Matthijs Koopmans
Dimitrios Stamovlasis *Editors*

Complex Dynamical Systems in Education

Concepts, Methods and Applications

Springer

Complex Dynamical Systems in Education

Matthijs Koopmans • Dimitrios Stamovlasis
Editors

Complex Dynamical Systems in Education

Concepts, Methods and Applications

Springer

Editors
Matthijs Koopmans
School of Education
Mercy College
Dobbs Ferry, NY, USA

Dimitrios Stamovlasis
Department of Philosophy and Education
Aristotle University of Thessaloniki
Thessaloniki, Greece

ISBN 978-3-319-27575-8 ISBN 978-3-319-27577-2 (eBook)
DOI 10.1007/978-3-319-27577-2

Library of Congress Control Number: 2016931314

Springer Cham Heidelberg New York Dordrecht London
© Springer International Publishing Switzerland 2016
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

Preface

Welcome to *Complex Dynamical Systems in Education: Concepts, Methods and Applications*. The application of the principles of complexity and dynamical systems in the social and behavioral sciences is a relatively new development, whose relevance to the field of education is only beginning to be appreciated. This book aims to further stimulate this advancement.

As our target audience, we see educational researchers as well as practitioners and policy-makers who take an active interest in the interface between educational research and their own practical work. The book appeals to their relatively sophisticated understanding of the complex interface between research, practice, and policy that motivates much of the current conventional research (and funding thereof). Our intended audience also includes scholars working in disciplines other than education who may take an interest in how, specifically, the complex dynamical systems paradigm that they know applies to the field of education in particular.

The book has the appropriate level of discourse to be used in graduate and advanced undergraduate educational research courses, particularly courses aiming to reflect the methodological diversity that currently exists in the field, or courses that seek alternative approaches to the convention of presenting experimental and quasi-experimental designs as the sole vehicle for legitimate causal inference in education.

The text assumes a readiness among its readership to engage in the substantive and methodological issues that present themselves when a complexity perspective is taken, but, contrary to quite a few other complex dynamical texts, will not require high level mathematical skills. We take pleasure in presenting these chapters to you and hope that they result in a fuller awareness of what the complex dynamical systems paradigm has to offer to the field.

Dobbs Ferry, NY, USA — Matthijs Koopmans
Thessaloniki, Greece — Dimitrios Stamovlasis

Contents

About the Contributors

Jeffrey W. Bloom is Professor Emeritus, Northern Arizona University. His specialties have been situated in science education and curriculum studies, with particular interests in the complexity sciences, qualitative research, children's learning and discourse, and teaching within communities of inquiry. E-mail: jeff@jeffbloom.net.

Umberto Cesar Corrêa is a professor at the School of Physical Education and Sport, University of São Paulo, and member of the Motor Behavior Laboratory for 16 years. He earned his undergraduate degree in Physical Education, Universidade de Mogi das Cruzes (1990); Certification in Gymnastics, Universidade de Mogi das Cruzes (1992); Certification in Sport Sciences, Universidade de Educação e Cultura do ABC (1993); Master's degree in Motricity Sciences, Universidade Estadual Paulista (1996); Ph.D. in Physical Education, Universidade de São Paulo (2001); and did Postdoctoral work at Queensland University of Technology (2011). His research interest is on motor skill learning and teaching of physical education for adolescents. He has investigated themes as practice schedule, feedback, instruction/cues, specificity and complexity of task, and ecological validity. He has been interested in understanding motor skill in physical education and sport contexts from an adaptive complex systems perspective. E-mail: umbertoc@usp.br.

Walter Roberto Correia is a professor at the School of Physical Education and Sport of the University of São Paulo for 10 years. He earned his undergraduate degree in Physical Education at the Faculdade de Educação Física de Santo André (1987); pedagogy, Universidade São Marcos (1992), certificate in School Physica Education, Universidade de Mogi das Cruzes (1992); certificate in Sport Sciences, Faculdade de Educação Física de Santo André (1994); a master's degree in Physical Education, Universidade de São Paulo (1999); Ph.D. in Education/Curriculum, Pontifícia Universidade Católica de São Paulo (2005); Training in Psychoanalysis, Psychoanalytic Studies Center (2010); Qualified Master of Ving Tsun/Kung Fu, Moy Yat Ving Tsun International Federation (2010). E-mail: wrcorreia@usp.br.

M.Jayne Fleener is currently the Dean of the College of Education at North Carolina State University. She has over 30 years of professional experience in K-12 and higher education, including teaching high school mathematics and computer science in North Carolina and serving as Associate Dean for Research and Graduate Studies at the University of Oklahoma and Dean at Louisiana State University. She has a bachelor's degree from Indiana University in philosophy and three graduate degrees from the University of North Carolina at Chapel Hill, including her Ph.D. in mathematics education. Her teaching and research have been in the areas of philosophy, computer science, mathematics, mathematics education, gender issues in mathematics and engineering, educational research and policy, and curriculum theory. She has over 100 publications, including her books *Curriculum Dynamics: Recreating Heart* and *Chaos, Complexity, Curriculum, and Culture: A Conversation* (edited, with others). Both books have been translated into Chinese. She has also participated in funded research in the areas of mathematics education, technology integration, gender studies, teacher professional development, internationalization of teacher education, and high school students' preparation and readiness for college and post-secondary experiences. E-mail: fleener@ncsu.edu.

Doris Pronin Fromberg is Professor Emerita and has served as chairperson of the Department of Curriculum and Teaching, Hofstra University. She served as a teacher and administrator in public and private schools as well as director of Teacher Corps projects. She is past president of the National Association of Early Childhood Teacher Educators (NAECTE), and has chaired the Special Interest Group on Early Education and Child Development of the American Educational Research Association as well as the Special Study Group on Elementary Education of the American Association of Colleges of Education. She was recipient of the Early Childhood Teacher of the Educator Award from NAECTE/Allyn & Bacon and the NYSNAEYC Champion for Children Leader Award. She serves on the NYS Governor's Early Childhood Advisory Council, the Defending the Early Years Advisory Board, and the editorial boards of professional publications. Among her publications are *The All-Day Kindergarten and Pre-K Curriculum: A Dynamic Themes Approach* (Routledge, 2012); *Play and Meaning in Early Childhood Education* (Allyn & Bacon, 2002); *Play from Birth to Twelve* (co-edited with D. Bergen, Routledge, 2015); and *The Encyclopedia of Early Childhood Education* (co-edited with L.R. Williams, Garland, 1992). She presented a 2012 TEDx talk: "Kindergarten Today: Is the Match between High-stakes Outputs and Low-impact Inputs Cost-effective?" She is an advocate for high-quality early childhood education, early childhood teacher education, and early childhood administrator education; and consideration for the dynamic nature of learning and education. E-mail: Doris.P.Fromberg@hofstra.edu.

Joanna K. Garner is Associate Director of Program Development and a Research Associate Professor at the Center for Educational Partnerships, Old Dominion University. Her professional training includes a B.Sc. (Hons.) Psychology and

M. Phil. in Psychology from the University of Surrey, United Kingdom, and a Ph.D. in Educational Psychology from the Pennsylvania State University. Dr. Garner's research interests focus on the translational space between theory and practice, spanning concepts and theories in teachers' and students' learning and motivation. Her recent work on the relations between executive function and self-regulated learning, and the relations between teachers' learning and professional development, is framed by concepts and methods drawn from assumptions of dynamical systems theory. E-mail: jkgarner@odu.edu.

Jeffrey A. Goldstein is Full Professor, Adelphi University; author/editor of numerous books and papers including *Complexity and the Nexus of Leadership*: *Leveraging Nonlinear Science to Create Ecologies of Innovation*; *Complexity Science and Social Entrepreneurship: Adding Social Value through Systems Thinking*; *Complex Systems Leadership Theory*; *Classic Complexity*; *Emergence—Annual Volumes*; *The Unshackled Organization*; and *Brainwaves*. Co-editor-in-chief of the journal *Emergence: Complexity and Organization* (since 2004); Board of Trustees of the journal *Nonlinear Dynamics, Psychology, and the Life Sciences*; Workshop and Seminar Leader and Lecturing at eminent universities in countries throughout the world including England, Canada, Russia, Israel, Sweden, Brazil, Norway, Italy, Cuba, Greece, China, Germany, Spain, and Austria. Consultant for many public and private organizations. E-mail: goldstein@adelphi.edu.

Porfirio Guevara is Senior Associate at INCAE Business School (Costa Rica). He holds a Ph.D. in Sustainability Management, University of Graz, Austria; M.Ph. in System Dynamics from the University of Bergen, Norway; M.Sc. in Economics from the University Pompeu Fabra, Barcelona (Spain); and B.Ph. in Economics, University of Costa Rica. His research interests include complex system dynamics modeling and simulation; economics of technological change and education. E-mail: porfirio.guevara@consultor.incae.edu.

Ton Jörg (1946) studied Physics and Mathematics with Chemistry (B.Sc., 1970). In 1970 he switched to the study of Psychology at the University of Amsterdam (MA, 1977). He received his Ph.D. in 1994. He was affiliated as educational scientist and evaluation researcher at the University of Utrecht, for different departments. In 2008, he retired and became an author. In 2011, his book on *New Thinking in Complexity for the Social Sciences and Humanities—A Generative, Transdisciplinary Approach* was published by Springer Publishers. Recently, he published several articles with "applications" of thinking in complexity: on topics like innovation, management of knowledge, and education. He is now writing a new book, titled *Towards A New Science of Complexity*, which is based on the new paradigm of generative complexity, which is of use for all disciplines, operating in the Age of Complexity. E-mail: agdjorg@gmail.com.

Matthijs Koopmans is Associate Professor and Assessment Coordinator at the School of Education at Mercy College. His professional interests include the application of complex dynamical systems approaches in education, cause and effect relationships, and nonlinear time series. He published numerous research articles and book reviews in peer-reviewed journals including *Evaluation and Program Planning*; *Nonlinear, Dynamics, Psychology and Life Sciences*; and *Complicity: An International Journal of Complexity in Education*. He is one of the editors of *Chaos and Complexity in Psychology: The Theory of Nonlinear Dynamics*, published by Cambridge University Press in 2009. He earned his doctorate in 1988 at Harvard Graduate School of Education. E-mail: mkoopmans@mercy.edu.

Tim Mainhard is associate professor at the Department of Education at Utrecht University, The Netherlands. Tim Mainhard obtained his doctorate at Utrecht University with a dissertation on the development of interpersonal teacher–student relationships in secondary education. His general research interests are social dynamics in the context of educational settings. He combines observation and questionnaire-based data, and his research is inspired by the Social Relations Model and nonlinear dynamics. Tim has published on the interpersonal dynamics of real-time classroom interactions, convergence between student and teacher perceptions of classroom interpersonal processes, and the development of teacher–student relationships during the first months in new classes. A list of publications is available via http://scholar.google.nl/citations?user=mgRXYMwAAAAJ&hl=en. E-mail: m.t.mainhard@uu.nl.

Russ Marion Professor of Leadership at Clemson University, has written numerous research and theoretical articles on complexity leadership, one of which was honored as best paper of the year by *The Leadership Quarterly* and the Center for Creative Leadership in 2001. An article he wrote with Uhl-Bien and McKelvey (2007) was the top-cited article in *The Leadership Quarterly* between 2007 and 2012, and his 2001 "best paper" article was one of the 15 most cited. He and Uhl-Bien served as guest editors of a special issue of *The Leadership Quarterly* on leadership and complexity in 2007. Marion is the author of the books *The Edge of Organization* (1999), *Leadership in Education* (now in two editions), and *Complexity Leadership* (with Mary-Uhl-Bien; 2007). Dr. Marion has co-organized workshops on complexity leadership at the Center for Creative Leadership and at George Washington University. He has lectured on complexity leadership at the India Institute of Technology, the Institute for Management Development in Switzerland, at Maasai-Mara University in Kenya, and in workshops on destructing complex movements at the US Department of Defense. Marion is currently working to develop unique strategies for evaluating outcomes in complex system. E-mail: marion2@clemson.edu.

Helena J.M. Pennings She is currently finishing her Ph.D. research in Educational Sciences at Utrecht University, The Netherlands. She recently started as a postdoctoral fellow at the Department of Education at Utrecht University and at the Eindhoven School of Education at the Eindhoven University of Technology. Her research topics are related to the interpersonal (social) dynamics in education. She uses an innovative method to observe teacher–student interactions, the computer joystick method, and is currently working on analyzing these data with time series analysis such as spectral analysis and state space grid analysis. She published several research articles on teacher interpersonal behavior and teacher–student interactions where she used State Space Grid analysis. A list of publications is available via http://scholar.google.nl/citations?user=P2ZdfrUAAAAJ&hl=en. E-mail: H.J.M.Pennings@uu.nl.

Emilio Porta is Senior Economist at Analitica and board member of FUNIDES where he provides advice on education statistics and planning to developing countries. Before, he was Senior Education Specialist at the World Bank. From 2007 to 2012 he led the Bank's work on education statistics, transforming EdStats and developing tools for analyzing education statistics. Between 2000 and 2005 he was Director of Planning at the Nicaraguan Ministry of Education. He has been consultant for UNESCO, IDB, and WB and has published extensively in the area of education statistics, equity, and learning outcomes. E-mail: eporta@gmail.com.

Daniel Russell is currently an Assistant Professor in the School of Physical Therapy and Athletic Training at Old Dominion University. His training has included earning a B.Sc. (Hons) Sport Science from Manchester Metropolitan University, UK, an MS in Kinesiology from Louisiana State University, and a Ph. D. in Kinesiology from the Pennsylvania State University. He has over 20 years of experience researching how humans control and learn movements and has published in well-regarded peer-review journals. Dr. Russell's work begins with the assumption that human movements emerge from a complex interaction of neurological, musculoskeletal, and cardiorespiratory systems acting between an individual and their environment. He employs nonlinear analyzes as part of a dynamic systems approach to address questions about how behavior arises from this complexity and how it changes through processes of adaptation and learning. E-mail: dmrussel@odu.edu.

Craig Schreiber's skill set includes change management and the effective coordination and collaboration within and between organizations to accomplish innovative and adaptive outcomes, the relation of organizations to societal and global issues within the hyper-connected context, and leadership processes that enable agile functioning. He uses social network analysis and systems thinking methodologies in most of his research and consulting practice. He has conducted research for the Center for Creative Leadership, Bank of America, Lockheed Martin, Stryker Medical, Booz Allen Hamilton, NASA, Office of Naval Research, Army Research

Laboratory, and National Science Foundation. Dr. Schreiber is also a Salzburg Global Fellow studying the social value of business practice. He has published his research in book chapters and refereed journal articles. E-mail: craigschreiber.acad@yahoo.com.

Georgios D. Sideridis is a Survey Methodologist at Boston Children's Hospital, and Assistant Professor of Pediatrics at Harvard Medical School. He is studying measurement error in traditional and contemporary psychometric theory, and motivation and learning in typical and special populations. He has published more than 120 articles in journals such as *Journal of Educational Psychology*, *Motivation & Emotion*, *Journal of Applied Measurement*, and *Educational and Psychological Measurement*, to name a few. E-mail: georgios.sideridis@childrens.harvard.edu.

Dimitrios Stamovlasis is Assistant Professor of Research Methodology and Applied Statistics in Social Science at the Aristotle University, Department of Philosophy and Education. His research interests are interdisciplinary and they focus on methodological and epistemological issues of contemporary social sciences that improve theory building; on nonlinear dynamics (complexity, catastrophe theory, entropy, and related fields) and their application to social, behavioral, and life sciences; on specific research endeavors in the area of educational research concerning neo-Piagetian theories, learning, science education, problem solving, creativity, and group dynamics. He published numerous research articles in peer-reviewed journals, and he served as a guest editor in the Special Issue of *Nonlinear Dynamics, Psychology and Life Science* (2014) focused on Nonlinear Dynamics in Education. He earned his Ph.D. at the University of Ioannina (2001) and he also holds an *M.Sc.* in Physical Chemistry from the University of Hawaii, an *M.B.A.*, and a *M.Sc.* in Statistics from the University of Athens. E-mail: stadi@edlit.auth.gr.

Henderien Steenbeek (1965) studied developmental psychology at the University of Groningen, where she defended her Ph.D. thesis in 2006. The subject of her thesis was modeling dyadic child–peer interactions during play. She currently works as associate professor at the department of Developmental Psychology at the same university. In addition, she works as a professor at Teachers College at the University of Applied Studies in Groningen. In her work she focuses on child–adult and child–child interaction in play-educational settings. Her research themes are children's learning processes in primary education (with a focus on children with special needs, and excellent performing children) and the dynamics of children's play. Most studies are focused on the observation of interaction behaviors in naturalistic circumstances. http://www.rug.nl/staff/h.w.steenbeek/research. E-mail: h.w.steenbeek@rug.nl.

Go Tani is professor at the School Physical Education and Sport of the University of São Paulo, and member of the Motor Behavior Laboratory for 30 years. He earned his undergraduate degree in physical education from the Universidade de São Paulo (1973); a master's degree in Education from Hiroshima University

(1978); a Ph.D. in Education from Hiroshima University (1982); and has held a Postdoctoral fellow in Psychology at the University of Sheffield (1995) and from Hiroshima University (1996). E-mail: gotani@usp.br.

Marijn van Dijk (1972) studied developmental psychology at the University of Tilburg. In 2004, she defended her Ph.D. thesis at the University of Groningen, on variability and ambiguity in early language acquisition. She currently works as an associate professor in developmental psychology. Her research theme is early social and cognitive development from a complexity approach. The focus is on teacher–child and parent–child interaction and the dynamics of learning in primary education. Most studies are focused on change processes and the observation of interaction behaviors in naturalistic circumstances. E-mail: m.w.g.van.dijk@rug.nl.

Paul van Geert (1950) holds a doctoral degree from the University of Ghent (Belgium) and is a professor of developmental psychology at the University of Groningen in the Netherlands since 1985. He has had a pioneering role in the application of dynamic systems theory to a broad range of developmental areas, including early language development and second language acquisition; cognitive development in the context of learning-teaching processes; and social development including social interaction and identity. His main aim is to better understand the general nature of developmental dynamics, i.e., nature of the mechanism(s) that drive and shape a developmental process in an individual, as the individual, given his or her biological properties and potentialities, interacts with his or her actively explored and transformed environment. He has been Fellow at the Center for Advanced Studies in the Behavioral Sciences and has held visiting professorships at the Universities of Torino (Italy), Paris V and Reims (France), Trondheim (Norway), and Harvard University (Mind-Brain-Education program). For his research and an overview of his artwork, see www.paulvangeert.nl. E-mail: p. l. c. van.geert@rug.nl.

Sabine van Vondel is currently working on her Ph.D. thesis in developmental psychology at the University of Groningen. She holds an *M.Sc.* in Education and Development of Behavioural and Social Sciences (research master) and an *M.Sc.* in Developmental Psychology, both at the University of Groningen. The focus of her thesis is studying the effects of a Video Feedback Coaching intervention for teachers on teacher–student interactions during science and technology education. Her research interests focus on the interface of educational science and developmental psychology, more specifically on teaching–learning processes that support (or hinder) student's learning in elementary education. E-mail: s. van. vondel@rug. nl.

Chapter 1
Introduction to Education as a Complex Dynamical System

Matthijs Koopmans and Dimitrios Stamovlasis

This book seeks to introduce educational researchers, practitioners, and policy makers to the theory of Complex Dynamical Systems (CDS), a novel perspective that has gained considerable ground in many scientific disciplines, but whose applicability to education remains underappreciated. The theory of complex dynamical systems (CDS) is concerned with the analysis of systems irrespective of how the unit of analysis of those systems is defined. These systems could be molecules, cells, words, people, or human organizations. In recent years, there has been a growing interest in the use of a complexity perspective in social science research as well as policy and practice, as the perspective provides a rich and widely applicable vocabulary to capture processes of change and the interaction between individuals and larger organizational constellations. This book focuses on educational processes in human systems.

Let us begin with a clarification of the terminology. When we say *complex*, we mean that the behavior of a larger systemic constellation cannot be readily reduced to that of its individual members. Consequently, the perspective inspires a holistic view where the behavior of individuals is understood in its larger context. For example, we can understand student learning in terms of collaborative behavior in the classroom in which it takes place, while a classroom climate conducive to learning cannot be readily reduced to the learning or interactive behavior of individual teachers and students. In other words, the whole is greater than the sum of its parts. When we say *dynamical*, we mean that current behavior is understood in terms of deviations from past behavior. As a result, the perspective focuses on behavioral change and its

M. Koopmans (✉)
Mercy College, School of Education, Mercy Hall, Rm. SE 31, 555 Broadway,
Dobbs Ferry, NY 10522, USA
e-mail: mkoopmans@mercy.edu

D. Stamovlasis
Aristotle University of Thessaloniki, Department of Philosophy
and Education, 54 124 Thessaloniki, Greece, Europe
e-mail: stadi@edlit.auth.gr

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_1

determinants, rather than on outcomes frozen in time. Thus, we might take an interest in the learning trajectories of individuals rather than whether students meet certain benchmarks or performance goals as a group. When we say *systems*, we refer to a constellation of individual members who are in a position to interact with each other as a coherent entity. In this sense, schools, districts, classrooms, and parent–teacher conferences are all examples of systems, and when we would like to understand the behavior of individuals within such systems, we also need to look at the behavior of other units at the same level of description within the same system.

This book seeks to provide a conceptual and methodological introduction to the use of complex dynamical systems (CDS) approaches in education, covering most of the basic dynamical concepts that can be found in the literature, such as emergence, complexity, self-organized criticality, attractors, catastrophe theory, chaos theory as well as recent innovations to the complexity field such as fractional differencing and power laws. As a field of inquiry, education has been slow to catch on to complex dynamical systems approaches, whereas, in other disciplines, such as psychology, econometrics, and theoretical biology, dynamical approaches have by now been largely integrated into the theoretical and empirical research agenda. Psychology, for example, has produced several edited volumes about the application of dynamical systems approaches to various subspecialties in the field (Abraham & Gilgen, 1995; Guastello, Koopmans, & Pincus, 2009; Robertson & Combs, 1995; Sulis & Combs, 1996; Tschacher & Dauwalder, 1999), but there is no similar book that is specific to the field of education. This book seeks to address this gap.

In education, work from a complexity perspective tends to be theoretical, and covers such topics as the exploration of the interface between dynamical systems, education, and post-modernism (e.g., Doll, 1993; Truiet, 2012), the use of complexity to characterize the political process in education (Osberg & Biesta, 2010), the implications for practice of complexity as a paradigm shift (Davis & Sumara, 2006), or it consists of retrospective interpretations in terms of complexity of research findings from studies utilizing conventional research paradigms (Morrison, 2006). While this work is valuable in its own right, it does not have the level of conceptual and methodological specificity that is required to capture the dynamical processes hypothesized in the dynamical literature, such as emergence, second order change, and sensitive dependence on initial conditions, nor does it speak to the specific gaps in our knowledge that result from the relative absence of dynamical perspectives in empirical research in education.

There needs to be greater clarity about how research into the dynamical aspects of the educational process can inform and supplement our knowledge obtained through more traditional research paradigms such as randomized control trial studies, quasi-experimental designs, and qualitative research. Recent progress in the field of dynamical systems includes significant empirical work to study the dynamical underpinnings of the educational process. A first inventory of this work was a special issue in *Nonlinear Dynamics, Psychology and Life Sciences* on education (Stamovlasis & Koopmans, 2014), which brought together significant new empirical studies in education that explicitly utilize a complexity perspective. This book further capitalizes on these developments by presenting some of the most recent path-breaking advances in this area.

The six chapters immediately following this introduction discuss the conceptual framework of complex dynamical systems and its applicability to educational processes. Chapters 8–10 translate some of these concepts into coherent research methodology. Chapters 11–17 report the results of empirical research illustrating the use of CDS research methods. This work aims to help the reader appreciate what we can learn about dynamical processes in education when this angle is taken. In Chap. 2, Fleener appreciates, at a theoretical level, the implications of CDS as a paradigm shift in education and its ability to address long-standing issues to which conventional research paradigms have failed to produce satisfying answers, such as how the complexities of school environment and individual differences contribute to learning outcomes, and to forge a new kind of link between research and practice. In Chap. 3, Bloom further explores the historical affinity between qualitative research and complexity research that dates back to the work of Gregory Bateson in the 1930s. Qualitative transformation is one of the central concerns in CDS, and the fine-grained observation that qualitative research permits make it possible to bring the dynamical underpinnings of causal processes to the surface in a way that randomized control trial studies cannot (Maxwell, 2004, 2012).

Currently, it is difficult to imagine how one can talk about change in dynamical terms without talking about *emergence*, the appearance of radical novelty in systemic behavior and the search for the origins of such novelty. Goldstein discusses the construct of emergence in Chap. 4 and points to the failure of most current dynamical literature to explain such novelty. He presents *self-transcending constructions* as one possible way to distinguish spontaneous transformation that may occur without any theoretically interesting antecedents from a change process where the propensity toward transformation is already built into the system. The identification of such propensities is both of theoretical and pragmatic interest to the field of education, because it will help us understand why change does occur or fails to transpire. This knowledge may, in turn, place findings of existing research into clearer perspective. It may even help us penetrate deeper into the metaphysical realm of questions about the origins of complex dynamical systems.

In Chap. 5, Jörg appreciates the magnitude of the paradigm shift produced in the field if a complexity perspective is taken, and he introduces the term generative complexity as a new way of looking at systemic behavior in terms of the processes through which systems maintain their integrity in the ongoing interrelationship with their constituent components. His contribution is unique in that it is grounded in a combination of the philosophical and early developmental literature (Vygotsky), rather than in the accomplishments in mathematics, physics, and chemistry (e.g., Bak, 1996; Prigogine & Stengers, 1984; Thom, 1975), engineering (Ashby, 1957; Wiener (1961), or anthropology (Bateson, 1972) as has been more common in the field of complexity.

Chapters 6 and 7 analyze, respectively, the dynamical processes underlying the acquisition of motor skills and children's play, which both require description of how integrated behavioral patterns occur over and above the individual elements that make up those patterns. In Chap. 6, Corrêa, Correia and Tani describe the complex processes that constitute psychomotor behavior, such as the fluency of movement based on identifiable behavioral elements, as well as the dynamical components of that behavior: consistency and flexibility are both present in the behavior. They address the question on motor skills acquisition as the main goal for teaching and coaching, based on a nonequilibrium model of motor learning, where psychomotor behavior can be understood as adaptive behavior in its spatiotemporal context. Likewise, in Chap. 7 Fromberg analyzes the contextual, transformative aspects of children's play as well as the complex relationship of the individual play episodes with the larger developmental outcomes to which the play activities bear a generative relationship. Play is the means through which children acquire their adaptive skills in the interface with the external environment, and the relationship between these developmental outcomes and individual play episodes illustrates complexity.

Koopmans in Chap. 8 focuses on an important methodological implication of taking a dynamical approach, namely the need to augment our knowledge grounded in rigorously sampled cross-sectional studies with an equally rigorous collection information about the behavior of individuals observed frequently over extended time periods. This focus on the changes in systemic behavior over time addresses a potentially very important aspect of cause and effect relationships in education, namely the extent to which behavior can be understood in terms of its own previous manifestations.

Complex dynamical systems approaches are grounded in a wide variety of mathematical models. One of the most important ones is a family of models known as catastrophe theory, a formulation of discontinuous changes based on sets of predictors that model the conditions under which discontinuity occurs. Stamovlasis in Chap. 9 provides a complete presentation of catastrophe theory starting with a brief history of its mathematical foundation and continuous with its mathematical formalism in deterministic and stochastic forms. Subsequently, he reviews all the current statistical methodologies that apply catastrophe theory to real data, focusing on cusp model, and discusses central epistemological issues associated with nonlinear dynamics in social and behavioral sciences. Furthermore, he demonstrates the applicability of catastrophe theory in educational research by presenting nonlinear models within the neo-Piagetian framework and science education. Marion and Schreiber in Chap. 10 discuss the recent advances in the use of network analysis and provide a primer of how these methods can be used in educational research. The main interest is to study networks of agents who share work-related experiences. For example, students and teachers in a given school, or informal leaders in a school community might constitute a relevant network. Network analysis has strong grounding in the mathematics of graph theory and it has a specific terminology in describing the system under consideration, associated with various network-level and agent-level measures. It is therefore a particularly useful approach to provide an empirical basis for our descriptions of how systems are organized.

A set of empirical studies follows in Chaps. 11–17. Each of these chapters provides examples of methodologies that are specific to the description of complex dynamical process and the exploration and confirmation of the hypotheses it generates. The section showcases several standard methodological approaches that are currently used in CDS: time series analysis, state space grid modelling, orbital decomposition, network analysis, and catastrophe theory, as well as simulation models. In addition, problem-specific approaches are discussed as well, such as van Vondel's macro-dynamical description of student reasoning skills and the temporal sequencing of types of teacher responses in Chap. 11. Van Vondel and her coworkers rightly argue that understanding development requires, at a minimum, a detailed understanding of how behavioral changes occur over time and what the environmental contingencies are of these changes. The authors developed a unique approach, and demonstrate the surplus value that a complex dynamic systems approach offers, based on new tools designed to answer questions about how the underlying processes affect students' performance and provide insights into how teachers can optimize their lessons.

The potential of CDS approaches to capture classroom interaction processes has been appreciated in the literature on at least several occasions recently. Pennings and Mainhart take the obvious next step in Chap. 12 by collecting and analyzing teacher interactions with students and the social climate in classrooms using a rigorous real-time data collection process as well as rigorous modelling practices, State Space Grids (SSG), to identify the attractors underlying these interactions. SSG is a powerful tool to examine the moment-to-moment nature of classroom interactions that could be correlated with teacher and teaching process characteristics. The authors in this chapter are making a remarkable contribution towards the new paradigm establishing classrooms as complex dynamical systems.

Two papers illustrate the use of orbital decomposition analysis (ODA), a method designed to study interaction processes and specifically to analyze time series measured at the nominal level. Stamovlasis in Chap. 13 illustrates the utility of the symbolic dynamics approach when looking at collaborative learning processes, where it is shown that discourse analysis of students' verbal interactions can reveal those dynamical characteristics that might have a decisive impact on outcomes; this exemplifies how to look closer, and thus, sheds light into the 'black box' of educational interventions; moreover it demonstrates that small group processes, under certain circumstances, behaves as a complex dynamical system driven by self-organization mechanisms, finding that is important for the theory of education regarding the emergent phenomena, such as learning and creativity. In Chap. 16, Garner and Russell demonstrate the use of the same approach to better understand self-regulated learning by looking at the interaction between learners and the learning materials they use. They use ODA to investigate the nature of gaze sequences during a self-regulated learning episode. They aimed to investigate research questions regarding the presence and the nature of patterned sequences in relation to global task strategies, and furthermore the degree to which these patterns of acting, responsible for directing attentional guidance during learning, are the fingerprints of an underling nonlinear dynamical system.

Scrutinizing ordered observations over a long period of time permits the detection of dynamical processes that otherwise remain hidden. Koopmans illustrates this point in Chap. 14 when discussing recent advances in time series analysis, such as fractional differencing and spectral power analysis to detect long-range dynamical features in high school daily attendance over a 7-year period (e.g., pink noise, self-organized criticality, self-similarity). Guevara and Porta in Chap. 15 reexamine the persistence of inequality in society and the questions it raises about the instrumentality of the educational system in perpetuating it. The authors build simulation models to better understand the relationships between critical variables and triangulate them against data compiled from the educational system of Nicaragua. It is important to note that simulation techniques are infrequently used in educational research, while they are particularly useful to address questions about the temporal evolution and dynamical complexity of the relationship between variables pertaining to educational outcomes. Within the CDS framework, simulation models track down the complex interactions of social inequalities that educational systems generate in the context of global trends, and permit the investigation of more complex causal models than those typically documented in studies using traditional linear methods.

Lastly, in Chap. 17 Sideridis and Stamovlasis discuss the complex interrelationships between motivation, arousal and achievement and they use a cusp catastrophe model to illuminate that relationship as well as providing empirical confirmation of the nonlinear character of the relationships between these variables. They combined nonlinear dynamics and self-organization theory in order to explain instabilities in arousal level in educational settings and thus they built bridges between psychology and physiology within the nonlinear dynamics and complexity framework.

In conjunction, we believe that these chapters illustrate the potential of CDS in providing a new perspective on some old and newer problems in education, as well as providing a new set of interests and priorities for the field to address. The book also presents a set of methodological innovations that are specifically tailored to the analysis of processes of stability and transformation in educational systems in particular, and they demonstrate how these new approaches can be used on real educational data collected in real educational settings.

The advent of chaos and complexity theory in the late 1980s and early 1990s (e.g., Gleick, 1987; Waldrop, 1992; West & Deering, 1995) has created a need among scientists as well as practitioners, policy makers and the business community for a deeper understanding about how these new perspectives can help them address the most persistent questions of their respective fields. Considering the enormous variety of disciplines in which these perspectives have been utilized (e.g., biology, organizational theory, physics and chemistry, psychology, education, art medicine), it is not hard to appreciate the difficulty in trying to find consistency in the language that we use when talking about complex dynamical processes. In the field of CDS, there has been some serious discussion about its terminological consistency, or the lack thereof (Abraham, 1995; Goldstein, 1995), resulting in the realization that we need to get our house in order regarding the definition of our critical constructs. In that spirit, this book provides a glossary of terms based on their use in its chapters. These definitions are not meant to be written in stone, but they provide explicitness about how we used our terms, and may help bring clarity to the field of complexity in education.

We hope that the contributions presented here will facilitate our discussions of education as a complex dynamical system and inspire the generation of new types of questions about educational processes and what makes them effective. The work presented in this book seeks to take a meaningful first step in that direction.

References

Abraham, F. D. (1995). A postscript on language, modeling and metaphor. In F. D. Abraham & A. R. Gilgen (Eds.), *Chaos theory in psychology* (pp. 311–342). Westport, CT: Praeger.

Abraham, F. D., & Gilgen, A. R. (1995). *Chaos theory in psychology*. Westport, CT: Praeger.

Ashby, W. R. (1957). *An introduction to cybernetics*. London: Chapman & Hall.

Bak, P. (1996). *How nature works: The science of self-organized criticality*. New York: Springer.

Bateson, G. (1972). *Steps to an ecology of mind: A revolutionary approach to man's understanding of himself*. New York: Ballantine.

Davis, B., & Sumara, D. (2006). *Complexity and education: Inquiries into learning, teaching, and research*. Mahwah, NJ: Erlbaum.

Doll, W., Jr. (1993). *A postmodern perspective on curriculum*. New York, NY: Teachers College Press.

Gleick, J. (1987). *Chaos: The making of a new science*. Harmondsworth: Penguin.

Goldstein, J. (1995). The Tower of Babel in nonlinear dynamics: Toward a clarification of terms. In R. Robertson & A. Combs (Eds.), *Chaos theory in psychology and the life sciences* (pp. 39–47). Mahwah, NJ: Erlbaum.

Guastello, S., Koopmans, M., & Pincus, D. (2009). *Chaos and complexity in psychology: The theory of nonlinear dynamical systems*. New York, NY: Cambridge University Press.

Maxwell, J. A. (2004). Causal explanation, qualitative research, and scientific inquiry. *Educational Researcher, 33*, 3–11.

Maxwell, J. A. (2012). The importance of qualitative research for causal explanation in education. *Qualitative Inquiry, 18*, 655–661.

Morrison, K. (2006). Complexity theory and education. Paper presented at the APERA conference, Hong Kong, China. Retrieved April 28, 2014 from http://edisdat.ied.edu.hk/pubarch/b15907314/full_paper/SYMPO-000004_Keith%20Morrison.pdf.

Osberg, D., & Biesta, G. (2010). *Complexity theory and the politics of education*. Rotterdam, Netherlands: Sense Publishers.

Prigogine, I., & Stengers, I. (1984). *Order out of chaos: Man's new dialogue with nature*. New York: Bantam.

Robertson, R., & Combs, A. (1995). *Chaos theory in psychology and the life sciences*. Mahwah, NJ: Erlbaum.

Stamovlasis, D., & Koopmans, M. (2014). Editorial introduction: Education is a dynamical system. *Nonlinear Dynamics, Psychology and Life Sciences, 18*(1), 1–4. Special Issue: Nonlinear Dynamics in Education.

Sulis, W., & Combs, A. (1996). *Nonlinear dynamics in human behavior*. Singapore: World Scientific.

Thom, R. (1975). *Structural stability and morphogenesis*. New York: Benjamin-Addison-Wesley.

Truiet, D. (2012). *Pragmatism, postmodernism and complexity theory: The fascinating imaginative realm of William Doll, Jr*. New York, NY: Routledge.

Tschacher, W., & Dauwalder, J. P. (1999). *Dynamics, synergetics, autonomous agents*. Singapore: World Scientific.

Waldrop, M. M. (1992). *Complexity: The emerging science at the edge of chaos*. New York: Simon & Shuster.

West, B. J., & Deering, B. (1995). *The lure of modern science: Fractal thinking*. Singapore: World Scientific.

Wiener, N. (1961). *Cybernetics, or control and communication in the animal and the machine* (2nd ed.). Cambridge, MA: MIT Press.

Chapter 2
Re-searching Methods in Educational Research: A Transdisciplinary Approach

M. Jayne Fleener

Academic educational research has been criticized for its inability to address the most intractable problems of public education. While critics point to the lack of impact educational research has had on policy and practice as evidence that the problem lies in a commitment of educational researchers to make a difference in the real context of schools, there is a more fundamental flaw with our ability to conduct meaningful educational research that requires a shift in our thinking about the goals and practices of educational research.

As a dean, I was always defending my faculty to policy makers and community leaders because they wanted to see research that was site based, scalable, and relevant to schools, practitioners, and policy makers. Even as I described some of the really outstanding research my faculty was doing and many of the innovations in which they were involved, community leaders felt the research being done was too "ivory tower" and not grounded in the real world.

This disconnect between the educational research being done by my faculty and the expectations of policy makers for definitive answers to significant challenges in education goes beyond a difference in purposes and goals of educational research. I know my faculty wanted to make a difference in the real context of schools. They wanted to impact and shape the future of education in positive ways. The disconnect points to the need for educational research to catalyze and sustain change in educational contexts. The drive to relevancy, however, does not require all educational research to be field based or empirical. The relevancy comes from a system of research, not separate research studies, that informs practice, promotes change, and makes a difference in meeting the goals of education.

This paper is an attempt to bridge the policy-researcher expectations gap by presenting a systems perspective of education research that addresses the

M.J. Fleener (✉)
Friday Institute for Education Innovation, College of Education, North Carolina State University, Raleigh, North Carolina, United States
e-mail: fleener@ncsu.edu

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_2

complexities of educational contexts, scalability of innovation, and sustained change. Interrogating the questions we pose and the research methods we employ supports a systems view of research that includes transdisciplinary application of complexity sciences approaches to educational research.

A systems perspective of educational research engages the “re” in research by creating a system of inquiry that is layered, recursive, self-reflexive, and conversational (interconnected). This multidimensional approach of re-searching involves a dynamic interplay across contexts, inquiry, and modes of inquiry. This re-searching process requires what Wittgenstein (1953) would refer to as a change in aspect (Fleener, 2002), specifically in this case, what Ton Jörg refers to as “thinking in complexity” (Jörg, 2011). Building on Morin’s notions of complex thought and method, this approach advocates for a more complex understanding of educational research as a system of re-searching. From the questions we ask to the methods we employ, our ability to address the challenges of education requires a system of research/inquiry that “reconnects that which is disjointed and compartmentalized” (Montuori, 2008, p. vii) and layers research and innovation across contexts and scales (Coburn, 2003).

The Question of Questions

The first issue of re-search is thinking about the kinds of questions that are asked. We have all experienced the unending litany of “why’s” from an inquisitive 5 year old. While we may ultimately end this type of recursive questioning with “because I said so,” the profundity of the child’s inquiry is shaping their world. Before we ever approach the “how” or “what” of research, we first need to question the “why” (Fleener, 2002, 2004).

Sometimes in exploring the “why” we discover even deeper questions that become even more central to the problems at hand. Reaching a point of impasse, as with the 5 year old, we are forced to create new solutions to our problems (or at least acknowledge defeat!). The biologist Humberto Maturana tells the story of problematizing the meaning of life as a pivotal point in his ultimate creation of the notion of “autopoiesis” or “self-creation” as a way to think about living systems (Maturana, 1980). As he explored attempts to answer the question “what is life” he discovered both internal and external contradictions with the approaches. Either attempts to answer the question would enumerate all of the characteristics of living systems, reaching a point where an artificial line ends up being drawn, or, as the list continues to be enumerated ad infinitum, the distinction between living systems and nonliving systems starts to become blurred. In either case, the ultimate answer to the question of a definition of life seemed to suggest we already knew the answer! The “why” that problematized assumptions (about the meaning of life) also exposed our limitations in understanding (of life) and opened up entirely new avenues of exploration. Maturana and his student Franscisco Varela created a notion of life that was self-reflective, self-reflexive, and self-generating. The “why” problematizes our thinking, allowing us to escape hidden assumptions and create new ways of thinking about problems in more complex ways.

There is another aspect of the "why" that is important in educational contexts. Sometimes we forget to provide opportunities for our students to interrogate their learning to open up new possibilities and engage them in expanding their world and their place in it. As an example, from my experience in teaching computer programming, I had one of those "take back" moments where I wished I had been more prescient about the kinds of questions computers can and cannot solve and more open to the possibilities of computer intelligence. The standard curricula for teaching introductory computer science detailed beginning programming instruction with definitions of an algorithm. I would assign students homework to define their algorithms for getting ready for school, preparing a meal, or going on a trip to initiate discussion in class about computer algorithms. I would lead discussions to probe students to think about what kinds of problems computers can solve and, importantly, not solve. Computers, we would decide, cannot solve complex problems that require intuition and insight. Computers need clear algorithms as step-by-step procedures, we decided, per the curriculum. Problems like war, poverty, and discrimination were not problems for the computer!

These discussions with my computer science students were occurring at the same time as an entirely new kind of mathematics was being developed. It was not until 1975 that Benoit Mandelbrot invented the word "fractal" to describe patterned relationships that embody unpredictability, indeterminacy, and "chaos" (Gleick, 1987). The next year, Kenneth Appel and Wolfgang Haken solved the four color problem using computers, raising issues of a new kind of computer intelligence and proof based on recursive problem solving and the ability to perform more calculations than a single human could in a lifetime. And just 13 years earlier, Lorenz developed analytic modeling tools that proved weather was not predictable beyond a few days and logistic functions provided new insights into unfolding patterns in chaotic systems (Gleick, 1987). These and other twentieth century scientific pioneers invented new approaches to inquiry that embraced rather than attempted to control for ambiguity and complexity, exploring patterned emergence, reorganization, and complex dynamics.

By failing to complexify the re-searching of educational problems, we also pass on our unexamined assumptions to our graduate students, the next generation of educational researchers. We tell our graduate students they must have clearly definable terms and constructs with answerable questions. As we probe their thinking about key constructs, we encourage them to go to the literature to find definitions of terms like "learning," "problem solving," and "knowing" that they can use. These very constructs, when pushed to their limits, invite multilayered discourse across multiple domains of inquiry including cybernetics, philosophy, sociology, anthropology, and the learning sciences. Too often, we fail to invite this complicated conversation across inquiry domains because we perceive the questions too irrelevant to educational contexts or the methods out of reach.

Complexifying our questioning exposes connections and relationships across intellectual domains and opens up the possibilities for new ways of thinking about problems. We have seen this process ebb, flow, and progress throughout the twentieth century in the sciences, for example, when Einstein first proposed the general theory of relativity (1915), the Copenhagen Conference debated the nature of quanta (1928), Gödel proved the incompleteness of mathematics (1931), the Macy Conferences (1944–1954) developed interdisciplinary approaches to study systems and invented the field of cybernetics (Umpleby & Dent, 1999), and the 1984 convening of physicists and economists in Santa Fe explored transdisciplinary approaches (Morin, 2008) and invented complexity research (Waldrop, 1992). This list of great twentieth century scientists and convenings, of course, is incomplete, as there are many pioneers who have shaped our understandings by interrogating the questions they were asking and looking outside of traditional boundaries to address significant problems in their fields.

Complexifying questions can often lead to the core of a problem, helping us arrive at a point where we have to reach outside traditional boundaries of thought. As we complexify educational research, we challenge the kinds of questions we might pose and need to extend our methods to include approaches to inquiry that address the inherent complexity of education as a social system. Education is also an important social system that impacts and shapes the vitality of any society. Educational innovation and reform, as an example, have their own set of implicit and explicit goals and assumptions that constrain how our work is done in schools (Hatch, 1998). Questions about curriculum, teaching, teacher preparation and development, school leadership, school organization, and so on, create a metaphorical Tower of Babel like scenario for educational researchers. To overcome the challenges to educational research, we need to interrogate our own "why" questions to understand how all of these different pieces of the educational research landscape come together, not as a puzzle that, when completed creates a clear picture, but as an ecosystem that is multidimensional, dynamic and is best understood by a systems approach examining all of its dynamic elements and interactions. And, as we interrogate our "why" questions of educational research, we open possibilities for complicated conversations across educational contexts and inquiry domains, efforts more likely to respond to and engage stakeholders.

The "Why" of Educational Research

I recall, as an early career teacher, I engaged in strategic planning at my school. We were asked to define the purpose of education and our goals for student outcomes. We debated issues of college and career readiness, the role prepared students would play in the future of society, the need for students to be lifelong learners, and the hope that students would become lovers of learning. Fortunately, these pre-No Child Left Behind (NCLB) discussions and requirements did not have to address the assessment and accountability challenges.

Regardless of where we stand on assessment and accountability, to meet the dynamic challenges of this mandate for universal and equitable education for all, educational research needs to be focused on studying and transforming how we prepare the next generation of thinkers and doers. This is a multidimensional challenge, as curriculum, instruction, learning theory, problem solving, teacher preparation, and all the rest are factors in ultimate student success for the future. If we can agree (and, of course, I invite thoughtful interrogation of this idea) that preparing students as the next generation of thinkers and doers is a fundamental purpose of education and therefore the central focus of educational research, if this is, indeed, the "why" of education and educational research, what are the "what's" and "how's" of educational research? These are the questions of methods.

Before transitioning to the question of methods, however, we need to tease out the "why" of education a bit more. What does it mean to be prepared for the future in our current societal context? Many States and school systems across the USA, as well as most state departments of education have some set of skills and competencies defined as twenty-first century learning, skills and dispositions for students upon which curriculum and instruction should be based. Although assessments are lagging behind these ideas of twenty-first century learners for which there is some overall acceptance, it is clear that, as a society, we recognize "reading, writing and 'rithmatic" are not sufficient and that unquestioned memorization will not prepare students of the future to be creative problem solvers, inventors, and adaptors in a world that is rapidly changing, technologically evolving, and economically globally intertwined.

The Framework for Twenty-First Century Learning developed by the P21 Partnership, the Partnership for Twenty-First Century Learning, is used by many states in the USA and provides a perspective of the purpose and goals of education (see Fig. 2.1). As seen in the figure below, the P21 (2009) emphasizes **Life and Career Skills** (including flexibility, adaptability, initiative and self-direction, social and cross-cultural skills, productivity and accountability, and leadership and

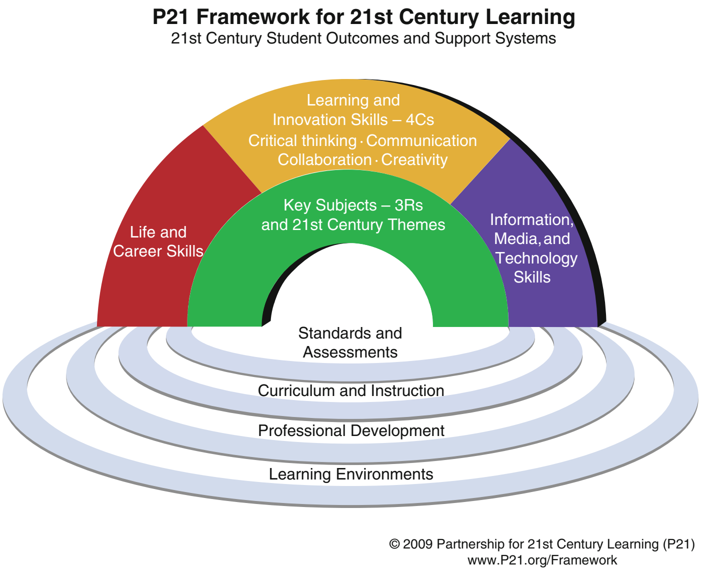

Fig. 2.1 Partnership for twenty-first century learning framework for twenty-first century learning

responsibility), **Information, Media, and Technology Skills** (including the ability to access, evaluate, and use information creatively to solve problems and share new understandings), **Learning and Innovation Skills** (including critical thinking, communication, collaboration, and creativity), and **subject matter knowledge** framed within **twenty-first century themes** (that include global awareness, financial, economic, business, and entrepreneurial literacy, civic literacy, health literacy, and environmental literacy).

Participating in a variety of businesses, public and private partnerships, and task forces for rethinking teacher education, I have observed how complex the conversation becomes when twenty-first century learning skills, for which there is basic agreement, are considered through the lenses of standards, assessments, curriculum, instruction, teacher and principal qualifications and development, and alternative approaches to education. Within the Twenty-First Century Framework, the idea of these fundamental supports for student learning as “pools of connectivity” provides a scaffold for educational research. From a complexity perspective, these “pools of connectivity” suggest a systems approach to educational research, what Bateson (1972, 1979) would refer to as ecologies of knowing.

Bateson’s notion of “schismogenesis” describes a process of inquiry through progressive differentiation, literally, “the birth of separation.” As described by Jewett (2005), Bateson’s application of schismogenesis in and over time revolutionized anthropological methods, placing, distancing, and re-placing the researcher within the context of the researched as both are subject to recursive scrutiny. The unfolding of research is a re-searching process that creates its own system subject to continually renewed inquiry, connectedness across contexts and time, and patterned emergence. The layering of contexts, symmetries, and differences provides an inquiry of the approaches to inquiry (recursively, an inquiry of inquiry approach), that complexifies and scaffolds research. Eschewing the goal of inquiry as final answers, this approach creates the opportunity for a “complicated conversation” across researchers and researched; a complicated conversation (Fleener, Carter, & Reeder, 2004; Lu, 2011) that is ongoing and transformative; re-search in its truest form as perpetual inquiry. The complicated conversation that is research as a system of inquiry embraces the ever-broadening and recursive understandings in concert.

Through this complex approach to inquiry, we have the opportunity to understand and to transform educational contexts in ways that invite revisiting and re-engaging the questions we explore while continually interrogating the “what’s” and “how’s” of educational research. This approach to inquiry is an approach that recognizes the recursive challenges of thinking about thinking. It invites an approach to research methodologies that is self-reflexive and dynamic. Such inquiry describes a questioning of questions and a method of methods whereby inquiry itself becomes part of that which is studied, adapted, and transformed.

By “complexifying” our methods to include these meta-loops of recursive inquiry, we open approaches to research that can engage in the “complicated conversation” of re-searching. Through “complexification” we create a “generative complexity” that recursively and dynamically interrogates method (see Jörg, 2011) and creates a system of research designed to address the “why’s” of education and expands the “what’s” and “how’s” of method.

Method of Methods

To interrogate the method of methods, our first step is to consider the limitations of traditional research methods. In his book, "Scientific Literacy and the Myth of the Scientific Method," Henry Bauer (1994) describes the dangers of applying the scientific method to social sciences research. Abstracting the researcher from the researched, and applying methods that assume objectivity and rationality are impossible in social science research, he argues, not only because of the reality and messiness of contexts, but because the notion of pure science, itself, is a myth. The "knowledge filter" of scientific inquiry, in an attempt to eliminate bias, subjectivity, and error removes the researcher-as-participant in the process; a human who has hunches, insights, makes mistakes, and disavows the role context and the researcher play in human discovery. This is not to say scientific research methodologies are worthless in educational contexts, nor that the scientific method cannot be applied to the social sciences, but, he warns, we need to engage in "reality therapy" that continually investigates our methods and our results. He intuits what Morin describes as the musical complexity of research, "construction in movement that transforms in its very movement the constitutive elements that form it" (Quoted by Montuori, 2008, p. vii).

To play, a bit, with this notion of musical complexity, one comes to understand a musical ensemble as both skilled application of musical technique and improvisation that captures the unique context of the moment (Forehand, 2005). The dance of the musical ensemble is one that continually plays off of structure and interpretation, form and function, global and emerging patterns, and recursive dynamics. The ensemble metaphor used in the context of our exploration of the method of methods validates the importance of traditional attempts to address objectivity, consistent application of methods, and clearly articulated goals. These are the backbones of inquiry. But our meta-method must also engage the improvisation, the differences that make a difference, the layers of complexity, the role of the researcher within the researched, and the complicated conversation that connects and reconnects across contexts, methods, and researchers. The musical ensemble is the system of the performance and multiple playings that become the complicated conversation of the arts.

The method of Cartesian doubt as the basis of modern inquiry and the culture of method (Doll, 2005) needs to be interrogated as its own limitations and boundaries become a part of the complicated conversation of research. The researched and researcher are within their own transformational dance that changes both through the process. By developing a culture of method that is open to possibilities and fearless in the face of ambiguity, uncertainty, and complexity, we have the opportunity to create an approach to methods that is multilayered, patterned, relational and connected. Such a method of methods allows for networked knowing across dimensions with intersections at key nodes of purpose to create a system of interrogation that solves complex, real-world problems in the context of education. As a system, the method of methods based on a culture of method that embraces complexity becomes an autopoietic system (Maturana & Varela, 1980) itself, a dynamic system much greater than a mere depository of discrete knowledge "chunks."

The culture of method that invites the complicated conversation of educational research also opens up the possibility of using methods designed to explore complex dynamics and the evolution of systems. Engaging research methodologies from the complexity sciences extends the capabilities to engage in the messiness of educational contexts and address issues of scale, complexity, and dynamics.

Edgar Morin (2008) advocates a transdisciplinary approach in educational research to avoid the pitfalls of unquestioning assumptions of method. Transdisciplinarity is driven not by methods, per se, not by "problem solving in the context of the agenda of a specific discipline . . . not in attempts to create abstract theoretical frameworks, or to further the agenda of a new discipline, but in the need to find knowledge that is pertinent for the human quest to understand and make sense of lived experience, and of the 'big questions'" (as quoted by Montuori, 2008, p. xii). Morin, as described by Montuori (2008), distinguishes interdisciplinary approaches whereby "methods of one discipline (are used) to inform another" from transdisciplinary research which "draws on multiple disciplines while actually challenging the disciplinary organization of knowledge," avoiding the pitfalls of "reductive/disjunctive way of thinking that makes up what Morin was to call the 'paradigm of simplicity'" (Montuori, 2008, p. xxi). Transdisciplinary approaches to research, according to Montuori (2005), are inquiry driven, meta-paradigmatic, connected, contextual, and transparent. These approaches are important for interrogating contexts that are complex and creating a system of inquiry that has the capacity to interrogate itself. Applying approaches to research developed in the complexity sciences is supported by the culture of transdisciplinary method, providing opportunity for the complicated conversation guided by Bateson's schismogenesis, and opening up inquiry to what Pierce referred to as the world of the probable (Truett, 2005) where knowledge is incomplete and open to infinite inquiry.

Re-searching the Culture of Method

As we think of educational research from this complex learning systems perspective and begin to engage a culture of method that is open to transdisciplinarity, we begin to see a layered or dimensional approach to the "what's" and "how's" of research. Educational research, as a system, then, engages the "why's" of education through a dynamic process, driven by the "why" of educational research, namely, to support the educational agenda.

Re-searching the questions of education interrogates and connects layers of complexity in educational contexts. Questions of curriculum, class size, uses of technology, and models of education (charters, magnets, autonomous networks, and traditional) have policy implications for funding that shape the educational experience and the future of society, at one dimension, and directly impacts students in a particular classroom in a particular context, at another dimension. Questions of teacher evaluation, teacher effectiveness, teacher preparation, teacher professional development, and teacher credentialing similarly shape the educational context across multiple dimensions of the educational landscape. The role of technology in education opens a series of questions that challenges across dimensions curricular decisions, educational organization, the role of the teacher, class size, and equity, among others.

Re-searching the culture of method engages a method of methods process that scales educational inquiry across contexts, psychic and social domains, and intellectual disciplines. The problem of scaling research to address the complex questions of education is similar to the challenge of scaling innovation in education. Cynthia Coburn (2003) describes a scale research framework for addressing the need for and challenges of studying education innovation across settings that is useful for framing educational research as a whole. Making educational research relevant for the real context of schools requires a system of research as a whole that provides useful information for teachers, administrators, policy makers, and community leaders in the same way that scale research of education innovation considers the multiple dimensions of implementation innovation that include "depth, sustainability, spread, and shift in reform ownership" (p. 4).

Deep change in instructional innovation, according to Coburn (2003), is multilayered and impacts practices, beliefs, classroom norms and values. Studies of classroom implementation of instructional reforms need to explore the depth of implementation and, therefore, these various dimensions of change. From a research perspective, depth of research includes research across the many dimensions of the implementation process. Implementation dynamics are both across time and across settings.

Sustainability of reform extends the question of depth of implementation to understand change over time and across implementation sites (Freeman, Corn, Bryant, & Faber, 2015). Clarke and Dede (2009) describe the challenges of scaling innovation when moving from the pilot phase, where additional resources and support are available, to replication of innovation at other sites. The same challenges exist for initial implementation sites as these resources are removed. Just as with sustainability of innovation, educational research needs to recursively reconnect research across contexts and time. Sustainability avoids the pendulum swings of innovation resulting from the lack of sustainable processes and resources being put in place after the initial innovation is tested. Beyond the sustainability of resources, however, Coburn (2003) describes that sustainability ultimately requires new ways of thinking, valuing, and interacting across multiple levels of the educational enterprise. This is apparent in diffusion of innovation studies where new technologies may be applied to classroom instruction, for example, but thinking about what it means to know in the context of a technology rich environment does not change and technology use is reduced to rote practice or enrichment of traditional instruction (Fleener, 1995).

The spread of educational reform is another dimension of innovation in education discussed by Coburn (2003) that considers how whole school contexts or district policies change to accommodate and support innovation. Policies and decisions that support change, including demonstrated values about where funds are allocated, decisions about professional development, and peer mentoring across classroom implementation of reform, reflect that deep change to the system has occurred. Sustaining and scaling innovation requires more than replication; it requires whole system commitment. Scaffolding research similarly requires spread from the perspective of policy changes and changes in school operations. We have seen, for example, how a few teachers using the Flipped Classroom approach (McCammon, 2011) to mathematics instruction ultimately influenced the entire school mathematics department to adopt Flipped classroom approaches. The principal's role in the spread of this innovation was instrumental in its success.

As educational innovations are scaled, we have already seen how replication is not sufficient to ensure significant change has occurred. Another dimension of the application of innovation is when practices, policies, and understandings are "owned" locally by those implementing the innovation. Adoption of innovation ultimately requires contextual adaptation and local ownership that creates a system of support for the innovation. Until our research can inform and change practice and those who change adopt the changes as their own, we have not had a true impact on schools.

Understanding education innovation and reform from the perspectives of depth, sustainability, spread, and shift in ownership requires inquiry over time and across many dimensions of the educational context. If we place these parameters for investigating success of educational reform efforts, or the implementation of innovation in educational settings, as focal points for organizing inquiry, we begin to see the complexity of research across domains and scales of educational contexts. Individual learners, teacher expertise, administrative commitment, redistribution of resources, changing beliefs about teaching and learning, and policy impact are just some of the dimensions of interconnectedness required for re-searching and impacting educational change.

Making a Difference Through Educational Research

The graphic below (Fig. 2.2) attempts to capture the layered and dimensional aspects of a system of educational research that is self-reflective, dynamic, and adaptive through time. Coherence is what gives the system identity, in this case, as the body of educational research. Within the dimensions of inquiry, transdisciplinary approaches are important to maintain system openness with intellectual domains and practices relevant to the complex social system that is education. Inquiry driven research ensures the "why's" of research are grounded in real problems of education. Relevance of individual research studies is layered across dimensions as part of the interrogation of the why's, what's, and how's of the research, adapting research findings to differing contexts. As a coherent system, divisions across methodologies are erased as all research is entered into the

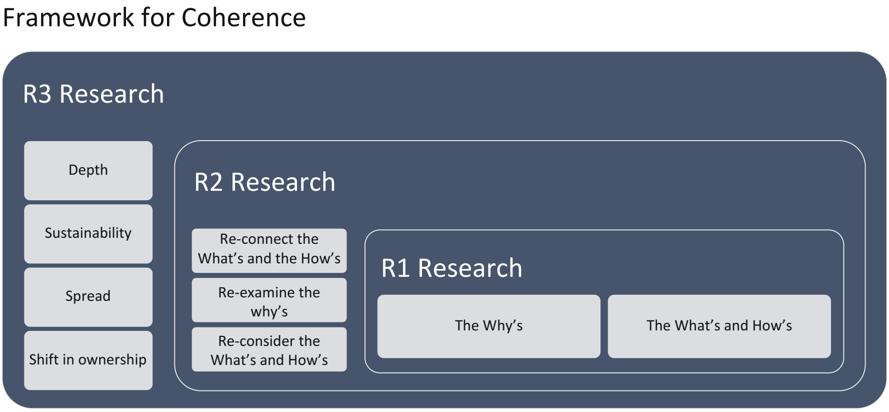

Fig. 2.2 A systems perspective of educational research

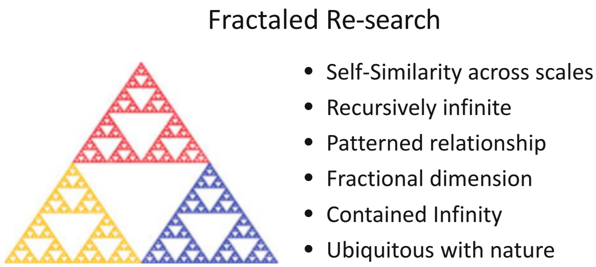

Fig. 2.3 Sierpinski triangle

complicated conversation of the purpose of education and change occurs through the complex approach of scaling.

For those who understand how fractals are created (or even the Ying and Yang of Eastern thought), the coherence of a system of educational research that is open, relational, recursive, and dynamic can also be represented by the Sierpinski triangle, depicted in Fig. 2.3 (Wikimedia Commons, 2015). Here, we see infinite layers of complexity within a defined space; the recursive process of the re-search approach, and scaffolding research across scales and contexts. Fractals, with the characteristics listed below, disrupt ideas about dimensionality, introducing the notion of fractional dimensions. Found in nature, fractals describe amazing complexity within finite spaces. The average size of human lungs, for example, has the surface area approximating the size of a tennis court (Gleick, 1987). This would not be possible were it not for the fractal relationships constituting the lungs.

We can imagine that at different levels of the Sierpinski triangle, R1–R3 reside as a framework for re-search that is ever present at all scale dimensions. The repeating patterns of the fractal suggest a local and global "intelligence" of the system where, in our case, the dimensions of research are ever present. And finally, the coherence of this system of research ensures relevance and connection to the real-world context of schools.

So what does this mean for schools, policy makers, community leaders, and educators who look to educational research for answers to preparing students for the twenty-first century? To our stakeholders, we are obliged to articulate the grounding of our research in the "why's" that matter to them; to engage them in the recursive "why" process to come to common understandings about the purposes of our research; and to change our own ideas about the role educational research should play in educational reform.

As we utilize transdisciplinary approaches to our research, there is another layering of complexity as our research methods engage in the complicated conversation across disciplines about the nature of knowing and knowledge. The patterns that connect across disciplines reveal important insights for knowing that have the potential to change ways of thinking, continuing the recursive process of inquiry to inform social understandings across social system domains.

References

Bateson, G. (1972). *Steps to an ecology of mind*. Chicago: University Chicago Press.

Bateson, G. (1979). *Mind and nature: A necessary unity*. New York: Dutton.

Bauer, H. H. (1994). *Scientific literacy and the myth of the scientific method*. Chicago: University of Illinois Press.

Clarke, J., & Dede, C. (2009). Robust designs for scalability. In L. Moller, J. B. Huett, & D. M. Harvey (Eds.), *Learning and instructional technologies for the 21st century* (pp. 27–48). New York, USA: Springer.

Coburn, C. E. (2003). Rethinking scale: Moving beyond numbers to deep and lasting change. *Educational Researcher, 32*(6), 3–12.

Doll, W. E. (2005). The culture of method. In W. E. Doll, M. J. Fleener, D. Truett, & J. St. Julien (Eds.), *Chaos, complexity, curriculum and culture: A conversation* (pp. 21–75). New York: Peter Lang.

Fleener, M. J. (1995). A survey of mathematics teachers' attitudes about the calculator: The impact of philosophical orientation. *Journal of Computers in Mathematics and Science Teaching, 14* (4), 481–498.

Fleener, M. J. (2002). *Curriculum dynamics: Recreating heart*. New York: Peter Lang.

Fleener, M. J. (2004). Why mathematics?: Poststructural topologies and negative spaces. In M. Walshaw (Ed.), *Mathematics education within the postmodern*. Greenwich, CT: Information Age Publishing.

Fleener, M. J., Carter, A., & Reeder, S. (2004). Language games in the mathematics classroom: Teaching a way of life. *Journal of Curriculum Studies, 36*(4), 445–468.

Forehand, C. (2005). *To journey into creativity*. Dissertation, University of Oklahoma.

Freeman, S., Corn, J., Bryant, L., & Faber, M. (2015). *Applying a scale research framework to an NSF math science partnership grant*. Paper presented at the Conference on Using Continuous Improvement to Integrate Design, Implementation and Scale-Up, National Center on Scaling Up Effective Schools, Retrieved October 7–9, 2015.

Gleick, J. (1987). *Chaos: Making a new science*. New York: Viking.

Hatch, T. (1998). The differences in theory that matter in the practice of school improvement. *American Educational Research Journal, 35*(1), 3–31.

Jewett, L. (2005). Minding culture. In W. E. Doll, M. J. Fleener, D. Truett, & J. St. Julien (Eds.), *Chaos, complexity, curriculum and culture: A conversation* (pp. 277–297). New York: Peter Lang.

Jörg, T. (2011). *New thinking in complexity for the social sciences and humanities: A transdisciplinary approach*. Berlin: Springer.

Lu, L. (2011). *Making transitions: A multiple case study of mathematics classroom teaching reform in China*. Dissertation, Louisiana State University.

Maturana, H. (1980). Introduction. In H. Maturana and F. Varela, *Autopoiesis and cognition: The realization of the living*. Boston: Reidel.

Maturana, H. & Varela, F. (1980). *Autopoiesis and cognition: The realization of the living*. Boston: Reidel.

McCammon, L. (2011). *Why I flipped my classroom*. (video, found at http://www.fi.ncsu.edu/selected-resources/why-i-flipped-my-classroom/).

Montuori, A. (2005). Gregory Bateson and the challenge of transdisciplinarity. *Cybernetics and Human Knowing*, *12*(1–2), 147–158.

Montuori, A. (2008). Foreword: Edgar Morin's path of complexity. In E. Morin (Ed.), *On complexity* (pp. vii–xliv). Cresskill, NJ: Hampton Press, Inc.. Robin Postel, Trans.

Morin, E. (2008). *On complexity*. Cresskill, NJ: Hampton Press, Inc.. Robin Postel, Trans.

P21. (2009). *Framework for 21st century learning*. Washington, DC: Author. Retrieved September 14, 2015 from http://www.p21.org/our-work/p21-framework.

Truett, D. (2005). Watercourses: From poetic to *poietic*. In W. E. Doll, M. J. Fleener, D. Truett, & J. St. Julien (Eds.), *Chaos, complexity, curriculum and culture: A conversation* (pp. 77–99). New York: Peter Lang.

Umpleby, S. A., & Dent, E. B. (1999). The origins and purposes of several traditions in systems theory and cybernetics. *Cybernetics and Systems: An International Journal, 30*, 79–103. Retrieved September 14, 2015 from http://www.gwu.edu/~umpleby/recent_papers/1998_origins_purposes_several_traditions_systems_theory_cybernetic_1.htm.

Waldrop, M. M. (1992). *Complexity: The emerging science at the edge of order and chaos*. New York: Simon and Schuster.

Wikimedia Commons. (2015). *Image of Sierpinski triangle*. Retrieved September 14, 2015 from https://commons.wikimedia.org/wiki/File:Sierpinski1.png.

Wittgenstein, L. (1953). *Philosophical investigations*. New York: MacMillan. G.E.M. Anscombe (Trans.).

Chapter 3
A Batesonian Perspective on Qualitative Research and Complex Human Systems

Jeffrey W. Bloom

Introduction

For years, we have been discussing how educational research in the complexity sciences is distinctive from other research, if it is at all. We need to continue this conversation to which I hope that this chapter will contribute. What we are talking about is an emerging paradigm of complexity (see Glossary) in education. However, in order to establish such a paradigm we need to look critically at the underlying assumptions of contemporary research approaches and of the research we have been doing and are planning to do. This chapter begins with an exploration of these three ideas of paradigms, assumptions, and complexity. The remaining parts of the chapter delve into the implications for how we might view and conduct complexity research (See Glossary for further explanation). The major emphasis throughout is on the work and ideas of Gregory Bateson.

The complexity sciences have roots in the early work on cybernetics and systems theories (Capra, 1996). Much of the development of cybernetics took place at the Macy Conferences, which occurred from 1946 to 1960. Gregory Bateson, Norbert Wiener, Heinz von Forester, Margaret Mead, George Evelyn Hutchinson, Warren McCulloch, and Kurt Lewin among many others were key participants in these conferences. These rather informal gatherings of some heavy hitting intellectuals from a variety of fields initially grappled with ideas that danced around the notion of cybernetics, including communication, learning, neural networks, teleological mechanisms, computers, neurophysiology, analog vs. digital brain functions, perception, etc. (a nice summary can be found at http://www.asc-cybernetics.org/foundations/history/MacySummary.htm). The first conference was entitled, "Feedback Mechanisms and Circular Causal Systems in Biological and Social Systems." As the conferences proceeded, they became more focused and formalized, but in the

J.W. Bloom (✉)
International Bateson Institute, Northern Arizona University, Flagstaff, USA
e-mail: jeff@jeffbloom.net

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_3

early days there were few records of what took place. Mary Catherine Bateson's (1972/2005) account of a later Macy Conference is quite illuminating in terms of the content discussed and the interpersonal dynamics. But what was unusual was the interdisciplinary makeup of the membership, which changed each year. The first year was composed of people with expertise in mathematics, physics, neurophysiology, medicine/physiology, anthropology, behavior, economics, psychology, psychiatry, philosophy, and sociology. The list of areas of expertise expanded each year. Overall, the Macy Conferences established cybernetics as at least a field of study, which in turn led to the development of the fields of systems and complex systems (Capra, 1996), as well as much of the current work in computers, artificial intelligence, information systems, and so forth.

Although Bateson and the others pursued their own intellectual agendas and wrote extensively, many of the ideas in the present chapter were more than likely discussed in depth by others in the group. Distinguishing the origins of these ideas may often be difficult, if not impossible. However, the emphasis in this chapter is on the ideas expressed by Gregory Bateson and his impact as I see it on how we might approach research from within a complexity sciences framework. Briefly, the major components of Bateson's work that I discuss in this chapter and that comprise a "Batesonian approach" include: (a) his emphasis on *nonlinear* ("nonlineal") patterns of causation, (b) not *confusing quantification and measurement* with what should be described, (c) primacy of *relationship* (see Glossary) over separation into entities and parts, (d) not confusing the *map* (an abstraction) for the *territory* (reality), (e) *epistemology* (see Glossary) as personal and social constructs, (f) *change* as a given, (g) the importance of *double description* and *multiple perspectives*, and (h) the critical importance of *context* for any kind of meaning.

Clash of Paradigms and Conflicting Assumptions

Those of us who are engaged in the complexity sciences face a number of challenges from those involved in establishing a new and emerging paradigm to those involving the baggage carried over from the dominant paradigms of the past few centuries. Among the top challenges are the ways in which we deal with the conflicting assumptions between complexity and positivism (including reductionist and mechanist assumptions). We certainly seem to be in the throes of a scientific revolution. As our particular field of complexity sciences in education and the social sciences continues to grow and develop towards an established paradigm, we need to pay close attention to what we do and how we do it, including how we think about the complexity sciences as a paradigm and the concomitant views, assumptions, and practices.

The notion of paradigm in this chapter can be situated within Kuhn's redefined concept of "paradigm." From an operational perspective he described the importance of paradigm in the following way:

> Without commitment to a paradigm there can be no science… the study of paradigms is what prepares a student for membership in a particular scientific community. Men whose research is based on shared paradigms are committed to the same rules and standards for scientific practice. That commitment and the apparent consensus it produces are prerequisites for normal science, i.e., for the genesis and continuation of a particular research tradition. …scientific revolutions are inaugurated by a growing sense that an existing paradigm has ceased to function adequately in the exploration of an aspect of nature (Kuhn, 1970, p. 11).

However, an expanded understanding of paradigm seems to include the following aspects that paradigms:

- Involve a **worldview** (see Glossary) (Cobern, 1991; Pepper, 1970) or set of reasonably compatible worldviews, including the values and assumptions associated with these worldviews. Some may argue that worldviews may be more fundamental to human experience than paradigms, which is likely true. However, for the present treatment of paradigms, the association of paradigms to worldviews may be useful for developing a feel for the interconnections and nature of the effects of paradigms.
- Involve a set of **theoretical and conceptual frameworks** (see Glossary) that comprise the particular domain of interest and inquiry.
- Are usually associated with one or more compatible **philosophical frameworks** (see Glossary).
- Involve **research methodologies** (see Glossary) that comprise the array of inquiry tools used within the paradigm, which also are consistent with the worldview(s) and theoretical frameworks.
- Involve the **practices and discourses** (see Glossary) characteristic of the particular paradigm.

Even with these descriptions and characteristics, the notion of paradigm is still quite slippery. Is "positivism" a paradigm? Is "feminist studies" a paradigm? Although various people may refer to both of these ideas (i.e., positivism and feminist studies) as paradigms, they are not equivalent. These two questions point to two different logical levels or types (Bateson, 1972/2000, 1979/2002, 1991; Bateson & Bateson, 1987/2005; Bateson, M. C., 1972/2005; Copi, 1971/2011; Korzybski, 1948/2010) or categories of thinking—acting (I am suggesting "Thinking—Acting" as a way of capturing the everyday aspect of paradigms in terms of how and what you think, and how you act and talk within your particular professional community.) Positivism is a higher level of categorization under which other ways of thinking—acting appear, such as behaviorism. And there are yet other ways of thinking—acting that appear at lower levels of categorization, such as classroom management.

A more useful way of discussing paradigms may involve a notion of levels of categorization, such as super-paradigms, paradigms, sub-paradigms, and even sub-sub-paradigms. Positivism seems to be at the level of super-paradigm in that it spans and includes many other more specific ways of thinking—acting that not only occur in research domains, but also occur across societies and cultures. At the same time, "behaviorism" may occur at the level of paradigm. Behaviorism falls within positivism, but also includes other more specific ways of thinking—acting, such as "classroom management," which may be a sub-paradigm. I am not sure that we can solidify such categorization schemes as absolute. The process of such categorization is more like a process of pattern thinking, where the utility of the categorization or pattern is in the justification or rationale for the categorization in the context or contexts (see Glossary) in which one is working. This approach to thinking about paradigms is still slippery, but the slippery-ness is acknowledged and addressed upfront. Typically, from our positivist heritage we want one right answer and one right way of doing things, which was the ultimate of Descartes' view of the world and of a Newtonian approach to science. However, even our notion of "paradigm" cannot fit nicely into a packaged definition. By "slippery," I am referring to the exceptions, the changeability, the hybridizing, the expanding, and the contracting of what we may think of "paradigms." We just need to take care in describing our paradigmatic orientations in ways that promote cohesiveness and consistency.

From the perspective of research, a careful alignment with a specific paradigm or set of compatible paradigms can provide a framework of consistency and cohesiveness. A significant danger arises when research or any sort of thinking involves conflicting paradigms. For instance, a teacher may be trying to deal with a situation involving a student's behavior in the classroom. A teacher considers herself a humanist, who values the cognitive and emotional aspects of her students. She encounters fundamental conflicts when she tries to address the problem using prescribed behaviorist approaches that do not take cognition and emotions into consideration and only use simplistic stimulus—response approaches to classroom management. Using such conflicting paradigms creates confusion for both the student and the teacher and may undermine the teacher's overall goals for student identity, relationships, and self-efficacy.

Since this chapter focuses on Batesonian and complexitivist implications for research, the primary paradigmatic conflicts involve those that undermine the emerging paradigm of complex systems. The paradigms (including super-, basic, and sub-paradigms) that seem to be particularly contradictory to complex systems are positivism, mechanism, reductionism, behaviorism, among many others. One of the major problems we face in our society and in research is that we live in a world that is deeply entrenched in positivism, mechanism, and reductionism. The underlying assumptions of almost everything we do and say have been molded by centuries of positivistic patterns. These underlying assumptions are so insidious that they work themselves into the way we think about complexity and complex systems. As a result, we risk creating further confusion and misconceptions and promoting views and approaches that undermine actions that can solve some of the major threats to schooling and to the very survival of humanity. I need to insert here that I do not view this situation of conflicting paradigms and conflicting assumptions as merely an academic exercise. In fact, I view these conflicts and their remediation as critical to understanding and addressing the major issues we face in and across multiple contexts and across scales from the individual to all of humanity and from the molecular to the biosphere.

The following list describes some of the more common conflicting paradigmatic assumptions that we encounter in research and society. In this list of binaries of conflicting assumptions, the ideas on the left side tend to be characteristic of positivism, mechanism, and reductionism. The ideas on the right side of the "vs." tend to be characteristic of complexity, complex systems, and Batesonian approaches.

- **Quantification and Measurement vs. Description.**
  We tend to try to quantify "things" that have no quantity, such as learning, behavior, and teaching abilities. The same issue applies to "measuring things" that have no dimensions, which include intelligence, learning, etc. Bateson (1979/2002, 1981, 1991) refers to such issues as epistemological errors, where such errors confuse quantity for quality and pattern. When I have brought up this issue of not being able to quantify or measure learning with a variety of different people, the responses range from looks of perplexity to angry refutations. The confusion is so deeply embedded that we cannot conceive of any alternative. Some people respond with, "so, how do we assess kids' learning?" But, more often than not, people stumble over what words to use instead of "measure." We do not value *description* as a meaningful and legitimate means of understanding learning or any other phenomenon.
- **Predictability vs. Unpredictability.**
  Maybe the focus on predictability is embedded in a primal desire for certainty, but predictability with any certainty only occurs in simple physical systems, such as objects colliding, planetary motion, and so forth. From the terms coined by Carl Jung, Bateson (1979/2002, 1991; Bateson & Bateson, 1987/2005) suggested that the nonliving world of *pleroma* is predictable and governed by linear cause and effect relationships. However, the living world of *cretura* is unpredictable and governed by multiple, nonlinear feedback loops and complex interrelationships with multiple interconnected causal factors. Descartes' and Newton's mechanical view of the world has perpetuated our desire to apply mechanistic predictability to all kinds of phenomena within living and ecological systems.
- **Complicated vs. Complex.**
  Complexity involves multiple interacting systems that perpetuate themselves through various nonlinear processes. In complex systems, the multiple interconnected processes cannot be explained separately from everything else. There are no separate parts. In fact, complex systems are generally interconnected with other complex systems. So a bear is a complex system, but cannot be understood separately from the forest (another complex system) in which it lives. A living organism, an ecosystem, the biosphere, and cognition—learning are all interrelated complex systems. However, we often describe events or objects as complex, when what we actually mean is that they are

complicated. A car is complicated, but not complex. An assessment approach may be complicated, but is not complex.

- **The Thing vs. Sets of Relationships.**

  Almost all educational research and pedagogy focus on “things” as separate entities and as extensions of Cartesian duality. Mind is separate from body. Children learn about a tree as a distinct object made of parts. We examine a teacher as an individual. We isolate “best practices” as distinctive and separate from context. From the perspectives of Bateson and complexity sciences, “things” as separate entities do not exist. Rather, everything is composed of sets of relationships, both within the thing itself, between the thing and context, and between things. Our tendency in research is to focus on things and not on the relationships. We look at the teacher and the role of the teacher, but do not look at the teacher within the contexts of classrooms, schools, communities, and so forth. When we focus on the role of the teacher, we exclude that role within the sets of roles of children, parents, and principals. As Bateson (Bateson, N., 2011) suggested, when we look at the role of someone, we are looking at a “half-assed” relationship. We do not look at classroom events, teacher thinking and practice, or children’s behavior and thinking, as sets of relationships within various overlapping contexts. In relating with students, rather than look at them as separate entities and label them with some sort of judgment, I have tried to look at them as bundles of relationships. What kinds of experiences have students had that made them who they are? What could account for this or that kind of attitude, behavioral characteristic, and so on? Such a process changed the way I saw and related to my students, but I still had to fight the tendency to judge. When teaching about some sort of content, I tried to take the same approach by emphasizing the interrelationships involved. If students were observing earthworms, at some point we would talk about gardens, similarities to ourselves, birds, soil, ecologies of forests and fields, foods, anatomy, behavior, sex, communication, beauty, and so forth. The relationships make up the contexts in which the meanings are embedded.

- **Objectivity vs. Subjectivity.**

  The positivistic notion of objectivity is generally dismissed among qualitative researchers, but the influence of this faulty assumption still affects how we conduct research. At very subtle levels, we take observational notes as if we were objective observers. Within this process, we rarely record our emotional reactions and explicit theoretical or belief reactions. We also rarely pay attention to how our presence in a classroom, school, or other context affects the dynamics of the setting and those individuals in this setting. A significant number of books and papers address concerns of validity and reliability. The qualitative versions of credibility, dependability, confirmability, and transferability (Lincoln & Guba, 1985) are utilized as if they are the equivalent of the positivist notions of validity and reliability. Qualitative researchers feel driven to take defensive postures against the predominant positivist paradigm by showing how their research is legitimate in ways that are understandable to positivist researchers. This position does not dismiss the value of credibility, dependability, confirmability, and transferability. Rather, I am suggesting here that our motivation to justify our work with these ideas is driven by more subtle drives to achieve recognition in positivist circles. As a Batesonian complexitivist (for the lack of a better descriptor), I do not dismiss positivism. It is useful in some contexts, such as some kinds of engineering. But I do not think it is at all useful in the social sciences and education, where we are dealing with living systems that are inherently complex and should be studied from that perspective.

- **Whole as Sum of Parts vs. Whole as Greater than Sum of Parts.**
  Reductionism contends that if we understand all of the parts, we can understand the whole. A Batesonian or complexitivist contends that wholes are much bigger than the sum of their parts. However, complexitivists often avoid looking at the parts and criticize anyone who looks at the parts. Such actions or reactions are problematic. The problem does not lie in the parts, but in thinking that the parts will lead to a complete understanding of the whole. Looking at the parts is necessary. However, a recursive process should involve zooming in to the depths of the parts, then zooming out to the whole, then back to the parts and so on (Bateson, 1972/2000, 1979/2002, 1991; Bateson, M. C., 1972/2005; Bateson, N., 2011).
- **Map vs. Territory.**
  Gregory Bateson (Bateson, 1979/2002) focused heavily upon Alfred Korzybski's (1933/1994) notion of "the map is not the territory" or that one's concepts and conceptual models are not the same as the objects or phenomena to which they refer. "Naming is always classifying, and mapping is essentially the same as naming" (Bateson, 1979/2002, p. 27). From Bateson's perspective, confusing the map for the territory is another fundamental epistemological error. In research, such error potentialities menacingly loom over every part of the process. As we construct explanations from observations, we may begin to believe that our explanations are the reality, rather than our interpretations of reality. Our explanations may be projections of our own theoretical and belief frameworks, which may not reflect the actually reality. A "map" is any level of abstraction, any representation, and is not the actual thing (Korzybski, 1948/2010). The Cartesian view of the natural world as a giant machine is a classic example of confusing the map (machine or mechanistic view) for the territory (natural, biological/ecological world). People actually thought the natural world worked like a machine. In fact, people still think this way.
- **Linearity** (see Glossary) **vs. Nonlinearity and Lineality** (see Glossary) **vs. Recursion (and Process—Outcome).**
  Bateson made a point of distinguishing between linear and lineal, although at present the notion of "lineal thinking" seems to have been appropriated by "linear thinking." Bateson distinguished between these two terms where "linear" is a mathematical relationship resulting in a straight line or a graph and "lineal" refers to sequential relations among causes or within an argument. "The opposite of linear is *non-linear*. The opposite of lineal is *recursive*" (Bateson, 1979/2002, p. 212). As Bateson contended, "lineal thinking will always generate either the teleological fallacy (that end determines process) or the myth of some supernatural controlling agency" (p. 56). Again, the lineal and positivistic—mechanistic

tendency is to assign a singular cause to a particular effect. From the perspective of complexity and recursive thinking, when we see a particular effect, we may consider that the effect is due to the interaction of many different factors, relationships, and contexts. In addition, such recursive thinking places greater emphasis on process than on the end product and sees that there are a variety of possible end products for any given process (Weinberg, 1975/2001). Much of educational thinking ascribes causes for particular issues and problems. Teachers are blamed for low student test scores. Lack of "time-on-task" is a cause of low learning outcomes. Children's lack of respect for teachers is the cause of classroom behavior problems. These types of linear cause and effect relationships are prevalent in both the popular and research literature in education, as well as in political speech and media reports.

- **Rigidity and Stasis vs. Variation and Change.**

  Both rigidity and stasis, as well as stability, often describe the state of some "thing", object, entity, or process. Once again, such notions contain epistemological errors. Even some seemingly rigid object or stable process is undergoing continual change. One of Bateson's (1979/2002) favorite examples, is the tight rope walker who is continually adjusting body and balancing pole positions in order to maintain balance. What may appear as stable, static, or rigid is actually undergoing continual change. Bateson (1979/2002) emphasized the importance of random variation and change as the characteristic of what he called the two great stochastic processes: (a) learning and (b) evolution. In contemporary practices, approaches to learning are rigidified, sequentialized, and controlled to the point where any random variation is excluded. In research, the methods of data collection and interpretation tend to disregard the importance of random variation by viewing and presenting events as stable and invariable. We make "conclusions" that this is the way it happened and this is what happened. We do not suggest that all kinds of variation and random events occurred and may occur. Rarely do we suggest that there were patterns in the way random occurrences and variation were handled or not handled.

- **Single Description vs. Double or Multiple Description or Multiple Perspectives.**

  Research commonly utilizes single vision. We examine particular phenomena from a single perspective. Within qualitative research, even the notion of "triangulation," which appears to address the issue of multiple perspectives, is still situated within a singular perspective. One set of triangulated data may include observations from the researcher, commentaries from subjects, and a variety of artifacts. Although these three sets of data provide information from different sources, they tend to be interpreted from a single conceptual or theoretical perspective. Such an approach is certainly useful, but a truly double or multiple description is most likely absent. One of Bateson's examples of double description involves relationships. Each component of a relationship describes that relationship. These descriptions are not the same, but they both describe and establish the relationship from different perspectives (Bateson & Bateson, 1987/2005). Our challenge as researchers is to find and elucidate these double or multiple descriptions. We also need to both describe "something" from our own epistemological frameworks and assumptions and describe the same "something" from one or more other epistemological frameworks and assumptions, in the same way Gregory and Catherine Bateson (1987/2005) wrote *Angels Fear* from theistic and nontheistic perspectives. The taking of multiple perspectives—from the arts, natural sciences, social sciences, philosophy... or from one's own perspectives as well as the perspectives of widely different people, societies, cultures—can provide possibilities for approaching some sense of bringing the map closer to the territory. Such approaches are powerful versions of transdisciplinarity (see Glossary) and transcontextuality (see Glossary). They play with contradictory assumptions and conflicting views in ways that prevent being trapped by those very assumptions and views, while allowing glimpses of accuracy and truth to emerge. In fact, such approaches provide the ability to see, describe, and utilize what Bateson (1979/2002) calls *metapatterns* (see Glossary) or patterns which connect: "It is that metapattern which defines the vast generalization..." (p. 10). These patterns are what we are trying to expose and understand through research. However, the fragmentation from specialization and the mechanization of our thinking has led us away from transdisciplinary inquiries and robust contextuality (Montouri, 2005).

