If the order parameter ξ_1 of the default pattern becomes finite in the stationary case, then $U = 0$.

Before exploiting the definition (3), it is useful to cast the order parameter equations (2) of the synergetic computer in yet another form. Eq. (2) can be written like

$$\frac{d}{dt} \xi_k = \xi_k \left(\lambda - g \sum_{m=1}^N \xi_m^2 - (1-g)\xi_k^2 \right), \tag{4}$$

where the mixed term contains the sum of all squared order parameters. Note that in Eq. (4) the cubic term ξ_k^3 actually has a positive coefficient because of $g > 1$ (or since $-(1-g) = B > 0$ holds using $C = 1$ again). Substituting the definition (3) into Eq. (4), we obtain

$$\begin{aligned} \frac{d}{dt} \xi_1 &= \xi_1 (\lambda - g[U + \xi_1^2] - (1-g)\xi_1^2), \\ \frac{d}{dt} U &= U (\lambda - g[U + \xi_1^2]) - (1-g) \frac{1}{U} \sum_{s \in I_U} \xi_s^4. \end{aligned} \tag{5}$$

In the stationary case, we have either $U = \xi_{k^*}^* > 0$ and $\xi_{j \neq k^*} = 0$ if a pattern $k^* \in I_U$ is selected or $U = 0$, $\xi_{k \in I_U} = 0$, $\xi_1 > 0$. In both cases, the dynamical system (5) for ξ_1 and U exhibits the same stationary fixed points as the coupled dynamical system

$$\begin{aligned} \frac{d}{dt} \xi_{1,a} &= \xi_{1,a} (\lambda - g[U_a + \xi_{1,a}^2] - (1-g)\xi_{1,a}^2), \\ \frac{d}{dt} U_a &= U_a (\lambda - g[U_a + \xi_{1,a}^2]) - (1-g)U_a^2 \end{aligned} \tag{6}$$

for the variables $\xi_{1,a}$ and U_a . Note that Eq. (6) assumes the form of the order parameter equations of the synergetic computer again. The question arises to what extent the variables $\xi_{1,a}$ and U_a can be regarded as useful approximations to the order parameter ξ_1 and the coarse-grained order parameter U .

In this context, we first note that the expression U^4 reads

$$U^4 = \left[\sum_{s \in I_U} \xi_s^2 \right]^2 = \sum_{s \in I_U} \xi_s^4 + \text{mixed terms of the form } (\xi_i^2 \xi_{j \neq i}^2)_{i,j \in I_U}. \tag{7}$$

Consequently, Eq. (5) reads

$$\begin{aligned} \frac{d}{dt}\xi_1 &= \xi_1 (\lambda - g[U + \xi_1^2] - (1 - g)\xi_1^2) , \\ \frac{d}{dt}U &= U (\lambda - g[U + \xi_1^2] - (1 - g)U^2) \\ &\quad + \text{mixed 3rd order terms of the form } \frac{1}{U} (\xi_i^2 \xi_{j \neq i}^2)_{i,j \in I_U} . \end{aligned} \tag{8}$$

As indicated the mixed terms are considered third order terms because the products of order 4 are divided with the variable U that depends linearly on the scales of the variables $\xi_{k \in I_U}$.

The dynamical systems (6) and (8) differ by the mixed terms occurring in the U -dynamics of the model (8). In order to assess the relevance of these terms, we apply a concept from psychophysics: the 'just noticeable difference' (JND) of sensations [31]. We assume that all patterns under consideration differ from each other by a distance measure D that will not be specified in detailed. For a human observer the patterns under consideration differ such that they can be distinguished from each other. In this sense, for all pairs of patterns the distance measure D is larger than a certain threshold that corresponds to the JND.

Mathematically speaking, we assume that the initial conditions are such that the order parameters ξ_k differ at $t = 0$ by a certain amount that reflects the distance D between the patterns and accounts for the aforementioned requirement that the sensation patterns (stimuli) under consideration differ at least by the JND. We distinguish between two cases.

Case I: It is assumed that patterns with a JND induce relative large differences between the initial values $\xi_k(0)$ of the order parameters. Accordingly, we assume that

$$\exists k^* : \forall j \neq k^* : \xi_{k^*}(0) \gg \xi_j(0) . \tag{9}$$

In this case, the order parameter ξ_{k^*} of the pattern k^* will not only win the selection process defined by Eq. (4) but the mixed terms in the U -dynamics of Eq. (8) will be negligibly small at all times relative to the U^3 term:

$$\forall t \geq 0 : U^3(t) \gg \text{mixed 3rd order terms of the form } \frac{1}{U(t)} (\xi_i^2(t)\xi_{j \neq i}^2(t))_{i,j \in I_U} .$$

If Eqs. (10) holds, then the dynamical systems (6) and (8) exhibit approximately the same transient and stationary solutions. Consequently, the model (6) involving the variable U_a is a good approximative model for the original order parameter model (4) of the synergetic computer. In particular, in the limiting case $\xi_{j \neq k^*}(0)/\xi_{k^*}(0) \rightarrow 0$ a point-wise convergence $\xi_{1,a}(t) \rightarrow \xi_1(t)$ and $U_a(t) \rightarrow U(t)$ holds at any time point t provided that we use the consistent initial conditions $\xi_{1,a}(0) = \xi_1(0)$ and $U_a(0) = U(0)$. An illustration is shown in Fig. 1AB.

