Flow Organization in Highly Turbulent Thermal Convection

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Dedication

Ladies and Gentlemen, dear colleagues and friends, it is with my greatest pleasure to address our Nestor and always stimulating academic advisor, Professor Hermann Haken, with my sincerest congratulations on the occasion of his 85th birthday. I wish to express my warmest thanks and appreciation for his leadership and his guidance.

Thank you, Hermann, for being our ideal over all the years, kindly accept all my, all our best wishes for you!

Abstract. Recent surprising results on very large Rayleigh-number thermal convection are presented and discussed. For Rayleigh numbers beyond about 10^{14} the scaling of the Nusselt number as well as the profiles are determined by turbulent boundary layers, though these are extremely thin. The theoretical interpretation is well consistent with the experimental data measured with the high pressure convection facility in Göttingen by Guenter Ahlers et al.

Keywords: Turbulence, Rayleigh-Bénard, thermal convection, log-layer, log-profiles, Nusselt-number, scaling

The results which I shall present have been obtained in close cooperation with the following colleagues: Detlef Lohse, Twente. – Experiment: Guenter Ahlers, Santa Barbara; Eberhard Bodenschatz, Göttingen; Denis Funfschilling, Nancy; Xiao-Zhou He, Göttingen; Ke-Qing Xia, Hong Kong; Quan Zhou, Shanghai. – Direct Numerical Simulation: Erwin van der Poel, Twente; Kazuyazu Sugiyama, Riken; Richard J. A. M. Stevens, Baltimore and Twente; Roberto Verzicco, Roma and Twente.

1 Introduction

Turbulent Rayleigh-Bénard convection has been the Drosophila of the physics of fluids for many decades, starting with the famous analytical calculation of

A. Pelster and G. Wunner (eds.), *Selforganization in Complex Systems: The Past, Present, and Future of Synergetics*, Understanding Complex Systems,
DOI: 10.1007/978-3-319-27635-9_1

the linear instability at the critical Rayleigh number $Ra_c = 1708$. The Rayleigh number is defined as usual, $Ra = g\beta_p L^3 \Delta/(\nu\kappa)$; g is the gravitational acceleration, β_p the isobaric expansion coefficient, L the height of the sample, and Δ the temperature difference between the hotter bottom and the colder top plates; ν and κ denote the kinematic viscosity and the temperature diffusivity, considered as temperature independent under present experimental conditions. Over the last decade experimental, theoretical, and numerical results have converged up to a Rayleigh number $Ra \sim 10^{12}$. – One of the quite unexpected findings is that even for such large Ra the boundary layers are still of Prandtl-Blasius, i. e., of laminar type, although time dependent. – In 2001 we had predicted that the transition to a turbulent boundary layer occurs around $Ra = 10^{14}$ (for gases), cf. [1]. Recently Ahlers et al. [2] indeed have experimentally found this laminar-turbulent transition at this very high Ra. Here due to a sufficiently large shear in the extremely thin boundary layers these eventually become turbulent, leading to a much stronger increase of the heat transfer with increasing Ra as in the laminar, the *classical* range below this turbulence onset. – In Grossmann and Lohse [3] we have calculated an effective scaling law $Nu \sim Ra^{0.38}$ for this ultimate regime by extending the unified scaling theory [4], [1], [5], [6], which determines the scaling behavior of the heat current Nu as well as the thermal wind amplitude Re as functions of the control parameters Rayleigh and Prandtl number Ra and Pr. Here the Prandtl number characterizes the fluid, $Pr = \nu/\kappa$, $Nu = Q/(\kappa\Delta L^{-1})$ describes the non-dimensionalized heat current density Q, and $Re = UL/\nu$ is the non-dimensionalized amplitude U of the convection (or wind) in the Rayleigh-Bénard container.

2 The Ultimate State of Thermal Convection for Very Large Rayleigh Numbers

Having explored strong thermal convection as described in the introduction we now also look at the local flow properties such as the (vertical) temperature profile. This turns out to show logarithmic dependence with distance z from the heated bottom and the cooled top plates [7]. This so called law of the wall and its properties as functions of the control parameters has been derived and analyzed in [8].

As a previously not yet studied surprise we have noticed the log-law in the classical regime below O(10^{14}) too, cf. above reference [7], apparently meaning that a turbulent bulk of thermal flow for Ra beyond the structure formation regime as observed at lower Ra, can well coexist with still laminar boundary layers. The notion *laminar* apparently has to be extended to time dependence on the gross convective time scale [9].

In the talk some of these self-organized flow structures in strongly driven thermal convection have been detailed together with some overview. The reader, who is interested in the development which lead to all the recent insight, is referred to reference [10]. A recent summarizing overview on the experimental details for the high pressure convection facility, known as the Göttingen Uboot, is provided in [11]. The measurement of the wind amplitude U, non-dimensionalized as the Reynolds number $Re = UL/\nu$, the second important response of the Rayleigh-Bénard heat flow experiment besides the heat current Nu, is described in [12]. As an overview on Rayleigh-Bénard flow reference [13] is recommended.

Some more details of the talk are: As in most, if not all, laboratory fluid flows the boundary layers are of utmost importance also in thermal convection. Although the flow – in particular also in the boundary layers – is time dependent for not too small Rayleigh numbers Ra, we have learned meanwhile, cf. [9], the surprising lecture that nevertheless the boundary layers have profile features of laminar flow; laminar in the sense that they satisfy the Prandtl boundary layer equations. Thus *laminar* does *not* mean *time independent*!