- **Disconnected vs. Contextualized.**

> Without context, words and actions have no meaning at all. This is true not only of human communication in words but also of all communication whatsoever, of all mental process, of all mind, including that which tells the sea anemone how to grow and the amoeba what he should do next (Bateson, 1979/2002, p. 14).

The word "context" is used frequently, but is rarely defined and probably has as many meanings as people using the word. Context can be referred to as a physical setting, as a social or cultural setting or framework, as a period of time, as social interaction, and so forth. Bateson (1979/2002) suggested that context is "pattern through time" (p. 13). But he also suggested that context is connected to the notion of story, and that contexts can be temporal, spatial, and formal. Formal contexts are those that focus on the sets of relations or patterns that underlie the particular phenomenon. A simplistic example that Bateson (1979/2002) liked to use was that of the trunk of an elephant. The trunk's location between the eyes designates a nose from the perspective of a spatial context. The function of the trunk as a nose for breathing relates to a temporal context. And the embryological history of the tissues of the nose relate to the formal context of ontological relations. The idea here is that the use of context, in fact, the use of multiple contexts, is necessary in providing a depth and extensiveness of meaning, which is generally rather thin in much research.

There are other conflicting assumptions, but this list comprises some of the more common assumptions that impact research practices. In general, the positivist, mechanist, and reductionist assumptions tend to insidiously work their way into our thinking and practices. Although we may align ourselves with subjectivity and decry objectivity, we still may operate under the influence of objectivism. We write as if we are taking an objective perspective. We design a study to address the suppositions of objective research. We discuss causation as if there is one cause for a specific effect. The effects of these assumptions can be subtle. They almost seem to operate like Andy diSessa's (1993) "phenomenological primitives." By "phenomenological" diSessa means that these understandings are based on our everyday experiences, such as seeing the moon rise in the east and set in the west. And they are "primitive" in that they do not necessarily operate at a conscious level. They can be so deeply embedded that they come into play automatically. So such a "p-prim" could result in a highly resistant to change notion that the moon moves around the Earth from east to west, just the way it appears to move. So many of our assumptions seem to operate in a similar way to these p-prims. They seem to be self-evident truths. However, the danger is that the effects of such assumptions can compromise or threaten the accuracy, depth or robustness, and usefulness of our understandings. We may place emphasis on the end product of some process (such as test results), while from a systems framework the product is not nearly as important as the process (such as the learning experiences) (Weinberg, 1975/2001). While emphasizing test results, rather than learning experiences, we also fall into thinking that we can quantify or measure learning, which is inherently immeasurable. As a result, we have created what Bateson (1972/2000) called an epistemological error or "muddle."

Systems Thinking and Complex Systems

Not all systems are equivalent. There are simple and complicated mechanical systems, such as bicycles, automobiles, planes, and planetary systems. Such systems operate according to specific physical laws and principles and are predictable. In contrast, ecological, biological, and social systems operate in different ways. Even though physical laws and principles continue to operate in such living systems, much more complex sets of interrelationships are at play. These sets of interrelationships operate in complex recursive pathways that help to maintain the systems of which they are a part. The processes involved in such self-maintaining systems are referred to as autopoiesis (see Glossary). This concept of self-maintaining, self-regulating, self-generating, and self-transcending systems are the defining features of "complex systems".

The kind of thinking required for investigating and understanding simple and complicated systems can be referred to as mechanistic thinking. We can think about simple cause and effect relationships, linear processes, and predictability. On the other hand, if we wish to understand ecological, biological, and social systems, we need to think in more complex ways. When I conceive of “systems thinking”, I am focusing on ways of thinking about complex systems. Such systems thinking has to focus on trying to understand the complex relationships and recursive processes. The following list highlights the primary foci of systems thinking:

- Nature and interactions of multiple interacting systems (not just how *one* system works).
- Relationships between parts and (a) processes, (b) wholes, and (c) other parts.
- Relationships between processes.
- Multiple perspectives of systems and processes.
- Contexts within which multiple systems operate and upon which the systems affect.
- Nature and dynamics of relationships.
- Patterns within and among the systems and their component parts.
- Function and nature of feedback loops and other nonlinear processes in terms of the flow of information and/or materials and in terms of their functions in regulation, adaptation, maintenance, and so forth.
- Nature of transformation and other change processes.
- Relevance and usefulness of processes and systems (Bateson, 1979/2002; Checkland, 1985; Daellenbachand & Petty, 2000; Paucar-Caceres & Pagano, 2009; Roberts, 1978; Ulrich, 2003; Weinberg, 1975/2001; Werhane, 2002).

These particular foci (and very likely additional foci) describe a different way of conceiving of research in education. Rather than looking at linear cause and effect relationships and at outcomes of various treatments, such foci can help us put more emphasis on the processes that affect other processes, etc. We can examine how the unique characteristics of a teacher affect the processes of her teaching and how these processes affect multiple other processes of student learning, thinking, talking, interacting, and so on. We can examine how educational systems affect teacher thinking, teacher practices, student learning, etc. Rather than focusing on rather simplistic relations and processes, we can begin to expand our vision to include the multiple interactive, interdependent, and interrelated systems that comprise children’s learning, thinking, and psychological development and well-being; teachers’ learning, thinking, practices, and psychological well-being; classroom and school community development and maintenance; parental participation and learning; local community functioning; and local and national political functioning. The extent of interrelatedness extends across contexts and levels of scale. Without recognition of such interrelations, we limit the relevance, meaning, and potential impacts of our research.

Research as and About Complex Systems

We, as human beings, are complex systems. We establish various scales of complex social systems. And we live within and affect complex ecological systems. Consequently, our approaches to research should be consistent with the nature of complex systems. In fact, our research is a complex system. The way we think and make sense of our world is a complex system. Our thinking and our research can serve to perpetuate and maintain our individual and social lives. We can adapt to emergent situations and changing conditions. We can refine and adjust the ways we live so that we can live in ways that are more in tune with the environments in which we live, which, by the way, has not happened with the reductionist, mechanistic, and positivistic research approaches of the last few centuries. In fact, although reductionist, mechanistic, and positivistic research has led to incredible scientific and technological advances, such research approaches have created the life-threatening crises we are now facing, including over-population, global warming, peak resources, and so forth. Our involvement with social contexts of learning, teaching, and education is no longer isolated and only relevant to specific contexts of schooling. Our work as educators and researchers must address the fundamental issues of our cultural and social survival, not to mention our survival as a species. Our work can no longer be an academic pastime. And we certainly should not perpetuate the assumptions and suppositions that have brought us to the brink of social and ecological collapse. As Gregory Bateson suggested, "the major problems in the world are the result of the difference between how nature works and the way people think" (Bateson, N., 2011). We have reached a critical point where we have to take Bateson's point seriously and change the way we think about research, learning, thinking, teaching, society, ecology, etc., and the way we do research.

Several years ago, Tyler Volk and I developed a model of research (which also can be applied to learning and teaching) based on Bateson's ideas (Bloom & Volk, 2007, 2012). There are three basic aspects to this model: (a) depth, (b) abstraction, and (c) extent or abduction (see Glossary). These three aspects interact recursively in ways that encapsulate the ideas that have been discussed thus far in this chapter. "Depth" involves examining the intricacies of the relationships, patterns, and processes within any particular system or sets of systems. "Abstraction" involves creating explanatory models or frameworks, the "maps" that describe the territory, and examinations of one's own and others' epistemologies. "Extent" or "abduction" refers to the processes of using and testing the concepts from "depth" and "abstraction" in other contexts. These contexts can involve levels of scale and contexts across various differences. For example, we may have examined teacher thinking from within a working group of teachers and have found certain areas of concern and how these areas are interrelated in various ways. Throughout this process we may have constructed various models of how these concerns can be explained by generalized patterns of relationship. And, concurrently, we may test out how these patterns of relationship and models seem to explain phenomena at various levels of schooling, from classrooms to schools to districts to states to the national institution of schooling. We also may find that these patterns of relationships and models explain phenomena in other contexts, such as businesses, organizations, and state and national political groups.

A Batesonian approach to research is in marked contrast to approaches that rely on false notions of objectivity with narrow foci and highly sequential series of predetermined steps. In addition, the typical separation of mind–body, self–other, or self–context and the separation of systems as distinct do not exist in a Batesonian approach. The entire approach can be seen as one that is rigorous (*note*: I use "rigorous" with great trepidation, since it implies a certain stiffness or rigidity, which is not descriptive of a Batesonian approach), yet relies on the complex knots of interrelated human propensities, such as rationality, emotion, aesthetics, perceptions, and belief frameworks (a "contexts of meaning" approach—Bloom, 1990, 1992) to provide multiple perspectives of the interactions between parts and wholes and between various wholes (systems) (Bateson, 1972/2000, 1979/2002, 1991).

Getting Past the Limitations of "The Researcher" and "The Research" as Separate and Special

The tendency over the past few centuries has been to make research and researcher appear to be inaccessible to the general population. Technical jargon among many other aspects of technical disciplines has created barriers to understanding. In school and in the media, we have represented science and other research oriented disciplines as something for particularly smart people and not for the general public. However, research as a way of using one's observations to create explanations and knowledge is a characteristic of being human. Of course, not all research is equivalent, but the processes of exploring, examining, questioning, abstracting, abducting, and so forth are common characteristics of research. We can refine these processes and the ways we think about these processes and the data we collect, but the fundamental approach is shared among people of all ages. Some groups may devalue or suppress these natural research processes, but, nonetheless, we use these processes from the time we are born.

The other major assumption that is problematic is this notion of research occurring in a specific location and during a discrete period of time. We assume that we do research in a lab or in some other setting, such as a classroom or school and that the research stops when we leave or when we are not analyzing data. However, if we consider that research is a natural propensity and that we do not turn on and turn off our brains, we begin to realize that research is occurring throughout a day, a week, a year, or for that matter our entire lives. Gregory Bateson did not turn off his research, then turn it back on again. He thought about these ideas throughout his days and throughout his life. All of his experiences became his laboratory. We can find many other scholars who operated in this way. The idea that we have to have approval from some human subjects committee to engage in research is very strange indeed.

How can we not observe others and our environments and utilize what we see and our insights in our continually developing explanatory frameworks? We teach about and represent research as some technical and mechanistic enterprise. Many people operate in this way. In such cases, there is a basic disconnect between ourselves as learning beings and the activities from which we learn. Experts talk about their research as if it is some external "thing" that has no connection to themselves, the way they think, and the way anything else works.

Research as a complex system is integrated into our own complex systems of living and learning. Research about complex systems is relevant and meaningful research that focuses on our biological, ecological, personal, social, and/or cultural contexts. Yet people are stuck in seeing the world through mechanistic, reductionist, and positivistic lenses. They see no other possibility. In a recent online conversation, I commented about the complex issues around a new technological development in solar panels. I posed questions about the multiple contexts that are not addressed, such as (a) shortages of resources that will be needed to produce and replace the technology on a regular basis, (b) energy costs, (c) net energy effects, (d) wear and tear, and (e) financial costs. People's responses ignored these complex issues. For them, technology was the answer and we'll find technological answers. Such lineal thinking is short-sighted and ignores the connections between various systems that are involved. And, by the way, if you missed it, this last snippet of an everyday event is an example of how our research as a way of living cannot ignore the events we experience throughout our lives.

We need to "flip," transform, or transmute our thinking about research. We need to move from a view of research as separate from our everyday lives, as exclusive to an elite group, as a mechanistic process, and as a lineal process. Research needs to span multiple contexts and disciplines and pay close attention to networks of relationships. And we all need to work at making the results of our research widely available to the general public. The media and politicians are not communicating accurate or relevant information, so we need to make that effort to help establish an informed public about issues in education and beyond.

References

Bateson, G. (1972/2000). *Steps to an ecology of mind*. Chicago: University of Chicago Press.

Bateson, M. C. (1972/2005). *Our own metaphor: A personal account of a conference on the effects of conscious purpose on human adaptation*. Cresskill, NJ: Hampton Press.

Bateson, G. (1979/2002). *Mind and nature: A necessary unity*. Cresskill, NJ: Hampton Press.

Bateson, G. (1981). Paradigmatic conservatism. In C. Wilder & J. H. Weakland (Eds.), *Rigor & imagination: Essays from the legacy of Gregory Bateson* (pp. 347–355). New York: Praeger.

Bateson, G. (1991). In R. E. Donaldson, (Ed.), *Sacred unity*. New York: A Cornelia & Michael Bessie Book/Harper Collins.

Bateson, N. (2011). *An ecology of mind: A daughter's portrait of Gregory Bateson* [DVD]. Retrieved from https://vimeo.com/ondemand/bateson.

Bateson, G., & Bateson, M. C. (1987/2005). *Angels fear: Towards an epistemology of the sacred*. Cresskill, NJ: Hampton Press.

Bloom, J. W. (1990). Contexts of meaning: Young children's understanding of biological phenomena. *International Journal of Science Education, 12*(5), 549–561.
Bloom, J. W. (1992). The development of scientific knowledge in elementary school children: A context of meaning perspective. *Science Education, 76*(4), 399–413.
Bloom, J. W., & Volk, T. (2007). The use of metapatterns for research into complex systems of teaching, learning, and schooling. Part II: Applications. *Complicity: An International Journal of Complexity and Education, 4*(1), 45–68. http://ejournals.library.ualberta.ca/index.php/complicity/issue/archive.
Bloom, J. W., & Volk, T. (2012). Metapatterns for research into complex systems of learning. In N. M. Seel (Ed.), *Encyclopedia of the sciences of learning* (pp. 2243–2247). Heidelberg, Germany: Springer.
Capra, F. (1996). *The web of life*. New York: Anchor Books/Doubleday.
Checkland, P. (1985). From optimizing to learning: A development of systems thinking for the 1990s. *Journal of the Operational Research Society, 36*(9), 757–767.
Cobern, W. (1991). *Worldview theory and science education research: Fundamental epistemological structure as a critical factor in science learning and attitude development*. Manhattan, KS: National Association for Research in Science Teaching.
Copi, I. M. (1971/2011). *The theory of logical types*. New York: Routledge.
Daellenbachand, H., & Petty, N. W. (2000). Using MENTOR to teach systems thinking and OR methodology to first-year students in New Zealand. *Journal of the Operational Research Society, 51*, 1359–1366.
diSessa, A. A. (1993). Toward an epistemology of physics. *Cognition & Instruction, 10*(2 & 3), 105–225.
Korzybski, A. (1933/1994). *Science and sanity: An introduction to non-Aristotelian systems and general semantics* (5th ed.). New York: Institute of General Semantics.
Korzybski, A. (1948/2010). *Selections from science and sanity: An introduction to non-Aristotelian systems and general semantics* (2nd ed.). Fort Worth, TX: Institute of General Semantics.
Kuhn, T. S. (1970). *The structure of scientific revolutions* (2nd ed.). Chicago: University of Chicago Press.
Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.
Montouri, A. (2005). Gregory Bateson and the promise of transdisciplinarity. *Cybernetics and Human Knowing, 12*(1/2), 147–158.
Paucar-Caceres, A., & Pagano, R. (2009). Systems thinking and the use of systemic methodologies in knowledge management. *Systems Research and Behavioral Science, 26*, 343–355.
Pepper, S. C. (1970). *World hypotheses*. Berkeley, CA: University of California Press.
Roberts, N. (1978). Teaching dynamic feedback systems thinking: An elementary view. *Management Science, 24*(8), 836–843.
Ulrich, W. (2003). Beyond methodology choice: Critical systems thinking as critically systemic discourse. *Journal of the Operational Research Society, 54*, 325–342.
Weinberg, G. M. (1975/2001). *An introduction to general systems thinking*. New York: Dorset House Publishing.
Werhane, P. H. (2002). Moral imagination and systems thinking. *Journal of Business Ethics, 38*, 33–42.

Chapter 4
Emergence, Self-Transcendence, and Education

Jeffrey Goldstein

> *Our world is so huge and complex that any model capable of accurately representing it would be so far beyond human cognitive resources that we could not use it ... All we can do is develop better and better inaccurate models to serve particular descriptive, predictive and explanatory purposes.*
>
> —Richard Healey

Introduction

Education devoid of creativity is oxymoronic. Wherever we turn on the educational landscape we find an intimate bond between learning and creativity. From the novelty of classroom methods and educational technologies to new findings in the cognition of learning; from new public–private partnerships supporting novel educational institutions to new programmatic configurations. . .it is the *alliance* of education with the creative process that promises success. Learning, after all, is about the inculcation of the *new*, the *not known before*, in a phrase, it is "creative in-*nova*-tion" ("nova" is the Latin for "new").

In recent years a revolutionary novel approach to creativity research has opened up within the field of *complex systems* known variously as complexity theory, complexity science, nonlinear dynamical systems, and cognate labels. One of the most exciting phenomena in this field is emergence, the arising of unexpected novel patterns, structures, dynamics, and entities in complex, nonlinear systems. Examples are literally boundless but certain prototypes standout (Goldstein, 2011b; see also the excellent new book on emergence by Lichtenstein, 2014):

J. Goldstein
Adelphi University, Garden City, NY 11530, USA
e-mail: goldstei@adelphi.edu

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_4

- "Self-organizing" physical systems that generate emergent phenomena in the form of "dissipative structures" such as hexagonally shaped Bernard convection cells (see the research programs established respectively by the Nobel Laureate Ilya Prigogine and the German physicist Hermann Haken);
- Computational emergence of artificial life that exhibits novel, moving patterns on a computer screen whose dynamisms of appearing, merging, generating "offspring," and disappearing suggests the kinds of activity usually associated with living organisms;
- Biological emergence of new speciation, symbiogenetic forms; collective colony behavior; and so forth;
- Quantum protectorates or novel collective states of matter with radically unexpected properties such as superconductivity and superfluidity;
- Social emergence of new social, organizational, community structures exhibiting cooperation, collective behavior, and unexpected cohesion/correlation;
- The emerging "social self" during infancy, childhood, adolescence, even adulthood during developmental processes and transformations.

As this list demonstrates, instances of emergence take place across a wide variety of fields, but our focus is on the last two since they have a direct bearing on education. We also delve at times into features from others on the list in order to get a better grasp of what emergence is all about.

Since all the items on the list are examples of emergence, they are certain common characteristics (see Goldstein, 1999, 2014):

- *Radical novelty* which refers to the unpredictability, non-deducibility, and irreducibility of emergent phenomena from the perspective of the substrates from which they emerge—the contemporary term which groups these three qualifications of radical novelty is *uncomputability*, i.e., emergent phenomena are not able to be computed from antecedent substrates;
- *Coherence/collectiveness/wholeness/integration* which refers to the novel manner by which the substrate components are related to one another in the new emergent structure;
- *Global- or macro-level* which refers to the "higher level" of the emergent phenomena in relation to the "lower level" of the substrates (this global or macro-level is constituted by a spanning across the micro-components of the lower level);
- *Ostensiveness* which refers to how emergent phenomena are unforeseeable until they show themselves (this is closely related to the first feature of unpredictability and non-deducibility);
- *Dynamical* which refers to how emergent phenomena emerge over time and hence are not something like pre-given wholes;
- *Self-transcending constructions* which refers to the dual nature of emergent phenomena as continuous with the substrate and at the same time transcending the properties of the substrate;

- *Explanatory gap* which refers to the gap in explanation that stems from the inability to completely explain emergent phenomena from knowledge of the lower level substrates alone.

Whereas in most other scientific and philosophical endeavors, a chief aim is to narrow and even eliminate explanatory gaps as much as possible, when it comes to emergence, it is actually the presence of such explanatory gaps which insures the viability of the idea since the presence of an explanatory gap functions as a marker indicating the need to shift inquiry away from the lower or micro-level substrates the such macro-level constructs as higher level organizing principles, multifarious constraints, and diverse novelty generating factors. Accordingly, in developing a "logic" for emergence we probe its explanatory gap as a key to unlock the potency possessed by those processes of emergence capable of generating emergent phenomena with the radically novel, unique characteristics listed above.

Our strategy assumes the following presuppositions:

- The idea of emergence hinges on the presence of explanatory gaps;
- Such gaps are by definition pointers as to where explanation breaks down;
- Exactly how explanation breaks down can reveal crucial insight into what must be going on in processes and operations of emergence that give it the potency to impede explanation;
- The most recent way of conceptualizing explanatory gaps is of uncomputability;
- By probing how uncomputable outcomes come about we are then put into a better position to comprehend how emergent phenomena can be uncomputable.

As mentioned, the aim of our inquiry is the development of a cogent "logic" for emergence" which, in aiding our understanding of emergence, will have pragmatic implications for educators in utilizing the concept of emergence. The term "logic" of emergence includes the conceptual infrastructure or blueprint by which emergence takes place, that is, explicit and implicit conditions, substrates, processes, operations, constraints, outcomes, and consequences of emergence. This use of "logic" is analogous to how the term "logical" was used by the physicist *cum* biologist Walter Elsasser (one of the first complexity-oriented biologists; quoted in Gilson, 1984, p. 109) to emphasize the disparity between the nature of biological phenomena and that of physics: "[biology is endowed] with a *logical* structure quite different from what we are accustomed to in physical science" (emphasis added). Similarly, we uncover how emergence has a logical structure differentiating it from other modalities of change.

Another example of what we mean by "logic" can be seen in the phrase the "logic" of painting developed, for example, by Piet Mondrian (http://www.theartstory.org/artist-mondrian-piet.htm). The logic of Mondrian's work covers his basic principles and how he saw the impetus behind his aesthetics which involved a threefold process of abstraction, simplification, and two dimension rectilinear geometrization (e.g., vertical and horizontal lines, squares and rectangles and so on). This threefold logic guided Mondrian (and his followers) to exhibit in his paintings an inherent, universal, and harmonious balance of two fundamental opposing forces. Moreover, the simplification and pairing down to essentials guided by the logic of Mondrian's paintings was held to insure the universal applicability of the harmony he sought behind and under all particular contexts.

When it comes to emergence, it is important to note that explicating its logic brings carries a somewhat daunting challenge precisely because of the explanatory gap which by definition precludes the use of typical reductionist explanations and thus provide scant conceptual space for the arising of the genuinely novel. One can discern this kind of *ban against novelty* in Western thought as far back as ancient Greece (see North, 2013). An example was Aristotle's pejorative labeling of novelties as aberrations for departing from his postulated ideal/normative ontology. This rejection of the novel often found expression in a turn away from purely naturalistic explanations toward appeals to supranatural sources of the radically new, e.g., the creative activity of the divine, a theme also taken up by certain emergentists over the past 100 years (see Blitz, 1992). In this chapter, our logic of emergence, though, will stay on a naturalistic course and leave immaterial speculations on how emergence works to others of that sentiment.

The Rise of the Idea of Emergence

The idea of emergence preceded complexity science by at least a century. The mid-nineteenth century, when the term "emergent" was coined, saw two closely related senses of an *explanatory gap*, both senses of which continued to guide the formulation of emergence into the twentieth and twenty-first centuries and thus can be said to ground the logic of emergence. The first sense was the basis of what the British philosopher and polymath John Stuart Mill (cited in Goldstein, 2014) called "heteropathic" in contrast to "homopathic" causation and what his student and follower, the American-English man of letters G. H. Lewes (cited in Goldstein, 2014) termed "emergent" in contrast to "resultant" effects. Inspired by the study of chemical reactions in which unanticipated outcomes were frequently observed, *heteropathic causation* and *emergent outcomes* referred to the radically novel, unexpected properties of the resultants of the reactions in contrast to the properties of the antecedent substrates involved in the reaction. Both Mill and Lewes' emphasized that whatever was going on in the reaction must be uniquely powerful to bring about "altogether new phenomenon" carrying no "traces" of the substrate components. Although both Mill and Lewes believed that science would eventually make headway in explaining emergence and its radically novel outcomes, they alluded to the need for changing our conceptualization of natural processes in order to allow for radical novelty generation. That this would amount to a deep and revolutionary change of perspective can be discerned in Mill's calling for a thoroughly novel type of causation, no mean feat. A logic of emergence, therefore, would need to incorporate a revision of traditional notions of causation.

The second sense of explanatory gap focused on the distinction between *mechanistic* explanations, framed in terms of mechanical interactions of mechanical parts, and *organicist explanations*, which aimed at explicating the arising of novel effects in terms of wholes, continuities and life forces (*elan vital*). Playing a key role in this distinction between organicist and mechanical explanations was consciousness or subjectivity, a subject that had been taken up by William James in the USA and Henri Bergson in France. Thus, Bergson propounded his own French version of the idea of emergence and emergents in his *Creative Evolution* (1911), utilizing the phrase, *fait jaillir* (literally, "springing up," "sprouting up," or "to bring forth out of") which became "upspringing" (a synonym for emergence) in English versions of the book.

On this side of the Atlantic, William James was developing his philosophy of "a stream of consciousness" whose laws of operation were decidedly not mechanical but more akin to life processes (Bauer, 2009). In particular, mechanical operations primarily involved two non-lifelike features: the need for parts from one system "touching" parts of another system so that the whole containing the parts is left out of the explanation; and the discontinuity incumbent upon the discrete nature of each instance of this mechanical causal touch, such as the repeated discrete motions of a steam engine driving a train. In contrast, life processes as seen in the subjective experience of the "Stream of Consciousness" (James' major trope in his philosophical psychology) take place by way of, first, wholes influencing other wholes with the parts playing only subsidiary roles and, two, a continuous flow of ideas, fantasies, sensations, perception and other mental contents. The continuity of the latter was conceived as an unbroken overlap of each content of consciousness with each succeeding one.

The most important point about emergence to be taken from both Poincare and James was that subjective experience, from an organistic point of view, could lead to unforeseen insights and inspirations which were not possible in a purely mechanistic interpretation of subjective experience. What Bergson and James had bequeathed to emergentist thinkers was both a focus on organicity and a placement of emergence in a context of experience. These two themes were greatly expanded a quarter of a century later in A. N. Whitehead's tour de force emergence-based metaphysics, e.g., his magnum opus *Process and Reality* (that Whitehead's later metaphysical system relied upon the idea of emergence is explored in Goldstein, 2004a, 2004b).

Whatever may be ultimately concluded about the success of the metaphysical systems devised by either Bergson or James, they had decidedly cut a deep wedge between mechanism and artifice on one side and life and the natural on the other (a conceptual stance that had to show itself in any logic for emergence at that time). Indeed, after Bergson and James, emergentists felt a pull to adopt either one side of the cut or the other. Yet a closer scrutiny into the explication of emergence shows that both reductionism and anti-reductionism can yield insights (Butterfield, 2001). Such a binary choice in philosophical and scientific conceptualization could only restrict inquiries into emergence. Emergent systems exhibit both continuity and discontinuity, and are composed of both parts and wholes. Leaving out one side or the other only results in a truncated explanation in danger of missing certain of the essential elements of emergence such as the relation of substrate parts to emergent wholes.

We can summarize the early logic of emergence as follows: it is a logic that recognized there were natural processes/operations resulting in unpredictable, even startlingly unanticipated outcomes not explainable by recourse to micro-level substrates alone, in other words, a logic that placed an explanatory gap at the heart of emergence. Along similar lines, it was a logic that rejected traditional understandings of causality since such traditional understandings did not attribute sufficient potency to causality with a capability to generate the special properties of emergent phenomena. It was a logic that favored an anti-mechanistic, even organic-like unfolding. And finally, it was a logic that tended to focus on consciousness and subjectivity as the place where emergence was most transparent to examination.

These early conceptions of emergence were later joined to Darwinian evolution in the formation of a loosely connected but significant intellectual movement on both sides of the Atlantic known as Emergent Evolutionism (from about 1915 to 1936). The main proponents were the philosopher/scientists C. L. Morgan, Samuel Alexander, C. D. Broad, W. Wheeler, Roy Wood Sellars, and others (see Blitz, 1992). Even though important differences can be detected among their approaches, in general they combined the two senses of explanatory gaps outlined above along with the notion that evolution proceeds by saltations or jumps of speciation, and thus did not follow a continuous path of descent. These saltations were momentous phenomena shown in the course of nature (and thus not restricted to Bergson's and James' subjective states), leaps of innovation exhibited in deep-seated and far-ranging shifts of qualitative properties. Such radical jumps, even reaching the cosmic in extent, could be seen in such examples of emergence selected by the Emergent Evolutionists as that of life from the inert, of consciousness from the merely alive, of new species out of already existing ones, even of such elusive metaphysical phenomena as Alexander's obscure claim that time itself emerged from space (see Gillett, 2006). Although it is not clear to this day, exactly what Alexander meant by this example of emergence, it did fall in with his general metaphysical search for the ultimate foundations of the ontology of the world. I find it surprising that one can come across equally arcane claims in modern theoretical physics which has appropriated the term "emergent" to describe many recondite matters in quantum mechanics, general relativity, quantum gravity, quantum field theory and other foci of investigations.

Steering between mechanism and vitalism, emergent evolution was mostly armchair speculation that contained not a few conceptual holes and plenty of mysterious happenings. That is why Morgan, for example, proclaimed some kind of enigmatic difference between causation and causality as a key to emergence (what this difference amounted to remained as enigmatic as Alexander's time space emergence), and Alexander himself said the best we could do in confronting emergence was to adopt a sentiment of "natural piety." The vagueness of such supposedly well-founded philosophical propositions is one of the reasons for the fact that emergence evolution did not fare well in the long term. After around 1940, we find emergence rising and falling in mini-movements spurred on by such scientific and philosophical luminaries as Karl Popper, Michael Polanyi, the Nobel Laureate Roger Sperry and others (see classic papers in each issue of the transdisciplinary complexity journal *Emergence: Complexity and Organization*).

It was the rise of complexity theory that greatly revived interest in emergence, picking up steam along with the burgeoning of interest in self-organizing physical systems, the computational emergence of the Game of Life and later "species" of artificial life, and macro-level collectivities such as "quantum protectorates" (all of which, as mentioned above, became prototypes of emergence). And, it was with the advent of complexity science that the process of self-organization became so closely entwined with emergence that the two words became nearly synonymous.

The Inadequacy of Self-Organization

Actually, self-organization is not a new idea but can be traced back to such preeminent German idealist natural philosophers as Kant and Schelling (see Keller's excellent and insightful history of the idea, 2008a, 2008b). During those early days, self-organization referred to the mutual influence of parts within organic wholes so that the nature of life itself became largely defined by the kind of circular causality by which self-organization was thought to operate. This continued as biology further developed, with self-organization becoming a way of expressing the organic regulation of one part to another, or whole to part, and the self-regulation of the whole organism itself.

As complexity science advanced out of such earlier systems sciences as cybernetics and related theoretical movements after WWII, self-organization was conceived of as not just playing a regulatory role but more emphatically as processes made up of interactions among micro-level substrate components resulting in macro-level adaptive functionalities. To the extent by which emergence became allied to self-organization, the logic of emergence was essentially the logic of self-organization.

A main thesis of this chapter (see also Goldstein, 2006) is that self-organization by itself as typically understood not only lacks a capacity for bringing about phenomena with the radically novel characteristics demanded for emergence listed above, it is not even the kind of explanatory construct that could be appropriate for emergence. Since my contention here considerably departs from tradition, I need to justify it.

First, let us unpack a few key elements of self-organization. To what does the "self" of "self-organization" refer? One candidate is as an indicator that the substrates are acting under their own efficacy and not being controlled by external imposition. Cognate terms here could include: "innate," "inherent," "automatic," "unplanned," and "natural." Closely related are the spontaneity connotations found in: "self-causing, "self-generating," "self-modifying," or some other "self-" prefixed term.

We find the eminent German physicist Hermann Haken (2011) recently defining self-organization as, ". . . the *spontaneous* often seemingly purposeful formation of spatial, temporal, spatio-temporal structures or functions in systems composed of few or many components . . . the action of the subsystem *without specific interference* from the outside." Along the same lines, in the other main school devoted to the study of self-organization in physical systems, that founded by the Nobel laureate Ilya Prigogine (Glansdorf & Prigogine, 1971: xx), we find, "Such 'symmetry breaking instabilities' are of special interest as they lead to a *spontaneous* 'self-organization' of the system both from the point of view of its space order and its function" (emphasis added). No doubt, the apotheosis of spontaneity in relation to self-organization is that found in one popularization of complexity theory (Kauffman, 1995) in which, according to a search through the book at "Google Books," the word "spontaneous" appears on 37 pages, with page 8 alone containing 8 repetitions of the "spontaneous" in discussing self-organization.

Both with "self" and "inner-directed" and "spontaneity" we can discern the sense of a *bottom-up* organizing process in that the higher level emergent order is thought to be the result of interactions of the lower level "self" or substrates and their inner resources leading to an upward organizing action and not a top-down imposition. It was these emphases on spontaneous, self-driven, and bottom-up processes which rendered the revolutionary implications to self-organization since previously it was widely held that change of a system required an external push. With the idea of self-organization, though, came the contrary but more compelling image of systems changing due to an inner adaptive response to internal interactions on the micro-level.

We can now put together the above remarks into the special logic of self-organization. First, it is a spontaneous and inner directed process. It is so spontaneous in nature that it can be brought about merely by eliminating or at least loosening those constraints operating to suppress this inner directedness. Self-organization is also a bottom-up process whereby lower level substrate components interact and through this interaction, higher level patterns and structures result.

It is important to note that once the idea of self-organization became an essential concept in the study of complex systems, its meaning had shifted from the earlier connotations of self-regulation which it acquired from, first Kant and Schelling, and then later the cyberneticians after WWII. Now, due to the respective research and theorizing of Prigogine and Haken, self-organization became associated with the lessening or outright dismantling of command and control mechanisms. The idea was disengaged from the equilibrium-seeking self-regulation sense towards proposing a lessening or even downright dismantling of command and control mechanisms that were interfering with the arising of novel properties. A prime example was how Prigogine's dissipative structure were supposedly prompted by *far-from-*equilibrium and not equilibrium. This was where the revolutionary nature of self-organization came to fore the since now the search was on how to facilitate the arrival of these new properties by reliance on the inner, supposedly "spontaneous" capabilities of complex systems and not through imposition novel from outside the system or even by means of the system's own internal "homeostatic" tendencies.

This is all well and good, yet we're left with the baffling question of exactly how lower level interactions lead to radically novel higher level emergent order.

Consider the following analogy. Place a bunch of fish hooks (the substrates) in a pail with a lid (container). Fasten the lid and shake the pail. Next, take the lid off and carefully look at the resulting structure of interlocked hooks. If this structure is then disentangled back to individual hooks and the process is repeated, each time the resulting structure will in fact be a novel different structure and thus this iterated action should satisfy being called a self-organizing process leading to a novel emergent outcome.

The novel fish hook structures will indeed display a kind of coherence or integration, one of the principal characteristics of emergent phenomena described above. This was called a "new relatedness" by the Emergent evolutionist C. L. Morgan. Yet one hesitates in calling this self-organizing operation emergence since the type of novelty ensuing hardly seems novel enough. Peering deeply into what goes on in self-organization reveals why: self-organization does not consist in potent enough novelty generating factors. Indeed, because of the ingrained bias against novelty and related conceptual assumptions, self-organization as it has been described in complexity literature lacks enough potency to bring forth radically novel outcomes. This does not mean, however, there is no place for self-organizing operations during emergence for surely there are. Rather it suggests that something else must be going on in addition which the logic of self-organization per se has not managed to include.

I contend the one of the main reasons why self-organization lacks a capacity for generating radical novelty is that it does not have a way for the substrates themselves to be transformed but instead they remain as they were before during processes of self-organization. This is so because according to the logic of self-organization all that is necessary is that the substrate components interact with one another which, to be sure, may lead to a new relatedness of the substrate components but such a new relatedness is not enough to count as emergence. The reason is that new relatedness can turn out to be nothing more than a rearrangement of an aggregate and not a genuinely radically novel new emergent integration (see Wimsatt, 1997, on the difference between aggregates and emergent wholes).

In an emergent whole, on the other hand, the substrate parts undergo transformation and it is this that conveys radical novelty to the emergent phenomena. A radically novel emergent whole must be constituted by radically transformed parts since a whole is precisely constituted by the parts in a congruity (see Bertoft, 1996; for a truly insightful account of this thesis, see Ganeri, 2011, 2012 on the requirement of transformation). Another way to put this is the dictum that for a radical novel product to be generated, the operations resulting in the generation of this product must themselves be radically novel. It is clear from the example of the fish hooks in the shaken pail, the interactions prompted by the shaking of the pail leave the fish hooks intact as they were before the operation. Certainly, the notion of self-organization has played a significant role in accounting for new systemic order in terms of the inner resources and direction of a complex system instead of the previous presumption that new order must be imposed from outside the system. But it has done little else besides that.

A New Logic for Emergence: Self-Transcending Constructions

Emergentists claim that emergence is a capacity nature has always possessed though it is only recently that this capacity has been recognized and come to the fore as a powerful new construct for exploring how radically novel and original outcomes can be produced. Because of both the newness of the construct and the inadequacies of processes like self-organization to explain emergence, a new approach is needed that departs in significant ways from self-organization. I have been developing such a new perspective on emergence called "self-transcending construction" (STC) for reasons I expound in this section (for the sources of STC, see the chart and descriptions in Goldstein, 2006). I hold that STC puts us in a better position to both understand and apply emergence in varied arenas of complex systems, more specifically for the purposes of this chapter, the arena of the "social self" mentioned in the beginning of the chapter as one of the prototypes of emergent phenomena. In fact, one of the originators of theorizing about the emergence of the emergent social self was the philosopher and social scientist George Herbert Mead (see Goldstein, 2007a, 2007b for an exposition of Mead's take on emergence) who intentionally and explicitly considered his work as expounding the idea of emergence.

Before we get to the social-self, we need to lay out the central features of the logic of emergence according to the notion of self-transcending construction. What originally struck me about this expression (as I say more about below) was a sense that it provided a more accurate description of emergence than self-organization. Self-organization does not so much describe emergence as one mechanism whereby emergence is supposed to take place, that is, through lower level interactions. STC, however, neutrally and with less implausible assumptions describes what emergence actually looks like, namely, the arising of radically novel order in a system that uses and then transcends some set of substrates in the complex system.

The "self" of "self-transcending constructions" refers to the substrates undergoing transformation but no claim is made that the action must be only spontaneous, innate, or self-directed, although to be sure these may indeed be seen in emergence. Rather, the emphasis is on how this substrate "self" is transcended during the processes of self-transcending constructions. We need to be careful with what this "transcendence" amounts to. Although it seems a hifalutin word, it is meant in the more prosaic sense of being transformed. For example, in the emergent prototype of superconductivity listed above in the introduction, the electrons which are the substrates, undergo radical transformation from being one type of elementary particle, i.e., fermions at a micro-level, to another at the macro-collective level, namely, bosons. Certainly there is a self-organizing like interactions of electrons during the emergence of superconductivity, but this is just a step and does not come close to explaining the transformation that ensues.

In describing emergence, the *self-transcending* facet of the term expresses the dual nature of emergence: one thrust expressing that there must be some sort of *continuity with* or *following/derivation from* the substrates, whereas the second refers the radical transcendence of the substrates at the same time. The first thrust of *continuity-with* substrates is what keeps emergence from being taken as "brute emergence," Strawson's (2006) term for the idea of emergence spontaneously arising "out of the blue," an idea which tends to evoke some immaterial force from above magically bringing about the emergent.

The self-transcending characteristic of STC implies there are natural capacities possessing a potency for radically transformative operations. The denial that nature could possess such capacities has been a conceptual stumbling block not only in the acceptance of the possibility of genuine emergence but has been a main factor behind calls for non-material, supranaturalistic sources for the production of radically novel outcomes The transformation involved in STC usually involves a complex set of operations on the substrates, e.g., higher level organizing constraints, emergence operations of mixing up, criticalization, and so forth (for my most recent detailed and technical account of these operations, see Goldstein, 2014). These act to transform the substrates so that the previous lower level is transcended in the production of the radically novel higher level order. It is here that the explanatory gap of emergence enters the picture, brought about by processes which have variously been called negation operators (adopted from logic and set theory), shifts, twists, and other cognate novelty operators which ensures the radicality of the emergent result. Self-organization builds the outcome from bottom-up processes of interaction but without the addition of higher level radical novelty generating and organizing factors and accordingly, self-organization cannot rise to the occasion of producing emergent outcomes. But the self-transcending *negation operator* negates previous structure and order in the direction of radically novel order and provides the potency for self-transcendence. This negation operation is the key component of criticalization which supplies the "plot twist" of the narrative of emergence, that is, pushes beyond self-organization to self-transcendence.

The phrase itself "self-transcending construction" came from a commentary on the proof of the existence of transfinite numbers devised in the late nineteenth century by the great German mathematician Georg Cantor (commentary by Felix Kaufmann—see Goldstein, 2014 for a description of why Kaufmann rejected Cantor's proof method). Kaufmann actually used the expression in a derogatory fashion believing that Cantor had illegitimately used a proof method that was *prima facie* impossible since nothing could transcend itself. When I came across the expression, I thought to myself, isn't self-transcendence exactly what happens in emergence when the substrates are radically transformed?

It must be pointed out that there was no sense on my part at that time that Cantor's work had anything to do with emergence, it was just that I thought the term had just the right connotations to evoke emergence. Later, however I found out that in a strange, rather convoluted manner Cantor's work did in fact have a great deal to do with the uncomputability characteristic of emergent phenomena. Uncomputability in relation to emergence was proved by Darley (1994) using an algorithmic complexity approach to emergent phenomena in artificial life, a prototype of emergence mentioned above. According to Darley, uncomputability meant that it was not possible to use a purely deductive, algorithmic method to derive the properties of emergent phenomena from the properties of the substrates. Hence, the characteristic of uncomputability has been added to the list of the properties of emergent phenomena listed above; which now are thought to be not just unpredictable, non-deducible, irrreducible to, radically novel with respect to, self-transcending, but also uncomputable from knowledge of the lower level substrates alone. Consequently, the explanatory gap of emergence was thereby made even stronger.

Darley did not stop there, he went on to apply his uncomputability idea through the framework of the mathematical mathematician Alan Turing's Halting Problem, a crucial element in Turing's own work on proving the existence of uncomputable numbers (this is, of course, the Turing who helped break the code of the Nazi's Enigma machine, and also the Turing who along with John von Neumann devised the first computers, see Goldstein, 2014). In his proof of uncomputable numbers, Turing relied on the previous theorems of Gödel which demonstrated a fundamental undecidability in formal logical/mathematical systems. It so happened that both Gödel and Turing used the same above mentioned Cantorian proof method disparagingly called self-transcending constructions. Thus, by this round-about way, Cantor's STC showed up in a path leading right to the uncomputability of emergent phenomena. Since, these mathematicians and logicians (and many others) showed how to generate uncomputability, I reasoned that I could reverse the chain of thinking from "emergents as uncomputable" to "what kinds of operations could produce uncompubility?" In this way, self-transcending construction became a formal logic demonstrating how the explanatory gap based on uncomputability could be produced (of course with suitable translational techniques that facilitated going from the rarefied world of mathematical logic to the more mundane world of prototypes of emergence in the natural world; all of this is gone into great detail in Goldstein, 2014).

Above, I used the phrase "plot twist" deliberately in order to introduce an analogy between mystery novels and emergence. In a mystery novel, the perpetrator of the crime remains unknown until the end. This functions as explanatory gap which motivates the reader to solve the mystery and close the gap. We can see this, e.g., in the numerous Sherlock Holmes stories, all of which can be viewed as fictionalized accounts of logical deduction in which Sherlock comes up with amazing insights to traverse the explanatory gap. The mystery consists of apparently incomplete or inconsistent evidence, which if put into syllogistic form would prompt a normal linear process of thinking not capable of closing the gap or as Holmes put it, "when you have eliminated the impossible, whatever remains, however improbable, must be the truth? We know that he did not come through the door, the window, or the chimney. We also know that he could not have been concealed in the room, as there is no concealment possible. When, then, did he come?" (A. Conan-Doyle, *The Sign of the Four*, chap. 6 (1890).

It is crucial to note, however, that although I use the mystery novel as an analogy, there is no implication that emergence involves anything ultimately mysterious or magical, nor does it necessitate appeal to any force more than natural, nor does it call for some special kind of radical logic that would enable the co-presences of seeming oppositions such as found in dialectical logic (as in Heraclitus, Hegel, Marx, dialectical materialism in general) or paradoxical/paraconsistent/dialethist logics (see e.g., Da Costa, 1992; Melhuish, 1973; Priest, 1994, 2002) or even the complementarity logic such as by Bohr with his the principle of complementarity in quantum mechanics.

Dialectical logic can be appealing for its apparent ability to show the conceptual undergirding of the logic of change/becoming/growth by following the classic threefold operations of thesis, antithesis (the negation operator in dialectical logic) and synthesis. Usually attributed to Hegel this tripartite logical scheme actually preceded Hegelian dialectics. The anti-thesis step is analogous to the negation operator/criticalization, plot twist "trigger" which triggers the transcendence of change. However, this dialectical scheme hardly does justice to all that is required to produce outcomes with the radical novelty properties bestowed on emergent phenomena.

Paradoxical logics, on the other hand, dispatch any problems of inconsistency or contradictoriness by building into logic itself at least two extra truth valuations, e.g., the addition of values that express the contradictory characterization of being true and false at the same time. But there is a serious problem with such a flippant introduction of inconsistency and contradictoriness. The Scholastics were well aware of this problem, seeing it as igniting a veritable unstoppable spread of permitted inconsistencies and contradictions, and thus introducing gibberish into normal conversation (Sainsbury, 2008).

The philosopher Graham Priest (1994, 2002) has done more than probably anyone in the contemporary scene to remedy the problem associated with paradoxical logics in producing a plague of contradictions in discourse. Yet his work leaves me with a sense of flippantly doing away with difficult problems by some conceptual sleight of hand that does not take seriously the potential for spreading inconsistencies. A much sounder approach is that of Simmons (1990) whose study of the paradox-seeming propositions of Cantor's famous proof of transfinite sets, which as mentioned above, was the indirect source of the phrase STC, showed how paradox can be handled without resorting to the fancy conceptual footwork of paradoxical logic. I have adapted Simmons' mathematical logical formulation of Cantor's proof in order to formalize my conception of self-transcending constructions (see Goldstein, 2002).

In any case, we do not need to turn to dialectical or paradoxical logics since STCs do the work they are supposed to in transforming, in a self-transcending manner, the substrates through their incorporation of some type of negation operator or other radical novelty generation process which does not require anything paradoxical. Instead, the logic of emergence via STC rests on, among other factors, the kinds of nonlinear operations of functional iteration, bifurcation, and other transformative processes formalized in nonlinear dynamical systems theory, none of which require paradox or dialectics.

Furthermore, the sources for the radically novel order displayed in emergent order also hinges on a plenum of constraints within and around the complex system wherein emergence operates. In Goldstein (2011a, 2011b), I pointed to two examples where we can see the required presence of constraints, namely, the hexagonal shape of Benard convection cells and scroll rings in excitable media. For the former I described D'arcy Thompson's masterful mathematical explanation of why these convection cells need to be hexagonally shaped, having to do with the constraining fact that six circles fit exactly around a central circle of the same side, a constraining demanded in such allied examples as optimal sphere packing and similar situations.

The second example I mentioned was Art Winfree's (and his student Steven Strogatz) theorem in differential geometry which set out the order constraints at work on the various possibilities for scroll ring formation in excitable media. As he put it (quoted in Goldstein, 2011a, 2011b), "Left to spontaneous processes, nothing much happens in an initial uniform excitable medium: it organizes itself in a featureless way, perpetually minimizing any concentration of gradients. But when prodded by a big enough, spatially structured stimulus, it reveals alternative stable modes, organizing itself periodically in space and time." Then, of course, there is the constraint afforded by the actual physical (or psychological) containers of complex systems undergoing emergence (see Goldstein, 2004a, 2004b). In both cases, mathematical constrains operative on the system guarantee specific types of order to characterize different types of emergent phenomena.

It is also the case that in their denigrating view of external constraints promoters of self-organization as the key to emergence have driven a conceptual wedge driven between the spontaneous/natural and what is otherwise constructional in nature. In point of fact, though, construction can be as natural as self-organization is supposed to be, a fact attested by such natural constructional phenomena, to mention just a few, as bone growth, turtle shells, beaver dams, bird nests, hurricanes, ant hills, termite cones, protein assemblies, and so forth.

The concept of construction and by implication structure can be found right at the beginning of contemporary neo-emergentist research when the Nobel Prize winning solid state physicist Philip Anderson (1972) offered his constructionist hypothesis as a response to the arch reductionism rampant at that time among particle physicists. This hypothesis proposes that although it might be possible to reduce nature to certain simple, fundamental laws, this did not then entail a similar ability for reconstructing the universe from these simple laws since each new level of complexity involved the emergence of entirely new properties and laws not appearing at the lower levels. Each new level of complexity, accordingly, can be said to exhibit the self-transcending construction of new structures with new properties that transcend lower level constructional characteristics and dynamics. Moreover, the construction of each new level does not necessarily imply an intentional designer or constructor behind the constructional activities since construction as such can arise in countless ways whenever lower level parts are constrained by each other and their environments, and/or interact in relation to each other to generate even more constructional constraints.

G. H. Mead and the Emergence of the Social Self

Psychological organization consists of all of the patterns, order, arrangements, and structures making up a person's psyche. Various candidates offered to explain the emergence of psychological organization have been offered including psychodynamic approaches (starting with Freud's compromise formation among ego, id, superego), personality theoretic constructs, Gestalt psychology, developmental theories, and so forth. Darwinian ideas on psychological morphogenesis have also been proposed to how mental characteristics have evolved. Since the study of emergence incorporates the various elements and stages that are found in the arising and changing of the organization of a complex system, research into emergence offers some intriguing hints into psychological morphogenesis.

An emergence-based approach to psychological morphogenesis though implies a creative emergence without the need to postulate any kind of innate drive such as in Freudian and other psychodynamic approaches (Goldstein, 2007a) happens innately and naturally when complex systems are in appropriate conditions, not in the sense of the spontaneity claimed for self-organization, but rather in the sense of a self-transcendence of the core psychological substrates. The novel order that is seen to emerge is a consequence of the system creatively transmuting the substrates of the nascent infant self within the social context of family and society.

This view on the emergence of the social self was the central conceptual theme of George Herbert Mead's emergentist theory of the social self. Mead's revolutionary work was, in the words of his colleague and friend John Dewey, "...the most original mind in philosophy in America of the last generation" who took the doctrine of emergence "much more fundamentally" than "most of those who have played with the idea" (Dewey quoted in El-Hani & Pihlström, 2002: 29). Yet Mead's highly original and even radical speculations on emergence as the linchpin of the social self are little discussed among complexity adherents.

Mead focused his emergentist-inspired speculations on how the personal self emerged out of a social nexi of interactions within the human community. As I pointed out in previous papers (Goldstein, 2007a, 2007b), for Mead, sociality was "the principle and form of emergence." This process occurred throughout nature, for the emergent higher level was equivalent to the sociality characterizing all of nature. For Mead, there was a double movement in which the lower level attributes of the individual were shaped by the higher level, emergent social whole and, simultaneously, the lower level attributes of the individuals were built up the from the higher level social whole.

Mead even considered consciousness itself as a reflective internalization of the social. That is why he held that any attempt at developing an adequate psychology of the self which neglected its social core would have to fall far short of an adequacy. According to the contemporary philosopher George Cronk's (2005) interpretation of Mead's point of view, "The world in which the self lives, then, is an inter-subjective and interactive world—a 'populated world' containing, not only the individual self, but also other persons. Inter-subjectivity is to be explained in terms of that 'meeting of minds' which occurs in conversation, learning, reading, and thinking."

This conceptualization of a social self emerging out of intersubjectivity can be clearly seen in Mead's famous discussion on the difference between the sense of "I" and "me" The "me" referred to that aspect of self-identity having to do with one's social self, the introjected social representations arising in the developing child through the mediation of family, friends, neighborhoods, and society at large. It is the "me" which enables social empathy and is organized according to "the attitude of the whole community" or the "generalized other." In terms of emergence, the "me" is the individual reflection of emergent sociality itself.

The "I," though, refers to the reaction of the organism to the ideas, values, and so forth of the "generalized other," particularly the reaction to the latter. For Mead, the "I" included that sense of efficacy or agency which a mature person possesses. However, because the "I" and the "me" are so intimately interrelated in a "creative balance," personal and social novelty can emerge as the "I" enacted in society and in so doing *reconstructs* it Thus, the individual self as a conscious, experiencing entity was not to be thought of as a mere passive recipient of a social whole's "downward" influence on the individual member, but rather an active shaper of what that very emergent sociality was and could become. This meant that even though logically the individual as a "lower" level entity and the social as a "higher" level collective occupied distinct realms, in the reality of conscious experience this distinction was mostly *confounded* in the ongoing actions of a person's life.

That the social self is a self-transcending emergent entity constituted by the transformation and melding of the substrate components was understood by Mead in terms of his *perspectivalism* involving the crucial notion of a frame of reference. Although that which may be viewed from one person's perspective can be quite different than that of another's perspective, the integration of these various perspectives, or their being grounded in an intersubjectivity, was what made up an emergent social whole, the different perspectives making up the different substrate components. The philosopher William Desmonde (1967) put it, "society is the ability to be in more than one system at a time, to take more than one perspective simultaneously. . . This phenomenon occurs in emergence, for here an object in the process of becoming something new passes from one system to another, and in the passage is in two systems at the same time. During this transition, or transmutation, the emergent entity exists on two levels of nature concomitantly" (p. 232, 233). Indeed, perspectivalism implies that there can be many more perspectives making up the social self than two. Instead, the emerging social self of an infant, child, toddler, adolescent, even adult are operated on by a whole slew of perspectives, this congeries becoming integrated into the wholeness of the self. Viewing this by way of self-organization would leave unanswered many questions about how exactly these interacting perspectives become integrated into a higher level unity since the narrative of higher level organizing principles and the novelty generating factors responsible for the radically novel emergent social self is left out of the account.

For Mead, sociality consists of a fundamental capacity for being several things at once or "the occupation of two or more systems by the same objects," and hence can be said to represent manifold relations of the emergent phenomena. Accordingly, the self's emergent sociality which organized the various clashing systems and perspectives, just as a particular society was the organization of diverse individuals so that sociality could be said to have a synthesizing function. Sociality then, operating according to a seeming paradox encompassed both permanence and change by allowing a process of adjustment made necessary by the new relations characterizing the emergent event. According to Jones (1969), society could do this through its dual capacity: first, to unite in a circularly reinforcing fashion was a member; and, second, to allow for the individual qua individual to actually express the social as a collective.

Conclusion: Education and the Emergence of the Social Self

Mead, of course, was well-known as an educator besides his other celebrated work. His understanding of the emergence of an integrated social-self, capable of thoughtful action and agency was tied into the unsurpassable role of education in generating, eliciting, shaping, directing, and channeling the social-self. From a complex systems vantage point, education functions as not only the place of higher level organizing which subjects the social self to a multitude of constraining influences, education should also be viewed as the place where diverse novelty generating factors are at work spurring the self-transcending transformation of diverse perspectives integrated into the social self. Mead's perspectivalism plays a key role here in providing the needed novelty generation through a mixing and integration of these diverse perspectives. Thus, for Mead, whatever educators could do in order to enable the exposure of students to different perspectives would be a main avenue for the emergence of a viable and well-functioning person whose social self was constructed out of these perspectives. Indeed, even exposure to one other social perspective could serve to start the process of transcendence of that substrate composed of the experience of being isolated within only one's own perspectives.

Some may object that what I have just described can be understood by appeal to self-organization alone. However, as I have tried to point out above, self-organization has neither the "muscle," the "potency," the "scope," or the "novelty generating capacity" to bring about the needed transformation of the substrates involved in emergence. When any particular action of education actually works well in facilitating human development, something more than self-organization is going on that leads to the transformation of the substrate components of the developing self. This something is what I have discussed above in the context of emergence as that which generates the explanatory gap. Remember the explanatory gap of emergence is a way of talking about the various higher level organizing dynamics which lead to an outcome not explainable by recourse to the lower level parts alone.

The radically novel outcomes studied in emergence requires the addition of a host of other factors beyond mere interaction. Certainly, the activities of interacting so emphasized in self-organization can and should also be done in educational settings. But these are not enough since although such interaction provides the sources of the various perspectives incorporated into the social self, the melding of such perspectives into a higher integration requires more than that.

It makes undeniable sense to link emergence closely to creativity in the sense that both aim at unanticipated, novel, unique, and original outcomes. Whatever is capable of producing the latter properties presumes the generation of an explanatory gap and thus calls for the processes leading to them to possess an impetus and quality substantially differing from that which produces anticipated outcomes. Hence, emergence provides further support for the role of creativity methods in education at all levels and contexts. Such methods should not be confined to art or writing classes but instead spread throughout all curricula. The key for an emergentist Mead-inspired approach is the immersion of students in a fecundity of perspectives out of which novel phenomena will emerge shaped by the multitude of constraining factors and rendered radically novel by facilitation of the transformation of the substrate perspectives into an integrated whole.

References

Anderson, P. W. (1972). More is different: Broken symmetry and the nature of the hierarchical structure of science. *Science, 177*(4047), 393–396.

Bauer, F. R. (2009). *William James on the stream of consciousness: All the evidence*. Bloomington, IN: Iuniverse.

Bergson, H. (1911). *Creative Evolution*. (from French *Evolution Créatrice*, 1908). Trans. A. Mitchell. NY: Henry Holt and Company.

Bertoft, H. (1996). *The wholeness of nature: Goethe's way toward a science of conscious participation in nature*. Lindisfarne, UK: Lindisfarne Press.

Blitz, D. (1992). *Emergent evolution: Qualitative novelty and the levels of reality*. Dordrecht: Kluwer Academic Publishers.

Butterfield, J. (2001). Emergence, reduction and supervenience: A varied landscape. *Foundations of Physics, 41*(6), 920–959.

Cronk, G. (2005). Herbert Mead (1863—1931). Internet encyclopedia of philosophy. Retrieved June 3, 2015 from http://www.iep.utm.edu/mead/.

Da Costa, N. (1974). On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic, 15*, 497–510.

Da Costa, N. (2000). Paraconsistent mathematics. In I. D. Batens et al. (Eds.), *Frontiers of paraconsistent logic* (pp. 165–180). Hertfordshire, UK: Research Studies Press.

Darley, V. (1994). Emergent phenomena and complexity. In R. A. Brooks & P. Maes (Eds.), *Artificial Life IV: Proceedings of the Fourth International Workshop on the Synthesis and Simulation of Living Systems* (pp. 411–416).

Desmonde, W. (1967). *H. Mead, George Herbert (1863-1931)*. In Donald M. Borchert (a cura di), *Encyclopedia of philosophy*, vol. 6, Macmillan Reference [1967], 2006, pp. 79–82.

El-Hani, C. N. & Pihlström, S. (2002). Emergence theories and pragmatic realism. *Essays in Philosophy, 3*(2), Article 3.

Ganeri, J. (2011). Emergentisms, ancient and modern. *Mind, 120*, 671–703.

Ganeri, J. (2012). *The self: Naturalism, consciousness, and the first-person stance*. New York: Oxford University Press.

Gillett, C. (2006). Samuel Alexander's emergentism. *Synthese, 153*(2), 261–296.
Gilson, E. (1984). *From Aristotle to Darwin and back again: A journey in final causality, species, and evolution. (Trans. J. Lyon)*. Notre Dame: University of Notre Dame Press.
Glansdorf, P., & Prigogine, I. (1971). *Thermodynamic theory of structure, stability and fluctuations*. New York: Wiley.
Goldstein, J. (1999). Emergence as a construct: History and issues. *Emergence, 1*(1), 49–62.
Goldstein, J. (2002). The singular nature of emergent levels: Suggestions for a theory of emergence. *Nonlinear Dynamics, Psychology, and Life Sciences, 6*(4), 293–309.
Goldstein, J. (2004a). *A New Metaphysics for Emergence, 2nd International Workshop on Complexity and Philosophy*. Rio de Janeiro, Brazil: Federal University. Retrieved November 18–19, 2004.
Goldstein, J. (2004b). *Experientialism and its limitations in whitehead's metaphysics of emergence: How the study of complex systems can help*. Paper presented at Society for Chaos Theory in Psychology & Life Sciences 14th Annual International Conference Marquette University, Milwaukee, WI. Retrieved July 15–18, 2004.
Goldstein, J. (2006). Emergence, creative process, and self-transcending constructions. In K. A. Richardson (Ed.), *Managing organizational complexity: Philosophy, theory, and application* (pp. 63–78). Greenwich, CT: Information Age Press.
Goldstein, J. (2007a). Emergence and psychological morphogenesis, In J. Muller, J. Brent, & C. Piers (Eds.), *Self-organizing complexity in psychological systems* (pp. 111–134). New York: Jason Aronson.
Goldstein, J. (2007b). 'Emergence and identity': The past is not what it used to be: An introduction to Mead's emergentism. *Emergence: Complexity and Organization*, *9*, 75–96.
Goldstein, J. (2011a). Probing the nature of complex systems: Parameters, modeling, interventions—part 1. *Emergence: Complexity and Organization, 13*(3), 94–121.
Goldstein, J. (2011b). Emergence in complex systems. In P. Allen, S. Maguire, & B. McKelvey (Eds.), *The Sage handbook on complexity and management* (pp. 65–78). London, UK: Sage Publications.
Goldstein, J. (2014). Reimagining emergence, part 3: Uncomputability, transformation, and self-transcending constructions. *Emergence: Complexity and Organization, 16*(2), 116–176.
Haken, H. (2011). Self-organization. Scholarpedia. Retrieved April 3, 2015 from http://www.scholarpedia.org/article/Self-organization.
Kauffman, S. (1995). *At home in the Universe: The search for the laws of self-organization and complexity*. New York: Oxford University Press.
Keller, E. F. (2008a). Organisms, machines, and thunderstorms: A history of self-organization, part one. *Historical Studies in the Natural Science, 38*(1), 45–75.
Keller, E. F. (2008b). Organisms, machines, and thunderstorms: A history of self-organization, part two. *Historical Studies in the Natural Sciences, 39*(1), 1–31.
Lichtenstein, B. (2014). *Generative emergence. A new discipline of organizational, entrepreneurial, and social innovation*. New York: Oxford University Press.
Melhuish, G. (1973). *The paradoxical nature of reality*. Rochester, NY: St. Vincent's Press.
Priest, G. (2002). *Beyond the limits of thought*. London, UK: Clarendon.
Sainsbury, R. M. (2008). *Paradoxes*. Cambridge, UK: Cambridge University Press.
Simmons, K. (1990). The diagonal argument and the liar. *Journal of Philosophical Logic, 19*, 277–303.
Strawson, G., et al. (2006). *Consciousness and its place in nature: Does physicalism entail panpsychism?* Exeter, UK: Imprint Academic.
Wimsatt, W. C. (1997). Aggregativity: Reductive heuristics for finding emergence. *Philosophy of Science*, *64*, Supplement. Part II: Symposia Papers (Dec., 1997), S372–S384.

Chapter 5
Opening the Wondrous World of the Possible for Education: A Generative Complexity Approach

Ton Jörg

Introduction

Notwithstanding the fact that we live in the so-called *Age of Complexity*, it may be stated that scholars of complexity in the field of education have no clear understanding of the complexity and its potential role in education yet. This state of the art is very much part of a more general state of the art in the science of complexity, of what Helga Nowotny[1] has called "the embarrassment of complexity" (Nowotny, 2013; emphasis added). She explains that this embarrassment of complexity "begins when we realize that old structures are no longer adequate and the new ones are not yet in place" (p. 1). She explains the embarrassment of complexity as follows: "when it dawns on us that the categories we normally use to neatly separate issues or problems *fall far short* of corresponding to the *real* world, with all its non-linear dynamical inter-linkages" (p. 1; emphasis added). Her position on the state of the art seems in agreement with other complexity scholars. Different scholars have noticed that complexity itself is still very much a contested concept. According to Melanie Mitchell "[M]any think the word complexity is not meaningful" (Mitchell, 2011). She also makes mention of the fact that to most complexity scholars there is not yet a science of complexity (see Mitchell, 2011, p. 299). Neither is a general theory of complexity yet available.[2] So, it may be concluded that *understanding* complexity is

[1] The former President of the European Research Council.

[2] See the grant of 2500.000 dollar for Santa Fe Institute, 2015, at http://www.santafe.edu/news/item/JTFgrant-2015-announce-comprehensive-theory-complexity/ to promote the development of a general theory of complexity, after so many years of bright people being active in this institute.

T. Jörg (✉)
University of Utrecht, Utrecht, The Netherlands
e-mail: agdjorg@gmail.com

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_5

still very much a problem in our twenty-first century of complexity.[3] Paul Cilliers links this problem with "a crisis of knowledge" (Cilliers, 1998, p. 121). Part of this crisis is the lack of an adequate language of complexity: that is, a language with an adequate vocabulary. One which is apt "to precisely describe what we're studying" (Mitchell, 2011, p. 301). A vocabulary which would describe the actual complexity of the *real* world (see Nowotny, 2013, p. 1, referred to above). This complexity is still very much a *hidden* complexity. The is the very complexity we cannot see, but which is very much present in this real world.

The analysis above does not offer a very positive description of the general state of the art around complexity in the *Age of Complexity*. This is, however, the state of the art the scholars of complexity in the field of education are active in. Complexity seems too complex to deal with. It may be no surprise therefore that complexity scholars are not yet able to offer a general theory of complexity *for* education, which is of use for those involved *in* education. That is, a theory of complexity about the complexity *of* education. A theory which may be used for *organizing* complexity as the fount of generative change, creativity, and novelty in education.

It may as well be no surprise that the *general* notion of complexity has not changed the field of education in a real sense. The dominant view of complexity is still very much a *restricted* view, being *insufficiently* complex. The complexity scholars in the field of education did not really open the new spaces of possible for the field of education (see Davis, Phelps, & Wells, 2005, with the start of the new journal of *Complicity*; and Osberg, 2009). They were not able to describe the link between complexity and the complexly *generative* nature of learning and development, lest the complex process of generative change (Ball, 2009). They could not link the processes of generative change with the process of generative emergence. They could not link the process of generative change with the concept of "generativity" and the corresponding "Zone of Generativity" (Ball, 2012a, 2012b). They were not able to conceive of the *transitory* nature of the child in its development over time (see Vygotsky, 1987, p. 91). It has remained very much unknown how this transitory nature of the child could be linked to the inherently complex concepts of generative change, generativity, and creativity. It has remained unknown how this transitory nature could be linked to the dynamic *generative* architecture of complexity, as being active *in* the child, thereby acquiring a "full *generative* power" of mastering the topic of concern (Bruner, 1996, p. 119). It has remained unknown how these complex concepts may be linked to the wondrous "world of the *possible*" in the field of learning and education.

Based on the critical analysis above, it may be concluded that the field is very much in need for a more adequate account of the complexity involved in education. This asks for a new general theory of complexity that may be linked to the very complex nature of learning and education, involving processes of complexly *generative* change. The new theory of complexity should therefore focus on the

[3] See "Understanding Complexity," Vienna 2015, at http://www.eventbrite.com/e/understanding-complexity-offering-solutions-to-problems-of-the-21st-century-tickets-14991336491?aff=esli.

very complex nature of processes of learning and development in the field of education. These processes involve processes of *generative* change; processes which are enabling for new ways of thinking *about* the very complexity *of* education. That is, of new thinking in terms of "*generative* complexity" as a scientific concept (Jörg, 2011). From this new thinking in generative complexity, new concepts may be derived for the field of learning and education: complex concepts like "generativity" and "Zone of Generativity," to be linked with complexly *generative* processes of change (see Ball, 2009, 2012a, 2012b). These are the very processes which are so much part of the *real* world. The problem is that we "simply" cannot see the generative nature of complexity of this world. We cannot see how complexity is actually *generated*. We "simply" cannot see that complexity is actually self-potentiating in the real world, which may be considered to be a *fact* (Rescher, 1998, p. 28). The concept of generativity itself may be understood as the general capability of an entity, like the human being, of "knowing how to go on" (Lord, 1994). This capability may be achieved *both* as an individual and as a collective capability in education (see Lord, 1994).

A New Framework of Complexity

To understand the (very) complex concept of generativity, linked to the very generative nature of complexity itself, a new framework of complexity is urgently needed. This implies a fundamental and foundational *reframing* of complexity (cf. Capra, Juarrero, Sotolongo, & Van Uden, 2007). This reframing implies a different view of the world: that is, of the *real* world. The real world may be very different from what we take it to be. This difference is part of what Nowotny (2013) has described as "the *embarrassment* of complexity" (see above). From understanding this embarrassing state of the art in the field of complexity science, as a fundamental failing state of description of the *real* world, it may be derived that we urgently need "an *altered* account of reality" (Kauffman, 2009; emphasis added). Opening the real world implies a kind of reclaiming reality (Bhaskar, 2011; see also Jörg, 2011). This opening will open the *real* world as a complex world of the *possible* (Kauffman, 1993, p. 375). The fundamental challenge for education is to show how this new reality, which is about this complex world of the possible, may be linked to the opening and enlargement of the possible around what it means to educate and be educated (Davis et al., 2005; Jörg, 2009; Osberg, 2009; Sumara & Davis, 1997). This link demands for a new way of thinking in complexity about the complexity of a complex world of the possible *for* education. It demands for a kind of *reframing* complexity itself (Capra et al., 2007): that is, of a *new* framework of complexity. This new framework will be the building stone for a new view and a new foundation of education.

To build a new framework of complexity, about *generative* complexity, it is necessary to link complexity with causality; that is, with a more viable concept of causality (Lincoln & Guba, 1985). One that encompasses the notion of *causal* complexity, in terms of cyclic causality, mutual or reciprocal causality, and emergent causality (see Grotzer, 2012; Jörg, 2011). Tina Grotzer argues convincingly that we need to *learn* causality in a *complex* world. Learning causality in a complex world implies the recognition of causation as a complexly generative process; a process which is thriving on the generative power of (causal) interaction (Bruner, 1996; Jörg, 2011).

Within the new framework of generative complexity, it will be possible to think of a new theory of generative change (Ball, 2009), which involves complexly generative processes (Jörg, 2011), thriving on the generative power of interaction (Bruner, 1996). A theory, which is opening a new way of thinking about complexity and education. It makes it possible to link the very processes of generative change with the concepts of "generative emergence" (Lichtenstein, 2014) and "emergent causality" (Grotzer, 2012), with emergent effects, which may be *nonlinear* effects over time.

The new theory of generative change may be taken as opening the field of complexity and education, as a very much *unexplored* territory of generative complexity. It will be possible to describe processes of learning as processes of *generative* learning: that is of "learning that enhances our capacity to create" (Senge, 1990, p. 14; see also Pellegrino, 1994; Pellegrino & Hilton, 2012; Fiorella & Mayer, 2015). Generative learning may be taken as a fundamental for acquiring so-called "twenty-first century skills," like creative problem solving, critical thinking, etc. (Fiorella & Mayer, 2015, p. 6). The challenge here is to describe and understand the underlying generative dynamics and mechanisms of change. The complexly generative *dynamics* and *mechanisms* involved may have complex nonlinear effects, hitherto very much *unknown*. This is all part of an unexplored territory of complexity in the field of complexity and education. It is through the *new lens* of generative complexity that new spaces of the possible may be opened and/or enlarged: about what it means to educate and be educated (see mission *Complicity*, above). The new lens may ultimately reveal a new world *for* the field of education. All of this may imply the opening of *the wondrous world of the possible* for education. This demands for a new foundation of education.

A New Foundation of Education

> Learning is *more* than the acquisition of the ability to think; it is the acquisition of *many* specialized *abilities* for thinking about a variety of things (Vygotsky, 1978, p. 83; emphasis added).

The aim is to offer a new foundation of education which is based on a new theory of generative complexity, based on new thinking in complexity (Jörg, 2011). This theory starts with a theory of complexly generative change (cf. Ball, 2009). The focus will be on the concept of "generative complexity," a concept which integrates dynamic *interlinkages* with complexly generative *processes*.

The key for a new complexity view of education may be grounded in a new generative theory of interaction, as a theory of generative learning *in* and *through* interaction. The shaping forces being "at work" in the interaction may be taken as thriving on generative processes of generative change. These are operating within the dynamic network of the composite unit of two actors and their particular reciprocal relations. This dynamic unit, involving a cycle from *each* entity to itself (see Jöreskog & Sörbom, 1993, p. 154), may be taken as a so-called "cyclic-helical unity" (Valsiner, 1998), to be linked to a spiral development of both entities involved in the interaction, exerting shaping generative forces on each other. These forces may be modelled as causal generative forces, operating within causal reciprocal relationships, showing potential nonlinear effects of strengthening small changes in the dynamic entities involved: that is, human individuals as learners in their interaction (cf. Vygotsky, 1978).

The causal complexity of the cyclic-helical unity, with the potential of spiral development, may now be linked to the potential nonlinear effects on the entities involved. It may be demonstrated how these effects are actually *generated* in the interaction. This demands for a new understanding of the causal dynamics and the generative dynamics involved in that interaction. This is the very causal dynamics Vygotsky was already aware of, but was not able to model, by lack of an adequate causal framework for modelling causal interaction (see Vygotsky, 1978, p. 62). Vygotsky was also fully aware of the *creative* nature of development (see p. 61), and the role of *transitional psychological systems* (p. 46; emphasis in original).

Based on the new thinking in complexity about the causal dynamics of causal complexity, it will be possible to open and enlarge the spaces of the possible for education. This may bring the description of generative processes of learning closer to the complex concepts of "transition" and "transformation" and their underlying transition mechanisms. These mechanisms may now be linked to the "transitional psychological systems," envisioned by Vygotsky (1978, p. 46). The learner, then, may finally be conceived of as a *transitory* human being (see Vygotsky, 1987, p. 91, on the "transitory child"). The transitional psychological systems, with their underlying *transition mechanisms*, may now be taken as responsible for the *transitory* nature of change in the learner as a complex human being. This is indeed *complexifying* the learner as a subject of study in the field of learning and education.

The above analysis of the very *problem of complexity* for the field of education may demonstrate that the concept of complexity "in use" in the field, falls indeed *far short* of describing the real complexity involved in learning and education. What is urgently needed is a new way of theorizing on complexity.

New Theorizing on Complexity

> How can we begin to help the next generation *build bridges* between their everyday causal reasoning and the forms of complexity that they will *need* to reason in a complex world? (Grotzer, 2012, p. 166; emphasis added).

To develop a more adequate theory of complexity demands a fundamental reframing of complexity, based on a new way of thinking in complexity about complexity (Jörg, 2011). Only then it will be possible to overcome the embarrassment of complexity in the field (Nowotny, 2013). The reframing of complexity needed implies the use of new tools of thinking (Capra et al., 2007). To develop a new theory of generative complexity, as being linked to a process of generative change, is not an easy task. Questions like "[W]hat is the true nature of complexity?" may not find an easy answer within the more regular framework in use. So, the development of a general theory of complexity for a science of education implies the development of a general language of complexity, with an adequate vocabulary, with adequate concepts, terms and metaphors. The challenge, then, is to link this general language to the complex concepts in use in the field of education. We may then better understand the very complexity of fundamental concepts like "generativity," as a complex capability, and the corresponding concept of "Zone of Generativity" (Ball, 2012a, 2012b). We may as well better understand the rather hidden complexity of a so-called "theory of generative change" (Ball, 2009), with the underlying generative mechanisms and dynamics of generative processes of propagating change. This new understanding of the complexity involved may lead to a new understanding of complexity as self-generating, self-sustaining, and self-potentiating (see Rescher, 1998, p. 28). The theory of generative change may then be taken as a self-potentiation theory (ibid., p. 9). This theory may also be taken as a bootstrapping theory about complexity.[4] A theory that can be based on the dynamics of self-generating, self-sustaining, and self-propagating processes, including processes of generative emergence in a self-amplifying loop, with self-enhanced loop effects (see below). This complex bootstrapping theory may be taken as the building stone to explain the process of bootstrapping in children, as f.i. in their process of learning and development of the number concept (Carey, 2009).

On Generative Complexity

> The *real* complexity, operating in the *real* world, is self-generating, self-sustaining, and self-potentiating: that is, a kind of *bootstrapping* complexity (see below).

It should be clear by now that we may understand complexity in a different way. To do so, we have to think different indeed! This new thinking demands for several steps to take. The first step is to understand complexity as *generative* complexity (Jörg, 2011). This new concept may then be linked to complex processes, described in literature:

– Processes of generative change (Ball, 2009).
– Processes of self-perpetuating change (Ball, 2009).
– Processes of self-propagating change (Arthur, 2013).
– Processes of self-amplifying (Phelps & Hase, 2002).

[4] See the general definition at http://www.thefreedictionary.com/bootstrap and as adjective, at http://dictionary.reference.com/browse/bootstrap.

– Processes in a self-amplifying loop (Lichtenstein, 2014).
– Causal loops with self-enhanced loop effects (Hayduk, 1996).
– Processes of self-potentiating (Rescher, 1998).
– Processes of (self-) bootstrapping (Bruner, 1996; Sloman, 2015).
– Deviation-amplifying mechanisms (Maruyama, 1963).
– Multiplier effects, like the so-called "Matthew effect"[5] and "Comenius effect"[6] (Jörg & Akkaoui Hughes, 2013).

The *fundamental challenge* for the *Age of Complexity* is to understand and to integrate all of these processes, with their complex dynamics, mechanisms, and ever-evolving architectures of dynamic interlinkages. To understand the real complexity of the real world, we need to *learn* causality anew in a complex world (see Grotzer, 2012). It may be stated that it is only possible to fully understand the complexity of the *real* world by understanding *causal* complexity. This implies a *new* understanding of causality, as a more viable concept (Lincoln & Guba, 1985). From such new understanding, we may understand that the *real* complexity, operating in the *real* world, may be a rather different kind of complexity, thriving on the causal interaction between the entities involved in that interaction. Complexity, then, may be taken as a kind of (self-) *generative*, self-sustaining, and self-potentiating complexity, thriving on the *generative* power of causal interaction (cf. Bruner, 1996, p. 119; and Jörg, 2011). This new understanding of (real) complexity may be linked and expanded into the concept of "bootstrapping complexity": that is, for understanding the bootstrap processes of learning and development in the field of education.

The new concept of "*generative* complexity" may be taken as a concept that integrates and *unifies* dynamic, generative *structures* with generative *processes*. This is already quite new. In literature, the notion of a generative nature of complexity is not very much used, with some exceptions, like Rescher (1998) and Lichtenstein (2014). Rescher, however, offers a very formal definition of generative complexity, as linked to "the length of the set of instructions that must be given to provide a recipe for *producing* the system at issue" (p. 9; emphasis added). Lichtenstein (2014) offers a more dynamic description of generative complexity as being linked to a system that "gains the capacity to *create* and capitalize on new opportunities" (p. 142; emphasis added). This is opening a new perspective on the role of complexity. Taking a further step, however, generative complexity is also about *dynamic*, nonlinear complexity, involving a generative structure (network) of "nonlinear dynamic interlinkages" (cf. Nowotny, 2013). Taking this structure into account, we may arrive at generative complexity as a self-generative process; that is, a complexly self-generative process; a process, which can be linked to the theory

[5] The Matthew effect describes how entities may bootstrap each other in their interaction, originally described as "the rich get richer and the poor get poorer," from the Bible.

[6] The Comenius effect describes how one can learn most by teaching the other (Gartner et al., 1971); that is, by the returning effects in and through interaction, enabled by increasing the influence on the other one interacts with (Jörg& Akkaoui Hughes, 2013).

of generative change, as proposed by Arnetha Ball (2009). She describes this concept as "a process of self-*perpetuating* change" (p. 48; emphasis added). One step further, this process of generative change may be linked to her use of the concept of "generativity" as a *complex* network concept. This rather unknown concept, in turn, may be linked with the corresponding concepts of "Zone of Generativity" (Ball, 2009; Ball, 2012a, 2012b), and the so-called "Space of Generativity" (Jörg, 2014). The dynamics involved is a kind of complexly generative dynamics, operating within networks of dynamic, generative structures or relationships as "nonlinear dynamic interlinkages" (Nowotny, 2013). As another step forward, with a focus on education, we may link generative complexity with the generative dynamics of acquiring the full *generative* power of mastering a topic of concern (Bruner, 1996, p. 119); that is, of acquiring *individual* and *collective* generativity through interaction within reciprocal relationships (see Lord, 1994). All of this new thinking in complexity is opening new spaces for describing a process "by which social entities come into being" as a process of generative emergence (Lichtenstein, 2014, p. 145). It may be shown how this process can be modelled within a causal framework with a more viable concept of causality, showing the possibility of emergent causality. This is not an easy task. It demands for *learning* causality anew in a complex world.

Learning Causality Anew in a Complex World

> For science to *explain scientifically* means to explain *causally* (Vygotsky, 1997, p. 240; emphasis added).

From the above description of complexity and the new complex concepts as linked to education, it may be derived that to understand generative complexity as a process, a new understanding of complexity as a generative process is needed. The generative nature of this process may be derived from the generative nature of causality, with causation as a generative process with potential nonlinear effects over time. To understand these very complex processes, we first need to learn causality anew in a complex world. This implies a more viable concept of causality that may be linked with complex notions of causality, like

– Cyclic causality,
– Reciprocal or mutual causality, and
– Emergent causality (see Grotzer, 2012).

All of these concepts focus on different aspects and perspectives of the causality involved in causal interaction. One can focus on the interactive nature of causality, on the forces exerted on one another which may be mutual or reciprocal. One can also focus on the fact that the relationship between the entities involved is a reciprocal relationship. The process of causal interaction involved in cyclic causality may show re-entrant, or cyclic effects, which may be nonlinear over time. These effects may lead to emergent effects which may *increase* over time. The nonlinear effects may be ascribed to the generative dynamics and generative mechanisms involved in causal interaction. This description of causality goes way beyond the description of linear causality with its standard linear version of cause and effect, leaving no room for entities causing themselves through interaction with other entities. This description of causal complexity and causation as a generative process demands for a new framework of causality, to become able to understand the very generative nature of causation and of emergent causality, involving complexly generative processes of emergence.

Understanding the Generative Nature of Causation

To understand how generative complexity is actually *generated* in the real world, thriving on interaction, one first needs to understand *causal* complexity. This is the really hard part of new thinking in complexity, about a neglected and unexplored territory of causality.[7] To understand how *generative* complexity actually operates in the real world, it is necessary to understand the generative nature of causation through causal interaction (Blossfeld, 2009; Jörg, 2011). This generative nature may be derived from the *multiplicative* nature of different causal effects of causal interaction. This *multiplicative* nature will be explained below.

For the introduction of causal interaction within reciprocal relationships, see Fig. 5.1. This figure shows the basic notion of causal interaction with β_1 and β_2 as parameters, showing the regular *direct* effects of A and B on each other. A and B are so-called "latent variables" within a so-called "structural causal relationship," which is a reciprocal relationship (see Jöreskog & Sörbom, 1993).

The direct effects exerted on one another may be different in value, showing different *strengths* of shaping causal forces exerted on the other.

The standard calculation of the total effects exerted on each other in causal interaction involves different effects (see Jöreskog & Sörbom, 1993). The two betas are the parameters for the *direct* effects on each other within their causal relationship(s). The entities A and B may also exert effects *on themselves* via the reciprocal causal relationship with the other entity (see Jöreskog & Sörbom, 1993, p. 154). These authors describe a *cycle* as "a causal chain going from the one Eta-variable,

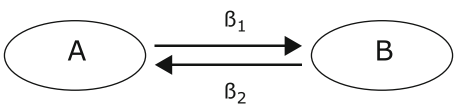

Fig. 5.1 A structural model of causal interaction between latent variables with corresponding parameters of *direct* causal effects

[7] The reader may skip the next part and read the conclusions of this paragraph only.

passing over some other Eta-variable and *returning* to the original variable" (p. 154; emphasis added). The effect of one cycle of A *on itself* is $\beta_1 \times \beta_2$. After two cycles the effect will be ${\beta_1}^2 \times {\beta_2}^2$ or $(\beta_1 \times \beta_2)^2$. After three cycles $(\beta_1 \times \beta_2)^3$ etc. These cyclic effects, taking place over time, are themselves *multiplicative* of nature. The total effect of A on itself will be the sum of the infinite geometric series (ibid., p. 154):

$$\beta_1 \times \beta_2 + (\beta_1 \times \beta_2)^2 + (\beta_1 \times \beta_2)^3 + \ldots\ldots\ (\beta_1 \times \beta_2)^n$$

This sum of infinite cyclic effects may be called the *interaction*-effect or the *loop* effect and is the *same* for A and B.

The geometric series of the loop effect above may be redescribed as

$$(\beta 1 \times \beta 2)/1 - (\beta 1 \times \beta 2).$$

This is the case if the *absolute* value of the product of $\beta_1 \times \beta_2$ is smaller than the value 1 (Jöreskog & Sörbom, 1993, p. 154). This formula represents the sum of all the cyclic effects. This formula shows to be *symmetric* in calculating the different effects on A and B (see below). This symmetric effect, called a "*loop-effect*," may increase over time in a *non*linear way. The loop effect is the key for understanding the complexity of emergent causality with nonlinear effects, as will be shown below. Emergent causality, in turn, is the key for understanding complexity as generated over time, and for complexity as self-potentiating (Rescher, 1998). These foundational concepts for understanding complexity anew will be explained further below. The focus for now is on the different effects involved in causal interaction, like the indirect effect and the total effects on A and B in interaction. These may show the unexpected complexity of causality, thriving on interaction.

The loop effect may be viewed as an *essential* part of the other effects of causal interaction between the two latent variables, like the *indirect* effects and the *total* effects on A and B (see Jöreskog & Sörbom, 1993, p. 154). The descriptive formula of these effects are as follows:

indirect effect **on B** = $\beta_1 \times$ (**loop** effect)
indirect effect **on A** = $\beta_2 \times$ (**loop** effect)
total effect **on B** = direct effect + indirect effect = $\beta_1 + (\beta_1 \times$ **loop** effect)
total effect **on A** = direct effect + indirect effect = $\beta_2 + (\beta_2 \times$ **loop** effect)

Table 5.1 shows the results of the calculations of these *indirect* and *total* effects for the various given values of the direct effects. These results show unusual increases of these effects, which may be understood as *emergent* effects resulting from the increasing loop effect. They show what emergent causality may actually be about in the *real* world, depending on the loop effect of causal interaction.

Table 5.1 shows more unusual effects. For instance, in case the direct effect β_1 of A on B increases, keeping the direct effect β_2 of B on A constant (=0.7), *not only* the indirect and total effect on B increase, but *also* the indirect and total effects on A itself. This is because of the increase in the loop effect, with increasing direct

Table 5.1 Calculation of the different causal effects on B and A

Direct effect β_1 on B	Direct effect β_2 on A	Symmetric loop effect	A-symmetric indirect effect on B	A-symmetric indirect effect on A	Total effect on B	Total effect on A
0.4	0.7	0.28/0.72 = 0.39	0.16	0.27	0.56	0.97
0.5	0.7	0.35/0.65 = 0.54	0.27	0.38	0.77	1.08
0.6	0.7	0.42/0.58 = 0.72	0.43	0.50	1.03	1.20
0.7	0.7	0.49/0.51 = 0.96	0.67	0.67	1.37	1.37
0.8	0.7	0.56/0.44 = 1.27	1.02	0.89	1.82	1.59
0.9	0.7	0.63/0.37 = 1.70	1.53	1.19	2.43	1.89

effects: see Table 5.1. This loop effect increases *non*linearly with linear increases of β_1. This increase shows the loop to be a self-amplifying loop and the corresponding loop effect as a self-enhanced loop effect (Hayduk, 1996). The increase of the direct effect β_1 on B gets *amplified* through the loop effect. Interestingly, the total effect on A is *also* amplified, even although the *direct* effect β_2 on A keeps *constant* (=0.7). So, the total effect of A on A itself increases through the increase of the direct effect β_1 on B! This is the case because of the increase in the loop effect. This unusual effect may be called the "Comenius effect" for education, after Jan Comenius, the Moravian pedagogue (1592–1670). He described a process of learning by teaching as very beneficial for the teacher himself! It may be easily derived that the increases of the total effects on both A and B may increase in nonlinear way if *both* direct effects increase, because of the amplifying loop effect. This effect may be called the "Matthew effect," as described by the disciple Matthew in the bible: "the rich get richer and the poor get poorer" (see also Robert Merton, 1968, who described this effect in science).