Case II: It is assumed that patterns with a JND induce differences between order parameters $\xi_k(0)$ that are scaled to the size of the set of patterns and are at least of the magnitude $\sqrt{N - 1}$. More precisely, we assume that

$$\exists k^* : \forall j \neq k^* : \xi_{k^*}(0) > \xi_j(0)\sqrt{N - 1} . \tag{10}$$

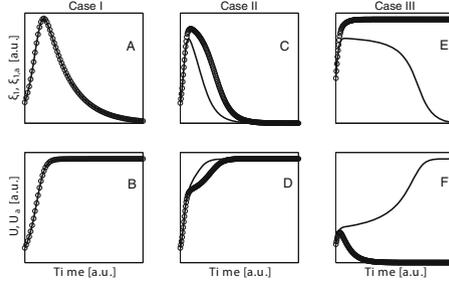


Fig. 1. Illustrations of solutions for case I (A,B), II (C,D), III (E,F) initial conditions. Solutions of Eq. (4) (solid lines) and Eq. (6) (circles) are shown under consistent initial condition: $\xi_{1,a}(0) = \xi_1(0)$, $U_a(0) = U(0)$. See text for details. Parameters: $N = 10$, $\lambda = 2.0$, $g = 1.3$. Case I initial conditions: $\xi_5(0) = 0.2$, $\xi_{j \neq 5}(0) = 0.2/\sqrt{(N-1)}/10/(1+0.1\epsilon)$. Case II: $\xi_5(0) = 0.2$, $\xi_{j \neq 5}(0) = 0.2/\sqrt{(N-1)}/(1+0.2\epsilon)$. Case III: $\xi_1(0) = 0.25$, $\xi_{j \neq 1}(0) = 0.2 + 0.01\epsilon$. In all cases, ϵ was uniformly distributed in $[0, 1]$.

Let us distinguish between the two sub-cases that $k^* \in I_U$ and $k^* \notin I_U$ (i.e., $k^* = 1$). If $k^* \in I_U$ then the original order parameter dynamics will converge to a fixed point with $\xi_{k^*} > 0$ such that in the stationary case $U(st) = \xi_{k^*}(st)$ holds. Moreover, it follows that $U(0) > \xi_{k^*}(0) > \xi_1(0)$. Consequently, if the dynamical system (6) is considered under consistent initial conditions (i.e., $\xi_{1,a}(0) = \xi_1(0)$ and $U_a(0) = U(0)$), then U_a converges to the finite stationary value $U_a(st) = U(st) = \xi_{k^*}(st) > 0$ of the original dynamical system (4) and $x_{1,a}(t)$ converges to zero consistent with the stationary behavior of ξ_1 : $\xi_{1,a}(st) = \xi_1(st) = 0$. In contrast, if $k^* = 1$ then the original selection equation dynamics (4) converges to the fixed point with $\xi_1 > 0$ and $U = 0$. In addition, it follows that

$$U^2(0) = \sum_{s \in I_U} \xi_s^2(0) < \sum_{s \in I_U} \frac{\xi_1^2(0)}{N-1} = \xi_1^2(0) \Rightarrow U(0) < \xi_1(0) . \quad (11)$$

If, again, the dynamical system (6) is considered under consistent initial conditions (i.e., $\xi_{1,a}(0) = \xi_1(0)$ and $U_a(0) = U(0)$), then U_a converges to the stationary value $U_a(st) = U(st) = 0$ and $x_{1,a}(t)$ converges to its finite fixed point value consistent with the stationary behavior of ξ_1 : $\xi_{1,a}(st) = \xi_1(st) > 0$. In summary, if condition (10) is satisfied, then the dynamical system (6) involving the variable U_a exhibits the same stationary behavior than the original selection equation model (4) provided that both dynamical systems are considered under consistent initial conditions. Figure 1CD exemplifies solutions of the dynamical systems (4) and (6) for this case.

In view of the fact that in the two aforementioned cases the performance of the dynamical model (6) is consistent in the stationary case with the original order parameter equation model (4) and given that both models exhibit formally the same mathematical structure, we will consider in what follows the coupled differential equations (6) involving the variables $\xi_{1,a}$ and U_a as the (approxima-

tive) coarse-grained order parameter equation model of the original synergetic computer model (4) (or (1)) involving the variables ξ_1, \dots, ξ_N .

