Chapter 13
Nonlinear Dynamical Interaction Patterns in Collaborative Groups: Discourse Analysis with Orbital Decomposition

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Introduction

Literature on learning-in-groups research, in all areas of education, frequently makes use of the term group dynamics to refer to a hypothetical dynamical process taking place when individuals are interacting within a group setting. This is not surprising because most researchers acknowledge the inherent dynamical character of human and social experience, expanding from the microlevel processes of mind functioning to the macro-level processes of collective and social life. Paradoxically, most of the research endeavors in this area have been carried out in the traditional way, ignoring the time aspect and any reference to dynamics is considered merely at a metaphorical level.

Nevertheless, focusing on some ontological aspects of group functioning one may recognize that the dynamics is more than a metaphor and acknowledge that a different methodological framework is needed for a profounder investigation. Considering the interactions among group members working towards a common goal that requires collective action, it is observed that individuals adapt their behavior according to other's actions. They respond and add iteratively to the ongoing process, the results of which cannot be reduced to the behaviors of individual group members. Interactions among participants give rise to an outcome that is not explicably understood as resulting merely from the individual actions, because it emerges from a complex dynamical process and it can be understood only in an evolutionary context. Thus, group interaction processes cannot be effectively studied with conventional linear approaches which are incompatible with the nature of the underlying phenomena.

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This chapter presents and exemplifies the application of nonlinear dynamics and complexity framework to the study of learning-in-groups studies focusing on science education. A discourse analysis is carried out by *orbital decomposition analysis* (ODA), a method designed for data that comprise categorical time series. In the beginning, a short reference to cooperative learning literature in science education is made. The section that follows discusses theoretical issues, such as the ontological characteristics of verbal interaction processes, and it reveals the nonlinear nature of the discourse taking place in a cooperative learning setting. Subsequently, fundamental theoretical concepts, such as *entropy*, *self-organization*, and *inverse power law distribution*, are explained in relation to discourse analysis; also, methodological assets of nonlinear dynamics and complexity, such as *Shannon entropy*, *topological entropy*, *dimensionality*, and *Lyapunov exponent*, are presented along with the basics of ODA. Results from experimental data and their interpretation are presented analytically in the following section, while a final discussion on methodological and epistemological issues along with implications for educational theory, practice, and research is provided.

Learning-In-Groups in Science Education

Educational sciences, in order to explain research findings and to guide practice, have fostered various psychological theories, such as Piaget's (1973) cognitive developmental theory, which focuses on the personal construction of knowledge; Vygotsky's (1978) theory, which emphasizes the social aspect of knowledge construction; and behavioral learning theories of Bandura (1977).

Learning-in-groups has traditionally been considered as an effective teaching approach and it belongs to cooperative learning methods, a generic group of educational procedures. In these settings, learners work together in small groups to accomplish shared goals, e.g., to understand a given topic or problem and arrive at a solution (e.g., Johnson & Johnson, 1991; Johnson, Johnson, & Maruyama, 1983; Lazarowitz & Hertz-Lazarowitz, 1998). These educational processes are relevant to Vygotsky's ideas emphasizing the construction of knowledge as social process. Within the social environment, the learner or novice negotiates the meaning of the matter to be learned with others, who could be either experts (e.g., a teacher) or peers. The process of negotiation results in a cognitive gain that is substantially higher than the anticipated achievement by one's own abilities. This learning environmental support is the *zone of proximal development*, which is modified and expanded when students interact within a learning-in-group setting. Relevant to social learning perspective is the *situation learning theory*, which emphasizes a local process depending on situational characteristics and being temporarily decoupled from individual differences. Situational characteristics include the means, the rules, and the setting climate that determine the function of the group under particular circumstances. It is imperative to mention here that specially for science education, which could be characterized by a synthesis of linguistic, mathematical/symbolic, and visual representations, (Lemke, 1998, 1999; Lynch & Woolgar, 1990), the role of language is crucial. This justifies why researchers attempt to understand learning outcomes by focusing on discourse analyses.

Typically, group interaction studies in learning science are designs which include recording and analyzing discourse material with the aim to identify dominant interactions and to correlate them with achievement. The data consists of verbal interactions occurring among members of small group of students who elaborate explanations about physical or chemical phenomena, and work together towards understanding the relative scientific concepts. The effectiveness of small-group process in learning has been studied as a function of various independent variables, such as the type and difficulty of the task or prior experience (Appleton, 1997; Bowen, 2000; Lazarowitz & Hertz-Lazarowitz, 1998; Lazarowitz, Hertz-Lazarowitz, & Baird, 1994; Shachar & Fischer, 2004; Zady, Portes, & Ochs, 2002), where it has been established that classroom interactions are correlated with students' performance; that is, enhanced learning outcomes are observed in group processes where certain types of interactions occur more frequently (Kempa & Ayob, 1991, 1995; Stamovlasis, Dimos, & Tsaparlis, 2006; Zady et al., 2002).

Research has been facilitated by recognizing various roles for the group members, which have been introduced for analytical convenience when doing empirical work, such as the *learner* or the *learner facilitator*. A member could also be a *leader* or a *follower*. The leader is someone who continuously takes the initiative to provide an idea or to develop an argument and leads in a way the unfolding discussion, while the follower is someone who merely responds to other's initiatives. Each individual engaged in the discourse interactions might play one or more roles successively. Some roles could be assigned in the group from the start, e.g., the leader; they might also appear or emerge during the course of interactions. In the latter case, these roles are rather correlated with some individual differences (Hall et al., 1988; Horn, Collier, Oxford, Bond, & Dansereau, 1998; O'Donnell, Dansereau, & Rocklin, 1991). For instance, a student with high cognitive skills and verbal ability attains the learning material faster and can provide support to his/her peers acting as a facilitator or leader. In most group settings members are encouraged to take initiatives to contribute to the process; however, not surprisingly, some members only demonstrate active participation and leadership. In science education, the active participation is encouraged and essentially it is presumed for effective outcomes; however, a considerable amount of silent learning is taking place as well (Stamovlasis et al., 2006).