All of these effects show the rather unusual *amplifying dynamics* of causal interaction in mutual or reciprocal causality, showing the possibility of *self-amplifying* loops, as a nonlinear phenomenon, based on the operation of direct effects, which may be *linear* themselves. This demonstrates the unexpected possibility of *emergent* causality with emergent, nonlinear increase of cumulative effects over time. The process of amplification takes place in time *without* being dependent on time itself as a variable!

This conclusion about amplification may be linked to the notions of interactive, relational and *emergent* causality, as described by Grotzer (2012). Emergent causality may now be taken as an example of causation as a generative process with amplifying potential. This possibility is opening new spaces for new thinking in complexity, which may be based on the causal complexity, as described above. It becomes possible to introduce new terms and concepts like a "self-amplifying loop" (Lichtenstein, 2014, p. 143), with a corresponding "self-enhanced loop effect" (Hayduk, 1996). The conclusion is that direct *linear* effects may actually be *amplified* by the loop effect, operating as a *multiplier* of the direct effects in the interaction, taking place over time. This amplification shows the possibility of causality as *emergent* causality, thriving on the interaction within the causal loop, with its potential multiplier effect. Causation as a generative process, then, shows the possibility of generative emergence of increasing total causal effects in causal interaction.

Conclusions About Causality

The formula and the calculation of the different effects above, depicted in Table 5.1, show the true nature of the causal effects involved in causal interaction; effects, which may show up as potential nonlinear effects of that interaction. Yet, they are derived from the causal modelling of the different effects of causal interaction, as sketched in literature (see Hayduk, 1987, 1996; Jörg, 2011; Jöreskog & Sörbom, 1993). It may be concluded from these effects that understanding complexity as based on causal complexity is opening new spaces for new theorizing on complexity. The causal complexity shown above demonstrates the possibility of a more viable concept of causality, based on new thinking about causal interaction (see Lincoln & Guba, 1985). This new thinking about causality opens up new spaces for new thinking in complexity about how complexity is actually *generated* in the real world. It is opening for thinking about complexity as generative complexity, to be linked to causation within networks of reciprocal relationships (cf. Barabási, 2003). The process of causation as a generative process may be taken as thriving on the full generative power of interaction (Bruner, 1996, p. 119). This new thinking may also address the possibility of complexity as self-potentiating through causal complexity, operating through self-amplifying causal loops, with self-enhanced loop effects. These loop effects may be taken as responsible for deviation-amplifying processes, with their amplifying dynamics and their underlying deviation amplifying mechanisms (cf. Maruyama, 1963).

The example with the effects of interaction, shown in Table 5.1 above, shows how causal complexity can be self-potentiating indeed: through the loop effect as amplifier of the direct effects of the interactors on each other.

The rather unusual complex notions about causality may now be taken as foundational for the building of a new theory of complexity, which is based on causal complexity, as described above. A theory of complexity which may be used as a foundation for a new theory of education as well. A theory of education which is based on the unexpected role of interaction in education in bringing about nonlinear effects on the partners involved in interaction.

The example above also demonstrates how the dynamics of the parts and the whole are interconnected, constitutive of the unity of the whole, as conceived already by Immanuel Kant:

> The parts of the things combine of themselves into the *unity of a whole* by being reciprocally cause and effect of their form (Kant, in his "Critique of Judgement"; see Taylor, 2001, p. 85).

Understanding the complexity of the causal dynamics of this unity of a whole is essential for new theorizing on education, as shown below (cf. Vygotsky, 1978, on the causal dynamics of interaction). Understanding this complexity, operating within the unity of a whole, is foundational for a new theory of complexity.

Building a New Theory of Complexity

Building a new theory, about generative complexity, may start with the new framework of causality, showing the very possibility of emergent causality and generative emergence of total causal effects in causal interaction. These possibilities may take place within a loop of a reciprocal relationship between two entities in interaction. The dynamics within the loop between two entities may operate as complexly generative processes, *generating* the different effects through the cumulative cyclic effects within the causal loop. These effects may cumulate and be described as the *loop* effect. Each of the entities involved in interaction may have potential *non*linear effects on itself through the loop effect. The loop effect operates as a kind of multiplier in bringing about these nonlinear effects. This loop effect may *generate* indirect and total causal effects as nonlinear effects over time: so-called "multiplier effects." These effects may increase when the values of the direct effects increase. The effects may become greater than 1 when the formula for the loop effect as multiplier gets higher than 1 (see the example in Table 5.1 above). They are foundational for new thinking about causality as a nonlinear concept and about its role in new thinking in complexity, based on causal complexity. It may show how reciprocal and emergent causality may be "at work" in the real world. The entities may bootstrap each other in the causal dynamics of their interaction, thriving on the generative power of the interaction, with its potential nonlinear multiplier effect. The generative mechanisms being "at work" in the causal dynamics may also be called "*bootstrapping* mechanisms" (Carey, 2009, p. 13). This opens the possibility of understanding these bootstrapping mechanisms as complex, causal "learning mechanisms" (ibid., p. 13). These mechanisms may be "at work" in the development of concepts, showing bootstrapping processes with potential nonlinear effects. A development that may be characterized by "conceptual discontinuity" (ibid., p. 20). Such kind of nonlinear development is foundational for processes of transition and transformation, as conceived by Vygotsky (1978). With these processes, it may become possible to conceive of the child as a "transitory child" (Vygotsky, 1987, p. 91). Understanding this kind of discontinuous development may be taken as the key for a new understanding of what education might be about. It is about learning and development as potential nonlinear processes with potential nonlinear effects, thriving on the generative, amplifying power of interaction within relations among peers, operating as self-amplifying loops. These processes may be taken as bootstrapping processes, with bootstrapping dynamics, enabled through bootstrapping mechanisms as *learning* mechanisms, responsible for bootstrapping processes of learning and development, characterized by *discontinuity* of processes and effects over time (cf. Carey, 2004, 2009).

From the new understanding and modelling of generative complexity, it will become possible to arrive at a fundamental scientific *theory of generative change* (see Ball, 2009, 2012a, 2012b; Phelps & Hase, 2002). The generative change may now be described as thriving on the causal dynamics of interaction, with potential nonlinear effects over time. The modelling of this generative change may be based on the causal dynamics of interaction. This modelling shows an unexpected complexity, with complexly generative processes and emergent effects; effects that have remained very much hidden in theories of interaction. It may be shown that the theory of generative change *in* and *through* interaction may be taken as a nonlinear process, with potential nonlinear effects. From this new theorizing we may finally arrive at a new theory, which is about the very *generative nature* of the dynamics and mechanisms involved in the complex causal dynamics of interaction. This theory is not only opening a new world of theorizing about learning and education. It also offers a new lens to view the field of learning and education as a very complex field, with learners as real-world entities in their interaction, showing complex dynamics of generative change, with discontinuities of learning and development.

With the new lens of generative complexity, it will be possible to open the *world of the possible* for the field of learning and education. By opening new spaces of possibility, this world may be taken as a *wondrous* world of the *possible* indeed. The new concept of generative complexity may be taken as opening new ways of theorizing on education as a fundamental complex subject of study.

All of this new theorizing on generative complexity will be supportive for the opening and enlarging of the space of the possible in a world of the possible in the field of learning and education.

New Theorizing on Education

Opening the world of the possible may start with a qualitative description of the complex dynamics of two human beings in interaction. It was the genius of Mary Parker Follett, who has convincingly described the complexity involved in that interaction. In her book about organizations, published in 1924 (!), she describes this interaction in terms of existing entities *creating* each other:

> In **human relations**, this is obvious:
> I *never* react to you but to **you-plus-me**;
> or to be more accurate,
> it is **I-plus-you** reacting to **you-plus-me**.
> 'I' can never *influence* 'you' because you have *already influenced me*;
> that is, in the very process of meeting, by the very process of meeting,
> *we* BOTH *become something* **different**.
> She continues as follows:
> It is I plus **the-interweaving-between-you-and-me** meeting you plus
> **the-interweaving-between-you-and-me**, etc, etc.
> If we were doing it mathematically we would work it out to the *n*th power.
> (Follett, 1924, pp. 62–63; see also Graham, 1995, p. 42).

So, the very process of human interaction, sketched here, is a very complex one: that of human individuals, reciprocally influencing each other, thereby *co-creating each other* over time, with the result of both partners becoming *different*. Both partners are ever-evolving *in* and *through* this interaction as a complex process of dynamic interweaving taking place over time. Both partners may become different in this process. Follett's description of human interaction may be combined with Martin Buber's description of human relations.

Martin Buber describes in his book "I and Thou" the role of "I—Thou" as the *origin*, the grounding notion, constitutive of the world of relations (Buber, 1970/1928, p. 6). Interestingly, he describes the real relation as follows: you "act" on me like I "act" on you (ibid., p. 11). He adds to this that relation implies *reciprocity*: "my Thou acts on me like I act on him" (p. 16). Buber takes the human being as fundamentally related to an *in-born* "Thou" (Other). Interestingly, this makes the reciprocity inborn too! Men/women are *born* to relate and communicate.

Mary Parker Follett was very right in her statement that "response is always to a relation" (see Graham, 1995, p. 42). Interestingly, Buber views the I as a *result* of the web of relationships (1970/1928, p. 29). From this description, we may derive the I and Thou in terms of *weaving each other* within a web of relations (see also Follett, in Graham, 1995, p. 43). More recently, the biologist Steven Rose (1997) described the process of *co-creating* organisms, operative as complex dynamic entities, in terms of (two) organisms which are "both the weaver and the pattern it weaves" (p. 171). The economist Brian Arthur (2013) described a dynamic unit like this as "a set of existing entities *creating* novel entities" (p. 19; emphasis added). Jerome Bruner (1996) described the complex unit of two learners in interaction in terms of "learners *bootstrapping* each other in small communities" (p. 21; emphasis added; cf. Fazio & Gallagher, 2009, p. 10).

All of these above descriptions are opening for a new interpretation of the dynamics of human interaction in terms of *co-creating dynamics*, with a focus on the very *mechanisms* underlying this complex dynamics. This new interpretation may now be linked to the *quantitative* causal modelling of the interaction, described above, which is based on the causal dynamics of interaction. In this interaction, the entities may indeed *change* each other (Illari & Russo, 2014, p. 113). The modelling above showed how these changes may increase over time, through the loop effect, operating as a multiplier effect, generated within self-amplifying loops. Small changes in the direct effects on the other showed to generate a potential large, nonlinear increase in the loop effect, operating as a multiplier effect. All of this new modelling could be related to complex processes of bootstrapping: of self-bootstrapping and mutual bootstrapping in interaction. Bootstrapping mechanisms may now be taken as underlying these bootstrap processes, operating as *learning* mechanisms (see Carey, 2009).

The new modelling of (causal) interaction, sketched above, may now be considered as the missing link in our knowledge about the complexity of interaction between *real-world* entities (Carey, 2009; emphasis added), operating as interactors, shaping each other by the shaping forces of their direct effects on each other.

The above modelling of the process of causal interaction within the unity of a whole makes it possible to understand the very complex learning mechanisms involved in complex processes of learning and development and cognitive functioning, as based on human interaction. This understanding of the complexity involved makes it possible to better understand the theory of Vygotsky about cognitive functioning, which is based on interaction and the causal dynamics involved in this interaction (Vygotsky, 1978, p. 62). It is through interaction, with interpersonal processes that intrapersonal processes of learning and development may take place. He was clear in his intention to develop a *general law of development* for the higher mental functions that could be applied to children's learning processes (Vygotsky, 1978, p. 90). He was also clear that learning is *not* the same as development. In his view, learning is "a necessary and universally aspect of the process of developing culturally *organized*, specifically human, psychological functions" (ibid., p. 90; emphasis added). About this process of learning, he stated the following:

> We propose that an essential feature of learning is that it *creates* the zone of proximal development, that is, the learning *awakens* a variety of internal *developmental* processes that are able to operate *only* when the child is *interacting with people* in his environment and *in cooperation with his peers* (ibid., p. 90; emphasis added).

So, his focus was on the very creation of developmental processes through learning, enabled through interaction with people, with peers. The zone of proximal development (ZPD) here is a *dynamic* zone to be *created* through learning. The changes taking place within this zone must be awakened through learning. For short, the learning *generates* the zone of proximal development as a dynamic zone of change: that is, of generative change. This generative change depends on the interaction with others. It is from the *interpersonal* relations with others that the *intrapersonal* processes of development within the ZPD may take place. In his own words: "[*A*]*n interpersonal process is transformed into an intrapersonal one*" (Vygotsky, 1978, p. 57; emphasis in original). This process describes the transfer inward, which is "linked with changes in the laws governing their activity; they are incorporated into a *new* system with its own laws" (ibid., p. 57). These changes may now be taken as complexly generative changes, involved in development, which can be linked with Vygotsky's description of school learning.

Vygotsky describes his aim of the analysis of development as follows: "to describe the *internal* relations of the intellectual processes *awakened* by school learning" (ibid., p. 91). He described his method as to be focused on "the analysis that *reveals* real, causal or dynamic *relations*" (ibid., p. 65; emphasis added). He was also clear about the difference between description and explanation: "[M]ere description does *not* reveal the actual *causal-dynamic* relations that underlie phenomena" (ibid., p. 62; emphasis added). So, it may be concluded that the focus should be more on how the processes are actually *generated* within these causal-dynamic relations. The processes of development may now be described as complexly generative processes, taking place within these dynamic, causal relations, thriving on the generative power of interaction within the internal relations of the intellectual processes.

From the above modelling of interaction within causal loops, it will now be possible to understand *how* these changes may be facilitated through interaction: by generative processes which may lead to generative emergence within the ZPD.

The possibility of multiplier effects in this interaction may turn the concept of ZPD into a *nonlinear* concept, with potential *generative* change and generative emergence of nonlinear effects. This new kind of theorizing on the nature of the concept of ZPD as a dynamic concept, with processes of complexly generative change, may be linked to the theory of generative change, proposed by Arnetha Ball (2009, 2012). Ball links this theory of generative change with the concept of "generativity" and the concept of "Zone of Generativity" as replacing the ZPD. This may bring an opening to understand the ZPD anew: as a Zone of Generativity, with generativity *generated* in and through the process of interaction with others. The challenge is to link the new concepts with a theory of generative change and the corresponding generative emergence of effects. It may be shown that this theory may enable to *enlarge* the spaces of the possible for education.

A Complex Theory of Generative Change

> Theory of generative change and the *unknown* link with the *emergent* nature of change (Phelps & Hase, 2002, p. 4; emphasis added).

> Emphasizing the importance of *relations* among beings, and of "becomings" as *generated* within these relations, *not* pre-existing them (Fenwick, 2009, p. 116; emphasis added).

From the understanding of causal complexity and the underlying causal mechanism of generative change, sketched above, it will better be possible to understand what a theory of generative change might actually be about. It may be shown how this theory can be linked to Vygotsky's analysis of causal-dynamic relations, and the corresponding causal dynamics, operating in the complex process of interaction. These relations may be related to human relations among human beings, with their processes of becoming, *generated* within these relations (Fenwick, 2009, p. 116; emphasis added). It may also be shown how this theory may be linked to the *emergent* nature of generative change (Phelps & Hase, 2002, p. 4; emphasis added). This theory can also be linked to the new concept of "generative complexity" and the new understanding of Vygotsky's way of thinking about the analysis of (school) learning and development. The theory of generative change may now be understood as a theory of *complexly generative change*, taking place within the dynamics of causal relations. This may involve processes of self-generative change and self-propagating change, showing generative emergence with potential nonlinear emergent effects. The generative nature of these processes may be taken as conditional for the self-amplifying nature of this generative change, demonstrating the *emergent* nature of change. The dynamics of generative change and the generative mechanisms involved may now also be understood as deviation-*amplifying* mechanisms (Maruyama, 1963), amplifying changes in the direct effects of interactors on each other (see Table 5.1, above). The generative dynamics of such complexly generative change may therefore be understood as a complex process of *self-potentiating*. This is what generative complexity is *actually* about in the *real* world, to be considered as a very *complex* world indeed, including the complex causal dynamics of generative change. So, it may become clear by now that to comprehend the *fact* that complexity is *really* self-potentiating (Rescher, 1998), one needs to understand the very nature of generative complexity and the emergent nature of (self-) generative change.

Generative complexity can now be understood as an *emergent* process, linked to *emergent* causality, with potential nonlinear effects over time, thriving on the causal dynamics of interaction. From such understanding of generative complexity, it will be possible to make a link with learning as an emergent, generative process. It will be possible to make a link with the concept of "generativity" and the corresponding concept of a "Zone of Generativity." The link may be expanded to the concept of a multidimensional "Space of Generativity," as proposed by (Jörg, 2014). All of these concepts may now be taken as complex, potential nonlinear concepts. These concepts can be linked to the nonlinear processes of self-amplifying and deviation-amplifying within loops, as manifested in processes of bootstrapping each other (Bruner, 1996). These are processes of generative change, thriving on the generative power of interaction through shaping forces exerted on each other in interaction. It is through this generative power that the generative power of (generative) complexity as self-potentiating may be understood (see Jörg, 2011). The challenge now is how to link this generative power of interaction with a new approach in education. Not about education-as-we-know-it but about education as to be *organized* from a different perspective: a complexity perspective that goes beyond "the very institutionalization of schooling" (Bruner, 1996, p. 21). This complexity perspective encompasses the concept of generative complexity, strongly based on a more viable concept of causality, showing emergent causality, with potential nonlinear effects of strengthening of direct effects within causal loops, as sketched above. This new perspective opens a different approach of learning and development, opening new spaces of possibility *for* education. This approach may be called a *possibility*-oriented approach.

A Possibility-Oriented Approach

Based on the new *understanding* of complexity as self-potentiating, it is possible to make a link with *organizing* complexity as the fount of learning as a complex, emergent process, based on complexly generative change. This process of learning may be called "generative learning," to be taken as a complexly generative process, showing the potential of transition and transformation (Vygotsky, 1978). Generative learning is linked to learners, thriving on the full generative power of their interaction within their causal-dynamic relations (Vygotsky, 1978, p. 62). Generative learning, then, may be linked to the corresponding processes of generative change and generative emergence through multiplier effects. These effects are enabling for emergent effects over time, which can be nonlinear. It is through these processes that "learners may bootstrap each other within a subcommunity" (cf. Bruner, 1996, p. 21). It is through these generative processes of change and development that learners may acquire their generativity as a real, complex capability. They may acquire this capability within the relations of a subcommunity: both as *individual* and as *collective* generativity (see Lord, 1994; emphasis mine). So, the concept of generative learning shows to be a complex concept linked with the complex concept of generativity. Both concepts are thriving on the generative power of (generative) complexity as self-potentiating. These concepts are opening new spaces of possibility. They may be taken as the complex building stones for a new, possibility-oriented approach in education. They may open the world of the *possible* for education, showing this world to be a wondrous world of the possible, with learning as an emergent, self-generative, self-amplifying, self-potentiating process, with potential nonlinear effects over time. Susan Carey (2009) describes the complex nature of the transitional process of development of the number concept in children as thriving on bootstrap *processes* and their underlying bootstrap *mechanisms*, showing the possibility of a sudden emergent change in this development over time. It is through such *emergent* change that the very acquisition of the number concept may be acquired by children around the age of three (see Carey, 2004, 2009).

Opening the World of the Possible

With the new complex concepts, developed above, we may offer an *alternative* account of what education might be about. The theory of generative change, sketched above, is foundational for reframing the complexity involved in education. The new lens of generative complexity may teach us to "see" education from a new perspective: as complexly generated processes of generative learning, encompassing emergent processes of generative change, generated within relations among learners. These processes take place within a relational unit, with relational complexity, thriving on the generative power of interaction within this unit. To show how education might be like in the real world, it is urgently needed to describe it in a way that facilitates understanding of the generative dynamics and mechanism involved (cf. Illary & Russo, 2014, p. 123). Based on such understanding, professionals may *organize* education differently. It may then be possible to organize complexity as the fount of new learning, of generative learning, based on a theory about the processes of complexly generative change. Inspired by the mantra of Padgett & Powell (2012, p. 2), it will be possible to conceive of education as *organizing* the complexity involved in building a dynamic unit of lasting relations among learners. Following their mantra, learners *create/acquire* relations in the short run; in the long run, these relations *create* actors. The dynamics involved may be the generative dynamics of self-generative, self-amplifying and self-potentiating processes. The unit of the dynamic relation may be taken as a subcommunity of "learners bootstrapping each other" over time (Bruner, 1996, p. 21).

From the complex theory of generative change, sketched above, we may derive a new language of complexity and a new vocabulary. This language is about existing entities *creating novelty* through generative change within loop-like networks in many dimensions. The novelty created through organizing complexity may ultimately lead to the acquisition of generativity for both learners in interaction, to be taken as a complex capability, and as a kind of emergent effects. All of this *organized* complexity may ultimately lead to transformation and transition taking place within the so-called "transitory child" (Vygotsky, 1978, 1987). This makes it possible to *re*-describe the description of education in terms of complex, emergent processes of generative learning and the acquisition of individual and collective generativity (see Lord, 1994). These are the new complex skills needed for the twenty-first century, which has become known as the *Century of Complexity* (Barabási, 2003; Hawking, 2000). This offers a description of education which is opening and enlarging new spaces of the possible around what it means to educate and be educated for the field of education.

The new challenge for the field of education is to open the wondrous world of the possible, with its unexplored territory of generative change and generative complexity within complex, multidimensional loop-like configurations. These configurations may possibly function as *bootstrapping* configurations, thriving on interaction. They may open a new landscape of the possible for education, with a new topology of the possible in an unexplored world of the possible. It may show an unexplored science domain in the field of education, which is about the generation of complexity as the source of novelty and complexly generative change, with the potential of transition and transformation (cf. Müller & Riegler, 2014). For sure, these complex notions about the real world as a nonlinear world demand for a new way of *thinking* and a new way of *seeing* the real world as a really complex world, in which complexity can be (self-) generated and self-potentiating.

All of this new thinking in complexity is opening a new science domain indeed (Müller & Riegler, 2014): the domain of the complexity of (generative) complexity. This domain may be taken as a domain "whose potential has not been sufficiently recognized and has been *insufficiently* explored so far" (Muller & Riegler, 2014, p. 11). A new domain that *brings complexity to life* for the domain of complexity and education. The new possibilities of the wondrous world of the possible for education are not *given*. Definitely *not*. They have to be *organized* that way. To open the wondrous world of the possible for education, one needs to bring complexity to life by showing how complexity can actually be *generated* in the field of education. This demands for a new theory of education which describes and explains the role of generative complexity in organizing education.

A New Theory of Education

> Generativity is the dynamic element of individualization. Education is a continuation of *procreation*, and often a kind of supplementary *beautification* of it (Sassone, 2002, p. 48; emphasis added).
>
> Most important is the notion that *one makes oneself through one's generativity* (ibid., p. 48; emphasis added).

Opening the wondrous world of the possible *for* education may now be taken as the opening and enlarging of the space of the possible around what it means to educate and be educated[8]. Based on the introduction of the concept of generative complexity, it becomes possible to link this concept to the complex concepts of "*generative* learning" and "*emergent* learning." These concepts may be based on complexly generative change as a form of *self-perpetuating change* (Ball, 2009, p. 48; emphasis added).

A new framework of complexity has been developed above, which demonstrates that understanding generative complexity is based on the understanding of the causal complexity of a dynamic unit of two entities or partners in their relation; that is, on the understanding of the underlying "complex causal dynamics" (Grotzer 2012, p. 173; and Vygotsky, 1978, p. 62), and of the complex mechanisms involved in this causal dynamics. These mechanisms may be taken as *generative* mechanisms of learning, with the power to operate as *bootstrapping* mechanisms (Bruner, 1996; Carey, 2009). All of these dynamics and mechanisms are thriving on the generative power of interaction. It is this generative power, developed within the reciprocal causal relationships, to be described as a loop, which is fundamental and foundational for generative learning and thinking as a process of complexly generative change. This generative change may be understood as an emergent process of emergent learning and thinking, with unexpected, complex, emergent outcomes, which can be nonlinear over time.

From the new framework of generative complexity, it may be understood that the theory of generative change, as proposed by Ball (2009), is a theory of *complexly* generative change. This theory can be linked with the "*emergent* nature of change" (Phelps & Hase, 2002; emphasis added). This complex theory of change may now be linked with the role of generative complexity in loop-like networks, with causal loops as dynamic interlinkages between the entities involved. So, the understanding of generative complexity is based on the understanding of the complex causal dynamics involved (Grotzer, 2012, p. 173). The causalities involved are about "extended and nonlinear causalities" (ibid., p. 173). From these nonlinear causalities we may understand generative complexity as a kind of self-generative, self-perpetuating, self-potentiating complexity, being operative within the causal dynamics of interaction within a relation or subcommunity of two learners. The understanding of a new framework of generative complexity is based on an understanding of the causal complexity involved in the generation of complexity within a dynamic relation of two learners as the smallest unit of a network. It will be possible to link the process of complexly generative change with processes of "bootstrapping each other" in small communities of learners (Bruner, 1996; Fazio & Gallagher, 2010). This is an example of existing entities *creating* each other, triggering novelty of these entities as emergent effects over time (cf. Arthur, 2013, p. 19). These novelties of entities may be taken as processes of

[8] Cf. the original mission of the International Journal of Complicity on Complexity and Education, above.

spiral development, enabling for phenomena of transition and transformation of the learning child, as described by Vygotsky (1978). These complex processes may be taken as foundational for the complex notion of "the transitory child" (Vygotsky, 1987, p. 91). The processes of complexly generative change may lead to the learning and development of the learner within his/her Zone of Proximal Development (ZPD). This demonstrates the complex *unity* of learning processes and developmental processes. It may then be understood that this ZPD is indeed a created or *generated* zone of development, generated by the child in and through interacting with people, i. c. in cooperation with his peers (Vygotsky, 1978, p. 90).

Linking *generative* learning with *emergent* learning may now be linked to the complex concepts of "*generativity*," the "Zone of Generativity" (Ball, 2009, 2012), and to the even more complex multidimensional concept of "Space of Generativity" (Jörg, 2014). Generativity is a very complex concept, not very much used in the field of education. Leslie Sassone views generativity as "the *cause and effect* of individualization" (Sassone, 2002, p. 42; emphasis added). William Wimsatt takes generativity from a different perspective in the field of biology, as "an extremely efficient way of *building complex adaptive structures*, while at the same time *locking in its own generators*" (Wimsatt, 1997, p. 137; emphasis added). Wimsatt describes these features as "*two sides of the same coin, their association is a deep fact of nature*" (ibid., p. 137; italics in original). So, generativity may be understood as closely linked to the very *fact* that complexity is *self-potentiating* (see Rescher, 1998, p. 28). Thinking about complex structures of self-amplifying loops and their multiplier effects may lead to this new concept of complexity as self-potentiating.

The complex concepts of generative complexity and bootstrapping complexity, as conceived above, may now be incorporated into a theory of education, with a new perspective on learners as "self-organizing beings" (Kant, in Mainzer, 2004, p. 97). The underlying processes may be taken as enabled and triggered by complexly *generative* processes of becoming. These processes may turn into bootstrapping processes: of self-bootstrapping and mutual bootstrapping. These processes may now be understood as fundamental complex *causal* processes. According to Kauffman (2013), "we live in both *a web of cause and effect* and a *web of enabling opportunities* that enable new *possible* directions of *becoming*" (p. 181; emphasis added). It may be understood from this and from the causal model of interaction, sketched above, that generativity, like fitness in evolution (see Wimsatt & Schank, 2002), is a *relational* property, to be acquired in and through interaction within relations with others. The concept of generativity may also be taken as a *web-like* property, with many dimensions.

The new thinking about the causal dynamics of a generative process of becoming may be linked to our methodological view of complexly (self-) generative change with emergent effects, thriving on the generative power of (causal) interaction. From a perspective of cognition, generativity may be taken as a complex capability, integrating knowing with acting in a complex process of enacting: of "knowing how to go on" (Lord, 1994). Generativity is a complex capability that may be viewed as a capability to be *created* in education (Nussbaum, 2011). In her book *Creating Capabilities*,[9] Martha Nussbaum views the complexity of capabilities as "*combined capabilities*" (pp. 20–21; italics in original). This is fully in line with Vygotsky's way of thinking about learning: "[L]earning is *more* than the acquisition of the ability to think; it is the acquisition of *many* specialized abilities for thinking about a variety of things" (Vygotsky, 1978, p. 83; emphasis added). Interestingly, Nussbaum links these combined capabilities with fluid and dynamic *internal capabilities*, as states of the person (see p. 21; italics in original). All of this new thinking about complex capabilities and states of a person may be taken as foundational for a complex theory of education.

Generativity may now be viewed as a *norm* for education *without being normative*, in the traditional sense. It may be taken as a norm in terms of opening a new concept of learning and education. Developing generativity as a complex capability and state of the person may now be viewed in terms of "the *power of constituting* the individual *directed by that individual* to becoming a being-*in*-and-*for*-self" (Sassone, 2002, p. 43; emphasis added). Sassone is clear that this is "a life's work" (p. 44). This is a description of a *self-realizing* being (see Kant, above). The complex *states of being* of learners as a human individual being may be triggered through complexly generative processes of becoming, enabling the full *generative power* of learners acquiring individual and collective generativity (cf. Bruner, 1996; Lord, 1994). Consequently, the theory of education may be more focused on "the achievement of individual and collective generativity" (Lord, 1994). The rather complex goal of *co-creating* each other over time, in terms of "existing entities *creating* novel entities" (Arthur, 2013, p. 19), may therefore be taken as the promise of complexity for the *future* of education. It shows education as learners *co-creating themselves*, perpetually creating new possibilities within new spaces of the possible. This co-creating of learners may take place through processes of mutual bootstrapping, showing how learners may *bootstrap* each other in sub-communities (see Bruner, 1996).

The world of education should be taken as a complex world, showing the unlimited enlargement of possibilities (cf. Maturana, 1980). This description is opening for a new, more complex *understanding* of education, with a description of education as a fundamental nonlinear phenomenon of complexly generative change. It is about education of learners *bootstrapping each other*, with potential nonlinear effects of generativity as a complex capability linked to the complex multidimensional Spaces of Generativity. They *create* their own Space of Generativity as a state space of complex being, with unlimited, self-generated possibilities through processes of co-creating in interaction.

Understanding education, then, is not only about *understanding* how the real world works: as a complex causal world, with causally generative complexity. It is also about a new way of *organizing* education, to be *based* on such complex understanding of how complexity may "work" in the real world; that is, *if well*

[9] Which has as subtitle *The Human Development Approach.*

organized. Education, then, may be organized as facilitating *lasting* relations among learners. First, they may create these relations as reciprocal relations. These are enabling for the development of a similar way of thinking and a converging of the language they speak in their communication. This new take of complexity involved in organizing education may then be taken as the *fount* of creativity and generativity, as a complex *creative* capability, complexly emerging through processes of generative and emergent learning. All of these complex processes are enabled by generative relationships operating as generative scaffolds for the potential *unlimited enlargement* of cognitive functioning (Maturana, 1980).

From the new understanding of organizing education, in terms of organizing complexly generative change, we may develop a new *generative pedagogy*. This is a pedagogy that aims at a new way of organizing education for enabling *generative* learning as a form of *emergent* learning, with potential nonlinear emergent effects (see Phelps & Hase, 2002; cf. Senge, 1990; Senge, Cambron-McCabe, Smith, Dutton, & Kleiner, 2000; Senge, Scharmer, Jaworski, & Flowers, 2004).

Ultimately the world of education, with a generative pedagogy about complexly generative processes of change, may reveal a new world *for* education. To open this complex world of education, we need not only learn how to "see" but also how to *learn* how generative complexity may actually operate as a self-generative, self-potentiating process in this complex world; a process which is enabling for self-realizing human beings, achieving their individual and collective generativity as co-created, co-generated states of being. This opening of the world of education may show what the real world of education may be about: about *the enlarging of the space of the possible* around what it means to educate and be educated. These spaces may be linked with complex *states* of being of persons as learners in their complex development: linked to fluid and dynamic *internal capabilities* (Nussbaum, 2011, p. 21; emphasis in original). This is in essence what a transitory human person might be about as learner in the field of education, showing the very possibility of transition and transformation (cf. Vygotsky, 1978, 1987).

Organizing education will then be about organizing complexity in terms of learners *creating* relationships and relationships. creating learners through processes of complexly generative change with emergent potential nonlinear effects. The focus is on acquiring individual and collective generativity within the complex Space of Generativity, involving combinations of complex capabilities (cf. Lord, 1994; and Nussbaum, 2011, p. x). Nussbaum sketches the complexity of capabilities in terms of "*combined capabilities*" (p. 21; emphasis in original), which she links with "substantial freedoms" of the individual to operate in the real world. She conceives of these freedoms not only as individual freedoms but also as "the freedoms or opportunities *created* by a combination of personal abilities and the political, social, and economic environment" (ibid., p. 20; emphasis added). These are the very freedoms for persons and their state of being, which may be *created* in the Space of Generativity: as a complex state of being, dynamic and ever-evolving over time. These are the freedoms to be achieved in education: as individual and collective generativity (Lord, 1994), *both* by teachers *and* by learners.

The new challenge for education is to *organize* education as a complexly generative system in which novelties are generated through complexly generative change within dynamic interlinkages between learners. These learners may then operate as existing entities, creating novelties within new spaces of the possible. These novelties may include learners bootstrapping each other through bootstrapping processes, complexly emerging within so-called "bootstrapping configurations" (cf. Fazio & Gallagher, 2009). These *interactive* configurations may be taken as "networks of scaffolding relations" (Wimsatt, 2014, p. 90). Wimsatt describes scaffolding as follows:

> [*S*]*caffolding* refers here to "structure-like dynamical interactions with performing individuals that are means through which other structures or competencies are constructed or acquired by individuals or organizations. Material or ideational entities that accomplish this are *scaffolds*." (Wimsatt, 2014, p. 81; emphasis in original; cf. Rossi, Russo, Sardo, & Whitford, 2010)

It may be understood that "all kinds of scaffolding are *relational*" (Caporael, Griesemer, & Wimsatt, 2014, p. 14; emphasis added). This scaffolding may easily be linked to the Vygotskian concept of "zones of proximal development" as target zones (ibid., pp. 14–15). The scaffolds described here may even turn into so-called "*transitional* scaffolds in Vygotsky's sense" (B. Wimsatt, 2014, p. 385; emphasis added). They may be taken as targeting to the acquisition and enlargement of generativity by agents, showing the acquisition of new complex capacities within the Space of Generativity of each agent in what Caporael et al. (2014) describe as "scaffolding *interactions*" (p. 15). The complex processes of cognition, involved in acquiring individual and collective generativity are "*generative* processes" (B. Wimsatt, 2014, p. 345; emphasis in original). She makes a link with "the mechanisms described by Vygotsky" (p. 345). The complexly generative processes and configurations may be part of a generative architecture of nonlinear dynamic interlinkages: of so-called "generative bootstrapping configurations" (see also Wimsatt, 2014, p. 77). It is within such configurations that complexly generative processes of "self-bootstrapping" may become possible in practice, also for education (ibid., p. 101; cf. Carey, 2009, on "bootstrapping" processes in cognitive development; and Sloman, 2015). These processes may actually *generate* "explosive growth" (Wimsatt, 2014, p. 102; emphasis added).

The theory of complexly generative change presented above may reveal the dynamics at play in generative processes of learning, growth and development. All of this may articulate the changing complexities and the dynamics of *generative* emergence involved in education (cf. Fenwick, 2012). Once you fully understand the changing complexities involved in education, educators may be able to *organize* these complexities for the sake of enlarging the spaces of the possible: showing the *unlimited enlargement* of cognitive functioning (cf. Maturana, 1980). This is the *key message* of the new theory of complexity and education presented here.

Reflecting on the state of the art in education, educators and those theorizing on education may now recognize that the dominant view of education is strongly *undertheorised* indeed (cf. Fenwick, 2012, p. 142; emphasis added). This is also very much the case for theorizing on the topic of complexity and education. The dominant view is ignoring the deep ignorance on the very generative nature of complexity involved in the processes of generative change, to be taken as self-propagating change (Ball, 2009): that is, of generative learning and thinking, of (explosive) growth and development into generativity as a complex, fluid and dynamic internal capability, of "knowing how to go on" (Lord, 1994; cf. Nussbaum, 2011, p. 21).

The new theory of education is foundational for building a new language of complexity, as a tool for an altered account of education as a complex topic of study, showing unexpected complexities involved in learning and development. With this tool it may better be possible to describe what we're studying in the field of education, with a clear focus on the very complex, transitory nature of the child, showing the complex possibility of transition and transformation, even of metamorphosis (Vygotsky, 1978, 1987). This transitory nature may be based on the complexly generative nature of change thriving on the generative power of interaction; a power which is based on the power of self-potentiating as resulting from emergent causality operating in interaction within a dynamic architecture of scaffolding relations. It is this generative power which is part of a *generative approach* of education (see Jörg, 2011). For sure, this is a possibility-oriented approach, opening new spaces of the possible for education.

Conclusions

The new theory of education, presented above, is a complex theory about the complexity involved in education, showing the *potential* of complexity *for* the field of education. This complexity shows to be a still very much *unexplored* territory for education. It is a territory about complexity operating as *generative* complexity, and about complexity as *self-potentiating*. Understanding these complexities is based on understanding *causal* complexity, operating within causal (reciprocal) relations, thriving on the complex generative power of (causal) interaction. Understanding causal complexity is about understanding the *causal dynamics* of interaction as a necessary condition for understanding transition and transformation within and through *transitional* psychological systems (Vygotsky, 1978). Modelling causal interaction is the key for understanding how complex interaction may enable for complexly generative processes with self-amplifying loop effects. This modelling shows the unlimited possibilities of strengthening effects on the entities involved in interaction. These possibilities include the complex possibilities of mutual bootstrapping and of self-bootstrapping of learners, taking place *in* and *through* interaction. These new possibilities may now be taken as foundational for the *unlimited* enlargement of cognitive functioning of these learners (Maturana & Varela, 1980).

The new theory, proposed here, shows the role of generative complexity in education in complexly generative processes in interaction within relations among peers. These generative processes are not given but need to be *organized* and facilitated by educators, by organizing the conditions for triggering these generative processes of change. Based on the causal modelling of (causal) interaction, these processes may be viewed as a kind of (self-) generative, self-amplifying, self-propagating, self-potentiating *processes*, operating within self-amplifying loops, with self-enhanced loop effects. They show “the highly *complex* dynamic relations between developmental and learning processes” (Vygotsky, 1978, p. 91; emphasis added). These processes and effects are thriving on interaction; that is, on the generative power of interaction with other people and/or peers. The new theory of education is very much in line with the view of Vygotsky, in his description of the role of learning and development in education. His view was based on a method that took the causal dynamics involved in the process of interaction very seriously. He was not able, however, to offer an account of the causal complexity involved in the interaction. So, he was not able to conceive of reciprocal causality and emergent causality, neither of causation as a complexly generative process, with potential nonlinear effects over time. *Organizing* these generative processes are the ultimate aim of a new generative pedagogy. They may be linked to *generative* learning, with the potential of *emergent* learning and the corresponding potential nonlinear effects. This is what the (generative) complexity of education might actually be about: the opening of new spaces of the possible *for* education.

The complexly generative processes involved in interaction may enable the complex capability of generativity, described above as a capability of “knowing how to go on” (Lord, 1994). This capability *integrates* the process of knowing with doing. This complex capability may now be taken as a complex aim of education. The achievement of individual and collective generativity may be facilitated in and through interaction with other people and/or peers (Vygotsky, 1978). This complex process can be linked with acquiring individualization through generative processes of *becoming* a human individual (Sassone, 2002). The concept of generativity may be linked to the human development approach, described by Martha Nussbaum (2011), encompassing the complexity of capabilities as *combined capabilities* (p. 21; emphasis in original). She describes capabilities in terms of to do and to be (p. 20), linked to states of being of the person. Nussbaum describes these states as fluid and dynamic *internal capabilities* (p. 21; emphasis in original).

The complex concept of generativity may now be linked to complexity as a (self-) generative and self-potentiating processes, taking place within a complex dynamic network of “nonlinear dynamic interlinkages” (Nowotny, 2013). A dynamic network that may show potential nonlinear emergent effects over time. The acquisition of individual and collective generativity takes place through a complex process of generative change within such a complex network of dynamic *relations* between learners. Education, then, may facilitate learners who operate as actors *creating* relations, which, in turn, *create* the actors through processes of complexly generative change, at both the individual and collective level, with generativity as emergent effect at both levels. This is opening and enlarging the space of the possible for the learners involved, showing their generativity as a complex capability within the multidimensional Space of Generativity of *each* learner. With generativity as the cause *and* effect of individualization (see Sassone, 2002). Generativity, then, is involved in the emerging process of this complexly generative process of individualization of these learners. It may be stated that learners "*make oneself* through one's generativity" (Sassone, 1996, p. 48). Learners may be conceived as *self-realizing* human beings operating within their dynamic networks of scaffolding relations and interactions, achieving their individual and collective generativity through complex processes of dynamic interweaving. Learners, then, may become able to *bootstrap* each other in these dynamic networks, to be taken as networks of communities of learners, thriving on the full generative power of interaction within the relations with other people and/or peers (Vygotsky, 1978). It is their generativity that opens and enlarges spaces of possibility: with the potential of *unlimited* enlargement of cognitive and other ways of complex functioning (cf. Maturana, 1980). All of this theorizing about learning and education has implications for practitioners in the field of education.

Educators may now be viewed in a rather new role: a role of so-called "multipliers" (Wiseman, 2010). In this role they may show the capacity "to extract all of the capability from people" (p. 11). This implies the fluid and dynamic internal capabilities of the learner as a person which comprise the complex state of *being* of a person, knowing what to do, "knowing how to go on" (Lord, 1994), knowing what choices to make and how to act (see Nussbaum, 2011, p. 20). Educators may fulfill their new role by *organizing* complexity within networks of learners, by organizing their relations and facilitating their *cooperation* with peers (Vygotsky, 1978). They may view the very possibility of so-called "*multiplier effects*" (Wiseman, 2010, p. 11), operating in interaction within relations. These dynamic networks may show how learners may *bootstrap each other into existence*: that is, into self-generative, *self-realizing* beings through complexly generative processes of *becoming* (cf. Sassone, 2002). The new role of educators may be based on a generative pedagogy, involving the organization of relations and the facilitation of the quality of interaction within these relations. This pedagogy is based on a theory of complexly generative change. The generative pedagogy can be linked to the bootstrapping mechanisms as *learning* mechanisms in cognitive development (Carey, 2009). These hitherto unexplored mechanisms may now be taken as conditional for transition and transformation in this development.

The challenge of a new generative pedagogy, with the new possibilities of self-realizing human beings through bootstrapping processes, goes way beyond what (Fenwick 2009) describes as "[T]he determined pedagogical impulse to control, to change, to rehabilitate." The new pedagogy is opening new spaces of the possible, with emergent effects of generativity within hitherto unknown Spaces of Generativity. Opening these complex, multidimensional spaces replaces the rather simplistic view of the *Zone* of Proximal Development. The focus, however, is very much the same: the very *creation* of these complex spaces in the process of learning and development (see Vygotsky, 1978). The new focus, then, is on individuals *creating* relations which, in turn, *create* individuals by response to the other: "as an awakening—both to the uniqueness of *me*, and to the relation and subjection to the *other* in which I am already, and have always been, constituted" (Fenwick, 2009,

p. 114; emphasis added). Fenwick is right that such a response "is opening to others." She describes this opening correctly as a *risk*: "[S]uch opening, such vulnerability, is a risk" (Fenwick, 2009, p. 114; emphasis added). She adds to this that *despite* this risk "we and others may *emerge* in educational relationships" (ibid., p. 117). It may be argued that it is *worth* to take this risk indeed, so to become able to open the spaces of the possible in a world of the possible (cf. Biesta, 2014, on the beautiful risk of education). The challenge now is to link this new thinking in complexity to the real world of education, taking account of the "causal connections between properties of *real-world* entities" (Carey, 2009, p. 8). These connections and properties may be taken as the causal relations between the learners operating as complex real-world entities, with their complex internal capabilities as complex properties. The complex dynamics involved is the causal dynamics of interaction operating within these connections (see Vygotsky, 1978, p. 62). The causal complexity described above showed the very possibility of complexity as generative and self-potentiating, enabled by the hitherto unexplored possibility of self-amplifying loops and the generative power of interaction, operating within these loops. This new way of thinking in complexity *about* education may reveal a new world *for* education, based on *new thinking in complexity* about the very complexity *of* education as a process.

The complex theory of education, proposed here, turns the description of education into "a 'complexified' educational vision" (Fenwick, 2009, p. 112): a vision about education that is "alive, ever-changing, organic, and full of messy vitality" (cf. Arthur, 2013, p. 19, on economics). This new description may show the very promise of complexity *for* education: that is, for "the enlargement of the space of the possible around what it means to educate and be educated" (see Davis et al., 2005; Jörg, 2009; Osberg, 2009; Sumara & Davis, 1997).

For educators the real challenge is to *organize* the complexity of education in terms of facilitating and organizing the very *conditions* of generative change and emergence for learners, so to enable the new possibilities, sketched above. The focus is on generative, emergent learning, based on bootstrapping processes, with their bootstrapping mechanisms operating as learning mechanisms (Carey, 2009), all for the sake of bringing about the complex capability of generativity. It is through the very *creation* of the Space of Generativity that is enabling for generativity as a complex capability to manifest itself within this complex multidimensional space of development. The focus here is clear, in that the learner "makes oneself through one's generativity" (Sassone, 2002, p. 48). This in line with Vygotsky's view of learning and development. Learners, then, may be viewed as *active* learners, able "to *organize* their own behavior" (Vygotsky, 1978, p. 74; emphasis added). They may be conceived as able to apply their own method, "*invented*" by them (ibid., p. 74; emphasis added). This may show what pedagogics may actually be about, according to Vygotsky (1997): "the *creation* of life in its infinite diversity" (p. 348; emphasis added). The complex capability of generativity, and the achievement of this complex capability, is the key for a *new* understanding of "the individual as a whole" (ibid., p. 348): that is, as a *complex* dynamic whole. The process of creation of the individual as a whole may happen *in* and *through* interaction within (lasting) relationships among learners, and with people in their environment. The quality of relationships may increase the quality of interaction, and vice versa. Both may increase the quality of the (emergent) effects of generative, emergent learning, showing the potential nonlinear effects for achieving growth and development within the complex Space of Generativity. Complexity as a generative, self-potentiating process, with its generative mechanism of change, may operate as the complex generative motor for this achievement. All of this exemplifies the very *quality* of dynamic, generative complexity as a self-generative, self-potentiating *process*, showing generative emergence of effects; a process that may be self-strengthening, self-amplifying, and self-propagating over time. All of these *new* complex possibilities may be related to the simple question, posed by Nussbaum: "what are people able to *do* and to *be*?" (ibid., p. x; emphasis added). The new possibilities may show the potential of *beautification* of education: as a continuation of procreation (see Sassone, 2002, p. 48). Ultimately, the new theory of education may demonstrate how complexifying the topic of education may actually *humanize* education and the view of the subject of education as a self-generative, ever-evolving, self-realizing human being, thriving on interaction with other people: "with people in his environment and in cooperation with his peers" (Vygotsky, 1978, p. 90). This view brings complexity to life and the focus of education on the *creation* of life, with its infinite possibilities. The pedagogy needed is a generative pedagogy, about increasing the capacities to *enhance* one's own creativity by the increase of one's generativity: through individual *and* collective activity (see Vygotsky, 1978, p. 88). This pedagogy goes way beyond the deficit mode of thinking, stressing what learners still do *not* know or are not able to do. Education, then, will be about human individuals *creating* new complex spaces of the possible for their own development as a complex human being, thereby generating the power *for* transition and transformation of the individual as a whole. This is opening the wondrous world of the possible as the real world *for* education.

The new framework of complexity and the corresponding new way of thinking in complexity, based on causal complexity, offer a new scientific understanding of learning and development for the field of education as a complex topic of study. The pedagogics needed for this understanding will be a complexity pedagogics: about complexity as a *process* being complexly generative and self-potentiating (cf. "complexity economics," by Arthur, 2013, 2015). About a complex pedagogics that, like economics, "can handle interactions more generally" (ibid., p. 19). All of this may be taken as being part of a larger shift in science (ibid., p. 19), enabling for a new generative, transdisciplinary approach for science (Jörg, 2011). It is this shift in science that makes it possible to open the wondrous world of the possible for all sciences.

References

Arthur, W. B. (2013). *Complexity economics*: *A different framework for economic thought*. SFI WORKING PAPER: 2013-04-012. Retrieved the 20th of January 2015 at http://www.santafe.edu/media/workingpapers/13-04-012.pdf.

Arthur, W. B. (2015). *Complexity and the economy*. Oxford: Oxford University Press.

Ball, A. F. (2009). Toward a generative theory of change. *American Educational Research Review, 46*(1), 45–72. Retrieved the 27th of July 2013 at http://aer.sagepub.com/content/46/1/45.full.pdf+html.

Ball, A. F. (2012a). To know is not enough. Presidential address AERA 2012. Retrieved from http://www.aera.net/tabid/13250/Default.aspx.

Ball, A. F. (2012b). To know is not enough: Knowledge, power, and the zone of generativity. *Educational Researcher, 41*(8), 283–293.

Barabási, A. L. (2003). *Linked. How everything is connected to everything else and what it means for business, science, and everyday life*. New York: Penguin Group.

Bhaskar, R. (2011). *Reclaiming reality*. New York: Routledge.

Biesta, G. (2010). *Good education in an age of measurement. Ethics, politics, democracy*. Boulder (CO): Paradigm Publishers.

Blossfeld, H.-P. (2009). Causation as a generative process. The elaboration of an idea for the social sciences and an application to an analysis of an interdependent dynamic social system. In H. Engelhardt, H.-P. Kohler, & A. Fürnkranz-Prskawetz (Eds.), *Causal analysis in population studies. Concepts, methods, applications* (pp. 83–109). Berlin: Springer.

Bohm, D., & Peat, D. M. (2000). *Science, order, and creativity* (2nd ed.). London: Routledge.

Bruner, J. (1996). *The culture of education*. Cambridge MA: Harvard University Press.

Buber, M. (1970/1928). *I and thou*. New York: Charles Scribner's Sons.

Caporael, L. R., Griesemer, J. R., & Wimsatt, W. C. (Eds.). (2014). *Developing scaffolds in evolution, culture, and cognition. The Vienna series in theoretical biology*. Cambridge (MA): The MIT Press.

Capra, F., Juarrero, A., Sotolongo, P., & Van Uden, J. (Eds.). (2007). *Reframing omplexity*. Mansfield (MA): ISCE Publishing.

Carey, S. E. (2004). Bootstrapping and the origin of concepts. *Daedalus*, 133 (1), 59-68.

Carey, S. (2009). *The origin of concepts*. Oxford: Oxford University Press.

Cilliers, P. (1998). Acknowledging complexity. Foreword. In G. Biesta & D. Osberg (Eds.), *Complexity theory and the politics of education* (pp. vii–viii). Rotterdam: Sense Publishers.

Davis, B. (2004). *Inventions of teaching. A genealogy*. Mahwah, NJ: Lawrence Erlbaum Associates.

Davis, B., Phelps, R., & Wells, K. (2005). Complicity: An introduction and a welcome. *Complicity: An International Journal of Complexity and Education, 1*(1), 1–7.

Fazio, X., & Gallagher, T. L. (2009). Supporting learning: An examination of two teacher development collectives. *Complicity: An International Journal of Complexity and Education, 6*(1), 1–19.

Fenwick, T. (2009). Responsibility, complexity science and education: Dilemmas and uncertain responses. *Studies in Philosophy and Education, 28*, 101–118.

Fenwick, T. (2012). Complexity science and professional learning for collaboration: A critical reconsideration of possibilities and limitations. *Journal of Education and Work, 25*(1), 141–162.

Fiorella, L. & Mayer, R. E. (2015). *Learning as generative activity: Eight learning strategies that promote understanding*. New York: Cambridge University Press.

Follett, M. P. (1924). *The creative experience*. See also P. Graham (1995). Also (partly) retrievable via http://www.follettfoundation.org/writings.htm.

Graham, P. (Ed.). (1995). *Mary Parker Follett. Prophet of management. A celebration of writings from the 1920s*. Boston: Harvard Business School Press.

Grotzer, T. (2012). *Learning causality in a complex world. Understandings of consequence*. Lanham: Rowman & Littlefield Education.

Hawking, S. (2000). *San Jose Mercury News*. Retrieved 20th January from http://www.comdig.com/stephen-hawking.php

Hayduk, L. A. (1987). *Structural equation modelling with Lisrel: Essentials and advances*. Baltimore, MD: John Hopkins University Press.

Hayduk, L. A. (1996). *LISREL issues, debates, and strategies*. London: John Hopkins University Press.

Illari, P., & Russo, F. (2014). *Causality. Philosophical theory meets scientific practice*. Oxford: Oxford University Press.

Jöreskog, K. G., & Sörbom, D. (1993). *LISREL8: A guide to the program and applications*. Chicago: SSI.

Jörg, T. (2004). A theory of reciprocal learning in dyads. *Cognitive Systems, 6*(2, 3), 159–170. European Society for the Study of Cognitive Systems (ESSCS), Groningen (The Netherlands): ESSCS.

Jörg, T. (2009). Thinking in complexity about learning and education – a programmatic view. *Complicity, 6*(1), 1–15. http://www.complexityandeducation.ualberta.ca/COMPLICITY6/Complicity6_TOC.htm.

Jörg, T. (2010). *The paradigm of complexity and the future of education*. Paper presented at the conference on theorizing on education, at the University of Sterling, 24–26 of June 2010. Sterling (UK).

Jörg, T. (2011). *New thinking in complexity for the social sciences and humanities – a generative, transdisciplinary approach*. New York: Springer Publishers.

Jörg, T., & Akkaoui Hughes, S. (2013). Harnessing the complexity of innovation. *International Journal of Knowledge and Systems Science, 4*(3), 1–13.

Jörg, T. (2014). The crisis of knowing in the age of complexity. In M. E. Jennec (Ed.), *Knowledge, discovery, transfer, and management in the information age* (pp. 1–19). Hershey, PA: IGI Global.

Jörg, T., Davis, B., & Nickmans, G. (2007). Towards a new complexity science of learning and education. *Educational Research Review, 2*(2), 145–156.

Kauffman, S. (1993). *The origins of order. Self-organization and selection in evolution*. Oxford: Oxford University Press.

Kauffman, S. A. (2009). Foreword. The open universe. In R. Ulanowicz (Ed.), *The third window. Natural life beyond Newton and Darwin* (pp. ix–xvii). West Conshohocken: Templeton Foundation Press.

Kauffman, S. A. (2013). Evolution beyond Newton, Darwin, and entailing law: The origin of complexity in the evolving biosphere. In C. H. Lineweaver, P. C. W. Davies, & M. Ruse (Eds.), *Complexity and the arrow of time* (pp. 162–190). Cambridge (UK): Cambridge University Press.

Lichtenstein, B. B. (2014). *Generative emergence. A new discipline of organizational, entrepreneurial, and social innovation*. Oxford: Oxford University Press.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills: Sage Publications.

Lord, B. (1994). Teacher's professional development: Critical colleagueship and the role of professional communities. In N. Cobb (Ed.), *The future of education* (pp. 175–204). New York: College Board Publications.

Mainzer, K. (2004/2007). *Thinking in complexity. The computational dynamics of matter, mind, and mankind*. Berlin: Springer.

Maruyama, M. (1963). The second cybernetics: Deviation-amplifying mutual causal processes. *American Scientist, 5*(2), 164–179.

Maturana, H. R. (1980), In: H.R. Maturana & F. J. Varela, *Autopoiesis and cognition. The realization of the living*. Dordrecht: Reidel Publishing Company.

Merton, R. K. (1968). The Matthew effect in science. Retrieved from http://www.unc.edu/~fbaum/teaching/PLSC541_Fall06/Merton_Science_1968.pdf.

Mitchell, C. M. (2011). *Complexity. A guided tour*. New York: Oxford University Press.

Müller, K. H., & Riegler, A. (2014). Second-order science: A vast and largely unexplored science frontier. *Constructivist Foundations, 10*(1), 7–15. http://www.univie.ac.at/constructivism/journal/10/1/007.introduction.

Nowotny, H. (2013). *The embarrassment of complexity*. Presentation at the 5th Global Drucker Forum, Vienna, 14–15 November 2013. Retrieved the 20th of January 2015 at http://helga-nowotny.eu/downloads/helga_nowotny_b143.pdf.

Osberg, D. (2009). "Enlarging the space of the possible" around what is means to educate and be educated. *Complicity: An International Journal of Complexity and Education, 6*(1), iii–x.

Padgett, J. F., & Powell, W. W. (2012). *The emergence of organizations and markets*. Princeton (NJ): Princeton University Press.

Palmer, P. (1998). *The courage to teach. Exploring the inner landscape of a teacher's life*. San Francisco: Jossey-Bass Publishers.

Pellegrino, J. W. (1994). Generative learning and anchored instruction: design, research and implementation issues. In P. M. Creemers & G. J. Reezigt (Eds.), *New directions in educational research* (pp. 33–61). Groningen: ICO-publications.

Pellegrino, J. W., & Hilton, M. L. (Eds.). (2012). *Education for life and work. Developing transferable knowledge and skills in the 21st century*. Washington, DC: The National Academies Press.

Phelps, R., & Hase, S. (2002). Complexity and action research: Exploring the theoretical and methodological connection. *Educational Action Research, 10*(3), 507–524.

Rescher, N. (1998). *Complexity. A philosophical overview*. New Brunswick: Transaction Publishers.

Rose, S. (1997). *Lifelines. Biology beyond determinism*. Oxford: Oxford University Press.

Rossi, F., Russo, M., Sardo, S., & Whitford, J. (2010). Innovation. generative relationships and scaffolding structures. Implications of a complexity perspective to innovation for public and private interventions. In P. Ahrweiler (Ed.), *Innovation in complex social systems* (pp. 150–161). London: Routledge.

Sassone, L. A. (1996). Philosophy across the curriculum: A democratic Nietzschean pedagogy. *Educational Theory, 46*(4), 511–524.

Sassone, L. A. (2002). *The process of becoming. A democratic Nietzschean philosophical pedagogy for individualization*. Chicago: Discovery Ass. Publishing House.

Senge, P. (1990). *The fifth discipline*. New York: Doubleday.

Senge, P., Cambron-McCabe, T., Smith, B., Dutton, J., & Kleiner, A. (2000). *Schools that learn*. New York: Doubleday.

Senge, P., Scharmer, C. O., Jaworski, J., & Flowers, B. S. (2004). *Presence. Exploring profound change in people, organizations and society*. London: Nicholas Brealy Publishing.

Sloman, A. (2015). *Meta-morphogenesis and toddler theorems: Case studies*. Retrieved from http://www.cs.bham.ac.uk/research/projects/cogaff/misc/toddler-theorems.html

Sumara, D., & Davis, B. (1997). Enlarging the space of the possible: Complexity, complicity, and action-research practices. *Action Research as a Living Practice, 67*, 299–312. http://www.jstor.org/discover/10.2307/42975255?uid=3738736&uid=2&uid=4&sid=21106640017293.

Taylor, M. (2001). *The moment of complexity: Emerging network culture*. Chicago: Chicago University Press.

Van Geert, P. (1994). *Dynamic systems of development. Change between complexity and chaos*. New York: Harvester Wheatsheaf.

Valsiner, J. (1998). *The guided mind*. Cambridge, MA: Harvard University Press.

Vygotsky, L. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Vygotsky, L. (1987). In R. W. Rieber, & A. S. Carton (Eds.), *Collected works, Vol. 1. Problems of general psychology*. New York: Plenum Press.

Vygotsky, L. (1997). In R. W. Rieber, & J. Wollock (Eds.), *Collected works, Vol. 3. Problems of the theory and history of psychology*. New York: Plenum Press.

Vygotsky, L. (1997). *Educational psychology*. Boca Raton, FL: St. Lucie Press.

Wimsatt, W. C. (2014). Entrenchment and scaffolding: An architecture for a theory of cultural change. In L. R. Caporael, J. R. Griesemer, & W. C. Wimsatt (Eds.), *Developing scaffolds in evolution, culture, and cognition. The Vienna series in theoretical biology* (pp. 77–105). Cambridge, MA: The MIT Press.

Wimsatt, W. C., & Schank, R. (2002). In G. Schlosser & G. Wagner (Eds.), *Modularity in development and evolution* (pp. 359–394). Chicago: University of Chicago Press. Retrieved from http://mechanism.ucsd.edu/~bill/ModelsAndPrediction/papers/wimsatt.Mod-evol-pardx-89POSTfin.pdf.

Wiseman, L. (2010). *Multipliers. How the best leaders make everyone smarter*. New York: HarperCollins Publishers.

Chapter 6
Towards the Teaching of Motor Skills as a System of Growing Complexity

Umberto Cesar Corrêa, Walter Roberto Correia, and Go Tani

Introduction

The twentieth century can be recognized as a period of "phase transitions" in science. Without being reductionist in the historical description, we posit that two events marked the scientific development on that period. The first event concerns the changes of classical Cartesian, Newtonian, or modern science to contemporary, new-paradigm, or systems science (Wiener, 1948). Classical science was based on the assumptions of simplicity, stability, and objectivity, which meant that, respectively, (1) from the parts, it would be possible to understand the whole; (2) the world would be stable, predictable, and reversible, with the possibility of control of phenomena; and (3) it would be possible to objectively know the world as it really is. Contemporary science, on the other hand, is concerned with complex phenomena that are apparently irreducible. It is based on the idea that a system tends to show new features not discernible from their units' components, but emerging from significant relationships between members of the team—that is, interaction (von Bertalanffy, 1952; Dupuy, 1996; Prigogine, 1997).

The second event is the change within the systemic thought from systems functioning based on negative feedback mechanisms to systems working based on the interplay of positive and negative feedback mechanisms. This change occurred during the second half of the twentieth century (von Foerster, 1960; Jantsch, 1980; Maruyama, 1963; Prigogine & Stengers, 1984; Weiss, 1967; Yates, 1987). This change was influenced mainly by applying of conceptions about thermodynamics of systems far from equilibrium and dissipative structures to living phenomena (von Bertalanffy, 1950, 1952; Schrödinger, 1945). This perspective brings about new themes such as order from noise, order by fluctuations, organized disorder,

U.C. Corrêa (✉) • W.R. Correia • G. Tani
University of São Paulo, São Paulo, Brazil
e-mail: umbertoc@usp.br

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_6

emergence, complexity, self-organization, chaos, and others as a basis of promising models that are oriented towards understanding and explaining phenomena from areas of knowledge, including educational ones (Chow et al., 2006, 2007; Koopmans, 2015; Steenbeek & van Geert, 2013; Stamovlasis & Koopmans, 2014). The present chapter was developed in this context.

We addressed questions on motor skills as a subject in education, their acquisition as the main goal for teaching/coaching, based on a nonequilibrium model of motor learning (Tani et al., 2014), which comprises two cyclical processes: functional stabilization towards a motor pattern formation whose structure reconciles order and disorder, and adaptation resulting in growing complexity.

Motor Skills as a Subject in Education

The motor skills phenomenon refers to those purposeful, efficient, adaptive, and learned movements that human beings have performed in order to survive (Corrêa, Walter, Torriani-Pasin, Barros, & Tani, 2014; De Paula Pinheiro, Marques, Tani, & Corrêa, 2015). The motor skills we are referring to here are those from sports, play, fight, dance, and exercise/gymnastics human cultural constructions, that is, which nowadays have been adapted and practised in order to fulfil biological, psychological, and social needs: health, competition, and leisure. For instance, while in the early days human beings threw darts, fought, and orienteered/navigated for utilitarian and survival reasons, today they do it to compete, improve health, and interact socially in their leisure time. It is through motor skills that human beings express their feelings and creativity and learn about themselves and the social environment in which they live (Tani, 1987). How human beings change their motor skills from one state of order to another thereby gaining in complexity poses an important question.

Due to the biological, psychological, social, cultural, and evolutionary roles that motor skills have had for human beings and their societies, they have been conceived of as part of the cultural heritage of many groups, which demonstrates their educational value (Tani & Manoel, 2004). To put it in another way, if sport, play, fight, dance, and exercise/gymnastics constitute sociocultural phenomena, enabling individuals to have access to them seems to be essential for certain educational processes. For example, in the school context, motor skills have been a subject because the knowledge related to them has been judged to possibly develop people's wellness and quality of life. It is thought that including motor skills in education allows individuals to do the following: (1) access and enjoy cultural heritage; (2) enhance and enrich their motor repertoire, which in turn improves interactions with the physical, social, and cultural environments in which they live; and (3) maintains and promotes health, providing them with opportunities to acquire knowledge, skills, and attitudes related to an active lifestyle (Tani & Manoel, 2004).

We suppose that in order for an individual to achieve the aforementioned educational goals, learning motor skills should occur by implementing three dimensions: the learning "of" motor skills, learning "through" motor skills, and learning "about" motor skills. The first refers to acquisition of ability to efficiently perform motor skills, the second is concerned with the acquisition of cognitive, affective and social skills by means of motor skills, and the third is related to the acquisition of knowledge about motor skills (Tani & Manoel, 2004). However, independently of the type of learning we believe that the teacher's knowledge about the nature of motor skills and on how they take place as a process of learning are a sine qua non of competence for effective teaching.

Motor Skills as a Complex System of Hierarchical Organization

There are a number of classifications of motor skills (Arnold, 1981; Fleishman, 1975; Gallahue, 2002; Gentile, 2000; Magill, 2000; Newell, 1989). For instance, they might be classified as open or closed according to their environmental stability; discrete, serial or continuous according to their starting and ending points and according to the order in which their components are performed; gross or fine according to the requirements of muscular groups and level of accuracy; closed or open circuit according to the use of feedback during or after execution; and cognitive or motor according to the demand for planning and memory. In fact, systems of classification for motor skills have not been made up as an end in themselves but as a tool for other purposes (Arnold, 1981).

For the purpose of the present chapter, motor skills are classified based on a systemic view of living phenomena (Laszlo, 2002), containing three important characteristics. First, motor skills can be conceived of as a complex system because in any level of analysis on which they are focused, they consist of the interaction of numerous components. A component refers to each part of a motor skill whose function in the skill as a whole is clearly identifiable (De Paula Pinheiro et al., 2015). For instance, in order to perform a pass in the game of futsal, an individual must integrate six components: (1) selecting a teammate target, (2) approaching the ball, (3) supporting a position with the non-kicking foot, (4) looking at the ball and holding the head steady, (5) contacting the ball, and (6) transferring weight forwards. Thus, we conceive motor skills as complex systems because they are emergent—that is, not discernible in their individual components but as a consequence of their interaction (Weiss, 1971).

A second important characteristic of motor skills is that they exist only in context. That is, motor skills involve a necessarily spatiotemporal relationship between performer and performance environment. This brings about a crucial implication for understanding the effectiveness of performances: the open nature of human beings. Similar to open systems, human beings interact continually with their environment, exchanging energy, matter, and information (von Bertalanffy, 1950, 1952). For instance, the selection, elaboration, and control of motor skills are made by continuous capture and utilization of environmental information.

An important theoretical issue related to the individual in a given environment is that the perception of environmental information is critical to the regulation of motor skills (Oudejans, Michaels, Bakker, & Dolné, 1996).

Finally, the third important characteristic of motor skills is that they simultaneously present consistency and flexibility. The first one is necessary to reliably reaches outcomes, and the second is essential for dealing with environmental instability (Bernstein, 1967; Cook, 1980; Glencross, 1980; Tani, 2005; Turvey, 1977). In the last few years these simultaneous characteristics have been explained by conceiving of motor skills as hierarchically organized systems with macrostructural and microstructural levels (Corrêa, Alegre, Freudenheim, Santos, & Tani, 2012; Corrêa et al., 2015; Corrêa, Davids, Silva, Denardi, & Tani, 2014; De Paula Pinheiro et al., 2015; Tani et al., 2014). The macrostructure refers to the motor skills' general spatiotemporal configuration, which emerges from the interaction between its components. It is constrained by the coupling of intention and task specificity, and it has been inferred based on relatively invariant measures of relative size, timing, and force, as well as sequencing.

The microstructure, in turn, refers to the components themselves. In complex open systems, the components simultaneously present autonomy and dependence because the macrostructure does not control the behavior of each part in detail but only constrains how they interact with each other (Lewin, 1999; Salthe, 1992; Waldrop, 1992; Weiss, 1971). As Laszlo (2002) writes, a macrostructure sets rules binding the parts among themselves. Thus, microstructure is responsible for the variability in the motor skills because of the performance options available within each component. The microstructure of motor skills has been accessed through measures of total movement time, overall force, and overall size.

In order to clarify the hierarchical organization of motor skills, let us consider the aforementioned example of the motor skills involved in futsal (Fig. 6.1): invariably, the numerous passes in a game of futsal are formed by the interaction

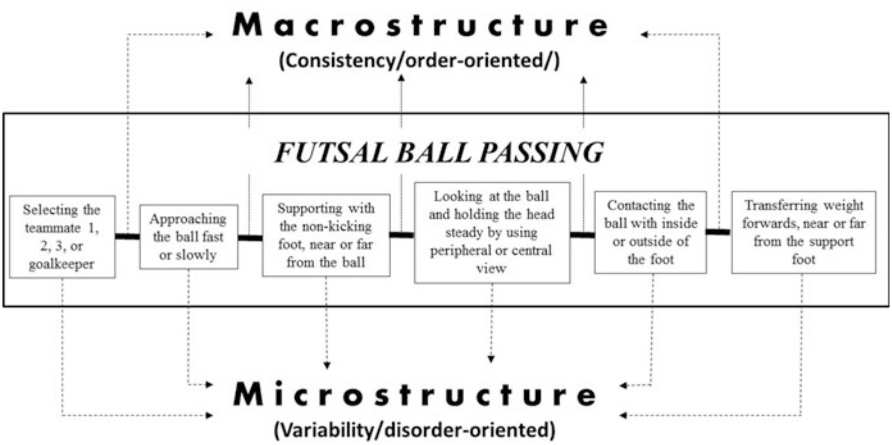

Fig. 6.1 Illustration of the hierarchical organization of passing of the sport of futsal

of six components mentioned above. This characterizes the macrostructure. Nevertheless, deciding which teammate to pass to, how fast to approach the ball, how to position the support foot, and which part of the body to touch the ball with vary according to the contexts of a game (e.g., displacement of teammates and opponents). One could say that such details emerge from specificity of context, which typifies the microstructure.

In sum, as a subject in education, motor skills can be considered as complex systems with a hierarchical organization. From this conception, the question we ask now is: what occurs when a given environment instability is greater than the flexibility of the skill microstructure? The answer to this question leads to the formulation of another important characteristic of motor skills: adaptability.

Motor Skills Learning as a Process of Growing Complexity

The currently adopted pedagogical models of motor skills can be characterized as equilibrium-oriented models. Equilibrium-oriented models explain the learning of motor skills as a process of reduction of inconsistency and incorrect responses through negative feedback mechanisms. The current movement pedagogy mainly emphasizes functional stabilization as a process of pattern formation and refinement (e.g., Mood, Musker, & Rink, 2012; Siedentop, 2009; Smith & Cestaro, 1998; Wrisberg, 2007). For instance, regarding the learning of futsal passing, teachers/coaches are advised to promote patterning in the interactions between the pass components through practice in a way the ball consistently reaches the intended teammate.

It is beyond doubt that the functional stabilization is a necessary state or condition for the life of human beings. One could say that its importance reflects the main definitions of motor skills: an acquired ability to achieve an environmental goal with the maximum certainty by organized and coordinated movements (Guthrie, 1952; Whiting, 1975). However, although models based on negative feedback mechanisms are able to explain the formation and maintenance of a stable pattern, they do not account for the open system nature of human beings, i.e., the adaptive human behavior.

This kind of system is able to become complex and elaborate by altering the content and organization of its contexts. This ability is possible because the system exchanges matter/energy and information with its environment (von Bertalanffy, 1950, 1952; Jantsch, 1980). Such exchanges allow humans to dismantle the acquired stability and change their internal state of organization. Thus, it is important to consider not only how human beings acquire and maintain stable patterns but also how such patterns are transformed into new ones—that is, adaptation.

From this perspective, a nonequilibrium view of motor learning has emerged as a promising alternative to elucidate the teaching of motor skills (Tani et al., 2014). It refers to an adaptive process model in which the acquisition of motor skills is considered as a cyclical and continuous process of stabilization and adaptation

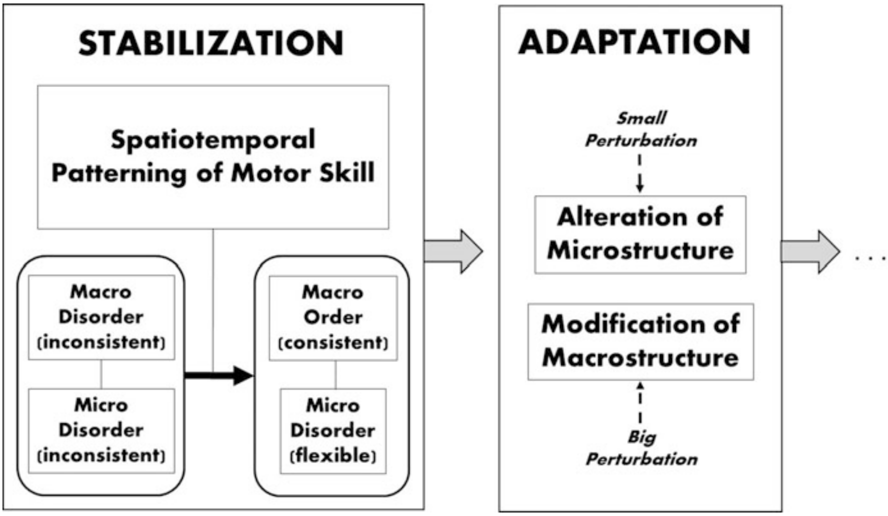

Fig. 6.2 Illustration of the nonequilibrium model of motor skill learning

(Fig. 6.2). The first refers to the functional stabilization phase that results in the spatiotemporal patterning of motor skills. As aforementioned, it occurs by mechanisms of diminishing errors and discrepancies (negative feedback), the main emphasis of the current pedagogical views.

However, as may happen in any open system, the acquired functional pattern may be perturbed by changes in the state of the individual (e.g., new intention), task (e.g., new rule), or environment (e.g., new space delimitation). In this case, the system functions based on influence of positive feedback (mechanisms of deviation amplification). Adaptation, the second learning phase, deals with how such perturbation can be used to reach a new stability regimen—that is, to generate order from disorder.

According to Tani et al. (2014), in order for adaptation to imply a new state of organization in the learning of motor skills, a reorganization of the acquired patterns in response to the perturbation must occur. For instance, in the game of futsal, those approaching a marking defender could require that a pass be performed faster. In this case, there would be the maintenance of stability since the perturbation would be eliminated by the system's flexibility (microstructure)—that is, by just altering a parameter (passing velocity). However, there may be a perturbation of such magnitude that would go beyond the reach of the system's flexibility. But the player could take advantage of it by reorganizing the skill structure. For instance, a shift in the velocity of approaching of a marking defender could make the player with the possession of the ball change the (1) support position from left to right foot and (2) contact the ball with the anterior instead of superior part of the foot. In this situation, the player would have changed two components in the skill structure. This has been named structural and self-organizational adaptation.

The main assumption here is that the reorganization of a previous structure into a new one implies growing in complexity, since it involves incorporating new information, components, and/or interaction modes (Tani et al., 2014) into existing structures. Furthermore, this growing in complexity also implies the acquisition of redundancy as an increase in the availability of resources to deal with perturbations (De Paula Pinheiro et al., 2015). Based on these statements, it seems reasonable to propose that the teaching of motor skills should be directed at adaptation of motor skills.

Final Remarks: Insights on the Teaching of Adaptation

As we wrote previously, motor skills are genuine educational content because they represent a significant part of the legacy of knowledge built by humankind throughout our existence. Motor skills are among the means by which humans interact with their environment by exchanging energy and information, which allow them to prevent an increase in entropy, remain in a state far from equilibrium, and develop towards growing complexity (Schrödinger, 1945). In sum, motor skills are the essence of life.

From the importance of motor skills as a subject in education, the comprehension of the open nature of humans and their capacity to continually learn allows us to conceive of learning as an adaptive process—that is, as continuum that involves cycles of functional stabilization and adaptation. The main assumption here is that the realization of each cycle implies states more complex of organization.

In this process, the basic question is: what kind of pattern or structure should be formed to account for the perturbation, or what type of competence is required from the motor system for perturbation to become an agent of change towards complexity? In response to this question, we have proposed that motor skills cannot be represented as a fixed pattern but as a pattern that reorganizes itself in the context of a new learning process. In other words, motor skills need to have some degree of stability, but they can reconstruct themselves through perturbation when they are destabilized. For this purpose, we have proposed that motor skills are organized hierarchically at the levels macrostructure and microstructure (Tani et al., 2014).

This view brings two main practical implications for teaching. First, teachers should emphasize the skill rather than technical. The technical has been considered as that best way to perform a motor action. It is closely related to rigidity and details in the performances. For other hand, the skill is related to efficiency in achieving goals, regardless of the form it occurs. Second, it seems crucial that teachers have clear how motor skills are composed, that is, which are their components (microstructure) and how are their interaction modes (macrostructure). For instance, while a pass of the sport of futsal and a spike of the sport of volleyball are open and discrete motor skills formed, respectively, by six and four components interacting sequentially, the front crawl and backstroke swimming refer to the closed and continuous motor skills, formed by three components that occur simultaneously. In the first example, instructional emphasis for formation of macrostructure should be on sequencing (e.g., running, jumping, hitting, and landing, for the volleyball spike). And, in the second example, the instructions should focus on the simultaneousness (e.g., breathing while perform the arm and leg strokes, for the backstroke swimming).

Furthermore, in order to manipulate the skill to cause adaptation teachers might change its spatial and/or temporal physical dimensions. For instance, temporal modifications (e.g., from faster to slower running) imply a low requirement for adaptation; spatial alterations (e.g., from larger to smaller steps during a running) imply moderate demand for adaptation; and spatiotemporal modifications in the skill (e.g., from larger and faster to slower and smaller steps during a running) imply higher requirements for adaptation (Corrêa, Ugrinowitsch, Benda, & Tani, 2010).

From this perspective, besides the structure of motor skills, factors related to disorder (e.g., uncertainty, variability, and error) should be considered by teachers in conjunction with others associated with order (e.g., information, consistency, and regularity) in order to promote the formation of a skill that contains both consistency and variability. This proposition has been supported by studies on practice schedule (Corrêa, Benda, Meira, & Tani, 2003; Corrêa et al., 2010; Gonçalves, Santos, & Corrêa, 2010). They have shown that the preceding skill structure might be formed from a combination of constant and varied practice schedules. According to these studies, the regularity of the constant regimen provides the formation of the macrostructure by patterning the interaction modes among the components. Also, the posterior varied condition increases the flexibility of the microstructure. Importantly, this proposition have been supported by studies with children, adults, and elderly (Gonçalves et al., 2010). Thus, teachers could organize practice initially in a constant way, and then in a varied regimen.

In order to adopt an ideal combination of constant-varied practices, teachers should consider the concept of self-organized criticality. When a system has self-organized criticality, it has achieved a degree of organization which places it at the border of chaos—prompting for creation, innovation, and/or evolution (Langton, 1992; Packard, 1988). This refers to a state in which a system reaches critical values that make it able to change; or, to a point of transition between order and disorder in which a small stimulus can generate a large change (Bak & Chen, 1991; Bak, Chen, & Creutz, 1989).

Recent evidence has shown that there is an optimal amount of constant practice to form the skill macrostructure (Corrêa, Barros, Massigli, Gonçalves, & Tani, 2007; Corrêa, Gonçalves, Barros, & Massigli, 2006; Corrêa, Massigli et al., 2010). These studies have shown that a minimum amount of practice is enough to prepare a motor skill for diversification. Furthermore, other evidence has shown that there is an optimal level of variability that provides the required flexibility for motor skill adaptation (De Paula Pinheiro et al., 2015). Thus, in order to develop ability for adaptation, teaching could involve the minimal amount of constant practice for macrostructure formation (e.g., until a pattern to be observed), and the minimal quantity of items in varied practice for skill diversification (e.g., three parameter values).

Finally, another important insight from the adaptive process studies is that the individual capability of the students should be considered in the practice schedule. Specifically, adaptation is beneficed when learner is provided with moderate freedom of choice in practice during the stabilization phase. According to Walter, Bastos, Araujo, Silva, and Corrêa (2008), constant practice with some freedom in the choice of components allows the learner to perform a sequence of movements more comfortably. It appears that the prior establishment of some components allows the learner understands the skill's macrostructure. Conversely, the freedom to choose some components enables the formation of flexible strategies, i.e., relative to the microstructure of the skill. For instance, considering the skill structure of the futsal passing previously described (Fig. 6.1), teachers could instruct the learner in relation to sequencing, selecting a teammate, and approaching the ball. Consequently, learner would choose how to perform the other components.

References

Arnold, R. K. S. (1981). *Developing sport skills: A dynamical interplay of task, learner, and teacher* (Monograph 2, motor skills: Theory into practice). Ewing Township, NJ: The College.

Bak, P., & Chen, K. (1991). Self-organized criticality. *Scientific American, 264*, 26–33.

Bak, P., Chen, K., & Creutz, M. (1989). Self-organized criticality in the "game of life". *Nature, 342*, 780–782.

Bernstein, N. A. (1967). *The coordination and regulation of movement*. London: Pergamon Press.

Chow, J.-Y., Davids, K., Button, C., Shuttleworth, R., Renshaw, I., & Araujo, D. (2006). Nonlinear pedagogy: A constraints-led framework for understanding emergence of game play and movement skills. *Nonlinear Dynamics, Psychology, and Life Sciences, 10*, 71–102.

Chow, J.-Y., Davids, K., Button, C., Shuttleworth, R., Renshaw, I., & Araujo, D. (2007). The role of nonlinear pedagogy in physical education. *Review of Educational Research, 77*, 251–278.

Cook, N. D. (1980). *Stability and flexibility: An analysis of natural systems*. Oxford, UK: Pergamon Press.

Corrêa, U. C., Alegre, F. A. M., Freudenheim, A. M., Santos, S., & Tani, G. (2012). The game of futsal as an adaptive process. *Nonlinear Dynamics, Psychology, and Life Sciences, 16*, 185–204.

Corrêa, U. C., Barros, J. A. C., Massigli, M., Gonçalves, L. A., & Tani, G. (2007). A prática constante-aleatória e o processo adaptativo de aprendizagem motora: efeitos da quantidade de prática constante. [The constant-random practice and the adaptive process of motor learning: effects of amount of constant practice]. *Revista Brasileira de Educação Física e Esporte [Brazilian Journal of Physical Education and Sport], 21*, 301–314.

Corrêa, U. C., Benda, R. N., Meira, C. M., Jr., & Tani, G. (2003). Practice schedule and adaptive process in the acquisition of a manual force control task. *Journal of Human Movement Studies, 44*, 121–138.

Corrêa, U. C., Benda, R. N., Oliveira, D. L., Ugrinowitsch, H., Freudenheim, A. M., & Tani, G. (2015). Different faces of variability in the adaptive process of motor skill learning. *Nonlinear Dynamics, Psychology, and Life Sciences, 19*(4), 465–487.

Corrêa, U. C., Davids, K., Silva, S. L., Denardi, R. A., & Tani, G. (2014). The influence of a goalkeeper as an outfield player on defensive subsystems in futsal. *Advances in Physical Education, 4*, 84–92.

Corrêa, U. C., Gonçalves, L. A., Barros, J. A. C., & Massigli, M. (2006). Prática constante-aleatória e aprendizagem motora: efeitos da quantidade de prática constante e da manipulação de exigências motoras da tarefa [Constant-random practice and motor learning: Effects of amount of constant practice and motor demands of task]. *Brazilian Journal of Motor Behavior, 1*, 41–52.

Corrêa, U. C., Massigli, M., Barros, J. A. C., Gonçalves, L. A., Oliveira, J. A., & Tani, G. (2010). Constant-random practice and the adaptive process in motor learning with varying amounts of constant practice. *Perceptual and Motor Skills, 110*, 442–452. doi:10.2466/pms.110.2.442-452.
Corrêa, U. C., Ugrinowitsch, H., Benda, R. N., & Tani, G. (2010). Effects of practice schedule on the adaptive process of motor learning. *Portuguese Journal of Sport Sciences, 10*, 158–171.
Corrêa, U. C., Walter, C., Torriani-Pasin, C., Barros, J. A. C., & Tani, G. (2014). Effects of the amount and schedule of varied practice after constant practice on the adaptive process of motor learning. *Motricidade Journal, 10*, 33–46.
De Paula Pinheiro, J., Marques, P. G., Tani, G., & Corrêa, U. C. (2015). Diversification of motor skills rely upon an optimal amount of variability of perceptive and motor task demands. *Adaptive Behavior, 23*, 83–96.
Dupuy, J. P. (1996). *Nas origens das Ciências Cognitivas. [In the origins of cognitive sciences]*. São Paulo: Editora Unesp.
Fleishman, E. A. (1975). Toward a taxonomy of human performance. *American Psychologist, 30*, 1127–1149.
Gallahue, D. L. A. (2002). Classificação das habilidades de movimento: um caso para modelos multidimensionais [Classification of movement skills: a case for multidimensional models]. *Revista da Educação Física, 13*, 105–111.
Gentile, A. M. (2000). Acquisition of skills: Action movement and neuromotor processes. In J. Carr (Ed.), *Movement science: Foundations for physical therapy in rehabilitation* (2nd ed., pp. 111–187). Gaithersburg: Aspen.
Glencross, D. J. (1980). Levels and strategies of response organization. In G. E. Stelmach & J. Requin (Eds.), *Tutorials in motor behavior* (pp. 551–566). Amsterdam: North-Holland.
Gonçalves, L. A., Santos, S., & Corrêa, U. C. (2010). Estrutura de prática e idade no processo adaptativo da aprendizagem de uma tarefa de timing coincidente [Practice schedule and age in the adaptive process of the learning of coincidente timing task]. *Revista Brasileira de Educação Física e Esporte, 24*, 433–443.
Guthrie, E. R. (1952). *The psychology of learning*. New York, NY: Harper & Row.
Jantsch, E. (1980). *The self-organizing universe: Scientific and human implications of an emerging paradigm of evolution*. Oxford: Pergamon Press.
Koopmans, M. (2015). A dynamical view of high school attendance: An assessment of short-term and long-term dependencies in five urban schools. *Nonlinear Dynamics, Psychology, and Life Sciences, 19*, 65–80.
Langton, C. G. (1992). Life at the edge of chaos. In C. G. Langton, C. Taylor, J. D. Farmer, & S. Rasmussen (Eds.), *Artificial life II: SFI studies in the sciences of complexity* (Vol. V). Redwood City: Addison-Wesley.
Laszlo, E. (2002). *The systems view of the word: A holistic vision for our time*. Cresskill, NJ: Hampton Press, Inc.
Lewin, R. (1999). *Complexity: Life on the edge of chaos* (2nd ed.). London: The University of Chicago Press.
Magill, R. A. (2000). *Aprendizagem motora: Conceitos e aplicações* (5th ed.). São Paulo: Editora Edgar Blücher Ltda.
Maruyama, M. (1963). The second cybernetics: Deviation-amplifying mutual causal process. *American Scientist, 51*, 164–179.
Mood, D. P., Musker, F. F., & Rink, J. (2012). *Sports and recreational activities* (15th ed.). New York, NY: McGraw-Hill.
Newell, K. M. (1989). On task and theory specificity. *Journal of Motor Behavior, 21*, 92–96.
Oudejans, R. D., Michaels, C. F., Bakker, F. C., & Dolné, M. A. (1996). The relevance of action in perceiving affordances: Perception of catchableness of fly balls. *Journal of Experimental Psychology, 22*, 879–891.
Packard, N. H. (1988). Adaptation toward the edge of chaos. In J. A. S. Kelso, A. J. Mandel, & M. F. Shlesinger (Eds.), *Dynamic patterns in complex systems*. Singapore: World Scientific Publishing.