Finally, let us consider the general case in which neither of the two conditions described above are satisfied.

Case III: If the conditions considered in cases I and II are not satisfied, then the dynamical model (6) may exhibit solutions that are inconsistent with the order parameter dynamics (4) even if both dynamical models are solved under consistent initial conditions. Let us prove this statement by an example. Let $\xi_1(0) = b > 0$ and $\xi_{k \in I_U}(0) = a > 0$ with $b > a$. For these initial conditions the original pattern recognition algorithm (4) converges to a fixed point with $\xi_1(st) > 0$ and $\xi_k = 0$ for $k \in I_U$ indicating that the default pattern $k = 1$ is recognized. Next, we consider the special case in which the distance D between the default pattern $k = 1$ and the other patterns $k \geq 2$ is not that large such that if the default pattern is presented we have $b > a$ but $b^2 < (N - 1)a^2$. That is, the condition of case II is violated. From $b^2 < (N - 1)a^2$ it follows that $U(0)^2 = (N - 1)a^2 > a^2 = \xi_1^2(0)$. In other words, although for $b > a$ and $b^2 < (N - 1)a^2$ the condition $\xi_1(0) > \xi_k(0)$ holds for any $k \neq 1$, we have $U(0) > \xi_1(0)$. Consequently, if we solve the coarse-grained selection equations (6) under consistent initial conditions ($\xi_{1,a}(0) = \xi_1(0)$ and $U_a(0) = U(0)$), then $U_a(t)$ converges to a finite stationary value $U_a(st) > 0$ and $x_{1,a}(t)$ converges to zero in the stationary case. The coarse-grained dynamical model (6) indicates that one of the patterns $k \geq 2$ was recognized, which is in contradiction with the recognition process described by the original selection equations (4). Figure 1EF illustrates this case.

In summary, we have considered the special case in which the set of N patterns under considerations exhibits a distinct default pattern and $N - 1$ patterns that constitute a class of non-default patterns. On a coarse-grained level, we considered the order parameters x_1 and U that describe whether a pattern is recognized as the default pattern ($\xi_1(st) > 0$) or as a pattern belonging to the class of non-default patterns ($U(st) > 0$). It was shown that for this special case a dynamical model for the variables $\xi_{1,a}$ and U_a can be derived (see Eq. (6)) that under certain circumstances behave approximatively in the same way as ξ_1 and U , respectively. More precisely, if patterns are considered that differ at least by a JND that induces (i) a relative large gap or (ii) at least a gap of $\sqrt{N - 1}$ in the spectrum of initial amplitudes $\xi_k(0)$, then in the stationary case the coarse-grained order parameter dynamics involving $\xi_{1,a}$ and U_a yields consistent results with the fine-grained dynamics of ξ_1, \dots, ξ_N . This implies that the pattern selection made by the two dynamical systems is consistent. Under the condition (i) the two dynamical models exhibit also approximatively the same transient solutions. If neither of the two gap conditions (i) and (ii) are satisfied, then the two models may yield inconsistent results. These considerations are summarized schematically in Table 1.

Importantly, the two dynamical models for the approximative coarse-grained order parameters $\xi_{1,a}$ and U_a and for the fine-grained order parameters ξ_1, \dots, ξ_N exhibit formally the same mathematical structure.

Table 1. Correspondence of fine- and coarse grained dynamics

Case	'JND' impact	Initial conditions	Fine- & coarse-grained
I	Large gap	$\exists k^* : \forall j \neq k^* : \xi_{k^*}(0) \gg \xi_j(0)$	Consistent transient and stationary solutions
II	Moderate gap	$\exists k^* : \forall j \neq k^* : \xi_{k^*}(0) > \sqrt{N-1} \xi_j(0)$	Consistent stationary solutions
III	Gap conditions I and II not satisfied		Stationary solutions may or may not be consistent

2.2 General case

Let us consider M levels of coarse-graining $L \in \{1, \dots, M\}$. The first level ($L = 1$) contains N_1 patterns. To each pattern an order parameter $\xi_{k,1}$ is assigned that is used to indicate whether the pattern is recognized. On the second level, patterns are grouped together such that there are $N_2 < N_1$ pattern classes. To each pattern class a coarse-grained order parameter $\xi_{k,2}$ is assigned that is used to indicate whether a pattern out of the class is recognized. In general, each level exhibits N_L pattern classes (with $N_1 > N_2 > \dots > N_M$) that are described by N_L coarse-grained order parameters $\xi_{k,L}$. For the sake of simplicity, the patterns of level $L = 1$ and the corresponding amplitudes $\xi_{k,1}$ will be treated as if they were pattern classes and pattern class amplitudes, respectively.