Research in science education has shown that the effectiveness of an interaction process in a group setting depends on a number of *factors*, some of which are individual differences of the group members, previous training, the nature of the task, and the interactive process itself (Johnson, Johnson, Ortiz, & Stanne, 1991; Johnson, Johnson, Stanne, & Garibaldi, 1990; Webb, 1989, 1991). On the other hand, properties which can characterize quantitatively and qualitatively the discourse are referred as the *features of interactions* and concern the type of information exchanged during group sessions. For example these could be of cognitive type or interpersonal interactions of social type. Some of the features may concern the group functioning as a whole, e.g., the *climate* which concerns explicit or tacit affective communication and/or the *cooperativeness* among members. These are characteristics, which in a structured session could be manipulated by the instructor in order to optimize the outcomes. The factors and the features of interactions are typically subjected to measurement and comprise the foremost independent variables in learning-in-group research. It has been pointed out that small-group processes contribute to productivity and to the development of higher order cognitive skills, provided that interactions with the appropriate features are developed (Noddings, 1989; Taggar, 2001; Vygotsky, 1978). Thus, the temporal patterns of verbal exchange developed in an evolutionary context, *ceteris paribus*, are the determinants ensuring learning and productivity.

A Note on Methodological Issues

Even though learning-in-groups has become a widely used instructional procedure at all levels of education and in all subject areas, and its effectiveness is well established, there are still theoretical and methodological issues that warrant for further examination. There is lack of a unified theoretical framework that could embrace all associated with learning-in-groups phenomena and provide a comprehensive description and explanation in terms of specific mechanisms underlying the interaction processes. In general, group research in behavioral sciences seems to be fragmented regarding the theoretical premise. There is a multiplicity of theoretical approaches and methodologies, which focus on different aspects and lead to a variety of perspectives, e.g., communication, psychoanalytic, social, developmental, or functional perspective (see Wheelan, 2005). Yet, no attempts have been made to formulate a unified theory.

The theoretical issues, however, are interrelated with the methodological ones. A sophisticated theory needs a robust methodology to be developed, and on the other hand, an effective methodology requires a coherent and intelligible theory to be founded on, while the epistemological issues are by far crucial. Regarding the present inquiry, putative dynamical processes put forward by the theory are in need of a methodology that is specifically tailored to measure those processes. To this end, nonlinear dynamics and complexity appear to be more than a distinct alternative perspective. There are substantial contributions at theoretical level that approach a general theory of group functioning (e.g., Arrow, McGrath, & Berdahl, 2000), and also research methodology assets and tools for extensive applications (e.g., Guastello, 1998, 2009, 2011; Guastello & Bond, 2007).

Returning to science education, research objectives and methodologies followed in collaborative group settings have been diverse and linked to the theoretical perspectives adopted by the researchers. Typically, when investigating the effectiveness of a relevant learning procedure, the quasi experimental design has been the dominant one in quantitative research. This, however, is a “black-box” approach, which possesses a series of disadvantages. It has not provided essential understanding about the underlying processes, while it has been severely criticized for scantiness on core issues, such as establishing causality (Koopmans, 2014a, 2014b).

A large body of research focusing on group-learning approach belongs to the perspective known as *process-product-studies* of peer interactions (e.g., Stamovlasis et al., 2006; Teasley, 1995; Webb, Troper, & Fall, 1995). In these studies, peer interactions are coded, analyzed statistically, and finally linked to group performance and learning outcomes. The coding schemes could be either predetermined or the categories/codes could be assigned inductively during the actual coding procedure. The latter approach to coding is considered to be grounded in the data and it takes into account the context in which the discourse occurs. Furthermore, the distinction between *content frames* and *interaction frames* has been introduced, focusing on how students bring their frame of reference to the interaction situation and how these frames are jointly negotiated and developed (e.g., Barnes & Todd, 1995).

In most studies, the attention has been focused on specific features of the interactions, measured at the nominal level, representing events/categories that occur successively and form patterns unfolding in time. In science education research these patterns have been characterized as interpretative or exploratory modes of interaction and on the basis of their frequencies they were shown to be indicative of certain quality features of the discourse. Certain patterns have been found to be the most effective and constructive in critical engagements, including argumentation and hypothesis testing (Mercer, 1996).

Other researchers have attempted to follow more process-oriented methods to group interactions, which are seen as socially and situationally developed in students' discourse (e.g., Kumpulainen & Mutanen, 1999). By concentrating on individual and group functioning, these methods aimed to highlight the situated dynamics of peer interactions and learning-in-groups. Data analysis, which was focused on three dimensions, the language function of verbal exchange, the cognitive interactions, and the social process, revealed stimulating interaction patterns, where, nonetheless, the time aspect was rather implicit in the analytical framework. The notion of *dynamics*, even though was evoked through microanalysis of interactions and the concepts and tools utilized, was the traditional linear means.

Moreover, while traditional methodologies applied to discourse analysis have yielded interesting findings, they have not been mathematically formalized to the extent that they can be meaningfully associated with a certain theoretical framework. This chapter seeks to address this gap by presenting ODA, a novel approach to the study of peer-interaction processes in educational settings; it adds to theoretical and epistemological development of the *situated learning* perspective, and sets the framework for the application of nonlinear dynamics and complexity to learning-in-groups methodology in science education.

Theoretical Issues

Discourse as a Nonlinear Dynamical System

A group of individuals, e.g., students working together and interacting with each other, form a system that possesses dynamical characteristics. Before developing any mathematical formalism on group interactions, it is imperative to attempt a narrative portrayal of the processes and their dynamical features in a physical language. This may seem trivial; however it provides an understanding of how the elementary actions or events are linked to the behavior outcomes at a higher level of complexity and contributes to the formation of macro-characteristics of a given discourse.

When students with a shared goal interact amongst themselves attempting, e.g., to solve a problem, to gain a common understanding, or to reach a consensus about an issue, collective action simultaneously or successively is required. In these processes, the group members adapt their behavior according to the actions of others. In discourse, verbal interactions are taking place as the participants are exchanging information, and in order to scrutinize it, one may have to track verbal exchanges and reveal their qualitative features that are patterns of sequential events unfolding over time. Of course, the focus is on the emerging interactions at the group level, whereas the individual dynamics unfolding in each one's mind are usually ignored; however, they are present at a lower level of complexity and a reference to them should be made when describing behavior at that level.

Within a single person, the cognitive and affective states and the goal-directed actions as well might evolve independently from external causes. The *intrinsic dynamics* of each individual is central to the characterization of his/her actions (Vallacher, Van Geert, & Nowak, 2015). Actions realized in time also have their own dynamics, and they typically have a hierarchical structure spanning in various *time scales*. Time scale is a fundamental notion in nonlinear dynamics and refers to the length of time during which an event occurs or develops; for example it could happen in the period of a few seconds or in the period of hours or days. Elementary actions being organized accordingly give rise to action at higher level, which could result in a qualitative change in the course of time (e.g., a decision to intervene or refrain from intervening in an ongoing discourse). The intrinsic dynamics are fundamental in understanding the dynamics of human experience overall, and human behavior at social level in particular (Vallacher & Nowak, 2007, 2009). *Coordination* of individuals' actions over time is a necessary condition in social interactions and collective behavior. At social level, research has showed that coordination dynamics are central to human behavior, and they include lower level actions such as speech and movement (e.g., Kelso, 1995), and synchronization phenomena at macro-social level, such as norms and public opinions (e.g., Nowak, Szamrej, & Latané, 1990; Vallacher & Nowak, 2006). Studying interpersonal dynamics of lower level action suggests that the *coordination* interplay exhibits features of nonlinearly coupled oscillatory processes, where the temporal pattern might include *in-phase* and *anti-phase* forms. These notions refer to synchronization effects of engaged vs. disengaged interacting parts, respectively, while phenomena such as *hysteresis* could also be present; the latter denotes the time-based lag between input and output and it is encompassed among the fundamental characteristics of nonlinear dynamical processes (Kelso, 1995).