Prigogine, I. (1997). *The end of certainty*. New York, NY: The Free Press.
Prigogine, I., & Stengers, I. (1984). *Order out of chaos: Man's new dialog with nature*. New York, NY: Bantam Books.
Salthe, S. N. (1992). Hierarchical non-equilibrium self-organization as the new post-cybernetic perspective. In G. V. Vijver (Ed.), *New perspectives on cybernetics: Self-organization, autonomy and connectionism*. Dordrecht: Kluwer.
Schrödinger, E. (1945). *What is life?* Cambridge: Cambridge University Press.
Siedentop, D. (2009). *Introduction to physical education, fitness, and sport* (7th ed.). Boston: McGraw-Hill.
Smith, T. K., & Cestaro, N. G. (1998). *Student-centered physical education: Strategies for developing middle school*. Champign, Il: Human Kinetics.
Stamovlasis, D., & Koopmans, M. (2014). Editorial introduction: Education is a dynamical system. *Nolinear Dynamics, Psychology, and Life Sciences, 18*, 1–4.
Steenbeek, H., & van Geert, P. (2013). The emergence of learning-teaching trajectories in education: A complex dynamic systems approach. *Nonlinear Dynamics, Psychology, and Life Sciences, 17*, 233–268.
Tani, G. (1987). Educação Física na pré-escola e nas quatro primeiras séries do ensino de primeiro grau: uma abordagem desenvolvimentista I [Physical education in preschool and the first four grades of elementary school education: A developmental approach I]. *Kinesis, 3*, 19–41.
Tani, G. (2005). Programação motora: organização hierárquica, ordem e desordem [Motor programming: hierarchical organization, order and disorder]. In G. Tani (Ed.), *Comportamento motor: aprendizagem e desenvolvimento [Motor behaviour: Learning and development]* (pp. 82–105). Rio de Janeiro: Guanabara Koogan.
Tani, G., Corrêa, U. C., Basso, L., Benda, R. N., Ugrinowitsch, H., & Choshi, K. (2014). An adaptive process model of motor learning: Insights for the teaching of motor skills. *Nonlinear Dynamics, Psychology, and Life Sciences, 18*, 47–65.
Tani, G., & Manoel, E. J. (2004). Esporte, educação física e educação física escolar [Sport, physical education and school physical education]. In A. C. Gaya, A. T. Marques, & G. Tani (Eds.), *Esporte para crianças e jovens: razões e finalidades [Sport for children and Young: reasons and purposes]* (pp. 45–60). Porto Alegre: Universidade Federal do Rio Grande do Sul.
Turvey, M. T. (1977). Preliminaries to a theory of action with reference to vision. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting and knowing* (pp. 211–266). Hillsdale, NJ: Erlbaum.
von Bertalanffy, L. (1950). The open systems in physics and biology. *Science, 111*, 23–29.
von Bertalanffy, L. (1952). *Problems of life*. London: Watts.
von Foerster, H. (1960). On the self-organizing systems and their environments. In M. C. Yovits & S. Cameron (Eds.), *Self-organizing systems* (pp. 31–50). Oxford: Pergamon.
Waldrop, M. M. (1992). *Complexity: The emerging science at the edge of order and chaos*. London: Penguin.
Walter, C., Bastos, F. H., Araujo, U. O., Silva, J. A. O., & Corrêa, U. C. (2008). Estrutura de prática e liberdade de escolha na aprendizagem de habilidades motoras. *Revista Portuguesa de Ciências do Desporto, 8*, 323–331.
Weiss, P. A. (1967). $1 + 1 \neq 2$ (One plus one does not equal two). In G. Quarton, T. Melnechuk, & F. O. Schimitt (Eds.), *The neurosciences: A study program* (pp. 801–821). New York, NY: The Rockefeller University Press.
Weiss, P. A. (1971). The basic concept of hierarchic systems. In P. A. Weiss (Ed.), *Hierarchically organized systems in theory and practice*. New York, NY: Hafner.
Whiting, H. T. A. (1975). *Concepts in skill learning*. London: Lepus.
Wiener, N. (1948). *Cybernetics*. New York, NY: Wiley.
Wrisberg, C. A. (2007). *Sport skill instruction for coaches*. Champign, Il: Human Kinetics.
Yates, F. E. (Ed.). (1987). *Self-organizing systems: The emergence of order*. New York, NY: Plenum Press.

Chapter 7
The Fractal Dynamics of Early Childhood Play Development and Nonlinear Teaching and Learning

Doris Pronin Fromberg

Introduction

How do children before 9 years of age actually learn about significant conceptual meanings, solve problems, and develop self-regulation? Educators who care to address this question—and are not content with rote memorization and children who parrot concrete verbalisms—can find some support in considering the dynamic, nonlinear processes by which young children learn. This chapter discusses the *relationship* between dynamic, nonlinear early learning and its implications for teaching. It makes sense to apprehend how young children learn in order to choreograph and coordinate how to teach in harmonious ways (that do no harm).[1]

Play. There is discussion in sections that follow of some current understandings of the dynamic, nonlinear ways that brains function as well as an explanation of the processes underlying young children's sociodramatic and construction play. Sociodramatic play typically involves two or more individuals who collaborate to improvise imaginative dramatizations that could be episodic. Construction play typically involves observable three-dimensional manipulation with materials such as blocks, clay, sand, and water.

Dynamic Themes Curriculum. The discussion can help to map the dynamic, nonlinear development of early learning that is meaningful to the players and relevant to the needs of today's world. There is also a description and discussion

[1] *To do no harm* suggests the sort of education that focuses on the adjustment of educators to the nonlinear dynamical ways in which young children develop and learn. Young children can experience harm when they feel rushed, pressured, or disrespected, and are expected to be passive and disempowered in educational settings.

D.P. Fromberg (✉)
Department of Curriculum and Teaching, Hofstra University, Hempstead, New York, USA
e-mail: Doris.P.Fromberg@hofstra.edu

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_7

within sections that follow of a nonlinear, content-rich, and meaning-based system of the early educational dynamic themes curriculum. Dynamic themes curriculum is the transdisciplinary teaching-learning practice of the *grammar of experience* with isomorphic dynamic themes (see also Fromberg, 2002, 2012).

The Brain and the Future. Throughout these strands, there appear to be similar nonlinear dynamical systems functioning within children's brains as they play, develop, and learn. Brain dynamics, play dynamics, and curricular dynamics are relevant and timely perspectives. Scholars, for example, identify the twenty-first century's need for people who can be versatile, think flexibly, enact their imaginations, collaborate, be resilient, and feel comfortable with the predictably unpredictable future (Bergen, Davis, & Abbitt, in press; Kaku, 1997, 2008; Yelland, Lee, O'Rourke, & Harrison, 2008). Young players already demonstrate these characteristics in their play and imagination, an observable venue in which to launch the narrative.

Sociodramatic Play

Sociodramatic play and imagination is one among seven *integrated* conditions of learning in early childhood (see Box 7.1) that begins to reveal the nonlinear dynamics of development that include language development, spatial and visual progress, social competence, and connection-making skills. In particular, sociodramatic play that involves two or more individuals reflects a kind of *script grammar* that functions as a basin of attractors. In effect, children collaborate in a kind of oral playwriting that is both immediately unpredictable while the underlying script theory/grammar of play is predictable.

Box 7.1. Seven Integrated Conditions for Learning in Early Childhood

- *Play*. Children use their imaginations to find out what they can do and can develop executive function skills (Diamond, Barnett, Thomas, & Munro, 2007). They explore the relationship between reality and fantasy. [The oscillations between reality and imagination have the potential to cascade into phase transitions/bifurcations.]
- *Induction*. The relationships between familiar variables, as compared or contrasted with a fresh variable, assist perception whereas isolated facts or sounds might be camouflaged within a rote environment (see also Fauconnier & Turner, 2002). [The oscillations between familiar and fresh variables have the potential to cascade into phase transitions/bifurcations.]
- *Cognitive Dissonance*. The relationships between (1) an expectation, (2) a real experience, and (3) the surprise contrast between the expectation and

(continued)

Box 7.1 (continued)

the experience are an instance of new learning. [The oscillation between expectation and experience has the potential to cascade into phase transitions/bifurcations.]

- *Social Interaction*. The relationships and comparisons between personal experience and recognition of another person's possibly different perspective occur within the growth of a theory of mind and self-regulation. [Children also come to appreciate that others may have perceptions and ideas different from their own, thereby strengthening their theory of mind (Astington & Pelletier, 2005; Blair & Razza, 2007; Harris & Kavanaugh, 1993; Leslie, 1996: Perner, 1991)]. Theory of mind is important to written as well as oral communication because effective communication requires appreciating that other people have thoughts, feelings, and beliefs. [The phase transition that defines the *relationship* between a personally centered view and a de-centered view reflects a growth of meaning and *social comp*etence (Piaget et al., 1965).]
- *Physical Interaction*. Experiences that begin within three-dimensional phenomena enhance the relationship between past experience and a dissonant or new perspective, and support the development of symbolic representations. [The relationship affords opportunity for a phase transition.]
- *Revisiting*. The relationship between prior experiences and possible fresh connections and fractal developments with subsequent activities can exercise the brain's plasticity (Bowman, Donovan, & Burns, 2001; Gopnik, 2009; Sylwester, 2000). [The brain's fractal, integrative functioning strengthens connections.]
- *A Sense of Competence*. The relationship between a sense of success and risk involves the integration of the brain's emotional and cognitive capacities (see Bergen et al., in press; Damasio, 2003; Kaku, 2014). A sense of competence develops out of the paradox of *challenges* which offer both a manageable risk and a reasonable chance for success. [The balanced relationship between risk and challenge supports children's development of executive functions.]

Sensitive Dependence on Initial Conditions. Play is the incarnation of a *sensitive dependence on initial conditions*. Each player's prior experiences introduce particular event possibilities that influence the capacity of other players to respond in a variety of different ways. The play framework itself involves a dynamic, nonlinear *relationship* between meta-communication and personal imagery, and then group imagery. In effect, the *script grammar* of play typically proceeds as follows:

1. Enplotment meta-communication: Outside the play framework's thematic content, young players briefly begin to plan the topic or direction of imaginative play, e.g., You be the father and I'll be the child who fell.
2. Enactment of imagery: Immediately entering the play framework, players interact in unpredictable ways, e.g., One "father-player" might offer comfort and a bandage. A different father-player might call for a physician. Still a different father-player might scold the child for doing dangerous activities.
3. Unpredictable trajectories of play content within bounded parameters: The play generates unpredictable trajectories which might range between retaining and elaborating the episodic nature of the play script or generating other topical directions.
4. There might be a reiteration process of moving outside the play framework to refocus the play. The players might begin new emplotment and enactment or simply role-play physician-patient activity or hospital-themed play or drunken father play or decide to create a baby-parent interaction or superhero character play or have one or both children simply abandon the sociodramatic play area. They might alternatively invite another player-emplotment partner or subject matter into their play.

Phase Transitions. "The predictability or grammatical structure of play constitutes a kind of 'attractor' in chaos theory. When weaker, the attractor (or underlying grammatical system) permits more random and predictable representations. Nevertheless, these representations still retain their relationship to the underlying attractor" (Fromberg, 2015, p. 425). Thus, phase transitions occur as the basins of attractors shift between meta-communication-emplotment and imagery-enactment. A particular action or comment by one player might serve as an attractor that leads to the self-organization of their play.

Phase transitions also occur when the episodic content oscillates or shifts unpredictably as each participating player contributes his or her enactment role. "The goal of the play, or the theme, serves as the attractor or driving point around which the play revolves and evolves. The dynamic nature of play implies that goals and themes can shift and, as they do, so will the attractor. These changes may be gradual or transformational, and then be governed by a periodic or even 'strange' attractor" (VanderVen, 2015, p. 415). Changes and transformations are related to periodic and strange attractors, respectively. Different children at different times are likely to have both shared and different experiences. Although the script components change, the play is identifiable as pretend play.

Relationships and Script Grammar. Relationships are important in sociodramatic play, beginning as early as the second year of life. A relationship takes place between the meta-communication in planning and the imagery within the development of the play script (Bateson, 1971, 1976, 1979). The *space between the relationships* defines the grammar of script theory. "Emergent behaviors … are all about living within boundaries defined by the rules, but also using that space to create something greater than the sum of its parts" (Davis & Samara, 2006, p. 148). The arc of the play emerges as the players interact to create fresh configurations.

Fractal Development. There is a *fractal progression* within sociodramatic play development. "Fractals describe self-similar patterns that appear on smaller to larger scales" (Fromberg, 2015, p. 426). Within the script framework, children learn from one another as well as from sensitive adult scaffolds. The fractal progression includes the (1) expanding development of language complexity and vocabulary; (2) increasing social competence; (3) increasing duration and coherence of thematic content; and (4) broadening event content knowledge. Each collaborative oscillating nonlinear progression incorporates a system of self-organization with a self-similar pattern at all scales/levels, far from equilibrium, and proceeds from simplicity and disorder toward complexity and order (Beran, Feng, Ghosh, & Kulik, 2014; Kurakin, 2011; Mandelbrot, 1983; see also Jadczyk, 2014). Thus, the fractal can serve as a heuristic for expanding memory, skills, and other developments through phase transitions. The fractal could also serve to predict patterns.

Oscillations Support Complexity Development. The oscillating movement between perception and collaborative construction of script development nourishes phase transitions toward increased complexity. The players' collaborative behavior creates a more complex drama than either of their individual contributions. At the same time, the underlying configurations of script grammar provide the organic medium, a basin of attraction, within which a variety of surface representations can emerge as bifurcations. The bifurcations result from phase transitions that grow out of the relationship between the shared imagery present in the players' meta-communication that, in turn, emerges as fresh imagery. The movement between enplotment and enactment involves the *relationships* and is essential for perception to occur. This process entails the unpredictability of events with the possibility of emergent bifurcation in new directions, an instance of chaos theory. For example, different children might each contribute a suggestion that might result in either a positive entrainment (synergy/the whole is more than the sum of its parts) or a destructive bifurcation. It may be the case that a fresh collaborative play configuration continues to develop and expand (synergy) or evolve into a schism that destroys the trajectory of the play so that the players disperse.

In these ways, sociodramatic play demonstrates that young children are capable of considerable self-direction and self-regulation. While play is a manifestation of children's imaginations, imagination is interdependent with thinking in concepts and reasoning (Vygotsky, 1987).

Construction Play

Construction play shares some of the play script grammar characteristics of sociodramatic play. However, young players often (1a) plan a structure, such as a three-dimensional construction with blocks; (1b) proceed silently to build; and (1c) only after the construction would they then engage in sociodramatic interactions (Gardner, 2006). Block play, for example, continues to develop with (2) repetitive linear building in vertical or horizontal rows; (3) followed by connecting and bridging; (4) then deliberately placing blocks to create enclosures; (5) followed by forming patterns or symmetrical structures; (6) engaging in role playing, pretense, and prop substitutions; (7) culminating in realistic representation; and (8) sometimes fantastic representations (see Bullard, 2010; Clements & Sarama, 2009; Cross, Woods, & Schweingruber, 2009; Erikson, 1977; Hanline, Milton, & Phelps, 2001; Hirsch, 1996; Johnson, 1933; Moyer & von Haller Gilmer, 1956 citing Krotsch; Piaget & Inhelder, 1976; Provenzo & Brett, 1983; Reifel, 1984; Schwartz & Copeland, 2010; Wardle, 2003).

Fractal Development. A *fractal progression* occurs as children's three-dimensional constructions expand and extend from basic, underlying structural variables and possibilities to increasingly complex embellishments and creations. They strengthen the underlying visual and spatial skills that are necessary for understanding concepts in science, technology, design engineering, and mathematics (STEM). Building with blocks, manipulation of three-dimensional objects or liquids, and games with objects contribute to building these learnings. Children also extend their three-dimensional representations to two-dimensional representations, a pathway along the continuum of symbolic development. In these ways, children at play create physical as well as mental models of their world.

The Relationship Between Assessment and Educator Scaffolds

The observable and physical products of construction play and sociodramatic play provide both a concrete opportunity for educators to celebrate the progression of complexity and an opportunity to support further development. *Scaffolds* (Vygotsky, 1978) refer to sensitive educator interventions that can support development. A scaffold is the relationship between an educator's "(1) assessment of a child's learning potential in relation to (2) a learning pathway, and (3)[an] invitation and challenge that provide a relevant next-step experience" (Fromberg, 2012, p. 61).

Educator scaffolds might include verbal harmony, adapting to each specific situation, as follows:

- Simply describe what children are doing
- Ask what they have done or noticed
- Ask what problems they might have encountered and how they resolved them
- Ask what help they might want
- Wonder if they might use a particular prop or what else they might use
- Ask what labels/signs they might use
- Ask about their plans

- Wonder what might happen if ...
- Wonder what a particular other child might contribute
- Enter the play framework with pretense
- Photograph/sketch the construction or interaction (see Ibid., pp. 62–66)

Scaffolds intend to keep opening children's capacities to experience phase transitions by focusing on the content of the play rather than judgments. For example, through sensitive scaffolding, an educator could introduce a new variable that children experience as a phase transition toward a fresh attractor; this process supports bifurcations by highlighting contrasts between variables or the *relationships* between expectations and actual experiences and possible next steps. Thus, *phase transitions help children to progress from non-meaning to meaning*. In this way teaching functions as an effective improvisational enterprise at its finest.

Brain Dynamics

"The brain is fundamentally a pattern forming self-organized system governed by potentially discoverable, nonlinear dynamical laws" (Kelso, p. 257). In particular, "The propensity to play is a biological system for promoting rapid adaptation to threats to survival that cannot be predicted. Playfulness, then, is characteristic of animals that make a specialty of being adaptable, and is a prime capability in changing and changeable settings (Ellis, 2015, p. 444)."

Interconnected Components and Processes. As a complex, dynamical system, the brain consists of billions of varied neurons and neuron networks that can interconnect across specialized regions by electrical and chemical transmissions modifiable by genetic and environmental influences (see Marcus & Freeman, 2015; Richardet, Chappelier, Telefont, & Hill, 2015). For example, the amygdala region monitors and helps to adapt to changes in threat levels, thereby providing emotional gatekeeping for cognitive processes. The parts of the brain that children use during play are integrated mainly in the *connections* between the amygdala (predominantly emotional center) and neocortex (predominantly thinking center). The same sections of the brain also are involved with attention, potential attitudes toward learning, self-regulation, creative thinking, problem solving, and the arts (see also Blair & Raver, 2012). Strengthening the amygdala strengthens these interrelated capacities. The term "emotional intelligence" (Goleman, 1995) has become a popular way to think about the importance of these connections. It is therefore significant that professional educators know how to maintain reasonable challenges and scaffolds in order to support optimal learning and young children's sense of competence.

Oscillations and Synapses—Pluripotentiality. Young children's enactment of their imaginations through observable play experiences can represent how the brain functions as a nonlinear dynamical system. The brain's physiology of communication and developmental trajectories involve *synapses*, the ongoing *relationships across chemical spaces between* the dendrites, and axons of the neurons in the brain. The *axon*(*s*) within the neuron transmit electrical signals and the many branching *dendrites* could receive signals from axons (Kelso, 1995, p. 231. See also Mitra & Bokil, 2008). The synapses are oscillating chemical or electrical signals that integrate across the various functional aspects of the brain, an oscillation between local and global regions, a process of *pluripotentiality*. Each neuron has the capacity to both receive and inhibit stimuli, a nonlinear oscillation process that re-balances the neuronal connections across synapses. Oscillations regulate neural thresholds and refractory periods.

Oscillations and Learning. The strength of synaptic coupling within the oscillation process (at varying rates) can change through learning. It is possible for new dendritic connections to form (Shelton, 2013). "[W]hen two or more nonlinear oscillators couple nonlinearly, the process of self-organization renders a wide variety of behaviors possible" (Kelso, p. 243).

Brain Grammar. Thus, similar to children's play, the brain functions as a self-organizing system of phase transitions with bifurcations "Brain maturation exhibits pattern and order that emerges from interactions of many different components, including those that are part of the play development dynamic system" (Bergen et al., 2016). Similar to the nonlinear functioning of children's play interactions, the underlying processes of the brain conform to a predictable *grammar* with unpredictable connections. Think about the enmeshed *intra*- (personal or intra-neuron), *inter*- (interpersonal or interneural and neural networks), and *extra*- (environmental context) processes.

Neuroplasticity and Self-organization. Neuroplasticity is the particular characteristic of the brain that is most active in young children as they play and make multitudes of connections. Neuroplasticity refers to the capacity for dendritic connections to form and networks to reorganize within the dynamic of experience that represents learning and the adaptation to new behavioral tasks (Mitra & Bokil, 2008; Shelton, 2013). The generative plasticity of the brain "is largely due to networks of [neurons] . . . rather than the sum of independent effects of individual [neurons]" (Mitchell, 2009, p. 275). The networks can serve as efficient attractors that support self-organization. Within these modifications, networks both absorb and generate complex brain functions (Cicurel & Nicolelis, 2015; Shelton, 2013). In turn, there are potentially rich and complex connections across the brain that support the development of myelination of the axons. Myelin layering, insulating the axons, improves the strength of connections and the speed of electrical connections (Chudler, n.d.; Hanline, 2008; NGIDD Consortium, 2010).

Pruning. There is a *paradox of plasticity*, however, in the process of *pruning* myelin. On the one hand, when children are in environments where there are rote instruction and limited opportunities for play and action-based learning experiences, there is pruning of the brain's less used rich network of myelinations. On the other hand, the brain benefits from pruning which supports and helps to focus the most often used connections that can support learning (Gopnik, Meltzoff, & Kuhl, 1999; Tang et al., 2014).

In effect, the brain's neural networks create models of the world and simulate them into the future; and along the way, the neural networks rewire themselves (Kaku, 2014). It is noteworthy that Lev Vygotsky wrote of imagination and creativity: "A person's creative activity does essentially this: it attends to the future, creating it, and changing the view of the present" (1990, p. 85). He saw imagination and creativity related to cultural context. Note that children's sociodramatic play develops unpredictably within the cultural event knowledge of the players.

In recent years, technological developments with functional MRIs and other technologies have added to the historical physical study of brain tissue. However, there are additional frontiers yet to confirm physiological dynamics (see Marcus & Freeman, 2015; Richardet et al., 2015). In the meanwhile, observing the nonlinear, dynamical functioning of young children's imaginative play provides an additional basis for illuminating how learning takes place. The narrative continues with discussion of the dynamic themes curriculum approach that mirrors the ways in which young children learn by participating in activity-based, content-rich, and meaningful environments.

Dynamic Themes Curriculum

Dynamic themes are predictable, isomorphic (self-similar) configurations/imagery patterns that underlie the multiple, surface forms that could represent them. For example, the dynamic theme *cyclic change* imagery appears in the history of society, animal, human, and plant growth; weather; evaporation; phases of the moon; and other objects that change across time. *Dialectical contrast/conflict* dynamic theme imagery appears in human conflicts; voting; economic scarcity; magnetism; and musical counterpoint. The dynamic theme *synergy* (the whole exceeds the sum of its parts) imagery appears in chemical processes; explosive events; growth and reproduction; economic processes; musical arts; square dancing; dramatic arts; collaborative constructions; and poetic arts (see Fromberg, 2012).

So, the underlying dynamic theme imagery is predictable across disciplines but the various surface forms are unpredictable. *Meaning takes place within the phase transition between the underlying and the surface forms*. In effect, the underlying dynamic theme imagery has the inherent potential to become represented in many surface forms as phase transitions connect by way of fractal attractors to create isomorphic imagery.

Grammar of Early Experience. Dynamic themes function as an analogical *grammar* of early experience and serve as underlying attractors; the underlying imagery of finite patterns can generate infinite possibilities. It is within the transformation (phase transition/bifurcation) between the deep and surface forms that meaning can occur. Isomorphism refers to "the generative process by which the underlying dynamic-theme relationships may take different surface forms. Analogies, built from cognitive connections based upon personal experiences, help humans to infer isomorphic connections" (Fromberg, 2015, p. 419).

Complexity theory explores nonlinear, dynamic, seemingly random experiences and phenomena that, different on the surface, manifest underlying regularities. From a psychologist's perspective, "Based on random interaction, [fractal construction] reveals the holistic quality of the underlying pattern .... The same fractal attractor inevitably emerges out of chaotic trajectories" (Marks-Tarlow, 2008, p. 260). Within this nonlinear framework, dynamic theme imagery provides foundational support for the ongoing educational development of phase transitions and bifurcations. Indeed, sensitive teaching involves the creation of potential phase transitions by the timely provision of resources, spaces, and opportunities.

Dynamic Theme Grammar and Brain Grammar. Paralleling these curricular concerns, neuroscientists contend that the neural networks of the human brain support these flexible and transformational processers (Payne & Kounios, 2009; Tognoli & Kelso, 2008). For example, "[T]he neural circuits must perform their functions locally, whereas the global distribution of activities is a collective function of the activities of the parts" (Kohonen, 1989, p. 255). The underlying dynamic theme imagery parallels the "local" whereas the multiple forms of cross-disciplinary experiences parallel the "global" functions.

Scaffolds Revisited. When educators provide exposure to experiences that represent a particular dynamic theme, children become receptive to perceiving that imagery within other diverse surface representations that can expand and deepen meaningful learning. In effect, dynamic themes are teacher scaffolds that adapt action-based learning experiences to young children's capacities to integrate challenges and fresh phase transitions. Thus, meaningful, quality teaching-learning and learning-teaching relationships are complex adaptive systems of coupled dynamics.

It is worth mentioning that meaningful activity-based experiences that represent concepts within, and tools of, the social sciences and sciences afford young children reasons to represent their grasp of meanings in two- and three-dimensional forms, in effect, speaking, writing, drawing, and constructions. Content-rich meanings also afford young children reasons to measure and calculate through the tools of mathematics.

Concluding Statement

Young children's sociodramatic and construction play can reveal the nonlinear dynamical system's grammar of their brain development. In turn, the underlying grammar of their play supports the educational development of dynamic themes as an active, meaning-based grammar of experience.

The discussion strands in this chapter embody the fractal nature of developmental trajectories along with the powerful value of oscillations between (1) integrative structures in the brain as well as within (2) the grammar of play and (3) the grammar of experience. Educators who envision these dynamical processes—and have strong grounding in meaningful content bases that are transdisciplinary—are well equipped to nurture phase transitions and bifurcations that expand and deepen children's knowledge bases and executive functions, including self-regulation. In effect, self-regulation grows out of experiences that are interpersonal as well as environmental-personal.

There are implications for educational practice that are based upon the nonlinear dynamics of children's development through play with objects and other children as well as from curriculum that includes other engaging, action-based, and meaningful experiences.

- "The challenge for professional educators is to create happenings/basins of attraction with children that balance both planning and adaptation to emerging events (Fromberg, 2015, p. 431)." Educators can meet the challenge by welcoming more than one interpretation of an issue or solution to a problem. They also can refine questioning skills that ask for children's perceptions, ideas, opinions, and reactions rather than merely isolated facts or yes-no responses. The focus is on the dynamics of children's learning and connection making as well as setting and solving problems.
- Educators who recognize that different children who engage in different activities at different times might have equivalent experiences have the power to be flexible. This principle suggests that educators could adjust to the variety of children's perspectives and their ways of learning by planning experiences and resources that welcome the *multiple ways* in which to represent underlying dynamic themes in activity-based formats. Formats could span concepts that use the tools of different disciplines as their representational forms.
- Thus, educators would focus on supporting children to make connections between events. They create cognitive dissonance when they create a basin of attraction, in effect, the opportunity for a comparison between expectation-experience-and-surprise comparison between the expectation and experience. Therefore, professional educators focus on nonlinear transactions, continuous communication (Davis & Sumara, 2006), and multiple forms of assessments to inform planning and instruction rather than uniform presentations and expectations. This focus supports children's capacity for self-organization and executive function. The focus is on children's exposure to isomorphic imagery and connections through vivid imagery and integrative concepts.
- The educator's role, furthermore, includes providing culturally relevant pedagogy with shared responsibility for planning. Such an educator engages in interpretive/ethical reflection (Van Manen, 1990). Flexible, extended time schedules, with mostly smaller groups, and collaborative as well as individual activities, could strengthen children's self-motivation, self-regulation, and positive attitudes toward education.

Moreover, a dynamic themes approach to curriculum could help both educators and children to feel empowered and to look forward to having great expectations for the future.

References

Astington, J. W., & Pelletier, J. (2005). Theory of mind, language, and learning in the early years: Developmental origins of school readiness. In B. D. Horner & C. S. Tamis-LeMonda (Eds.), *The development of social cognition and communication* (pp. 205–230). Mahwah, NJ: Lawrence Erlbaum.

Bateson, G. (1971). The message "This is play.". In R. E. Herron & B. Sutton-Smith (Eds.), *Child's play* (pp. 261–266). New York: Wiley.

Bateson, G. (1976). A theory of play and fantasy. In J. S. Bruner, A. Jolly, & K. Sylva (Eds.), *Play—Its role in development and evolution* (pp. 119–129). New York: Basic Books.

Bateson, G. (1979). *Mind and nature*. New York: E.P. Dutton.

Beran, J., Feng, Y., Ghosh, S., & Kulik, R. (2014). *Long-memory processes*. Berlin: Springer-Verlag.

Bergen, D., Davis, D. R., & Abbitt, J. T. (2016). *Technology play and brain development*. New York: Taylor and Francis/Routledge.

Blair, C., & Raver, C. C. (2012). Child development in the context of adversity: Experiential canalization of brain and behavior. *American Psychologist, 67*(4), 309–318.

Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief to emerging math and literacy ability in kindergarten. *Child Development, 78*(2), 647–663.

Bowman, B. T., Donovan, M. S., & Burns, M. S. (Eds.). (2001). *Eager to learn: Educating our preschoolers*. Washington, DC: National Academy Press.

Bullard, J. (2010). *Creating environments for learning: Birth to age eight*. Upper Saddle River, NJ: Merrill/Pearson.

Chudler, E. H. (n.d.). Brain plasticity: What is it? *Learning and memory*. Retrieved March 30, 2015, from http://www.faculty.washington.edu/chudler.plast.html

Cicurel, R., & Nicolelis, M. N. (2015). *The relativistic brain: How it works and why it cannot be simulated by a Turing machine*. Authors.

Clements, D. H., & Sarama, J. (2009). *Early childhood mathematics educational research: Learning trajectories for young children*. New York: Routledge.

Cross, C. G., Woods, R. A., & Schweingruber, H. (Eds.). (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: The National Academies Press/National Research Council.

Damasio, A. (2003). *The feeling of what happens: Body and emotion in the making of consciousness*. New York: Harvest.

Davis, B., & Sumara, D. (2006). *Complexity and education: Inquiries into learning, teaching, and research*. New York: Lawrence Erlbaum Associates/Taylor and Francis.

Diamond, A., Barnett, W. S., Thomas, J., & Munro, S. (2007). Preschool program improves cognitive control. *Science, 318*, 1387–1399. Supplemental online material. Retrieved May 28, 2010, from www.devcogneuro.com/publications/science%20article&20-%20Diamond%20et%20al.pdf

Ellis, M. J. (2015). Play and the origin of species. In D. P. Fromberg & D. Bergen (Eds.), *Play from birth to twelve* (3rd ed., pp. 443–444). New York: Routledge (Reprinted and originally published in 1988).

Erikson, E. H. (1977). *Toys and reason*. New York: W.W. Norton.

Fauconnier, G., & Turner, M. (2002). *The way we think: Conceptual blending and the mind's hidden complexities*. New York: Basic Books.

Fromberg, D. P. (2002). *Play and meaning in early childhood education*. Boston, MA: Allyn & Bacon.

Fromberg, D. P. (2012). *The all-day kindergarten and pre-k curriculum: A dynamic-themes approach*. New York: Taylor and Francis/Routledge.

Fromberg, D. P. (2015). How nonlinear systems inform meaning and early education. In D. P. Fromberg & D. Bergen (Eds.), *Play from birth to twelve: Contexts, perspectives, and meaning* (pp. 419–434). New York: Taylor and Francis/Routledge (Reprinted and originally published in 2010).

Gardner, H. (2006). *Multiple intelligences: New Horizons*. New York: Basic Books.

Goleman, D. (1995). *Emotional intelligence*. New York: Bantam.

Gopnik, A. (2009). *The philosophical baby*. New York: Farrar, Straus, & Giroux.

Gopnik, A., Meltzoff, A. N., & Kuhl, P. K. (1999). *The scientist in the crib: What early learning tells us about the mind*. New York: Harper Collins.

Hanline, D. (2008). What is myelin? *Neuron glia biology, 4*(2), 153–163.

Hanline, M. F., Milton, M. F., & Phelps, P. C. (2001). A longitudinal study of the predictive relation among construction play and mathematical achievement. *Early Childhood Development and Care, 167*, 115–125.

Harris, P. L., & Kavanaugh, R. D. (1993). Young children's understanding of pretense. *Monographs of the Society for Research in Child Development No. 231, 58* (1).

Hirsch, E. S. (1996). *The block book*. Washington, DC: National Association for the Education of Young Children.

Jadczyk, A. (2014). *Quantitative fractals: From Heisenberg's certainty to Barnsley's fracticality*. Hackensack, NJ: World Scientific.

Johnson, H. M. (1933). *The art of block building*. New York: Joshua Day.

Kaku, M. (1997). *Visions: Science revolution for the twenty-first century*. New York: Basic Books.

Kaku, M. (2008). *Physics of the impossible*. New York: Doubleday.

Kaku, M. (2014). *The future of the mind: The scientific quest to understand, enhance, and empower the mind*. New York: Doubleday.

Kelso, J. A. S. (1995). *Dynamic patterns: The self-organization of brain and behavior*. Cambridge, MA: MIT Press.

Kohonen, T. (1989). *Self-organization and associative memory* (3rd ed.). New York: Springer-Verlag.

Kurakin, A. (2011). The self-organizing theory of a universal discovery method: The phenomenon of life. *Theoretical Biology and Medical Modelling, 8*, 4. Retrieved June 20, 2015. doi:10.1186/1742-4682-8-4.

Leslie, A. M. (1996). Pretending and believing: Issues in the theory of ToMM. In J. Mehler & S. Franck (Eds.), *COGNITION on cognition* (pp. 193–220). Cambridge, MA: MIT Press.

Mandelbrot, B. B. (1983). *The fractal geometry of nature*. New York: W.H. Freeman.

Marcus, G., & Freeman, J. (Eds.). (2015). *The future of the brain*. Princeton, NJ: Princeton University Press.

Marks-Tarlow, T. (2008). *Psyche's veil: Psychotherapy, fractals and complexity*. New York: Routledge.

Mitchell, M. (2009). *Complexity: A guided tour*. New York: Oxford University Press.

Mitra, P. P., & Bokil, H. (2008). *Observed brain dynamics*. New York: Oxford University Press.

Moyer, F., & von Haller Gilmer, B. (1956). Experimental study of children's preferences and blocks in play. *Journal of Genetic Psychology, 89*, 3–10.

NGIDD Consortium. (2010). *Neuron-glia interactions in nerve development and disease*. Retrieved March 30, 2015, from http://222.ngidd/eu/public//myelinated.html

Payne, L., & Kounios, J. (2009). Coherent oscillatory networks support short-term memory retention. *Brain Research, 1247*, 126–132.

Perner, J. (1991). *Understanding and the representational mind*. Cambridge, MA: MIT Press.

Piaget, J., & Inhelder, B. (1976). *The child's conception of space*. (F. J. Langdon, & J. L. Lunzer, Trans.). London: Routledge & Kegan Paul. (Originally published in 1956)

Piaget, J., (1965). *The moral judgment of the child*. (M. Gabain, Trans.). New York: Free.

Provenzo, E. F., Jr., & Brett, A. (1983). *The complete block book*. Syracuse: Syracuse University Press. Photos M. Carleback.

Reifel, S. (1984). Children's developmental landmarks in representation of space. *Young Children, 40*(1), 61–67.

Richardet, R., Chappelier, J. -C., Telefont, & Hill, S. (2015). *Large-scale extraction of brain connectivity from the neuroscientific literature*. Retrieved June 15, 2015, from http://bioinformatics.oxfordfournals.org/content/early/2015/02/16

Schwartz, S. L., & Copeland, S. M. (2010). *Connecting emergent curriculum and standards in early childhood classrooms: Strengthening content and teaching practice*. New York: Teachers College Press.

Shelton, C. D. (2013). *Brain plasticity: Rethinking how the brain works*. E-book: Author.

Sylwester, R. (2000). *A biological brain in a cultural classroom*. Thousand Oaks, CA: Corwin.

Tang, G., (2014). Loss of mTOR-dependent macroautophagy causes autistic-like synaptic pruning deficits. *Neuron, 83*(6), 1482.

Tognoli, E., & Kelso, J. A. S. (2008). Brain coordination dynamics: True and false faces of phase synchrony and metastability. *Progress in Neurobiology, 87*, 31–40.

Van Manen, M. (1990). *Researching lived experience: Human science for an action sensitive pedagogy*. Albany, NY: State University of New York Press.

VanderVen, K. (2015). Protean selves, trading zones, and nonlinear dynamical systems: The role of play in future progress. In D. P. Fromberg & D. Bergen (Eds.), *Play from birth to twelve* (3rd ed., pp. 409–418). New York: Taylor and Francis/Routledge.

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. In E. M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.) Cambridge, MA: Harvard University Press. (Originally published in 1934)

Vygotsky, L. S. (1987). The role of play in development. In R. W. Reiber & A. S. Carton (Eds.), *The collected works of L.S. Vygotsky* (Vol. 2, pp. 92–104) (N. Minick, Trans.). New York: Plenum. (Originally published in 1932)

Vygotsky, L. S. (1990). Imagination and creativity in childhood. *Soviet Psychology, 28*(1), 84–96.

Wardle, F. (2003). *An introduction to early childhood education: A multidimensional approach to child care and learning*. Boston, MA: Allyn & Bacon.

Yelland, N., Lee, L., O'Rourke, M., & Harrison, C. (2008). *Rethinking learning in early childhood education*. Maidenhead: McGraw-Hill/Open University Press.

Chapter 8
Ergodicity and the Merits of the Single Case

Matthijs Koopmans

Introduction

While dynamical scholarship has played a significant role in the twentieth century in the development of theories describing the underpinnings of development and instruction (Piaget, 1967; Vygotsky, 1978), the impact of recent understandings in dynamical theory, such as fractals, chaos, catastrophe, and complexity, has been relatively modest up this point. There have been some major attempts to accommodate the new insights from those dynamical models to our existing knowledge. For example, van der Maas and Molenaar (1992) used catastrophe theory to describe the dynamical underpinnings of Piagetian stage transitions, and Stamovlasis and Tsaparlis (2012) similarly applied catastrophe models to problem solving in science education. Steenbeek, Jansen and van Geert (2012) used a complexity perspective to describe the real-time scaffolding dynamics in teacher interactions with students with emotional behavioral disorders, and children's play has been conceptualized and described as an emergent developmental process by Fromberg (2010) and Laidlaw, Makovichuk, Wong and O'Mara (2013).

There have also been a number of more broad based discussions about the need to consult models of complexity when studying educational change (Jörg, Davis, & Nickmans, 2007; Koopmans, 2014b; Lemke & Sabelli, 2008). While significant, these developments have not yet clearly positioned educational research from a dynamical systems or complexity perspective as an alternative paradigm to negotiate the relationship between theory and practice in education. In fact, in the policy arena, educational research has evolved more or less in the opposite direction. There has been an increasing reliance on quantitative information in the service of school and teacher accountability models, and on large scale randomized control trial (RCT)

M. Koopmans (✉)
Mercy College, School of Education, Mercy Hall, Rm. SE 31, 555 Broadway,
Dobbs Ferry, NY 10522, USA
e-mail: mkoopmans@mercy.edu

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_8

studies to examine the effectiveness of educational interventions and school reform efforts. Especially the use of RCT designs has been put forward as the preferred method for establishing causal links between educational interventions and their outcomes (Raudenbush, 2005; Slavin, 2002), because it removes the ambiguity that affects the inference of such causality in quasi-experimental and traditional correlation and regression designs (Murray, 1998; National Research Council, 2002).

Several investigators have taken issue with this methodological disposition, most notably Maxwell (2004; 2012), who argues from a qualitative research standpoint that we should scrutinize the processes that generate causality, because they help understand *how* certain outcomes were obtained, rather than merely establishing *that* they were obtained, thereby leaving it up to theory to provide an answer to the *how* question, as RCT often does. From the vantage point of the complex dynamical systems (CDS) paradigm, one would concur with Maxwell's critique because the investigation of the dynamical processes underlying cause and effect potentially offers important qualifications to the knowledge obtained through RCT and related designs (Koopmans, 2014a). Because of their strictly inductive nature (Bogdan & Biklen, 1982), qualitative research methodologies are well suited to uncover such dynamical processes, and they have been successfully used to address a wide range of complex processes in education, such as school reform (White & Levin, 2013), literacy (Laidlaw et al., 2013) and leadership development (Combes & Patterson, 2013). Studies such as these illustrate the usefulness of the qualitative perspective addressing complexity questions. The richness of the data makes it possible in these instances to observe and describe the dynamical processes in detail.

Establishing a unidirectional causal relationship between interventions and outcomes addresses only a narrow part of the causal process. It relies on aggregated information (comparison of group averages) without typically providing a detailed description of the evolution of the behavioral changes that constitute an effect or the self-organizing process through which change manifests itself systemically. CDS argues that the behavior of agents in the educational system (teachers, students, administrators, policy makers) needs to be understood in relation to that of other agents in the same systemic context. The isolation of the behavior of individual units that is needed for aggregation of findings discourages consideration of the systemic interactions that define those larger units. Examples of such interactions are teacher–student interactions, student–student interactions, and the interactions between teachers and the principal's office. Information is also needed about recursive processes that tie the behavior of individuals to that of the systemic constellations in which they interact. Causal attribution requires an understanding of the mutual contingency of the behaviors of individuals in relation to those of larger systemic constellations of which those individuals are part (Koopmans, 1998; Sawyer, 2005). Examples are the relatedness of students and their classroom systems, and school administrators within the systemic confinements of their districts and school buildings.

It is important to note that to meet the external validity needs that come with policy research, the large scale data collection and the generation of replicable findings is required to the same extent in complexity research as it is elsewhere, because a high degree of resolution is needed in the data to detect the dynamical processes of interest. This situation has created a need for rigorous statistical methods that are specific to the discovery and description of complex dynamical processes such as self-organized criticality (Bak, 1996; Jensen, 1998), qualitative transformations (Watzlawick, Weakland, & Fish, 1974); sensitive dependence on initial conditions (Sprott, 2003) and emergence (Goldstein, 1988, Chap. 4). In many academic disciplines, such designs have been fully incorporated by now into the research practice (e.g., Guastello & Gregson, 2011). However, they have been underutilized in education until recently, when a cascade of highly promising work appeared utilizing a wide variety of empirical approaches to uncover the dynamical underpinnings of teacher–learner interaction (Steenbeek et al., 2012), the interaction between the learner and the task (Garner & Russell, 2014), the impact of economic conditions on schooling outcomes (Guevara, López, Posh, & Zúñiga, 2014), the dynamics of collaboration among school administrators (Marion et al., 2012), interpersonal teacher–student dynamics (Pennings et al., 2014), the impact of arousal and motivation on academic achievement (Stamovlasis & Sideridis, 2014), self-similarity in high school attendance (Koopmans, 2015), and the adaptive process of motor learning (Tani et al., 2014). This research illustrates a high level of methodological sophistication in the empirical work done in this area that was virtually absent as recently as a decade ago.

Merits of the Single Case

In a *New York Times* article entitled *Why Doctors Need Stories*, Peter Kramer, the author of *Listening to Prozac* expresses his concern about the disregard of the individual case study as a legitimate source of evidence in medicine (Kramer, 2014). He argues for a revitalization of the case study as a necessary complement to aggregated data that are used in randomized control trial (RCT) studies. In a climate that was still quite hostile to case-based field research (Campbell & Stanley, 1963; Scriven, 1967), Smith and Geoffrey (1968) likewise advocated for the use of such designs in the social sciences to help generate hypotheses to be verified through experimental or correlational studies, and it has often been argued since that using ethnographic research in conjunction with comparative designs can help rule out alternative explanations to RCT-based findings (National Research Council, 2002; Yin, 2000), and strengthen explanations of observed effects that fall outside of the purview of the confirmatory study (National Research Council, 2002).

In education, the study of the particularities of individual cases has traditionally been relegated to qualitative researchers using ethnographic designs to immerse themselves into the system to observe the processes of interest unfold in real time and provide detailed description of the interaction between the various components of the system. The use of ethnography provides “thick descriptions” that may uncover the processes through which transformation takes place in classrooms and school buildings (e.g., Bogdan & Biklen, 1982; Lincoln & Guba, 1985). Ethnographic designs also allow for a triangulation of findings from experimental designs and other designs with thick descriptions of the implementation story, allowing for a deeper understanding of what makes given interventions effective, and what processes ultimately explain the outcomes of an experiment. Ethnographic designs can also be utilized by themselves to investigate causal processes (Maxwell, 2004, 2012), and examine the way antecedent and consequent events play out over time (Miles & Huberman, 1994). There is a long-standing affinity between ethnography and the dynamical literature going back to Gregory Bateson's (1935/1972) anthropological work, and Kurt Lewin's (1947) description group dynamics in terms of stability and change in the systems, a development that has formed the basis for the infusion of qualitative research with more updated dynamical systems concepts (Bloom, Chap. 3; Bloom & Volk, 2007; Hetherington, 2013; Laidlaw et al., 2013).

Nomothetic Versus Idiographic Perspectives

The literature sometimes describes research in the behavioral sciences as representing either one of two distinct epistemological perspectives, often referred to as *nomothetic* and *idiographic* (Burns, 2000; Burrell, 1979). The nomothetic perspective represents the search for generally applicable principles and regularities in the relationship between variables that can be used to make inferences about the population based on what is observed in a sample. It is therefore often equated with quantitative research. In education, conventional RCT designs and quasi-experimental designs would fall under the nomothetic approach. This method derives its utility from the representativeness of the sample to the population. However, information about the particularities of the individual cases gets lost in the aggregation that is required to estimate population characteristics. Hence, there is a trade-off between the rigor derived from conducting observations over a large number of individuals, allowing for a generalization from sampled groups to the populations they represent, and the rigor derived from the accumulation of a large number of sequentially ordered observations of an individual case permitting the detailed estimation of the dynamics of stability and transformation of behavior. This latter approach represents the idiographic approach (Allport, 1960; 1961), which capitalizes on the richness of detail that can be accessed through detailed study of the individual case, and is therefore often equated with qualitative research. Allport presented the idiographic approach in the context of personality psychology, a field of inquiry that takes great interest in the search for stability in the personality traits of individuals over time in the face of shifting environmental conditions, or *stimulus fields*.

Stimulus fields can vary a great deal as the number of variables is large enough to yield an ecologically valid description of the types of influences affecting behavioral outcomes. Stouffer (1941) invites us to an instructive "thought experiment" involving a contingency table with multiple categorical dimensions and notes that as the number of variables increases, the number of cells in the contingency table increases very rapidly as well. Imagine, for instance, how students from lower SES or non-lower SES backgrounds (2 categories) who are male or female (4 categories), take instruction under treatment or control conditions (8 categories), with effective, somewhat effective, or non-effective teachers serving both conditions (24 categories), who all have been classified as belonging in one of the following reading proficiency categories: below basic reading proficiency, approaching proficiency, proficient, advanced (96 categories). If one distinguishes the home environments for each student as supportive or non-supportive, the number of possible ratings an individual can obtain on the variable constellation doubles to 192. In that light, it is not hard to appreciate how little individuals might have in common when it comes to the environmental and other baseline conditions under which learning and instruction takes place. This makes the individual case an understandable and intuitively appealing methodological choice, when the stimulus field harboring these environmental influences comes under closer scrutiny.

The Temporal Dimension

The idiographic perspective carries two major implications. One is that it identifies the need to focus on the situational specificity of behavior of in relation to stimulus fields, which dynamical scholars would refer to *exogenous processes*. The second implication is that "one must account for the *recurrences* and *stabilities* in personal behavior" (Allport, 1961, p. 312) over time. This temporal aspect of behavior, i.e., behavior relative to itself at a previous point time, is referred to in the dynamical literature as *endogenous* processes, and studying them facilitates understanding of its connection to a constantly evolving antecedent stimulus field.

In a linear causal model, causes are assumed to precede effects (Pearl, 2009). In a recursive causal model, causes and effects are assumed to precede each other in an ongoing interrelationship (Sawyer, 2005). In both instances, time is a critical aspect of our understanding of behavior. The argument also been made in the psychological literature that behavioral measures that ignore the temporal dimension are potentially misleading as they disregard the uniqueness of responses to contingencies that are time dependent, as well as the periodicity in behavioral variability both at the individual and the collective level of description. In psychology, periodicity has been measured in such variables as muscle activity cycles (electrocardiograms) and electrical activity recorded from the scalp (electroencephalograms) and eye movements (Barrett, Johnston, & Pennypacker, 1986). In education, studying the temporal dimension has productively informed our understanding of the effectiveness of behavior modification processes in the classroom in terms of the impact of teacher actions on student behavior, linking behavioral outcomes to their antecedent conditions (Hall et al., 1971; Neef, Shade, & Miller, 1994; O'Leary, Becker, Evans, & Saundargas, 1969).

Dewey (1929) describes the temporal span as the most important aspect of the educational process, yet the contributions of time to education have not often been systematically investigated. Glass (1972) laid out time series analysis as a methodological framework for doing so, to deal specifically with the correlations between in time-ordered within-subject observations, the estimation of the constancy of statistical properties of a time trajectory over a longer time period, as well as the perturbation of interventions on a trajectory of outcome measurements. There has been little follow-up in educational research to meet the challenges he put forward (Koopmans, 2014a).

Ergodicity

In 2004, the Dutch psychologist Peter Molenaar published a paper entitled *A Manifesto on Psychology as Idiographic Science: Bringing the Person Back into Scientific Psychology, This Time Forever*. It appeared in the journal *Measurement* (Molenaar, 2004) and it was accompanied by seven peer responses. The article takes note of the fact that an orientation toward the individual case ($N = 1$) is almost completely absent from psychology, where conventional research typically observes and analyzes the behavior of large numbers of individuals, and computes central tendency and variability measures to characterize the group (interindividual variation). If a sample is randomly drawn from a population of interest, the sample results permit generalization to that population. There is a shortage of work, however, that takes it as its mission to observe and analyze the behavior of a particular individual over a single time series (intraindividual variation) in order to learn about the particularities of its behavior and how it evolves across the time spectrum of interest. The description of the individual case permits a level of detail in the description of processes and behaviors that would not be possible if those processes and behaviors are aggregated across groups.

As in psychology, applied researchers in education tend to investigate phenomena cross-sectionally, compute measures of central tendency across individuals to characterize group, and estimate variability in terms of the degree to which individual observations deviate from the means in their group. Thus, the effectiveness of educational interventions is estimated in terms of their impact on mean group outcomes. It is implicitly assumed in such designs that the measures used to estimate outcomes and predictors duly characterize the full time spectrum to which the measurements purport to apply. For example, in the course of a school day, or a school year, it is expected that *if* a large number of observations had been taken across the time spectrum, the statistical properties of the trajectory of ordered observations across time would be constant or at least predictable. Moreover, it is typically assumed that individual variation (error) is randomly distributed around the mean of the entire span of observations, as it would be in the measurements across individuals after a successful statistical modeling effort.

Both in the cross-sectional and in the longitudinal case, variances are traditionally defined in terms of the sum of squared differences of individual observations from their sample means, adjusted for sample size. In cross-sectional research, a population mean is then estimated from the observed sample mean computed across individuals, and the reliability of this estimation is said to increase as the number of individuals who are sampled gets larger. Variability, in this context, is a quantification of the extent to which between subjects outcomes vary from the means of their groups (measurement errors). In the longitudinal case, variability quantifies the extent to which within subjects individual observations vary from the mean of the trajectory of measurements for that particular individual. In both cases, it is typically assumed that these errors are randomly distributed and independent of one another. In either case, it is an empirical question whether this assumption is justified.

In its more general form, the implication of the putative equivalence between the distribution of measurements across cases and within cases across the time spectrum is known as the *ergodic assumption*, which states that the latent variance structure across individuals corresponds to the latent variance structure within individual across the time spectrum. Another way of stating this idea would be to say that the data behavior we can assume in the individual case is somehow captured in the description of the group. The problem with this assumption is that the conclusions that are drawn based on group means do not necessarily apply to all individual cases in that group, and studying the individual cases may reveal idiosyncratic patterns over time that may or may not conform to the causal structure inferred from the cross-sectional group descriptions (Gu, Preacher, & Ferrer, 2014). While it may seem obvious that the ergodic assumption would therefore require empirical confirmation (Molenaar, 2004), the assumption is rarely discussed explicitly in our discipline (but see Gu et al., 2014 and Molenaar, Sinclair, Rovine, Ram, & Corneal, 2009).

The discussion of ergodicity speaks to the important methodological concern of how to account for sources of variability in observed data and the assumptions we are making about relevant data that we typically fail to collect. In part, however, the ergodic question also reflects a substantive concern. Given that traditional fixed effects models may not generalize to anyone in particular (Molenaar, 2004), perhaps the question “Is the program effective?” is the wrong one to ask, and should be rephrased as “For what type of student is what type of program effective?” The latter emphasis allows for different causal structures underlying the behavior of different individuals (Curran & Wirth, 2004).

In its purest form, idiographic science implies that no knowledge can be generalized beyond the single individual, and that the analysis of individuals on a case-by-case basis is the only legitimate basis for the acquisition of knowledge. In their response to Molenaar’s manifesto, Curran and Wirth (2004) rightly note that this position undermines one of the key goals of empirical science, namely external validity, i.e., the applicability of research findings to other people and/or settings than those that were studied. Both Curran and Wirth (2004) and Rogosa (2004) argue in their responses for a conditional between-subjects modeling approach “moving us into the interior of the space demarcated on one side by the strict study of the individual, and on the other side by the sole focus on the characteristics of the overall group (Curran & Wirth, 2004, p. 221).” However, if you define external validity in terms of places and people, as well as time, the challenge of attaining external validity extends to the adequate sampling in all three areas.

We may therefore just not be able to resolve all aspects of external validity within the confines of a single research design.

The use of within-subjects findings to fortify generalizations from between-subjects studies addresses an important external validity issue in conventional RCT designs. A fully reliable randomization of units to various treatment conditions may ensure that causal inferences about the effects of those conditions may be drawn (internal validity, Murnane & Willett, 2011; Murray, 1998). However, since the behavior of individuals or clusters within the sample for those studies may not necessarily be governed by a uniform causal structure, information coming from individual cases may provide important qualifiers to the group results, either as counterexamples, or salient illustrations of the group level processes (Cook & Campbell, 1979).

The possibility of using more refined cross-sectional models to study time dependencies deserves careful consideration. Latent growth curve modeling (e.g., Bryk & Raudenbush, 1992; Willett, 1988) allows for the estimation of nonlinear within-subjects processes (e.g., quadratic, cubic models), and it uses individual growth trajectories as input, and therefore, it can be argued that in many instances, growth modeling allows for valid inferences about both within and between subjects processes. The method also permits an analysis of individual growth trajectories before the cross-sectional components are added to the model. However, the growth modeling currently typically found in the educational research literature does not allow for the more refined estimation of periodicity, a-periodicity and non-periodic error dependencies, which would require the incorporation into the hierarchical regression framework of autoregressive, frequency domain or state space modeling features to estimate error dependencies over the short term and the long term of the trajectory (Koopmans, 2015; Shumway & Stoffer, 2011). In such cases, we would need a sufficient number of input data points per trajectory to ensure representativeness of findings across the time spectrum. The aggregation of findings across cases also results in the loss of information that could potentially be useful for the provision of counter examples to the interpretation of results from cross-sectional comparisons. This individual-level information cannot be recovered from the aggregated information (Rogosa, 2004). This situation calls for an analytical approach that incorporates a within subjects component to between subjects designs to solidify inferences to the population based on samples (Nesselroade, 2004; Rogosa, 2004).

One could argue that studying education and human development almost by definition calls for a within-subjects approach, notwithstanding our methodological predisposition toward group level analyses. Questions of causality in education almost automatically invoke an RCT framework (Raudenbush, 2005) and thereby also the way in which we tend to phrase our research questions. Complex dynamical systems perspectives are also concerned with the causality question, but they are not similarly inclined to articulate their research questions in terms of differences between group averages. Instead, the complexity angle is more likely to focus on how a behavioral trajectory operationalizes self-organizing processes within the system, and how those processes are affected by events that are external to this process. This interest expresses the causal question in a different way.

An Example of Data Suggesting a Non-Ergodic Structure

Can we assume, based on a single snapshot taken at an arbitrary point on the time spectrum, that all microstates within a given time range are equiprobable and that there would be no dependence between observations, had they actually been measured? Upon further empirical scrutiny, what kind of violations can we expect to encounter to this assumption? Let us take a cross-sectional result with a random distribution of individual observations around the group mean, and then play out two scenarios on the time spectrum, one of which would show such randomness, as the ergodic assumption would lead you to expect, and the other which illustrates non-randomness. Figure 8.1 shows these two scenarios in a simulation of $N = 1000$ successive observations, here called Y_t. The top panel on the left (Fig. 8.1a) shows a time series with a mean of zero and a random distribution of measurement error (white noise). The corresponding histogram (Fig. 8.1b) shows that the distribution

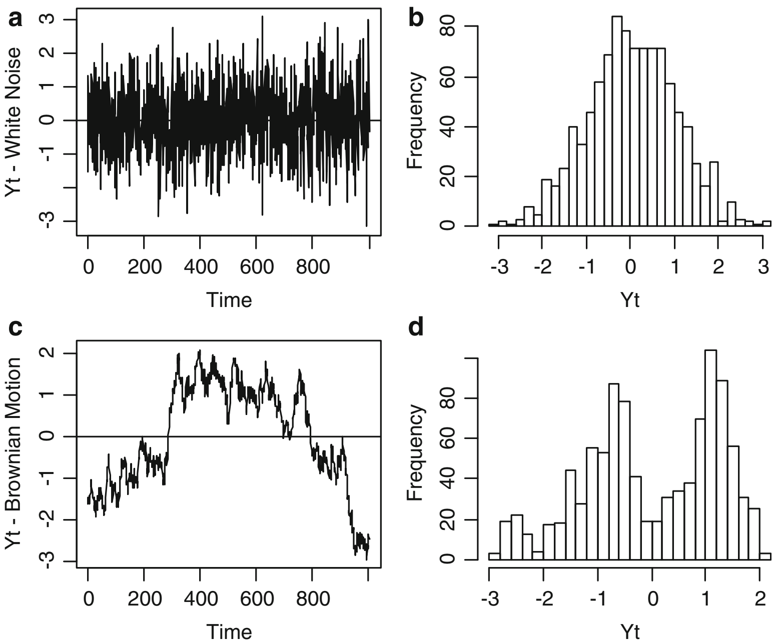

Fig. 8.1 (**a**) Simulated time series with a zero mean, a standard deviation of 1 and randomly distributed errors (white noise), (**b**) corresponding frequency distribution, (**c**) simulated time series with a zero mean, a standard deviation of 1.23 and non-randomly distributed errors (Brownian motion), (**d**) corresponding frequency distribution. $N = 1{,}000$ for both simulations

of errors approximates normality quite well. It can also be shown that the measurement errors are uncorrelated in such cases, and that the pattern revealed by this trajectory carries little information about what the trajectory might look like in the future. If we assume cross-sectional white noise, then, these data would be ergodic in relation to it.

Now consider the simulation result shown in the bottom two panels of the figure. While the mean in these distributions equals zero as well, the time series plot in Fig. 8.1c indicates that the errors are not random, but clearly show a pattern in the way individual observations deviate from the mean of the series. The tight clustering of observations to their immediate neighboring ones contrasts with what is shown in Fig. 8.1a, and it indicates a strong correlation between measurement errors. The histogram (Fig. 8.1d) shows that in this simulation, the mean does not characterize the distribution very well due to the bimodality that is also on display in the time series plot. Therefore, if we assume white noise in a collection of cross-sectional data on Y_t, these data are not ergodic in relation to it, as they display a non-corresponding variance structure, and a cross-sectional mean would actually misrepresents the mean of the observations that distributed this way across the time spectrum.

The simulated error scenario shown in the time in Fig. 8.1c is actually known in the dynamical literature as *Brownian motion*, or the *random walk*, and it characterizes unstable systems in which a high level of dependency between individual observations in close proximity is coupled with a high degree of volatility in the trajectory overall. As a result, there is no constancy in the statistical properties (mean, variance) characterizing the series in its entirety, as those properties heavily depend on the location of the observations on the trajectory. Therefore, conducting valid outcome measurements would require in this situation that the status of the ergodic assumption be addressed (Molenaar, 2004; Molenaar et al., 2009) by empirically establishing the variance structure underlying the sequence of the measurements.

The Dynamics of Daily High School Attendance Rates

One of my currently ongoing research projects is concerned with daily attendance in New York City public schools, which has recorded the daily attendance rates of all of its schools starting in 2004, and continuing up to the time of this writing. This research strictly follows a case study approach, albeit at the school level rather than at the level of individual students. Koopmans (Chap. 14) provides a detailed justification of the research agenda as well as its methodology. It will suffice here to say that the inspection of daily attendance trajectories over a longer time period allows us to discern patterns of non-randomness in temporal educational data that would remain hidden if conventional summary statistics are used to aggregate results across schools. Simply reporting mean daily attendance rates and their standard deviations is insufficient if the distribution of individual observations

Table 8.1 Demographic characteristics of School A, B, and C: Academic year 2013–2014

	School A	School B	School C
Enrollment	315	105	184
Gender			
% Female	50.2	77.3	59.8
Ethnicity			
% Asian	3.8	1.0	2.7
% Black	30.5	37.1	26.6
% Hispanic	64.1	60.2	65.2
% White	1.3	0.0	4.3
% Other	1.0	1.0	1.1
Other			
% English Language Learners	9.5	1.9	7.1
% Students with Special Needs	31.1	3.8	31.5
% Eligible for Free or Reduced Priced Lunch	100.0	90.5	83.2

Table 8.2 Summary statistics for the daily attendance rates in 2013–2014: School A, B, and C

	School A	School B	School C
Mean	.77	.55	.87
Standard deviation	.09	.14	.08
Minimum	.26	.17	.39
First quartile	.74	.47	.87
Median	.78	.53	.88
Third quartile	.82	.63	.90
Maximum	1.00	.99	.99

(i.e., attendance measures on any given day) is not random. A look at the attendance trajectory over time allows for an empirical verification of this assumption. A sample of approximately 180 observations in a year provides sufficient resolution to look closely at many aspects of the dynamics of school attendance as an ongoing process, and develop hypotheses about the susceptibility of these patterns to external influences (Koopmans, 2015). Below, I would like to illustrate this point by discussing the daily attendance trajectories in three New York City high schools.

Table 8.1 summarizes the basic enrollment and demographic information for the three schools in the 2013–2014 school year. It can be seen that the schools are similar in three important respects. All three are small high schools with a total enrollment ranging from 105 to 315. Moreover, the schools are demographically similar with an overwhelming majority of students being Black or Hispanic and about twice as many Hispanic as Black students. Furthermore, all three schools serve students from predominantly poor socioeconomic backgrounds, as can be seen by the high percentage of students eligible for free or reduced priced lunch.

Table 8.2 shows the traditional summary statistics for the attendance rates in these schools in the 2013–2014 school year. A total of 187 daily attendance rates were recorded in that year. The mean attendance rates vary considerably, ranging

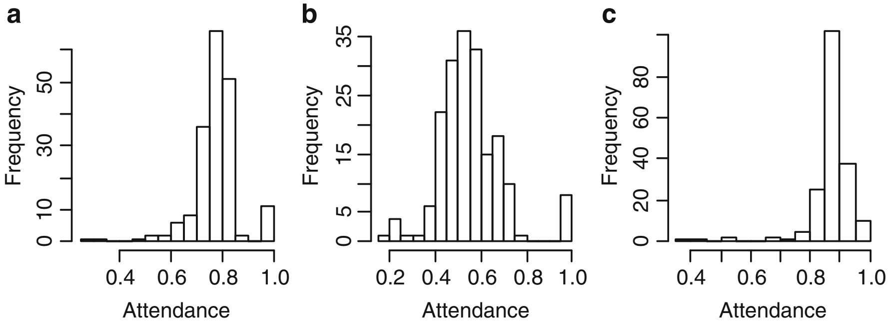

Fig. 8.2 (**a**, **b**, and **c**) Histograms of the daily attendance rates for the 2013–2014 School Year in School A, B, and C, respectively

from 55 % in School B to 87 % in School C. The corresponding medians are 53 and 88 %. Inspection of the other statistics in the table indicate a negatively skewed distribution in Schools A and C: the first and third quartile in those two schools are close to the median and to each other, while the minimum value in the distribution falls far below the first quartile. The quartiles are farther apart in School B than in the other two schools. The histograms shown in Fig. 8.2a, b, c and d further illustrate these features for the three schools.

Turning to the single-case dynamics, Fig. 8.3a, b, c and d show the attendance trajectories for the 2013–2014 school year in Schools A, B, and C. A straight line is superimposed in each plot to represent the average daily attendance for that school for the entire school year. In School A, daily attendance rates appear to be fairly stable across the time spectrum, except for some turbulence toward the middle of the trajectory, possibly reflecting the ramifications of inclement weather during the winter months. This turbulence is shown by the dips into lower values, but also their off-set with peaks into higher values during the same period. In School C the low-range dips are even more pronounced, though not compensated for by instances of unusually high attendance rates. It should be noted, however, that mean attendance rates are much higher on the average in School C to begin with, and very stable as well as can be seen by the low level of variability between observations.

In School B, attendance rates show less stability over time than those in the two other schools, and there is a steady decline in those rates as the school year progresses toward the middle of the year, and a slight recovery in the second half of that year. It can also be seen that in School A and B, the attendance rates become highly variable toward the end of the school year, reflecting in all likelihood the end of year celebrations and wrap-up. In School C the trajectory does not appear perturbed to the same extent toward the end of the year, but simply shows above average attendance for the last string of school days.

School A and C illustrate a stationary process, which is to say that there are no clear signs of an upward or downward trend, heteroscedasticity, or other transformations in the outlook of the series that depend on the position of the data points

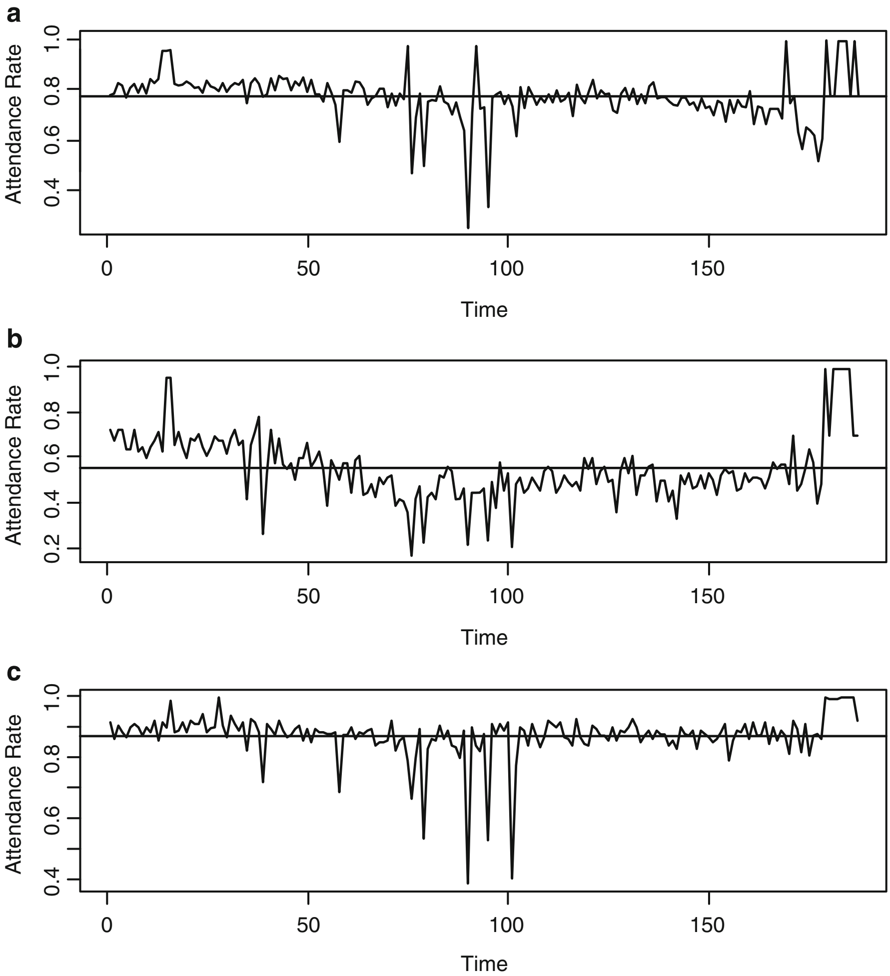

Fig. 8.3 (**a**, **b**, and **c**) Daily attendance rates for the 2013–2014 School Year in School A, B, and C, respectively, as a time series

on the timeline. The peaks and valleys are outlying observations in an otherwise consistent trend (and would be treated as such in the statistical analysis, see Koopmans, 2014c for an example). The trajectory shown for School B in Fig. 8.3b, on the other hand shows a mild resemblance to the Brownian motion shown in Fig. 8.1c. The mean of the series underpredicts the observed data for extended periods in the course of the school year (roughly corresponding to fall and spring), while it overpredicts in the winter months.

In all three instances discussed here, however, traditional measures of central tendency do not characterize very well what goes on in these data, and they conceal important time-dependent features. A case-by-case scrutiny of the observations and their dependency over time provides an altogether different appreciation of daily attendance in these three schools. Educational practitioners are likely to be aware of the fluctuations in attendance rates in their classrooms and school buildings, as well as their seasonal dependence, but these conventional measures do not operationalize this aspect of the variability in those rates (Koopmans, 2015), and therefore, the study of daily attendance rates over time is a useful endeavor in its own right.

Endogenous Processes and Feedback Loops

Successful causal attribution requires that baseline conditions in the system are measured in sufficient detail to estimate the propensity toward change in the system, as well as the change processes that actually occur in response to given interventions (Koopmans, 2014b). In conventional research, the measurement of the baseline typically involves at most a limited number of pretest observations and a comparison between pretest and posttest to determine whether change has been produced by intervention. The change process is then inferred a-posteriori. A convincing description of the stability in a system with respect to certain measurement outcomes requires a reliable estimation of the intra-subject variability in those outcomes. Such estimation, in turn, requires a much larger number of measurement occasions than is typically provided in traditional pretest-posttest designs.

Moreover, given that establishing a cause-effect relationship at the aggregate level does not necessarily carry over to all individuals, the individual case may prove to be a source for instructive counter-examples. From the vantage point of complex dynamical systems, there are two interrelated aspects to causality that are not captured in conventional research designs: the *endogenous process* and the *recursive feedback loop* between different parts of the system. The endogenous process explains observed outcomes at a given point in time in terms of previous measurements of that same outcome and thus can be seen as an indicator of the adaptive behavior of the system with respect to that particular outcome (e.g., learning) irrespective of a particular impulse from sources external to the system. An educational intervention, in this context, can be seen as an *exogenous* process that may or may not have an impact on the endogenous process. The analytical framework for conducting such assessments has been in place for a long time, and has been part and parcel of the empirical behavior modification literature referred to above, that measures the impact of given interventions on the endogenous process.

The measurement of recursive feedback loops is a challenge of an altogether different magnitude, as it requires an estimation of the behavior of individual members of a given system in relation to the behavior of the system at large. While there is a rich literature to appreciate such feedback loops in theory (Koopmans, 1998; McKelvey, 2004; Minuchin & Fishman, 1981; Sawyer, 2005), the empirical literature has not yet risen to the challenge of connecting these two levels of description within a single analytical framework (but see Salem, 2013). In the linear paradigm, there is a cross-over between levels of description in the hierarchical multilevel designs equipped to deal with nested data structures (Bryk & Raudenbush, 1992; Gelman & Hill, 2007). However, these approaches do not deal with the reciprocal nature of the influences between the behavior of systems and that of the individual members making up those systems, nor do they provide great detail about how this process plays out over time. Both aspects are central to the interests of complex dynamical scholarship because it describes the process of self-regulation through which systems maintain their structure, composition and integrity as a distinct functional unit. The analytical focus on endogenous processes does not at all preclude an analysis of external influences that may perturb the system (McDowall, McCleary, Meidinger, & Hay, 1980). In fact, the impact of those external influences on the behavior of the system is better understood if we have knowledge of the endogenous processes through which the system maintains itself.

Concluding Remarks

The single case presents an alternative perspective on the question of cause and effect in education. It can provide us with a fine-grained description of the transformations that constitute the effects of interest. In the attendance data discussed above, for example, one could speculate about a "winter effect" on the attendance trajectories, where inclement weather impacts the transportation options for students and these options may vary depending on the location of the school building. Likewise, these influences can be estimated relative to other causal factors such as parental support and effective school building leadership (Koopmans, 2015). The confirmation of such causal processes requires a triangulation of the data presented here with those from other sources. The detailed sampling of observations across the time spectrum enables us to contemplate these causal processes to begin with. They remain hidden in cross-sectional summary statistics.

Traditional summary statistics such as means and standard deviations, as well as ordinary least squares regression, make assumptions about the data that do not carry over very well to time series data. When ordered observations over time are analyzed, two issues invariably come up: error dependency between observations and the constancy of the statistical properties of the series across the entire time spectrum. The dependency between observations across the time spectrum is referred to in the time series literature as *autocorrelation*. The constancy of statistical characteristics across the time spectrum is called *stationarity* and the detection of these two features is a central part in the description and analysis of most time-ordered data (Box & Jenkins, 1970). Comparison of Fig. 8.1a and c above illustrates how drastically the appearance of outcome trajectories can differ depending on whether the data are stationary (Fig. 8.1a) or not (Fig. 8.1c). Stationary time series are not necessarily random in the sense of white noise, as they can also contain clustering and dependencies between observations i.e., autocorrelation, that needs to be modeled as well. Koopmans (Chap. 14) provides further elaboration on that aspect of the sequential ordering of the data.

Dealing with the temporal aspects of data is common practice in education, as in traditional pretest-posttest comparisons and the use of growth modeling to describe changes in student achievement over time (Bryk & Raudenbush, 1992; Rogosa, 2004). However, these designs are not equipped to address the dynamical features of changes over time, nor are they typically used to address dynamical questions about those changes. Rogosa, Floden, and Willett (1984) provide an interesting exception to this latter point by examining the stability in teacher behavior over time. However, while very dynamical in the way it is framed, with four to six measurement occasions the study does not sample the time dependent process adequately to make meaningful inferences about the stability of teacher behavior as a temporal phenomenon. This lack representativeness of sampling across the time spectrum is characteristic of most of the cross-sectional work done in educational research.

It is far beyond the scope of the work presented here to summarize all we have learned over the years from the study of single cases in education. Suffice to say here that the case study has many times been productively used in the field, such as for example in the school district reform literature (e.g., Cuban, 2010; Reville, 2007), the developmental literature (e.g., Bassano & van Geert, 2007; Brown, 1973), or the aforementioned behavior modification studies. Sometimes, this work is explicitly concerned with hypothesized dynamical processes (e.g., Bassano & van Geert, 2007; Johnson, 2013; Laidlaw, Makovichuk, Wong, & O'Mara, 2013; Molenaar et al., 2009), but often it is not. This chapter argues that there is clear potential in the study of the single case to obtain a deeper understanding of the dynamical underpinnings of cause and effect, without the information loss that comes with the aggregation of information across cases (Rogosa, 2004). The particular contribution of statistics to this area lies in the detailed analyses of the temporal ordering of the information elements. Ethnographic research, on the other hand, can provide the thick descriptions that may one day form the basis for the development of meaningful mathematical models underlying interactive behavior (Dobbert & Kurth-Schai, 1992).

In the realm of quantitative research here is a trade-off between sampling rigor across cases between subjects and across observations within subjects. The single case design attains sampling rigor in this latter respect, and thus address different types of questions that are of interest to the field. While ethnographic research may not require this type of sampling rigor for inferential purposes, its thick descriptions are very suitable for the description of the temporal features of behavior. Miles and Huberman (1994) explain how qualitative approaches can be used to analyze the temporal features of the behavior of individuals and organizations. Both the qualitative and the quantitative single case study are well-equipped to address the question of the underlying dynamics of stability and change in greater detail, but have been used infrequently for that purpose.

In its most uncompromised reading, the idiographic approach altogether rejects the aggregation of information for purposes of statistical summary as a matter of principle (Molenaar, 2004), as a result of which study of human development would boil down to the accumulation of all of our life stories. This would leave us without an accessible empirically based knowledge structure about what we can learn from those stories (Curran & Wirth, 2004). Rather than viewing the single case as a radical alternative to nomothetic science, it may be more productive to view it as a necessary supplement to it. The triangulation of data from large scales studies with those from single case designs can enrich those studies with added detail about causal mechanisms, and challenge the interpretation of aggregated findings through the provision of individual counterexamples (Flyvbjerg, 2006).

Perhaps it is possible one day to deal with complex cross-sectional data structures and detailed temporal information about those structures simultaneously. In the methodological field, the discussion about integrating state space techniques and structural equation modeling (Browne & Zhang, 2007; Molenaar, 2009; Molenaar, van Rijn, & Hamaker, 2007) is a highly promising development. However, this work seems to be in its early stages and does not yet provide a clear answer to the question how we can meaningfully reduce the sheer volume of the information from individual cases to manageable proportions without losing the level of resolution that is needed to study the finer details of dynamical complexity, the representativeness of observations across cases, or both.

Do we need an ergodic argument to make a case for single case designs? Perhaps this argument unnecessarily complicates a relatively straightforward justification for using such designs (Thum, 2004). Rather than framing the case for the single case in terms of underlying assumptions about data distributions, we can also promote the intrinsic interest of the particularities of the single case for the sake of learning and scholarship (Stake, 1994). The readiness to investigate the particularities of the individual case through statistical means would represent an interesting point of departure in the field of education, which lacks a time series analysis tradition at this point.

Within the field of complex dynamical systems, the increasing reliance on the investigation of complex processes through statistical means is a favorable development as well, as it takes us away from the reliance on woolly inspirational metaphors, such as "the edge of chaos" (Dodds, 2012; Goldstein, 1995; Koopmans, 2009; Waldrop, 1992) to the development of observation and measurement strategies specifically designed to identity the specific empirical referents of such hypothesized processes and constructs (Bak, 1996; Jensen, 1998; Koopmans, 2009, 2015). Such work would generate knowledge that is falsifiable as well as useful to educational practitioners and policy makers, as it gives the dynamical aspects of their thinking about education the representation it deserves in our research efforts.

References

Allport, G. W. (1960). *Personality and social encounter: Selected essays*. Boston, MA: Beacon Press.

Allport, G. W. (1961). *Pattern and growth in personality*. New York, NY: Holt, Rinehart & Winston.

Bak, P. (1996). *How nature works: The science of self-organized criticality*. New York: Springer.

Barrett, B. H., Johnston, M. M., & Pennypacker, H. S. (1986). Behavior: Its units, dimensions and measurement. In R. O. Nelson & S. C. Hayes (Eds.), *Conceptual foundations of behavioral assessment* (pp. 156–200). New York: Guildford.

Bassano, D., & van Geert, P. (2007). Modeling continuity and discontinuity in utterance length: A quantitative approach to changes, transitions and intra-individual variability in early grammatical development. *Developmental Science, 10*, 588–612.

Bateson, G. (1935/1972). Culture contact and schismogenesis. In G. Bateson (Ed.), *Steps toward an ecology of mind: A revolutionary approach to man's understanding of himself* (pp. 61–71). [Reprinted from Man, XXXV, Article 199].

Bloom, J. W., & Volk, T. (2007). The use of metapatterns for research into complex systems of teaching, learning, and schooling. *Complicity: An International Journal of Complexity and Education, 4*, 45–68.

Bogdan, R. C., & Biklen, S. K. (1982). *Qualitative research for education: An introduction to theory and methods*. Boston, MA: Allyn & Bacon.

Box, G. E. P., & Jenkins, G. M. (1970). *Time series analysis, forecasting and control*. San Francisco, CA: Holden-Day.

Brown, R. W. (1973). *A first language: The early stages*. Cambridge, MA: Harvard University Press.

Browne, M. W., & Zhang, G. (2007). Repeated time series models for learning data. In S. M. Boker & M. J. Wenger (Eds.), *Data analytic techniques for dynamical systems* (pp. 25–45). Mahwah, NJ: Erlbaum.

Bryk, A. S., & Raudenbush, S. W. (1992). *Hierarchical linear models: Applications and data analysis methods*. Newbury Park, CA: Sage.

Burns, R. B. (2000). *Introduction to research methods* (4th ed.). London: Sage.

Burrell, G. (1979). *Sociological paradigms and organizational analysis*. London: Heinemann.

Campbell, D. T., & Stanley, J. C. (1963). Experimental and quasi-experimental designs for research in teaching. In N. L. Gage (Ed.), *Handbook for research in teaching*. Chicago, IL: Rand-McNally.

Combes, B. H., & Patterson, L. (2013, April). *Leadership in special education: Using human systems dynamics to address sticky issues*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Cook, T., & Campbell, D. (1979). *Quasi-experimentation: Design and analysis issues for field settings*. Boston, MA: Houghton Mifflin.

Cuban, L. (2010). *As good as it gets: What school reform brought to Austin*. Cambridge, MA: Harvard University Press.

Curran, P. J., & Wirth, R. J. (2004). Interindividual differences in intraindividual variation: Balancing internal and external validity. *Measurement, 2*, 219–227.

Dewey, J. (1929). *The sources of a science in education*. New York: Liveright.

Dobbert, M. L., & Kurth-Schai, R. (1992). Systematic ethnography: Toward an evolutionary science of education and culture. In M. D. LeCompte, W. L. Millroy, & J. Preissle (Eds.), *The handbook of qualitative research in education* (pp. 93–159). San Diego, CA: Academic Press.

Dodds, J. (2012, March). *Ecology and psychoanalysis at the edge of chaos*. Presented at the International Nonlinear Science Conference, Barcelona, Spain.

Flyvbjerg, B. (2006). Five misunderstandings about case-study research. *Qualitative Inquiry, 12*, 219–245.

Fromberg, D. P. (2010). How nonlinear systems inform meaning and early education. *Nonlinear Dynamics, Psychology and Life Sciences, 14*, 47–68.

Garner, J., & Russell, D. (2014, April). *The symbolic dynamics of self-regulated learning: Exploring the application of orbital decomposition*. Paper presented at the annual meeting of the American Educational Research Association, Philadelphia, PA.

Gelman, A., & Hill, J. (2007). *Data analysis using regression and multi-level/hierarchical models*. New York: Cambridge University Press.

Glass, G. V. (1972). Estimating the effects of intervention into a non-stationary time-series. *American Educational Research Journal, 9*, 463–477.

Goldstein, J. (1988). A far-from-equilibrium systems approach to resistance to change. *Organizational Dynamics, 17*, 16–26.

Goldstein, J. (1995). The Tower of Babel in nonlinear dynamics: Toward the clarification of terms. In R. Robertson & A. Combs (Eds.), *Chaos theory in psychology and the life sciences* (pp. 39–47). Mahwah, NJ: Erlbaum.

Gu, F., Preacher, K. J., & Ferrer, E. (2014). A state space modeling approach to mediation analysis. *Journal of Educational and Behavioral Statistics, 39*, 117–143.

Guastello, S. J., & Gregson, R. (2011). *Nonlinear dynamical systems analysis for the behavioral sciences using real data*. New York: CRC Press.

Guevara, P., López, L., Posh, A., & Zúñiga, R. (2014). A dynamic nonlinear model for educational systems: A simulation study for primary education. *Nonlinear Dynamics, Psychology and Life Sciences, 18*, 91–108.

Hall, R. V., Fox, R., Willard, D., Goldsmith, L., Emerson, M., Owen, M., et al. (1971). The teacher as observer and experimenter in the modification of disputing and talking-out behaviors. *Journal of Applied Behavior Analysis, 4*, 141–149.

Hetherington, L. (2013). Complexity thinking and methodology: The potential of 'complex case study' for educational research. *Complicity: An International Journal of Complexity in Education, 10*, 71–85.

Jensen, H. J. (1998). *Self-organized criticality: Emergent complex behavior in physical and biological systems*. Cambridge: Cambridge University Press.

Johnson, L. (2013). *A case study exploring complexity applications to qualitative inquiry*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Jörg, T., Davis, B., & Nickmans, G. (2007). Position paper: Toward a new complexity science of learning and education. *Educational Research Review, 2*, 145–156.

Koopmans, M. (1998). Chaos theory and the problem of change in family systems. *Nonlinear Dynamics in Psychology and Life Sciences, 2*, 133–148.

Koopmans, M. (2009). Epilogue: Psychology at the edge of chaos. In S. J. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: The theory of nonlinear dynamical systems* (pp. 506–526). New York: Cambridge University Press.

Koopmans, M. (2014a). Nonlinear change and the black box problem in educational research. *Nonlinear Dynamics, Psychology and Life Sciences, 18*, 5–22.

Koopmans, M. (2014b). Change, self-organization and the search for causality in educational research and practice. *Complicity: An International Journal of Complexity and Education, 11*, 20–39.

Koopmans, M. (2014c, May). *Using ARIMA modeling to analyze daily high school attendance rates*. Presented at the Modern Modeling Methods (M3) conference, Storrs, CT.

Koopmans, M. (2015). A dynamical view of high school attendance: An assessment of short-term and long-term dependencies in five urban schools. *Nonlinear Dynamics, Psychology and Life Sciences, 19*, 65–80.

Kramer, P. D. (2014, October 19). Why doctors need stories. *New York Times Sunday Review*, p. 1, 7.

Laidlaw, L., Makovichuk, L, Wong, S., & O'Mara, J. (2013, April). *Complexity, pedagogy, play: On using technology within emergent learning structures with young learners*. Presented at the American Educational Research Association, San Francisco, CA.