At this stage, it is useful to introduce the index sets $I_{k,L+1} \subset \{1, \dots, N_L\}$. The index set $I_{k,L+1}$ contains all the pattern class indices j from the coarse-grained level L that are grouped together to the class k of the level $L + 1$. For example, the index set I_U discussed in Section 2.1 becomes $I_{k=2,L=2} = \{2, \dots, N\}$. The sets satisfy $\forall k \neq j, k, j \in \{1, \dots, N_{L+1}\} : I_{k,L+1} \cap I_{j,L+1} = \emptyset$ and $\cup_{k=1}^{N_{L+1}} I_{k,L+1} = \{1, \dots, N_L\}$. In words, all sets belonging to a particular level of coarse-graining are mutually disjoint and the unification of all sets of a coarse-graining level $L + 1$ gives the index set of all pattern classes of the previous level L . In analogy to Eq. (3), coarse-grained order parameters $\xi_{k,L+1}$ are defined iteratively by

$$\xi_{k,L+1} = \sqrt{\sum_{s \in I_{k,L+1}} \xi_{s,L}^2} \tag{12}$$

Let us assume that for a particular level L of coarse-graining the selection equations for $\xi_{k,L}$ assume the form of the order parameter equations of the synergetic computer. In analogy to Eq. (4), we consider the selection equations

$$\frac{d}{dt} \xi_{k,L} = \xi_{k,L} \left(\lambda - g \sum_{m=1}^{N_L} \xi_{m,L}^2 - (1-g)\xi_{k,L}^2 \right) \tag{13}$$

for $k \in \{1, \dots, N_L\}$. Proceeding as in Section 2.1, we use

$$\sum_{m=1}^{N_L} \xi_{m,L}^2 = \sum_{k=1}^{N_{L+1}} \left(\sum_{s \in I_{k,L+1}} \xi_{s,L}^2 \right) = \sum_{m=1}^{N_{L+1}} \xi_{m,L+1}^2. \tag{14}$$

Differentiating Eq. (12) with respect to time t and substituting Eqs. (13) and (14) into the resulting equation, we obtain in analogy to Eq. (5) the following result

$$\frac{d}{dt} \xi_{k,L+1} = \xi_{k,L+1} \left(\lambda - g \sum_{m=1}^{N_{L+1}} \xi_{m,L+1}^2 \right) - (1-g) \frac{1}{\xi_{k,L+1}} \sum_{s \in I_{k,L+1}} \xi_{s,L}^4 \tag{15}$$

for $k \in \{1, \dots, N_{L+1}\}$. Using the same line of arguments as in Section 2.1, the most right standing term in Eq. (15) can be expressed in terms of $\xi_{k,L+1}$ and mixed terms of the form $\xi_{i,L}^2 \xi_{j,L}^2$ with $i \neq j$. Consequently, in analogy to Eq. (8), Eq. (15) can be cast into the form

$$\begin{aligned} \frac{d}{dt} \xi_{k,L+1} = & \xi_{k,L+1} \left(\lambda - g \sum_{m=1}^{N_{L+1}} \xi_{m,L+1}^2 - (1-g) \xi_{k,L+1}^2 \right) \\ & + \text{mixed 3rd order terms} \frac{1}{\xi_{k,L+1}} (\xi_{i,L}^2 \xi_{j \neq i,L}^2)_{i,j \in I_{k,L+1}}. \end{aligned} \tag{16}$$

Neglecting the third order mixed terms, we obtain a coupled set of approximate selection equations of the coarse-grained level $L + 1$ that read

$$\frac{d}{dt} \xi_{k,L+1} = \xi_{k,L+1} \left(\lambda - g \sum_{m=1}^{N_{L+1}} \xi_{m,L+1}^2 - (1-g) \xi_{k,L+1}^2 \right) \tag{17}$$

and just assumes the form of the order parameter equations of the previous level L , see Eq. (13).

Finally, we assume that the patterns under consideration exhibit a JND that induces gap conditions as discussed in cases I and II of Section 2.1 for the initial amplitudes $\xi_{k,L}(0)$ on all coarse-grained levels L . Under these conditions, the mixed third order terms in Eq. (16) can be neglected (Case I) or affect the transient dynamics only to a relatively small degree which implies that the approximate selection equations (17) of the level $L + 1$ yield consistent results with the selection equations (13) of the level L (Case II).

Let us exemplify the relationship between the selection equations (13) and (17) on subsequent levels L and $L+1$ of coarse-graining. For illustration purposes it is sufficient to consider just two levels $M = 2$ and a set of $N_1 = 4$ patterns on $L = 1$ that is reduced to $N_2 = 2$ pattern classes on the level $L = 2$. Furthermore, the patterns $k = 1, 2$ and $k = 3, 4$ on $L = 1$ are assumed to constitute the pattern classes $k = 1$ and $k = 2$ on $L = 2$. That is, we have $I_{1,2} = \{1, 2\}$ and $I_{2,2} = \{3, 4\}$.