Returning to the discourse analysis, the process where the elementary actions give rise to macrostructure of temporal communication patterns, coordination dynamics are decisive for the process evolution and coherence. In a cooperative learning setting, temporal coordination dynamics of internal states such as feeling, mood, and dispositions also occur, and are rather the prerequisites to the coordination of actions within the group. The *coordination* dynamics in a discourse include temporal patterns of in-phase and anti-phase forms of synchronization such as competition/cooperation or agreement/disagreement; that is, they encompass the *complementary opposites* that function in self-organized fashion and shape the evolving information flow (Kelso & Engstrøm, 2006).

The observable traces of the *coordination* interplay in a discourse are sequence of utterances/categories unfolding in time that convey information about the evolving scenario, which however cannot be reduced to the individual's dynamics of lower level action. In such sequence, each step is a function of the previous steps and the trajectory in time possesses characteristics that may resemble to nonlinear or even chaotic time series; this implies sensitive dependence on the initial conditions and on the parameters shaping the unfolding discourse. A different order of utterances, a different pattern, induces different dynamics and it might yield to a different outcome. A leader in the group often imposes his/her thesis to their peers, the process, then, might be halted, and the discourse comes to conclusion; however the process goes on if the intrinsic dynamics of another individual allows an action that intervenes with an objection and/or different proposition. The peer's intervention feeds back the process, which continues in an unpredictable way since the present state depends on the previous one and the evolving scenario becomes history dependent; multiple scenarios are likely to emerge. The discourse evolution is not determined by certain features or properties of the interacting elements (group members), but it seems to be self-regulated by the coordinated actions of the participating agents.

Therefore, both the initial conditions and the evolution of the process do play a role. In the language of nonlinear dynamics, it is said that the trajectory of the process follows a complex pattern, which on the course of time might possess thresholds, *bifurcations*, and/or *attractors*. If the coordination pattern does not converge to a certain point of consensus (an attractor), it might be trapped to a limit-cycle attractors that characterize a system evolving in time, being unable to shift towards a desirable conclusion (for the attractor concept see also Chap. 9 in this volume).

The self-regulation mentioned above implies that the system is not driven by an external cause, but it shapes its own dynamics via self-organization mechanisms. The irreducibility of the system's behavior as a whole (discourse in the group) to that of its elementary components (members' actions) can define the discourse process as a complex adaptive system (CAS). The ontology of such system requires the epistemological shift towards the new science of nonlinear dynamics and complexity.

Discourse and Self-Organization

Having provided a theoretical description of discourse interactions, an epistemological step towards the regime of nonlinear dynamics and complexity has been made. Further investigation on the discourse interaction process and under this meta-theoretical framework requires the application of the relevant mathematical formalism and for this some core concepts are elucidated next. Relevant to the present inquiry are the notions of *entropy*, *self-organization*, and *fractal distribution*. General introduction to nonlinear dynamics and complexity theory can be found in Nicolis and Nicolis (2007), while relevant comprehensive outlines for psychology and life sciences could be found in Guastello, Koopmans, and Pincus (2009). In educational research literature, besides the present volume, relevant introductions have been sporadically reported in a number of papers (e.g., Koopmans, 2014a; Stamovlasis, 2006, 2011).

Within nonlinear dynamics and complexity theory, a significant descriptor of a system's state is its *entropy*. The concept of *entropy* originates from classical thermodynamics. Its statistical definition was developed by Ludwig Boltzmann in the 1870s. Entropy was introduced in social science applications with the development of *Information Theory* by Claude E. Shannon in 1948. In general, entropy stands for disorder (-order) or uncertainty and in the complex system sciences it appears as a significant variable associated with the information needed to describe the system, and thus it is related to system's complexity. Basic formalism of the entropy concept and its applications could be found in Nicolis and Prigogine (1989) and Nicolis and Nicolis (2007). A related entropy measure is *information entropy*, or *Shannon entropy* (H_S), which concerns a system or a set of categories with unequal odds of occurrence (see next section).

Self-organization concerns the corresponding theory which focuses on the study of open systems that operate at far-from equilibrium conditions, exchanging information, energy, or matter with their environments. Such systems, known as CAS, are self-regulated through complex feedback mechanisms, so that they can tune their dynamics and their own evolutionary characteristics, thus being adaptive to their environment (Nicolis & Prigogine, 1989; Prigogine & Stengers, 1984).

Self-organization means that the system is driven neither by any external intervention or control nor by any internal "demon" or a homunculus-like agent. It is the complex feedback processes, the temporal microscale fluctuations, and the underlying dynamics that determine the system's evolution. Under certain conditions, a discourse might exhibit such self-organizing behavior, when coordination among individuals leads to the organization of verbal interaction into dynamic patterns that emerge as a global structure from the local elementary actions with no predetermined scenario.

One characteristic property of a *self-organization* process is that the output variables or other measured quantities do not follow a Gaussian—normal distribution as it happens with independent measurements. There is a high degree of dependency among observations, which obey the *inverse power law* (iPL), a distribution that mathematically is expressed by the equation

$$S(f) \propto f^{-\beta} \tag{13.1}$$

where S is the size of an event (object or attribution) and f is the frequency of the event (object or attribution). The iPL is also called *fractal* distribution. The iPL in the case of event time series suggests that a large number of small events are expected, while exponentially very fewer large events occur. The exponent β can be calculated as the slope of the distribution curve at the log-log scale; it is called the *fractal dimension* and it is a measure of the system's complexity. Values $1 < \beta < 2$ denote dynamical characteristics (Schroeder, 1991; West & Deering, 1995). Higher values of β, that is steeper curves, denote that there are more small events, while lower values corresponding to relatively flat curves denote more large-scale events. Note that in discourse and group interaction phenomena, lower fractal dimensions are associated with greater structure or coherence (Guastello, 2005; Pincus & Guastello, 2005).