Lemke, J., & Sabelli, N. H. (2008). Complex systems and educational change: Toward a new research agenda. *Educational Philosophy and Theory, 40*, 118–129.
Lewin, K. (1947). Frontiers in group dynamics. *Human Relations, 1*, 5–41.
Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.
Marion, R., Klar, H., Griffin, S., Brewer, C., Baldwin, A., & Fisk, W. (2012, March). *A network analysis of complexity dynamics in a bureaucratically pressured organization*. Presented at the international nonlinear science conference, Barcelona, Spain.
Maxwell, J. A. (2004). Causal explanation, qualitative research, and scientific inquiry. *Educational Researcher, 33*, 3–11.
Maxwell, J. A. (2012). The importance of qualitative research for causal explanation in education. *Qualitative Inquiry, 18*, 655–661.
McDowall, D., McCleary, R., Meidinger, E. E., & Hay, R. A., Jr. (1980). *Interrupted time series*. Thousand Oaks, CA: Sage. Sage university series on quantitative applications in the social sciences.
McKelvey, B. (2004). Complexity science as an order-creation science: New theory, new method. *Emergence, Complexity and Organization, 6*, 2–27.
Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook* (2nd ed.). Thousand Oaks, CA: Sage.
Minuchin, S., & Fishman, H. (1981). *Family therapy techniques*. Cambridge, MA: Harvard.
Molenaar, P. C. M. (2004). A manifesto on psychology as in idiographic science: Bringing the person back into psychology, this time forever. *Measurement, 2*, 201–218.
Molenaar, P. C. M. (2009). *State space techniques in structural equation modeling: Transformation of latent variables in and out of latent variable models*. Retrieved November 12, 2014, from http://www.hhdev.psu.edu/media/dsg/files/StateSpaceTechniques.pdf
Molenaar, P. C. M., Sinclair, K. O., Rovine, M. J., Ram, N., & Corneal, S. E. (2009). Analyzing developmental processes on an individual level using non-stationary time series modeling. *Developmental Psychology, 45*, 260–271.
Molenaar, P. C. M., van Rijn, P., & Hamaker, E. (2007). A new class of SEM model equivalences and its implications. In S. M. Boker & M. J. Wenger (Eds.), *Data analytic techniques for dynamical systems* (pp. 189–211). Mahwah, NJ: Erlbaum.
Murnane, R. J., & Willett, J. B. (2011). *Methods matter: Improving causal inference in educational and social science research*. New York: Oxford University Press.
Murray, D. M. (1998). *Design and analysis of group-randomized trials*. New York: Oxford University Press.
National Research Council (2002). *Scientific research in education*. Committee on Scientific Principles for Educational Research. Center for Education. Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.
Neef, N. A., Shade, D., & Miller, M. (1994). Assessing influential dimensions of reinforcers on choice in students with serious emotional disturbance. *Journal of Applied Behavior Analysis, 25*, 555–593.
Nesselroade, J. R. (2004). Yes, it's time: Commentary on Molenaar's manifesto. *Measurement, 2*, 227–230.
O'Leary, K. D., Becker, W. C., Evans, M. B., & Saundargas, R. A. (1969). A token reinforcement program in a public school: Replication and systematic analysis. *Journal of Applied Behavior Analysis, 2*, 3–13.
Pearl, J. (2009). *Causality: Models, reasoning and inference* (2nd ed.). New York: Cambridge University Press.
Pennings, H. J. M., Brekelmans, M., Wubbels, T., van der Want, A. C., Claessens, L. C. A., & van Tartwijk, J. (2014). A nonlinear dynamical systems approach to real-time teacher behavior: Differences between teachers. *Nonlinear Dynamics, Psychology, and Life Sciences, 18*, 23–46.
Piaget, J. (1967). *Six psychological studies*. New York: Vintage.
Raudenbush, S. W. (2005). Learning from attempts to improve schooling: The contribution of methodological diversity. *Educational Researcher, 34*, 25–31.

Reville, S. P. (2007). *A decade of school reform: Persistence and progress in the Boston Public Schools*. Cambridge, MA: Harvard University Press.

Rogosa, D. (2004). Some history on modeling the processes that generate the data. *Measurement, 2*, 231–234.

Rogosa, D., Floden, R., & Willett, J. B. (1984). Assessing the stability of teacher behavior. *Journal of Educational Psychology, 76*, 1000–1027.

Salem, P. (2013). The complexity of organizational change: Describing communication during organizational turbulence. *Nonlinear Dynamics, Psychology, and Life Sciences, 17*, 49–65.

Sawyer, R. K. (2005). *Social emergence: Societies as complex systems*. New York: Cambridge University Press.

Scriven, M. (1967). The methodology of evaluation. In R. W. Tyler, R. M. Gagné, & M. Scriven (Eds.), *Perspectives of curriculum evaluation* (pp. 39–85). Chicago, IL: Rand-McNally.

Shumway, R. H., & Stoffer, D. S. (2011). *Time series and its applications: With R examples*. New York: Springer.

Slavin, R. E. (2002). Evidence-based education policies: Transforming educational practice and research. *Educational Researcher, 31*, 15–21.

Smith, L. M., & Geoffrey, W. (1968). *The complexities of the urban classroom*. New York: Holt, Rinehart, & Winston.

Sprott, J. C. (2003). *Chaos and time series analysis*. New York: Oxford University Press.

Stake, R. E. (1994). Case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 236–247). Thousand Oaks, CA: Sage.

Stamovlasis, D., & Sideridis, G. D. (2014). Ought-avoidance: Nonlinear effects on arousal under achievement situations. *Nonlinear Dynamics, Psychology and Life Sciences, 18*, 67–90.

Stamovlasis, D., & Tsaparlis, G. (2012). Applying catastrophe theory to an information-processing model of problem solving in science education. *Science Education, 96*, 393–410.

Steenbeek, H., Jansen, L., & van Geert, P. (2012). Scaffolding dynamics and the emergence of problematic learning trajectories. *Learning and Individual Differences, 22*, 64–75.

Stouffer, S. A. (1941). Notes on the case study and the unique case. *Sociometry, 4*, 349–357.

Tani, G., Corra, U. C., Basso, L., Benda, R. N., Ugrinowitsch, H., & Choshi, K. (2014). An adaptive process model of motor learning: Insights for the teaching of motor skills. *Nonlinear Dynamics, Psychology and Life Sciences, 18*, 47–66.

Thum, Y. M. (2004). Some additional perspectives on representing behavioral change and individual variation in psychology. *Measurement, 2*, 235–240.

van der Maas, H. L. J., & Molenaar, P. C. (1992). Stagewise cognitive development: An application of catastrophe theory. *Psychological Review, 99*, 395–417.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Waldrop, M. M. (1992). *Complexity: The emerging science at the edge of order and chaos*. New York: Simon & Schuster.

Watzlawick, P., Weakland, J., & Fish, R. (1974). *Change: Principles of problem formation and problem resolution*. New York: Norton.

White, D. G., & Levin, J. A. (2013, April). *Navigating the turbulent waters of school reform guided by complexity theory*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Willett, J. B. (1988). Questions and answers in the measurement of change. *Review of Research in Education, 15*, 345–422.

Yin, R. K. (2000). Rival explanations as an alternative to reforms as 'experiments.'. In L. Bickman (Ed.), *Validity and social experimentation: Donald Campbell's legacy* (pp. 239–266). Thousand Oaks, CA: Sage.

Chapter 9
Catastrophe Theory: Methodology, Epistemology, and Applications in Learning Science

Dimitrios Stamovlasis

Introduction

Catastrophe theory is a mathematical theory that addresses discontinuities and qualitative changes in dynamical systems. It states that in a complex dynamical system changes could be smooth and linear, but that they could also be nonlinear, and contrary to the common sense anticipation, they might be surprisingly large even though the input is quite small. In reality, we observe that except human constructions, straight lines do not exist in nature neither in social and human experience. The assumption of linearity in social science research, in both qualitative and quantitative approaches, has been a philosophical convention, since it is the simplest one to examine, by the methodological tools available thus far. Moreover it facilitated the cause-and-effect notion of classical reductionistic interpretations. Catastrophe theory is acknowledged for its descriptive and interpretative modeling power and its uniqueness to be the most applicable methodological approach that infers nonlinearity from cross-sectional empirical data. This chapter begins with a brief history of its mathematical foundation and continues with the presentation of catastrophe theory in its deterministic and stochastic forms. Subsequently, all the current statistical methodologies are presented and the epistemology associated with catastrophe theory and nonlinear dynamics is extensively discussed. Finally, applications within the neo-Piagetian framework and science education research are presented.

D. Stamovlasis (✉)
Aristotle University of Thessaloniki, Department of Philosophy and Education, 54124 Thessaloniki, Greece
e-mail: stadi@edlit.auth.gr

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_9

A Brief History of Catastrophe Theory

The history of catastrophe theory begins in the decade of 1880s, when the famous French mathematician Henri Poincaré founded *bifurcation theory* while working on a qualitative analysis of systems by means of nonlinear differential equations. Poincaré was interested in answering questions concerning the structural stability of the solar system. The main question was whether the planets would escape to infinity or crash into each other if they experience an external shock. He found that small perturbations would either leave the system relatively unchanged or would cause it to move in a very different mode. This signified the onset of bifurcation theory, which led to *singularity theory*, as a special case of which catastrophe theory appeared decades later. A substantial contribution on the development of the above mathematical theories is credited also to the Russian mathematician Vladimir Arnol'd (1988, 1992). However, the basic notions and the formulation were Poincaré's work: Bifurcation theory considers a dynamical system described by ordinary differential equations. Certain points where the first derivative equals zero characterize equilibrium states. At a critical point, called a *singularity*, this set of equilibria bifurcates into separate branches. Note that such critical points are the degenerate ones and they are not associated extrema.[1] This splitting is a bifurcation of the degenerate equilibrium and since it concerns equilibrium solutions it makes the connection between the singularity of mapping and structural stability (Morse, 1931). A crucial step in the history of catastrophe theory was the invention that there were many types of such functions and two of them are stable in all their forms; later they become known as the fold and the cusp catastrophe (Whitney, 1955). The discovery of these two types of structurally stable singularities for differentiable mappings was the first element of *catastrophe theory*, even though at that time the emerging theory was not referred with this name.

In 1950s René Thom, a French mathematician, working on structural stability introduced the notion of *transversality* and stated the corresponding theorem in order to describe the transverse intersection properties of smooth maps (Thom, 1956). According to Thom's theorem any smooth map may be deformed by an arbitrary small amount into a map that is transverse to a given sub-manifold.[2] The *transversality theorem* facilitated the classification of singularities or elementary catastrophes and Thom (1972) managed to define seven types of singularities, which can be described by up to six dimensions and named them as the "elementary" catastrophes. For systems with dimensionality greater than eleven,

[1] In mathematics, a critical point of a differentiable function is a point where the derivative is zero (or undefined). Degeneracy refers to a property of a case in which an element of a class of objects is qualitatively different from the rest of the class belonging to a different, usually simpler, class. A singular point is a degenerate one and is not associated with usual non-degenerate extrema, maximum or minimum, where the first derivative is also zero.

[2] Transversality in Thom's theorem refers to a generic property of the maps according to which any smooth map f: $X \to Y$ may be deformed by an arbitrary small amount into a map that is transverse to a given $Z \subseteq Y$ sub-manifold (Arnol'd, 1988).

singularities were difficult to be classified, because, as it was shown later by Arnol'd and his coworkers, the number of categories becomes infinite (Arnol'd, Gusein-Zade, & Varchenko, 1985).

Thom specified the basic mathematical formulation of the elementary catastrophe theory considering the behavior of a *deterministic* dynamical system that is described by n state variables y_j and r control variables b_i. A potential function is assumed to be operating on this set of state and control variables so that for all y_j the first derivative is zero, while the set of points where the derivative equals zero constitutes the equilibrium manifold. The first catastrophe which attracted attention was the cusp catastrophe with a three-dimensional equilibrium surface described by one state variable as a function of two control variables. The topology suggests that when the control variables change, even slowly, the state variable adjusts quickly on the equilibrium manifold. The topological characteristics of the response surface of catastrophe model exhibit a number of features, such as hysteresis, bimodality, inaccessibility, sudden jumps, and divergence, which are presented in the following section.

While the formulation of catastrophe theory was being developed in the area of mathematics, in 1960s and 1970s, a number of applications appeared in the literature of economics, psychology, and other behavioral and social science (see Poston & Stewart, 1978; Woodcock & Davis, 1978). Most of the very early applications were with low-dimensional catastrophes, in the sense of having a few predictors, and their onset brought up methodological and epistemological considerations with a plethora of concerns. One fundamental question in a continuing debate over catastrophe theory was the existence of system's potential function. A potential function posits a symmetry condition that all cross-partial derivatives are equal, which again singularity theory does not require. Within the ongoing discussion it seemed also that the mathematics of Arnol'd had departed from Thom's original formulation and this became a further controversy which appeared in late 1970s. Another issue of debate that appeared during the early discussions was the issue of time. Since singularity theory is about mappings, unfolding in space, and it might not involve time at all, the question arises about whether catastrophe theory has to involve the time dimension. Thom strongly associated catastrophe theory with dynamical systems, where time might be explicit a dimension as well. Since years earlier, Thom had argued that an elementary catastrophe form might be embedded in a larger system, which incorporates time as variable. He stressed that the discussion concerns dynamical systems evolving in $S \times t$ space, where S is the structural characteristics and t is time. If the larger system is *transversal* to the catastrophe set in the enlarged space, then time could be control variable; however the argument was a theoretical one and hard to demonstrate in empirical applications. Moreover, some crucial details were brought up in the discussion when considering transitions between stable states. These are associated with the notion of *discontinuity* and led to various misconceptions and furthermore to criticism (Zahler & Sussman, 1977). On this matter, two conventions regarding the way that the system moves between multiple equilibria were stated: the *Maxwell* and the *delay convention*. Note that the choice of one or the other convention might exclude a range of applications (i.e., in real systems behavior). Thom (1972) clearly fostered Maxwell convention from the beginning, but this later proved to be a problem, mainly because it appeared to be a weak point concerning the definition of catastrophe theory itself. The intense dispute on this issue led Zeeman to state later "*there is strictly speaking no* '*catastrophe theory*, *but then this is more or less true for any non-axiomatic theory in mathematics that attempts to describe nature*" (Zeeman, 1974, p. 623). Obviously, until that period the foundation of catastrophe theory at mathematical level was not a completed issue; nevertheless, active researcher in other fields rather intuitively had acknowledged a merit to it.

Catastrophe theory started to become popular around 1970s and a plethora of applications appeared in many fields of research, which however followed a rather descriptive and qualitative approach. It seemed a fascinated premise that could aid to understand unforeseen changes in nature and society. The unstable sociocultural environment that existed during that period, with radical political movements, facilitated its dissemination and appeal among mainly intellectuals (Rosser, 2007). This explosion of popularity triggered criticism and counteractions against the emerging theory, which, at that time, existed only in its deterministic version. A lot of theoretical, epistemological, and ultimately methodological questions were raised.

Catastrophe theory faced a severe condemnation mainly by Kolata (1977), Zahler and Sussman (1977), and Sussman and Zahler (1978a, 1978b). Their criticism was centered on the mathematical formulation and indirectly on its epistemology, which was unclear at that period. The most striking points of criticism referred to (1) the descriptive and qualitative approaches that were implemented; (2) the incorrect ways of quantification; (3) the existence of potential function; (4) the exclusion of time as a control variable in many applications; (5) the limited set of possible elementary catastrophes; and (6) the incorrect verification of global forms from local estimates (i.e., any surface can be fit to a set of points). The criticizing group also focused the disapproval on basic mathematical concepts associated with nonlinear behavior. For example (7) they claim that no real discontinuous jumps exist and cusp or fold model could be inferred with a few points. Extrapolation tells nothing about predicted behavior and, due to observational error, any surface could be arbitrarily close to a surface that Thom's theorems examine; (8) they also criticized Zeeman for incorrectly using the concept of genericity in his frontier example; (9) the predictions based on catastrophe theory are not testable and are unverified expectations, while many underlying hypotheses are often ambiguous; (10) the cusp models used (e.g., Zeeman's) were based on hypotheses carefully chosen in order to facilitate it, and the critique was talking about "*mystifying*" terms.

Some of the points of criticism, such as those regarding the use of qualitative methods, made sense, because the way bifurcation theory was founded by Poincaré had a qualitative character. Quantitative deterministic models had been demonstrated in physical sciences, but they seemed inappropriate for the social (soft) sciences. Most points of criticism for inappropriate ways of model design and quantification challenged mainly Zeeman's work. Thom, who had already acknowledged Whitney's work on singularity theory, agreed to some extent with the criticism on the qualitative character of catastrophe theory. Interestingly, on this issue, Thom appeared to be more a theoretician and philosopher, rather than as a mathematician in his responses towards defending the emerging theory. Thom wrote:

> On the plane of philosophy properly speaking, of metaphysics, catastrophe theory cannot, to be sure, supply any answer to the great problems which torment mankind. But it favors a dialectical, Heraclitean view of the universe, of a world which is the continual theatre of the battle between 'logoi,' between archetypes ... Just as the hero of the Iliad could go against the will of a God, such as Poseidon, only by invoking the power of an opposed divinity, such as Athena, so shall we be able to restrain the action of an archetype only by opposing to it an antagonistic archetype, in an ambiguous contest of uncertain outcome. Thom (1975, p. 384):

The above expresses Thom's intention to demonstrate metaphorically his *dialectical view* on uncertain outcomes upon the operation of two opponent processes or actions. Similarily to Hegel's dialectics, his position was that catastrophe theory was the means to demonstrate how qualitative changes could emerge from quantitative changes. Arnol'd referred to the "*mysticism*" of catastrophe theory showing his disagreement on Thom's "metaphysical" turn; however he admitted that "*in mathematics always there is an mysterious element: the astonishing concurrences and ties between objects and theories, which at first glance seem far apart*" (Arnol'd, 1992, p. 103).

The consequences of the criticism were *καταστροφικές* (disastrous) for catastrophe theory. Research showed a declined interest in applying catastrophe theory, and finally it became out of fashion for some years. Despite its temporary overthrow, catastrophe theory came back restored and more rigorous in 1980s, due to the work of Cobb (1978) in statistics, Oliva and Capdeville (1980) in economics, and Guastello (1981) in psychology. They defended the emergent theory by responding to the points of criticisms, while they made substantial contributions to the development of methodology for application of catastrophe theory in social sciences. With their pioneer work they maintained and showed that finally "*the baby was thrown out with the bathwater*" (Oliva & Capdeville, 1980). For more than a decade strong-minded scholars in various fields, who were convinced that catastrophe theory could become a valuable asset in research for social sciences (i.e., Cobb & Zacks, 1985; Cobb, Koppstein & Chen, 1983; Guastello, 2002; Lorenz, 1989; Puu, 1981; Rosser, 1991), worked for its development. Strong and clear responses to all points criticism were given also by van der Maas and Molenaar (1992); Wagenmakers, Grasman, and Molenaar (2005); and Wagenmakers, Molenaar, Grasman, Hartelman, and van der Maas (2005), while catastrophe theory has gained its reputation among scientist. Presently, it has been understood that the criticism was based mainly on confusions and conceptual misunderstanding of core ideas of the new theory and the only weak point, at the earliest times, which has now been overcome, was the lack of the proper statistical methodology applied to real-world research.

The return of catastrophe theory ensued in late 1980s where it stayed in the stage of social sciences with the development of its stochastic version, which permitted testing research hypotheses related to discontinuous changes in empirical data. Overviews of the theory and some applications across disciplines can be found in Arnol'd (1992), Castrigiano and Hayes (2004), Gilmore (1981), Saunders (1980), Poston and Stewart (1978), Thompson (1982), and Woodcock and Davis (1978).

Catastrophe Theory

Deterministic Catastrophe Theory

Catastrophe theory based on the initial work of Thom (1956, 1972, 1983) and Arnol'd (1988, 1992) is concerned with the classification of equilibrium behavior of systems in the neighborhood of singularities. The mathematical foundation of catastrophe theory includes the proof that the dynamics of systems in such singular points can be locally modeled by seven elementary catastrophes, which implement up to four independent variables. These elementary behaviors of systems in the neighborhood of singularities depend only on the number of predictors, the *control factors*. The seven elementary catastrophes are namely *fold catastrophe, cusp catastrophe, swallowtail catastrophe, butterfly catastrophe, elliptic umbilic catastrophe, hyperbolic umbilic catastrophe,* and *parabolic umbilic catastrophe*. The first four, known as cuspoids, have one behavioral axis while the last three have two behavioral axes; the formers are the most common and pertinent to social science. The fold, the cusp, the swallowtail, and the butterfly catastrophe have one, two, three, and four control variables, respectively. Each catastrophe is associated with a potential function in which the control parameters are represented as coefficients (a, b, c, or d), while one state variable, y, describes the behavior of the system. The behavior surface is the geometrical representation of all points where the first derivative of the potential function is zero (Zeeman, 1976). The cuspoids, which are the most applicable, are summarized in Table 9.1.

Deterministic catastrophe theory has been applied in physics and engineering for modeling various phenomena, such as the propagation of stock waves, the minimum area of surfaces, or nonlinear oscillations. Moreover interesting applications have been developed for conceptual formulation of thermodynamics, scattering, elasticity, and in the predictions of van der Waals equation in the transition between the liquid and the gaseous phase of matter using a cusp catastrophe, where temperature and pressure were implemented as two conflicting control factors, while density is the behavioral variable (Gilmore, 1981; Poston & Stewart, 1978). On the other hand, in social and human systems, where nonlinear effects and sudden changes are ubiquitous, with the development of its stochastic form, the perspectives for catastrophe theory became by far promising.

In order to attain a conceptual understanding of the core idea in catastrophe theory models, consider an analogy from a physical system that is moving toward an equilibrium state. The system in Fig. 9.1 comprises a hypothetical "one-dimensional surface" on which a sphere is moving driven by gravitational forces. The sphere represents the state of the system that can be at local minima or maxima, which are the equilibrium states. The minimum is the stable state where the system will stay or return when permutated by an external cause. The maximum is an unstable state, that is, small perturbations cause system's shift to another state. The above qualitative behaviors of changing states can be characterized according to the configuration of the corresponding positions, which are critical points, local

Table 9.1 The four *cuspoid* elementary catastrophes describing all possible discontinuities in phenomena controlled by no more than four factors

Catastrophe	Control dimensions	Potential function	First derivative
Fold	1 (a)	$U(y) = \frac{1}{3}y^3 - ay$	$\frac{\partial U}{\partial y} = y^2 - a$
Cusp	2 (a, b)	$U(y) = \frac{1}{4}y^4 - \frac{1}{2}by^2 - ay$	$\frac{\partial U(y)}{\partial y} = y^3 - by - a$
Swallowtail	3 (a, b, c)	$U(y) = \frac{1}{5}y^5 - \frac{1}{3}cy^3 - \frac{1}{2}by^2 - ay$	$\frac{\partial U(y)}{\partial y} = y^4 - cy^2 - by - a$
Butterfly	4 (a, b, c, d)	$U(y) = \frac{1}{6}y^6 - \frac{1}{4}dy^4 - \frac{1}{3}cy^3 - \frac{1}{2}by^2 - ay$	$\frac{\partial U(y)}{\partial y} = y^5 - dy^3 - cy^2 - by - a$

Each catastrophe is associated with a potential function in which the control parameters are represented as coefficients (a, b, c, or d), while one state variable, y, describes the behavior of the system. The behavior surface is the geometrical representation of all points where the first derivative of the potential function is zero (Zeeman, 1976)

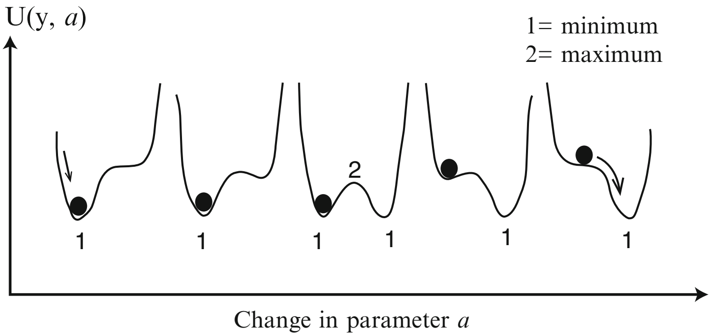

Fig. 9.1 The *sphere* represents the state of the system and can be at local minima or maxima—the equilibrium states. It is demonstrated how the sphere "jumps" from one position to another as the configuration of surface changes gradually

maxima or minima, with first and/or second derivative equal to zero. Next, observing across the five representations (Fig. 9.1), it is demonstrated how the sphere "jumps" from one position to another (local minimum) as the configuration of surface changes gradually (Castrigiano & Hayes, 2004; Gilmore, 1981).

This behavior may be described mathematically by postulating that the state of the system, y, will change over time t according to the equation

$$dy/dt = -\partial U(y/a)/\partial y \tag{9.1}$$

where $U(y/a)$ is the potential function and a is a vector of the control variables that affects the state of the system. The above equation characterizes a *gradient dynamical system*, which is at an equilibrium state, if the Eq. (9.1) equals zero (Feraro, 1978).

The equilibrium behavior of singular systems leads to multiple equilibria (multimodal distributions); thus abrupt changes in behavior might be expected as the system shifts from one equilibrium state to another. This "strange" behavior reflects *discontinuity* in mathematical sense. The concept of *discontinuity* is a fundamental issue in catastrophe theory and from the beginning it was a source of misconceptions that induced the criticism mentioned in the first section.

Catastrophe models become extremely complex, and less applicable, when number of the state and control parameters increase. However, the simplest and the most eminent one, the cusp catastrophe has numerous applications, and it is the best representation of the catastrophe theory models to be used also for didactic purposes. The cusp model describes the discontinuous behavior of a state variable as a function of just two independent variables. Considering that in traditional approaches a large number of independent variables are usually implemented when attempting to model changes, the choice of the cusp with merely two candidates has certainly an advantage; this justifies the widespread use and the applicability of the cusp catastrophe.

C*usp* describes the behavior as a function of the two control variables: asymmetry (a) and bifurcation (b). The potential function of the cusp catastrophe is expressed by the deterministic equation

$$U(y, a, b) = \frac{1}{4}y^4 - \frac{1}{2}by^2 - ay \tag{9.2}$$

The first derivative with respect to y is given by the equation

$$\frac{\partial U(y, a, b)}{\partial y} = y^3 - by - a \tag{9.3}$$

Setting $\partial U(y, a, b)/\partial y = 0$ gives rise to equilibrium function, which is geometrically represented by the three-dimensional surface. Note that Eq. (9.3) is cubic, a point with important consequences: small variation of the control variables can lead to abrupt shifts or jumps in the behavior y. This is an exclusive characteristic of the above function and the changes in the dependent variable are qualitatively different from other cases such as those in models with quadratic terms, where small continuous variation of independent variables is just accelerating y. Moreover, the changes in the cusp function are different from any sudden changes implied in a model, for instance, with a threshold function and also they are distinct from the shifts in logistic type functions, such as Rasch models, and from Markov models as well. Discontinuous changes in the cusp are sudden jumps occurring between regions of a smooth surface. This is a very important mathematical feature linked with primary epistemological issues related to nonlinearity. In real-world research, these discontinuous changes might imply a *qualitative* change within the system under investigation.

Further examination of the cusp model via its response surface reveals certain unique qualitative features, known as the *catastrophe flags*, which could be used to identify the presence of cusp catastrophe (Gilmore, 1981):

Bimodality: Refers to the probability distribution of the dependent variable, where two distinctly different modes exist or two simultaneously present sates.

Hysteresis: Is the effect, where cases with the same values of the two controls, asymmetry (a) and bifurcation (b), can be found in both distributional modes; that is, they can exhibit two types of behavior corresponding to both behavioral attractors; for a dynamical system hysteresis effect denotes memory for the path through the phase space of the system, in the sense that some point or areas of the system keep values from the preceded states.

Inaccessibility: The region on the response surface existing in between the two behavioral modes. This area is inaccessible in the sense that the corresponding behavior is unlikely to occur. The points within this area are pulled towards either attractor.

Divergence: Deviation from a linear relationship between the response and predictors demonstrated by two diverging response gradients—deviating paths towards the upper or the lower part of the surface.

Bifurcation point: The two divergent paths are joined at the bifurcation point at which the behavior is ambiguous, and beyond this point the system enters the *bifurcation set*, the area where discontinuous changes take place.

Sudden jumps: Abrupt changes between attractors, representing distinct behavioral modes, occurred even with slight changes in the control variables.

Among the above, sudden jumps in the value of the state variable, hysteresis, and bimodality are the most common flags constituting indicators for the presence of a cusp catastrophe in empirical data (Figs. 9.2 and 9.3). The identification of such flags encompasses a *qualitative* approach in evaluating the cusp as a model for data (see also Gilmore, 1981; Stewart & Peregoy, 1983; van der Maas & Molenaar, 1992; van der Maas, Kolstein, & van der Pligt, 2003).

Stochastic Catastrophe Theory

Catastrophe theory was developed initially for deterministic dynamical systems, whose basic processes entail change towards states of extrema (maximum or minimum), and it is perfectly applied to physical systems, e.g., to a pendulum

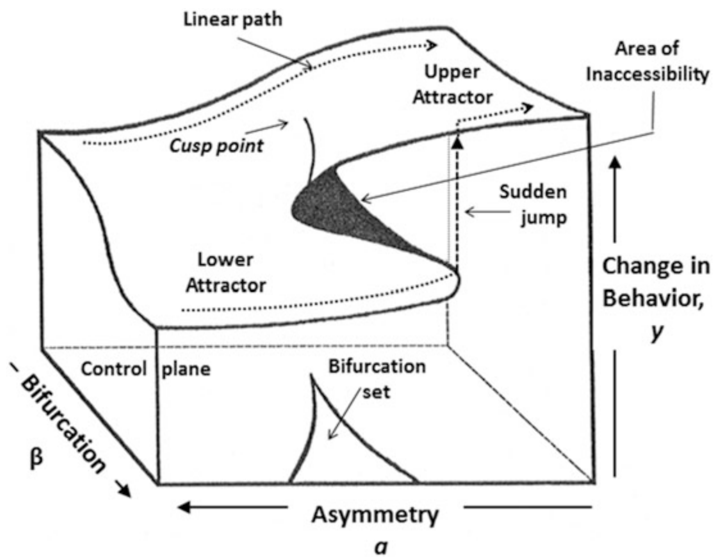

Fig. 9.2 Response surface of the cusp catastrophe model

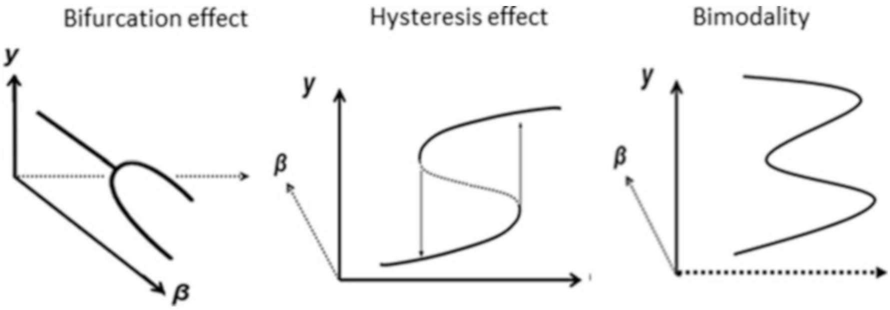

Fig. 9.3 Schematic representations of *bifurcation*, *hysteresis* effects, and *bimodality*

moving towards states of minimum potential energy. Theoretically, at least, the central idea seems to be applicable to human systems as well, by considering that the underlying processes of any social system always attempt to optimize some kind of "function," e.g., to maximize support or to minimize conflict.

Thinking stochastically, and focusing on the differential equation (9.1) holding for gradient dynamics, if the change in the dependent variable y is probabilistic rather than deterministic, then there is a probability density function over the rate of changes in y. On this idea Cobb (1978) set the basis for the development of *statistical catastrophe theory*. He restated catastrophe models using stochastic differential equations, where the assumed stochastic processes have stationary probability density functions of topological interest, which are receptive to statistical analysis.

The construction of stochastic catastrophe models starts by considering a deterministic system controlled by smooth potential function $U(y)$ and the relation (9.1)

$$dy/dt = -\partial U(y)/\partial y$$

The *singularities* of $U(y)$ are the points for which $\partial U/\partial y = 0$, while if they are *degenerate* ones the relation $\partial^2 U/\partial y^2 = 0$ also holds. In order to get a stochastic equation, a white noise term $dw(t)$ is added, so the differential equation becomes

$$dy = (-\partial U/\partial y)dt + \omega(y)dw(t) \tag{9.4}$$

The function $w(t)$ corresponds to standard Wiener process (Brownian motion), while the $\omega(y)$ modulates the intensity of the random input $dw(t)$ (Cobb, 1978). The increments of a Wiener process, $w(t + \Delta t) - w(t)$, are normally distributed with variance Δt. The function $\omega(y)$ determines the size of the variance of the noise and is called the *diffusion function*, which could be set to be constant. It was shown that the probability density function of the state variable y ultimately converges to a stationary one. Placing an error term in equation, the model becomes stochastic and the concept of persistence replaces the concept of stability in the deterministic one. Moreover, distinction between and within subject variability is allowed; thus stochastic catastrophe models can provide the means for investigating systems driven by underlying nonlinear processes (Cobb, 1978; Stewart & Peregoy, 1983).

Applying stochastic calculus and using Ito-Wright formulation finally a general equation is derived, which expresses that any differential equation can be presented as a probability density function *pdf*:

$$pdf(y) = \xi \cdot \exp\left[2\int^{x}(-\partial U/\partial y)ds/\varepsilon\right] \qquad pdf(y) = \xi \cdot \exp[2U(y)/\varepsilon] \tag{9.5}$$

where ε is the value of the variance function assuming to be constant and ξ is a constant introduced to ensure unity density.

In Cobb's stochastic catastrophe theory the derived stochastic differential equation is associated with a probability density that describes the distribution of the system's states in time. Thus, there is a unique relation between the potential function and the *pdf*. The stable and unstable equilibria of the potential function correspond to modes and antimodes of the *pdf*, respectively. A stochastic bifurcation occurs when the number of modes and antimodes changes as the control variables vary. By choosing a potential function one formulates the corresponding model. For instance using the canonical potential function for cusp catastrophe (Table 9.1) the corresponding probability density function is

$$pdf(y) = \xi\exp\left[-\frac{1}{4}y^4 + \frac{1}{2}by^2 + ay\right] \tag{9.6}$$

For empirical research, the next step was the development of statistical procedures to make estimates for the parameters for a specified hypothetical model, given a random sample of observations. Over the last decades various methods were developed based on maximum likelihood or least square optimization methods, so that given a set of empirical data, it becomes possible to test statistically hypotheses concerning the existence of degenerate singularities within the data.

Statistical and Methodological Issues

In this section, some crucial issues that appeared during the development of the stochastic catastrophe theory are highlighted, along with comments on the various methodological approaches and solutions. It is important to realize that catastrophe theory models, compared to the linear ones, are not easily workable and there are difficulties in developing evaluation procedures due mainly to the probability density functions, that is, the idiosyncrasy of the bimodality (or multimodality) and the non-triviality of the error variance. In addition, there are some strictly mathematical impediments concerning the nonlinear diffeomorphic transformation of the measurement, which however are not addressed here.[3]

From mathematical point of view the development of catastrophe theory involved primarily understanding of the critical points, that is, to determine how critical points behave via an, e.g., "equation of motion," which actually does not exist. Thus, the state of the system can be determined by fostering certain assumptions about the dynamics of the system (Gilmore, 1981, p 143). There are two *conventions* associated with these underlying assumptions, the *Maxwell convention* and the *delay convention*. The *Maxwell convention* considers that the system immediately jumps to a new equilibrium area. The state of the system is determined by the global minimum of the potential function. As the control parameters change, the state remains at the minimum as long as the current minimum remains the global minimum of the potential. When this minimum stops to be the global minimum, then the system state jumps to a new global minimum. The *delay convention* assumes that the system remains in the old equilibrium zone until the last possible point before it passes to the new equilibrium area. The state of the system is determined by the local minima of potentials. As the controls change, the state remains at the local minimum as long as the minimum exists. When the current minimum disappears, then the system's state jumps to a new local one. For the stochastic catastrophe theory, the above have crucial impact on the way the expected value of the bimodal distribution is estimated and affect the computation of the error variance and *scale*. It is recommended and worth trying to proceed with both conventions.

The various modeling techniques developed for testing catastrophe theory in empirical data are based on different assumptions and statistical approaches. A difference could be based on aforementioned *conventions*. Another difference lies in the presumed nature of variables; that is, they could be considered as univariate or multivariate. The univariates are measured directly as observable, while the multivariates are treated as *latent* variables with multiple indicators. Differences could also be based on the modeling formula, which could be the system's potential function or the derivative of the potential function.[4] Different optimization methods, such as the least squares or the maximum likelihood method, could also be implemented. Accordingly, different statistical tests and indexes are used for model evaluation; for example in the maximum likelihood method, BIC and AIC

[3] Another mathematical issue of concern is that the classification scheme of the systems developed by Thom presupposes that the systems under consideration must be transformed to its canonical form using diffeomorphism transformations. Thus, the invariance under diffeomorphic transformation should hold. For the deterministic case it does. The stochastic version as developed by Cobb based on pdf is *not* invariant under nonlinear diffeomorphic transformation of the measurement. Statistical problems related to diffeomorphic transformation have not been addressed, while solution has been proposed for some cases, e.g., time series data (Wagenmakers, Molenaar et al., 2005).

[4] There are pros and cons to that choice, since as it has been pointed out that methods based on the derivative of potential function might reward the presence of unstable equilibrium states, while those based on the *pdf* might punish their presence, as these correspond to points in an area of the density function of low probability that lies in between two high-probability states.

criteria are implemented, while in least squares method the percent variance explained (R^2) is used as the effect size criterion for comparing a catastrophe model with the linear competitors. Besides the above criteria, a nonlinear model has to have all parameters statistically significant, while special attention should be given to certain parameters, e.g., the bifurcation factor in the cusp model, which plays a crucial role in the model specification and its interpretation. Practically wise, the different methods and the corresponding calculations could be performed either with popular software (e.g., IBM-SPSS, Statistica, Stata, SAS, Minitab) or with more specialized ones (e.g., GEMCAT, cuspfit in R). Specific concerns about the methodological choices, pros and cons, critiques, and debates could be found elsewhere (e.g., Alexander, Herbert, DeShon, & Hanges, 1992; Guastello, 1992; Guastello, 2011a, p. 275; van der Maas et al., 2003).

Finally, it is imperative to single out that the researcher be aware of the fact that in catastrophe theory analyses, like in any other methodological approach and stochastic procedure, assumptions and conventions always are made, which might inevitably limit the anticipated results and conclusions. Ergo, it is suggested that analyses might be strengthened by a combination of methods. Encouraging, however, is that the methodological assets of catastrophe theory nowadays support high-quality research, and thus are promising for the advancement of theory and practice in educational research, as it has been realized in other social sciences. Methodologically, when new research endeavors are initiated, it is important that statistical procedures are not merely applied to available data with a curve fitting philosophy, but rather, a research design is followed in model specification, which is sourcing out from a deeper understanding of the underlying mechanism and the dynamics of the system.

Sample Size and Research Design Philosophy

The sample size issue is in general an unexplored territory for nonlinear regression modeling. It is related to *statistical power*, which is the *odds* of rejecting the null hypothesis (*Ho*) given that it is actually false. Note that the issue arises from cases with very large samples that result in statistical significance, while the effects are very small. In the linear regime and for bivariate tests the statistical power analysis is rather a straightforward procedure, whereas for multivariate analysis, e.g., multiple regressions, the determination of sample size for a given power is a more complicated matter, since it depends on a number of factors, such as the intended effect size, overall R^2, the number of independent variables, the degree of correlation among them, and assumptions on their equal or unequal weights. Therefore, a lot of different procedures have been developed for determining the proper sample size.

For catastrophe theory models a concerned researcher has to rely on rubrics that developed for linear models with the same number of variables. For example, for a linear regression with three independent variables, medium effect, and intended power of 0.80, 55 cases might be the sample size (Maxwell, 2000). Recently, a Monte Carlo simulation-based method was reported, which was used to calculate statistical power and sample size for Guastello's polynomial regression cusp catastrophe model. A power curve is produced under different model specifications (e.g., different error term) and then it was used to determine sample size required for specified statistical power (Chen. Chen, Lin, Tang, Lio & Guo, 2014). Interestingly, sample size varies with measurement error. For power 0.85 and $\sigma = 1$ the sample size is 36 and becomes 100 for $\sigma = 2$. Thus, for this statistical approach, a moderate sample size is adequate for cusp analysis. Moreover, as far as the statistical significance is concerned for small samples, the results can be strengthened by implementing bootstrapping techniques (Stamovlasis, 2014a).

The sample size and the sampling adequacy in nonlinear analysis and modeling have been an issue of debate for some time where the "myth of million data points" has been untangled (Gregson & Guastello, 2005). A fundamental notion related to the issue in question is the *restriction of topological range*, which concerns the full ranges of data which the hypothesized dynamics are unfolding in. It is of paramount importance that the available data should cover the proper spectrum of values in order to capture the nonlinear effect associated with hypothesized model (Guastello, 1995). Given that nonlinear phenomena are manifested along with linear dependences, it is the researcher's responsibility not to just seek for merely a good curve fitting, but to also build first a theory-laden model, which satisfies aspects of the anticipated behavior in the context of system's dynamics.

Statistical Methods in Cusp Catastrophe Analysis

The contemporary stochastic catastrophe theory permits testing related hypotheses and examining the type of catastrophe structure that a set of observational data might possess. In this section, the cusp model analysis will be examined as the most eminent and applicable to behavioral sciences. In practice, when analyzing data one may start with the *qualitative* approach, seeking for catastrophe "flags," such as sudden jumps, hysteresis effects, and bimodality. For example, bimodality increases at higher values of bifurcation variable and it can be observed using the graphical representation showing the frequency distributions of the state variable at different levels of the bifurcation. However, the *quantitative* approach, which includes statistical procedures, merely, provides the sound evidence that the model fits the observational data. A number of methods and techniques have appeared in the literature based on different assumptions and statistical modeling. Some of them are more established, popular, or applicable; it is worth presenting, however, all the most contributing to development of the stochastic catastrophe theory and its application to behavioral sciences.

Model with Probability Density Function

First, Cobb (1978, 1981) starting from stochastic differential equations demonstrated that the cusp catastrophe can be represented by the cusp family of probability density function, such as the *pdf* in equation (9.6). The state variable is corrected for location and scale, $z = (y - \lambda)/\sigma$, while it is assumed to be *univariate*, but the control variables a and b, the asymmetry and the bifurcation factors, respectively, are assumed to be *multivariate* (latent). The canonical parameters a and b in the model depend on the two observed and measured control variables, i.e., c_1 and c_2, and they are expressed with the equations

$$a = a_0 + a_1 c_1 + a_2 c_2 \qquad b = \beta_0 + \beta_1 c_1 + \beta_2 c_2$$

The cusp catastrophe fitting procedure then involves the estimation of the parameters α_0, α_1, α_2, β_0, β_1, β_2, λ, and σ using maximum likelihood method (Cobb & Watson, 1980). A reliable computer program was developed for data analysis, which later had undergone some computational improvements (see Hartelman, van der Maas, & Molenaar, 1998) and it is free on the Web.

Based on the probability function a direct method has also been proposed for fitting the cusp model using nonlinear regression with least square procedures (Guastello, 2011b). The cusp model is compared with its linear alternatives and it has to be superior in terms of R^2. The method is easy to perform and the related statistics can be carried out with a usual software.

The GEMCAT Methodology

Oliva and his coworkers (1987) developed the GEMCAT methodology, primarily for cusp, but also for swallowtail and butterfly catastrophe. The mathematical formalization for the cusp assumes that the response Z and the two controls, asymmetry and bifurcation X and Y, are defined as *latent* variables, each measured by a number of observables:

$$Z = \sum_{k=1}^{k} \gamma_k Z_k \qquad X = \sum_{i=1}^{i} a_i X_i \qquad Y = \sum_{j=1}^{j} \beta_j Y_j$$

The equation $f(Z, X, Y) = \frac{1}{4}Z^4 - \frac{1}{2}YZ^2 - XZ$ defines the cusp function and its first derivative set equals to zero: $Z^3 - YZ - X = 0$. The estimation problem then is stated as

$$\min\left(a_i, \beta_j, \gamma_k\right) = \Phi = \left\|\varepsilon^2\right\| = \sum_{1}^{N} \left[Z^3 - YZ - X\right]^2 \tag{9.7}$$

where ε = error and the summation is over the N observations. Given a set of empirical data for the response Z and the two controls, asymmetry X and bifurcation Y, one may estimate the impact of coefficients (α_i, β_j, γ_k) that define the corresponding latent variable, which minimize the function Φ. A modified control random search (CRS) algorithm was developed to estimate the desired parameters. The procedure, which is an *MLE* method, is equivalent to finding the best cusp catastrophe surface fitting to the empirical data. Analogous methodology and similar optimizing algorithms are followed for the other of catastrophe models. The GEMCAT program which is free on the Web provides a series of options, such as constraints on the coefficients (α_i, β_j, γ_k), standard errors for the parameters, a utility for testing competed nested models, chi-square statistics, standard likelihood ratio tests, and AIC statistic for fitting indices. In the latest version of the method (GEMCAT II, Lange, Oliva, & McDade, 2000), the technique was improved and inference is based on resampling techniques (jackknife and nonparametric bootstrap). The present program has been popular mainly among economic researchers.

Method of Difference Equations and Polynomial Regression Techniques

This model was developed by Guastello (1982, 1987, 2002, 2011), who followed a different approach. Starting from the deterministic equation $dz = (z^3 - yz - x)\, dt = 0$ by setting $dt = 1$ and inserting beta coefficients one gets the statistical formula:

$$\Delta z = z_2 - z_1 = \beta_1 {z_1}^3 + \beta_2 y z_1 + \beta_3 x + \beta_0 + \varepsilon \tag{9.8}$$

where ε is the error term. The polynomial regression technique approximates Cobb's stochastic form of Eq. (9.4) by a difference equation, which essentially results in a polynomial regression equation. The above equation is used to model the behavioral change $z_2 - z_1$ between two points in time, *Time 1* and *Time 2*, with behavioral outcomes z_1 and z_2, respectively. The difference equation in this formalism is assumed to imply a differential equation. Practically the equation implemented in data analysis contains often a quadratic term $\beta_4\, {z_1}^2$, which serves as a correction term associated with location, and it could be dropped if it is not significant or if it does not improve the model (Guastello, 2002). Data analysis with model includes testing the following alternative linear models:

$$\text{Linear 1} \quad \Delta z = \beta_1 x + \beta_2 y + \beta_0 \tag{9.9}$$

$$\text{Linear 2} \quad \Delta z = \beta_1 x + \beta_2 y + \beta_3 xy + \beta_0 \tag{9.10}$$

$$\text{Linear 3} \quad z_2 = \beta_1 x + \beta_2 y + \beta_3 z_1 + \beta_0 \tag{9.11}$$

z is the normalized behavioral variable, while x and y are the normalized asymmetry and bifurcation, respectively. The normalization procedure involves transformation of raw scores λ to z scores corrected for location and scale σ_s:

$$z = (\lambda - \lambda_{\min})/\sigma_s \tag{9.12}$$

Location correction is made by setting the zero point at $\lambda_{\min}$, the minimum value of λ, and the scale σ_s is the ordinary standard deviation of λ. The normalization is applied to the control variables as well. In some cases, the scale could represent the variability around the modes rather than around the mean (Guastello, 2002). The most competitive model is usually the *pre-post* linear model in Eq. (9.11).

In the above model the least square (OLS) method is used as optimization procedure. The distribution of the dependent measure at *Time 2* is expected to possess larger variance and it might exhibit bimodality. In order to demonstrate that a cusp catastrophe is the appropriate model to describe the outcome, its regression equation should account for a larger percent of the variance (R^2) in the dependent variable than the linear models. In addition, both the cubic and the product terms in Eq. (9.8) must have significant weights and/or the confidence intervals (95 % CI) should not span the zero point. The regression slopes, standard errors, *t*-tests, confidence intervals, and model fit for the cusp and the control linear models should be reported.

When modeling nonlinear phenomena, the inclusion of a nonlinear function in the model affects basic assumptions of standard measurement theory. In classical psychometric theory a measurement Y consists of a true score, T, and error term e. The percent unexplained variance is considered as error, while errors are assumed to be normally distributed and uncorrelated to each other and to true scores (*iid*). However, when a nonlinear function is included *dependent errors* (*de*) are expected to appear in the residuals. It has been shown that such non-*iid* errors (residuals) are indicative of nonlinear processes (Brock, Hseih, & Lebaron, 1990). The residual analysis could suggest that this might be the case. In nonlinear dynamical processes the score variance has four components:

$$\sigma^2(z) = \sigma^2(\text{linear}) + \sigma^2(\text{nonlinear}) + \sigma^2(\text{de}) + \sigma^2(\text{iid}) \qquad (9.13)$$

The four components are the linear, the nonlinear, the dependent errors, and the *iid*. A linear model treats the last three components as errors [($\sigma^2(e)$)], while the dependent errors are captured only by the proper and well-defined nonlinear model and could increase the variance explained (Guastello, 2002).

The difference equation model is affected by the restrictions and disadvantages of the OLS, e.g., under suboptimal condition the empirical coefficient may not be significant, while the bivariate correlations are. In those cases a cross-validation strategy is suggested by investigating collinearity effects among the control variables or other components of the model. Also, the order that the variables are entered in the OLS procedure could make a difference. It is recommended that all variables are entered simultaneously. In principle, the method considers the asymmetry and bifurcation as observables; however combination of candidate variables could be tested (e.g., Stamovlasis & Tsaparlis, 2012).

For enhanced generalization, *bootstrap* estimates have been recommended to cross validate the significance of the beta coefficients and the overall fitness of the model (Stamovlasis, 2014a). Note also that a large explained variance that might appear in some cases due to high linear correlations is not adequate to ensure a cusp structure. The fundamental components, such as the cubic term and especially the bifurcation term, have to be statistically significant (Guastello, 2011a, pp. 276).

The Cuspfit in R

Latest advances in catastrophe theory literature have presented methodological improvement and sophisticated software supported the analyses. The cusp package in R (Grasman, van der Maas, & Wagenmakers, 2009) combines the maximum likelihood approach of Cobb and Watson (1980) and the subspace fitting method proposed by Oliva et al. (1987).

The state-dependent variable y and the control variables of the cusp are considered as canonical variables, that is, they are smooth transformation of the actual state and control variables of the system. If there are n measured dependent variables $Y_1, Y_2, \ldots, Y_n$, then y is a linear weighted sum of them:

$$y = c_0 + c_1Y_1 + c_2Y_2 + \cdots + c_nY_n$$

Similarly the latent controls a and b are linear functions of the k measured independent control variables $X_1, X_2, \ldots, X_k$:

$$a = a_0 + a_1X_1 + a_2X_2 + \cdots + a_kX_k$$
$$b = b_0 + b_1X_1 + b_2X_2 + \cdots + b_kX_k$$

The fitting routine in R package performs maximum likelihood estimation of all the parameters in the above equations. The cusp program using one built-in optimization routine minimizes the negative log-likelihood L for a given set of experimental data, with respect to parameters, $\alpha_0, \alpha_1, \ldots, \alpha_k, b_0, b_1, \ldots, b_k, c_0, c_1, \ldots, c_n$:

$$L = \sum_{i=1}^{n} \log\Psi_i - \sum_{i=1}^{n} \left[-\frac{1}{4}y_i^4 + \frac{1}{2}b_iy_i^2 + a_iy_i \right] \tag{9.14}$$

In order to preserve stability and to control collinearity among predictors, standardized data are used.[5] A problem might arise from non-convergence of the optimization algorithm, which is overcome by providing alternative starting values (Grasman et al., 2009).

For statistical model fit evaluation, a number of diagnostic tools are provided. One is the pseudo-R^2 which is defined by the equation

$$\text{pseudo } R^2 = 1 - \frac{\text{ErrorVariance}}{\text{Var}(y)} \tag{9.15}$$

[5] The standardization is performed with QR decomposition, which is a mathematical procedure for obtaining accurate matrix decomposition using the modified Gram Schmidt re-orthogonalization method. It accounts for collinearity in the design matrix and the stability of the estimation algorithm (Press, Teukolsky, Vetterling, & Flannery, 2007).

The concept is analogous to the squared multiple correlation coefficient; however in the cusp catastrophe model the pseudo-R^2 is not the same as the measure of explained variance. It can take negative value if the error variance exceeds the variance of y, and for this reason it is not a reliable fit index. This is because the error variance is nontrivial and it is calculated based on predictions of *delay* or *Maxwell* rules. Recall that these estimation rules (conventions) are relevant to the concept of discontinuity as it was discussed earlier. Thom had from the beginning fostered the *Maxwell convention*, while in Cobb's method the *delay convention* was suggested. The *cuspfit* in R offers both conventions with the delay convention as the default.

Additional criteria typically used for evaluating the model fit of the cusp catastrophe are the following:

- The coefficients in the model should be statistically significant.
- Cusp model compared to the linear counterparts should be significantly better in terms of its likelihood.
- Cusp model could also be compared to the logistic function below:

$$y_i = \frac{1}{1 + \mathrm{e}^{-a_i/b_i^2}} + e_i \quad i = 1, \ldots, n$$

which does not possess degenerate critical points, but it can model steep changes mimicking abrupt transitions similar to the cusp (Hartelman, 1997). Besides the statistical part, however, it is important to note here that even though the logistic function is co-examined as an alternative model, it is not associated with rigorous theoretical interpretations as the cusp catastrophe (see the epistemology section).

- The use of AIC, AICc, and BIC for all alternative models should be in favor to cusp model. Especially the BIC can be used to compute approximation of the posterior odds for the cusp relative to the logistic curve, assuming equal prior probabilities (Wagenmakers, van der Maas, & Molenaar, 2005).

When analyzing with the *cuspfit* in R a difficulty arises if there are two or more dependent variables because in these cases the counterpart antagonistic linear regression model is not uniquely defined. Additional limitations are the absence of the alternative linear model with the interaction term and the lack of an effect size index, such as the R^2 in least square approach that could serve as a basis for comparison.

The advantage of the cuspfit method is that it can implement control variables as multivariate latent constructs and can be used in confirmatory analysis. When it is used in an exploratory approach the independent variables should not be assigned arbitrarily to the controls because the results might be very peculiar and uninterpretable. In order to improve estimations and get better results, it is recommended that before using the *cuspfit*, a factor analysis (e.g., PCA) should be applied, in order to identify the sets of potential candidates for control parameters.

Theoretical and Epistemological Issues

The following epistemological discussion focuses on the cusp catastrophe and its main features, which are fundamentally related to nonlinear dynamics. The cusp model reveals the pattern of behavior as a function of the two control variables, the asymmetry a and the bifurcation β, and it states that both linear and nonlinear changes in behavioral variable are expected depending on the values of the two controls.

The model interpretation via Fig. 9.2 suggests that at low values of β changes are smooth and a linear relation can better describe the relationship between the asymmetry and the response. At low values of a, changes occur over the lower mode and are relatively small. At high values of a, changes occur around the upper mode and are again small. At high values of β, however, changes are discontinuous and abrupt shift can be observed between the two modes or behavioral attractors. At the control surface we can observe the bifurcation set mapping in the unfolding of the surface in two dimensions. The cusp bifurcation set induces two diverging response gradients, which are joined at the *cusp point*. At the cusp point the behavior is ambiguous, while the two diverging gradients represent varying degrees of probability that a point be in the one or in the other behavioral mode (Guastello, 2002).

The three-dimensional response surface entails the *geometry of behavior*, which explicates that for certain values of the asymmetry a and the bifurcation β, a point, the *bifurcation point*, exists, beyond which the system enters the *bifurcation set,* the area where discontinuous changes occur. Points within the area of *inaccessibility* are unlikely to be observed, since they are pulled towards either behavioral attractor, and this is what introduces nonlinearity and uncertainty in the system, which, it is said, enters the chaotic regime. This behavior is also depicted on the other fundamental feature disclosed in the cusp structure, the *hysteresis effect*; that is, cases with the same values on control variables could be found either in the upper or the lower mode of the response surface.

The above geometry of behavior, which seems quite complicated to ordinary linear thought, is obviously *phenomenological*; that is, it apparently does not explain, but merely it describes the behavior. Thus, the crucial question, which entails explanation, is what kind of mechanism might force the state of the system to follow the response surface. This is a fundamental epistemological question to be answered (Zeeman, 1977).

Catastrophe theory models in science involve dissipating systems or potential-minimizing systems. The mathematical formalism using a potential function for a mechanical system, e.g., Zeeman's catastrophe machine, seems appropriate since by nature it is expected to obey some sort of deterministic type natural law. Epistemological questions arise, however, when attempting the application of catastrophe theory to "soft" science dealing with human behavior and related systems. Recall here that one of the points of criticism of catastrophe theory was the existence of potential function, which seems to arbitrarily appear in order to describe the sudden shifts in the system. The issue is related to argument originated from the confusion about the conception of discontinuous jumps (Zahler & Sussman, 1977), which ignores the *attractor notion*, a fundamental concept in nonlinear dynamics. The cusp model describes the shifts between stable states or distinct modes of behavior (behavioral attractors). This behavioral change might imply or be a *qualitative change*. This description is founded on the operation of a potential function, which mathematically is the proper tool to model shifts between attractors. The mathematical formalism of the cusp model assumes that the system is controlled by a "potential" function with two stable equilibria (Poston & Stewart, 1978). This assumption, for behavioral sciences, is not as arbitrary as it seems to be. Note that the assumption of linearity is also arbitrary, to the extent that there are no reasons for the behavior to follow straight lines; however the assumption of linearity being seemingly the simpler one is easier to accept.

Within complexity and nonlinear dynamics, epistemological arguments concerning human systems are advocates to the existence of attractors and dissipating mechanisms. One is that for human systems' behavior, an optimization process, analogous to energy dissipation or potential minimization process, can be reasonably assumed. A psychological system for instance could be sought as seeking to minimize cognitive dissonance, or to maximize the degree of adaptation (Saari, 1977). The concept of energy minimum is closely related to and it is a special case of the *attractor* concept, which by definition represents the stable state of a system operating in a dynamical equilibrium. Moreover, attractors at the psychological level can be assumed that originate from the brain functioning, which operates as nonlinear dynamical system possessing multiple coexisting attractors (Kelso, 1995; Freeman, 2000a, b; Freeman & Barrie, 2001). In addition, theoretical models on brain functioning based on neuropsychological evidences have provided mathematical description of its dynamics in perception and action, using the language of nonlinear dynamics. According to Nicolis and Tsuda (1999), brain functions as dissipative dynamical system, which is characterized by sensitive dependence on the initial conditions and the control parameters. These are manifested as chaotic behavior including bifurcations, braking symmetry, and multiplicity of behaviors beyond an instability point. In compensation to unpredictability due to the nonlinear character of the underlying process, the following hold for the system: (1) the existence of multiple attractors possessing invariant measures in the dynamical system governed by the interplay among the order parameters and (2) drastic reduction of degrees of freedom in the vicinity of a bifurcation and the emergence of essentially only a few dominant order parameters. These parameters may subsequently interact in a nonlinear fashion, giving rise to low-dimensional dissipative chaos. (3) Within such systems *information* is produced (Nicolis & Tsuda, 1999). The latter, the potential to produced information, is a property of nonlinear dynamical processes and it will be seen again in a later discussion on learning and creativity.

Answers to epistemological questions on phenomena, such as a bifurcation and hysteresis effects (Fig. 9.3), the interpretation of which seems too complicated for linear and reductionist ways of thought, are given by *self-organization theory*. The important feature of complex dynamical systems is the *emergent* properties that appear through *self-organization* processes. A cusp catastrophe for instance, when detected, is by virtue a state transition, and it is an emergent discontinuity. This finding at the behavior level has important philosophical implications targeting to ontological questions, since a bifurcation is the phenomenology of complex adaptive systems; it is in fact the signature of complexity and indicative of *self-organization* mechanisms (Nicolis & Nicolis, 2007). The notion *self-organization* has supported the development of the major scientific theories of nonlinear dynamics: Prigogine's non-equilibrium thermodynamics (Nicolis & Prigogine, 1977; Prigogine, 1961), Haken's synergetics (1983, 1990) and Thom's catastrophe theory (1975), even though they were grown with different rationales.

Self-organization can provide a *causal interpretation* of the bifurcations and state transitions within a nonlinear dynamical system and it is the process that occurs when a system is at a state of high *entropy* and far-from-equilibrium condition (Prigogine & Stengers, 1984). The structure that is taken on, which is an ordered state, allows the system to operate more efficiently and interestingly it does not require any outside intervention; this is the notion of "*order for free*" pointing out by Kauffman (1995, p. 17). *Self-organization* has been implemented for physical and biological systems as an explanatory theory; however it could be transferred to human system as well, for explaining emergent patterns observed in psychological processes (Grigsby & Osuch, 2007; Hollis, Kloos, & van Orden, 2008). It has been fostered for a causal interpretation of Piaget's theory of stepwise cognitive development (Molenaar & Raijmakers, 2000) and for interpreting the emergence of creativity (Stamovlasis, 2011).

A final point to be singled out is that the phenomenology of nonlinear systems is due to *self-organization* mechanism and on the other hand to the operation of coexisting attractors and the dynamics of the system. Bifurcation mechanism in a physical system such as Zeeman's catastrophe machine is nested in the operation of a potential function and the dynamics of the system (Zeeman, 1976). Similarly, when examining a cognitive or human system, its dynamic behavior is the formative cause of the ensuing bifurcation and the emergence of the new topological pattern in the state space of the system.

Note also that in psychological and educational sciences, the *processes* under examination regarding cognitive and human systems are more likely non-*ergodic*, and the hypothesized underlying *evolution equation* that describes the system over time is unknown. These two points are where catastrophe theory is filling the gap: it concerns sudden changes and it exemplifies that for studying these state transitions in a system, the evolution equation does not have to be known in advance; the description and the explanation of local observed behaviors can be attained with a small number of control parameters (Castrigiano & Hayes, 2004; Gilmore, 1981; Poston & Stewart, 1978; Thom, 1972, 1975, 1983). The above are also in accordance with primary postulates of nonlinear dynamical systems, where the principle of *dynamical minimalism* is assumed; that is, complex behaviors can be produced by simple rules and/or a few interacting variables. Thus, in constructing nonlinear models it is always sought to identify the simplest realistic set of assumptions and variables that finally produce theories that provide the simplest explanation of phenomena (Nowak, 2004; Vallacher & Nowak, 2009).

The above epistemological discussion concerns and applies to any process in educational research. The application of catastrophe theory, as a part of the meta-theoretical framework of nonlinear dynamics to a specific domain and discipline, does not ignore, but it essentially requires a local theory, which can provide the variables to implement as state and control factors.

Catastrophe Theory and Neo-Piagetian Premises in Learning Sciences

The Piagetian and Neo-Piagetian Theories

A requisite local theory that could serve as the bridge between science education research and nonlinear dynamics is the Piagetian and neo-Piagetian premises (Case, 1985; Pascual-Leone, 1970; Piaget, 1967; Piaget & Inhelder, 1969). They have been exceptionally appealing to educational sciences and they are the first on which catastrophe theory and nonlinear dynamics have been applied in a remarkable way. At earlier times, catastrophe theory has been connected to Piagetian stagewise development (Molenaar & Oppenheimer, 1985). A few interesting models had been proposed with the implementation of some core Piagetian concepts, such as the *assimilation* and *accommodation* processes, which were considered as controls determining the abrupt shifts between developmental stages, while discontinuities in the children responses in the vicinity of a transition from preoperational to concrete operational thought have been shown (Preece, 1980; Saari, 1977). A few decades ago, it has been pointed out that catastrophe theory analysis could embrace the traditional methodological approaches concerning stagewise cognitive development, and later the dynamic systems theory has been proposed as the unified framework of development (van Geert, 1991; van der Maas & Molenaar, 1992; van der Maas & Raijmakers, 2009).

The fundamental connection points between Piagetian and catastrophe theory are the notion of *equilibration* as applied to the former and the concept of *equilibrium* to the latter. Both are expressed mathematically by setting the first derivative of the dynamic system equation to zero. As it was pointed out in the epistemological section of this chapter, the *equilibrium* is implied by an *optimization process*, which is taking place within a dissipating system. This process allows the cognitive system to choose its internal states so that it maximizes the degree of adaptation, given the environmental inputs. Thus, from the beginning it was recognized that the inherent compatibility with catastrophe theory holds also for the neo-Piagetian theories, which can make available all the prerequisite psychological constructs for a catastrophe model specification.

The most representative within the neo-Piagetian premises is the *theory of constructive operators* (TCO), founded by Pascual-Leone (1970, 1987) as an account of individual differences in performance on mental tasks. According to TCO, cognitive processes involve a variety of *constructive operators*, each of which performs a specific function: the *M-operator* deals primarily with mental capacity, the *C-operator* with content knowledge, the *L-operator* with logical operations such as conservation and formal logic, the *F-operator* with field dependence/independence, and so on. The development of psychometric tests operationalizing the above mental resources allowed an array of applications in learning and educational sciences.

The merit of the neo-Piagetian framework as a scientific program with the Lakatotian sense (Lakatos, 1974) has demonstrated by its continuing evolution through the last decades (Pascual-Leone, 1970, 2000, 2013). Furthermore, it has supported a considerable amount of research at the behavioral level which has become the basis for further progress on theories of cognitive organization and growth that has determinedly added to our understanding about the architecture and function of mind (e.g., Demetriou, Efklides, & Platsidou, 1993; Demetriou & Efklides, 1994).

The neo-Piagetian theories emphasize the importance of a match between subject's mental operators and certain characteristics of mental tasks, for instance, the relation between *M*-operator and the mental demand of a task or between *F*-operator and the existing misleading information or "noise" in the data. Based on the above and given that numerous types of mental tasks or problems could be designed, a considerable amount of research has been carried out in the area of learning sciences, where individual differences associated with neo-Piagetian constructs have been shown to play a decisive role. The most known are the information processing capacity (*M*-capacity), the field dependence/independence or disembedding ability, the logical thinking (developmental level), and the convergent and divergent thinking. Note also that the information processing models (Baddeley, 1986) offer an analogue to *M*-capacity construct, the working-memory capacity, which has been linked to the well-known working memory *overload hypothesis* (Johnstone & El-Banna, 1986; Stamovlasis & Tsaparlis, 2001, 2005; Tsaparlis & Angelopoulos, 2000). It has been shown that the effect of the above variables is apparent in different types of mental tasks, such as algorithmic problems (Johnstone & Al-Naeme, 1991; Johnstone & El-Banna, 1986; Niaz, 1989), non-algorithmic problem solving (Lawson, 1983; Niaz, de Nunez, & de Pineda, 2000; Tsaparlis, 2005; Tsaparlis & Angelopoulos, 2000), and conceptual understanding (Danili & Reid, 2006; Kypraios, Stamovlasis, & Papageorgiou, 2014; Tsitsipis, Stamovlasis, & Papageorgiou, 2010, 2012; Stamovlasis, Tsistipis & Papageorgiou, 2010). Moreover, it has been shown that the effect of these individual differences is present at different ages from elementary school to the upper secondary education (Stamovlasis & Papageorgiou, 2012). Thus, the relationships between these individual differences and performance in learning sciences are well established, at least, in the linear regime.

Nonlinear Dynamics and Learning Science

In this section, the development of the framework for the application of catastrophe theory and nonlinear dynamics in science education is presented. It includes findings of inductive and deductive endeavors and implications for theory and the practice.

The first attention of nonlinear dynamical thinking to issues in science education was on problem solving, the most intriguing areas where neo-Piagetian constructs have been proved predictive variables. However, these effects have not been consistently observed across topics and ad hoc explanations were given to various contradictions, such as the unexpected failure of highly skilled students. On the other hand, it was clear that success could not be attributed to merely one variable and that some other individual differences interfere and cover up the effect of the hypothetical main predictor (Johnstone & Al-Naeme, 1991; Tsaparlis & Angelopoulos, 2000). The moderator role of some variables, e.g., field dependence/independence on information processing capacity, was evident, but there was lack of a comprehensible model that joins the synergetic role of these two mental resources. A response to this inquiry was the proposition of a cusp catastrophe model with the two above variables as controls. The effect of the two independent variables operationalizing two opponent processes is visualized as *force field dynamics*, where the outcome cannot be merely estimated as their weighted linear sum. Analysis of empirical data showed that for some cases the cusp catastrophe model was superior to its linear alternatives explaining a large portion of the variance of students' performance in chemistry problem solving (Stamovlasis, 2006). The above cusp structure, however, was not identified in every type of problem-solving data. Nonlinear models are not always better; that is, nonlinearity is not manifested everywhere.

The explanation to this was sought in the nature of mental processes and the differences that might exist among various tasks. There was need for reasonable justification, rooted, however, to fundamental theoretical premises. In science teaching, there are two types of cognitive tasks: The first are known as *exercises*. The students by applying a well-known solution path reach the answer successfully. The algorithm has usually been practiced, while the subjects are not necessarily aware about the strategy followed. On the other hand, there are "*real*" *problems*, where students cannot apply a learned procedure and the challenge is to find the solution path. Often it is said that those non-algorithmic problems require conceptual understanding and *high-order cognitive skills* (Tsaparlis & Zoller, 2003), implying an effective synergy of mental resources (e.g., neo-Piagetian constructs). Of course, in the school context all the above depend on what has been taught.

The answer to the question regarding nonlinearity manifested at the behavioral level is hidden in the differences between the two above categories of cognitive tasks; they correspond to two different processes, with distinct qualitative characteristics that determine the observed behavioral outcomes. A note of statistical interest is that the differences between the two types of problem solving are reflected in the empirical data and might become apparent in the briefing descriptive statistics. In easy and algorithmic problem solving, students' achievement scores are more likely distributed normally around the mean and most of the basic assumptions for linear modeling hold. A second statistical remark is that practically, in everyday school evaluation, students' scores conform to Gaussian distribution because they have to; that is, following *procrustean rationality*, which all the traditional evaluation theories propose, teachers and/or researchers tailor the assessment tests, so that they purposely produce bell-shaped curves in order to proceed with linear statistical analyses.

However, contrary to the ordinary thought, when exploring really challenging tasks, achievement scores are not recorded as normally distributed around the expected value, but deviation from normality, strong skewness, or even bimodality is often observed. Frequently, observations in these asymmetric distributions conform to the *inverse power law* or the *fractal distribution*. These are indicative for underlying dynamic processes where multiplicative rather than additive effects are taking place (West & Deering, 1995). Bimodality quite often appears also in very demanding tasks denoting bifurcation in a nonlinear process rather than the existence of two district subpopulations. In these cases, the implementation of conventional linear approach is proved inadequate and a nonlinear model, e.g., the cusp, arises then as a potential candidate. The reasons, however, for applying a nonlinear model to these empirical data are not merely statistical, but primarily are relevant to theoretical and philosophical issues. Algorithmic and non-algorithmic mental tasks belong to different categories as far as the nature of the underlying process is concerned. Algorithmic problem solving is a linear process, where predetermined and learned steps are followed. Non-algorithmic problem solving is a process with no predetermined scenario; each step is determined by the previous steps and there isn't a unique path to follow. The solution (if any) *emerges* from an iterative and recursive process, which is nonlinear and dynamic in nature. In this type of problems, nonlinearity at the behavioral level is more likely to be observed. Methodologically wise, yet, exploring empirical data obtained from such processes with linear models is an *epistemological fallacy* because the method is incompatible with the nature of the phenomenon being investigated (Stamovlasis, 2010, 2014b).

Based on the above theoretical premise, deductive endeavors have further supported the nonlinear hypothesis. A series of investigations have provided evidences for nonlinearity by the application of catastrophe theory in empirical data taken from science education research. Cusp catastrophe models explained students' achievement scores in chemistry and physics problem solving as a function of neo-Piagetian constructs that operationalize mental resources associated with the task execution. Those constructs were the information processing capacity (*M*-capacity or working memory capacity), logical thinking, disembedding ability, and divergent and/or convergent thinking. The dependent measure was the difference between the achievement scores in the prerequisite theoretical knowledge (z_1) and the problem-solving performance (z_2), while the least square technique (Guastello, 2002) was implemented. R^2 values were higher in the cusp compared to the linear alternatives. The nonlinear models were also supported by maximum likelihood estimates using cuspfit in R (Grasman et al., 2009), with fit criteria, such as AIC, AICc, and BIC (Stamovlasis, 2014a). The cusp structures do not appear in data originated from algorithmic problem solving and simple mental tasks. They are learned predetermined procedures, where the solution is actually known and nested in the algorithm. These are linear processes.

The crucial question for learning sciences is what the implications are. What have these nonlinear endeavors offered to science education, in theory and practice? Have they just provided an additional support to neo-Piagetian theories with new methodological tools? This is obviously true, but the main message is the crucial epistemological issues that challenge the dominant paradigm in educational research and practice.

In the epistemological section it was discussed that bifurcation and hysteresis effects are the signature of complex adaptive systems (CAS) and *self-organization* mechanisms. The findings via catastrophe theory models provide direct links to *self-organization* theory, and connect the behavioral level in education sciences with psychology and neuroscience, where the paradigm shift has already been attained. Thus, the above empirical research signified the departure from the mechanistic view of educational settings and set the framework for reconsidering, under the new perspective, the epistemological assumptions and the methodological issues in the existing local theories. It should be emphasized that the cusp models cited above are not advocates to the *reductionist view* for the role of individual differences and in general for any independent variables selected for describing and predicting phenomena in education. On the contrary, what the cusp models explicate is that given the protagonist role of decisive components in a nonlinear process the outcome might be ambiguous, due to the dynamics of the system and the sensitivity of the parameters.

Moreover, nonlinear dynamics and complexity challenge the conventional notion of *causality*, emphasizing the *emergent* nature of the outcomes through self-organization mechanism. The above concern the existing theories in educational sciences and in science education particularly, e.g., *constructivism* or *conceptual change* theories, which totally ignore, at least at the methodological level, the actual phenomena under investigation. Crucial debates and unanswered questions, such as those concerning the nature of conceptual change, could be resolved. For instance, the question, whether *conceptual change* is an outcome of a linear additive process modeled on the "architecture metaphor" or it is the outcome that *emerges* from a nonlinear dynamical process, could be addressed by implementing catastrophe theory. It is obvious that a new area of investigation opens that could elucidate crucial disputes and incoherent theoretical perspectives.

Coming to practical implications, based on rational explanations of students' failure, teaching strategies could be developed with the aid of the cusp response surface as a qualitative/metaphorical guide for manipulation of variables; for instance by reducing the "noise"-to-"signal" ratio one might induce "catastrophic success" (avoiding failure) for field-dependent students (see Stamovlasis, 2006). In addition, the identification of potential bifurcation variables in different cognitive tasks is crucial in learning sciences because these variables are more sensitive to the parameters and induce nonlinearity, turbulence, and uncertainty in the outcomes. For instance the moderating role of disembedding ability or logical thinking deficiencies beyond a threshold value might have a severe impact, leading abruptly to the overload phenomena (Stamovlasis & Tsaparlis, 2012). Catastrophe theory models could be applicable also to other educational processes at different level of complexity, e.g., at classroom or school level, where other variables, such as *motivation* or *performance climate*, under certain conditions, could operate as bifurcation variables for students' academic behavior (e.g., Sideridis & Stamovlasis, 2014; Sideridis, Stamovlasis, & Antoniou, 2015; Stamovlasis & Sideridis, 2014). It fact, a plethora of variables, individual, collective or environmental ones (Vygotsky, 1978), associated with educational process are potential candidates to be tested in a nonlinear context.

The new paradigm of nonlinear dynamics and complexity encourages further deductive endeavors and it signifies the departure from mechanistic views of the cognitive and educational processes and specifically of learning. Returning to the distinction between the two types of mental tasks, it was pointed out that the execution of algorithms and memorizing procedures are linear processes, which do not actually produce *information* (Nicolis, 1986, 1991). On the contrary, in *real* problem solving, where the system proceeds step by step in an iterative and recursive process without predetermined scenario, the solution *emerges* from the course of a nonlinear dynamical process driven by self-organization mechanisms. This theoretical remark affects obviously the definition of *learning*; algorithmic problem solving, like raw or parrot learning, is not "learning" per se (Stamovlasis, 2011). Novices attain learning outcomes if they involved in cognitive tasks mimicking processes that are nonlinear and dynamical in nature: the processes that produce *information*. Thus, educators and the scholars who develop curricula should be aware about this significant knowledge and should act accordingly. In science education the dominant and traditional instructing methodology stands on the opposite thesis, and persists in teaching algorithms, contributing essentially nothing to the issue of learning.[6]

It is noteworthy that most teaching practices have been developed on the computer view for mind, and it is rather amazing that they are still active, even though the theory has been proved flawed. The nonlinear dynamical nature of brain functioning as complex adaptive system operating far from equilibrium is the inherent property of mind that permits development that is not restricted by the

[6] A characteristic example is the plethora of problem-solving techniques taught in the Greek education system (and perhaps elsewhere) focusing on how to succeed in examinations in chemistry and physics, while students remain ignorant about the strategy followed or how to turn the implicit into explicitly. Behind this educational policy are wrong theoretical premises, that of computer metaphor for mind, and the hope that teaching problem solutions will enhance students' repertoire. This actually does not happen and rather it leads to *functional fitness*. The computer metaphor as theory of mind, applied to education, has been catastrophic for a novice's mind.

repertoire of the contributed components. This feature is a core element for nonlinear theories in psychology and behavioral sciences addressing human development, learning, and motor skill acquisition (e.g., Corrêa, Alegre, Freudenheim, Santos, & Tani, 2012; Molenaar & Oppenheimer, 1985; van Geert, 1991). It is relevant here to recall a debate and the criticism on Piaget's constructivist theory of stagewise cognitive development, around 1980s. Reductionist views (e.g., Fodor, 1980), refuting the nonlinear dynamical nature of human development, stated the alternative with the notion of "nativism", that is, certain features are "native" in the brain at birth, thus setting strictly programmed limitations to learning and development. The response to the criticism was decisive at that time, showing the possibility of acquiring more powerful structures, by fostering the nonlinear dynamical view of human development (Molenaar, 1986). That was merely a theoretical conjecture, and at that period along with the "adventures" of catastrophe theory and due to deficits in research methodology, the advancement of the new ideas delayed for two decades. Today the nonlinear dynamics and complexity framework returned in the scene with vigorous epistemological and methodological assets, as the new paradigm, alternative to linear and reductionist view of cosmos. Regarding educational issues in science teaching and in general the social and academic behavior, nonlinear dynamics is also filling the gap between genetic and environment dilemmas and offering a holistic view of reality that could amalgamate Piagetian and Vygotskian interpretations to a unified theory.

References

Alexander, R. A., Herbert, G. R., DeShon, R. P., & Hanges, P. J. (1992). An examination of least-squares regression modeling of catastrophe-theory. *Psychological Bulletin, 111*(2), 366–374.

Arnol'd, V. I. (1988). *Geometrical methods in the theory of ordinary differential equations*. Berlin: Springer.

Arnol'd, V. I. (1992). *Catastrophe theory* (3rd ed.). Berlin: Springer.

Arnol'd, V. I., Gusein-Zade, S. M., & Varchenko, A. N. (1985). *Singularities of differentiable maps* (Vol. I). Boston, MA: Birkhäuser.

Baddeley, A. D. (1986). *Working memory*. Oxford: Oxford University Press.

Brock, W. A., Hseih, D. A., & Lebaron, B. (1990). *Nonlinear dynamics, chaos and instability: statistical theory and economic evidence*. Cambridge, MA: MIT Press.

Case, R. (1985). *Intellectual development: Birth to adulthood*. San Diego, CA: Academic Press.

Castrigiano, D. P. L., & Hayes, S. A. (2004). *Catastrophe theory*. Boulder, CO: Addison-Wesley.

Chen, D.-G., Chen, X. G., Lin, F., Tang, W., Lio, Y., & Guo, Y. Y. (2014). Cusp catastrophe polynomial model: Power and sample size estimation. *Open Journal of Statistics, 4*, 803–813.

Cobb, L. (1978). Stochastic catastrophe models and multimodal distributions. *Behavioral Science, 23*, 360–374.

Cobb, L. (1981). Parameter estimation for the cusp catastrophe model. *Behavioral Science, 26*, 75–78.

Cobb, L., Koppstein, P., & Chen, N. (1983). Estimation and moment recursion relations for multimodal distributions of the exponential family. *Journal of the American Statistical Association, 78*, 124–130.

Cobb, L., & Watson, B. (1980). Statistical catastrophe theory: An overview. *Mathematical Modelling, 1*, 311–317.

Cobb, L., & Zacks, S. (1985). Applications of catastrophe theory for statistical modeling in the biosciences. *Journal of the American Statistical Association, 80*, 793–802.

Corrêa, U. C., Alegre, F. A. M., Freudenheim, A. M., Santos, S., & Tani, G. (2012). The game of futsal as an adaptive process. *Nonlinear Dynamics, Psychology, and Life Sciences, 16*, 185–204.

Danili, E., & Reid, N. (2006). Cognitive factors that can potentially affect pupils' test performance. *Chemistry Education Research and Practice, 7*, 64–83.

Demetriou, A., & Efklides, A. (1994). Structure, development, and dynamics of developing mind: A meta-Piagetian theory. In A. Demetriou & A. Efklides (Eds.), *Intelligence, mind, and reasoning: Structure and development* (pp. 75–110). Amsterdam: North-Holland.

Demetriou, A., Efklides, A., & Platsidou, M. (1993). The architecture and dynamics of developing mind: Experiential structuralism as a frame for unifying cognitive developmental theories. *Monographs of the society for research in child development* (Vol. 5, Serial no. 234).

Ding-Geng, C., Xinguang, C., Feng, L., Wan, T., Yuhlong, L., & Yuanyuan, G. (2014). Cusp catastrophe polynomial model: Power and sample size estimation. *Open Journal of Statistics, 4*, 803–813.

Feraro, T. (1978). An introduction to catastrophes. *Behavioral Science, 23*, 291–316.

Fodor, J. (1980). Methodological solipsism considered as a research strategy in cognitive science. *Behavioral and Brain Sciences, 3*, 63–73.

Freeman, W. J. (2000a). *Neurodynamics: An exploration in mesoscopic brain dynamics*. Berlin: Springer.

Freeman, W. (2000b). *How brains make up their minds*. New York: University Columbia Press.

Freeman, W., & Barrie, J. M. (2001). Chaotic oscillations and the genesis of meaning in cerebral cortex. In W. Sulis, & I. Trofimova (Eds.), *Nonlinear dynamics in the life and social science* (pp. 45–62). NATO Science Series, Vol. 320.

Gilmore, R. (1981). *Catastrophe theory for science and engineers*. New York: Wiley.

Grasman, R., van der Maas, H., & Wagenmakers, E. (2009). Fitting the cusp catastrophe in R: A cusp package primer. *Journal of Statistical Software, 32*(8), 1–27.

Gregson, R. A. M., & Guastello, S. J. (2005). Introduction to nonlinear methodology, part 1: Challenges we face and those that we offer. *Nonlinear Dynamics, Psychology, and Life Sciences, 9*, 371–374.

Grigsby, J., & Osuch, E. (2007). Neurodynamics, state, agency, and psychological functioning. In C. Piers, J. P. Muller, & J. Brent (Eds.), *Self-organizing complexity in psychological systems* (pp. 37–83). New York: Jason Aronson.

Guastello, S. J. (1981). Catastrophe modeling of equity in organizations. *Behavioral Science, 26*, 63–74.

Guastello, S. J. (1982). Moderator regression and the cusp catastrophe: Application of two-stage personnel selection, training, therapy, and program evaluation. *Behavioral Science, 27*, 259–272.

Guastello, S. J. (1987). A butterfly catastrophe model of motivation in organization: Academic performance. *Journal of Applied Psychology, 72*, 165–182.

Guastello, S. J. (1992). Clash of the paradigms—A critique of an examination of the polynomial regression technique for evaluating catastrophe-theory hypotheses. *Psychological Bulletin, 111*(2), 375–379.

Guastello, S. J. (1995). *Chaos, catastrophe, and human affairs: Application of nonlinear dynamics to work, organizations, and social evolution*. Mahwah, NJ: Lawrence Erlbaum Associates.

Guastello, S. J. (2002). *Managing emergent phenomena: Nonlinear dynamics in work organizations*. Mahwah, NJ: Lawrence Erlbaum Associates.

Guastello, S. J. (2011a). Discontinuities and catastrophe models with polynomial regression. In S. J. Guastello & R. A. M. Gregson (Eds.), *Nonlinear dynamical systems analysis for the behavioral sciences using real data* (pp. 251–280). Boca Raton, FL: CRC Press.

Guastello, S. J. (2011b). Catastrophe models with nonlinear regression. In S. J. Guastello & R. A. M. Gregson (Eds.), *Nonlinear dynamical systems analysis for the behavioral sciences using real data* (pp. 305–318). Boca Raton, FL: CRC Press.

Haken, H. (1983). *Synergetics, nonequilibrium phase transition and self-organization in physics, chemistry, and biology*. Berlin: Springer-Verlag.

Haken, H. (1990). *Information and self-organization: A microscopic approach to complex systems*. Berlin: Springer-Verlag.

Hartelman, P. (1997). *Stochastic catastrophe theory*. Unpublished doctoral dissertation.

Hartelman, P. A. I., van der Maas, H. L. J., & Molenaar, P. C. M. (1998). Detecting and modeling developmental transitions. *British Journal of Developmental Psychology, 16*, 97–122.

Hollis, G., Kloos, H., & van Orden, G. (2008). Origins of order in cognitive activity. In S. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: The theory of nonlinear dynamical systems* (pp. 206–241). New York: Cambridge University Press.

Johnstone, A. H., & Al-Naeme, F. F. (1991). Room for scientific thought? *International Journal of Science Education, 13*, 187–192.

Johnstone, A. H., & El-Banna, H. (1986). Capacities, demands and processes—A predictive model for science education. *Education in Chemistry, 23*, 80–84.

Kauffman, S. (1995). *At home in the universe: The search for laws of self-organization and complexity*. New York: Oxford University Press.

Kelso, J. A. S. (1995). *Dynamic patterns: The self-organization of brain and behaviour*. Cambridge, MA: MIT Press.

Kolata, G. B. (1977). Catastrophe theory: The emperor has no clothes. *Science, 196*(287), 350–351.

Kypraios, N., Stamovlasis, D., & Papageorgiou, G. (2014). The role of some individual differences in understanding chemical changes: A study in secondary education. *International Journal of Environmental and Science Education, 9*(4), 413–427.

Lakatos, I. (1974). Falsification and the methodology of scientific research programmes. In L. Lakatos & A. Musgrave (Eds.), *Criticism and the growth of knowledge* (pp. 100–101). Cambridge: Cambridge University Press.

Lange, R., Oliva, T. A., & McDade, S. R. (2000). An algorithm for estimating multivariate catastrophe models: GEMCAT II. *Studies in Nonlinear Dynamics and Econometrics, 4*, 137–168.

Lawson, A. E. (1983). Predicting science achievement: The role of developmental level, disembedding ability, mental capacity, prior knowledge, and beliefs. *Journal of Research in Science Teaching, 20*(2), 117–129.

Lorenz, H.-W. (1989). *Nonlinear dynamical economics and chaotic motion*. Heidelberg: Springer.

Maxwell, S. E. (2000). Sample size and multiple regression analysis. *Psychological Methods, 5*, 343–458.

Molenaar, P. C. (1986). On the impossibility of acquiring more powerful structures: A neglected alternative. *Human Development, 29*, 245–251.

Molenaar, P. C., & Oppenheimer, L. (1985). Dynamic models of development and the mechanistic-organismic controversy. *New Ideas in Psychology, 3*, 233–242.

Molenaar, P. C., & Raijmakers, M. E. (2000). A causal interpretation of Piaget's theory of cognitive development: Reflections on the relationship between epigenesis and nonlinear dynamic. *New Ideas in Psychology, 18*, 41–55.

Morse, M. (1931). The critical points of a function of n variables. *Transactions of the American Mathematical Society, 33*, 72–91.

Niaz, M. (1989). Relation between Pascual-Leone's structural and functional M-space and its effect on problem solving in chemistry. *International Journal of Science Education, 11*, 93–99.

Niaz, M., de Nunez, S. G., & de Pineda, R. I. (2000). Academic performance of high school students as a function of mental capacity, cognitive style, mobility-fixity dimension, and creativity. *Journal of Creative Behavior, 34*, 18–29.

Nicolis, S. J. (1986). Chaotic dynamics as applied to information processing. *Reports on Progress in Physics, 49*, 1109–1187.

Nicolis, S. J. (1991). *Chaos and information processing*. Singapore: World Scientific.

Nicolis, G., & Nicolis, C. (2007). *Foundations of complex systems*. Singapore: World Scientific.

Nicolis, G., & Prigogine, I. (1977). *Self-organization in non-equilibrium systems*. New-York: Wiley.