The validity of the Prandtl boundary layer equations quantitatively means that the velocity ("kinetic") boundary layer thickness δ scales with the wind amplitude U or $Re = UL/\nu$ in the container as $\delta = a/\sqrt{Re}$. The empirical constant a in Rayleigh-Bénard flow in containers of width to height ratio (so called aspect ratio) of order 1 has been found as $a = 0.5$. Since the wind along the plates fluctuates locally and temporally in strength, so does δ fluctuate; but experiment as well as theory have confirmed that on the respective local δ-scale the profile is excellently of Prandtl-type.

The shear across the boundary layer can be quantified by a shear Reynolds number $Re_s = U\delta/\nu$. This then for laminar boundary layers is $Re_s = a\sqrt{Re}$. Now, if this boundary layer shear exceeds a certain range of size, say an interval around some Re_s^*, the boundary layer becomes turbulent. Re_s^* is not sharp, since onset of turbulence in shear flow depends on the type of disturbances. Empirical results for Re_s^* for various macroscopic flows give values in the range of about 320 to 420, meaning that Re has to exceed a $Re^* = (Re_s^*/a)^2 = 4.1 \times 10^5 - 7.1 \times 10^5$. The wind before transition according to [4] to [6] is $Re = 0.346 Ra^{4/9} Pr^{-2/3}$. Therefore the onset of turbulence in the boundary layers of thermal flow in gases, having $Pr = 0.84$, is expected (and has been predicted cf. [1] !) in the range $Ra^* = 3.7 \times 10^{13} - 1.3 \times 10^{14}$. This is well confirmed meanwhile by the Ahlers et al. experiments.

To give some numbers: The Prandtl boundary layer thickness at turbulence onset is (using above formulas) $\delta^*/L = 8 \times 10^{-4} - 6 \times 10^{-4}$, which in the Göttingen high pressure convection facility, the Uboot device, of $L = 2.24$ m height is $\delta^* = 1.8$ mm to 1.3 mm, very small indeed. Also after turbulence onset there is a linear layer in the immediate vicinity of the plates, known as the linear viscous sublayer, followed – after a transitional buffer range – by the log-law profile, called the "law of the wall". The viscous sublayer width $z_* = \nu/u_*$ is determined by the turbulent fluctuation amplitude u_*, defined (and measured) by the kinematic shear stress or drag at the wall (plate).

$$u_*^2 = \sigma_{xz}(0) = p_{xz}(0)/\rho = \nu \partial_z U_x(0) \ . \tag{1}$$

The turbulent fluctuation amplitude u_* is the key quantity for turbulent flows. E. g., u_*^2/U^2 is the friction coefficient; u_* also determines the turbulent transport coefficients $\nu_{turb} = \bar{\kappa} z u_*$ and $\kappa_{turb} = \bar{\kappa}_\theta z u_*$ as well as the local turbulent dissipation rate $\epsilon_u(z) = u_*^3/(\bar{\kappa} z)$. The empirical constant $\bar{\kappa}$ is called the von Kármán constant, whose value in many flows is $\bar{\kappa} = 0.4$. – We have calculated u_* in [3] for a homogeneous plate to be the solution of the transcendental equation

$$\frac{u_*}{U} = \frac{\bar{\kappa}}{\ln(Re \frac{u_*}{U} \frac{1}{b})} \ . \tag{2}$$

Here $B_u = \bar{\kappa}^{-1} \ln(b^{-1})$ is called the profile constant of the log-law of the wall. – Calculating from this equation the thickness of the linear sublayer we obtain that z_*/L is of order 0.5×10^{-4} and thus an order of magnitude smaller than the laminar Prandtl width δ. In particular in the high Ra Uboot device it is $z_* \approx 0.11$mm.

The measured profile, see [7], i. e., the log-law of the wall, can be parametrized in the form

$$\frac{\langle T(z) \rangle - T_m}{\Delta} = A \cdot \ln\left(\frac{z}{L}\right) + B \ , \tag{3}$$

with T_m the arithmetic mean temperature between the bottom and top plates. This profile in vertical, z direction has been measured at about 10.1 cm off the side wall. Direct numerical simulations allow to calculate the profile also for all other wall distances but yet for smaller Ra, up to $O(10^{13})$. In [8] we have succeeded to evaluate the parameters A and $B = \ln 2 \cdot A$ and find

$$A = -\frac{\kappa}{\bar{\kappa}_\theta} \frac{Nu}{u_* L} \approx -\frac{1}{2\bar{\kappa}} \frac{u_*}{U} \approx -0.038 \ . \tag{4}$$

A depends on Ra very weakly only, $A \propto Ra^{-0.043}$, and it depends on the distance r from the wall center, its magnitude $|A|$ increasing with distance r. We explain this by the decrease of the plate parallel velocity component $U \hat{=} U_x(r)$ with distance r from center; this interpretation is quantitatively well consistent with the experimental data.

All these theoretical results originate from a Reynolds stress plus mixing length ansatz in the time averaged Boussinesq equation, as detailed in [8]. The main issue is $\overline{\boldsymbol{u}'\theta'} \approx -\kappa_{turb}(z)\partial_z \theta$. Surprisingly enough the numerical values of the characteristic parameters as $\bar{\kappa}$ and B_u are rather near to those, which have been determined for flows along plates, in channels, and through pipes.

As I have demonstrated, there is quite a lot of exciting and surprising new insight into thermal convection at very large Rayleigh numbers Ra and its flow organization, but still much more has to be explored.

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