If a system's distributional characteristics exhibit fractal structure, then the underlying process evolves through a series of discontinuous shifts, a state that manifests itself as an iPL distribution, and/or through more global transformations as the system is being adjusted between different degrees of relative chaos (disorder) and order. In the language of nonlinear dynamics and complexity the above denote that the system is working at the dynamic regime, being at the *edge of chaos* (EOC) (Waldrop, 1992). Systems at the EOC demonstrate high capability of adaptation without annihilation or stagnation. Such properties of adaptive behavior are observed in complex adaptive systems across the sciences. Related examples are the distribution of total acts in social interaction systems initiated based on rank (Bales, 1999), the in-degree distributions in Web (Broder et al., 2000), and the distribution of verbal turn-taking interaction in family therapy sessions (Pincus & Guastello, 2005), to mention a few.

Returning to the current study and the discourse analysis, if the process under investigation is driven indeed by self-organization mechanisms, then the information flow or the evolving exchange of utterances within the group are expected to conform to the above type of temporal fractal structure as evidenced by the existence of an iPL distribution in the magnitude of recurrence of the various patterns.

Method

Experimental Settings, Data Collection, and Measurements

A common practice in science education involves small groups working together in order to carry out a task, such as physics or chemistry problem solving, explaining phenomena, elaborating and understanding science concepts, or even experimentation aiming to develop practical skills. The results presented in a following section are derived from experimental settings with groups of three members aimed to investigate how students' verbal interactions evolved during a discourse segment, when developing explanations of physical phenomena and the relevant concepts, such as gravity, velocity, and acceleration. The subjects were secondary school students in tenth grade, taking compulsory classes in the sciences. The assignment of the groups was based on two criteria: academic achievement and performance in a psychometric test of developmental level (Lawson's test, 1978), so that the group synthesis preserved heterogeneity within each group and thus equivalence. The design included pre- and posttest (which are not used here) and in addition a *group test*, which was an evaluation test on questions that had to be answered after negotiation and consensus, at the end of the session. This was a measure of the *group performance*, reflecting the amount of learning emerged from the discourse. Group performance was measured and recorded as a three-level ordinal variable: high performance (successful), intermediate (partial success) performance, and low performance (failed). No specific time limit was imposed on these sessions, which however had by design two important features: First, these tasks were relatively novel to the students; this was chosen in order to pose an intellectual challenge to the students and allow emerging phenomena, e.g., brain storming. Second, the groups were unstructured in terms of role taking and they were let to function spontaneously, allowing so the manifestation of pronounced dynamic effects. Students' verbal interactions through their negotiations in all groups were audiotaped and transcribed.

After the coding procedure and the identification of verbal interactions, a variable named *group activity* was defined to measure the total contributions or actions occurred during the interaction process. Ordinarily, the *group activity* is measured by the number of utterances brought up in the discussion and it has been correlated with high performance (Kempa & Ayob, 1991, 1995; Stamovlasis et al., 2006). It has been acknowledged however that not only the number of contributions enriches the discourse and enhances the probability of an ultimately successful outcome, but the multiplicity and variety of utterances as well. To this end, within the present methodology *information entropy* (H_S) is proposed as a measure of the group activity, because it has two advantages. First, H_S is a theoretically suitable measure to reflect the degree of *novelty* in terms of new categories and/or new patterns. Second, it is a concept of complexity theory and thus it can be co-examined along with other nonlinear measures (see next section).

Coding Procedure: The Key of Inquiry

The first and crucial step in a discourse analysis is the coding procedure. Spoken conversations produce utterances which can be coded according to the theoretical framework of interest and create a series of events/categories unfolding in time. It should be emphasized that the coding procedure does not necessarily implement predetermined categories; the categories/codes could be inductively emerged from the coding procedure, e.g., see qualitative approaches (Denzin & Lincoln, 2005). Thus, the coding procedure is not different from a typical one followed in traditional inquiries. ODA focuses on the ensued symbolic sequences and analyzes them accordingly. The categorical time series analysis could be applied with various

Table 13.1 Example categorization systems

Coding system I		Coding system II	
Cognitive interactions		Interpersonal interactions	
R=	Reflection on the problem	Y=	Expressing approval
E=	Explanation with a physical law	N=	Expressing disagreement
H=	Hypothesis	D=	Expressing doubt
A=	Argument	A=	Asking for approval
T=	Thesis	H=	Asking for help
S=	Skeptical	G=	Providing guidance or help
C=	Recall a physical law		

methodological approaches of data collection and coding procedures, depending on the research questions or hypotheses posted. The coded data resemble the following stings of symbols:

AABBBDCAABAEAABBBEAABAEAABBBABABABEED. . ..

where A, B, D, or E are coded utterances. An utterance is defined as a simple, complete or incomplete phrase or a chain of phrases, which possess recognizable and interpretable elements of communication. When the interaction process involves written messages, e.g., in e-mail communications, the interacting agents have the opportunity to express much more ideas and greater variety of utterances can be recorded, and the coding scheme becomes richer. Note that the content of the coded utterances is not related to dynamics; it is the pattern structure that is associated with the dynamical characteristics. Also in ODA discrete event sequences are recorded regardless of the length of time required to complete the event or the time that elapses between the events. Time length is an interesting feature, which deserves a special focus in the time series analysis; however, it is not examined in this chapter.

A discourse could be coded with category systems of various forms simultaneously and analyzed accordingly with ODA. In Table 13.1, examples of coding systems are presented. Coding systems I and II include codes of two types of interactions, cognitive and social-interpersonal interactions, respectively. Another simple coding scheme might assign a symbol to each participant, so that the turn-taking pattern can be followed and recorded. Different coding schemes facilitate different hypothesis testing of theoretical or practical interest. For instance, it might be desirable to compare the level of cognitive activity to the level of social interaction process evolving simultaneously through a given discourse. Another interesting example of coding scheme might include categories characterizing language function, such as "informative," "evaluative," and "affectional," (e.g., Kumpulainen & Mutanen, 1999). In the latter case the application of ODA may serve testing hypotheses concerning the evolution of discourse at linguistic level and determining potential relationships between language functions and discourse outcomes.