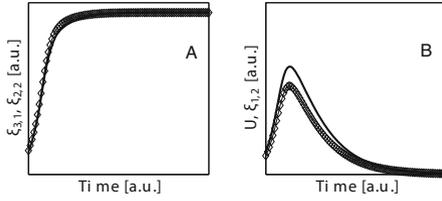


Fig. 2. $\xi_{1,2}$ (panel A) and $\xi_{2,2}$ (panel B) computed from Eqs. (18) and (19) (solid lines) and Eq. (20) (circles) for consistent Case II conditions with $\xi_{3,1}(0) = 0.2$, $\xi_{j \neq 3}(0) = 0.2/\sqrt{(N-1)/(1+0.2\epsilon)}$, ϵ uniformly distributed in $[0, 1]$, and $N = 4$, $\lambda = 2$, $g = 1.3$.

For $L = 1$ the order parameter equations for $\xi_{k,1}$ with $k = 1, 2, 3, 4$ read

$$\frac{d}{dt}\xi_{k,1} = \xi_{k,1} \left(\lambda - g \sum_{m=1}^{N_L} \xi_{m,L}^2 - (1-g)\xi_{k,1}^2 \right) \tag{18}$$

and the coarse-grained order parameters on the level $L = 2$ are defined by

$$\xi_{1,2} = \sqrt{\xi_{1,1}^2 + \xi_{2,1}^2}, \quad \xi_{2,2} = \sqrt{\xi_{3,1}^2 + \xi_{4,1}^2}. \tag{19}$$

Under Case I and II initial conditions, the coarse-grained order parameters on level $L = 2$ satisfy at least approximately the selection equations

$$\begin{aligned} \frac{d}{dt}\xi_{1,2} &= \xi_{1,2} (\lambda - g[\xi_{1,2}^2 + \xi_{2,2}^2] - (1-g)\xi_{1,2}^2), \\ \frac{d}{dt}\xi_{2,2} &= \xi_{2,2} (\lambda - g[\xi_{1,2}^2 + \xi_{2,2}^2] - (1-g)\xi_{2,2}^2). \end{aligned} \tag{20}$$

Under case I initial conditions $\exists k^* : \xi_{k^*,1}(0) \gg \xi_{j \neq k^*,1}(0)$ the solutions of the coarse-grained differential equations (20) are good approximations to the exact solutions calculated from Eqs. (18) and (19) provided consistent initial conditions $\xi_{2,1}(0) = \sqrt{\xi_{1,1}^2(0) + \xi_{2,1}^2(0)}$ and $\xi_{2,2}(0) = \sqrt{\xi_{3,1}^2(0) + \xi_{4,1}^2(0)}$ are used. In order to illustrate this correspondence, we solved Eqs. (18), (19), and (20) numerically, see Figure 2.

3 Motion-induced blindness and schizophrenia

Motion-induced blindness is an optical illusion produced by a visual stimulus composed of a fixed stationary foreground pattern and a rotating background pattern. Typically, the foreground pattern consists of three yellow dots arranged in a triangle, whereas the background pattern is a rotating array (or grid) of blue dots. A human observer exposed to the MIB stimulus typically reports that some of the target dots disappear for a while. In this sense, the motion of the background pattern induces a temporary blindness with respect to the target pattern [32].

3.1 Modeling of fine- and coarse-grained order parameter dynamics

We distinguish between 8 spatio-temporal patterns on the level $L = 1$ that fall into two classes on the coarse-grained level $L = 2$. There is one perceptual pattern not subjected to a MIB effect (i.e., the three yellow target dots are perceived), which is regarded as the default pattern indexed by $k = 1$ on $L = 1$. The default pattern constitutes its own class on $L = 2$. Moreover, there are 7 different patterns that are subjected to a MIB effect (i.e., at least one dot is perceived as being absent). They are indexed by $k = 2, \dots, 8$ on $L = 1$. and constitute the class of 'incomplete patterns' on $L = 2$. On $L = 2$ the default pattern is index by $k = 1$ and the incomplete patter class is index by $k = 2$. Following earlier work on selective attention phenomena [1, 4], certain oscillatory phenomena of the perceptual [19, 20] and auditory system [21, 22], priming [23, 24], grasping [25, 26], and child development [27, 28], we assume that in general the attention parameters of the two classes are different from each other. In this case, the evolution equations for $L = 1$ and $L = 2$ read

$$\frac{d}{dt}\xi_{k,1} = \xi_{k,1} \left(\lambda_{k,1} - g \sum_{m=1, m \neq k}^N \xi_{m,1}^2 - \xi_{k,1}^2 \right) , \quad k = 1, \dots, 8 \quad (21)$$

and

$$\frac{d}{dt}\xi_{1,2} = \xi_{1,2} (\lambda_{1,2} - gU^2 - \xi_{1,2}^2) , \quad \frac{d}{dt}U = U (\lambda_U - g\xi_{1,2}^2 - U^2) \quad (22)$$

with $\lambda_{1,1} = \lambda_{1,2}$ and $\lambda_{k,2} = \lambda_U$ for $k = 2, \dots, 8$. The coarse-grained order parameter variables $\xi_{1,2}$ and U are related to the fine-grained order parameters $\xi_{1,1}, \dots, \xi_{8,1}$ as discussed in the Section 2 with $\xi_{1,2} \leftrightarrow \xi_{1,1}$ and $U \leftrightarrow \sqrt{\sum_{k=2}^8 \xi_{k,1}^2}$.