Nicolis, S. J., & Tsuda, I. (1999). Mathematical description of brain dynamics in perception and action. *Journal of Consciousness Studies, 6*, 215–228.

Nowak, A. (2004). Dynamical minimalism: Why less is more in psychology? *Personality and Social Psychology Review, 2*, 183–193.

Oliva, T. A., & Capdeville, C. M. (1980). Sussman and Zahler: Throwing the baby out with the bathwater. *Behavioral Science, 25*, 229–230.

Oliva, T. A., Desarbo, W., Day, D., & Jedidi, K. (1987). GEMCAT: A general multivariate methodology for estimating catastrophe models. *Behavioral Science, 32*, 121–137.

Pascual-Leone, J. (1970). A mathematical model for the transition rule in Piaget's developmental stages. *Acta Psychologica, 32*, 301–345.

Pascual-Leone, J. (1987). Organismic processes for neo-Piagetian theories: A dialectal causal account of cognitive development. *International Journal of Psychology, 22*, 531–570.

Pascual-Leone, J. (2000). Reflections on working memory: Are the two models complementary? *Journal of Experimental Child Psychology, 77*, 138–154.

Pascual-Leone, J. (2013). Can we model organismic causes of working memory, efficiency and fluid intelligence? A meta-subjective perspective. *Intelligence, 41*, 738–743.

Piaget, J. (1967). *Six psychological studies*. New York: Vintage.

Piaget, J., & Inhelder, B. (1969). *The psychology of the child*. London: Routledge.

Poston, T., & Stewart, I. (1978). *Catastrophe theory and its applications*. New York: Dover.

Preece, P. F. W. (1980). A geometrical model of Piagetian conservation. *Psychological Reports, 46*, 142–148.

Press, W. H., Teukolsky, S.A. Vetterling, W.T., & Flannery, B. P. (2007). Section 2.10. QR decomposition. *Numerical recipes: The art of scientific computing* (3rd ed.). New York: Cambridge University Press.

Prigogine, I. (1961). *Introduction of thermodynamics of irreversible processes*. New York: Wiley.

Prigogine, I., & Stengers, I. (1984). *Order out of chaos: Man's new dialogue with nature*. New York: Bantam.

Puu, T. (1981). Catastrophic structural change in a continuous regional model. *Regional Science and Urban Economics, 11*, 317–333.

Rosser, J. B., Jr. (1991). *From catastrophe to chaos: A general theory of economic discontinuities*. Boston, MA: Kluwer.

Rosser, J. B., Jr. (2007). The rise and fall of catastrophe theory applications in economics: Was the baby thrown out with the bathwater? *Journal of Economic Dynamics & Control, 31*, 3255–3280.

Saari, D. G. (1977). A qualitative model for the dynamics of cognitive process. *Journal of mathematical Psychology, 15*, 145–168.

Saunders, P. T. (1980). *An introduction to catastrophe theory*. Cambridge: Cambridge University press.

Sideridis, G. D., & Stamovlasis, D. (2014). The role of goal orientations in explaining academic cheating in students with learning disabilities: An application of the cusp catastrophe. *Ethics and Behavior, 24*, 444–466.

Sideridis, G. D., Stamovlasis, D., & Antoniou, F. (2015). Reading achievement, mastery, and performance goal structures among students with learning disabilities: A nonlinear perspective. *Journal of Learning Disabilities*. doi:10.1177/0022219415576524.

Stamovlasis, D. (2006). The nonlinear dynamical hypothesis in science education problem solving: A catastrophe theory approach. *Nonlinear Dynamics, Psychology and Life Science, 10*, 37–70.

Stamovlasis, D. (2010). Methodological and epistemological issues on linear regression applied to psychometric variables in problem solving: Rethinking variance. *Chemistry Education, Research and Practice, 11*, 59–68.

Stamovlasis, D. (2011). Nonlinear dynamics and Neo-Piagetian theories in problem solving: Perspectives on a new epistemology and theory development. *Nonlinear Dynamics, Psychology and Life Science, 15*, 145–173.

Stamovlasis, D. (2014a). *Catastrophe theory in education*. Paper presented in the 6th International Nonlinear Science Conference—INSC 2014, 20–23 March, Faculty of Social Sciences, Radboud University, Nijmegen, The Netherlands.

Stamovlasis, D. (2014b). Bifurcation and hysteresis effects in students' performance: The signature of complexity and chaos in educational research. *Complicity: An International Journal of Complexity and Education, 11*(2), 51–64.

Stamovlasis, D., & Papageorgiou, G. (2012). Understanding chemical change in primary education: The effect of two cognitive variables. *Journal of Science Teachers Education, 23*, 177–197.

Stamovlasis, D., & Sideridis, G. (2014). Ought approach–ought avoidance: Nonlinear effects under achievement situations. *Nonlinear Dynamics, Psychology and Life Science, 18*(1), 67–90.

Stamovlasis, D., & Tsaparlis, G. (2001). An application of complexity theory to an information model in science education. *Nonlinear Dynamics, Psychology and Life Science, 5*(3), 267–286.

Stamovlasis, D., & Tsaparlis, G. (2005). Cognitive variables in problem solving: A nonlinear approach. *International Journal of Science and Mathematics Education, 3*, 7–32.

Stamovlasis, D., & Tsaparlis, G. (2012). Applying catastrophe theory to an information-processing model of problem solving in science education. *Science Education, 96*(3), 392–410.

Stamovlasis, D., Tsitsipis, G., & Papageorgiou, G. (2010). The effect of logical thinking and cognitive styles on understanding the structure of matter: An analysis with the random walk method. *Chemistry Education, Research and Practice, 11*, 173–181.

Stewart, I. N., & Peregoy, P. L. (1983). Catastrophe theory modeling in psychology. *Psychological Bulletin, 94*(2), 336–362.

Sussman, H. J., & Zahler, R. (1978a). A critique of applied catastrophe theory in applied behavioral sciences. *Behavioral Science, 23*, 383–389.

Sussman, H. J., & Zahler, R. (1978b). Catastrophe theory as applied to the social and biological sciences. *Synthése, 37*, 117–216.

Thom, R. (1956). Les singularité s des applications différentiables. *Annales Institute Fourier, 6*, 43–87.

Thom, R. (1972). *Stabilité Structurelle et Morphogenése: Essai d'une Théorie Générale des Modé les*. New York: Benjamin.

Thom, R. (1975). Catastrophe theory: its present state and future perspectives. In: Manning, A. (Ed.), Dynamical Systems-Warwick 1974. *Lecture Notes in Mathematics* (Vol. 468, pp. 366–389). Springer, Berlin (with response by Zeeman, E. C.).

Thom, R. (1983). *Mathematical models of morphogenesis*. Chichester: Ellis Harwood.

Thompson, J. M. T. (1982). *Instabilities and catastrophes in science and engineering*. New York: Wiley.

Tsaparlis, G. (2005). Non-algorithmic quantitative problem solving in university physical chemistry: A correlation study of the role of selective cognitive factors. *Research in Science and Technological Education, 23*, 125–148.

Tsaparlis, G., & Angelopoulos, V. (2000). A model of problem solving: Its operation, validity and usefulness in the case of organic-synthesis problems. *Science Education, 84*, 131–153.

Tsaparlis, G., & Zoller, U. (2003). Evaluation of higher *vs.* lower-order cognitive skills-type examinations in chemistry: Implications for university in-class assessment and examinations. *University Chemistry Education, 7*, 50–57.

Tsitsipis, G., Stamovlasis, D., & Papageorgiou, G. (2010). The effect of three cognitive variables on students' understanding of the particulate nature and the changes of state of matter. *International Journal of Science Education, 32*(8), 987–1016.

Tsitsipis, G., Stamovlasis, D., & Papageorgiou, G. (2012). A probabilistic model for students' errors and misconceptions in relation to three cognitive variables. *International Journal of Science and Mathematics Education, 10*, 777–802.

Vallacher, R. R., & Nowak, A. (2009). The dynamics of human experience: Fundamentals of dynamical social psychology. In S. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: Theory of nonlinear dynamical systems* (pp. 370–401). Boston, MA: Cambridge University Press.

van der Maas, H. L. J., Kolstein, R., & van der Pligt, J. (2003). Sudden transitions in attitudes. *Sociological Methods & Research, 23*(2), 125–152.

van der Maas, H. L. J., & Molenaar, P. C. (1992). Stagewise cognitive development: An application of catastrophe theory. *Psychological Review, 99*, 395–417.

van der Maas, H. L. J., & Raijmakers, M. E. J. (2009). Transitions in cognitive development: Prospects and limitations of a neural network approach. In J. Spencer, M. S. C. Thomas, & J. J. McClelland (Eds.), *Toward a unified theory of development: Connectionism and dynamic systems theory reconsidered* (pp. 299–312). Oxford: Oxford University Press.

van Geert, P. (1991). A dynamic systems model of cognitive and language growth. *Psychological Review, 98*, 3–53.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wagenmakers, E. J., Grasman, R. P., & Molenaar, P. C. M. (2005). On the relation between the mean and the variance of a diffusion model response time distribution. *Journal of Mathematical Psychology, 49*, 195–204.

Wagenmakers, E. J., Molenaar, P., Grasman, R. P. P. P., Hartelman, P. A. I., & van der Maas, H. L. J. (2005). Transformation invariant stochastic catastrophe theory. *Physica D: Nonlinear Phenomena, 211*, 263–276.

Wagenmakers, E. J., van der Maas, H. L. J., & Molenaar, P. C. M. (2005). Fitting the cusp catastrophe model. In B. S. Everitt & D. C. Howell (Eds.), *Encyclopedia of statistics in behavioral science* (Vol. 1, pp. 234–239). Chichester: Wiley.

West, B. J., & Deering, B. (1995). *The lure of modern science: Fractal thinking. Studies of nonlinear phenomena in life sciences* (Vol. 3). New Jersey: World Scientific.

Whitney, H. (1955). Mappings of the plane into the plane. *Annals of Mathematics, 62*, 374–410.

Woodcock, A., & Davis, M. (1978). *Catastrophe theory*. New York: E.P. Dutton.

Zahler, R., & Sussman, H. J. (1977). Claims and accomplishments of applied catastrophe theory. *Nature, 269*, 759–763.

Zeeman, E. C. (1974). On the unstable behavior of the stock exchanges. *Journal of Mathematical Economics, 1*, 39–44.

Zeeman, E. C. (1976). Catastrophe theory. *Scientific American, 234*(4), 65–83.

Zeeman, E. C. (1977). *Catastrophe theory: Selected papers, 1972–1977*. Reading, MA: Addison-Wesley.

Chapter 10
Evaluating Complex Educational Systems with Quadratic Assignment Problem and Exponential Random Graph Model Methods

Russ Marion and Craig Schreiber

Introduction

The dynamic, changing nature of complex systems poses unique challenges for researchers. This is because complex systems are interactive, constantly changing networks in which agents are driven by personal perceptions and local rules of behavior yet are simultaneously embedded in, and engaging in interdependent relationships with, larger networks of agents. Described differently, the theory of complexity is multilevel theory (integrates multiple levels of analysis; Dansereau, Yammarino, & Kohles, 1999; Gupta, Tesluk, & Taylor, 2007; Hogue & Lord, 2007); its components interact interdependently and are changed by that interaction (Cilliers, 1998), and complex systems evolves in response to environmental pressures (Coveney, 2003).

Several research methodologies can capture elements of such complex processes. Qualitative analysis provides methods for understanding the collective constructionist nature of complex dynamics and how different processes interact to shape a collective (Chiles, Meyer, & Hench, 2004; Plowman et al., 2007). Mathematical modeling (Solow, Burnetas, Piderit, & Leenawong, 2003), agent-based modeling (Bonabeau, 2002), and systems dynamics (Sterman, 1994) can analyze evolving or emergent outcomes in complex systems. Traditional statistical designs examine variable relationships (measures of independent cases) and are generally inadequate for the estimation of complex processes. However, there are emerging statistical procedures tailored specifically for evaluating relationships among complex networks rather than among variables, and two such methods are the foci of this chapter.

R. Marion (✉)
Clemson University, Clemson, South Carolina, USA
e-mail: marion2@clemson.edu

C. Schreiber
Lenoir-Rhyne University, Hickory, North Carolina, USA
e-mail: craigschreiber.acad@yahoo.com

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_10

By way of introduction to these methods, there are two branches of statistical network analysis: actor-focused and tie-focused (van Duijn & Huisman, 2011; actors are also referred to as agents.) Actor-focused models distinguish between actor groups with the goal of explaining or predicting actor attributes. For more information on actor-focused models see van Duijn and Huisman (2011). Tie-focused models, on the other hand, seek to explain or predict ties and tie patterns within networks (or matrices). The emphasis of this chapter is on the two most widely used tie-focused models: multiple regression quadratic assignment problem (MR-QAP) and Exponential-family random graph models (ERGM). The social relations model (SRM) is another tie-focused model that is much less used and therefore is not covered in this chapter. More information about SRM is available in Kenny & La Voie, 1984.

MR-QAP (Krackhardt, 1987) is a statistical method for regressing output networks (matrices) on input networks. For example, a researcher might regress social capital (measured as each networked agent's level of access to resources) onto network density (a measure of the density of interactive relationships in a network), centrality (the degree to which information is channeled to central agents), and a trust network (who trusts whom). MR-QAP produces coefficients of determination, effect coefficients, and probability levels for each input matrix—the results are recognizable to anyone conversant with variable-based linear regression techniques. The interpretation, however, is somewhat different. For example, a significant positive coefficient for social capital on trust network indicates that capital tends to distribute to interdependent, high trust agent-relationships across a network rather than to low trust relationships. MR-QAP methods can perform analyses using standard variables like wealth or job position, but its real strength is in understanding how peoples' positions in a network of relationships affect given outcomes.

ERGM (also referred to occasionally as p*, or p-star) is likewise a regression procedure for networks but its focus is somewhat different. ERGM examines how network patterns "arise from the internal processes of the system of network ties" (White, Currie, & Lockett, 2014, p. 736). Robins, Pattison, Kalish, and Lusher (2007) stated more simply that ERGM examines how patterns of ties in a network are related to other patterns of ties. White et al. (2014), for example, used ERGM to argue that weak network hierarchy is associated with distributed patterns of information exchange in an organization while strong professional or strong managerial hierarchy is associated with LMX-like (leader-member exchange relationships) direct exchanges between leader and follower (see, for example, Graen & Uhi-Bien, 1995). ERGM results are presented as effect coefficients and standard errors.

We begin by describing salient issues regarding network analysis, particularly how to collect and setup networks for MR-QAP or ERGM analysis. We follow by describing MR-QAP analyses, its basic premises and interpretation; we then discuss possible research questions appropriate for QAP analysis and describe sample research studies utilizing this process. We follow a similar outline to describe ERGM procedures later in the chapter.

Network Analysis

Network analysis procedures study characteristics of networks of relationships, such as networks that are described by complexity theory. Network analysis can, for example, identify informal leaders in a network, describe the robustness of a network (such as its ability to move and use information), simulate what might happen to a network under various conditions, or model how a network will evolve over time. Networks analyzed by network analyses are constructed of paths through which information flows. Like complex systems, they are dynamic in that their structures change over time (a process that network analyses can simulate). Agents in such networks influence one another because they interact and are interdependent (e.g., they can represent common dependence on resources). Network analysis data is collected from organizations but that original data can be modified to demonstrate how that organization would respond to varying conditions.

Data for network analyses are collected by determining such things as who is social with whom (or who one interacts with over work, lives near, trusts, or is related to, etc.), who is associated with what tasks, resources, or knowledge, who is located where, and so on. This data is then formatted as matrices (e.g., an agent-by-agent (A×A) matrix of who consults with whom at work, an A×A trust network, etc.), typically by using spreadsheet software. Some analysis programs evaluate only A×A networks (e.g., UNICET); the software package ORA permits inclusion of such things as agent-by-tasks, or agent-by-resources, knowledge, or location matrices.

Participants in network analyses are typically (but not exclusively) selected because they are united in a common network of interactive and often interdependent relationships, and anyone who is not a part of the interactive, interdependent network would not be considered participants. School bus drivers, for example, are not usually considered part of a professional school staff network because they have little professional interaction with that staff.

A given pair of agents in a network need not have direct dyadic relationships with one another to be included but if not, they should be indirectly related through other relationships. Logically, of course, this indirect relationship principle could extend networks to include every person on this globe—the 6-degrees of separation, or Kevin Bacon, principle (the notion that everyone is connected by, on average, six intervening relationships, also called the Small World phenomenon). In practice, however, this is tempered by the nature of the design: One may only be interested, for example, in studying networks of agents who share work-related experiences and would not be interested in the familial relatives or club relationships of those agents. Teachers in a given schools or informal leaders in a school community might constitute a relevant network while teachers or community leaders in different schools or different school zones might not. That is, networks should be bounded by commonalities that are relevant to the researcher's questions.

This discussion of networks leads us to an important issue that differentiates network analysis from traditional variable-based methodologies: Variable-based

Table 10.1 How network matrices are represented for network analysis

The Agent 4 column represents the other agents in the network who selected Agent 4 as a relationship; this is Agent 4's in-degree

Rows represent who each agent selected, or their out-degree choices

	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5
Agent 1		0	1	1	0
Agent 2	0		0	1	1
Agent 3	1	1		1	0
Agent 4	0	0	0		1
Agent 5	0	1	1	1	

This table represents an agent-by-agent matrix, and thus the names in the first column mirror names in the first row, and the number of rows equals the number of columns

This table might be generated by asking respondents who they regularly go to for advice about instructional issues. 1s represent choices, 0s are non-choices

designs presume that variables may be correlated but that cases (participants in the analysis) should not be; the violation of this assumption is called autocorrelation (Fields, 2009). Consequently, variable-based researchers must be particularly attentive when evaluating samples in which participants are interactive and interdependent. Network analysis is attentive to just the opposite concern: Agents *should be* interactive and interdependent (also a basic principle of complexity theory). Variable-based research operates on randomly selected participants from across a population, participants who ideally do not influence one another's responses to any significant degree; network analysis operates on participants who do influence one another. Consequently, the selection of participants for network analysis should be carefully considered and participants who lie outside a given interactive, interdependent network are typically rejected.

Human interaction networks used in network analyses are agent-by-agent matrices and as such they must be square (number of rows equal number of columns) and the order of the names for the rows must be the same as the order of names in the columns (see Table 10.1). Other types of networks (e.g., agent-by-resources, knowledge-by-task) are not necessarily square. Data usually is represented in binary form representing the presence or absence of a link but may be recorded as weighted ordinal or scale data.

Data for network analyses can be collected in any of a number of ways. One might, for example, use existing documents such as the headers of emails in a given organization, which would reveal who is communicating electronically with whom.

Alternatively, one might build network matrices from direct observations. Educational researchers may find it more convenient, however, to collect data with surveys.

There is a sample survey for network analysis in the appendix. Several things about it should be noted. First, it is commonly necessary to obtain the names of respondents, which are needed to build matrices—one must know who has selected which other agents, tasks, etc., and matrices typically need considerable name-related manipulation before they are usable (rows and columns should be alphabetized so they can be ordered and the researcher must determine that all names in the columns have corresponding entries in the rows). Respondents may, of course, be provided assurance that their names will be anonymized once matrices are built and prior to analysis (this helps with ethics board's approval as well). Second, as noted, names (and the codes that ultimately may replace them) should be alphabetized. The program one uses to analyze network data will likely provide alphabetized output measurements and the researcher will often need to generate correspondences between the original matrix and these outputs. One can save much time by simply alphabetizing names in rows and columns to start with.

Third, note that respondents are given a choice of names (or tasks, resources, etc.) in the response scales for each question in the survey. If respondents were free to add names to an "other" write-in field, then any additional names would not likely be available for other respondents to select. Consequently, measurement outputs will be misrepresented (there are strategies for snowball selection but the snowballing process should precede collection of data for the network analysis).

The researchers should carefully develop the response scales used for data collection. Network participants, along with tasks, skills, and resources (if appropriate), must be defined and the network fully bounded. Task, skills, and resources are selected that relate to the research question. Further, tasks, skills, and resources should usually represent pathways that can link respondents (e.g., common experiences teaching grade 9 math, a task, disposes agents to interact with one another).

Network Measures

Network analyses describe networks with a number of useful network-level and agent-level measures. Network level measures include *density* (the number of actual links in a network divided by the total possible links) and *speed* (a measure based on the shortest paths between each agent and every other agent; networks with high scores move information quickly). From a practical perspective, density represents how intensely interactive a network is and speed represents how fast information flows through a network (see Fig. 10.2; the density of this network is 0.136, and the speed at which information flows through this network is 0.463 (moderate speed); both measures are scaled 0–1).

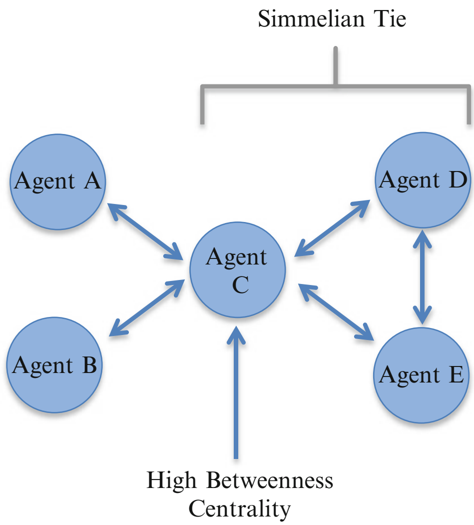

Fig. 10.1 Representation of Simmelian ties and betweenness centrality. Simmelian ties are triads of agents with reciprocal ties (they select one another as relationships, i.e., agents C, D, and E). Agents with high betweenness centrality connect two or more groups, i.e., agent C

Useful agent-level statistics include betweenness centrality (the degree to which an agent links other agents; see Fig. 10.1), resource competence (the degree of access an agent has to resources), Simmelian ties (the degree to which agents belong to three-way reciprocal relationships; Fig. 10.1), and hub centrality (the degree to which an agent has many out-degree links to agents who have many in-degree links). More practically defined, betweenness centrality (e.g., individuals who are connected, or "stand between," two or more groups) identifies the degree to which individuals are in position to intercept, thus influence, information flow and information content in a network; resource competence (the level of each individual's access to resources) can be a useful measure of social capital; Simmelian ties (triads of agents, all of whom identify one another as relationships) identify individuals who tend to be more affectively associated to the network than are others; and hub centrality identifies individuals who send information to highly connected others—the principal of a school likely has high hub centrality, for example. Figure 10.2 illustrates several measurements in the context of a full network. For a comprehensive overview of social network analysis see the *SAGE Handbook of Social Network Analysis* (Scott & Carrington, 2011) and *Social Network Analysis: Methods and Applications* (Wasserman & Faust, 1994).

MR-QAP

For the remainder of the chapter we explore the potential of two methods (MR-QAP and ERGM) for studying complexity dynamics and we broadly overview how these methods work. Complex networks are characterized by any number of interacting structures and processes. For example, groups of three interacting agents influence

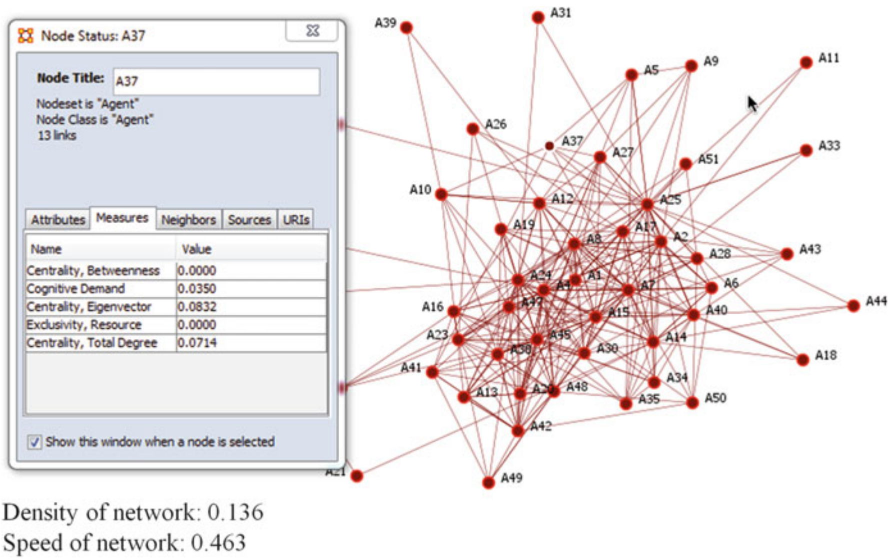

Fig. 10.2 A network created from survey data. *Circles* represent agents and *lines* represent ties. Select statistics for Agent 37 (A37) are shown; each ranges between 0 and 1. This individual has zero betweenness centrality (see text) but exhibits moderate cognitive demand (0.35; Agent 37 has a fair amount of access to knowledge and tasks performed by each agent, thus enabling him or her to perform moderate levels of work). Eigenvector centrality is a measure of well-connected individuals who are linked to well-connected others; resource exclusivity represents the number of resources to which a given agent has exclusive access; and total degree centrality is a measure of the total links a given agent has. Density of network: 0.136. Speed of network: 0.463

one another in unpredictable ways, but this relational dynamic also shapes the group's affective relationship with one another and with the system as a whole. Consequently, such triads (or Simmelian ties) tend to have significant influence in a system and help to shape and strengthen the complex system as a whole (Krackhardt, 1998, 1999; Tortoriello & Krackhardt, 2010). Cliques are another such interactive process, and we have learned that they absorb and process massive amounts of information in a complex knowledge system (Marion, Schreiber, Klar, Christiansen, & Reese, 2014), thus modifying well-known conclusions about moderate coupling and information flow by Stu Kauffman (1993) and Karl Weick (1976). Further, we have learned that another interactive dynamic, informal leadership, exerts important influence over the capacity of a network to perform effectively (Marion et al., 2014). Such insights are available because researchers now have tools for analyzing complex networks; MR-QAP and ERGM are two such tools.

The multiple regression quadratic assignment problem (MR-QAP) was first introduced by Koopmans and Beckmann (1957) to solve the problem of assigning a set of companies to a set of locations such that transportation costs between them are minimized (Burkard, Çela, Karisch, & Rendl, 2013). MR-QAP is a regression procedure that feels in many ways like variable-based regression except that it acts

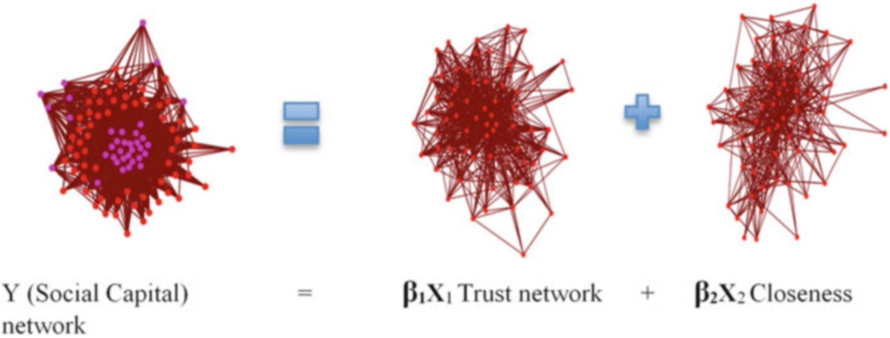

Fig. 10.3 Visual representation of QAP matrix regression; the error matrix is omitted for simplicity. **Y**, $\mathbf{X}_1$, and $\mathbf{X}_2$ represent matrices. Based on Blackwell (2014). Y (Social Capital) = $\boldsymbol{\beta}_1\mathbf{X}_1$ Trust network + $\boldsymbol{\beta}_2\mathbf{X}_2$ Closeness network

instead on networks, or matrices of interacting agents. MR-QAP regression can be visualized as shown in Fig. 10.3. Here networks are represented as circles (nodes, or agents) and lines (sides, or links between nodes). The figure shows a dependent network on the left that is hypothesized to be affected by two independent networks on the right. A regression equation similar to that produced by variable regression is shown below the networks, except that X and Y are matrices of social networks instead of variables.

Regression coefficients are relatively simple to produce in MR-QAP, and their calculation is quite similar to that for rank order correlation. The calculation of probability levels is more complex, for there is no variance inherent in network data from which to calculate confidence intervals. Instead, MR-QAP utilizes a bootstrapping permutation method. Imagine the face of a Rubrik's cube that is the same size as the matrix under investigation. A regression coefficient is calculated between the original observed matrix and the expected matrix. Then, the cube is randomly altered to produce a new configuration of relationships for the original matrix such that rows and columns are randomly permuted synchronously, and a new regression coefficient is calculated. The rows and columns are permuted synchronously so that the autocorrelation that exists in the observed relationships is maintained. This is repeated numerous times (our personal experience is that 10,000 such permutations produce stable results) to calculate numerous, randomly generated regression coefficients. The probability level is determined by the relative position of the original coefficient within these randomly generated coefficients; if the original is greater or less than 95 % of the random coefficients, then it is significant at the 5 % level.

Permutations can be performed in several ways. Each differs in its ability to deal with network (as opposed to variable) skewness, collinearity (which exists when vectors are equal to, or multiples of, other vectors), and autocorrelation (an agent's value or property is dependent on the value or property of others in their relational sphere, i.e., my productivity depends on your productivity). A recent permutation procedure called the Dekker permutation is robust against such network issues.

Table 10.2 Repeated vector techniques, used to create A×A networks when the researcher has only a single measure for each respondent

	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5
Agent 1	0.25	0.25	0.25	0.25	0.25
Agent 2	0.34	0.34	0.34	0.34	0.34
Agent 3	0.46	0.46	0.46	0.46	0.46
Agent 4	0.16	0.16	0.16	0.16	0.16
Agent 5	0.14	0.14	0.14	0.14	0.14

The first column is copied into each of the subsequent columns

Another popular permutation procedure, the Y-permutation, is a bit less robust against violations of assumptions (Dekker, Krackhardt, & Snijders, 2007).

MR-QAP acts on square matrices, thus one might use it to determine (for example) whether an agent-by-agent trust matrix is a function of an agent-by-agent social network plus an agent-by-agent physical proximity network. However, while valuable, this restriction to square matrices is a bit limiting: It would be useful, for example, to know how the principal's engagement in the school's advice network or agents' degree of betweenness centrality also contributes to explaining trust. The problem with doing this is that such measures are single vector matrices rather than square—each agent has one measure and the measure is represented as a single column matrix. Each agent has only one measure of centrality, and the principal advice engagement is represented as the single vector in-degree column for that person that is lifted out of an agent-by-agent advice network (Friedkin & Slater, 1994).

There are two primary ways for dealing with this. The first we may call the repeated vector method. Using closeness centrality as an example, one enters agent names and their measures in contiguous columns of a spreadsheet. Then one copies the names column to the top row of the spreadsheet (use Paste Transpose) to produce a mostly blank (except for the first column vector) agent-by-agent network. Finally, the agent's scores for closeness centrality measures are copied from column 2 to each remaining column in the agent-by-agent matrix (see Table 10.2). The network is now square and can be used in a QAP analysis (see Borgatti, Everett, & Johnson, 2013).

The second procedure is useful when data points can be interpreted as distances. For example, one may have a single vector that identifies whether the given agent is low, middle, or high socioeconomic status (SES). The distance between low and middle is 1 and that between low and high is 2, and to characterize distances between dyadic pairs of agents in this manner makes conceptual sense. A square, symmetrical agent-by-agent matrix, then, is calculated as the distance (non-absolute) between the statuses of each pair of agents.

MRQAP results for difference matrices should be interpreted as indicating differences between different regions of a dependent network. For example, a researcher might use difference matrices to determine if various regions in a dependent matrix differ by socioeconomic status and to what degree they differ.

Interpretation

MR-QAPs involving repeated vector matrices are interpreted differently than are relational matrices (e.g., who associates with whom). Consequently we look at three conditions: MR-QAPs involving just repeated vector matrices, those involving a mixture of vector and relational matrices, and analyses of relational matrices only.

Interpretation of relationships among repeated vector matrices of agent–level measures only (such as the effect of betweenness centrality and Simmelian ties on resource capability; see above for definitions) is straight forward, for coefficients in these cases can be understood just as relationships among variables in traditional statistics methods are understood. Indeed, were one to treat the measures as variables rather than as matrices, and if autocorrelation were not a problem, standard variable-based multiple regression should produce results close to those produced by MR-QAP.

Regressions of repeated vector and relational networks are a little more complicated to interpret. A positive relationship between, say, a betweenness centrality repeated vector matrix and an A×A social network indicates that agents with high betweenness centrality measures map onto highly connected areas of the A×A social matrix and vice versa. Thus individuals who exhibit high betweenness centrality are likely to be socially well connected.

Regressions of relational on relational networks are interpreted similarly. A positive relationship between A×A social and A×A advise networks, for example, indicate that agents in highly interactive regions of one network map onto highly interactive regions in the other network. Positive coefficients between two networks indicate that highly vibrant interactions in the two networks tend to map onto each other (and vice versa, of course) and negative coefficients indicate that high vibrant areas in one network map with low vibrant areas in the other.

MR-QAP Illustrated

Table 10.3 is produced by a MR-QAP analysis using the ORA software package. This particular analysis illustrates how certain agent-level measures and relationship networks affect access to resources. The measurements were created by converting the agent measures to matrices with repeated vector procedures. Ten thousand permutations were run to obtain significance levels; we started at 1000 then tried increasingly larger numbers until the probability levels stabilized.

The second table of results in Table 10.3 report correlations between each independent network and the dependent network. The Hamming distance in this table is the total number of ties that must be changed (new links added or old ones deleted) in the independent matrix to convert it into a duplicate of the dependent network; i.e., it measures how different the two networks are (Borgatti, Carley, & Krackhardt, 2006). The Euclidean distance is the square root of the sum of squares of distances between every pair of dyads in two matrices. This is roughly equivalent to how mean squares are calculated in linear statistics.

Table 10.3 MR-QAP analysis of betweenness, centrality, principal engagement, social network, and work network on access to political resources

Random seed	0			
Number of permutations	10,000			
Dependent network	Agent x Agent Resource Capability			
Independent network names	Agent x Agent Betweenness Total, Agent x Agent Hub Central Total, Agent x Agent Principal Engagement, Agent x Agent Trust, Agent x Agent Work			
Number of independent networks	5			
Correlation results				
Network	Correlation	Significance	Hamming distance	Euclidean distance
Agent x Agent Betweenness Total	0.203	0.080	2550	24.322
Agent x Agent Hub Centrality Tot	0.158	0.137	2550	17.071
Agent x Agent Principal Engagement	0.245	0.037	2550	22.573
Agent x Agent Social	0.067	0.138	2550	27.116
Agent x Agent Work	0.132	0.011	2550	20.812
Regression results				
R-Squared: 0.144875839392				
Variable	Coef	Std. Coef	Sig. Y-Perm	Sig. Dekker
Constant	−0.623		0.020	
Agent x Agent Betweenness	1.498	0.517	0.032	0.035
Agent x Agent Hub Centrality Total	−0.819	−0.627	0.068	0.072
Agent x Agent Principal Engagement	0.717	0.777	0.024	0.023
Agent x Agent Social	−0.012	−0.022	0.039	0.028
Agent x Agent Work	0.037	0.054	0.039	0.025

The third table, the regression results, reveals that about 14.5 % of the variance in the dependent network is explained by the independent networks. The table itself reports unstandardized and standardized coefficients plus significances using Y-permutations and the Dekker permutation. Only hub centrality fails to affect resource capability at the 5 % level, but since the probability level is 0.07 and permutations are calculated using Monte Carlo procedures, it is safe to conclude that the hypothesis for hub centrality is at least promising (this is particularly appropriate when the analysis is exploratory; Friedkin & Slater, 1994). As observed earlier, the Dekker permutation is the more robust of the permutations.

The dependent matrix, resource capability, was generated from a single vector measure called resource capability, a measure of access to resources (in this study, political resources). Each agent had one measure for resource capability thus producing a single vector; this vector was copied repeatedly into each column of a square A×A matrix as described earlier, the repeated vector method. Betweenness, hub centrality, and principal's engagement in the advice network (this represents the teachers who seek advice or do not seek advice from the principal) were created similarly using the repeated vector method. The interpretation of their impacts is straightforward: Betweenness and principal engagement in the advise network have positive coefficients and consequently map directly with correspondingly high or low regions of the resource capability network; hub centrality exhibits a negative, hence inverse, effect on resource capability. That is, the degree to which teachers are informal leaders because they stand in the communication links between groups and the degree to which they seek advice from the principal are positively related to access to school resources. The degree to which teachers communicate to powerful clusters of other teachers in the school is inversely related to resource access.

Agent-by-agent social and agent-by-agent work are matrices of dyadic relationships that were generated by asking agents with whom they socialized and with whom they worked. The work matrix is positively related to the dependent network and we conclude that regions of high work relationships on the A×A work landscape map onto areas of high access to resources on the resource capability matrix. The negative coefficient for social indicates that regions of high sociability in the social landscape are associated with low resource access and vice versa. That is, agents with closer social relationships have less access to resources than do agents with fewer social relationships.

Research Possibilities Using MR-QAP

The repeated vector method or the distance (between pairs of agents) method enables matrices to be created for just about any attribute. One could, for example, create A×A matrices from student test scores or attitudinal scales. Blackwell (2014), for example, examined the effects of closeness centrality, Simmelian ties, a trust network, social network, and work interaction network on an independent, attribute measure of creativity, and found significances for closeness centrality, trust network, and the social network. Tsai and Ghoshal (1998) similarly found that trust and social networks are related to resource interaction, which in turn affect creativity.

We found relatively little in the education literature that used MR-QAP, but the procedure has seen greater application in the business, psychology, and sociology literature. Brewer and Webster (1999), for example, found that forgetting past friends is related to density, cliques, centralizations, and individual centralities; this has implications for research designs that analyze variables based on memory. Bowler and Brass (2006) related interpersonal organizational behavior to the strength of network ties. Marineau and Labianca (2010) looked at the effects of personal and work-related conflict on the flow of information in an organization. The interested reader can find other examples in various business, psychology, sociology, and methodology journals, but we would recommend especially the Academy of Management journals, the journal *Social Networks*, and the journal, *Connections*.

ERGM

QAP predicts one network from one or more other networks. ERGM, or exponential random graph models, predict different sorts of network outcomes. From a basic standpoint, an ERGM identifies the micro-structures, called configurations, within an observed network that are influenced by social processes that generate network ties. ERGM then compares these configurations to a random network (or graph) in which these social processes are not at play to determine if the observed configurations in the actual social (or observed) network appear more often than one would expect by chance (Borgatti et al., 2013). For example, one might explore network features such a cliques or gender to explain the emergence and patterns of bullying in schools. ERGMs belong to the general linear family of models in standard statistics, but ERGMs differ from these standard models in that ERGMs have been modified to deal with the dependence of observations, or ties or relations between people in a network.

To explain, we will discuss how ERGM developed over time to show how dependence has been incorporated into the models. The evolution of ERGM can be arranged into four increasingly complex categories of models: simple random graph models, dyadic independence models, dyadic dependence models, and higher-order dependence models (Harris, 2014). We begin with the least complex of these models, the simple random graph models.

A *random* graph is a network that has *n* nodes and each independent tie between all pairs of nodes has the same common probability of being chosen (Frank & Strauss, 1986). The probability of a tie being chosen is equal to the density of the separate, observed network. Patterns of relationships, like three-way ties or stars configurations (a central actor connects four other actors) among such random, independent ties have equal chances of occurring and are not dependent on any other relationship in the network. Therefore, any attributes of the nodes (e.g., gender or tenure) or any social forces (e.g., numerous three-actor friendships or patterns of trust) influencing relationship formation are ignored.

This randomness, of course, is unlikely to occur in actual human networks, particularly in the bounded, interdependent networks described earlier in this chapter. Network researchers have identified several structural features based on influencing forces that commonly occur in observed networks; these features include nonuniform distribution, homophily, transitivity, and reciprocity (Harris, 2014). Nonuniform distribution refers to the tendency for some people to form a high degree of ties while others form a moderate degree of ties and still others form only a few ties. Homophily refers to the reality that people with similar attribute (s) tend to form ties with one another. Transitivity is the tendency for the friends of friends to become one's friends. Reciprocity means that a relationship, which is directed from one person to another person, is reciprocated by that other person more often than would be expected by random chance (if I identify you as a friend, you are likely to identify me as a friend as well; Robins & Lusher, 2013).

ERGM compares the patterns of relationships, or configurations, in an observed structure against those in random graphs that represent other possible ways that a network of the same size can be arranged. It does this by generating large numbers of random graphs using Monte Carlo procedures, which produces a sample space. It then determines if the patterns of relationships in the observed network occur more often than would be expected by chance. If a relational pattern does occur more often than by chance then it can be kept in the model as an explanatory social process underlying the observed network's structural formation. For example, if the number of same gender ties occurs significantly more often in the observed network as compared to the random networks then it can be concluded that gender homophily is a social process influencing the observed network structure. This is repeated several times for different relational patterns that are theoretically or hypothetically expected to have an influence on network formation. The process of using ERGM is like model-building and the baseline model obtained from simple random graphs provides a way to assess model improvement as different relational patterns are input into the model.

The next innovation in the development of ERGM is called, dyadic independence models, and was introduced by Holland and Leinhardt (1981). Their p_1 model, as they labeled it, modified the basic model described above to capture a larger variation of in-degree ties plus the reciprocity existing in observed networks (e.g., agent A trusts agent B and agent B reciprocates that trust). As a result, networks of various sizes and densities could be directly compared. This was a major breakthrough for statistical network modeling. Despite this breakthrough, dyadic independence models were not able to capture transitivity, homophily and other influencers of social structure and therefore were still of limited use.

Dyadic dependence models, the third modification to ERGM, were first presented by Frank and Strauss (1986). Frank and Strauss improved the p_1 dyadic independence models of Holland and Leinhardt (1981) by incorporating the exponential family of distributions and adding a Markov dependence assumption. Markov dependence assumes that two ties with a node in common are dependent (e.g., if A and B both relate to C, they are likely to become related to each other). With this assumption, dyadic dependence models began to capture transitivity (e.g., a friend of my friend will likely become my friend as well) and improved the capturing of nonuniform degree distribution (some agents form numerous relationships and others form few). While this expanded the representation of observed network characteristics, the model did not account for node attributes such as gender or tenure, which are often significant influencers of network formation.

Wasserman and Pattison (1996) extended the dyadic dependence model by developing their p* model. The p* model uses a more general assumption of dependence whereas the probability of a tie existing is conditionally dependent upon all other ties in the network and not just ties that share a common node. In addition, the p* model can incorporate the attributes of nodes. This made p* models extremely useful, as node attributes are often related to the structural characteristics that emerge in observed networks. All in all, the p* model improved ERGM by allowing for a better representation of the structural features of observed networks and accounting for social process influencers of network formation.

Despite the improvement, dyadic dependence models often had problems with degeneracy. Degeneracy occurs when some simulated random networks are almost void of ties or almost full of ties. When these degenerate networks are averaged with the other simulated networks in the Monte Carlo process, the resulting estimate may look to be reasonable whereas in actuality it is not an appropriate baseline against which the observed network can be compared. Degeneracy is due to the configuration patterns of observed networks not being adequately captured.

Higher-order dependence models were developed to help address the issues of degeneracy. Hunter and Handcock (2006), building on work by Pattison and Robbins (2002) and Snijders, Pattison, Robbins, and Handcock (2006), added three terms to the p* model to better account for conditional dependence patterns and thus reduce degeneracy. The three terms are:

- Geometrically weighted degree distribution (GWD)—accounts for the decreasing degree distribution (variance) found within observed networks, i.e., fewer members have a high degree of ties and many members have a few degree of ties.
- Geometrically weighted edgewise shared partnerships (GWESP)—accounts for the transitivity patterns of clusters found within observed networks.
- Geometrically weighted dyad-wise shared partnerships (GWDSP)—accounts for dyads with shared partners, which is another cluster characteristic often found in observed networks.

Progress on the ability to statistically capture observed network characteristics has made ERGM useful and popular. At the moment, ERGM only handles binary network data, or data that indicates the existence/nonexistence of a relationship. Current efforts are seeking ways to allow ERGM to handle valued data that indicates not only the existence of a relationship but also the strength of the relationship (Krivitsky, 2012). For a thorough explanation of ERGM see *Exponential Random Graph Models for Social Networks* (Lusher, Koskinen, & Robins, 2013).

ERGM Illustrated

In this section we illustrate the model-building process of ERGM. We begin with a null (random) model and then add variables. As we add variables we assess model improvement and determine what to keep in the model. A tie is an outcome, and node attributes plus structural characteristics are used to explain or predict the probability of a tie (Hunter, Goodreau, & Handcock, 2008).

Prior to building a model, it is advisable to explore one's data to see if there are any patterns that will give insight about the structure of the network. These insights will help develop the model. One way to explore the data is to visualize it with a network program like ORA or UNICET and to color-code the nodes by specific attributes (e.g., see Fig. 10.2). If, for instance, gender is color-coded and gender-based clusterings are observed, it would be a good idea to use gender as a control variable in the model. The hypothesis is that the attribute-based clustering is occurring more than would happen by chance. Any given pattern-identified attribute may not necessarily result in a statistically significant outcome for that attribute or, even if statistically significant, it may not result in model improvement, but at this point in the analysis, the intent is to focus the model building with theory-based and hypothesized variables that show promise for explaining patterns.

Another way to explore the data is to look at the effects of descriptive network statistics like density, degree frequency and triads. The density of the observed network (number of actual ties/total possible ties) enables the researcher to match the random graphs generated during model building to the observed network. Degree frequency indicates whether there is declining frequency across ties in the observed network that simple random graphs do not reflect. Examining triads (agents, or nodes, that are linked by reciprocal ties) will show if triads are an element of the underlying structure of the network. Of course, one would want to look at other statistics and potential influencers in the observed network structure for possible inclusion in the model. The terms included in the model should, however, represent a theoretical and hypothesized statement of what is affecting network formation.

There are a few software packages available for running ERGM. The most widely used currently are Statnet (Goodreau, Handcock, Hunter, Butts, & Morris, 2008; Handcock, Hunter, Butts, Goodreau, & Morris, 2008) and PNet (Wang, Robins, & Pattison, 2009). The software used in this illustration is Statnet, which is a package in the R statistical computing environment.

The dataset used in this illustration is a study seeking to explain collaborative innovation ties within an organization. A collaborative innovation tie exists when two people interact and work on an innovative idea. In addition to the collaborative innovation network, data for separate shared leadership and adaptive leadership networks were collected to determine if they have an effect on the collaborative innovation network. Data on agents' demographics were also collected to account for agent attributes that may influence the formation of collaborative innovation ties.

To begin model building, the hypothesized antecedent networks (shared leadership and adaptive leadership) and agent attributes were entered as variables in the model (see Table 10.4). These antecedent networks, which represent tie (also called edge) covariates, are the main network tie-variables of interest and agent attributes are added because these variables are hypothesized to influence network formation. Social networks are complex and there are many simultaneous explanations of a particular structure in observed networks. Several variables are included in this first iteration to determine if the combined presence of these predictors provides

Table 10.4 Effects of shared leadership, adaptive leadership, education, and gender on the likelihood of collaborative innovation ties

Maximum likelihood results:			
	Estimate	Std. Error	*p*-value
Edges	−2.17170	0.09161	<1e-04 ***
edgecov.sharedlead	0.98789	0.05193	<1e-04 ***
edgecov.adaptlead	1.06109	0.08597	<1e-04 ***
nodematch.edu	0.49363	0.12805	0.000118 ***
nodematch.gender	0.19876	0.10060	0.048250 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
AIC: 2646.5 BIC: 2677.4

explanation for the collaborative innovation ties. An examination of the effects of each predictor independently may overestimate the influence of that predictor (Robins & Lusher, 2013).

The results of the ERGM are shown in Table 10.4, which includes parameter estimates that specify the direction and strength of the network patterns. In addition, the standard errors and *p*-value are reported. One thing to note is that the strength of different network patterns is not comparable because scaling varies for each statistic. Results are interpreted below:

Edges: Edges are a measure of tie frequency; it indicates whether density is greater or less than 50 %. Its interpretation is similar to that for the intercept of linear regression. An edge coefficient of 0 indicates a density of 50 % whereas a positive or negative edge coefficient indicates a density above or below 50 %, respectively. In this case, the edge coefficient is negative indicating a density below 50 %. This is quite common as observed networks normally do not have densities above 50 %.

Shared Leadership: The shared leadership network (edgecov.sharedlead) is considered exogenous to the collaborative innovation network and is therefore fixed in the ERGM modeling process. The estimate associated with this covariate network of shared leadership is positive and significant. This indicates that shared leadership and collaborative innovation ties co-occur—shared leadership is a predictor of collaborative leadership.

Adaptive Leadership: The adaptive leadership network (edgecov.adaptlead) is also considered exogenous and is fixed in the modeling process. The covariate network of adaptive leadership is positive and significant indicating that the adaptive leadership and collaborative innovation ties co-occur.

Education: Education is a measure of the tendency for agents to interact in the collaborative innovation network based on similar education levels. This is a homophily effect. The node attribute education (nodematch.edu) is positive and significant.

Gender: The node attribute gender (nodematch.gender) is both positive and significant. This indicates the tendency for agents to interact in the collaborative innovation network based upon gender. This is another homophily effect.

In addition to the parameter estimates, two measures of model fit, AIC and BIC, are reported. AIC, or the Akaike information criterion, takes the form, deviance $(-2\text{LL}) + 2p$. BIC is the Bayesian information criterion and takes the form, deviance $(-2\text{LL}) + p^{*}\ln(N)$, where p refers to the number of parameters and N refers to the sample size (Harris, 2014).

ERGM can have many terms, and deviances (AIC and BIC) will become smaller as more terms are added to the model. However, AIC and BIC penalize models for adding terms that do not contribute enough explanation to provide a better fit. This is similar to adjusted R^2 in linear regression. The BIC measure penalizes additional parameters more so than AIC. At this point, we merely note the AIC and BIC statistics for comparison with subsequent model runs.

Now, we run another model to see if other variables can further explain the collaborative structure. Additional node attributes relating to service at the organization were input as variables into the model along with structural terms accounting for triads. The node attributes relating to organizational service included tenure (nodematch.orgyr), professional affiliation (background; nodematch.profaff), and subunit (nodematch.subunit).

The structural terms for triads were included to account for transitivity (a relationship of someone I know is likely to become another relationship of mine). There were, additionally, two terms used: GWESP and balance. GWESP, as noted before, accounts for transitivity found in clusters (also called cliques). We also used a curved exponential family model indicated by the gwesp.alpha. GWESP and the curved exponential family model were calculated using a sub-routine within Statnet. The calculation of GWESP requires the input of a degree weighting parameter alpha (α), which influences the value of this statistic. Typically, one would start with a low α and then increase it until the log-likelihood stopped improving. Running an ERGM can take a very long time and so running several models until the best log-likelihood is reached could be quite time intensive. A curved exponential family model estimates the α resulting in the best log-likelihood during model estimation thereby not requiring an a priori α.

Balance refers to balanced triads. Balanced triads are ones where sets of two actors have reciprocal ties. More about balanced triads and transitivity can be found in Davis and Leinhardt (1972). This is included to test for another way that triad structure and transitivity may be present in the observed network. One needs to use a directed network (indicating who is selecting whom) to test for balance since it measures reciprocity.

The results of this ERGM are shown in Table 10.5. The model includes the original parameter estimates since they were significant plus the new variables and terms. Results are interpreted below:

Original Model: The edges coefficient is still negative and significant. The shared leadership network, adaptive leadership network, education attribute and gender attribute are all still positive and significant.

Tenure: The tenure coefficient, represented by orgyr, is negative and significant. The number of years an agent worked in the organization was categorized into

Table 10.5 Table 10.4 with addition of tenure, professional affiliation, subunit, and triads (transitivity) and balance triads

Maximum likelihood results:			
	Estimate	Std. Error	*p*-value
edges	−6.506121	0.053030	<1e-04 ***
edgecov.sharedlead	0.800741	0.016539	<1e-04 ***
edgecov.adaptlead	0.814074	0.030644	<1e-04 ***
nodematch.edu	0.176544	0.051671	0.000641 ***
nodematch.gender	0.134263	0.027396	<1e-04 ***
nodematch.orgyr	−0.151436	0.047942	0.001598 **
nodematch.profaff	0.136785	0.045288	0.002543 **
nodematch.subunit	−0.116729	0.052493	0.026231 *
gwesp	2.965689	0.199404	<1e-04 ***
gwesp.alpha	0.492072	0.067252	<1e-04 ***
balance	0.038541	0.001522	<1e-04 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
AIC: 2294.2 BIC: 2362.1

bins such as 0–5, 5–10, etc. The negative and significant result indicates the tendency for agents to interact in the collaborative innovation network regardless of years worked in the organization. They worked with others who were within their bin and with others who were located in bins that were either higher or lower in years than their own. In other words, there is collaboration across organizational generations.

Professional Affiliation: The professional affiliation coefficient is positive and significant indicating the tendency for agents to interact more with others of the same professional affiliation within the collaborative innovation network.

Subunit: The subunit coefficient is negative and significant indicating the tendency for agents to interact in the collaborative innovation network regardless of the subunit one belongs to. In other words, there is collaboration across subunits of the organization.

GWESP: The gwesp coefficient is positive and significant indicating the existence of triad structures beyond what one would expect by chance.

Balance: The balance coefficient is positive and significant indicating the existence of balanced triads beyond what one would expect by chance.

The model fit is then compared with that of the original model (Table 10.4) to see whether the model was improved by adding the new variables and terms. A lower AIC or BIC is indicative of improvement. The original AIC and BIC were 2646.5 and 2677.4, respectively. The new AIC and BIC is 2294.2 and 2362.1, respectively. Both measures show model improvement and the decision is to keep the new model with the additional variables and terms.

The model of fit presented above only compares the models that were produced. There are more advanced goodness-of-fit methods for determining how well the model represents the data. These methods are beyond the scope of this chapter. For a comprehensive discussion of goodness-of-fit methods for ERGM see Lusher et al.(2013).

Research Possibilities with ERGM

As was the case with MR-QAP, we found no study in the educational literature that used ERGM methods; however, a search in the journal, *Social Networks*, found studies that suggest potential directions for educational research. These studies examine questions such as: How does the nature of ties among agents affect social status formation in workplaces (Yap & Harrigan, 2015): How does the status of individuals affect emergence of positive and negative gossip at work (Ellwardt, Labianca, & Wittek, 2012)? How does popularity and changes in popularity affect substance abuse among middle school students (Moody, Brynildsen, Osgood, Feinberg, & Gest, 2011)? How do patterns of network relationships influence the emergence of ad hoc work teams in organizations (Zhu, Huang, & Contractor, 2013)? How do communication ties affect the social integration of immigrant children (Windzio, 2015)? How does ethnicity relate to friendship groups in schools (Smith, Maas, & van Tubergen, 2014)? How do network dynamics explain segregation in a system (Bojanowski & Corten, 2014)? We should also add the study from *The Leadership Quarterly* by (White et al., 2014) (referenced earlier in this chapter), who found that weak network hierarchy is associated with distributed patterns of information exchange in an organization while strong professional or strong managerial hierarchy is associated with LMX-like (leader-member exchange relationships) direct exchanges between leader and follower. The common feature of all these studies is that they examine how micro network dynamics emerge in a network.

Taking cues from these studies, educational researchers might apply ERGM to explore the emergence of peer group dynamics in schools (such as bullying, influence on student learning, fads, etc.), the nature and movement of negative information among teachers, informal emergence of work groups such as collaborations or professional learning communities, social barriers to the integration of ethnic and racial minorities in schools, teacher informal groups and how inter-influence patterns in such group limit or enable administrative use of authority, what structures influence emergence of positive and negative cliques, how creativity emerges, rumor formation and transmission in schools, and the principal's level of engagement in advise networks and its effect on teacher group dynamics. The common thread through these is the formation of ties and group structures—bullying, cliques, peer groups, for example—and, arguably, there is little in social life that is not influenced by such structures.

Conclusions

Educational researchers have written extensively in recent years about such things as collaborative teams, distributed leadership, and professional learning communities. Such structures are interactive and interdependent, hence subject to description by complexity theory and analysis with methods such as those described in this chapter.

Network analysis enables researchers to describe the structure and dynamics of networks and to simulate their dynamics under varying conditions. MR-QAP explores relationships among these networks and ERGM describes how network processes emerge. These methods permit unique research questions, such as whether student performance is influenced by the degree of teachers' engagement as informal leaders plus network vibrancy in general. They can help researchers explore how peer processes among students influence the emergence of negative interactive processes. They can help principals foster stronger, more effective collectivist behaviors in their schools (collaborative activities, distributed leadership, etc.). It can help administrators better understand the structure of informal leadership and cliques among parents and residents in their attendance zones, what dynamics influence those cliques and informal leadership, and how to leverage these dynamics to support the school. Summarizing, these techniques focus researchers and practitioners on the potency of complex dynamics in schools, and this is a refreshing new direction for study in education.

Appendix: Sample Survey for Collecting Network Data—Structured for use in ORA

What is your name? (this is very important; your name will be deleted as soon as the data is formatted and before analysis).

[DROPDOWN LIST WORKS WELL]

1. From the following list, identify the people with whom you regularly talk about work-related issues (choose all that apply).

   [LIST ALL PROFESSIONALS BOUNDED BY THE RESEARCH NETWORK; this question, with the drop-down list above, enables construction of an agent-by-agent matrix]
2. Which of the following tasks do you perform on a regular basis at this school (Choose all that apply)? This data can be used to create an agent-by-task matrix.

Teach pre-k	Teach Gr 4	Teach Special Ed	Teach Art	Administration
Teach k	Teach Gr5	Teach remedial lessons	Coordinate Title I Activities	Other support services
Teach Gr1	Teach Art	Teach computers	Teach, other	Financial monitoring
TeachGr2	Teach PE	Teach music	Counseling/Psychology	
Teach Gr3				

3. Which of the following knowledge would someone *most* need to perform your tasks at this school (choose all that apply)? Data for an agent-by-knowledge matrix.

Budgeting	Finding resources	Differentiating instruction	Music	Using technology
Community partnerships	Subject area content	Child growth/ development	Organizational management	Clerical
Student testing	Subject area content standards	Motivating students	Using data to assess learning	Nursing
Writing IEPs	Developing curriculum	Classroom management	Standardized test statistics	Psychology
Implementing IEPs	Pedagogy/teaching styles	Recreation/physical development	School rules- policies- procedures	Using technology

References

Blackwell, B. (2014). *South Carolina public high schools: Leadership, network dynamics and innovation*. PhD, Clemson University, Clemson, SC.

Bojanowski, M., & Corten, R. (2014). Measuring segregation in social networks. *Social Networks, 39*, 14–32. doi:10.1016/j.socnet.2014.04.001.

Bonabeau, E. (2002). *Agent-based modeling: Methods and techniques for simulating human systems*. Paper presented at the Colloquim of the National Academy of Sciences.

Borgatti, S. P., Carley, K., & Krackhardt, D. (2006). On the robustness of centrality measures under conditions of imperfect data. *Social Networks, 28*(2), 124–136. doi:10.1016/j.socnet.2005.05.001.

Borgatti, S. P., Everett, M. G., & Johnson, J. (2013). *Analyzing social networks*. Thousand Oaks, CA: Sage.

Bowler, W. M., & Brass, D. J. (2006). Relational correlates of interpersonal citizenship behavior: A social network perspective. *Journal of Applied Psychology, 9*(1), 70–82.

Brewer, D., & Webster, C. (1999). Forgetting of friends and its effects on measuring friendship networks. *Social Networks, 21*, 361–373.

Burkard, R. E., Çela, E., Karisch, S. E., & Rendl, F. (2013). Quadratic assignment problem. *Journal of Global Optimization*. Retrieved from http://www.neos-guide.org/content/quadratic-assignment-problem doi: 10:391–403, 1997.

Chiles, T. H., Meyer, A. D., & Hench, T. J. (2004). Organizational emergence: The origin and transformation of Branson, Missouri's musical theaters. *Organization Science, 15*(5), 499–519.

Cilliers, P. (1998). *Complexity and postmodernism: Understanding complex systems*. London: Routledge.

Coveney, P. (2003). *Self-organization and complexity: A new age for theory, computation and experiment*. Paper presented at the Nobel symposium on self-organization, Karolinska Institutet, Stockholm.

Dansereau, F., Yammarino, F. J., & Kohles, J. C. (1999). Multiple levels of analysis from a longitudinal perspective: Some implications for theory building. *Academy of Management Review, 24*(2), 346–357.

Davis, J. A., & Leinhardt, S. (1972). The structure of positive interpersonal relations in small groups. In J. Berger (Ed.), *Sociological theories in progress* (Vol. 2, pp. 218–251). Boston: Houghton Mifflin.

Dekker, D., Krackhardt, D., & Snijders, T. (2007). Sensitivity of MRQAP tests to collinearity and autocorrelation conditions. *Psychometrika, 72*(4), 563–581. doi:10.1007/s11336-007-9016-1.

Ellwardt, L., Labianca, G., & Wittek, R. (2012). Who are the objects of positive and negative gossip at work?: A social network perspective on workplace gossip. *Social Networks, 34*(2), 193–205. doi:10.1016/j.socnet.2011.11.003.

Fields, A. (2009). *Discovering statistics using SPSS* (3rd ed.). Thousand Oaks, CA: Sage Publishing.

Frank, O., & Strauss, D. (1986). Markov graphs. *Journal of the American Statistical Association, 81*(395), 832–842.

Friedkin, N. E., & Slater, M. R. (1994). School leadership and performance: A social network approach. *Sociology of Education, 67*(2), 139–157.

Goodreau, S. M., Handcock, M. S., Hunter, D. R., Butts, C. T., & Morris, M. (2008). A Statnet tutorial. *Journal of Statistical Software, 24*(9), 1–26.

Graen, G. B., & Uhi-Bien, M. (1995). The transformation of professionals into self-managing and partially self-designing contributors: Toward a theory of leadership-making. *Journal of Management Systems, 3*(3), 25–39.

Gupta, A. K., Tesluk, P. E., & Taylor, M. S. (2007). Innovation at and across multiple levels of analysis. *Organization Science, 18*(6), 885–897.

Handcock, M. S., Hunter, D. R., Butts, C. T., Goodreau, S. M., & Morris, M. (2008). Statnet: Software tools for the representation, visualization, analysis and simulation of network data. *Journal of Statistical Software, 24*(1), 1–11.

Harris, J. K. (2014). *An introduction to exponential random graph modeling*. London: Sage.

Hogue, M., & Lord, R. G. (2007). A multilevel, complexity theory approach to understanding gender bias in leadership. *The Leadership Quarterly, 18*(4), 370–390.

Holland, P. W., & Leinhardt, S. (1981). An exponential family of probability distributions for directed graphs. *Journal of the American Statistical Association, 76*(373), 33–50.

Hunter, D. R., Goodreau, S. M., & Handcock, M. S. (2008). Goodness of fit of social network models. *Journal of the American Statistical Association, 103*(481), 248–258.

Hunter, D. R., & Handcock, M. S. (2006). Inference in curved exponential family models for networks. *Journal of Computational and Graphical Statistics, 15*(3), 565–583.

Kauffman, S. A. (1993). *The origins of order*. New York: Oxford University Press.

Kenny, D. A., & La Voie, L. (1984). The social relations model. In L. Berkowitz (Ed.), *Advances in experimental social psychology* (Vol. 18). New York: Academic.

Koopmans, T. C., & Beckmann, M. J. (1957). Assignment problems and the location of economic activities. *Econometrica, 25*, 53–76.

Krackhardt, D. (1987). QAP partialling as a test of spuriousness. *Social Networks, 9*(2), 171–186.

Krackhardt, D. (1998). Simmelian ties: Super strong and sticky. In R. M. Kramer & M. A. Neale (Eds.), *Power and influence in organizations* (pp. 21–38). Thousand Oaks, CA: Sage.

Krackhardt, D. (1999). The ties that torture: Simmelian tie analysis in organizations. *Research in the Sociology of Organizations, 16*, 183–210.

Krivitsky, P. N. (2012). Exponential-family random graph models for valued networks. *Electronic Journal of Statistics, 6*, 1100–1128.

Lusher, D., Koskinen, J., & Robins, G. (2013). *Exponential random graph models for social networks*. New York, NY: Cambridge University Press.
Marineau, J. E., & Labianca, G. J. (2010). Work and personal based conflict and advise and knowledge seeking relationships. *Academy of Management Proceedings, 2010*(1), 1–6. doi:10.5465/ambpp.2010.54501343.
Marion, R., Schreiber, C., Klar, H., Christiansen, J., & Reese, K. (2014). *Collectivist analysis of adaptive leadership, interaction, and cliques on organizational capacity*. Paper presented at the Academy of Management, Philadelphia, PA.
Moody, J., Brynildsen, W. D., Osgood, D. W., Feinberg, M. E., & Gest, S. (2011). Popularity trajectories and substance use in early adolescence. *Social Networks, 33*(2), 101–112. doi:10.1016/j.socnet.2010.10.001.
Pattison, P. E., & Robbins, G. L. (2002). Neighborhood-based models for social networks. *Sociological Methodology, 32*(1), 301–337.
Plowman, D., Solansky, S., Beck, T., Baker, L., Kulkarni, M., & Travis, D. (2007). The role of leadership in emergent, self-organization. *The Leadership Quarterly, 18*, 341–356.
Robins, G., & Lusher, D. (2013). Simplified account of an exponential random graph model as a statistical model. In D. Lusher, J. Koskinen, & G. Robins (Eds.), *Exponential random graph models for social networks* (pp. 29–36). New York, NY: Cambridge University Press.
Robins, G., Pattison, P., Kalish, Y., & Lusher, D. (2007). An introduction to exponential random graph (p*) models for social networks. *Social Networks, 29*(2), 173–191. doi:10.1016/j.socnet.2006.08.002.
Scott, J., & Carrington, P. J. (Eds.). (2011). *The Sage handbook of social network analysis*. London, UK: Sage.
Smith, S., Maas, I., & van Tubergen, F. (2014). Ethnic ingroup friendships in schools: Testing the by-product hypothesis in England, Germany, the Netherlands and Sweden. *Social Networks, 39*, 33–45. doi:10.1016/j.socnet.2014.04.003.
Snijders, T. A. B., Pattison, P. E., Robbins, G. L., & Handcock, M. S. (2006). New specifications for exponential random graph models. *Sociological Methodology, 36*(1), 99–153.
Solow, D., Burnetas, A., Piderit, S., & Leenawong, C. (2003). Mathematical models for studying the value of motivational leadership in teams. *Computational and Mathematical Organization Theory, 9*(1), 61–81.
Sterman, J. D. (1994). Learning in and about complex systems. *System Dynamics Review, 10*, 291–330.
Tortoriello, M., & Krackhardt, D. (2010). Activating cross-boundary knowledge: The role of Simmelian ties in the generation of innovations. [Article]. *Academy of Management Journal, 53*(1), 167–181.
Tsai, W., & Ghoshal, S. (1998). Social capital and value creation: The role of intrafirm networks. *Academy of Managemenl Journal, 41*(4), 464–476. doi:10.2307/257085.
van Duijn, M. A. J., & Huisman, M. (2011). Statistical models for ties and actors. In J. Scott & P. J. Carrington (Eds.), *The Sage handbook of social network analysis* (pp. 459–483). London, UK: Sage.
Wang, P., Robins, G. L., & Pattison, P. E. (2009). *Pnet: Program for the simulation and estimation of p* exponential random graph models*. Retrieved from http://sna.unimelb.edu.au/
Wasserman, S., & Faust, K. (1994). *Social network analysis: Methods and applications*. Cambridge, UK: Cambridge University Press.
Wasserman, S., & Pattison, P. (1996). Logit models and logistic regression for social networks: I. An introduction to Markov graphs and p*. *Psychometrika, 61*(3), 401–425.
Weick, K. E. (1976). Educational organizations as loosely coupled systems. *Administrative Science Quarterly, 21*, 1–19.
White, L., Currie, G., & Lockett, A. (2014). The enactment of plural leadership in a health and social care network: The influence of institutional context. *The Leadership Quarterly, 25*(4), 730–745. doi:10.1016/j.leaqua.2014.04.003.

Windzio, M. (2015). Immigrant children and their parents: Is there an intergenerational interdependence of integration into social networks? *Social Networks, 40*, 197–206. doi:10.1016/j.socnet.2014.11.002.

Yap, J., & Harrigan, N. (2015). Why does everybody hate me? Balance, status, and homophily: The triumvirate of signed tie formation. *Social Networks, 40*, 103–122.

Zhu, M., Huang, Y., & Contractor, N. S. (2013). Motivations for self-assembling into project teams. *Social Networks, 35*(2), 251–264. doi:10.1016/j.socnet.2013.03.001.

Chapter 11
"Looking at" Educational Interventions: Surplus Value of a Complex Dynamic Systems Approach to Study the Effectiveness of a Science and Technology Educational Intervention

Sabine van Vondel, Henderien Steenbeek, Marijn van Dijk, and Paul van Geert

Introduction

There is no doubt that a classroom can be conceived of as a complex dynamic system, in that it consists of many interacting components —the students and the teacher— that influence each other's behavior and characteristics over a wide variety of nested time scales (Lewis, 2002; Smith & Thelen, 2003; Van Geert & Steenbeek, 2005). If one takes, for instance, a science lesson in a classroom consisting of 11-year-old students, then the teacher's questions during a science activity influence the reactions of the students. The interactions during this activity influence the interaction during the next activity or next lesson.

As this is an example of an educational system, the interactions at the behavioral level of the system are explicitly aimed at durably changing particular properties —such as the students' knowledge, skills, or understanding about science. Note that at the same time other properties, such as the order in the class or the level of involvement of the students, should be maintained. Modern schools that promote the lifelong learning of the teacher make decisions about programs for teacher professionalization, which are either reluctantly or enthusiastically received by the teachers (Wetzels, Steenbeek, & Van Geert, 2015). Such professional interventions are often presented as fixed protocols, but in reality they unfold as highly idiosyncratic processes. In fact they are emergent processes in which many components —including the written intervention protocol, the coach's capacities, the unique circumstances of the school and the time and effort invested by the teachers— are dynamically intertwined. Such interventions are in fact forms of perturbation in an existing, self-sustaining pattern of activities which takes place during real-time learning situations. Asking a particular type of questions, performing a particular

S. van Vondel (✉) • H. Steenbeek • M. van Dijk • P. van Geert
University of Groningen, Netherlands
e-mail: s.van.vondel@rug.nl

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_11

type of activities or typical reactions of students are examples of such self-sustaining patterns. The aim of perturbations, i.e., the intervention, is to durably change these self-sustaining patterns and replace them by new, more adequate patterns that, once they are established, should also be self-sustaining (Van Geert, 1994; 2003). From a dynamic systems point of view, changing these patterns of action and thinking of the teacher is quite similar to changing the patterns of action and thinking of the students, i.e., those can be indicated as teaching-learning processes.

In order to fully understand the effect of educational interventions on students' performance, insight is needed in the properties of these teaching-learning processes in individual teacher–student pairs. However, the progress of individual students as a result of an intervention is hardly reflected in effectiveness studies. This is because the effectiveness of interventions is usually studied using standard research practices. This methodological study aims to demonstrate how properties of a complex dynamic systems approach can help gain insight into change in teaching-learning processes due to educational interventions. This will be illustrated by examining a science education intervention, Video Feedback Coaching for teachers (VFCt), aimed at improving the quality of teachers' questions, and by doing so increasing students' scientific reasoning levels.

Standard Research on Educational Interventions

Assuming that the description given above provides a reasonably realistic picture of education as a complex dynamic system, we may ask ourselves what kind of picture teachers, parents and policymakers get from the standard research on education. Although probably few teachers will read the scientific journals on education science, the standard approach trickles down via various sources, such as via policymakers who have been trained in the standard practice of educational research, or the news media who report about scientific findings on education.

What the standard research practice implicitly or explicitly conveys to educators is, to begin with, the idea that influences of one variable onto another —such as motivation on school science performance— can be meaningfully separated from other influences and then in a sense stitched together again to provide a picture of individual educational processes.

Another idea that educators can get from the research is that effectiveness of an intervention (a curriculum, a teacher training program and so forth) resides in the intervention itself, i.e., that effectiveness is like an intrinsic causal force present in the intervention. In addition, the effectiveness of an intervention is something that is seen as applicable to particular kind of persons, i.e., to particular populations, such as the population of primary school teachers.

Another idea that the standard research practice in education conveys to educators is that knowledge and skills are internal properties of individuals, internal representations, internally represented schemes of action and so forth that are transmitted from a teacher or a curriculum to an individual student. These internal skills or levels of knowledge can best be measured by validated, normed and relatively objective measurement instruments that express the internal skill or knowledge by means of a single number, i.e., a test score on a science test (Borman, Gamoran, & Bowdon, 2008 Penuel, Gallagher, & Moorthy, 2011; Şimşek & Kabapinar, 2010). Though, a more proximal measure, at the behavioral level, like the quality or complexity of the answers may be a better indicator of, for instance, a student's scientific reasoning level compared to a more distal measurement, like paper and pencil tests —as paper-and-pencil tests require other skills like reading as well (Van der Steen, Steenbeek, Van Dijk, & Van Geert, 2015). In addition, several studies report that interaction is essential to stimulate students' performance (Vygotsky, 1986). More specific, both Chin (2006) and Oliveira (2010) state that asking thought-provoking, student-centered, questions is a key element to stimulate students to reason with longer sentences and on higher levels of understanding.

Standard educational research also conveys the idea that what actually matters is the real or true skill, level of knowledge or ability, and that this real or true skill or ability can best be represented by averaging over individual fluctuations or individual variability (for more information see Rosmalen, Wenting, Roest, De Jonge, & Bos, 2012). The message is that these fluctuations or variability are in fact purely random variations around the true skill, level of knowledge or ability, and that they reflect purely accidental influences. For that reason, such fluctuations or variability within individuals are not intrinsically interesting, and should thus be averaged out. Preferably this is done by averaging over many individuals who, together, constitute a representative sample of the unit of analysis that really matters, namely the unit of populations characterized by a particular natural property, such as "typically developing students" or "dyslexia."

In this standard approach, there is of course room for interaction, for context, for individual variation, for change over time and so forth. These aspects are, however, clearly viewed from a perspective that is different from the perspective of complex dynamic systems. In the latter, they are like the primary givens, the starting point of theory formation and research (Fogel, 2011; Thelen, 1992; Van Geert, 2003), whereas in the more standard picture they are like secondary aspects, inferred from the primary aspects of research as discussed above.

How should educational research be transformed in such a way that it can convey to educators a picture of education that comes closer to the reality of education as a complex dynamic system? In the remainder of this chapter, we shall first discuss how properties of a complex dynamic systems approach can be applied to study the effect of educational interventions, such as the Video Feedback Coaching program for teachers. This approach will then be further illustrated by discussing an example of educational research, which uses properties from complex dynamic systems thinking in order to examine the effect of an intervention.

Intervention Assessments

In order to assess the effectiveness of such interventions several guidelines are frequently used. Veerman and van Yperen (2007), for instance, describe an often used classification scheme for assessing the effectiveness of youth care interventions as evidence-based practice. This scheme consists of four stages from potential effective interventions to efficacious interventions. An intervention is considered effective when the causality between the intervention and the outcome can be determined. Large-scale experimental research, multiple case-studies and norm related research are considered as ways to accomplish these causal relations.

Another way to establish the effectiveness of an intervention has been described by Boelhouwer (2013; as adapted from Lichtwarck-Aschoff, Van Geert, Bosma, & Kunnen, 2008). Boelhouwer proposes a taxonomy using four dimensions —which are grounded in the complex dynamic systems approach— to address the effectiveness of an intervention. Boelhouwer stresses the importance of using observational data and studying mutual causality. The four dimensions are:

1. *The static versus dynamic dimension* pertains to the dimension of analysis. Respectively, data are aggregated over many individuals versus data are displayed as a process over many time points. The *static dimension* can be used to analyze science performance as a combination of factors in a large sample. The effect of an intervention can, for instance, be assessed by focusing on the difference-score between pre measure and post measure, in which half of the participants receive an intervention while the other half does not (control group). The *dynamic dimension*, on the other hand, can be used to depict the process of change. Time series are used to depict how the changes emerge in and over time (Velicer, 2010).
2. *The micro versus macro time-scale* refers to the time-dimension. Respectively, a student's performance in real-time (i.e., the micro time-scale of seconds, minutes or hours (Lewis, 1995)) versus learning and development over several lessons or years (i.e., the macro time-scale of weeks, months or years (Lewis, 1995)). Analysis can be situated on different time scales at which the micro level is at the one end of the continuum and the macro level on the other end of that continuum. At the *micro level*, scientific reasoning skills can be captured in one specific situation, in which action sequences are studied. An example is a conversation during a science and technology lesson, consisting of one or several action–reaction sequences. At the *macro level* scientific reasoning skills can be captured over a longer period of time, for instance a series of science and technology lessons. The change in students' scientific reasoning skills due to the implementation of an intervention can also be interpreted as an example of a macro time-scale.
3. *The distinction between direct and indirect assessment* refers to the dimension of information sources, respectively the assessed person him or herself or a third-party assessor. A researcher can use several sources of information when evaluating an intervention program. One way is using *direct measures*, which

means information from those persons who actively participate in the intervention. In a professionalization trajectory for teachers, the teacher would be a direct source of information when (s)he is observing own behavior and reports about that, for instance by means of a questionnaire. *Indirect assessment* might refer to scientists who report about behavioral change.

4. *The distinction between short-term effects versus long-term effects* refers to the dimension of behavioral change due to —the effects of— an intervention. The short-term effects of an intervention can be seen as a change in observable behavior right after or eventually during the intervention lessons. The long-term effects refer to maintaining effects that are still observable a long time after the intervention, which can be visualized at follow-up or post-measurements (Boelhouwer, 2013; Steenbeek & Van Geert, 2015).

Using a Complexity Approach to Map Change: How to Apply the Properties

The complex dynamic systems approach offers tools to focus on properties of development and learning as dynamic processes (Steenbeek & Van Geert, 2013; Van Geert, 1994), which lie beneath the aforementioned dimensions. Using this approach is a way to study how learning occurs in interaction with the material and social context by focusing on those processes during real-time and frequent observations, i.e., during actual lessons (Granott & Parziale, 2002; Van Geert, 1994; Van Geert & Fischer, 2009). In order to understand the dynamics of a complex system, such as a teacher's behavior in the context of a group of developing students, the assessment should also focus on the dynamic character of learning, i.e., how a student's performance emerges in interaction with the context (see Steenbeek & Van Geert, 2013; Wetzels, Steenbeek, & Van Geert, in press). Observational methods, i.e., video recordings, are considered essential to be able to capture the developments on these real-time (micro) timescales and to preserve the complexity of the process of learning. Several properties of learning —such as change, nonlinearity, iteration and self-organization, variability, and the transactional nature of learning— as a result of an intervention must accordingly be taken into account. Mapping these properties is important to explain average group-based findings and provide insight into the underlying processes of learning and subsequent performance of individual students (Van Geert, 2004) and the quality of a science education intervention (Wetzels et al., 2015). The relevance of a complex dynamic systems approach, for intervention studies, demonstrates itself in offering possibilities for answering different research questions.

In the next section we will discuss three important properties of a complex dynamic system for the context of learning. This is a background for understanding the need for a process-based methodology. For this reason, we will describe how underlying properties of Boelhouwer's (2013) dimensions can be integrated in educational intervention studies as an essential addition to group-based analyses.

The role of time in change has a prominent role in Boelhouwer's taxonomy: in the time-dimension (micro versus macro) as well as the behavioral change-dimension (short-term versus long-term intervention effects). Velicer (2010) states that a time series analysis can help to understand the underlying naturalistic process and patterns of change over time, or to evaluate the effects of an intervention. For instance, time provides valuable information about the dependency between all measurements. As Steenbeek and Van Geert (2005) state, behavior of the student — which can be as small as an utterance— at a certain point in time affects the subsequent activity of the teacher —also known as *iteration*.[1] Since changes in the micro-timescale —short-term effects— are intertwined with long-term effects, analyzing student's actions during real-time interactions might be helpful in understanding change (Steenbeek, Jansen, & Van Geert, 2012).

As an illustration, let us return to a science class in an upper grade elementary classroom. The teacher's questions influence the reactions of the students in the form of answers, signs of interest or of avoidance, which on their turn influence the subsequent questions and reactions of the teacher following the reactions of the students. Students hear other students giving an answer, or see them performing particular activities, and this influences their own potential answers to questions asked by the teacher. The effect of the interactions takes place on various, nested timescales (e.g., Van der Steen, Steenbeek, & Van Geert, 2012). There is, for instance, the short-term time scale of a particular science class, which involves the dynamics on the level of activities, solving problems and formulating explanations. There is also the long-term timescale of changes in the nature of the answers or the probabilities of high-level reasoning that develops as a consequence of the short-term interactions. As is typical of a complex dynamic system, events on these various timescales affect one another, that is to say there is mutual influence and reciprocal causality (Steenbeek et al., 2012). Another example is the short-term timescale of asking a particular kind of questions by the teacher and the long-term timescale of eventual changes in the nature of the questions asked by the teacher, for instance as a consequence of an intervention aimed at teacher professionalization (e.g., Wetzels et al., 2015). A class of students with their teacher tend to evolve towards particular, class-specific patterns of activity, that is to say towards a typical pattern of asking questions, giving assignments, giving answers, showing interest or

[1] Dynamic processes are *iterative* in nature. Iteration refers to "a procedure that operates on an input that is in fact its preceding output" (Van Geert, 1997). This means that over time, the teaching-learning process (the current state) is a product of the previous state, and serves as input for the next state. Teacher and student mutually influence each other over time; the current action of the teacher influences the next (re)action of the student, which influences the next (re)action of the teacher, and so on.

boredom, and many other properties. These patterns form some sort of complex *attractor state*[2] (e.g., Steenbeek & Van Geert, 2005) that is typical of the teacher-classroom system in question. These attractor patterns are in a sense self-sustaining, for instance the nature of the questions habitually asked by the teacher influences the nature of the answers habitually given by the students, and these answers are likely to sustain the nature of the questions asked by the teacher. In addition, the attractor patterns, i.e., few variability is visible in the teacher–student interaction patterns, are relatively resistant to change.

A focus on *variability*[3] provides information about interindividual variability and intraindividual variability. Bassano and Van Geert (2007) state that "variability is informative on the nature of developmental change". The dynamic dimension in Boelhouwer's taxonomy (2013) allows further for possibilities to map *interindividual variability*[4], variability among students, teachers, or groups. This might be done to compare several individual teachers to find out whether one teacher's intervention trajectory is more effective compared to a similar intervention trajectory of another teacher. Questions might focus on whether the pathways of all students are equivalent, i.e., did they develop in similar ways? A change in student's science performance might be found in trajectories in which a teacher seems capable of adjusting his/her questions to a student's level of functioning and thinking, while the less effective trajectories remain in a fixed pattern of non-differentiating interactions (Ensing, Van der Aalsvoet, Van Geert, & Voet, 2014). Variability at the micro level (adjusting to the level of students) might, in this case, be an important element accounting for the variability between the teachers. Interindividual variability can provide important information about underlying dynamics of (less) effective intervention trajectories. Each trajectory —either an intervention or another developmental trajectory— takes the form of a dynamic pathway, constructed as real-time iterative processes, which emerges through interaction with the context (Fischer & Bidell, 2006). As each student starts an

[2] An *attractor state* is a temporarily stable state that recurs over time: "the state to which systems are attracted, that is, towards which they spontaneously evolve as a consequence of the underlying dynamic principles that govern their behavior" (Van Geert, 2003). For instance, in a classroom a teacher may routinely ask knowledge-based questions. This mode of interaction becomes a self-sustaining comfortable state for both teachers and students, making this type of questioning and students' reactions an attractor state for this particular classroom. If the teacher, under influence of the intervention, begins to change her questioning strategies towards open-ended questions, the students might first resist. However, if the teacher persists in using these open-ended questions and the students start to engage in critical thinking, the classroom system (teacher–student interaction) might change permanently over time—resulting in a new attractor state.

[3] *Variability* is defined as the "coexistence of many different patterns of development" (Van Geert, 1998). Two types of variability can be distinguished: 1. *Interindividual variability*: differences in the behavior between—groups of—individuals at some point in time. 2. *Intraindividual variability*: Van Geert and Van Dijk (2002) have defined intraindividual variability as "differences in the behavior within the same individuals, at different points in time" (p. 341).

[4] Note that 'individual' does not necessarily refer to a single person. It refers to the level at which a particular process actually occurs, which can be an individual person, but also a classroom.

intervention at their own level and masters science and technology to the best of his/her capabilities, each trajectory is unique and should be analyzed as such to provide insight in the variability.

Intraindividual variability is defined as "differences in the behavior within the same individual, at different points in time" (Van Geert & Van Dijk, 2002). By looking at multiple measures of individuals it is possible to see how the change and development proceeds (e.g., Van der Steen, Steenbeek, Van Dijk, & Van Geert, 2014). Van der Steen, et al. (2014), for instance, showed that a student's performance changed over several science activities. By focusing on intraindividual variability, a change in interaction between a student and a researcher was found. At the start of the learning trajectory, the teacher took initiative by asking thought-provoking questions during inquiry activities (state 1); the student followed the level of the teacher. At the third lesson, a change in interaction pattern was found, in that the student took initiative (state 2) and seemed to have initialized the process of inquiry. In between these two states, some form of "chaos", in this case increased variability, was found in which the researcher and student did not seem to adapt to each other as well as before (state 1) and after (state 2). Transitions from one state to another are often accompanied by qualitative indicators, but also by increased variability or critical slowing down of variability (e.g., Bassano & Van Geert, 2007).

The surplus value of focusing on variability is that it yields information about the differences in underlying characteristics leading to differences between lessons or participants. Specifically, this might show whether there are behavioral characteristics accounting for why a trajectory seems to yield more positive change for one subgroup than another or how one state changes into another (concerning development - Lichtwarck-Aschoff, Van Geert, Bosma, & Kunnen, 2008; education - Steenbeek et al., 2012; sports - Den Hartigh, Gernigon, Van Yperen, Marin, & Van Geert, 2014).

The *transactional nature* provides insight into how the learning gains of students can be understood, i.e., how is performance (co-) constructed during actual lessons, why is the intervention for some classrooms or students more effective than others? Learning can be seen as a dynamic and distributed, transactional process (Steenbeek, Van Geert, & Van Dijk, 2011). Students do often not come to a conclusion spontaneously. Teacher support is essential to reach a higher level of performance (Van de Pol, Volman, & Beishuizen, 2011). Teaching and learning are dynamic processes that are constantly adapting to changing needs and opportunities. It is therefore important to focus on the dynamics of reaching a performance by studying interactions, i.e., what the teacher's contribution is in students' performance. The unit of analysis ought to be the dyad of a teacher and the students, and not the individual student on its own. Knowing more about how teachers stimulate students toward higher levels of science performance might provide valuable information about how to optimize inquiry-based learning situations (Van der Steen et al., 2014).

Note that although the three properties describe distinct mechanisms, during the process of learning, they all work simultaneously. Boelhouwer's (2013) observational dimensions might be seen as different levels of analyses and can show increasingly detailed information about how well the averaged findings (static) represent the variability in individual trajectories (dynamic) in (micro) and over time (macro). For the purpose of this article, the properties are presented in such a way that the surplus value compared to the classical approach is stressed (Table 11.1). However, we do not intend to give the impression that this classification is *the* ultimate way to study interventions. The principles of variability can, for instance, be very well applied at the micro level to find change points in the transactional nature of learning trajectories over several lessons (e.g., Steenbeek et al., 2012).