Coding procedures are fundamental parts of the analysis since the results concern the theoretical premise that is behind the coding scheme and drive the

Table 13.2 Coding scheme with cognitive utterances and participants' psychometric measures

Digit	Description	Evaluation
First digit	Actor's level of a psychometric variable (e.g., *M*-capacity)	1 = Low
		2 = Intermediate
		3 = High
Second digit	Cognitive utterance (nominal scale)	1 = Reflection on the problem
		2 = Explanation with a physical law
		3 = Hypothesis
		4 = Argument
		5 = Thesis
		6 = Skeptical
		7 = Recall a physical law
Third digit	Correctness (ordinal scale)	1 = Incorrect
		2 = Partially correct
		3 = Correct

hypotheses posted. It is also possible to include multiple categorical variables in ODA (Pincus, 2001; Pincus, Fox, Perez, Turner, & McGee, 2008). A complex coding that includes multiple categorical variables applied to an educational setting is depicted in Table 13.2. A three-digit code for each utterance includes the following: the first for the speaker, the second for the type of cognitive category, and the third digit evaluates the content correctness at an ordinal scale. Moreover, in a multiple coding scheme, certain individual difference might also be coded and their role in the peer interaction process could be examined.

Finally, after a set of mutually exclusive and exhaustive categories have been derived their reliability should be established by two or more raters. Cohen's Kappa statistic could be used for measuring inter-rater reliability. Typically, values above 0.65 are considered satisfactory.

Symbolic Dynamic Analysis with Orbital Decomposition Method

ODA is based on symbolic dynamics and it is specially designed for the analysis of time series with data measured at the nominal level (Guastello, 2000; Guastello, Hyde, & Odak, 1998). The basic idea of ODA originates from a methodological approach involving calculations of entropy with scale variables, applied primarily to a physical system when characterizing an experimental strange attractor with periodic orbits (Lathrop & Kostelich, 1989). In these systems periodic orbits presuppose basins of attraction, and thus if more basins exist, the more chaotic the motion becomes (Newhouse, Ruell, & Takens, 1978). Analogously, in an interaction process of verbal exchange, the concept *proximal recurrences* of a repeated pattern plays the role of neighboring orbits and thus the greater the variety of these orbits, the more unpredictable the conversation flow will be, and more nonlinear or even chaotic the dynamic character of the evolving process might be. This analogy between periodic orbits and pattern recurrence in a categorical time series allows the application of similar concepts and formulas to the latter and the description of the process under investigation via quantitative means.

The primary form of a symbolic sequence under investigation is a string of symbols: e.g., AABEDBDEAABAEAABBBEA, where A, B, D, and E are the codes for the events occurring during the course of time. Patterns are combination of at least two symbols with varying length. A single symbol is not considered as pattern; however it is included in the analysis. The first two steps of the procedure involve two calculations: a likelihood χ^2 and φ^2 test for a string sequence or pattern of responses with varying length (C), and topological entropy (H_T). For $C=1$ a single utterance (e.g., A) is considered as the unit of analysis, while if $C=2$, two utterances in the row (e.g., AB or DB) are taken together as the unit of analysis. The calculations include all string lengths starting with $C=1$ and continuing with $C=2$, $C=3$, $C=4$, etc.

For each increasing string length, a likelihood χ^2 test provides the statistical significance; this is to exclude the pattern that occurred by chance. For a given string length C (e.g., $C=3$, A-B-D) and N_c strings of length C in the data, the expected frequency of the string is

$$F_{\text{exp}} = P_{\text{A}}P_{\text{B}}P_{\text{D}}N_c \tag{13.2}$$

where P_{A}, P_{B}, and P_{D} are the probabilities for A, B, and D, respectively. The corresponding likelihood χ^2 is given by the formula

$$\chi^2 = 2\sum\left[F_{\text{obs}}\ln\left(\frac{F_{\text{obs}}}{F_{\text{exp}}}\right)\right] \tag{13.3}$$

Note that for $C=1$ equal probability is considered for the null hypothesis, while for $C\geq 2$ the H_0 is that the odds of the string are equal to the a posteriori combinatorial probabilities of the states. The φ^2 test provides a correction to the χ^2. Moreover, φ^2 test is a measure analogous to the proportion of variance accounted for this string length and it is given by the equation

$$\varphi^2 = \frac{\chi^2}{N_c} \tag{13.4}$$

χ^2 and φ^2 are used to determine the optimal length at which the analysis of the symbolic sequence should be carried out. The optimum length corresponds to the maximum φ^2.

Topological entropy (H_T) describes the deterministic *nonrandom complexity* for the time series and it is the upper bound for the metric entropy, which is equal to the positive Lyapunov exponent (Lathrop & Kostelich, 1989). The latter is a measure of the chaoticity of the dynamical process. The calculation of H_{T} is based on the diagonal entries or trace of a hypothetical transition matrix at each string length ($\boldsymbol{M}^C$). Each cell entry is binary and indicates whether a particular symbolic sequence is followed in time by any other symbolic sequence. The trace tr$\boldsymbol{M}^C$ of this matrix represents instances in which a pattern is followed by itself in a consecutive period of time. This is the *proximal recurrence* (Guastello et al., 1998). The topological entropy (H_{T}) measure based on the trace of the matrix $\boldsymbol{M}^C$ is given by the equation

$$H_{\mathrm{T}} = \lim_{c \to \infty} (1/C) \log_2 \mathrm{tr}\left(\boldsymbol{M}^C\right) \tag{13.5}$$

The trace tr$\boldsymbol{M}^C$ is the sum of the diagonal elements, which consists of 0 and 1 s. *Proximal recurrences* become less likely for longer patterns based on simple combinatorial probabilities of single utterances; thus, as C increases, H_{T} is expected to decrease and eventually drops to zero. The longest optimal string length for analyzing the discourse corresponds to the string length C, when the tr$\boldsymbol{M}^C$ becomes zero at $C+1$, and at which, under optimum conditions, the value of φ^2 is maximized.

As the string length approaches infinity, assuming that the dynamics of the system is described by the transition matrix $\boldsymbol{M}^C$, H_{T} approaches the base-2 logarithm of the maximum eigenvalue of the matrix, which is the Lyapunov exponent, a measure of the chaoticity of the dynamical process described by the matrix $\boldsymbol{M}^C$ (Lathrop & Kostelich, 1989), and it also reflects its complexity that is not due to chance. The Lyapunov dimensionality then is calculated by the equation

$$D_{\mathrm{L}} = \mathrm{e}^{H_{\mathrm{T}}} \tag{13.6}$$

The second entropy measure is the *information entropy* or *Shannon entropy* (H_{S}). For a set of categories with unequal odds of occurrence it is defined by the following equation

$$H_{\mathrm{S}} = \sum_{i=1}^{r} p_i [\ln(1/p_i)] \tag{13.7}$$

where p_i is the probability associated with each ($i = 1$ to r) categorical outcome (Shannon, 1948; Shannon & Weaver, 1949). Shannon entropy is not related to dynamics; however it is a measure of complexity since it reflects the information content needed to describe the system. H_{S} has been proposed as a measure of the degree of novelty present within a categorical time series (Attneave, 1959). It indicates the degree to which a categorical time series contains relatively rare patterns, that is, those with low probabilities of occurrence. Topological entropy, on the other hand, does not reflect this degree of novelty because it relies on proximal recurrences.