The stability of the winner-takes-all fixed points $\xi_{k^*,1} = \sqrt{\lambda_{k^*,1}} \wedge \xi_{j \neq k^*,1} = 0$ of Eq. (21) and $(\xi_{1,2} = \sqrt{\lambda_{1,2}} , U = 0)$, $(\xi_{1,2} = 0 , U = \sqrt{\lambda_U})$ for Eq. (22) depend on the attention parameter spectrum. The stability of fixed points of the synergetic computer in the case of an inhomogeneous attention parameter spectrum has been discussed in detail in a series of studies [7, 23–27]. From these studies it follows that for the default pattern the stability depends on $\lambda_{1,2}$, λ_U , and g like

$$\left. \begin{aligned} \xi_{1,1} = \sqrt{\lambda_{1,2}} \wedge \xi_{k \geq 2,1} = 0 \\ \xi_{1,2} = \sqrt{\lambda_{1,2}} \wedge U = 0 \end{aligned} \right\} = \begin{cases} \text{stable} & \text{if } \lambda_{1,2} > \lambda_U/g \\ \text{unstable} & \text{if } \lambda_{1,2} < \lambda_U/g \end{cases} . \quad (23)$$

By analogy, for the incomplete patterns we have

$$\left. \begin{aligned} \exists k^* \geq 2 : \xi_{k^*,1} = \sqrt{\lambda_U} \wedge \xi_{j \neq k^*,1} = 0 \\ \xi_{1,2} = 0 \wedge U = \sqrt{\lambda_U} \end{aligned} \right\} = \begin{cases} \text{stable} & \text{if } \lambda_U > \lambda_{1,2}/g \\ \text{unstable} & \text{if } \lambda_U < \lambda_{1,2}/g \end{cases} . \quad (24)$$

In what follows, we will primarily focus on the coarse-grained model. The oscillatory switching between the default pattern and a pattern out of the class of incomplete patterns can be modeled by assuming that the attention parameters

$\lambda_{1,2}$ and λ_U vary in time [18–22]. More precisely, we assume that if the default pattern is perceived then $\lambda_{1,2}$ decays gradually until the critical ratio $\lambda_{1,2} = \lambda_U/g$ is reached at which the percept becomes unstable, see Eq. (23). Consequently, the perceptual dynamics is subjected to a bifurcation and the perceptual experience of the default pattern is replaced by the percept of one of the incomplete patterns. However, the percept is assumed to induce again a decay in the corresponding attention parameter. That is, λ_U is assumed to decay gradually, while $\lambda_{1,2}$ relaxes back to a ‘rest level of attention’. Among various possible dynamical systems that are able to capture these mechanisms, we will use the following evolution equations for the attention parameter dynamics:

$$\frac{d}{dt}\lambda_{1,2} = -\frac{1}{\tau}(\lambda_{1,2}(t) - b_{1,2}) , \quad \frac{d}{dt}\lambda_U = -\frac{1}{\tau}(\lambda_U(t) - b_U) \quad (25)$$

with

$$\begin{aligned} b_{1,2} = 0 \wedge b_U = b_0 & \text{ if } \xi_{1,2} = \sqrt{\lambda_{1,2}} \wedge U = 0 \\ b_{1,2} = b_0 \wedge b_U = 0 & \text{ if } \xi_{1,2} = 0 \wedge U = \sqrt{\lambda_U} , \end{aligned} \quad (26)$$

where b_0 denotes the aforementioned rest level and $\tau > 0$ is a time constant.