Present Study

In this study, we aim to demonstrate the contribution of a complex dynamic systems approach when assessing the effectiveness of a science education intervention. We illustrate this by presenting data from the Curious Minds Video Feedback Coaching program for teachers (Van Vondel, Steenbeek, Van Dijk, & Van Geert, 2015). By doing so, we intent to provide a more thorough and multifaceted view of the process of studying the effectiveness of an intervention, compared to standard evaluations. By starting with the more classical group-based analysis, we aim to demonstrate that each remaining analysis —increasingly more process-based— can provide more understanding of the effectiveness. Hence, information about students' performance (static and dynamic) and the development of students' scientific reasoning skills during one lesson (micro) and over several lessons (macro) will be presented. In addition, the role of the teacher in this process (micro-dynamic) can be shown during the intervention (short-term effects) and a few weeks after the intervention (long-term effects).

Method

Rationale for the Teaching Intervention

The Video Feedback Coaching program for upper grade teachers is a professionalization trajectory designed to support teachers in improving the quality of science education lessons in their classroom. More specifically, this pedagogical-didactic intervention was developed to stimulate change in teacher–student interactions, i.e., changing the discourse from mostly teacher-centered into a more stimulating student-centered discourse (Wetzels et al., in press). By doing so, teachers enhance the quality of students' scientific reasoning skills by establishing a series of inspiring teachable science moments (Bentley, 1995; Hyun & Marshall, 2003). The way teachers interact with students was regarded as a key to quality of the

Table 11.1 Combination of complexity properties and dimensions as formulated in Boelhouwer's

			Dimensions			
	Goal	(possible) Research question	Information source	Analysis	Time	Behavioral change
Pre versus post-measure	Generalization Insight into whether there is an effect or not	What is the group-based effect of the intervention?	Indirect group	Static aggregated	Macro	Long term effects
Role of time in change	Map development and/or change Help to understand the underlying naturalistic process and patterns of change over time, important to evaluate the effects of an intervention	How can we characterize short-term and long-term change in students' scientific reasoning on group level during the intervention trajectory?	Indirect group	Dynamic	Macro	Short- and long-term effects
(Intraindividual) variability	Map temporal change Information about the quality of interventions	How does this classroom level change over the time covered by the intervention?	Indirect classroom	Dynamic	Macro	Short- and long-term effects
Transactional nature	Map co-construction Insight into how the learning gains of students can be understood	How is scientific reasoning co-constructed and how does this co-construction process change over time under influence of the intervention?	Indirect individual dyad	Process (dynamic)	Micro	Short-term effects

science lessons. The intervention contained the following evidence-based key elements: (1) improving teachers' knowledge about teaching science and scientific skills, (2) establishing behavioral change by improving teachers' instructional skills by means of (a) VFCt and (b) articulating personal learning goals.

The first element was reflected in an interactive educational session about knowledge of teaching science and scientific skills for participating teachers. Osborne (2014) defined these skills as knowledge about the process of science — including knowledge about the empirical cycle— and the skills needed for performing an actual scientific inquiry —such as higher order thinking skills. During this educational session information was provided and the features important for science learning were discussed: the use of the empirical cycle (De Groot, 1994), use of thought-provoking questions (Chin, 2006; Oliveira, 2010), scaffolding (Van de Pol et al., 2011), and science and technology-education in general (Gibson & Chase, 2002). According to Lehmann and Gruber (2006) expertise can best be acquired through case-based learning, including authentic cases which are embedded in naturalistic contexts. Therefore, several best-practice video fragments of teacher–student interactions during science lessons were shown to illustrate the transactional nature of performance; i.e., the importance and effect of high quality interactions during science and technology-activities.

The second element referred to the aim to establish —durable— behavioral change. A promising method for implementing evidence-based instructional strategies, i.e., establishing behavioral change is providing feedback on real-time behavior (Noell et al., 2005; Reinke, Sprick, & Knight, 2009). Teachers instructional quality can be greatly increased by offering video feedback on own classroom behaviors (see also Mortenson & Witt, 1998; Seidel, Stürmer, Blomberg, Kobarg, & Schwindt, 2011; Wetzels et al., 2015). As a rule, the effect of feedback is best when a 3/1 ratio is used (Fredrickson, 2015), i.e., three positive fragments were discussed and one fragment which could be improved. In order to stimulate teachers to fully understand the behavioral patterns and consequences of those interactions for students' performance, the coaching focused on the transactional nature of learning by reflecting on teacher's own specific behaviors and interactions at the micro-timescale and was conducted immediately after each lesson, as immediate feedback is most beneficial for learning (Fukkink, Trienekens, & Kramer, 2011). Note that aside from this practical application, these videotapes were used as the primary source to evaluate the effectiveness of the intervention.

In addition, goal setting at the beginning of a coaching trajectory is an effective way to achieve results (Hock, Schumaker, & Deschler, 1995), i.e., behavioral change, as they ensure feelings of autonomy (Pintrich, 2000). By formulating learning goals that reflect teacher's personal professionalization trajectory, teacher's feelings of autonomy were respected and teachers were provided with opportunities to monitor and control their motivation and behavior. Another way to ensure teacher's feelings of autonomy and thus to create more responsibility for their own learning process, was by encouraging them to prepare science and technology-lessons to his or her own liking (Table 11.2). Teachers were allowed to choose a topic and an instructional method (for instance experiments or a design

Table 11.2 Type and topic of lessons as provided by each teacher

	Pre-measure	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Post-measure
Classroom 1	Experiments: air pressure	Classical experiment: air pressure	Classical experiment: air pressure	Classical experiment: air pressure	–	Classical experiment: air pressure
Classroom 2	Experiments: air pressure	Experiments: surface tension	Design: planetarium	Drawing: rainbow	Experiments: gravity	Experiments: air pressure
Classroom 3	Experiments: air pressure	Experiments: air pressure	Experiments: gravity	Experiments: gravity	Experiments: balance	Experiments: air pressure
Classroom 4	Experiments: air pressure	Classical experiment: air pressure	Classical experiment: air pressure	Classical experiment: air pressure	Classical follow-up discussion: air pressure	Classical experiment: air pressure
Classroom 5	Design: barometer	Experiment: air pressure	Classical experiment: air pressure	Experiment: water	–	Classical experiment: air pressure
Classroom 6	Experiment: air pressure	Laptop: satellite	Design: balloon rocket	Experiment: blending	Design: "Techniektoren"[a]	Experiments: air pressure

[a]The "Technique Towers" are lockers shaped as towers with 80 lesson-boxes inside—10 lessons for each year of Dutch elementary education. Each lesson box is focused on a specific aspect within a domain of technology, for instance construction or making soap. Each box has a step-by-step manual which can be used by a small group of students, without a teacher. This manual guides them through the activity that is in the box

assignment) suiting their own and students' interest. The table shows that the first lesson of classroom 6 mainly focused on experiments and the topic was air pressure. The main focus of the next lesson was on using laptops to search for information about satellites.

Participants

Six upper grade teachers (two men and four women) and their students (M_{age}: 11.2, 9–12 year olds) from the North of the Netherlands participated in the study in school year 2013/2014. Their teaching-experience ranged from 6 to 18 years in regular elementary education. The average classroom consisted of 28 students (49 % girls, 51 % boys).

Procedure and Materials

Six science and technology lessons and an educational session were conducted in a period of 3 months: one pre-measure, four lessons immediately followed by a VFC session led by a trained coach (first author) and one post-measure, on average 4.5 weeks, after the end of the VFCt.

Although the intervention was intended as adaptive support and was highly idiosyncratic, some standardization was implemented during data collection. That is, the same coach provided identical information during the introductory session, videotaped all lessons, and was responsible for the guided reflection after each lesson. In addition, teachers were asked to use the following guidelines: provide six lessons using a fixed format: introduction (plenary introduction), middle part (students work on their own or in groups), and end (plenary discussion). Furthermore, they were asked to teach lessons about the "earth and space" system —such as weather, air pressure, gravity, or the positions of the moon. Lastly, the teachers were instructed to focus on air pressure and aim at learning students about high and low pressure during the pre-measurement and post-measurement.

Data Analysis

Ten minutes of the middle part of the lessons were coded, because in this part a relatively larger amount of rich, interactive interaction was present. For further data analysis, the classroom of students as a whole was taken as the unit of analysis, which means that the individual case is always consisting of a group of individuals. However, in contrast with the classical group approach of looking at the performance of independent individuals, which most studies use to calculate averages, this group is conceived of as a collection of interdependent individuals interacting with each other. In line with that, the previous utterance of the teacher or a fellow student was taken into account when scoring students' level of complexity.

Students' scientific reasoning skills were measured by quantifying verbal utterances, using a scale based on skill theory (Meindertsma, Van Dijk, Steenbeek, & Van Geert, 2012; Parziale & Fischer, 1998; Van der Steen, Steenbeek, Wielinski, & Van Geert, 2012). The dynamic skill theory (Fischer, 1980) is a cognitive developmental theory focusing on how skills —which are considered complex and variable— are constructed in specific domains. These skills can be captured by focusing on those skills as they emerge in interaction with the context. This scale has proven useful for task-independent measures in the analysis of student's scientific explanations. Student utterances were scored on complexity using a 10-point scale, divided in three tiers (sensory-motor, representations, and abstractions). The first tier (level 1–3) consists of sensorimotor observations and explanations, which mean simple observable connections are given. Level 1 means the least complex utterance, a single sensorimotor aspect (e.g., an expression of what they see; the student says: "It [the balloon] is white"). At level 2, the sensorimotor mapping level, the student is able to combine to single sensorimotor aspect into one mapping (e.g., the student says: "It is white and that one is yellow."). The second tier (level 4–6) comprises representational predictions and explanations, which means that students use higher order thinking skills to go beyond simple perception-action couplings. The student understands that an object has a specific characteristic, outside the present situation. (S)he can, for instance, make a prediction about what is going to happen when you put salt into a water/oil fluid —without directly seeing it. The third tier (level 7–9) constitutes abstract explanations; students are capable of generalizing ideas about the object outside specific situations. A student might for instance explain that "the molecules in the water are strongly drawn towards each other... probably leading to surface tensions... the water and oil cannot blend because of that" or "the density of the water is higher compared to the density of the oil, the fluid with lower density floats". Level 10 could be scored when students expressed understanding about global laws and principles (e.g., the abstract principles of thermodynamics can be applied to the situation at hand). Ten to twelve-year olds are expected to be "capable" of reaching the seventh level of understanding (Fischer & Bidell, 2006). They could express abstract thinking skills (e.g., relate abstract concepts to the situation at hand, as showing in the following utterance "the air pressure pushes the paper towards the table").

Coding was done by means of the program "Mediacoder"[5] (Bos & Steenbeek, 2009). To establish the interobserver reliability for the application of the coding scheme, the interobserver agreement was determined in advance by the first author

[5] Mediacoder can be obtained free of charge by sending an e-mail to one of the developers: h.w.steenbeek@rug.nl or j.bos@rug.nl. Mediacoder is a simple application for coding behaviors within media files. A media code is a moment in time in a recorded video when a particular event occurs. The meaning of the coding is determined by the specified character (which the user can choose him/herself). The point in time is determined by the time within the recorded media. Each media

and an independent coder. With an agreement ranging from 79 to 83 %, Cohen’s kappa of .76, the interobserver agreement was considered substantial.

Excel was used for descriptive analysis and to display patterns in the data. As the collected data consisted of a small group of participants, dependency between variables, and multiple measures, a nonparametric test was used to test differences in students’ scientific reasoning level over several lessons. This random permutation test was used to test the empirical results in relation to a statistically simulated baseline of random patterns, using Poptools (Hood, 2004). This means that the nonparametric test statistically simulated the null hypothesis that the probability of the relationship or property was based on chance alone. For instance, the scientific reasoning level data were randomly shuffled (values were randomly drawn from the data without replacement), and the same average and difference score was calculated for the statistical simulation of the null hypothesis. This random shuffling, i.e., data generated on the basis of the null hypothesis model that there was no effect of the intervention, was permutated 1000 times in order to calculate whether the empirically found difference between pre- and post-measure could be expected to occur on the basis of chance. When the finding was smaller than 0.05, the test statistic was considered significant. This means that when we speak about significantly different, we mean a considerable difference that has applied meaning (for instance a difference that is big enough, one complexity level, for the teacher to be observed in the real world). A significance score between 0.05 and 0.1 is considered as a trend, i.e., non-randomness (see for a discussion about cut-off scores of p-values and the use of confidence intervals: Kline, 2004; Lambdin, 2012; Cumming, 2014).

Pre-measure versus Post-measure

All task-related student utterances were coded on complexity. Subsequently, we calculated the average complexity level of all students over all classes at pre-measure and post-measure, and computed the difference between the two. In addition, as significance scores are not directly linked to practical significance (Sullivan & Feinn, 2012) the effect size was calculated using Cohen’s D. Following Sullivan and Feinn, an effect size of 0.2 is considered small, 0.5 medium, 0.8 large, and 1.3 or higher very large.

The Role of Time in Change

The long-term effects were operationalized as the effects that were still observable, 4.5 weeks, after the intervention. These were assessed by comparing students’ scientific reasoning level at the intervention-lessons with students’ scientific

code can be supplemented with an explanation. After coding the file can be exported to excel or SPSS for further analyses.

reasoning at the post-measure. Therefore, we calculated the average complexity score of each lesson. The same was done for the statistical simulation of the null hypothesis. Short-term effects were assessed by focusing on scores during the intervention.

Variability

Again, all students in the classroom were taken as our unit of analysis, and focused on the classroom performance level. While doing so, the focus was on variability in the sense of differences between the various classrooms (interindividual variability) and of differences over time within classrooms (intraindividual variability). The variability of each classroom was computed and compared with the variability between lessons of that classroom. The same analysis was done on the group level, in that the variability was computed of all classrooms and the variability of each classroom was compared with the overall —averaged— variability. This analysis can be the basis to find intraindividual variability which might show the properties of effective and less effective trajectories. In order to actually study the process, you must study the process on the individual case level. Second, in an attempt to generalize, or more precisely to find similarities between individual cases, clustering techniques may be used (e.g., clustering of students working on science activities; Van der Steen et al., submitted). As an illustration a simple example of looking for groups of cases, of which the averages are clearly different, will be presented. The quantitative findings were supplemented with qualitative findings, derived from video fragments, to show possible explanations for variability between and within classrooms (mixed method; Johnson, Onwuegbuzie, & Turner, 2007). Significant differences were used as a starting point for examining the data in a qualitative manner.

Transactional Nature of Learning

In order to be able to make a comparison with the first, group-based analysis, the focus of this representative case was again on the pre-measures and post-measures. Variables which were assessed (over time) concerned task-related utterances: the number and types of questions asked by the teacher, the complexity of student utterances, and the occurrence of coherent "action–reaction chains" in teacher–student interaction. Therefore, for the teacher variable the utterances were coded on an ordinal scale of "level of stimulation" (based on the "openness-scale" of Meindertsma, Van Dijk, Steenbeek, & Van Geert, 2014); i.e., utterances intended to evoke students' (higher order) scientific reasoning skills. The scale ranged from giving instructions, providing information, asking a knowledge-based question, asking a thought-provoking question, posing encouragements, to posing a task-related follow-up. Giving an instruction is considered as least stimulating, i.e., the smallest possible chance of evoking a high level of reasoning as an answer. With a Cohen's kappa of .72 the interobserver agreement was considered substantial. First, the interactional space, i.e., the amount of utterances, covered by the teacher and students was computed to gain insight into the general distributions of turns during the lesson. Note that the non-task related utterances are removed from this graph. Next, a graph showing the temporal sequence of the interaction is displayed (with the program Excel), as an alternative to the state space grid method (Hollenstein & Lewis, 2006). Both the graph and a state space grid use two axes to display the interaction between variables. A state space grid is a useful way to depict attractor states. However, for the purpose of answering the research question about how scientific understanding is co-constructed an excel graph is, in this particular case, a more accessible application. Lastly, a transition diagram (e.g., Ensing et al., 2014; Steenbeek et al., 2012) was used to study the micro dynamics of the transaction between students (as a class) and the teacher. Transition diagrams were made to reveal pattern characteristics, which provide insight into the number and types of questions asked and potentially how the difference between pre- and post-measure can be explained. These diagrams show the succession of variables. The observed differences between the pre- and post-measure regarding the percentages were statistically tested based on the null hypothesis that the observed differences were accidental. For the transition diagrams the follow-ups were summarized in non-stimulating reactions —instructions, providing information— and stimulating reactions —thought-provoking questions and comments and encouragements.

Results

Pre-measure versus Post-measure: Static-Macro Dimension

In order to answer the research question on whether there is an effect of the VFCt on students' performance, the observational data of the pre- and post-measure is aggregated over all classrooms. Note that the pre-measure and post-measure had the same teaching goal in all groups, i.e., teaching students about high and low (air) pressure. The scores during these lessons can therefore be compared validly.

Students performed on average better during the post-measure, $M = 4$, compared to the pre-measure, $M = 3.25$ ($p < 0.05$; Cohens $d = 1.6$, very large). Results show an expected intervention effect, i.e., students' science performance increased. This static macro dimension is the standard answer to questions about effectiveness of an intervention; most researchers are confining themselves to this single static macro evaluation. However, more insight can easily be gained by knowing how these average classroom complexity levels are constructed. In this particular case, the lower levels of scientific reasoning (1, 2, 3) are, for instance, more apparent during the pre-measure (PreM = 52; PostM = 25), while the higher levels (5, 6, 7) of scientific reasoning (PreM = 17; PostM = 36) are manifested more during the post measure (resp. $p < 0.05$ and $p < 0.01$). Looking at all measurements provides more information about the question what happens during the intervention-lessons.

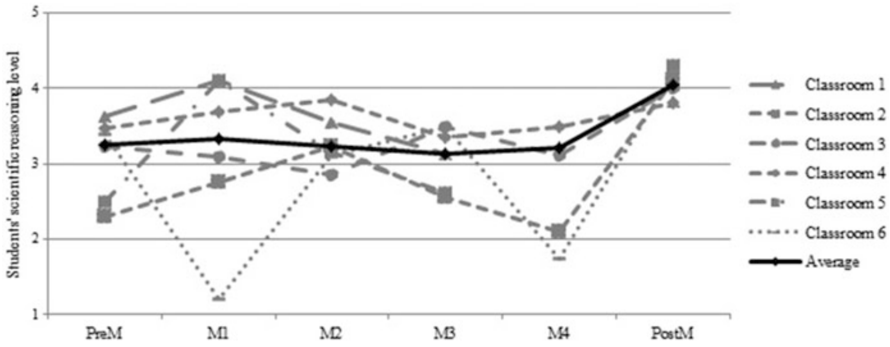

Fig. 11.1 Dynamic-macro scores of students' scientific reasoning skills of all classrooms during all measurements

Time: Short- and Long-Term Effects

In order to answer the question about development; how can we characterize students' scientific reasoning on the group level during the intervention trajectory, the solid black-diamonds line in Fig. 11.1 represents the average score of students' scientific reasoning level over all classrooms over time.

The solid line in Fig. 11.1 depicts that students display higher levels of scientific reasoning at the post-measure compared to the other measurements (preM = M1 = M2 = M3 = M4 < postM, $p < 0.01$). We thus see a long-term effect for this variable and the level of scientific reasoning seems rather stable on group level from the pre-measure to the lessons during the intervention.

This is already one step forward in comparison to the static macro comparison of the pre- and posttest. However, since the black line represents the average of the levels for all the classes, it is still the representation of a pseudo process (as a sequence of averages over independent cases it is not a real process). Based on this notion of a pseudo process, in order to actually see the process of change, analysis should focus at the process on the individual level, which in this case is the classroom level. Note that this is, in turn, a pseudo-process for the individual trajectories.

Variability: Dynamic-Macro Dimension

Next, there is a need to know the performance level of each classroom and how this changes (dynamic) over time (macro) under influence of the VFCt. Figure 11.1 depicts considerable variation in the level of scientific reasoning between classrooms (dashed lines), but also within a classroom over time.

With regard to interindividual variability: In Fig. 11.1, all observations over the six clasrooms in the post-measurement case are very close to one another, whereas almost all the preceding measurements show quite considerable variation between individual classrooms. This shows, for instance, that during post-measure the average complexity level of students' level of scientific reasoning of all classrooms is closer to each other compared to the pre-measure ($p = 0.1$). Furthermore, quite considerable differences were found in the amount of task-related utterances among classes. For instance, classroom 6's first scientific reasoning level is based on five task-related utterances ranging from complexity level 1 to 4, while classroom 1's level is based upon 54 task-related utterances ranging from complexity level 1 to 7. In addition, as an illustration of the clustering of individual cases: two subgroups were found in the level of variability ($M1_{variability} = 0.4$ and $M2_{variability} = 1.3$, $p < 0.01$). Classroom 1, 3, and 4 showed a rather stable level of scientific reasoning level over the lessons ($M_{variability} = 0.4$), while classroom 2, 5, and 6 showed considerable variability ($M_{variability} = 1.3$).

With regard to intraindividual variability: Intraindividual variability is visible in all classrooms (see Fig. 11.1, dashed lines), but most clearly in classrooms 2, 5, and 6 (note that this is one of the two subgroups mentioned above). When we zoom in at the development of classroom 6, the difference between the first and second lesson in students' scientific reasoning level is 1.91 complexity level. Measurement 1 ($p < 0.01$) and measurement 4 ($p < 0.01$) are different from the other lessons in that the average scientific reasoning level is lower. During both lessons only a handful task-related utterances could be scored, and 75–80 % of those utterances were on the lowest complexity level.

Looking back, these results may be explained by the content of lesson 1 and 4 (Table 11.2—method section). In both cases, the students were not allowed to experiment and the material was less provoking (note that the same variation in lessons applies for classroom 2 and 5). This suggests that the type of lesson and material used influences the —amount of— emergent complexity level of students' utterances.

Transactional Nature of Learning: Micro-dynamic and Long-Term Effects

Due to the labor-intensive nature of the observations, the following illustrations focus on one representative case; one teacher and her students. Classroom 3 could be used as a representative case in that preliminary analyses of teacher behavior showed that the behavior of the teacher represented the general interactional patterns in the classrooms best —i.e., starting the intervention by predominantly using instruction towards a more thought-provoking teaching style at the end of the intervention— the teacher neatly followed the guidelines, students' average age closely resembled the average age of all participating students, and all measures were available of this classroom.

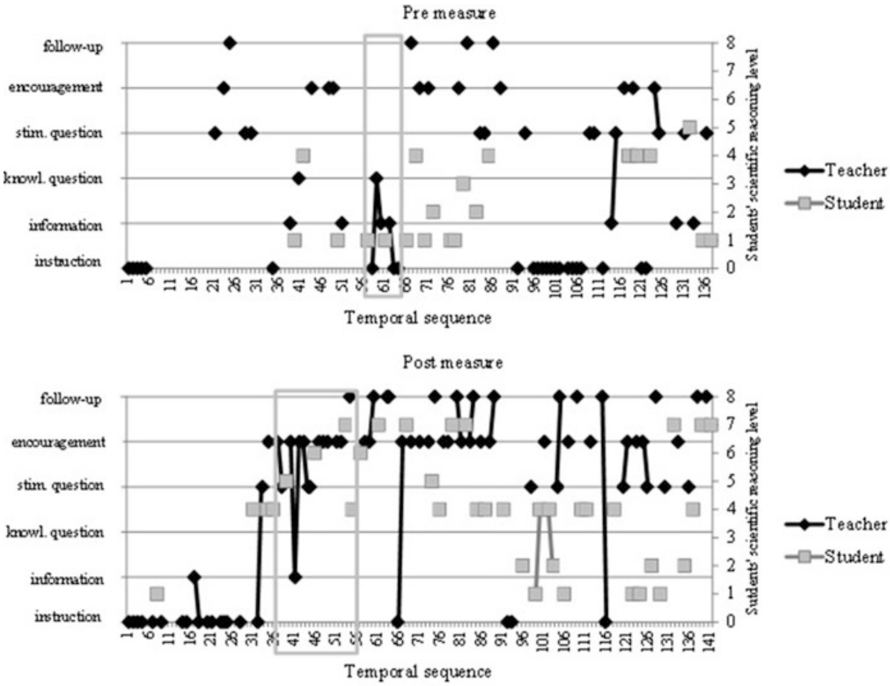

Fig. 11.2 Dynamic-micro scores during pre- (*top*) and post-measure (*bottom*) for classroom 3 *Note*: the teacher (*left*) axis depicts an ordinal scale from less stimulating to more stimulating utterances to provoke scientific reasoning skills: 0 = instruction, 1 = providing information, 2 = knowledge question, 3 = thought-provoking question, 4 = encouragement, 5 = follow-up. The student (*right*) axis depicts the ordinal complexity scale based on skill theory. The *grey boxes* are illustrated in the text

Figure 11.2 depicts the quantified interaction during 10 minutes of the middle part of the pre- and post-measure of classroom 3. The figure depicts different interaction patterns during pre-measure and post-measure. During post-measure there is in general much more interaction, mainly at the higher (more stimulating and complex) side of the graph. This type of display is a way to represent the nature of the process of interaction between the teacher and the students. On the x-axis the temporal sequence of the interaction is displayed. Each number represents an utterance of either the teacher or the student. On the left y-axis the task-related teacher utterances (diamonds) are categorized according to the degree of stimulation, while on the right y-axis the complexity level of task-related student utterances (squares) are depicted. Blank spaces represent a non-task related utterance. For purposes of illustration and as a guide how to read the graph, part of a literal translated transcript of an experiment "blow a paper wad in a bottle" will be described. Starting from utterance 57 (the grey square in Fig. 11.2, on the top): the teacher starts with a knowledge-based question: "*I think… What's in there?*"

followed by self-iterated information giving "*There is still moisture in it*." Next the student answers by formulating what he sees: "*Yes, it is red*." The teacher continues with providing information "*And then the paper sticks, that's a shame*." She offers a possibility for why the moisture has an effect on the outcome "*This bottle is dry...*" and offers a new bottle with the instruction to retry the experiment: "*Try this [dry] one*."

The level of stimulation: Figure 11.2, on the top, depicts that the teacher occupies most interactional space (75 %) during the lesson, more specifically most of her utterances are on the lowest stimulation level, namely to instruct students (41 % of her utterances). The transcript described above is an example of that type of interaction. During the pre-measure most of the utterances were teacher-centered (56 %), i.e., focusing on what students need to do and on knowledge acquisition by instructing, providing information, and asking knowledge-based questions, while 44 % were student centered utterances, i.e., stimulating utterances focusing on students thinking process—thought-provoking questions, encouragement, and substantive follow-up.

In contrast, although the teacher still occupies most of the interactional space (67 %) during post-measure, we can now see reciprocity between teacher utterances and student utterances which seem to emerge in higher levels of complexity (Fig. 11.2, bottom). Compare for this the upper side (on the teacher axis stimulating question, encouragement and follow-up) of the pre-measure graph with the upper side of the post-measure graph. During post-measure there is much more interaction at the higher (more stimulating and complex) side of the graph. The teacher asks more questions, poses more encouragements and students reason on higher levels of complexity (4, 5, 6, and 7). In addition, compared to the pre-measure a reversed pattern was found in teacher style, meaning that 29 % was teacher-centered (least stimulating) and 71 % consisted of stimulating utterances during post-measure. Table 11.3 describes an interaction during the post-measure showing how this was seen during the activity. Here, the teacher starts the interaction with a thought-provoking question, followed by a student answer that shows understanding of the experiment. The teacher continues with encouragements and rephrases student answers.

To conclude, by comparing the pre-measure and post-measure, the quantitative data shows an emerging pattern in which the teacher uses higher levels of stimulation during post-measure. The teacher asks more stimulating questions or poses encouragements to reason further (compared to preM; $p < 0.05$), students answer more often (preM = 20; postM = 36) and on a higher level of complexity ($p < 0.01$).

Action—reaction sequences: Figure 11.3 shows transition diagrams of both lessons. Both the type and number of teacher and student utterances change. During the pre-measure, students answer a teacher initiation question in only 31 % of the cases and the teacher answers her own question or continues herself in 15 % of the utterances. A student answer is in 20 % of the cases followed by a non-stimulating teacher response (like providing information or instruction) and in 44 % of the cases by a stimulating follow-up (encouragement, question, or an utterance to encourage

Table 11.3 Literal translated transcript of the experiment: "candle and lemonade," starting from utterances 38 (grey square) and further during the post-measure

Teacher	Student(s)	Comment
What do you think [will happen] [***]?		Thought-provoking initiation question
	When you put the glass over [the candle]. . . the water comes up and. . . because of the water the candle goes out	Student is capable of making a representation about what he expects to happen
Ok. . . hmm. . .		Encouragements (without directing to the "right" answer)
Ok, you think the candle extinguishes because of the water.		Rephrasing student's answer
Who has another idea?		Invite other students to formulate a hypothesis
	When you put the glass. . . the fire causes vapor. . . when that comes down the candle stops burning	Student is capable of formulating a representation in which insight into a natural phenomenon is represented
Hmm. . . Basically you make rain. . .		Rephrasing student's answer — providing information about how it could compare to daily life situations
What do you think [***]?		Invite another student to formulate a hypothesis
	I think there will be no more oxygen	Student is capable of formulating a hypothesis using abstract language
No more oxygen. . . Where?		Teacher uses a follow-up question to make the student elaborate on her answer

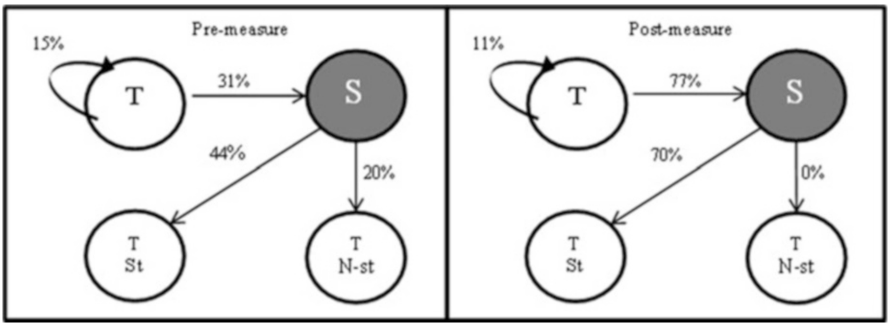

Fig. 11.3 Transition diagrams pre-measure (*left*) and post-measure (*right*) of Teacher initiation (T), Student task-related utterance (S), Teacher's stimulating response (T st), and Teacher's non-stimulating response (T N-st)

reflection). A significantly different interaction pattern is found between pre- and post-measure ($p < 0.01$) in that during the post-measure an initiation question of the teacher is often (in 77 % of the cases) directly followed by a task-related student utterance. Next, a student utterance is most often followed by a stimulating follow-up of the teacher. This seems to indicate better attuned interactions, i.e., stimulating interactions, possibly emerging into higher levels of student complexity.

Conclusion and Discussion

From a *content-based perspective*, the surplus value of a complex dynamic systems approach was illustrated by analyzing the (effect of) the Video Feedback Coaching program for teachers intervention, in which complexity properties were intertwined in design, data collection, and analysis.

When looking at the aggregated and static data, the results showed a positive intervention effect on the macro level of students’ science performance. The question arose about the practical significance of this result. An average increase of 1 complexity level seemed trivial. The effect size ($d = 1.6$) showed that this effect can be considered very large. However, this number does not provide practical tools for teachers. By using a process-based intervention study the surplus value of applying the properties became clear:

1. By incorporating time serial aspects of change, the intervention effect could be further explained. The average trajectory of all classrooms over several lessons (dynamic) showed a rather stable level during the intervention. The effect of the intervention on students’ performance only became apparent at post-measure.
2. By focusing on intra-individual variability, however, it became clear that the average trajectory underestimated the variability present in individual trajectories. Half of the classrooms showed a rather stable trajectory, while the other half represented great variability. None of the groups showed a clear positive intervention effect on students’ scientific reasoning level *during* the intervention sessions. However, previous research indicated that before a new state (i.e., higher level of performance) can be reached, a period of “increased variability” appears (Bassano & Van Geert, 2007; Van Der Steen et al., 2014; Van Geert & Van Dijk, 2002). These suggestions can be further analyzed by focusing on micro-dynamic processes in all lessons, in order to find out whether there is more variability leading to a new state at the micro level during the lessons of the intervention period. Another explanation for the, in this case rather high, variability might be found by focusing on the lesson characteristics. When a teacher provides a lesson mainly focussing on following the steps on a worksheet, a different interactional quality might be expected compared to a lesson in which students have more degrees of freedom to experiment. Note that the transactional nature might be used to further interpret this qualitative finding.

3. By examining the transactional nature, it became apparent that the higher performance seems to be achieved by a mutual investment of teacher and students and that a change in interaction patterns seems to underlie this phenomenon. The representative case showed that an increase in students' understanding is accompanied by a change in interactional quality and that the students' scientific reasoning level fluctuates in interaction with the teacher. During the post-measurement, teacher and students seem more attuned to each other, in that a teacher's question is twice as often followed by a student answer compared to the pre-measurement. Students seem more capable of using complex terms to express their thinking processes, as is expressed in the higher complexity scores. In addition, during post-measurement, the student utterance is only followed by a stimulating response, while during pre-measure, non-stimulating utterances were apparent. Based on the micro-dynamic data, we therefore suggest that the higher performance during the post-measurement can be explained by interactions of higher quality in which the teacher poses more stimulating questions and that the students reason on higher scientific reasoning levels. The point of this type of analysis is not to pretend that these percentages apply to the population, as an average level. We aimed to depict a technique of representation that shows the time serial nature of the process. It goes without saying that the structure of these processes may be quite different for one case in comparison to another, but the nature of the representation, in terms of a transition diagram, in principle applies to all possible forms of interaction in classrooms. By choosing a different way of representing the interaction in the classroom, namely by means of these transition diagrams, the emphasis which is traditionally put on static measures, is now replaced by a dynamic representation, which in some cases may be of quite considerable complexity. Especially for teachers, the latter might be a more accurate reflection of the teacher's real time experiences as teachers are "aware" —usually without being familiar with the technical terms— that they are working within a complex dynamic system.

To summarize, the surplus value of the analysis is that it illustrates how a complex dynamic systems approach can be used to describe the processes underlying static group-based educational intervention effects, and provide information about the quality of that intervention. By using a process-based methodology, we were able to show that average results can be deepened by focusing on several complexity properties. We suggested answers to the question of why the VFCt intervention worked and why it seemed to work better during some lessons compared to other lessons within one classroom (i.e., type of questions, attuned interactions, using active participation during experiments versus classical experiment lessons). In addition, insight was provided into the actual changes during lessons and how interaction proceeded. This information cannot be found in conventional longitudinal studies, but are essential for teachers as this might more accurately reflect what they experience during their lessons and gives insight into how teachers can optimize their lessons—compared to standard evaluations.

Of course, when assessing the effectiveness of an intervention the use of a control group will primarily provide information about differences between the actual processes; especially the micro-process differences (see for instance Wetzels et al., 2015). Veerman and van Yperen (2007) state that the use of a control group is a prerequisite for analysis of the effectiveness. Therefore, the next step is to analyze classrooms that did not participate in the VFCt, but did provide science and technology lessons (Van Vondel et al., 2015).

From a *methodological point of view*, we would like to make a distinction between "hard" complex dynamic systems research and "soft" complex dynamic systems research in education. The distinction might be somewhat exaggerated and is rather a matter of degree, but we think it is important to discuss it in order to put much of the complex dynamic systems research that is currently being done in education in the right perspective.

By "hard" complex dynamic systems research, we mean the research that focuses on typical complex dynamic systems properties and which is based on very dense time series. Examples are studies of attractors and discontinuities, for instance by means of cusp catastrophe models (Van Der Maas & Molenaar, 1992), or studies of the statistical structure of time series revealing properties such as pink noise in rower's coordination of ergometer strokes (Den Hartigh, Cox, Gernigon, Van Yperen, & Van Geert, 2015) or studies using techniques such as recurrence quantification analysis that try to reconstruct the complexity of the state space that underlies the attractors of the system (Wijnants, Bosman, Hasselman, Cox, & Van Orden, 2009).

By "soft" complex dynamic systems research, in contrast, we mean educational research inspired by basic, qualitative features of a complex dynamic systems view on education and which is rooted in educational practice, as the VFCt. Some examples which would typically qualify as "soft" complex dynamic systems research are presented by Steenbeek, et al. (2012): research on learning that focuses on individual trajectories and on intraindividual variability, on the transactional and iterative nature of the teaching-learning process and on the relationship between the short-term time scale of learning activities and the long-term time scale of development. It is a kind of research that describes how such patterns are self-sustaining and hard to change, i.e., tends to show considerable resistance to change and thus have the qualitative properties of attractor states.

Scientific implications for intervention studies: Especially evaluation studies of —applied— educational interventions are fruitful areas for a "soft" complex dynamic systems approach. As performance is usually constructed in interaction between a more knowledgeable partner and a student (Steenbeek & Van Geert, 2013; Van De Pol et al., 2011), observational classroom studies provide rich information. Analyses on the micro-level show whether the effect of an intervention can be found on the level where interventions focus at, in this case on interactions of higher quality. For a complete understanding of the process of teaching students a particular way of reasoning, an intensive study of a teacher's —in combination with the students'— behavior over several lesson will reveal important insights. Focusing on "how" an intervention works is a way of describing why one state changes into another, and in fact implies a way of describing what can be done to make the state change into another one (Van Geert & Steenbeek, 2005). Furthermore, the case study findings can be supported by findings of a multiple case study. These findings can then be used to generalize findings and by that strengthen evidence-based practice.

Practical implications: The results of process analysis can be used in two different ways, as both scientific and practical purposes can be highlighted. First, the results add to fundamental knowledge about how scientific reasoning skills are (co-) constructed in real-time (Meindertsma et al., 2014) and how the effect of a teaching intervention emerges during actual science and technology lessons. Second, the results can be used for educational purposes. This approach provides accessible practice-based tools for best practice, or perhaps more importantly, familiar examples which can be used for (in-service) teacher professionalization (Wetzels et al., 2015). The micro-dynamic analysis might map the most interesting information for educational practitioners as it yields practice-based results.

Further analyses: An important next step for the study of interventions is to map the teacher–student interactions of individual teachers in order to study whether interindividual variability can be further explained on the micro-level (Van Vondel, Steenbeek, Van Dijk, & Van Geert, in preparation). The analyses of the empirical example as presented in this paper may be not more than only the first steps towards a complex dynamic systems approach. More information can be extracted by repeating similar analyses for teacher variables, by focusing on all lessons of individual teachers, by comparing micro and macro findings, or by comparing two extreme cases on the micro level (e.g., Steenbeek et al., 2012).

To conclude, interventions should be studied as emerging processes on various, intertwined time scales taking place in individual cases, and not as isolated causal factors, with an intrinsic effectiveness, applying to a specific population category. We, therefore, stress the importance of using variables that capture the transactional character of interventions, specifically when they are aimed at improving interaction patterns in the naturalistic classroom situation. For future research we like to state that it is essential to look more closely at what the intervention is aiming at and what the role of the immediate context/proximal factors are in this process. When more understanding is gained about what happens during the intervention, for instance about stability or change in interaction patterns, intervention programs can be specifically attuned to supporting high quality interaction patterns in the classroom and students can thus be stimulated to perform optimally.

References

Bassano, D., & Van Geert, P. L. C. (2007). Modeling continuity and discontinuity in utterance length: A quantitative approach to changes, transitions and intra-individual variability in early grammatical development. *Developmental Science, 10*(5), 588–612.

Bentley, M. L. (1995). Making the most of the teachable moment: Carpe diem. *Science Activities, 32*(3), 23–27.

Boelhouwer, M. D. (2013). *Tussen weerstand en weerbaarheid, en andere recepten; Een effectevaluatie van het WIBO-lesprogramma met behulp van vragenlijsten, dagboeken en observaties* (Between resistance and empowerment; the evaluation of the WIBO-program prevention program). Groningen, The Netherlands: University of Groningen.

Borman, G. D., Gamoran, A., & Bowdon, J. (2008). A randomized trial of teacher development in elementary science: First-year achievement effects. *Journal of Research on Educational Effectiveness, 1*, 237–264. doi: 10.1080/19345740802328273.

Bos, J., & Steenbeek, H. W. (2009). *Mediacoder; software voor het coderen van video- en audiomaterialen*. Groningen: Internal publication, IDP Department, University of Groningen.

Chin, C. (2006). Classroom interaction in science: Teacher questioning and feedback to students' responses. *International Journal of Science Education, 28*(11), 1315–1346. doi: 10.1080/09500690600621100

Cumming, G. (2014). The new statistics why and how. *Psychological Science, 25*(1), 7–29. doi: 10.1177/0956797613504966

De Groot, A. D. (1994). *Methodologie: Grondslagen van onderzoek en denken in de gedragswetenschappen*. Assen: Van Gorcum.

Den Hartigh, R. J. R., Cox, R. F. A., Gernigon, C., Van Yperen, N. W., & Van Geert, P. L. C. (2015). Pink noise in rowing ergometer performance and the role of skill level. *Motor Control, 19*, 355–369. doi: 10.1123/mc.2014-0071

Den Hartigh, R. J. R., Gernigon, C., Van Yperen, N. W., Marin, L., & Van Geert, P. L. C. (2014). How psychological and behavioral team states change during positive and negative momentum. *PLoS One, 9*(5), e97887. doi: 10.1371/journal.pone.0097887

Ensing, A., Van der Aalsvoet, D., Van Geert, P.L.C., & Voet, S. (2014). Learning potential is related to the dynamics of scaffolding. An empirical illustration of the scaffolding dynamics of five year olds and their teacher. *Journal of Cognitive Education and Psychology, 13*(3), 1–18. doi: 10.1891/1945-8959.13.3.375

Fischer, K. W. (1980). A theory of cognitive development: The control and construction of hierarchies of skills. *Psychological Review, 87*, 477–531. doi: 10.1037/0033-295X.87.6.477.

Fischer, K. W., & Bidell, T. R. (2006). Dynamic development of action, thought, and emotion. In R. M. Lerner (Ed.) & W. Damon (Series ed.), *Handbook of child psychology: Vol. 1. Theoretical models of human development* (pp. 313–399). New York: Wiley.

Fogel, A. (2011). Theoretical and applied dynamic systems research in developmental science. *Child Development Perspectives, 5*(4), 267–272. doi: 10.1111/j.1750-8606.2011.00174.x.

Fredrickson, B. L. (2015). The broaden-and-build theory of positive emotions. *Philosophical Transactions, 359*(1449), 1367–1377. doi: 10.1098/rstb.2004.1512

Fukkink, R. G., Trienekens, N., & Kramer, L. J. (2011). Video feedback in education and training: Putting learning in the picture. *Educational Psychology Review, 23*(1), 45–63.

Gibson, H. L., & Chase, C. (2002). Longitudinal impact of an inquiry-based science program on middle school students' attitudes toward science. *Science Education, 86*(5), 693–705. doi: 10.1002/sce.10039

Granott, N., & Parziale, J. (2002). Microdevelopment: A process-oriented perspective for studying development and learning. In N. Granott & J. Parziale (Eds.), *Microdevelopment: Transition processes in development and learning* (pp. 1–28). New York, NY: Cambridge University Press.

Hock, M. F., Schumaker, J. B., & Deschler, D. D. (1995). Training strategic tutors to enhance learner independence. *Journal of Developmental Education, 19*, 18–26.

Hollenstein, T., & Lewis, M. D. (2006). A state space analysis of emotion and flexibility in parent-child interactions. *Emotion, 6*(4), 656. doi: 10.1037/1528-3542.6.4.656

Hood, G. (2004). Poptools [Computer software]. Pest Animal Control Co-operative Research Center.

Hyun, E., & Marshall, J. D. (2003). Teachable-moment-oriented curriculum practice in early childhood education. *Journal of Curriculum Studies, 35*(1), 111–127. doi: 10.1080/00220270210125583

Johnson, R. B., Onwuegbuzie, A. J., & Turner, L. A. (2007). Toward a definition of mixed methods research. *Journal of Mixed Methods Research, 1*(2), 112–133. doi: 10.1177/1558689806298224

Kline, R. B. (2004). *Beyond significance testing. Reforming data analysis methods in behavioral research*. Washington, DC: APA Books.

Lambdin, C. (2012). Significance tests as sorcery: Science is empirical—Significance tests are not. *Theory and Psychology, 22*(1), 67–90. doi: 10.1177/0959354311429854

Lehmann, A. C., & Gruber, H. (2006). Music. In K. A. Ericsson, N. Charness, P. J. Feltovich, & R. R. Hoffman (Eds.), *The Cambridge handbook of expertise and expert performance* (pp. 457–470). New York, NY: Cambridge University Press.

Lewis, M. D. (1995). Cognition-emotion feedback and the self-organization of developmental paths. *Human Development, 38*(2), 71–102. doi: 10.1159/000278302

Lewis, M. D. (2002). Interacting time scales in personality (and cognitive) development: intentions, emotions, and emergent forms. In N. Grannot & J. Parziale (Eds.), *Microdevelopment: Transition processes in development and learning* (pp. 183–212). New York, NY: Cambridge University Press.

Lichtwarck-Aschoff, A., van Geert, P. L. C., Bosma, H., & Kunnen, S. (2008). Time and identity: A framework for research and theory formation. *Developmental Review, 28*(3), 370–400. doi: 10.1016/j.dr.2008.04.001.

Meindertsma, H. B., Van Dijk, M. W. G., Steenbeek, H. W., & Van Geert, P. L. C. (2012). Application of skill theory to compare scientific reasoning of young student in different tasks. *Netherlands Journal of Psychology, 67*, 9–19.

Meindertsma, H. B., van Dijk, M. W. G., Steenbeek, H. W., & van Geert, P. L. C. (2014). Assessment of preschooler's scientific reasoning in adult–child interactions: What is the optimal context? *Research in Science Education, 44*, 215–237. doi: 10.1007/s11165-013-9380-z.

Mortenson, B., & Witt, J. (1998). The use of weekly performance feedback to increase teacher implementation of a prereferral academic intervention. *School Psychology Review, 27*(4), 613–627.

Noell, G. H., Witt, J., Slider, N., Connell, J., Gatti, S., Williams, K., et al. (2005). Treatment implementation following behavioral consultation in schools: A comparison of three follow-up strategies. *School Psychology Review, 34*(1), 87–106.

Oliveira, A. W. (2010). Improving teacher questioning in science inquiry discussions through professional development. *Journal of Research in Science Teaching, 47*(4), 422–453.

Osborne, J. (2014). Teaching scientific practices: Meeting the challenge of change. *Journal of Science Teacher Education, 25*, 177–196. doi: 10.1007/s10972-014-9384-1.

Parziale, J., & Fischer, K. (1998). The practical use of skill theory in classrooms. In R. Sternberg & W. Williams (Eds.), *Intelligence, instruction, and assessment: Theory into practice* (pp. 95–110). Mahwah: Lawrence Erlbaum Associates Publishers.

Penuel, W. R., Gallagher, L. P., & Moorthy, S. (2011). Preparing teachers to design sequences of instruction in earth systems science: A comparison of three professional development programs. *American Educational Research Journal, 48*(4), 996–1025. doi: 10.3102/0002831211410864

Pintrich, P. R. (2000). The role of goal orientation in self regulated learning. In M. Boekaerts, P. R. Pintrich, & M. Zeidner (Eds.), *Handbook of selfregulation* (pp. 451–502). San Diego: Academic.

Reinke, W. M., Sprick, R., & Knight, J. (2009). Coaching classroom management. In J. Knight (Ed.), *Coaching: Approaches & perspectives* (pp. 91–112). Thousand Oaks: Corwin Press.

Rosmalen, J. G., Wenting, A. M., Roest, A. M., de Jonge, P., & Bos, E. H. (2012). Revealing causal heterogeneity using time series analysis of ambulatory assessments: Application to the association between depression and physical activity after myocardial infarction. *Psychosomatic Medicine, 74*(4), 377–386. doi: 10.1097/PSY.0b013e3182545d47

Seidel, T., Stürmer, K., Blomberg, G., Kobarg, M., & Schwindt, K. (2011). Teacher learning from analysis of videotaped classroom situations: Does it make a difference whether teachers observe their own teaching or that of others? *Teaching and Teacher Education, 27*(2), 259–267. doi: 10.1016/j.tate.2010.08.009

Şimşek, P., & Kabapinar, F. (2010). The effects of inquiry-based learning on elementary students' conceptual understanding of matter, scientific process skills and science attitudes. *Procedia - Social and Behavioral Sciences, 2*(2), 1190–1194. doi: 10.1016/j.sbspro.2010.03.170.

Smith, L. B., & Thelen, E. (2003). Development as a dynamic system. *Trends in Cognitive Sciences, 7*(8), 343–348. doi: 10.1016/S1364-6613(03)00156-6

Steenbeek, H. W., Jansen, L., & Van Geert, P. L. C. (2012). Scaffolding dynamics and the emergence of problematic learning trajectories. *Learning and Individual Differences, 22*(1), 64–75. doi: 10.1016/j.lindif.2011.11.014

Steenbeek, H. W., & Van Geert, P. L. C. (2005). A dynamic systems model of dyadic interaction during play of two student. *European Journal of Developmental Psychology, 2*(2), 105–145. doi: 10.1080/17405620544000020

Steenbeek, H. W., & Van Geert, P. L. C. (2013). The emergence of learning-teaching trajectories in education: A complex dynamic systems approach. *Nonlinear Dynamics, Psychology, and Life Sciences, 17*(2), 233–267.

Steenbeek, H. W., & Van Geert, P. L. C. (2015). Assessing young student's learning and behavior in the classroom; a complexity approach. In G. M. Van der Aalsvoort (Eds.), *International handbook on early childhood education, Volume 2—Curriculum and assessment.*

Steenbeek, H. W., Van Geert, P. L. C., & Van Dijk, M. W. G. (2011). The dynamics of student's science and technology talents: A conceptual framework for early science education. *Netherlands Journal of Psychology, 66*(3), 96–109.

Sullivan, G. M., & Feinn, R. (2012). Using effect size-or why the P value is not enough. *Journal of Graduate Medical Education, 4*(3), 279–282. doi: 10.4300/JGME-D-12-00156.1

Thelen, E. (1992). Development as a dynamic system. *Current Directions in Psychological Science, 1*(6), 189–193. doi: 10.1111/1467-8721.ep10770402.

Van de Pol, J., Volman, M., & Beishuizen, J. (2011). Patterns of contingent teaching in teacher–student interaction. *Learning and Instruction, 21*(1), 46–57. doi: 10.1016/j.learninstruc.2009.10.004

Van der Maas, H., & Molenaar, P. (1992). Stagewise cognitive development: An application of catastrophe theory. *Psychological Review, 99*, 395–417. doi: 10.1037/0033-295X.99.3.395

Van der Steen, S., Steenbeek, H. W., Van Dijk, M. W. G., & Van Geert, P. L. C. (2015). How to characterize the development of student's understanding of scientific concepts: A longitudinal microgenetic study. Manuscript submitted for publication.

Van der Steen, S., Steenbeek, H. W., Van Dijk, M. W. G., & Van Geert, P. L. C. (2014). A complexity approach to student's understanding of scientific concepts: A longitudinal case study. *Learning and Individual Differences, 30*, 8–91. doi: 10.1016/j.lindif.2013.12.004

Van der Steen, S., Steenbeek, H. W., Wielinski, J., & Van Geert, P. L. C. (2012). A comparison between young students with and without special needs on their understanding of scientific concepts. *Educational Research International, 2012*, 1–12. doi: 10.1155/2012/260403.

Van der Steen, S., Steenbeek, H. W., & Van Geert, P. L. C. (2012). Using the dynamics of a person-context system to describe student's understanding of air pressure. In H. Kloos, B. J. Morris, & J. L. Amaral (Eds.), *Current topics in student's learning and cognition* (pp. 21–44). Rijeka: InTech.

Van Geert, P. L. C. (1994). *Dynamic systems of development: Change between complexity and chaos*. New York, NY: Harvester Wheatsheaf.

Van Geert, P. L. C. (1997). Time and theory in social psychology. *Psychological Inquiry, 8*, 143–153.

Van Geert, P. L. C. (1998). A dynamic systems model of basic developmental mechanisms: Piaget, Vygotsky, and beyond. *Psychological Review, 105*(4), 634. doi: 10.1037/0033-295X.105.4.634-677

Van Geert, P. L. C. (2003). Dynamic systems approaches and modeling of developmental processes. In J. Valsiner & K. J. Conolly (Eds.), *Handbook of developmental psychology* (pp. 640–672). London: Sage.

Van Geert, P. L. C. (2004). Dynamic modelling of cognitive development: Time, situatedness and variability. In A. Demetriou & A. Raftopoulos (Eds.), *Cognitive developmental change: Theories, models and measurement* (pp. 354–379). Cambridge, MA: Cambridge University Press.

Van Geert, P. L. C., & Fischer, K. W. (2009). Dynamic systems and the quest for individual-based models of change and development. In J. P. Spencer, M. S. C. Thomas, & J. McClelland (Eds.), *Toward a unified theory of development? Connectionism and dynamic systems theory reconsidered* (pp. 313–336). Oxford: Oxford University Press.

Van Geert, P. L. C., & Steenbeek, H. W. (2005). Explaining after by before: Basic aspects of a dynamic systems approach to the study of development. *Developmental Review, 25*(3–4), 408–442. doi: 10.1016/j.dr.2005.10.003

Van Geert, P. L. C., & van Dijk, M. W. G. (2002). Focus on variability: New tools to study intra-individual variability in developmental data. *Infant Behavior and Development, 25*(4), 340–374.

Van Vondel, S., Steenbeek, H. W., Van Dijk, M. W. G., & Van Geert, P. L. C. (2015). The effects of Video Feedback Coaching for teachers on elementary students' scientific knowledge. Manuscript submitted for publication.

Van Vondel, S., Steenbeek, H.W., Van Dijk, M.W.G., & Van Geert, P.L.C. (in preparation). Ask Don't Tell; Improving science education by focusing on the co-construction of scientific understanding.

Veerman, J. W., & van Yperen, T. A. (2007). Degrees of freedom and degrees of certainty: A developmental model for the establishment of evidence-based youth care. *Evaluation and Program Planning, 30*(2), 212–221.

Velicer, W. (2010). *Applying idiographic research methods: Two examples*. In 8th International Conference on Teaching Statistics, Ljubljana, Slovenia.

Vygotsky, L. (1986). *Thought and language*. Cambridge: Cambridge University Press.

Wetzels, A. F. M., Steenbeek, H. W., & Van Geert, P. L. C. (2015). Primary science teaching: Behavior of teachers and their pupils during and after a coaching program. Manuscript submitted for publication.

Wetzels, A. F. M., Steenbeek, H. W., & Van Geert, P. L. C. (in press). A complexity approach to the effectiveness of an intervention for lower grade teachers on teaching science. *An International Journal of Complexity and Education.*

Wijnants, M. L., Bosman, A. M., Hasselman, F., Cox, R. F., & Van Orden, G. C. (2009). 1/f scaling in movement time changes with practice in precision aiming. *Nonlinear Dynamics, Psychology, and Life Sciences, 13*(1), 79.

Chapter 12
Analyzing Teacher–Student Interactions with State Space Grids

Helena J.M. Pennings and Tim Mainhard

Introduction

In educational research there is a growing body of knowledge on the general classroom and teacher characteristics that enhance or hamper student learning. This body of knowledge is primarily based on a product oriented research approach (Lavelli, Pantoja, Hsu, Messinger, & Fogel, 2005), that is, relying on global perceptions and measures, which summarize any development, rather than focusing on how classroom processes unfold in time. Another, more process oriented focus, may be more suited to understand how classroom interventions or specific teacher behavior take their effect in class during teaching. In this chapter we describe how we use a research tool called State Space Grid analysis, originated in Complex Dynamic Systems theory, to take a more process oriented rather than product oriented approach.

Our research mainly focuses on interpersonal teacher behavior and teacher–student interactions as the building blocks of teacher–student relationships and the quality of the classroom social climate. In this chapter we present four illustrations of studies that share this focus and in which State Space Grid analysis was used to examine the moment-to-moment nature of classroom interactions. First, interpersonal theory is introduced, which we use to frame classroom interaction. Then we discuss classrooms as complex dynamical systems and introduce the State Space Grid technique.

H.J.M. Pennings (✉) • T. Mainhard
Department of Education, Utrecht University, The Netherlands, Heidelberglaan 1, 80140, 3508, TC, Utrecht, Netherland
e-mail: h.j.m.pennings@uu.nl

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_12

An Interpersonal Perspective on Classroom Social Processes

The classroom social climate is the social aspect of the classroom environment; In product oriented terms, it refers to the overall quality of interpersonal relations in a classroom (Mainhard, Pennings, Wubbels, & Brekelmans, 2012). These relations can be conceptualized in terms of the generalized interpersonal meaning students and teachers attach to their own and each other's behavior in class. The basic processes that underlie the classroom social climate are virtually all interactions that occur in a classroom.

To describe interactions between teachers and students (or classes of students), we use Interpersonal Theory (Horowitz & Strack, 2011). In interpersonal theory a two-dimensional circular model called the *InterPersonal Circle* (IPC) is used to describe interpersonal styles and interpersonal behavior of people (Fournier, Moskowitz, & Zuroff, 2011; Gurtman, 2009; Horowitz & Strack, 2011; Kiesler, 1996). The basic premise of this theory is that every behavior can be positioned in the IPC as a specific blend of the two dimensions *agency* (i.e., power) and *communion* (i.e., warmth) (Fournier et al., 2011; Locke & Sadler, 2007). In essence an IPC always consists of the two basic dimensions, however how these dimension are called may vary depending on the context in which the model is applied (Fournier et al., 2011). The IPC can be divided into octants that describe prototypical interpersonal behavior located in that part of the IPC.

To study interpersonal behavior of teachers and students we use two IPCs (Fig. 12.1), one to describe interpersonal teacher behavior, the *IPC-T*, and one to describe interpersonal student behavior, the *IPC-S* (Pennings, Mainhard, & Brekelmans, 2015; Prins & Mainhard, 2012).

For decades interpersonal theory has been used in educational research to study the quality of the classroom social climate (for an overview see Wubbels,

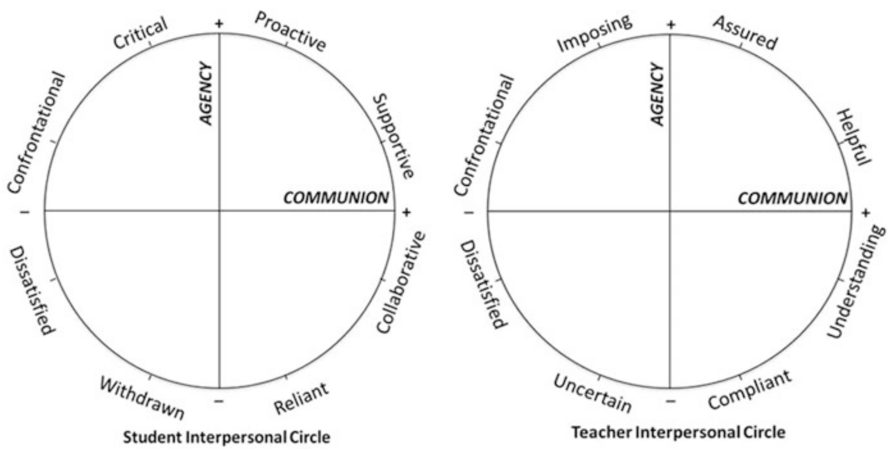

Fig. 12.1 Teacher and student interpersonal circle (Pennings et al., 2015; Prins & Mainhard, 2012)

Brekelmans, Den Brok, & Van Tartwijk, 2006), mainly through the applications of the Questionnaire on Teacher Interaction (QTI; Wubbels et al., 2006), which measures teacher agency and communion as perceived by students. By completing the QTI, students provide their general interpersonal perception of their teacher in class. To describe the general classroom social climate the individual student scores can be aggregated per class or teacher (Wubbels et al., 2006). Throughout the years nine general types of classroom social climates have been distinguished (Pennings et al., 2015). Eight types correspond to the IPC octants and the ninth is located in the center of the IPC.

Throughout the years ample knowledge has been gathered on how the quality of the classroom social climate, also from perspectives other than interpersonal theory, is related to student motivation and achievement (e.g., Cornelius-White, 2007; Henderson, 1995; Henderson & Fisher, 2008; Maulana, Opdenakker, Den Brok, & Bosker, 2011; Roorda, Koomen, Spilt, & Oort, 2011; Wentzel, 2012), but also to teacher motivation, self-efficacy, well-being and quality of teaching (e.g., Spilt, Koomen, & Thijs, 2011; Van Petegem, Creemers, Rossel, & Aelterman, 2005; Wubbels et al., 2014). For example, classroom social climates that are characterized by high levels of agency and communion in teacher behavior are most desirable for student motivation and achievement, but also for teacher well-being (Wubbels et al., 2006). A problematic classroom social climate is often related to classroom management issues (Mainhard, Brekelmans, & Wubbels, 2011) and can even be a reason for teachers to leave the profession (De Jong, Van Tartwijk, Verloop, Veldman, & Wubbels, 2012).

Since, daily interactions are the building blocks of relationships (Granic & Hollenstein, 2003), many scholars, including Kiesler (1996), Thomas, Hopwood, Woody, Ethier, and Sadler (2014) and Wubbels et al. (2012), advocate to study the dynamical process of interpersonal interactions as they unfold in time instead of focusing solely on the static products of these interactions (such as the quality of the classroom social climate). Ultimately, doing so might help us to understand better how teachers who experience problems in creating and maintaining a classroom climate conducive to learning can be supported. The theoretical framework and its accompanying methods that guide us in this process oriented endeavor is Complex Dynamical Systems (CDS) theory.

Classrooms as Complex Dynamical Systems

CDS theory describes how complex processes unfold in time (Guastello, Koopmans, & Pincus, 2009), and how *change* occurs gradually or dramatically (Guastello & Liebovitch, 2009). CDS theory originates from physics and mathematics, where it is used to study complex processes within and between systems (Guastello et al., 2009; Hollenstein, 2013). For example in thermodynamics the study of how temperature and energy are related to each other can be explained using CDS theory. There are two types of systems, *closed* and *open* systems. Closed systems are systems that cannot interact with other systems in its environment, whereas open systems develop through interactions with other systems in their environment (Hollenstein, 2013). Humans are open systems, because they interact with other systems in their environment, such as other humans or animals.

Interactions and development simultaneously take place on various time-scales. In real-time from second to second (i.e., micro-level time-scale), from hour to hour (i.e., meso-level time-scale), or in developmental time like month to month or year to year (i.e., macro-level time-scale) (Hollenstein, 2013). Development is, therefore, hierarchically nested in time (Hollenstein, 2007; Thelen & Smith, 1998). Defining the specific measurement level needed depends on the research question and the phenomenon that is studied. What makes development complex is that interactions occur *within* time-scales but also *between* time-scales (Hollenstein, 2013) and that behavior on one time-scale may affect behavior on another. For example, friendly teacher behavior from moment-to-moment may result in a supportive and warm classroom climate (a higher level time-scale), which in turn makes disruptive student behavior less likely (the lower-level timescale).

We have described how humans are considered to be individual complex dynamic systems and that interactions between humans foster development of systems. In the educational context teachers and students resemble individual systems as well, while classrooms (or the classroom social climate) can be seen as higher order social systems in which multiple individual systems (i.e., teacher and students) interact with each other. Interactions and development within classrooms also occur on multiple time-scales, from second to second within lessons (micro-level), from lesson to lesson (meso-level), month to month (meso/macro-level), and in some cases from year to year (macro-level). To complicate this, all individual systems within a specific classroom social system are also individual systems within other social systems (e.g., other classrooms, families, and sports teams). In these other social systems which different interactions might lead to differences in development of, for example, relationships (Bronfenbrenner & Morris, 2006). For teachers, this means that experiences in one classroom may transfer to other classrooms, also in subsequent years. Guided by CDS theory, we assume that these interactions are necessary for teachers to improve in their profession and that they drive teacher professional development.

To understand the illustrations of our research we provide in this chapter, it is necessary to grasp the meaning of several terms that are commonly used in CDS theory; terms such as state(s), state space, attractors, circular causality, and entropy. We elaborate on these terms in the next section. For a complete and comprehensive overview of these concepts and the CDS terminology we refer the reader to Guastello et al. (2009).

Using State Space Grids in Educational Research

Now that we have explained the two main theoretical perspectives that guide our research we turn to State Space Grid (SSG) analysis, which is the focus of the remainder of this chapter. The SSG tool is rooted in CDS theory and is used to examine the *content* (e.g., the level of friendliness) and *structure* (e.g., the variability in friendliness) of real-time (micro-level) interactions in systems. SSG studies can be found in various areas of social sciences research, such as family studies (e.g., Granic, Hollenstein, Dishion, & Patterson, 2003) where they originated, peer relationship studies (e.g., Lavictoire, Snyder, Stoolmiller, & Hollenstein, 2012), studies on cognitive styles in solving jigsaw puzzles (e.g., Hong, Hwang, Tam, Lai, & Liu, 2012), coach–athlete interactions (e.g., Turnnidge, Cote, Hollenstein, & Deakin, 2013), or clinical psychology (e.g., Bento, Ribeiro, Salgado, Mendes, & Gonçalves, 2014). The last couple of years SSG analysis has found its way into educational research (e.g., Mainhard et al., 2012; Pennings, Brekelmans et al., 2014; Turner, Christensen, Kackar-Cam, Trucano, & Fulmer, 2014; Vauras, Kinnunen, Kajamies, & Lehtinen, 2013).

First, we explain what SSGs are; second, we discuss the measures that can be derived from SSG analysis; third, we describe how attractors and information about the structure of interactions can be derived from these measures; and finally we provide some illustrations from our own research in which we used various approaches to construct and use SSGs to study real-time interactional processes in class.

In 1999 Lewis, Lamey, and Douglas developed SSG analysis (Fig. 12.2) and GridWare (www.statespacegrids.org; Lamey, Hollenstein, Lewis, & Granic, 2004) the software needed to built and analyze SSGs, because they needed methods to study complex dynamical processes in child parent interactions (Hollenstein, 2013). In this chapter we largely draw on ideas and research that originated from this group and specifically on the Gridware manual (Lamey et al., 2004) and Hollenstein's book on SSG analysis (Hollenstein, 2013).

The foundation of SSG analysis lies within the CDS term *State Space*. A SSG is a graphic representation of a state space that consist of at least two orthogonal dimensions that describe the states a system might reside in and all possible states a social system can adopt are graphically represented as cells in a grid. Hence, these cells together represent the state space of a social system. Thereby, SSGs provide an intuitively appealing way to view the structure of complex interactional, which makes SSGs also very suitable for exploratory analysis (Granic & Hollenstein, 2003; Hollenstein, 2013).

The dimensions underlying the SSGs usually consist of categorical observations of behavior states. It is important that these categories are mutually exclusive and exhaustive on each dimension (Granic & Hollenstein, 2003; Hollenstein, 2013; Hollenstein & Lewis, 2006). For example, in our studies one dimension may represent interpersonal teacher behavior, while the other represents interpersonal student behavior. Yet the dimensions and the number of categories underlying the dimensions need not to be similar (Hollenstein, 2013). For example, in one of our

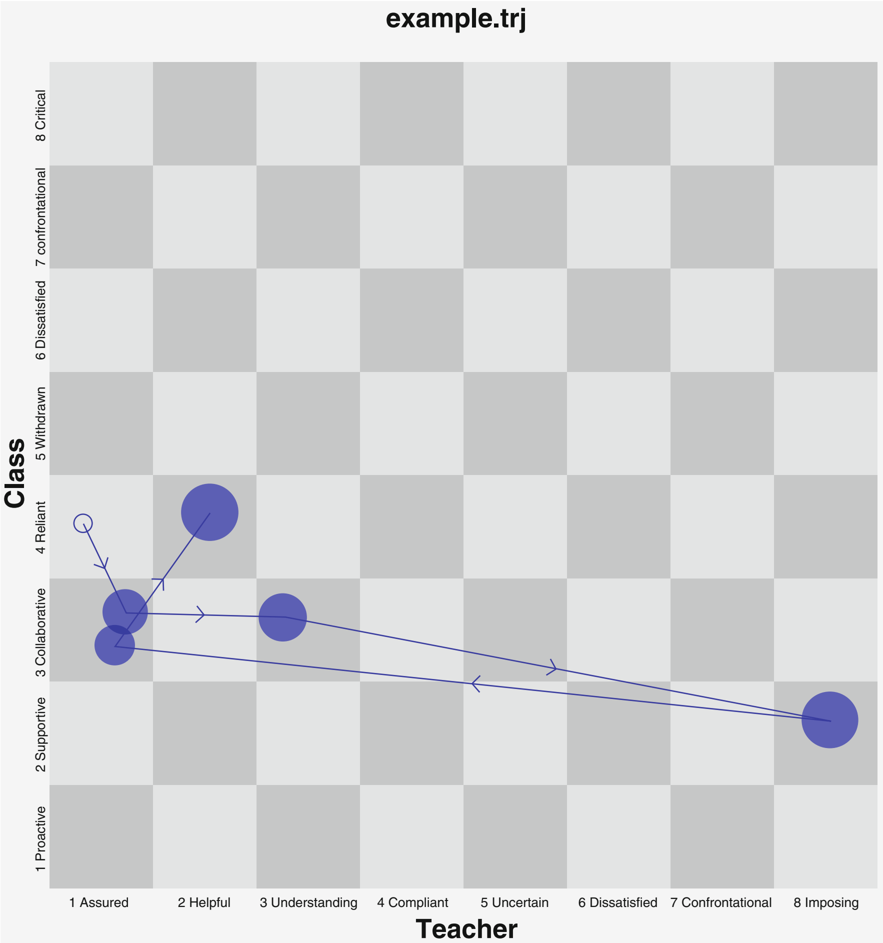

Fig. 12.2 Example of a state space grid of teacher–class interaction. *Note*. The horizontal axis shows the teacher's behavior and the vertical axis the class's behavior. The *arrowed line* represents the change in behavior in the interaction over the course of a few minutes (i.e., the interaction trajectory), and the thickness of the nodes indicates the duration of each interaction state. Note that the position of a node in a cell is arbitrary. The *opaque node* marks the start of the interaction

first studies (Pennings, Van Tartwijk, Vermunt, & Brekelmans, 2012) we combined eight categories of interpersonal teacher behavior (i.e., the categories reflect the octants of the IPC-T) with four categories of student behavioral engagement (i.e., passive/active on-task or off-task behavior; see Illustration 4).

In several studies (i.e., Mainhard et al., 2012; Pennings et al., 2015; Pennings, Van Tartwijk et al., 2014) the state space of teacher and class behavior is comprised of all the possible joint states of agency and communion that teachers and students might adopt in interaction. Thus, in these studies every cell in the SSG represents a specific *dyadic* state, that is a typical combination of the interpersonal behaviors the teacher and the class show at a certain moment during the lesson. Every time the behavior of the teacher or the class changes a new event occurs and a new point is plotted in a SSG cell (i.e., the dyadic state changes), this is often referred to as online or real-time coding. In this way all changes in level of agency/communion in both teacher and student behavior are visualized as a change in the interaction trajectory within the SSG.

In order to explain how a SSG is used as a visual representation of a given interaction an example SSG is included in Fig. 12.2. Note that this SSG represents a fictional trajectory of classroom interaction.

The state space in Fig. 12.2 consists of an 8×8 grid. Teacher behavior is displayed on the x-axis and student behavior on the y-axis. The eight categories correspond to the octants of the IPC-T and IPC-S. Therefore each cell in the grid represents the intersection of interpersonal teacher behavior and interpersonal student behavior observed in the students. For the analysis it is arbitrary on which axis whose behavior is displayed.

When a particular combination of behavior, let us say teacher assured behavior and student reliant behavior, is observed at a given point in time a so-called *node* is drawn in the corresponding cell. Teacher assured behavior corresponds to the first cell on the x-axis, thus $x1$, and student reliant behavior corresponds to fourth cell on the y-axis, the $y4$. To refer to a specific cell we follow xy convention, and thus this cell is called 14.

The start of the interaction can be marked with a hollow node. In this example the interaction starts in cell 14. Let us say that this state reflects a lecturing situation where students listen rather quietly to what the teacher says. Then, some students start to chat unvoiced with each other, and the level of agency in their behavior increases; the interaction trajectory thus moves to cell 13. Next the teacher may notice that some students chat, but lets the students talk for a while and thus the level of agency in the teacher's behavior decreases and the trajectory moves to cell 33. The students' talk becomes louder and more engaged, and after a while the teacher restricts the students, that is, the teacher's level of agency increases and the level of communion decreases and thus the trajectory moves to cell 82. In this hypothetical scenario, the students react to the teachers' imposing behavior and their behavior becomes more collaborative and eventually reliant again and at the same time the teacher becomes assured and eventually helpful, and the teacher resumes her lecture in an assured manner. Thus from cell 82 the trajectory moves to cell 13 and eventually to cell 24. To visualize how the observed interactional trajectory changes chronologically in time, it is possible to add a line that connects the nodes in the SSG. For the purpose of clarity we also added arrows to the line in this example SSG. The combination of these nodes and lines is called the interaction trajectory.

It is important to note that the interpretation of the observed behaviors in each cell entirely depends on the observational system used. In most of our own studies the unit of analysis represented in the SSGs is the dyadic behavior of teachers and students in moment-to-moment interactions. To study patterns in these interactions Granic and Hollenstein (2003) recommend looking at the content and structure of interaction.

Content of Interaction

The locations of attractors (a single cell or several adjacent cells) provide information about the *content* of the micro-level interactions as they indicate *what* states occur most frequently (Granic & Hollenstein, 2003; Hollenstein & Lewis, 2006), for example mutually friendly behavior. As specific interactional patterns between teacher and students become apparent (i.e., attractors emerge), the macro-level classroom social climate may become more constrained and defined. For example, a poorly organized classroom lesson might evoke distraction and chatting amongst students, which in turn may lead to dissatisfied teacher behavior; the more often lessons are poorly organized, the more easily students may become distracted, and the more easily aversive teacher behavior may be triggered, i.e., a negative interaction attractor develops (see for an example from teacher practice Créton, Wubbels, & Hooymayers, 1989). Moreover, an attractor may become stronger through *feedback loops* and *circular causality* between those negative interactions on the micro-level time-scale and the poor classroom social climate on the macro-level time-scale. Such processes could explain why in classrooms with less positive social climates even minor student misbehavior may trigger repressive teacher reactions with a high intensity (Créton et al., 1989). On the other hand, a teacher that introduces project based work in class for the first time may struggle to structure and support student activities. Students however may get engaged by this kind of work and may be more responsive to the teacher's efforts the next time project based assignments are used. An, in interpersonal terms, positive (i.e., mutually warm or communal) interaction attractor emerges. In more positive classrooms, corrections with a low intensity may be sufficient to return students' attention to class related activities (Wubbels et al., 2006). Indeed, such processes of stabilization seem to occur within only one or just a few lessons (Mainhard, Brekelmans, den Brok, & Wubbels, 2011).

Based on how long interactional behavior is located in a particular cell or cell region attractors and their strength can be identified. Hollenstein (2013) explains several methods to identify attractors in detail; some methods are more rigorous than others. It is for example possible to select the cell or cells with (a) the highest mean durations, (b) the highest total duration, or (c) the highest number of visits as attractors (Hollenstein, 2013). It is also possible that, based on theory, the researcher has defined an attractor or attractor region beforehand. One may then calculate a measure called *perseverance*, the mean duration that interaction remains in that specific state. A higher value for a cell represents a stronger perseverance. It is also possible to calculate perseverance for a specified area in the SSG, which is defined by the researcher (see Illustration 1). Yet a more empirical procedure to identify attractors is the *Winnowing procedure* (Lewis, Lamey, & Douglas, 1999). Using this method, the cell or cell region with the highest probability of being an attractor is identified based on a heterogeneity criterion. This procedure iteratively (step-by-step) eliminates the cells with the lowest durations (i.e., perseverance). Then a heterogeneity score is calculated using the following formula:

$$\text{Heterogeneity}_j = \frac{\sum \left(\text{Observed}_i - \text{Expected}_j\right)^2 / \text{Expected}_j}{\#\text{ of Cells}_j}$$

where i represents the specific cell targeted in iteration j. The observed value is the duration that the interaction trajectory resided in the target cell. The expected value in each cell is calculated by the total duration of the observed interaction divided by the number of cells included in the iteration.

The heterogeneity scores corresponding to each cell are quantified as a proportion of the heterogeneity score in the first iteration by dividing heterogeneity$_j$ by heterogeneity$_\mathrm{i}$. The value after the largest drop in proportions (i.e., Lewis et al., 1999 defined large as $\geq$50 %) indicates that the target cell in that iteration may be regarded as an attractor cell (Hollenstein, 2013). If multiple adjacent cells are turn out to be attractors, this can be referred to as an attractor region. Please refer to Hollenstein (2013) who describes this procedure in a comprehensible and straightforward way.

Thus, with SSGs it is possible to identify attractors by tracking how long interactions remain in some states but not others or how quickly interaction returns to or stabilizes in particular states (Granic & Hollenstein, 2003). However, an interaction often does not remain in only one state, even though that one state might be an attractor. It is therefore also possible to study changes from state to state, how often these occur and how predictable these state-to-state changes are. These changes are what Granic and Hollenstein (2003) refer to as *structure*.

Structure of Interaction

An interaction trajectory may remain in one or a few states for a large part of the time, which would indicate a stable or inflexible system. On the other hand, if the dyadic trajectory includes many different states and there are a lot of state-to-state changes, that indicates a more chaotic or flexible system (Granic & Hollenstein, 2003). The terms someone chooses to describe the degree of variability (flexible versus chaotic) ultimately depends on external criteria. Because it has been found that in Mother-child interactions variability is positively associated with the child's social adjustment later, Granic and Hollenstein (2003) used the more positive term "flexible." In the classroom, however, higher variability in teacher behavior or teacher–student interactions is associated with less desirable classroom social climates (i.e., in terms of learning outcomes), and therefore, Mainhard et al. (2012) used the term "chaotic" rather than flexible to describe highly variable systems.

Gridware (Hollenstein, 2013; Lamey et al., 2004) provides several so-called whole-grid measures to study variability or the structure of interaction. In our own research (Claessens et al., 2014; Mainhard et al., 2012; Pennings, Brekelmans et al., 2014; Pennings, Van Tartwijk et al., 2014) we have used the following grid measures: (1) the number of uniquely visited cells (i.e., cell range), (2) total cell transitions (i.e., number of visits or state-to-state changes), (3) the average duration per cell, (4) the average duration per visit, (5) dispersion, and (6) visit entropy. Before turning to a more detailed description of our own work, we first provide a more general explanation of these whole-grid measures. All these measures are related as they all tap the structure of a trajectory, but each concerns a specific aspect of how interaction moves across the state space.

The two measures *number of events* and *number of visits* may seem similar, but can yield very different figures. The *number of events* corresponds to the number of nodes in the SSG whereas the *number of visits* is the number of nodes transitioning to a new cell. In our example in Fig. 12.2 the number of events and the number of visits are both 6. In some studies it is possible that there is a change in behavior observed and is counted as another event, but that event remains in the same cell. This is completely dependent on the observation method and scheme used.

The *number of unique cells visited* (i.e., cell range) is the number of unique behavioral states that occur in an interaction trajectory. The example SSG provided in Fig. 12.2 consists of 64 cells (i.e., based on the octants in the IPC-T and IPC-S) and the interaction moved between only 5 out of these 64 cells, then the number of unique cells visited is 5. Of course it is likely that some cells are visited multiple times by an interaction-trajectory, in the example SSG there was one cell that was visited twice. A higher value is one indicator of more variability in the interaction.

Total cell transitions (*TCT*) is the number of movements between cells. *TCT* is calculated as the *number of visits*—1 (i.e., the first visit is not counted as a transition). A lower value indicates less frequent changes of system states, and therefore less variability. *TCT* may be high while the number of unique visited cells is low. In our example in Fig. 12.2 *TCT* is 5 (6 cells—1).

The *average duration of visits* is the duration of the observed interaction trajectory divided by the *number of visits*. The *average duration of visits* indicates the overall variability of behavior, which Hollenstein (2013) calls “the overall stuckness or rigidity of the trajectory” (p. 72). When the interaction trajectory during the observed period remains in one specific cell (i.e., the *number of visits* is low), the average duration of visits is extremely high (i.e., the average duration equals the total duration of the interaction). When the interaction trajectory continuously switches from one cell to another (i.e., the *number of visits* is high), the average duration of visits is low.

The *Average duration per visited cell* is the duration of the observed interaction trajectory divided by the number of uniquely visited cells (rather than total visits, see above). When the interaction trajectory during the observed period remains in one specific cell (i.e., the *cell range* is low), the *average duration per visited cell* will be extremely high, then the average duration equals the total duration of the observation period. Also, note that if multiple events within that single cell occur, the *average duration per visited cell* remains the same. When the interaction trajectory continuously switches from one cell to other cells (i.e., the cell range is high), the *average duration per cell* is low.

Dispersion describes the extent to which interactional states are scattered across the state space. This measure is based on the *number of visited cells* while controlling for the proportional *average duration per cell*. It is calculated by taking the sum of the squared proportional average duration per cell across all visited cells corrected for the total number of cells and inverted (Hollenstein, 2013). Thus, dispersion is expressed in a value between 0 (no variability) and 1 (maximum variability).

Visit entropy represents the degree of predictability of an interaction trajectory. It is calculated by summarizing the conditional probabilities of cell visits (Dishion, Nelson, Winter, & Bullock, 2004; Hollenstein, 2013) in order to do so the Shannon and Weaver (1949) formula[1] for entropy was built into GridWare (Dishion et al., 2004; Hollenstein, 2013). When visit entropy is high, the system's behavior changes frequently between many cells, indicating that the pattern of interaction is unpredictable. Low visit entropy means behavior remains in only a few states, returns to the same states often, or constantly visits a few states in the same order; this indicates a highly organized and predictable pattern of interaction (Dishion et al., 2004; Hollenstein, 2013; Lunkenheimer & Dishion, 2009).

Areas of Interest and Specific Cells

In some cases researchers may study predefined grid regions, or *areas of interest*, which can be based on theory or previous studies. For example, with regard to the most desirable classroom social climate, we know that assured and helpful teacher behavior in combination with reliant and collaborative student behavior is good for learning outcomes and a positive social climate (Wubbels et al., 2006). We could therefore, define a specific area in the grid and study to what extent the teacher–class interaction trajectories visit this specific area. To study such areas of interest Gridware (Lamey et al., 2004) allows selecting specific cells or cell regions of the SSG, which makes it possible to derive several measures related to those specific cells or the cell region. These measures are called *cell or region measures* (Granic & Hollenstein, 2003). We have already explained some of these measures, such as *perseverance*, because this measure is needed to identify attractors with the winnowing procedure. Remember that p*erseverance* is the mean duration an interaction remains in a specific cell. It is however also possible to select multiple cells and to calculate the *perseverance* within that entire grid area. Another *cell* or *region* measure is the *return latency*. *Return latency* equals the time it takes before an interaction returns to a specific cell or area of interest and is an additional measure

[1] The Shannon and Weaver (1949) formula for entropy that is built into Gridware to calculate visit entropy: $\sum(P_i \times \ln(1/P_i))$. In which i is an index of each cell on the grid and P_i is the probability in cell i. Thus, for visit entropy, P_i is the number of visits to cell i divided by the total number of visits in the entire trajectory.

for the strength of an attractor. A lower *return latency* indicates a stronger attractor and a high *return latency* may indicate a repellor (opposite of attractor) or weak attractor. A return is defined as a sequence of events starting with the exit from the cell or region and ending with the return to the cell or cell region. The duration of this sequence is the *return latency*. For example, the interaction in a disorderly classroom may be mutual positive at times, but with long intermediate states including unfriendly behavior. This would result in relatively long return latencies for more positive interpersonal states (e.g., friendly behavior) or areas of interest.

Applications of SSG to Study Interpersonal Processes in Classrooms

In this section we provide some examples of how we used SSGs in our research focusing on classroom interaction. We present four illustrations of how we used SSGs and in each example we first sketch the question of the specific study and explain the global method that was followed. Across the studies different types of state spaces have been built, which are explained for each illustration separately. It is also explained which grid measures were chosen and finally, the general conclusion of each study is summarized. The illustrations discussed are all based on research in Dutch secondary education classrooms.

The illustrations we provide focus on how teacher behavior (an intra-personal process) or teacher–student interaction (a dyadic process), which occur in real-time, can be captured with SSGs. In Illustration 1, a specific area of interest, which was predefined to reflect more favorable teacher–student interaction, was used to facilitate the comparison of two classrooms taught by teachers with rather distinct classroom social climates. In this study three consecutive lessons were included. In Illustration 2 SSGs are used to plot *intra*personal processes, here the behavior of the teacher, in terms of agency and communion is examined to illustrate that it is also possible to study real-time processes within persons. In Illustration 3 we studied teacher–student interactions of 35 teachers during the lesson start and studied how content and structure of those interactions are related to the general classroom social climate. In Illustration 4 we looked at interpersonal teacher behavior and student behavioral engagement in four classrooms. In this study we also predefined an area of interest and sampled three classroom situations (i.e., lesson start, a positive interaction episode, and a negative episode).

Before we turn to our own research, we would like to emphasize that there are other examples of educational studies that used SSG analysis. Vauras et al. (2013) conceptualized teacher–student interaction in terms of scaffolding (Van de Pol, Volman, & Beishuizen, 2010), that is, from a cognitive perspective. Their question was whether and how teachers offer opportunities to learn in class, and how (e.g., whether or not) students respond to these opportunities (i.e., student up-take). Turner et al. (2014) chose yet another way to employ SSGs. First, they combined observations of teacher motivational support with student engagement in a single grid, that is, they combined "cause and effect" in one trajectory. They crafted grids that summarized interactions in three activity settings as the unit of analysis across 12 lessons per year (for 3 years), instead of micro-level second-to-second interactions. All these studies underpin the versatility of SSG analysis.

Illustration 1: Favorable Interaction States

The goal of this study (Mainhard et al., 2012) was to explore the value of SSG for research on the quality of the classroom social climate by comparing classroom interaction in a classroom of a teacher characterized as drudging (Teacher A), that is with according to students a considerably lower agency and communion, with a class characterized by high levels of both teacher agency and communion (i.e., a positive climate conducive to learning; Teacher B). A cluster of cells was defined as reflecting favorable states of classroom interaction to facilitate the comparison of the two classrooms (see bordered cells in Figs. 12.3 and 12.4). These interpersonally favourable states reflect what Woolfolk Hoy and Weinstein (2006) refer to as a *warm demander*.