Having found the optimal string length using the procedure described above, then the calculation of *fractal dimension* can be carried out using the iPL distribution graph (Eq. 13.1). If S is the magnitude of the recurrent pattern and f is the frequency at which each particular pattern occurs, the slope of the $1/f$ curve (Eq. 13.1) can be used as an estimate of fractal dimension.

Nonlinear Hypotheses for Discourse Analysis

The nonlinear analysis applied to categorical time series might be driven by various types of research questions and hypotheses. The identification of repeated patterns of different size, proximally or distantly, is a key feature to be sought. ODA can provide this information along with frequency distributions. By applying ODA to verbal interactions in learning-in-groups settings, the structure within turn-taking patterns can be examined, and a characterization of discourse sessions could be achieved, based on the amount of structure within the discourse patterns. Such macrostructure of a given discourse might be a qualitative emergent property that could be used as a classification criterion. The emerged macrostructure, along with its nonlinear quantitative measures, such as exponents and entropy of a given discourse, might also be correlated to the outcomes and the effectives of the interaction process.

Assigning codes for each person's name initials can accommodate the investigation of whether the interacting group is balanced as far as each member's contribution in the turn-taking patterns. In multiple coding schemes individual differences might be included so that hypotheses regarding their role in the interaction process might be investigated. For example, in Table 13.2, the coding scheme where each member's personal code is replaced with a code representing levels of a psychometric variable, e.g., M-capacity, facilitates testing the hypothesis that students with high information processing capacity demonstrate influential contributions to the peer interaction process.

Moreover, two fundamental interrelated theoretical hypotheses can be tested: (1) Verbal interaction processes in learning-in-group sessions display complex dynamic characteristic of *self-organization*. (2) Learning outcomes from group-member interactions are *emergent* phenomena from nonlinear dynamical processes.

Methodologically, the two measures, Lyapunov dimensionality (a measure of turbulence) and fractal dimension (a measure of complexity), are the means of demonstrating when the signature of nonlinearity and complexity is present in a group interaction process. Shannon's entropy is not a dynamic measure per se; however it indicates whether the discourses encompass novel patters regarding spoken utterances. Information entropy might be related to other nonlinear characteristics and is a valuable tool for evaluating complex patterns.

An interesting endeavor is the investigation of the conditions under which dynamical characteristics are present and how these might be associated with effectiveness and learning outcomes. Special cases, such as brain storming situations with emerging phenomena, attract special attention and are potential candidates for the application of ODA. In this inquiry, an additional hypothesis posted was that the *information entropy* of the resulting symbolic sequences, which reflects the group *activity* in each session, is correlated with the group performance.

Discourse Analysis with Orbital Decomposition

The application of ODA to symbolic time series and the related calculations can be carried out with ORBDE software (Peressini & Guastello, 2010). The provided tables and results are explained in the following paragraphs.

Table 13.3 presents the ODA results for a students' discourse, where the time series comprised of cognitive type interactions. All the relevant statistical indices were calculated for $C = 1$ to $C = 5$. The tr$\boldsymbol{M}^C$ becomes zero at $C = 5$, and between $C = 3$ and $C = 4$ strings, the former was chosen as the optimum string length for analysis based on the greater φ^2 value. The anomaly of φ^2 values greater than 1.0 has been described as resulting from a violation of the assumption of a 2×2 matrix, which however does not affect comparison, and the value of φ^2 reflects the proportion of variance accounted for this string length (Guastello et al., 1998). At $C = 3$ the entropy measure is $H_S = 4.137$ and Lyapunov exponent $D_L = 1.390$, indicating a nonlinear complex process. The measures D_L and H_S might be used to compare two categorization systems. For instance, an interpersonal interaction coding scheme that results to values $H_S = 3.252$ and $D_L = 1.134$ shows lower degree of novelty and less turbulence or chaos at the level of social interactions.

The most frequently recurring patterns are listed in Table 13.4. The first and second columns show the most repeated patterns (e.g., HES, SAS and RAC), while in the next columns the expected frequency along with the observed probability is given. In the last column the contribution of each triplet to the total information entropy value is calculated. The findings suggest that certain patterns or structure dominate in the evolution of the discourse; that is, triplets of utterance that express skepticism or doubt on preceding propositions or combine reflection with argumentation appeared more frequently and they might have a decisive contribution to

Table 13.3 Complexity and entropy indicators from orbital decomposition analysis of cognitive type interactions

C	tr$\boldsymbol{M}^C$	H_T	D_L	χ^2	df	N*	φ^2	H_S
1	4	2.00	7.389	68.85	8	114	0.604	1.895
2	7	1.404	4.070	67.937	22	113	0.601	3.324
3	2	0.333	1.390	153.725	27	112	1.373	4.137
4	1	0	1.000	123.534	14	111	1.112	4.483
5	0	–						

String length (C), number of proximal recurrences (trace of binary matrix C), topological *entropy* (H_T), Lyapunov dimensionality (D_L), Shannon entropy (H_S), χ^2, φ^2, and number of strings for $C = 1$–4

Table 13.4 Primary strings of cognitive utterances identified at $C = 3$

Code ($C = 3$)	Utterances' pattern	Frequency	Expected frequency	P_{obs}	Shannon $p \log (1/p)$
HES	Hypothesis-explanation-skeptical	4	0.542	0.036	0.119
SAS	Skeptical-argument-skeptical	4	1.703	0.036	0.119
RAC	Reflection-argument-recall	4	0.210	0.036	0.119
ESR	Explanation-skeptical-reflection	3	0.348	0.027	0.097
STS	Skeptical-thesis-skeptical	3	0.310	0.027	0.097
CAS	Recall-argument-skeptical	3	0.745	0.027	0.097
EAE	Explanation-argument-explanation	3	0.426	0.027	0.097
AEA	Argument-explanation-argument	3	0.585	0.027	0.097
ESH	Explanation-skeptical-hypothesis	3	0.542	0.027	0.097
ACH	Argument-recall-hypothesis	3	0.326	0.027	0.097
HSE	Hypothesis-skeptical-explanation	3	0.542	0.027	0.097

the final outcomes. Table 13.5 depicts the patterns of multiple coding; it suggests that crucial contributions of decisive utterances with correct contents were made by members of high information processing capacity (*M*c), who are essentially undertaking the role of facilitator. Analogously, a number of similar hypotheses regarding the effects of other individual differences in collaborative groups could also be tested.