Our aim is to investigate the oscillatory dynamics (22), (25), (26) in a special case which allows for a semi-analytical approach. To this end, we note that the parameter τ defines the characteristic time scale of the attention parameter dynamics. Likewise, $1/\lambda_{1,2}$ and $1/\lambda_U$ define the characteristic time scale of the dynamics of $\xi_{1,2}$ and U . Let $\lambda_{c,low}$ and $\lambda_{c,high}$ with $\lambda_{c,low} > \lambda_{c,high} = g\lambda_{c,low}$ denote the critical attention parameters at which percept-switching occurs. Then $\lambda_{1,2}$ and λ_U oscillate between these levels. Consequently, $\xi_{1,2}(t)$ and $U(t)$ evolve on a time scale at least as fast as given by $1/\lambda_{c,low}$. If $\lambda_{c,low}$ is chosen large enough (the value of $\lambda_{c,low}$ depends on the model parameters b and g) such that $1/\lambda_{c,low}$ is much shorter than τ , then the $\xi_{1,2}(t)$ and $U(t)$ are fast evolve variables, whereas the attention parameters $\lambda_{1,2}(t)$ and $\lambda_U(t)$ are slowly evolving variables. Figure 3 illustrates this case. In this case, the oscillation period can be calculated from the attention parameter dynamics alone. Moreover, differences in the transient behavior of the fine- and coarse-grained dynamics become irrelevant as long as both levels of consideration yield consistent results in the stationary case (case II, see Table 1).

In order to derive an expression for the oscillation period, we consider the case in which λ_U decays from $\lambda_{c,high}$ towards zero and $\lambda_{1,2}$ relaxes back towards b_0 :

$$\begin{aligned} \lambda_{1,2} &= \lambda_{c,low} \exp\left\{-\frac{t}{\tau}\right\} + b_0 \left(1 - \exp\left\{-\frac{t}{\tau}\right\}\right) , \\ \lambda_U &= \lambda_{c,high} \exp\left\{-\frac{t}{\tau}\right\} . \end{aligned} \quad (27)$$

This phase will be terminated when $\lambda_{1,2} = \lambda_{c,high}$ and $\lambda_U = \lambda_{c,low}$. The duration of the phase corresponds to half of the oscillation period. Therefore, at $t = T/2$

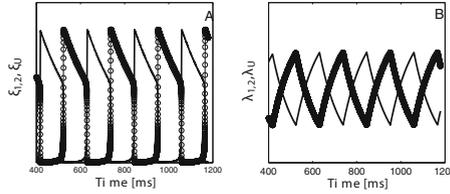


Fig. 3. Oscillatory behavior of the order parameters (panel A) $\xi_{1,2}$ (solid), ξ_U (circles) and attention parameters (panel B) $\lambda_{1,2}$ (solid), λ_U (circles) for $\tau = 100\text{ms}$, $g = \exp\{1\}$, $b_0 = g + 1$ as computed from Eqs. (22), (25), (26). Note that the observed period is $T \approx 2\tau$ as expected.

we have

$$\begin{aligned} \lambda_{c,high} &= \lambda_{c,low} \exp\left\{-\frac{T}{2\tau}\right\} + b_0 \left(1 - \exp\left\{-\frac{T}{2\tau}\right\}\right), \\ \lambda_{c,low} &= \lambda_{c,high} \exp\left\{-\frac{T}{2\tau}\right\}. \end{aligned} \tag{28}$$

Substituting $\lambda_{c,high} = g\lambda_{c,low}$ into the second relation of Eq. (28), we can determine T as a function of g and τ like $T = 2\tau \ln(g)$. Substituting $\lambda_{c,high} = g\lambda_{c,low}$ into the first relation of Eq. (28), we then obtain a relationship between the parameters b_0 and g :

$$b_0 = g + 1. \tag{29}$$

This relation tells us that the scenario described above can not be realized for any arbitrary values of g and b_0 . Rather, the model parameters must satisfy the matching condition (29). Eliminating b_0 by means of Eq. (29), the model defined by Eqs. (22), (25), (26) involves two unknown parameters g and τ . If we fix one of the two parameters, then the remaining parameter can be estimated from the experimentally observed oscillation period T_{obs} . In this context, a particular simple model can be constructed if we put $g = e$ (where $e = \exp\{1\}$). In this case, we have $T = 2\tau$, and the model parameter τ can be estimated from the observed oscillation period T_{obs} like $\tau_{estim} = T_{obs}/2$.

Let us generalize the model in order to account for the fact that in MIB experiments the default percept and the incomplete percepts are not necessarily perceived for the same amount of time. That is, in general, the MIB paradigm involves perceptual oscillations composed of two phases with unequal durations. In order to introduce two phases with different phase durations, we include a bias in the dynamical model defined by Eqs. (22), (25), (26). To this end, Eq. (22) is replaced by

$$\frac{d\xi_{1,2}}{dt} = \xi_{1,2} \left(\lambda_{1,2} + \frac{\delta}{2} - gU^2 - \xi_{1,2}^2 \right), \quad \frac{dU}{dt} = U \left(\lambda_U - \frac{\delta}{2} - g\xi_{1,2}^2 - U^2 \right). \tag{30}$$

For $\delta > 0$ the duration of the phase with $\xi_{1,2} > 0$ and $U = 0$ becomes longer than the duration of the phase with $\xi_{1,2} = 0$ and $U > 0$. According to our

interpretation of the model, we say that for $\delta > 0$ there is a bias towards perceiving the default pattern. Likewise, for $\delta < 0$ the model reflects a perceptual systems exhibiting a bias towards the perception of an incomplete pattern. If we consider the parsimony model defined by Eqs. (25), (26), and (30) with fixed parameters $g = \exp\{1\}$ and $b_0 = g + 1$, then we have two parameters τ and δ at our disposal to model experimentally observed durations of the phase of default pattern perception and the phase of incomplete pattern perception.