One of the main concerns expressed in the hypotheses frequently posted is the relation (if any) between group performance and effective dynamic patterns unfolding in the discourse. In the current study, group performance is measured by the group test, which accounted for the correct answers received after negotiation by the group members, and it reflects the amount of learning resulted from the collaborative session. On the other hand, *group activity*, which traditionally is operationalized by the number or frequencies of utterances, is a prerequisite for high-level outcomes. A relation between group performance and group activity was sought by implementing *information entropy* (H_S) as a measure for the latter. H_S proved to be a suitable index to characterize discourse based on certain categorization scheme, since it reflects the degree of novelty of utterance patterns in regard to category/code scheme of the choice. Successful sessions, that is, those of high group-performance, appeared to display higher information entropy, compared to the unsuccessful ones. Even though causality between H_S and group performance cannot be directly established, a probabilistic relation might be derived from empirical data analyzed by means of ordered logistic regression. Figure 13.1 depicts the proposed relation showing the probability of attaining low and high group performance level (effective and ineffective sessions) as a function of *information entropy* H_S (calculated values) encompassed in the utterance patterns of the discourse. The probability of attaining a successful session increases as the information entropy increases, while the probability of attaining an unsuccessful session decreases as the information entropy increases.

Table 13.5 Patterns of multiple coding

Code		Multiple pattern
HES	333 223 363	High Mc-Hypothesis-correct, Int.Mc-Explanation-correct, High Mc-Skeptical-correct
	333 222 362	High Mc-Hypothesis-correct, Int.Mc-Explanation-partially cor., High Mc-Skeptical-correct
	232 223 363	Int.Mc-Hypothesis-correct, Int.Mc-Explanation-partially cor., High Mc-Skeptical-correct
	333 121 263	High Mc-Hypothesis-correct, Low Mc-Explanation-incorrect, High Mc-Skeptical-correct
SAS	262 342 363	Int. Mc-Skeptical- partially cor., High Mc-Argument-partially cor., High Mc-Skeptical-correct
	262 342 363	Int.Mc- Skeptical- partially cor., High Mc-Argument-partially cor., High Mc-Skeptical-correct
	362 141 363	Int.Mc-Skeptical-partially cor., Low Mc-Argument-incorrect, High Mc-Skeptical-correct
	262 342 162	Int.Mc-Skeptical-partially cor., High Mc-Argument-partially cor., Low Mc-Skeptical-incorrect
RAC	313 243 171	High Mc-Reflection-correct, Int.Mc-Argument-correct, Low Mc, Recall-incorrect
	313 243 172	High Mc-Reflection-correct, Int.Mc-Argument-correct, Low Mc, Recall- partially cor.
	313 243 373	High Mc-Reflection-correct, Int.Mc-Argument-correct, High Mc -Recall-correct
	313 243 373	High Mc-Reflection-correct, Int.Mc-Argument-correct, High Mc -Recall-correct
ESR	323 263 313	High Mc-Explanation-correct, Int.Mc-Skeptical-correct, High Mc-Reflection-correct
	323 262 313	High Mc-Explanation-correct, Int.Mc-Skeptical-par.correct, High Mc-Reflection-correct
	121 263 313	Low Mc-Explanation-incorrect, Int.Mc-Skeptical-correct, High Mc-Reflection-correct
HSE	333 263 131	High Mc-Hypothesis-correct, Int.Mc-Skeptical-correct, Low Mc-Explanation-incorrect
	233 363 233	Int.Mc-Hypothesis-correct, High Mc-Skeptical-correct, Int.Mc-Explanation-correct
	333 263 333	High Mc-Hypothesis-correct, Int.Mc-Skeptical-correct, High Mc-Explanation-incorrect

Processes at the Edge of Chaos

One of the main hypotheses is whether the propagation of verbal interactions or the time series of verbal turn-taking patterns conforms to *inverse power law*. It was found that some of the analyzed symbolic sequences followed the iPL distribution. The iPL for one session is demonstrated in Fig. 13.2 showing the log of the

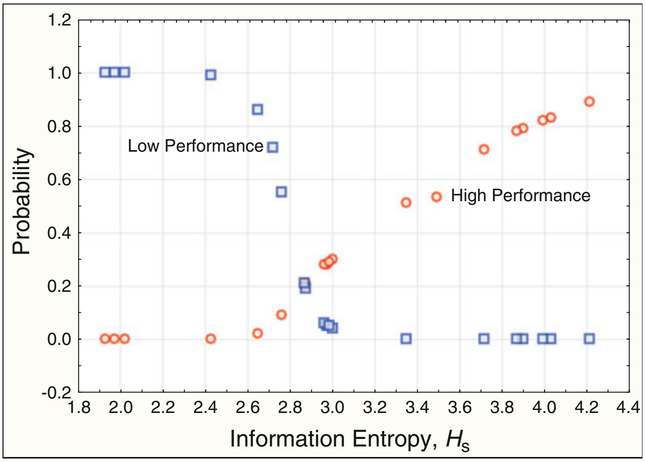

Fig. 13.1 Plot of the probability of attaining low and high group performance as a function of *information entropy* (H_S) encompassed in the dynamical utterance patterns [calculated values using a logistic function]

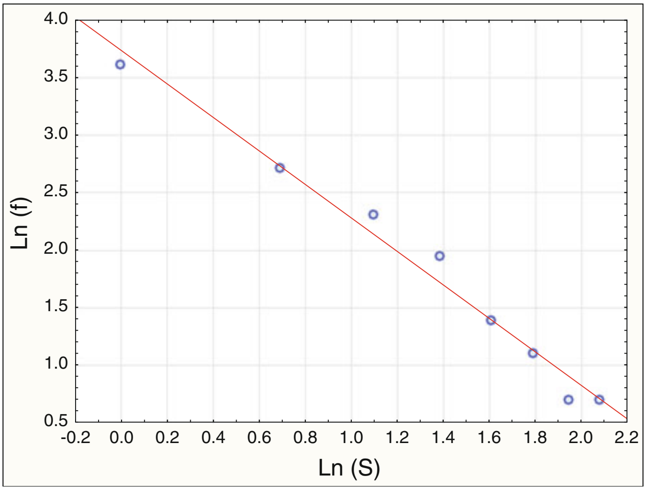

Fig. 13.2 A log-log scale plot of the number of different patterns that occur at various frequencies. The *fitted line* ($R^2 = 0.97$) suggests an iPL distribution with $\beta = 1.46$ ($t = -17.7$, $p < 0.001$)

frequency [ln(f)] at which each of these recurrence phenomena occur as a function of the log of the number of recurrences [ln(S)] for a given pattern at the optimum string length. The distribution has a negative slope which is the *fractal dimension*. The fitted line ($R^2 = .97$; $F = 313.30$; $p < 0.001$) provides the value of $\beta = -1.46$, with 95 % CI [−1.68; −1.26], which is within the typical range ($1 < b < 2$) for EOC processes (Bak, 1996; Kauffman;, 1995). The presence of an iPL denotes that the system is working within the dynamic regime, being at the *EOC* (Waldrop, 1992), a state characterized by both complexity and coherence, and even though the categorical time series is unpredictable on a moment-by-moment basis, it could be somewhat predictable on a global level (fractal distribution). This finding, along with the other nonlinear indices, supports the central hypothesis on the *emergence* of learning outcomes and the creative nature of interactive processes.

Discussion and Overview

This chapter presented the ODA, a novel method for studying dynamical properties of patterns in categorical time series. ODA is based on symbolic dynamics and it was used to identify patterns of interactions in discourses taking place within collaborative group sessions. Symbolic dynamics is an area of mathematics that studies series of entities or categories forming regularities or patterns unfolding in space or time, whereas they can be further examined for structures of higher order. The identification of regularities and hierarchical structures within symbolic sequences is an analogous endeavor to cryptographic analysis, where meaningful patterns of symbols are sought, and it is motivated by similar philosophy as the Turing's computational machine (Hodges, 2012). The main question that challenges this inquiry in a discourse analysis is if, at the optimum unit of analysis (string length), there are certain combinations of utterances, events, or multiple patterns of them, which are the more prevailing or the more creative contributions to the process under investigation.

Research has shown that discourse verbal interactions are not randomly organized in time (Pincus & Guastello, 2005). They possess dynamical structures of nonlinear character with varying dimensionality, order, or entropy. Typical mathematical tools, such as Markov chains used in symbolic dynamic analysis, cannot identify emerging and recurring patterns of utterances. Moreover, the various discourse analysis techniques, which have been applied to psychological and educational research for testing specific hypotheses, have not been grounded on any mathematical formalism or coherent theoretical premise. ODA is filling this gap in the literature of methodology by providing a general philosophy to measuring dynamical properties and unfolding patterns in time series measured at the nominal level. It provides quantitative indices of patterning, information, complexity, entropy, or chaos that can characterize the systems generating these series. The ODA method originates from an orbital decomposition method applied to chaotic time series; however it does not require the presence of chaos per se, but it can distinguish systems of sufficient complexity and quantify them based on measures comparable to chaotic indices, such as topological entropy and Lyapunov dimensionality. D_L is a dynamic measure and it is informative for the degree of turbulence or chaos in the categorical time series; higher values of D_L denote higher degree of complex patterning over the course of conversation that is not due to chance. In addition, *information entropy*, H_S, which increases with longer strings and richer combinations, reflects the decree of novelty characterizing the time series. D_L and H_S at the optimum string length correspond to the most probable structure conveying the dynamical characteristics and the information content, and are used for comparisons and further analyses.

The present study demonstrates that group interactions in cooperative learning settings can be studied effectively with ODA. Methodologically, it challenged the traditional approaches, which due to epistemological fallacies ignore what is between the input and output. It shed light into the "black box" by implementing the proper methodological tools and revealed the determinative role of dynamics, while it opens a new area of investigation for education research. The method could be extended to discourses of various topics in science and other disciplines as well. ODA is an appropriate mean of analysis for any relevant to education processes, such as attention, reading, studying, or playing. Moreover, it is applicable to any time series of qualitative attributes, actions, or events taking place within the school system, such as class attendance, accomplishments, episodes of decent or antisocial behavior (e.g., bulling), to name a few.

Returning to group-interactions inquiry, it must be said that groups are not always functioning as nonlinear systems and discourses do not always display emergent patterns. In the experiments presented in this work, group settings were designed so that activated members were involved in a free interaction process. Discourses, under certain circumstances, show special features of nonlinearity, nonrandom complexity, and novelty as measured by information entropy, which are associated with group performance and productivity. These cases are more likely to occur within unstructured settings where the discourse is allowed to evolve spontaneously without preexisting scenario. The findings support a central hypothesis that the learning outcomes from interactive groups *emerge* from nonlinear dynamical processes. This is in line with theoretical premises and empirical evidences from chaos and complexity research. The identification of iPL and fractal dimensionality supports the hypothesis that in certain cases discourse in a group interaction process could be functioning at the EOC, indicating creative processes and emergent phenomena. The connection between creativity and nonlinear processes has been elaborated in a special issue of *Nonlinear Dynamics Psychology, and Life Sciences* (issue 2, April, 2011). At the individual level, and focusing on the interactive mental resources in task executions, it was pointed out that the effective cognitive processes, those which lead to learning outcomes, are nonlinear dynamical processes. On the contrary, there are linear processes, such as "raw learning" procedures and algorithmic problem solving, which are not associated with learning and creativity (Stamovlasis, 2010, 2011, see also Chap. 9 in his volume). At a theoretical level, any mental process and inductive-type complex problem-solving procedure, where the solution is not hidden in the initial conditions, but is generated via an iterative and recursive process, conform to nonlinear dynamical processes. These are the processes that produce *information* (Nicolis, 1991). That is, these are the creative processes. In this chapter, the central notion of emerging learning or creativity through nonlinear dynamical processes has been extended to processes at social level, referring to a constellation of individuals/students who interact with each other as a coherent unity. It is of paramount significance that irrespective to the unit of analysis, at individual or collective level, the same principles can be demonstrated, and this is the advantage of the theory of complex dynamical systems.

The above findings have also important implications for education. At the group level, a productive interaction process differs from a traditional instructional session, which is based on behavioristic "transfer of knowledge" assumptions. The latter represents a linear process having a predetermined scenario, in which "learning" (if any) is considered to occur as a passive reception of emitted information. In those cases, active involvement is not taking place and the participating minds do not contribute to construction of meaning in the classroom. On the contrary, within interactive groups, learning outcomes *emerge* through an iterative and recursive process. The nonlinear perspective for the *situated learning theory* suggests that collaborative construction of knowledge requires an "activated" group involved in a dynamical interplay. The term activated implies strong interventions and contribution to the evolving session. Given that the outcome is not nested in any of the member's initial repertoire, it has to be created through the interaction process. Thus, creativity is associated with *emergence*, and this is the fundamental element that nonlinear dynamics offers to educational theory and practice. Learning-in-group approaches should encourage and train novice for active participation, in a way that nonlinearity is induced in the interactive process. An evolving discourse—a categorical time series portrayed by ODA—following a trajectory which possesses low-dimensional chaos and operating at the EOC, is more likely to be creative process.

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