3.2 Schizophrenia patients data versus controls

Schizophrenia patients frequently show deficits in the perceptual processing of visual stimuli. In particular, perceptual processes are affected that involve higher cognitive functions such as feature binding [33–36]. On the other hand, there is evidence that the MIB phenomenon involves such higher cognitive processes and does not arise from low hierarchical processes like retinal suppression. For example, visual aftereffects that are assumed to emerge on a relative low hierarchical level of sensory processes are induced by the target dots of the MIB stimulus although these dots are not perceived by the observers [37, 38]. In other words, there is experimental evidence that when a target dot is not perceived by an observer then the sensory stimuli of the target dot is still processed in low hierarchical levels of the perceptual system but it is not processed (‘correctly’) on higher cognitive levels involved in consciousness and sensory experiences that are explicit to the observer. This point of view is also supported by experimental studies that point out the similarity between the MIB phenomenon and other Gestalt theoretical phenomena such as perceptual filling-in [39]. In summary, higher cognitive functions are relevant both for the MIB phenomenon and our understanding of schizophrenia, which makes the MIB phenomenon a promising paradigm to investigate schizophrenia [40].

In a study by Tschacher et al. [40] controls and schizophrenia patients were tested on the MIB phenomenon. Both groups were exposed to three trials of 60 seconds. On the average, the number of total MIB experiences within these three minutes was about 42 for controls and 29 for patients (see Table 3 in [40]). In what follows we distinguish between total and single event durations. The total durations of the MIB experiences was about 42 seconds for controls and 33 seconds for patients. From these data we can obtain a crude measure for the duration of a single MIB event. For controls we obtain a single MIB duration of about $T_{\text{MIB}} = 1.0\text{s}$ (i.e., $42\text{sec}/42$). For patients we obtain a single MIB event duration duration of about $T_{\text{MIB}} = 1.1\text{s}$ (i.e., $33\text{sec}/29$). Likewise, we can calculate a crude measure for how long on average the perception of a default pattern was experience before it became unstable (single event duration). Controls perceived the default pattern on the average for a total period of 138 seconds. Assuming (in line with our simplified model) that there were on average 42 switches to the default percept, we obtain an estimated single event duration of $T_{\text{default}} = 3.3\text{s}$ (i.e., $138\text{sec}/42$). Likewise, for patients we obtain a single event duration of the default pattern of about $T_{\text{default}} = 4.5\text{s}$ (i.e., $147\text{sec}/33$). In view of Eq. (30), we anticipate that a model-based analysis of the data should reveal that the

Table 2. Descriptive experimental data and model parameters

Group	Data		Model	
	T_{default} [ms]	T_{MIB} [ms]	τ [ms]	δ [1/ms]
Controls	3300	1000	1500	1.6
Patients	4500	1100	1700	1.8

parameter δ is larger for schizophrenia patients because the asymmetry of the durations of the MIB and non-MIB phases is more pronounced.

We fitted the model parameters δ and τ to reproduce the duration data T_{MIB} and T_{default} for controls and patients. To this end, τ was varied in the interval $[T_{\text{default}}, T_{\text{MIB}}]$ in steps of 100 ms, while δ was varied in the interval $[0, 2.0]$ 1/ms in steps of 0.1. The results of this fitting procedure are summarized in Table 2. As expected, we found that the asymmetry parameter δ is larger for schizophrenia patients than for controls.

4 Discussion

We studied coarse-graining of order parameter equations of the synergetic computer and followed in part earlier studies on hierarchical generalizations of the synergetic computer concept [8]. In particular, we showed that under certain conditions the coarse-grained order parameter equations exhibit the same mathematical structure as the corresponding fine-grained order parameter equations. In this sense, self-organizing artificial and natural systems, whose dynamics can be described (at least to some approximation) by the synergetic computer equations, exhibit a scale free system dynamics. A model-based analysis of literature data on multistable perception of schizophrenia patients tested in an MIB experiment was carried out. The observation that the frequency of MIB experiences is lower for schizophrenia patients than for controls corresponds in the model to a time scale parameter τ that is larger for schizophrenia patients than for controls. In addition, the model-based analysis highlights a second perceptual characteristics of schizophrenia patients that has so far received only little attention. The two different perceptual phases in MIB experiments seem to be less symmetric in duration under schizophrenia. This shows up as a symmetry breaking parameter δ which is larger for schizophrenia patients than for controls.

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