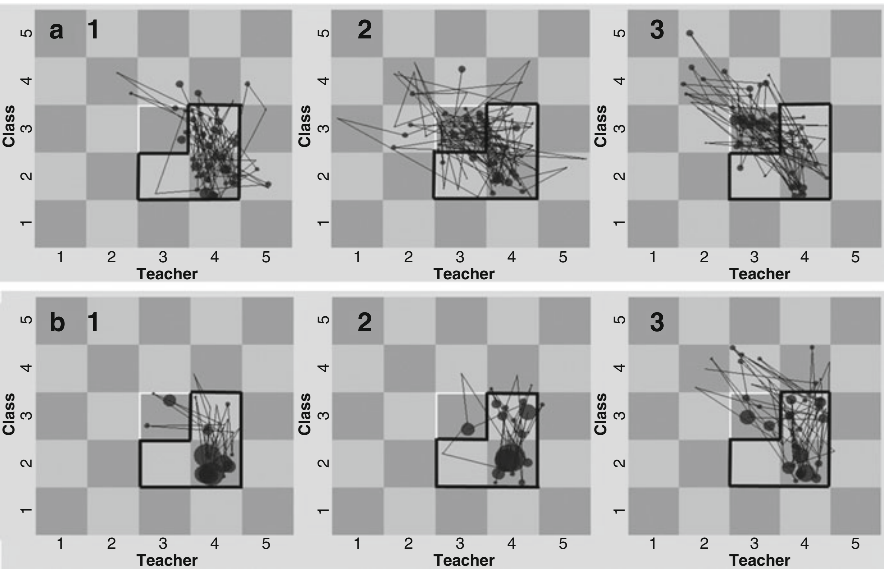

Fig. 12.3 Agency State Space Grids for the two classrooms per lesson. *Bordered cells* represent the favorable interaction area. The *upper panel* (**a**) refers to the classroom of the drudging teacher, the *lower panel* (**b**) refers to the classroom with the more positive climate

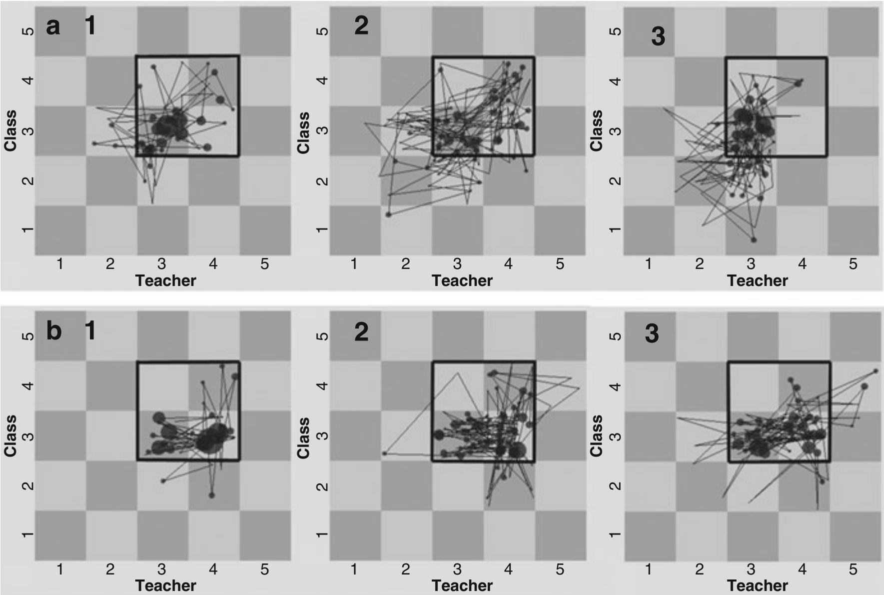

Fig. 12.4 Communion State Space Grids for the two classrooms per lesson. *Bordered cells* represent the favorable interaction area. The *upper panel* (**a**) refers to the classroom of the drudging teacher, the *lower panel* (**b**) refers to the classroom with the more positive climate

Approach

Three consecutive lessons in two different classrooms taught by two different teachers were videotaped and coded for teacher agency and communion and class agency and communion. In this study the two interpersonal dimensions were directly coded on a scale running from 1 to 5 (i.e., 1 = very low vs. 5 = very high interpersonal agency). Following an online coding procedure every time either the teacher's or the class's behavior changed a new code was added. Teacher and class were coded separately and subsequently codes were combined into SSG trajectories.

Trajectories and Grid Measures

In this study we chose to craft two 5 × 5 SSGs which represented the interpersonal teacher–classroom state space: one representing the interactional trajectory in terms of agency (i.e., combining teacher and class agency in one SSG) (Fig. 12.3) and one representing a communion trajectory (Fig. 12.4).

As working hypothesis and in order to compare the two classrooms we defined a *favorable interaction area* reflecting interactional states that we thought of being relatively more positive or constructive than other states. This area of interest was defined based on findings of previous research described by Wubbels et al. (2006).

We used the *perseverance* and *return latency* measures to further explore the areas of interest for these two classrooms' interactional trajectories. To study variability the whole-grid measures *TCT* and *dispersion* were calculated. As higher values of agency and communion are more conducive to learning, the areas we defined here encompass states including relatively high teacher and low student values on agency (i.e., cells 32, 42, and 43), and "neutral" to friendly teacher and student values for communion (i.e., cells 33, 34, 43, and 44). The more favorable areas in the SSG are represented by the bordered cells in the grids (Figs. 12.3 and 12.4). States that include the highest teacher values on agency in combination with the lowest possible student values (e.g., cells 41 or 51, a combination of obedient students and a very strict teacher) were considered as less desirable. Likewise, combinations with very high teacher and student Affiliation values (e.g., cells 54 or 55, the teacher is or tries to be "one of the crowd") were also regarded as less desirable. Note however, that occasional projections of a trajectory into less favorable areas were not deemed unwanted. On the contrary, occasional interaction in the less favorable areas might sometimes be necessary or beneficial, for example, when a teacher is restricting incidental deviant student behavior.

Findings and Conclusion

Already a first visual inspection of the interactional trajectories shows that the two classrooms differ. Interaction remained longer in a specific state (see the larger dots in the lower panel of Fig. 12.4) in classroom B with the more favorable overall climate, whereas interaction in the upper panel, representing the classroom of the drudging teacher, consists of smaller dots (short durations in a specific state) and more projections in various areas of the grid. Nonetheless, it seemed that the interaction trajectories in both classrooms were rooted within roughly comparable, central regions of the grid. Figures 12.5 and 12.6 summarize the *perseverance* and *return latency* measures for the favorable areas of the agency and communion SSGs as histograms.

For agency, cell 42 had the largest *perseverance* in both classrooms (somewhat higher teacher than student agency) as is indicated by the relatively large *perseverance* bars in Fig. 12.5.

Since a strong *perseverance* may be regarded as an indicator of an *attractor* in a system cell 42 could be regarded as an attractor for both teachers. Yet the interaction of classroom B was more strongly attracted to this specific agency state than classroom A, taking the three lessons together, *perseverance* of cell 42 was twice as large in classroom B (A = 0.35; B = 0.65). Furthermore, the *return latencies* of the cells included in the favorable agency area indicated that the interaction of classroom B was much faster to return to these favorable states than classroom A. Taking the three lessons together, the shortest *return latency* in classroom A (0.11, cell 43) was about four times longer than the shortest *return latency* of classroom B (0.03, cell 43). Thus the attraction to cell 43 was much stronger for classroom B than for classroom A.

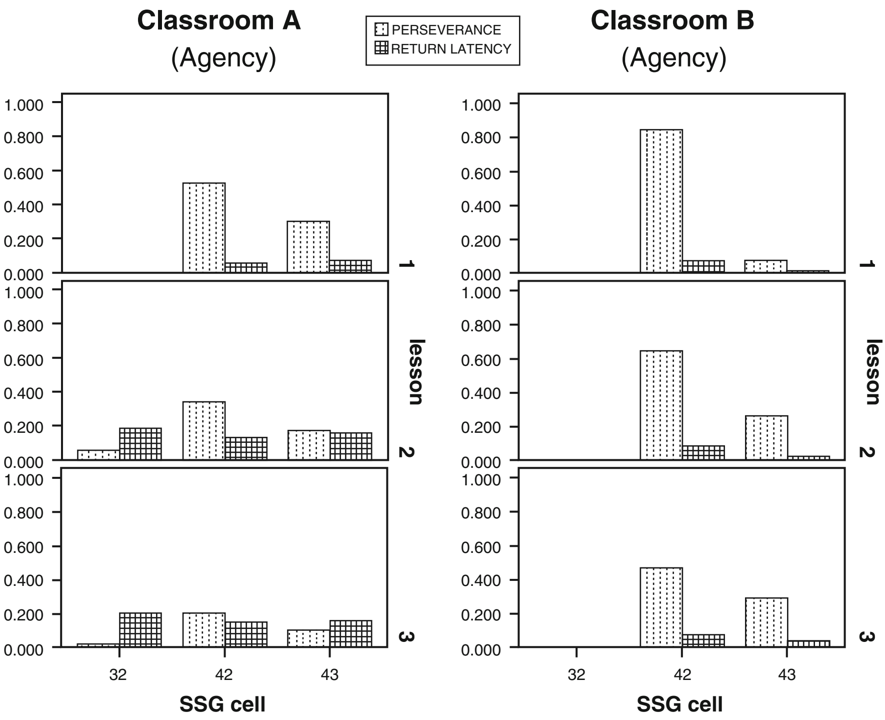

Fig. 12.5 Histograms of perseverance and return latency for agency

The state occurring most frequently in the interaction trajectories for communion of both classrooms was reciprocated "neutral" interaction (cell 33; Fig. 12.4). For example, the teacher goes through a homework assignment without much enthusiasm while students cooperate, but do not contribute spontaneously. Interestingly, in classroom A with the less desirable climate this state had the highest perseverance of all states in this study (0.63, Fig. 12.6). Also, *return latencies* of the states included in the favorable areas were markedly longer for the communion interaction trajectories of classroom A (see Fig. 12.6). In the more positive classroom B however, from lesson two on a state including warm teacher behavior showed the highest *perseverance* (cell 43, perseverance = 0.56). Thus, interaction in the more positive classroom was more strongly attracted to states including warmer interaction.

In Table 12.1, the whole-grid measures *TCT* and *dispersion* are summarized. Both the agency and communion interaction trajectories of classroom A with the less positive classroom climate were more dispersed and fluctuating than those interaction trajectories of classroom B.

The higher variability of the interaction in classroom A was most obvious in terms of the transitions between interpersonal states (i.e., *TCT*), especially for agency. Overall, it appeared that interaction in classroom B was more consistent and visited more positive interpersonal states.

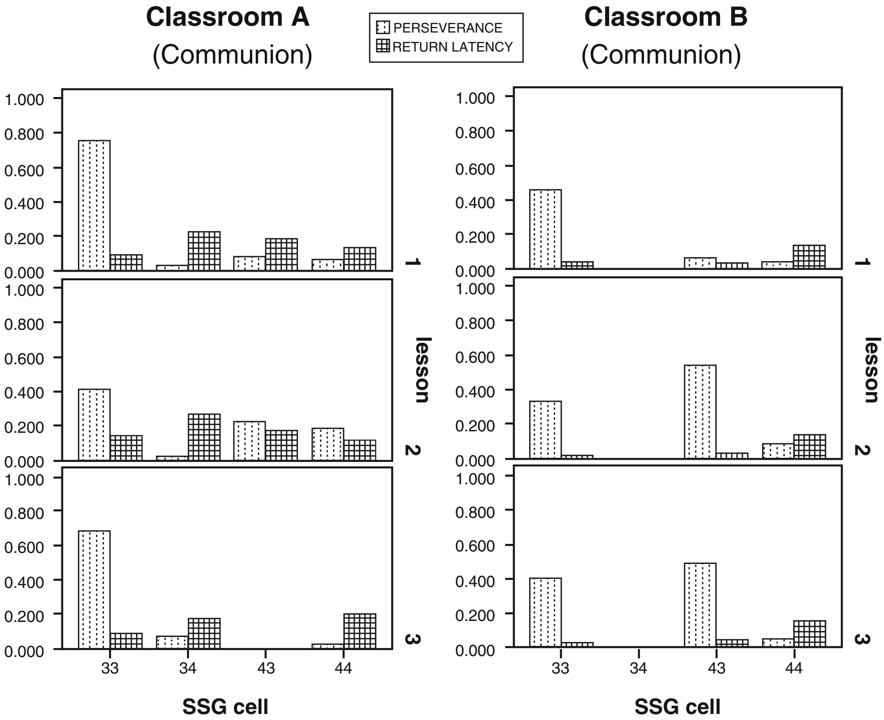

Fig. 12.6 Histograms of perseverance and return latency for communion

Table 12.1 Grid-measures for agency and communion trajectories of the three lessons

	Agency		Communion	
	Total cell transitions	*N* of visited cells (dispersion)	Total cell transitions	*N* of visited cells (dispersion)
Classroom A				
Lesson 1	1.62	10	0.97	6
Lesson 2	3.04	10	2.72	11
Lesson 3	1.88	8	1.77	10
Total	2.12	13	1.77	12
Classroom B				
Lesson 1	0.49	4	0.72	5
Lesson 2	0.63	7	1.75	7
Lesson 3	1.11	7	1.44	9
Total	0.74	9	1.30	10

Thus, although interaction in both classrooms was primarily characterized by a positive interpersonal valence, greater variation seemed to be linked to movements away from more favorable interpersonal states, and lower variability seemed to indicate more balanced interpersonal interaction and less "need" for projections into less favorable states.

Classroom A resided relatively longer in less favorable agency and communion states (43 % and 15 % of the total time respectively), while the trajectory of classroom B seemed to just shortly tap these less favorable states (13 % and 4 % of the time), returning quickly to more favorable states, which is also indicated by the smaller and less frequent dots in the B-grids outside the favorable areas (see Figs. 12.3 and 12.4).

An example of a projection into less favorable interaction areas in classroom B is a situation where the teacher was confused about his notes and tried to figure out what he wanted to do; meanwhile the students started to chat rather loudly. However, as soon the teacher had reorganized, the students were back on track immediately. Notably, the agency trajectory of classroom A projected into interpersonal states including the highest teacher values for teacher agency (i.e., cells 52, 53, and 54), which the trajectory in classroom B never did. In classroom A the teacher for example restricted students with a high intensity for relatively minor disruptions, to which the students occasionally responded indignantly.

In classroom A projections of the communion trajectories covered states representing both relatively high-, and low-reciprocated communion (e.g., cells 44 and 22, see Fig. 12.4). In contrast to classroom B, states with the highest teacher communion scores were not covered by the trajectory of classroom A at all.

To sum up, in both classrooms only one clear attractor was found, rather than multiple stable states of interaction. This might reflect the commonly assumed social hierarchy in the classroom, with legitimate teacher power in combination with a basically non-oppositional attitude of both teacher and students towards each other. However, although in both classrooms the same agency attractor existed, it was stronger in the classroom with the more positive social climate. Thus, the differences between the two classrooms were especially apparent in the strength of the attractor and not in its position in the state space. The findings suggest further that variability in interaction is a potent variable in explaining differences in the quality of classroom social climates. In the more negative classroom not only four times more time was spent in less favorable interpersonal states, interaction also shifted far more often between different states (Table 12.1).

Illustration 2: Intrapersonal SSG of Teacher Behavior

In another study (Pennings, Brekelmans et al., 2014) we used a different application of SSGs. We did not study the content and structure of dyadic interactions but instead used SSGs to build intrapersonal trajectories depicting how teacher behavior differs for teachers with different general types of classroom climates.

Approach

To observe sequences of real-time or micro-level teacher behavior we used Sadler's Computer Joystick method (Box 12.1; Sadler, Ethier, Gunn, Duong, & Woody, 2009). This method enabled us to observe teacher behavior (1) as a blend of agency and communion and (2) continuously over time (behavior is coded every half second).

As in Illustration 1, we defined the macro-level *classroom social climate* as aggregated student perceptions of interpersonal teacher behavior, tapped with the Questionnaire on Teacher Interaction (QTI), also in terms of agency and communion. These aggregated perceptions represent relatively stable and predictable patterns of teacher behavior resulting from frequent interpersonal behavior exchanges between a teacher and his or her students (cf., Mainhard et al., 2011).

Assuming that the quality of the classroom social climate is based on student perceptions of actual interactions in class, our goal was to study the correspondence between the location of attractors in teacher behavior and the degree of agency and communion characterizing the general quality of the social climate. Thus, we expected that differences in general climate would be reflected in differences in the content of teachers' micro-level interpersonal behavior as indicated by (a) differences in the strength or existence of attractors and/or by (b) the location of attractors in the SSG. We formulated specific criteria to assess the correspondence between micro-level behavior (i.e., location of attractors) and macro-level classroom social climate. First, we expected that teachers with a classroom social climate characterized by high levels of agency and communion (teacher A and B; Fig. 12.7) would have attractors in the upper right part of the SSG: e.g., frequent occurrences of laughing, helping, and explaining in a friendly manner. Second, we expected that teachers with a classroom social climate characterized by low levels of agency and high levels of communion (teacher C and D) would have attractors in the lower right part of the SSG: frequent occurrences of for example tolerant or understanding behavior. Third, we expected that teachers with a classroom social climate characterized by low levels of agency and communion (teacher E and F) would have attractors in the lower left part of the SSG we used in this study: frequent occurrences of aggressive, hesitating, and uncertain behavior. Finally, we expected teachers with a classroom social climate characterized by high levels of agency and low levels of communion (Teacher G and H) to have attractors in the upper left part of the SSG, reflecting frequent occurrences of, for example, sarcasm or confrontational and enforcing teacher behavior.

Our expectations concerning structure were based on previous findings about which social climate types are most productive in terms of student or teacher outcomes and classroom atmosphere (see for an elaborate overview Wubbels et al., 2006) and also findings from our previous SSG case studies (Mainhard et al., 2012; Pennings, Van Tartwijk et al., 2014). We expected that teachers with climates characterized by lower levels of agency and communion would have higher variability (more visited cells, lower mean durations of visits) and less predictable trajectories (higher entropy) in real-time interpersonal teacher behavior

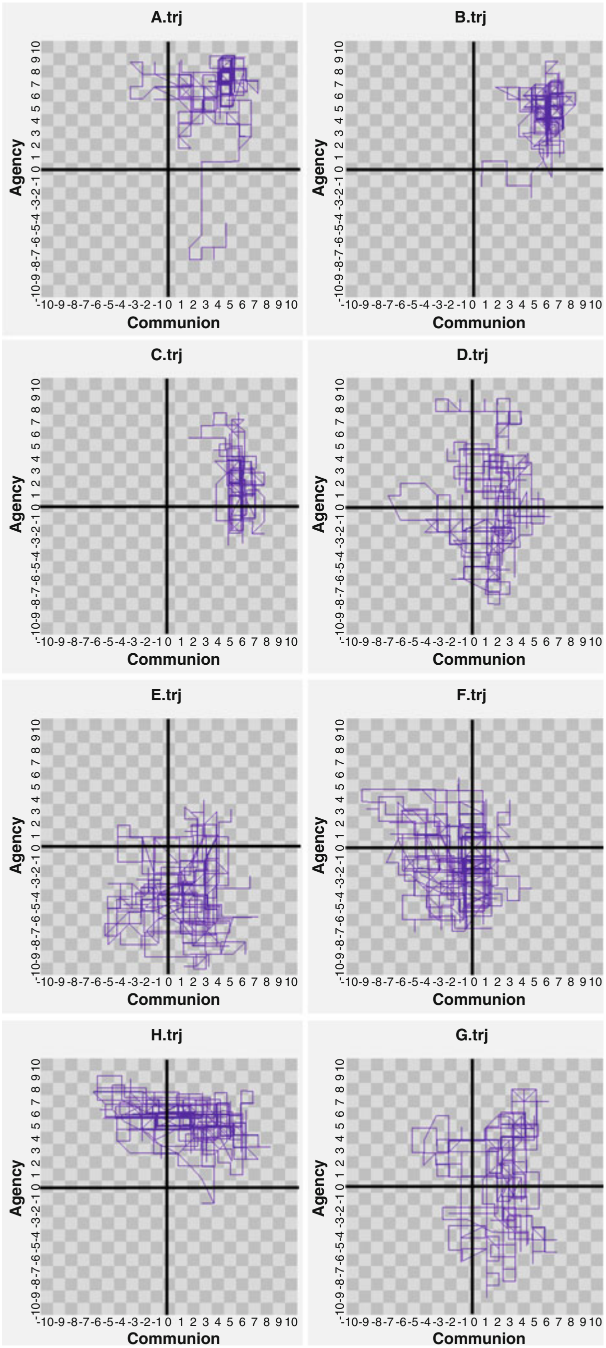

Fig. 12.7 SSGs for eight teacher's (A–H) interpersonal behavior. Agency is represented on the *y*-axis and Communion on the *x*-axis. The *black lines* are included to illustrate how the SSGs represent the IPC as an interpersonal grid

than teachers with climates characterized by higher levels of agency and communion. More specifically we expected a negative association between agency and communion with the number of visited cells and with visit entropy, and a positive association between agency and communion and the mean duration per cell visit.

Trajectories and Grid Measures

To analyze the content and structure of micro-level interpersonal teacher behavior, a combined agency and communion SSG was built with Gridware (Lamey et al., 2004). The with the Joystick coded behavior was represented with 20 categories and 1 category included just the 0 value, resulting in 21 categories per dimension ranging from -10 = *Very low Agency/Communion* (0 = *Neutral*) to 10 = *Very high Agency/Communion*. In this study we used what Hollenstein (2013) refers to as the more simple criteria for the identification of attractors. We selected a cell or cluster of cells if (a) the *average duration per cell* was longer than 100 s (i.e., based on 30 min coding and 441 cells) and (b) the *number of visits per cell* was larger than two times the *average number of visits* of all eight teachers.

This study used three whole grid measures concerning the structure of teacher behavior: the *number of unique cells visited*, the *average duration per cell*, and *visit entropy*.

We defined our criterion for differences in structure in real-time interpersonal behavior between teachers as larger than 1 SD difference regarding number of visited cells and mean duration per visit. For visit entropy, we followed the procedure used by Dishion et al. (2004) using boxplots to identify teachers with highly predictable versus highly unpredictable behaviors. Visit entropy values within the first quartile (i.e., 25th percentile) were regarded as low, thus highly predictable. Visit entropy values in the second quartile (i.e., the median split) were regarded as average predictability, and visit entropy values in the third quartile (i.e., 75th percentiles) were regarded as highly unpredictable.

For structure in real-time interpersonal behavior, we explored whether differences between teachers were related to the level of agency and communion of their macro-level classroom social climate. In order to do so we calculated Spearman's rank-order correlations between the grid measures and the continuous scores for agency and communion characterizing the macro-level classroom social climate.

Findings and Conclusion

The SSGs representing the interpersonal behavior of the eight teachers are presented in Fig. 12.7. It can be seen that the trajectories depicted are somewhat different from the example in Fig. 12.2 and the SSGs in Illustration 1. GridWare provides two options for visualization, the random and the diagonal layout of SSGs. The SSGs in Figs. 12.2, 12.3, and 12.4 are examples of the random layout, but for the SSGs in Fig. 12.7 (and also Fig. 12.8) we used the diagonal layout. Also, the

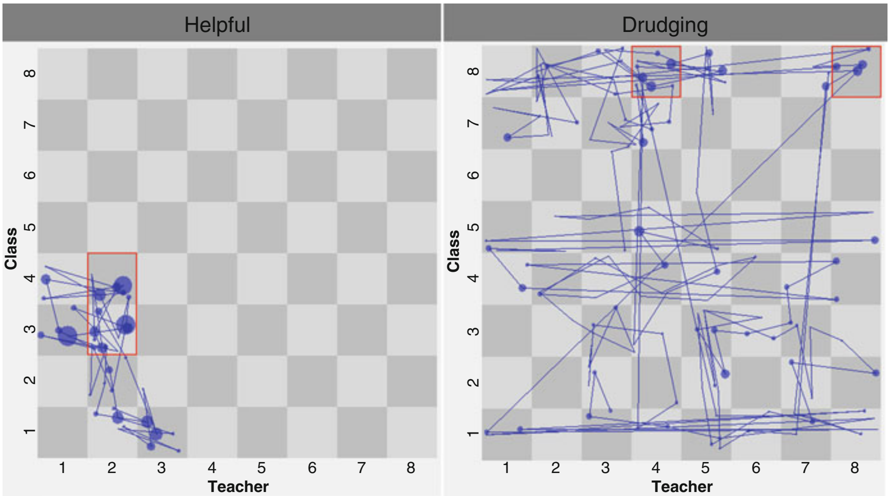

Fig. 12.8 SSG for a helpful and a drudging teacher in this study. The number 1–8 correspond to the octants of the IPC-T/IPC-S 1 is the *upper right* octant (i.e., assured/pro-active) and 8 is the *upper left* octant (i.e., imposing/critical). The *red lines* mark the identified attractor cells

nodes are not really visible in these SSGs, the reason for this apparent lack of nodes is that the interaction trajectory is 30 min long and the number of possible cells is very high, transitions to adjacent cells occur more frequently and as a result the nodes are very small and almost invisible.

Visual inspection of the different SSGs in Fig. 12.7 already shows some obvious differences between the selected teachers in terms of their real-time behavior trajectories. The trajectory of Teacher B for example was characterized by almost entirely highly communal and agentic interpersonal states. Compared to Teacher E's behavior, who seemed very submissive and frequently switching between unfriendly and friendly behavior, the behavior of Teacher B seemed much more predictable. The grid measures as presented in Table 12.2 confirmed this (i.e., 43 versus 112 visited cells, an average of 42.57 versus 16.26 s visit duration, and the lowest versus one of the highest values for visit entropy).

The identified attractor cell(s) with corresponding values for *perseverance* and number of visits are presented on the right in Table 12.2. The level of agency in teacher behavior corresponds to the *y*-part and the level of communion in teacher behavior corresponds to the *x*-part of the cell(s).

Figure 12.7 and Table 12.2 indicate that the majority of the attractors in teacher behavior trajectories were found in the SSG area corresponding to the levels of general agency and communion that characterized the macro-level social climate of that teacher. Note that for teachers D, E, and G, no specific attractors could be identified.

Most results for structure were in line with our hypothesis, yet for teacher A, D and H the findings were only partly in line with the hypotheses. For example, we

Table 12.2 Results for content and structure in interpersonal teacher behavior

	Whole grid measures			Attractor cells		
	Number of visited cells	Average cell duration	Visit entropy	Cells	Perseverance	Number of visits
A	75	24.98	3.54	48	107.0	41
				74	166.5	48
				95	110.5	18
				58	209.0	43
				57	166.5	55
B	43	42.57	3.07	55	143.50	28
				66	172.5	46
				65	199.5	56
				64	212.5	62
C	44	41.56	3.29	5−1	101.0	15
				63	115.5	34
				62	124.5	35
				6−1	360.5	20
D	103	17.51	4.35	NAS	–	–
E	112	16.26	4.39	NAS	–	–
F	114	15.82	4.23	−1−2	108.5	41
G	113	15.62	4.46	NAS	–	–
H	88	20.34	3.99	46	102.0	26

Note. *NAS* no attractor specified

expected that the number of visited cells would be lower for teacher A and higher for teacher H, both numbers of visited cells were indeed lower than the number of visited cells for teacher D, E, F, and G. Yet based on the desirableness of the classroom social climates we expected that the number of visited cells for teacher A would be more similar to teacher B, and for teacher H we expected that the number of visited cells would be more similar to teacher C. The result showed that teacher A and H were more similar and teacher B and C were more similar in the number of visited cells.

We calculated Spearman's rank correlations to explore whether differences between teachers were related to the level of agency and communion characterizing the teachers' macro-level social climate. For *Number of visits* and *Visit entropy*, we found negative correlations with agency. Respectively Spearman's Rho were −0.36 and −0.05. For *Duration in visited cells* we found a positive correlation, Spearman's Rho was 0.33. However, none of the correlations were significant.

For communion we also found very high and significant correlations with all three whole grid measures. For *Number of visits* and *Visit entropy*, we found negative correlations with communion. Respectively Spearman's Rho were −0.90 and −0.73, and for *duration in visited cells* we found a positive correlation, Spearman's Rho was 0.86. Thus, we could only confirm our hypothesis about the relation between macro-level communion and micro-level structure of interpersonal teacher behavior.

Overall, using SSGs in this study allowed us to visualize and study how the content and structure of interpersonal teacher behavior and to show how they differentiate between teachers with different types of classroom social climates.

Illustration 3: Interpersonal SSG of Teacher–Class Interaction

In a recent study (Pennings et al., 2015) we studied the content, structure, and degree of complementarity in teacher–student interactions during the lesson start of 35 teachers with different classroom social climates. In order to do so we included observations of both the teacher's and the students' (coded as whole class) behavior.

Approach

Our observational approach was similar to our approach in Illustration 2. Again we used the joystick method (Box 12.1) to observe interpersonal teacher behavior, yet for this study we also included observations of class interpersonal behavior. Since we included 35 teacher–class dyads we were able to use some additional statistical analyses to study differences between teachers.

Also, we introduce a new concept in this study, the concept of *complementarity* which is rooted in interpersonal theory. The principle of complementarity defines how the interpersonal behaviors of both participants fit together, mutually adjust to each other, and how this dynamically changes during interactions. Complementarity in terms of agency is called *reciprocity*, and denotes the tendency to pull an interaction partner towards *oppositeness*. Complementarity in terms of communion is called *correspondence*, and denotes the tendency to pull a partner in interaction towards *sameness* (Sadler et al., 2009).

Trajectories and Grid Measures

The SSGs in this illustration represent dyadic behavior states (combinations of teacher and class behavior) and are therefore comparable to the SSGs presented in Illustration 1. The difference is that in the present illustration we combined the level of agency and communion in teacher and class behavior into one SSG. In order to do so we recoded (following a procedure described by Gurtman, 2011) the joystick data into eight categories corresponding to the octants of the IPC-T and IPC-S and collapsed those on the *x*-axis (teacher) and *y*-axis (class) of the SSG. This yielded SSGs similar to the example SSG provided in Fig. 12.2.

In this study we used the winnowing procedure (Hollenstein, 2013; Lewis et al., 1999) to empirically derive attractors to study the content of the interactions. We expected that the octant representing the teacher behavior component of the attractor cell(s) (micro-level data) would correspond to the octant that characterized the classroom social climate (macro-level data). Given the complementarity principle we expected that octants corresponding to the class behavior component of the attractor cell(s) would represent opposite behavior for teacher agency and similar behavior for teacher communion. Thus, if the attractor cell corresponds to the first octant for teacher behavior (assured; i.e., which is characterized by high levels of agency and moderately high levels of communion), we expected that the octant for class behavior would be octant 4 (reliant; i.e., which is characterized by low levels of agency and moderately high levels of communion).

For structure we formulated the following expectations based on our previous studies; we expected that interactions of teachers with desirable classroom social climates would be less variable than interactions of teachers with less desirable classroom social climates. To study the structure of the teacher–class interactions we used the g*rid range*, the *duration per cell,* the *number of transitions*, *duration per visit, dispersion,* and *visit entropy*.

Findings and Conclusion

The SSGs for the teacher–class interaction of two of the 35 teachers are presented in Fig. 12.8. One for a assured teacher and one for a drudging teacher. These two SSGs show differences between the interactions trajectories of both teachers.

In Table 12.3 the results of the winnowing procedure are provided. For most teachers one or two attractor cells are identified. The perseverance values show that for those teachers the attractors are strong. For some teachers more than two attractors are identified, for these teachers the attractors are weaker. Also for both the helpful (16) and drudging teacher (27), of whom the SSGs are provided in Fig. 12.8, two attractor cells are identified. However, the location and strength of these attractors is quite different. For the helpful teacher the attractor shows that the teacher mainly shows helpful behavior and the students mainly are reliant or collaborative (i.e., cell 24 and 23). The two cells are adjacent to each other and together form an attractor region. For the drudging teacher two quite different cells are identified as attractors, the drudging teacher shows imposing and compliant behavior and the students mainly show critical behavior (i.e., cell 48 and 88). That the strength of the drudging teacher's attractors is weaker than those of the helpful teacher, which can also be seen in the visualizations of the SSG as well as in the grid measures that represent structure, provided in the next section (Table 12.3).

From Table 12.3 it can also be seen that 32 out of 35 teachers have at least one attractor where teacher behavior is characterized by assured or helpful behavior. Of these 32 teachers 10 teachers have attractors with students also showing more agentic (i.e., critical, proactive or supportive) behavior. Most of these teachers have a drudging classroom social climate according to their students. The other three teachers have attractors of compliant or imposing behavior, it can also be seen that students of these teachers are mainly critical, confrontational or dissatisfied. These three teachers all have drudging classroom social climates.

Table 12.3 Attractor cells (and regions) with corresponding total duration (perseverance)

Teacher ID	Attractor 1 (perseverance)	Attractor 2 (perseverance)	Attractor 3 (perseverance)	Attractor 4 (perseverance)	Attractor 5 (perseverance)	Attractor regions (perseverance)
1	34 (140.50)	24 (137.50)	48 (86.50)	–	–	34/24 (278.00)
2	24 (128.50)	23 (100.00)	–	–	–	24/23 (228.50)
3	36 (124.00)	37 (91.00)	–	–	–	36/37 (215.00)
4	14 (79.00)	37 (59.00)	17 (54.50)	–	–	
5	13 (107.00)	32 (101.50)	–	–	–	
6	14 (164.50)	38 (116.00)	–	–	–	
7	22 (179.50)	32 (166.50)	–	–	–	22/32 (346.00)
8	13 (212.50)	12 (194.50)	28 (99.00)	–	–	13/12 (407.00)
9	24 (260.50)	14 (138.50)	–	–	–	24/14 (399.00)
10	24 (235.50)	–	–	–	–	
11	14 (257.50)	–	–	–	–	
12	14 (142.50)	13 (83.00)	–	–	–	14/13 (225.50)
13	14 (166.50)	11 (107.00)	21 (94.00)	–	–	11/21 (201.00)
14	14 (324.00)	–	–	–	–	
15	22 (318.50)	–	–	–	–	
16	23 (174.00)	24 (130.00)	–	–	–	23/24 (304.00)
17	24 (244.50)	–	–	–	–	
18	23 (298.50)	–	–	–	–	
19	14 (289.00)	–	–	–	–	
20	21 (92.00)	13 (88.50)	–	–	–	
21	14 (235.50)	–	–	–	–	
22	24 (214.50)	–	–	–	–	
23	48 (233.00)	–	–	–	–	
24	14 (141.50)	13 (105.50)	84 (78.00)	–	–	14/13 (247.00)
25	22 (121.50)	21 (105.00)	28 (84.50)	–	–	22/21/28 (311.00)

26	13 (238.00)	14 (232.50)	–	–	–	13/14 (470.50)
27	48 (80.50)	88 (56.50)	–	–	–	
28	13 (226.00)	23 (132.50)	–	–	–	13/23 (358.50)
29	24 (161.00)	22 (124.00)	–	–	–	
30	24 (160.00)	21 (126.00)	–	–	–	
31	22 (117.00)	21 (113.00)	23 (88.50)	28 (69.50)	–	22/21/23/28 (388.00)
32	14 (113.00)	15 (83.50)	–	–	–	14/15 (196.50)
33	14 (301.00)	24 (229.50)	–	–	–	14/24 (530.50)
34	14 (102.00)	38 (100.00)	24 (99.50)	13 (54.50)	28 (53.00)	14/24/13 (256.00) 38/28 (153.00)
35	14 (343.00)	–	–	–	–	

Note. Perseverance is the total duration in the attractor cell, provided in seconds. The attractor cells are named by first giving the octant number for the teacher and then for the student. Thus teacher octant 1 and student octant 4 give results in cell number 14

Table 12.4 Means and standard deviations for the grid measures

Style	Grid range	Number of transitions	Dispersion	Visit entropy	Duration per cell	Duration per visit
1	12.50 (3.27)	43.83 (7.73)	0.73 (0.10)	2.16 (0.46)	50.11 (14.48)	13.77 (2.53)
2	12.84 (4.02)	43.23 (14.46)	0.80 (0.08)	2.26 (0.32)	49.80 (14.75)	14.13 (4.07)
3	15.33 (5.51)	61.00 (39.15)	0.78 (0.10)	2.40 (0.30)	42.95 (19.36)	143.80 (12.71)
4	16.50 (0.71)	54.50 (7.78)	0.89 (0.04)	2.59 (0.08)	35.64 (1.53)	10.89 (1.55)
8	18.33 (3.27)	77.00 (30.80)	0.90 (0.01)	2.66 (0.06)	32.19 (2.62)	8.76 (4.29)
9	24.13 (12.56)	76.50 (32.89)	0.87 (0.08)	2.83 (0.52)	30.40 (14.35)	8.94 (3.67)
Total	16.26 (7.98)	56.74 (25.75)	0.82 (0.09)	2.44 (0.44)	42.51 (15.80)	12.30 (5.13)

Note. Standard deviations are presented within brackets

In Table 12.4 the overall means and standard deviations of the grid measures of the 35 teachers and the means and standard deviations per classroom social climate are provided. We concluded that there are large variations between the teachers for most grid measures.

In this study we carried out quantitative statistical analyses to study differences in teachers' grid measures. Six separate ANOVAs were carried out to compare grid measures between teachers with different classroom social climates. The results showed that all grid measures except *duration per visit* [$F(5, 29) = 1.75, p = 0.156$] showed significant differences between teachers with different classroom social climates. Post hoc tests showed that: (1) For *Grid range* [$F(5, 29) = 3.06, p = 0.025$] teachers with a drudging classroom social climate visited significantly more cells than teachers with assured ($p < 0.01$) and helpful ($p < 0.01$) classroom social climates. (2) The *Number of transitions* [$F(5, 29) = 2.69$, $p = 0.041$] was significantly higher for teachers with an imposing classroom social climate compared to teacher with a helpful classroom social climate ($p < 0.05$). Teachers with a drudging classroom climate switched cells significantly more often than teachers with an assured ($p < 0.05$) or helpful ($p < 0.01$) climate. (3) For *dispersion* [$F(5, 29) = 2.99, p = 0.027$] teachers with an assured classroom social climate had significantly lower dispersion than teachers with a compliant ($p < 0.05$), imposing, or drudging ($p < 0.01$) climate. (4) For *Visit Entropy* [$F(5, 29) = 3.03, p = 0.024$] teachers with a drudging climate had significantly higher visit entropy values than teachers with an assured or helpful classroom social climate ($p < 0.01$. (5) For *Duration per cell* [$F(5, 29) = 2.60$, $p = 0.047$] teachers with a drudging social climate had significantly lower cell durations than teachers with an assured ($p < 0.05$) or helpful ($p < 0.01$) climate. Thus, these results showed that especially teachers with a drudging social climate, which is relatively less desirable, show significantly more variability in their interactions with the class than teachers with assured and helpful social climates, which are the most desirable social climates in terms of student and teacher outcomes.

In sum, the study showed that the number, the strength, and the location of attractors varies between teachers with different classroom climates. Also, a quantitative comparison of the interaction trajectories in terms of the grid measures showed, for example, that teachers with less desirable classroom climates changed their behavior more often and were relatively more unpredictable.

Illustration 4: Interpersonal Teacher Behavior and Student Behavioral Engagement

This example was adapted from an unpublished study (Pennings et al., 2012). In this study we observed in four teacher–class dyads interpersonal teacher behavior and student behavioral engagement (Fredricks, Blumenfeld, & Paris, 2004) in terms of active or passive on- and off-task behavior (Skinner, Kindermann, & Furrer, 2009). The idea of this study was that students of teachers with classroom social climates characterized by higher levels of agency and communion have better academic results (Wubbels et al., 2006) and show more behavioral engagement (Birch & Ladd, 1997; Valeski and Stipek (2001).

Skinner and Belmont (1993) for example, found that students' emotional and behavioral engagement was not only influenced by their perception of the classroom social climate, but also by the teachers actual behavior. Therefore we wanted to observe interpersonal teacher behavior in connection with student behavioral engagement. In this illustration we stick to what can be seen in the SSGs. We included this example for the purpose of illustrating that it is also possible (1) to create SSGs with different kinds of behavior for the two parties in the interaction (i.e., interpersonal behavior vs. student behavioral engagement), and (2) to create asymmetrical SSGs (i.e., 8×4 SSGs).

Approach

Participants were four secondary school teachers with their students. These teachers were chosen based on their classroom social climate (based on their agency and communion scores measured with the QTI). Figure 12.9 the SSGs of the four teachers are ordered following the quality of the social climates of these teachers (i.e., teacher A in the upper right quadrant; teacher B in the lower right quadrant; teacher C in the lower left quadrant; and teacher D in the upper left quadrant of the IPC-T).

We used the joystick method (Box 12.1) to observe teacher behavior the same way we described in Illustration 2 and 3. Yet we also used the joystick method to observe student/class engagements as two dimensions. The horizontal axis was used to observe on-task (+) vs. off-task (−) student behavior, and the vertical axis was used to code whether on/off-task behavior was active (+) or passive (−).

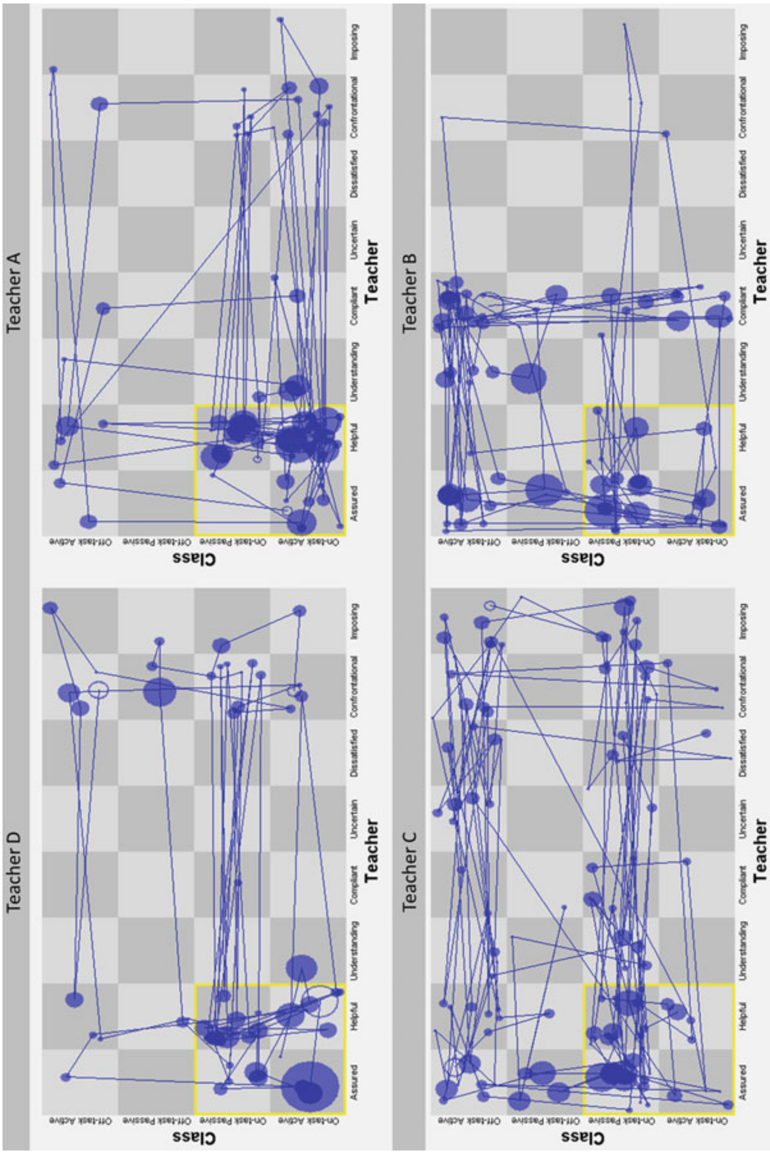

Fig. 12.9 SSGs for teacher–class A–D showing teacher interpersonal behavior on the x-axis and class behavioral engagement on the y-axis

Gurtman's (2011) procedure was used to recode teacher interpersonal behavior coordinates into the octants of the IPC-T. The student engagement coordinates were also recoded following this procedure, but an additional computation was used to combine the octants into quadrants, resulting into four categories representing the observation categories defined by Skinner et al. (2009): (1) On-task active (upper right quadrant), (2) On-task Passive (lower left quadrant), (3) Off-task Passive (lower left quadrant), and (4) Off-task Active (upper left quadrant).

Findings and Conclusion

As can be seen in Fig. 12.9, the four lower left cells (11, 12, 21, and 22) in the SSGs are marked with a thicker line.

This area is the predefined area of interest where teacher behavior is assured and helpful and student behavior is active or passive on-task. All four interactions regularly visit the cells in this area. The nodes in this area of interest are larger for teacher A and D, than for teacher B and C. This means that the duration of individual visits to these states for teacher A and D were longer than for teacher B and C. The SSGs also show that the interactions of teacher A and D less often and for shorter durations of time visited areas of the grid where students are off-task than teacher B and C. In addition, the interactions of teacher C and D show more behavior that is characterized by lower levels of communion (i.e., uncertain, dissatisfied, confrontational or imposing behavior) than teacher A and B.

Thus, even without using any grid measures the visual information provided by the SSGs provides information about the interactions in these classrooms. It also shows that although classroom social climates (macro-level) of these four teachers are different with some being more favorable, that students in all classrooms still show some degree of on-task behavior, and that teachers show some degree of assured and helpful behavior.

Box 12.1: Sadler's Computer Joystick Method for Observation of Interpersonal Teacher and Student Behavior

Interpersonal behavior of students and teachers was coded continuously within the IPC following an online-scoring procedure and using Sadler's joystick tracking method (Fig. 12.10) (Sadler et al., 2009).

(continued)

Box 12.1 (continued)

Fig. 12.10 Sadler's computer Joystick observation (this picture is adapted from Pennings, Brekelmans et al., 2014)

First teacher behavior and then student behavior was coded in separate observation sessions. The joystick tracking device is designed to observe verbal and nonverbal behaviors that have clear interpersonal meaning (Markey, Lowmaster, & Eichler, 2010). By moving the joystick in a certain direction the behavior of people can be observed (a) continuously in time (online observation) and (b) represented as a degree of both agency and communion (Markey et al., 2010). Thus, an observer moves the joystick to code the teacher's or the students' ongoing interpersonal behavior, while watching a video recording of a lesson. The joystick device enabled us to observe behavior as a specific blend of agency and Communion, instead of coding behavior separately (and arbitrarily) for both dimensions.

Joymon

This joystick method comes with a computer program (Joymon.exe; Lizdek, Sadler, Woody, Ethier, & Malet, 2012) that numerically records the exact location (based on *X*- and *Y*-coordinates) of the joystick within a two dimensional space, meant to represent the IPC (Markey et al., 2010; Sadler et al., 2009). During the observation, a dot in the IPC (i.e., presented in a separate

(continued)

Box 12.1 (continued)

screen) marks the exact location of the joystick. These behavior coordinates ranged from −1000 (i.e., very low agency/Communion) to +1000 (very high agency/Communion). This range is a default setting of the joymon-progam and ensures maximum sensitivity of the computer joystick device. Also, by default the program is set to record the joystick cursor's location twice per second. We also used this default setting to record teacher and student behavior twice per second.

Thus, in a study where 10 minute interactions are observed, about 600 behavior coordinates were provided for agency and Communion, per teacher and class. For a more elaborate description of this computer joystick procedure see Lizdek et al. (2012).

Joystick Training and Interrater Reliability

To learn how to observe teacher–student interactions with the computer joystick two of the researchers (first author and last author of the study presented in illustration 3) participated in a computer joystick training provided by Pamela Sadler. Four trained observers independently coded the videos. Every video was coded by two out of these four observers. Interrater reliability was established for the observations by calculating intraclass correlations (ICC(K); Markey et al., 2010; Thomas et al., 2014). Resulting in ICC(K = 2) values of 0.72 for teacher agency, 0.84 for teacher communion, 0.82 for student agency, and 0.89 for student communion. This indicated strong agreement between the observers (LeBreton & Senter, 2008).

General Discussion

Our goal for this chapter was to illustrate a process oriented way of doing classroom research. We wanted to show how moment-to-moment or *real-time* classroom interaction can be captured and studied with State Space Grids (SSGs) (Hollenstein, 2013; Lewis et al., 1999). We think that SSGs are a suitable tool for many issues that arise when classroom or educational processes are approached from a CDS perspective. SSGs make it possible to visualize and capture many features of real-time interaction. This in turn allows us to study how higher levels developmental outcomes, like the quality of the classroom climate or for instance student engagement, are grounded in real-time processes but also how they restrain those real-time processes at the same time. Therefore, SSGs offer a way to move away from solely product oriented research that summarizes entire lessons or even larger time units in single measures. An approach that is much needed in order for educational scientists to make a contribution to educational practice (Koopmans, 2014; Wubbels et al., 2012).

We provided four illustrations of how we approached the study of the classroom social climate and its connection with teacher interpersonal behavior and teacher student interactions. We think that using SSGs has advanced our understanding of teaching in terms of what teachers seem to have in common regarding interpersonal processes, but also in terms of differences between teachers and types of classroom climates.

One basic observation is that visualizations of interactions in different classrooms are already compelling in the way they convey differences and sameness between teachers. In exploratory studies this helps to formulate hypotheses about the content and structure of interactions, which can be tested with more advanced methods and larger sample studies. For example, in all of the illustrations the visual inspection of the SSGs shows that there seems to be a rather common base in the interpersonal state space all teachers share and that in most classrooms only this one general attractor exists. This common base consists of states where teacher and class are both friendly (or on-task for students), for example by showing assured and helpful behavior. Yet variation on the agency dimension in combination with friendliness (high communion) is possible. Note that for classrooms in which this area is not an attractor as such, the interaction still visits this area quite often. Therefore, differences between teachers seem to be rooted more in the way they move in and out of this area or an attractor, rather than in were an attractor is located. Indeed, as the illustrations included in this chapter show, this first observation is confirmed when more sophisticated grid- and cell-measures are employed. Specifically in Illustration 3, which uses relatively more advanced techniques and the largest sample, it becomes clear that there is something like a commonly assumed social hierarchy in the classroom, with legitimate teacher power in combination with a basically non-oppositional attitude of both teacher and students towards each other (moderately high agency and communion or in other words assured or helpful teacher behavior).

Notwithstanding these common features of classrooms, a lot of differences in interaction were detected. The main theme seems to be that the more favorable the general classroom climate (also in terms of student and teacher outcomes; Wubbels et al., 2006), the more firmly interaction is rooted in a moderately high agency and communion attractor, and the more classrooms divert from this more favorable states, the more variable or chaotic classroom interaction becomes. This is apparent in all of the four illustrations used here and specifically reflected in grid- and cell-measures like the number of transitions between states, duration per cell, but also more sophisticated measures like visit entropy. Not only teachers that have a what we have called drudging social climate in class change more often between interpersonal states, also interactions in classrooms of teachers where the classroom social climate is generally imposing or compliant, interactions showed relatively more variability. Indeed, as Illustration 2 shows, correlations between agency and communion and many of the measures that indicate variability are negative, indicating that less social influence and less warmth go together with more variability in classroom interaction.

Not only variability indicated less favorable interactions, interactions may also visit more “extreme” cells of the grid (for example states including the highest scores for teacher dominance) from time to time (see for example discussion of Illustration 1). As these extreme episodes of interaction seem to be very short, it is questionable whether such a characteristic of classroom interaction could have been captured with more product oriented approaches.

We think that the use of SSG is attractive also because of the intuitive way of visualizing data and the versatile way in which grids can be built and employed. Indeed, there are many ways in which SSGs can be build, varying the combinations of dimensions, methods of observation, study individual behavior (e.g., Pennings, Brekelmans et al., 2014), dyadic interactions (e.g., Granic et al., 2003; Mainhard et al., 2012), and even triadic interactions (e.g., Lavictoire et al., 2012) and one can chose to conceptualize development and visualize trajectories over time (e.g. Granic et al., 2003; Turner et al., 2014). Also the possibilities for the interpretation of the resulting data are virtually infinite. Possibilities for analysis start from mere visual inspection of SSGs, include qualitative comparison of cell and whole grid measures (e.g., Illustration 1), but also allow researchers to conduct more advanced analysis, for example by using the winnowing procedure for attractor identification. Of course it is also possible to use any measure resulting from SSG-analysis in more "classical" statistical analyses (see Illustration 3) and in multilevel or structural equation modeling to test hypotheses. Thus, researchers with various degrees of statistical knowledge should be able to profit from this tool. Bear in mind, however, that the SSG technique is merely as good as the data that is used. It totally depends on the theoretical rigor that underlies the decisions made by the researcher who builds the grids.

Overall, SSGs and CDS thinking are very promising in educational research, because they provide the means to study individual teachers, teacher–student/class dyads, teacher teams, teachers with parents, or student–student interactions. It is possible to generalize results across, for example, teachers but also to focus on individual development of teachers or students. The insights in the differences in content and structure that we have found in our studies can easily be incorporated in professional development courses for teachers (e.g., to create awareness on the effect of behavior in interactions with students on the teacher–student relationship or the general classroom social climate; Pennings, Van Tartwijk et al., 2014). The interested reader should turn to Hollenstein (2013) for a more comprehensive introduction to SSG analysis or should consult the GridWare manual (Lamey et al., 2004).

References

Bento, T., Ribeiro, A. P., Salgado, J., Mendes, I., & Gonçalves, M. M. (2014). The narrative model of therapeutic change: An exploratory study tracking innovative moments and protonarratives using state space grids. *Journal of Constructivist Psychology, 27*(1), 41–58.

Birch, S. H. & Ladd, G. W. (1997). The teacher–child relationship and children's early school adjustment. *Journal of School Psychology*, 35, 61–79.

Bronfenbrenner, U., & Morris, P. A. (2006). The bioecological model of human development. In W. Damon & R. M. Lerner (Eds.), *Handbook of child psychology, Vol. 1: Theoretical models of human development* (6th ed., pp. 793–828). New York: John Wiley.

Claessens, L. C. A., Pennings, H. J. M., Van Der Want, A. C., Van Tartwijk, J., Brekelmans, M., & Den Brok, P. (2014). De docent en het sociaal klimaat in de klas: een benadering vanuit interpersoonlijk perspectief [The teacher and the classroom social climate: From an interpersonal perspective]. *Pedagogische Studien, 91*, 300–315.

Cornelius-White, J. (2007). Learner-centered teacher-student relationships are effective: A meta-analysis. *Review of Educational Research, 77*(1), 113–143. doi:10.3102/003465430298563.

Créton, H. A., Wubbels, T., & Hooymayers, H. P. (1989). Escalated disorderly situations in the classroom and the improvement of these situations. *Teaching and Teacher Education, 5*(3), 205–215.

De Jong, R. J., Van Tartwijk, J., Verloop, N., Veldman, I., & Wubbels, T. (2012). Teachers' expectations of teacher-student interaction: Complementary and distinctive expectancy patterns. *Teaching and Teacher Education, 28*, 948–956. doi:10.1016/j.tate.2012.04.009.

Dishion, T. J., Nelson, S. E., Winter, C., & Bullock, B. (2004). Adolescent friendship as a dynamic system: Entropy and deviance in the etiology and course of male antisocial behavior. *Journal of Abnormal Child Psychology, 32*, 651–663. doi:0091-0627/04/1200-0651/0.

Fournier, M. A., Moskowitz, D. S., & Zuroff, D. C. (2011). Origins and applications of the interpersonal circumplex. In L. M. Horowitz & S. Strack (Eds.), *Handbook of interpersonal psychology* (pp. 37–56). New York: Wiley.

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*, 59–109. doi:10.3102/00346543074001059.

Granic, I., & Hollenstein, T. (2003). Dynamic systems methods for models of developmental psychopathology. *Development and Psychopathology, 15*(3), 641–669. doi:10.1017.S0954579403000324.

Granic, I., Hollenstein, T., Dishion, T. J., & Patterson, G. R. (2003). Longitudinal analysis of flexibility and reorganization in early adolescence: A dynamic systems study of family interactions. *Developmental Psychology, 39*, 606–617.

Guastello, S. J., Koopmans, M., & Pincus, D. (2009). *Chaos and complexity in psychology: The theory of nonlinear dynamic systems*. New York: Cambridge University Press.

Guastello, S. J., & Liebovitch, L. S. (2009). Introduction to nonlinear dynamics and complexity. In S. J. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: The theory of nonlinear dynamic systems* (pp. 1–40). New York: Cambridge University Press.

Gurtman, M. B. (2009). Exploring personality with the interpersonal circumplex. *Social Psychology Compass, 3*, 1–19. doi:10.1111/j.1751-9004.2099.00172.x.

Gurtman, M. B. (2011). Circular reasoning about circular assessment. In L. M. Horowitz & S. Strack (Eds.), *Handbook of interpersonal psychology* (pp. 209–312). New York: Wiley.

Henderson, D. G. (1995). *A study of the classroom and laboratory environments and student attitude and achievement in senior secondary biology classes*. Unpublished doctoral dissertation, Curtin University of Technology, Perth.

Henderson, D. G., & Fisher, D. L. (2008). Interpersonal behaviour and student outcomes in vocational education classes. *Learning Environments Research, 11*, 19–29. doi:10.1007/s10984-007-9034-z.

Hollenstein, T. (2007). State space grids: Analyzing dynamics across development. *International Journal of Behavioral Development, 31*, 384–396. doi:10.1177/0165025407077765.

Hollenstein, T. (2013). *State space grids: Depicting dynamics across development*. New York: Springer.

Hollenstein, T., & Lewis, M. D. (2006). A state space analysis of emotion and flexibility in parent–child interactions. *Emotion, 6*(4), 663–669. doi:10.1037/1528-3542.6.4.656.

Hong, J., Hwang, M., Tam, K., Lai, Y., & Liu, L. (2012). Effects of cognitive style on digital jigsaw puzzle performance: A GridWare analysis. *Computers in Human Behavior, 28*, 920–928.

Horowitz, M. L., & Strack, S. (2011). *Handbook of interpersonal psychology*. New York: Wiley.

Kiesler, D. J. (1996). *Contemporary interpersonal theory and research*. New York: Wiley.

Koopmans, M. (2014). Change, self-organization and the search for causality in educational research and practice. *Complicity: An International Journal of Complexity and Education, 11*(1), 20–39. Retrieved from: http://wigan-ojs.library.ualberta.ca/index.php/complicity/article/view/19523/16145.

Lamey, A., Hollenstein, T., Lewis, M. D., & Granic, I. (2004). GridWare (Version 1.1). [Computer software]. http://statespacegrids.org

Lavelli, M., Pantoja, A. P. F., Hsu, H., Messinger, D., & Fogel, A. (2005). Using microgenetic designs to study change processes. In D. M. Teti (Ed.), *Handbook of research methods in developmental science* (pp. 40–65). Malden, MA: Blackwell Publishing.

Lavictoire, L., Snyder, J., Stoolmiller, M., & Hollenstein, T. (2012). Affective dynamics in triadic peer interactions in early childhood. *Nonlinear Dynamics in Psychology and the Life Sciences, 16*(3), 293–312.

LeBreton, J. M., & Senter, J. L. (2008). Answers to twenty questions about inter-rater reliability and inter-rater agreement. *Organizational Research Methods, 11*, 815–852. doi:10.1177/1094428106296642.

Lewis, M. D., Lamey, A. V., & Douglas, L. (1999). A new dynamic systems method for the analysis of early socio-emotional development. *Developmental Science, 2*(4), 458–476. doi:10.1111/1467-7687.00090.

Lizdek, I., Sadler, P., Woody, E., Ethier, N., & Malet, G. (2011).Capturing the stream of behavior: A computer joystick method for coding interpersonal behavior continuously over time. *Social Science Computer Review*, 1–9. doi:10.1177/0894439312436487.

Locke, K. D., & Sadler, P. (2007). Self-efficacy, values and complementarity in dyadic interactions: integrating interpersonal and social-cognitive theory. *Personality and Social Psychology Bulletin, 33*(1), 94–109. doi:10.1177/0146167206293375.

Lunkenheimer, E. S., & Dishion, T. J. (2009). Developmental psychopathology: Maladaptive and adaptive attractors in children's close relationships. In S. J. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology* (pp. 282–306). New York: Cambridge University Press.

Mainhard, M. T., Brekelmans, M., den Brok, P., & Wubbels, T. (2011). The development of the classroom social climate during the first months of the school year. *Contemporary Educational Psychology, 36*(3), 190–200.

Mainhard, M. T., Brekelmans, M., & Wubbels, T. (2011). Coercive and supportive teacher behaviour: within- and across-lesson associations with the classroom social climate. *Learning and Instruction, 21*(3), 345–354. doi:10.1016/j.learninstruc.2010.03.003.

Mainhard, M. T., Pennings, H. J. M., Wubbels, T., & Brekelmans, M. (2012). Mapping control and affiliation in teacher-student interaction with state space grids. *Teaching and Teacher Education, 28*, 1027–1037. doi:10.1016/j.tate.2012.04.008.

Markey, P. M., Lowmaster, S. E., & Eichler, W. C. (2010). A real-time assessment of interpersonal complementarity. Personal relationships. *Personal Relationships, 17*, 13–25. doi:10.1111/j.1475-6811.2010.01249.x.

Maulana, R., Opdenakker, M.-C., Den Brok, P., & Bosker, R. (2011). Teacher-student interpersonal relationships in Indonesia: Profiles and importance to student motivation. *Asia Pacific Journal of Education, 31*, 33–49. doi:10.1080/09243453.2011.619198.

Pennings, H. J. M., Brekelmans, M., Wubbels, T., Van Der Want, A. C., Claessens, L. C. A., & Van Tartwijk, J. (2014). A nonlinear dynamical systems approach to real-time teacher behavior: Differences between Teachers. *Nonlinear Dynamics, Psychology, and Life Sciences, 18*(1), 23–45.

Pennings, H. J. M., Mainhard, T., & Brekelmans, M. (2015). *Momentary teacher-student interactions and teacher interpersonal styles*. Paper (to be) presented at the 18th annual meeting of the Society for Interpersonal Theory and Research (SITAR) May 29–30, Toronto, Canada.

Pennings, H. J. M., Van Tartwijk, J., Vermunt, J., & Brekelmans, M. (2012). *Leren en ontwikkeling van competenties van aanstaande, beginnende en ervaren docenten: Interpersoonlijk docentgedrag en leerlingbetrokkenheid* [Learning and developments of competenties of novice and expert teacher: Interpersonal behavior and student engagement]. Paper presented at the Onderwijs Research Dagen (ORD), juni 20-22, Wageningen, Nederland.

Pennings, H. J. M., Van Tartwijk, J., Wubbels, T., Claessens, L. C. A., Van Der Want, A. C., & Brekelmans, M. (2014). Real-time teacher-student interactions: A dynamic systems approach. *Teaching and Teacher Education, 37*, 183–193.

Prins, F. J., & Mainhard, M. T. (2012). *Development of a questionnaire on interpersonal student behavior (QISB) during feedback dialogues*. Paper presented at the meeting of EARLI SIG Assessment, 28–31 August, Brussels, Belgium.

Roorda, D. L., Koomen, H. M. Y., Spilt, J. L., & Oort, F. J. (2011). The influence of affective teacher-student relationships on student's school engagement and achievement: A meta-analytic approach. *Review of Educational Research, 81*, 493–529. doi:10.1007/s10648-011-9170-y.

Sadler, P., Ethier, N., Gunn, G. R., Duong, D., & Woody, E. (2009). Are we on the same wavelength? Interpersonal complementarity as shared cyclical patterns during interactions. *Journal of Personality and Social Psychology, 97*(6), 1005–1020. doi:10.1037/a0016232.

Shannon, C. E., & Weaver, W. (1949). *The mathematical theory of communication*. Urbana, IL: University of Illinois Press.

Skinner, E. A., & Belmont, M. J. (1993). Motivation in the classroom: Reciprocal effect of teacher behavior and student engagement across the school year. *Journal of Educational Psychology, 85*, 571–581.

Skinner, E. A., Kindermann, T. A., & Furrer, C. J. (2009). A motivational perspective on engagement and disaffection. Conceptualization and assessment of children's behavioral and emotional participation in academic activities in the classroom. *Educational and Psychological Measurement, 69*, 493–525. doi:10.1177/0013164408323233.

Spilt, J. L., Koomen, H. M. Y., & Thijs, J. T. (2011). Teacher well-being: The importance of teacher–student relationships. *Educational Psychology Review, 23*, 457–477. doi:10.1007/s10648-011-9170-y.

Thelen, E., & Smith, L. B. (1998). *A dynamic system approach to the development of cognition and action* (3rd ed.). Cambridge, MA: MIT Press.

Thomas, K. M., Hopwood, C. J., Woody, E., Ethier, N., & Sadler, P. (2014). Momentary assessment of interpersonal process in psychotherapy. *Journal of Counseling Psychology, 61* (1), 1–14. doi:10.1037/a0034277.

Turner, J. C., Christensen, A., Kackar-Cam, H. Z., Trucano, M., & Fulmer, S. M. (2014). Enhancing students' engagement report of a 3-year intervention with middle school teachers. *American Educational Research Journal, 51*, 1195–1226.

Turnnidge, J., Cote, J., Hollenstein, T., & Deakin, J. (2013). A direct observation of the dynamic content and structure of coach-athlete interactions in a model sport program. *Journal of Applied Sport Psychology, 26*(2), 225–240. doi:10.1080/10413200.2013.821637.

Valeski, T. N., & Stipek, D. (2001). Young children's feelings about school. *Child Development, 73*, 1198–2013. doi:0009-3920/2001/7204-0016.

Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher-student interaction: A decade of research. *Educational Psychology Review, 22*, 271–297. doi:10.1007/s10648-010-9127-6.

Van Petegem, K., Creemers, B. P., Rossel, Y., & Aelterman, A. (2005). Relationships between teacher characteristics, interpersonal teacher behaviour and teacher wellbeing. *Journal of Classroom Interaction, 40*(2), 34–43.

Vauras, M., Kinnunen, R., Kajamies, A., & Lehtinen, E. (2013). Interpersonal regulation in instructional interaction: A dynamic systems analysis of scaffolding. In S. Volet & M. Vauras (Eds.), *Interpersonal regulation of learning and motivation: Methodological advances* (pp. 125–146). London, UK: Routledge.

Wentzel, K. (2012). Teacher-student relationships and adolescent competence at school. In T. Wubbels, P. Den Brok, J. Van Tartwijk, & J. Levy (Eds.), *Interpersonal relationships in* *education: An overview of contemporary research* (pp. 19–36). Rotterdam, The Netherlands: Sense Publications.

Woolfolk Hoy, A., & Weinstein, C. S. (2006). Students' and teachers' perspectives on classroom management. In C. Evertson & C. S. Weinstein (Eds.), *Handbook for classroom management: Research, practice, and contemporary issue* (pp. 181–220). Mahwah, NJ: Lawrence Erlbaum.

Wubbels, T., Brekelmans, M., Den Brok, P., Levy, J., Mainhard, T., & Van Tartwijk, J. (2012). Let's make things better. In T. Wubbels, P. Den Brok, J. Van Tartwijk, & J. Levy (Eds.), *Interpersonal relationships in education: An overview of contemporary research* (pp. 225–250). Rotterdam: SENSE Publishers.

Wubbels, T., Brekelmans, M., Den Brok, P., Wijsman, L., Mainhard, T., & Van Tartwijk, J. (2014). Teacher-student relationships and classroom management. In E. T. Emmer & E. J. Sabornie (Eds.), *Handbook of classroom management* (pp. 363–386). New York: Taylor & Francis.

Wubbels, T., Brekelmans, M., Den Brok, P., & Van Tartwijk, J. (2006). An interpersonal perspective on classroom management in secondary classrooms in the Netherlands. In C. Evertson & C. Weinstein (Eds.), *Handbook of classroom management: Research practice and contemporary issues* (pp. 1161–1191). New York: Lawrence Erlbaum Associates.

Chapter 13
Nonlinear Dynamical Interaction Patterns in Collaborative Groups: Discourse Analysis with Orbital Decomposition

Dimitrios Stamovlasis

Introduction

Literature on learning-in-groups research, in all areas of education, frequently makes use of the term group dynamics to refer to a hypothetical dynamical process taking place when individuals are interacting within a group setting. This is not surprising because most researchers acknowledge the inherent dynamical character of human and social experience, expanding from the microlevel processes of mind functioning to the macro-level processes of collective and social life. Paradoxically, most of the research endeavors in this area have been carried out in the traditional way, ignoring the time aspect and any reference to dynamics is considered merely at a metaphorical level.

Nevertheless, focusing on some ontological aspects of group functioning one may recognize that the dynamics is more than a metaphor and acknowledge that a different methodological framework is needed for a profounder investigation. Considering the interactions among group members working towards a common goal that requires collective action, it is observed that individuals adapt their behavior according to other's actions. They respond and add iteratively to the ongoing process, the results of which cannot be reduced to the behaviors of individual group members. Interactions among participants give rise to an outcome that is not explicably understood as resulting merely from the individual actions, because it emerges from a complex dynamical process and it can be understood only in an evolutionary context. Thus, group interaction processes cannot be effectively studied with conventional linear approaches which are incompatible with the nature of the underlying phenomena.

D. Stamovlasis (✉)
Aristotle University of Thessaloniki, Department of Philosophy and Education, 54124 Thessaloniki, Greece
e-mail: stadi@edlit.auth.gr

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_13

This chapter presents and exemplifies the application of nonlinear dynamics and complexity framework to the study of learning-in-groups studies focusing on science education. A discourse analysis is carried out by *orbital decomposition analysis* (ODA), a method designed for data that comprise categorical time series. In the beginning, a short reference to cooperative learning literature in science education is made. The section that follows discusses theoretical issues, such as the ontological characteristics of verbal interaction processes, and it reveals the nonlinear nature of the discourse taking place in a cooperative learning setting. Subsequently, fundamental theoretical concepts, such as *entropy*, *self-organization*, and *inverse power law distribution*, are explained in relation to discourse analysis; also, methodological assets of nonlinear dynamics and complexity, such as *Shannon entropy*, *topological entropy*, *dimensionality*, and *Lyapunov exponent*, are presented along with the basics of ODA. Results from experimental data and their interpretation are presented analytically in the following section, while a final discussion on methodological and epistemological issues along with implications for educational theory, practice, and research is provided.

Learning-In-Groups in Science Education

Educational sciences, in order to explain research findings and to guide practice, have fostered various psychological theories, such as Piaget's (1973) cognitive developmental theory, which focuses on the personal construction of knowledge; Vygotsky's (1978) theory, which emphasizes the social aspect of knowledge construction; and behavioral learning theories of Bandura (1977).

Learning-in-groups has traditionally been considered as an effective teaching approach and it belongs to cooperative learning methods, a generic group of educational procedures. In these settings, learners work together in small groups to accomplish shared goals, e.g., to understand a given topic or problem and arrive at a solution (e.g., Johnson & Johnson, 1991; Johnson, Johnson, & Maruyama, 1983; Lazarowitz & Hertz-Lazarowitz, 1998). These educational processes are relevant to Vygotsky's ideas emphasizing the construction of knowledge as social process. Within the social environment, the learner or novice negotiates the meaning of the matter to be learned with others, who could be either experts (e.g., a teacher) or peers. The process of negotiation results in a cognitive gain that is substantially higher than the anticipated achievement by one's own abilities. This learning environmental support is the *zone of proximal development*, which is modified and expanded when students interact within a learning-in-group setting. Relevant to social learning perspective is the *situation learning theory*, which emphasizes a local process depending on situational characteristics and being temporarily decoupled from individual differences. Situational characteristics include the means, the rules, and the setting climate that determine the function of the group under particular circumstances. It is imperative to mention here that specially for science education, which could be characterized by a synthesis of linguistic, mathematical/symbolic, and visual representations, (Lemke, 1998, 1999; Lynch & Woolgar, 1990), the role of language is crucial. This justifies why researchers attempt to understand learning outcomes by focusing on discourse analyses.

Typically, group interaction studies in learning science are designs which include recording and analyzing discourse material with the aim to identify dominant interactions and to correlate them with achievement. The data consists of verbal interactions occurring among members of small group of students who elaborate explanations about physical or chemical phenomena, and work together towards understanding the relative scientific concepts. The effectiveness of small-group process in learning has been studied as a function of various independent variables, such as the type and difficulty of the task or prior experience (Appleton, 1997; Bowen, 2000; Lazarowitz & Hertz-Lazarowitz, 1998; Lazarowitz, Hertz-Lazarowitz, & Baird, 1994; Shachar & Fischer, 2004; Zady, Portes, & Ochs, 2002), where it has been established that classroom interactions are correlated with students' performance; that is, enhanced learning outcomes are observed in group processes where certain types of interactions occur more frequently (Kempa & Ayob, 1991, 1995; Stamovlasis, Dimos, & Tsaparlis, 2006; Zady et al., 2002).

Research has been facilitated by recognizing various roles for the group members, which have been introduced for analytical convenience when doing empirical work, such as the *learner* or the *learner facilitator*. A member could also be a *leader* or a *follower*. The leader is someone who continuously takes the initiative to provide an idea or to develop an argument and leads in a way the unfolding discussion, while the follower is someone who merely responds to other's initiatives. Each individual engaged in the discourse interactions might play one or more roles successively. Some roles could be assigned in the group from the start, e.g., the leader; they might also appear or emerge during the course of interactions. In the latter case, these roles are rather correlated with some individual differences (Hall et al., 1988; Horn, Collier, Oxford, Bond, & Dansereau, 1998; O'Donnell, Dansereau, & Rocklin, 1991). For instance, a student with high cognitive skills and verbal ability attains the learning material faster and can provide support to his/her peers acting as a facilitator or leader. In most group settings members are encouraged to take initiatives to contribute to the process; however, not surprisingly, some members only demonstrate active participation and leadership. In science education, the active participation is encouraged and essentially it is presumed for effective outcomes; however, a considerable amount of silent learning is taking place as well (Stamovlasis et al., 2006).

Research in science education has shown that the effectiveness of an interaction process in a group setting depends on a number of *factors*, some of which are individual differences of the group members, previous training, the nature of the task, and the interactive process itself (Johnson, Johnson, Ortiz, & Stanne, 1991; Johnson, Johnson, Stanne, & Garibaldi, 1990; Webb, 1989, 1991). On the other hand, properties which can characterize quantitatively and qualitatively the discourse are referred as the *features of interactions* and concern the type of information exchanged during group sessions. For example these could be of cognitive type or interpersonal interactions of social type. Some of the features may concern the group functioning as a whole, e.g., the *climate* which concerns explicit or tacit affective communication and/or the *cooperativeness* among members. These are characteristics, which in a structured session could be manipulated by the instructor in order to optimize the outcomes. The factors and the features of interactions are typically subjected to measurement and comprise the foremost independent variables in learning-in-group research. It has been pointed out that small-group processes contribute to productivity and to the development of higher order cognitive skills, provided that interactions with the appropriate features are developed (Noddings, 1989; Taggar, 2001; Vygotsky, 1978). Thus, the temporal patterns of verbal exchange developed in an evolutionary context, *ceteris paribus*, are the determinants ensuring learning and productivity.

A Note on Methodological Issues

Even though learning-in-groups has become a widely used instructional procedure at all levels of education and in all subject areas, and its effectiveness is well established, there are still theoretical and methodological issues that warrant for further examination. There is lack of a unified theoretical framework that could embrace all associated with learning-in-groups phenomena and provide a comprehensive description and explanation in terms of specific mechanisms underlying the interaction processes. In general, group research in behavioral sciences seems to be fragmented regarding the theoretical premise. There is a multiplicity of theoretical approaches and methodologies, which focus on different aspects and lead to a variety of perspectives, e.g., communication, psychoanalytic, social, developmental, or functional perspective (see Wheelan, 2005). Yet, no attempts have been made to formulate a unified theory.

The theoretical issues, however, are interrelated with the methodological ones. A sophisticated theory needs a robust methodology to be developed, and on the other hand, an effective methodology requires a coherent and intelligible theory to be founded on, while the epistemological issues are by far crucial. Regarding the present inquiry, putative dynamical processes put forward by the theory are in need of a methodology that is specifically tailored to measure those processes. To this end, nonlinear dynamics and complexity appear to be more than a distinct alternative perspective. There are substantial contributions at theoretical level that approach a general theory of group functioning (e.g., Arrow, McGrath, & Berdahl, 2000), and also research methodology assets and tools for extensive applications (e.g., Guastello, 1998, 2009, 2011; Guastello & Bond, 2007).

Returning to science education, research objectives and methodologies followed in collaborative group settings have been diverse and linked to the theoretical perspectives adopted by the researchers. Typically, when investigating the effectiveness of a relevant learning procedure, the quasi experimental design has been the dominant one in quantitative research. This, however, is a “black-box” approach, which possesses a series of disadvantages. It has not provided essential understanding about the underlying processes, while it has been severely criticized for scantiness on core issues, such as establishing causality (Koopmans, 2014a, 2014b).

A large body of research focusing on group-learning approach belongs to the perspective known as *process-product-studies* of peer interactions (e.g., Stamovlasis et al., 2006; Teasley, 1995; Webb, Troper, & Fall, 1995). In these studies, peer interactions are coded, analyzed statistically, and finally linked to group performance and learning outcomes. The coding schemes could be either predetermined or the categories/codes could be assigned inductively during the actual coding procedure. The latter approach to coding is considered to be grounded in the data and it takes into account the context in which the discourse occurs. Furthermore, the distinction between *content frames* and *interaction frames* has been introduced, focusing on how students bring their frame of reference to the interaction situation and how these frames are jointly negotiated and developed (e.g., Barnes & Todd, 1995).

In most studies, the attention has been focused on specific features of the interactions, measured at the nominal level, representing events/categories that occur successively and form patterns unfolding in time. In science education research these patterns have been characterized as interpretative or exploratory modes of interaction and on the basis of their frequencies they were shown to be indicative of certain quality features of the discourse. Certain patterns have been found to be the most effective and constructive in critical engagements, including argumentation and hypothesis testing (Mercer, 1996).

Other researchers have attempted to follow more process-oriented methods to group interactions, which are seen as socially and situationally developed in students' discourse (e.g., Kumpulainen & Mutanen, 1999). By concentrating on individual and group functioning, these methods aimed to highlight the situated dynamics of peer interactions and learning-in-groups. Data analysis, which was focused on three dimensions, the language function of verbal exchange, the cognitive interactions, and the social process, revealed stimulating interaction patterns, where, nonetheless, the time aspect was rather implicit in the analytical framework. The notion of *dynamics*, even though was evoked through microanalysis of interactions and the concepts and tools utilized, was the traditional linear means.

Moreover, while traditional methodologies applied to discourse analysis have yielded interesting findings, they have not been mathematically formalized to the extent that they can be meaningfully associated with a certain theoretical framework. This chapter seeks to address this gap by presenting ODA, a novel approach to the study of peer-interaction processes in educational settings; it adds to theoretical and epistemological development of the *situated learning* perspective, and sets the framework for the application of nonlinear dynamics and complexity to learning-in-groups methodology in science education.

Theoretical Issues

Discourse as a Nonlinear Dynamical System

A group of individuals, e.g., students working together and interacting with each other, form a system that possesses dynamical characteristics. Before developing any mathematical formalism on group interactions, it is imperative to attempt a narrative portrayal of the processes and their dynamical features in a physical language. This may seem trivial; however it provides an understanding of how the elementary actions or events are linked to the behavior outcomes at a higher level of complexity and contributes to the formation of macro-characteristics of a given discourse.

When students with a shared goal interact amongst themselves attempting, e.g., to solve a problem, to gain a common understanding, or to reach a consensus about an issue, collective action simultaneously or successively is required. In these processes, the group members adapt their behavior according to the actions of others. In discourse, verbal interactions are taking place as the participants are exchanging information, and in order to scrutinize it, one may have to track verbal exchanges and reveal their qualitative features that are patterns of sequential events unfolding over time. Of course, the focus is on the emerging interactions at the group level, whereas the individual dynamics unfolding in each one's mind are usually ignored; however, they are present at a lower level of complexity and a reference to them should be made when describing behavior at that level.

Within a single person, the cognitive and affective states and the goal-directed actions as well might evolve independently from external causes. The *intrinsic dynamics* of each individual is central to the characterization of his/her actions (Vallacher, Van Geert, & Nowak, 2015). Actions realized in time also have their own dynamics, and they typically have a hierarchical structure spanning in various *time scales*. Time scale is a fundamental notion in nonlinear dynamics and refers to the length of time during which an event occurs or develops; for example it could happen in the period of a few seconds or in the period of hours or days. Elementary actions being organized accordingly give rise to action at higher level, which could result in a qualitative change in the course of time (e.g., a decision to intervene or refrain from intervening in an ongoing discourse). The intrinsic dynamics are fundamental in understanding the dynamics of human experience overall, and human behavior at social level in particular (Vallacher & Nowak, 2007, 2009). *Coordination* of individuals' actions over time is a necessary condition in social interactions and collective behavior. At social level, research has showed that coordination dynamics are central to human behavior, and they include lower level actions such as speech and movement (e.g., Kelso, 1995), and synchronization phenomena at macro-social level, such as norms and public opinions (e.g., Nowak, Szamrej, & Latané, 1990; Vallacher & Nowak, 2006). Studying interpersonal dynamics of lower level action suggests that the *coordination* interplay exhibits features of nonlinearly coupled oscillatory processes, where the temporal pattern might include *in-phase* and *anti-phase* forms. These notions refer to synchronization effects of engaged vs. disengaged interacting parts, respectively, while phenomena such as *hysteresis* could also be present; the latter denotes the time-based lag between input and output and it is encompassed among the fundamental characteristics of nonlinear dynamical processes (Kelso, 1995).

Returning to the discourse analysis, the process where the elementary actions give rise to macrostructure of temporal communication patterns, coordination dynamics are decisive for the process evolution and coherence. In a cooperative learning setting, temporal coordination dynamics of internal states such as feeling, mood, and dispositions also occur, and are rather the prerequisites to the coordination of actions within the group. The *coordination* dynamics in a discourse include temporal patterns of in-phase and anti-phase forms of synchronization such as competition/cooperation or agreement/disagreement; that is, they encompass the *complementary opposites* that function in self-organized fashion and shape the evolving information flow (Kelso & Engstrøm, 2006).

The observable traces of the *coordination* interplay in a discourse are sequence of utterances/categories unfolding in time that convey information about the evolving scenario, which however cannot be reduced to the individual's dynamics of lower level action. In such sequence, each step is a function of the previous steps and the trajectory in time possesses characteristics that may resemble to nonlinear or even chaotic time series; this implies sensitive dependence on the initial conditions and on the parameters shaping the unfolding discourse. A different order of utterances, a different pattern, induces different dynamics and it might yield to a different outcome. A leader in the group often imposes his/her thesis to their peers, the process, then, might be halted, and the discourse comes to conclusion; however the process goes on if the intrinsic dynamics of another individual allows an action that intervenes with an objection and/or different proposition. The peer's intervention feeds back the process, which continues in an unpredictable way since the present state depends on the previous one and the evolving scenario becomes history dependent; multiple scenarios are likely to emerge. The discourse evolution is not determined by certain features or properties of the interacting elements (group members), but it seems to be self-regulated by the coordinated actions of the participating agents.

Therefore, both the initial conditions and the evolution of the process do play a role. In the language of nonlinear dynamics, it is said that the trajectory of the process follows a complex pattern, which on the course of time might possess thresholds, *bifurcations*, and/or *attractors*. If the coordination pattern does not converge to a certain point of consensus (an attractor), it might be trapped to a limit-cycle attractors that characterize a system evolving in time, being unable to shift towards a desirable conclusion (for the attractor concept see also Chap. 9 in this volume).

The self-regulation mentioned above implies that the system is not driven by an external cause, but it shapes its own dynamics via self-organization mechanisms. The irreducibility of the system's behavior as a whole (discourse in the group) to that of its elementary components (members' actions) can define the discourse process as a complex adaptive system (CAS). The ontology of such system requires the epistemological shift towards the new science of nonlinear dynamics and complexity.

Discourse and Self-Organization

Having provided a theoretical description of discourse interactions, an epistemological step towards the regime of nonlinear dynamics and complexity has been made. Further investigation on the discourse interaction process and under this meta-theoretical framework requires the application of the relevant mathematical formalism and for this some core concepts are elucidated next. Relevant to the present inquiry are the notions of *entropy*, *self-organization*, and *fractal distribution*. General introduction to nonlinear dynamics and complexity theory can be found in Nicolis and Nicolis (2007), while relevant comprehensive outlines for psychology and life sciences could be found in Guastello, Koopmans, and Pincus (2009). In educational research literature, besides the present volume, relevant introductions have been sporadically reported in a number of papers (e.g., Koopmans, 2014a; Stamovlasis, 2006, 2011).

Within nonlinear dynamics and complexity theory, a significant descriptor of a system's state is its *entropy*. The concept of *entropy* originates from classical thermodynamics. Its statistical definition was developed by Ludwig Boltzmann in the 1870s. Entropy was introduced in social science applications with the development of *Information Theory* by Claude E. Shannon in 1948. In general, entropy stands for disorder (-order) or uncertainty and in the complex system sciences it appears as a significant variable associated with the information needed to describe the system, and thus it is related to system's complexity. Basic formalism of the entropy concept and its applications could be found in Nicolis and Prigogine (1989) and Nicolis and Nicolis (2007). A related entropy measure is *information entropy*, or *Shannon entropy* (H_S), which concerns a system or a set of categories with unequal odds of occurrence (see next section).

Self-organization concerns the corresponding theory which focuses on the study of open systems that operate at far-from equilibrium conditions, exchanging information, energy, or matter with their environments. Such systems, known as CAS, are self-regulated through complex feedback mechanisms, so that they can tune their dynamics and their own evolutionary characteristics, thus being adaptive to their environment (Nicolis & Prigogine, 1989; Prigogine & Stengers, 1984).

Self-organization means that the system is driven neither by any external intervention or control nor by any internal "demon" or a homunculus-like agent. It is the complex feedback processes, the temporal microscale fluctuations, and the underlying dynamics that determine the system's evolution. Under certain conditions, a discourse might exhibit such self-organizing behavior, when coordination among individuals leads to the organization of verbal interaction into dynamic patterns that emerge as a global structure from the local elementary actions with no predetermined scenario.

One characteristic property of a *self-organization* process is that the output variables or other measured quantities do not follow a Gaussian—normal distribution as it happens with independent measurements. There is a high degree of dependency among observations, which obey the *inverse power law* (iPL), a distribution that mathematically is expressed by the equation

$$S(f) \propto f^{-\beta} \tag{13.1}$$

where S is the size of an event (object or attribution) and f is the frequency of the event (object or attribution). The iPL is also called *fractal* distribution. The iPL in the case of event time series suggests that a large number of small events are expected, while exponentially very fewer large events occur. The exponent β can be calculated as the slope of the distribution curve at the log-log scale; it is called the *fractal dimension* and it is a measure of the system's complexity. Values $1 < \beta < 2$ denote dynamical characteristics (Schroeder, 1991; West & Deering, 1995). Higher values of β, that is steeper curves, denote that there are more small events, while lower values corresponding to relatively flat curves denote more large-scale events. Note that in discourse and group interaction phenomena, lower fractal dimensions are associated with greater structure or coherence (Guastello, 2005; Pincus & Guastello, 2005).

If a system's distributional characteristics exhibit fractal structure, then the underlying process evolves through a series of discontinuous shifts, a state that manifests itself as an iPL distribution, and/or through more global transformations as the system is being adjusted between different degrees of relative chaos (disorder) and order. In the language of nonlinear dynamics and complexity the above denote that the system is working at the dynamic regime, being at the *edge of chaos* (EOC) (Waldrop, 1992). Systems at the EOC demonstrate high capability of adaptation without annihilation or stagnation. Such properties of adaptive behavior are observed in complex adaptive systems across the sciences. Related examples are the distribution of total acts in social interaction systems initiated based on rank (Bales, 1999), the in-degree distributions in Web (Broder et al., 2000), and the distribution of verbal turn-taking interaction in family therapy sessions (Pincus & Guastello, 2005), to mention a few.

Returning to the current study and the discourse analysis, if the process under investigation is driven indeed by self-organization mechanisms, then the information flow or the evolving exchange of utterances within the group are expected to conform to the above type of temporal fractal structure as evidenced by the existence of an iPL distribution in the magnitude of recurrence of the various patterns.

Method

Experimental Settings, Data Collection, and Measurements

A common practice in science education involves small groups working together in order to carry out a task, such as physics or chemistry problem solving, explaining phenomena, elaborating and understanding science concepts, or even experimentation aiming to develop practical skills. The results presented in a following section are derived from experimental settings with groups of three members aimed to investigate how students' verbal interactions evolved during a discourse segment, when developing explanations of physical phenomena and the relevant concepts, such as gravity, velocity, and acceleration. The subjects were secondary school students in tenth grade, taking compulsory classes in the sciences. The assignment of the groups was based on two criteria: academic achievement and performance in a psychometric test of developmental level (Lawson's test, 1978), so that the group synthesis preserved heterogeneity within each group and thus equivalence. The design included pre- and posttest (which are not used here) and in addition a *group test*, which was an evaluation test on questions that had to be answered after negotiation and consensus, at the end of the session. This was a measure of the *group performance*, reflecting the amount of learning emerged from the discourse. Group performance was measured and recorded as a three-level ordinal variable: high performance (successful), intermediate (partial success) performance, and low performance (failed). No specific time limit was imposed on these sessions, which however had by design two important features: First, these tasks were relatively novel to the students; this was chosen in order to pose an intellectual challenge to the students and allow emerging phenomena, e.g., brain storming. Second, the groups were unstructured in terms of role taking and they were let to function spontaneously, allowing so the manifestation of pronounced dynamic effects. Students' verbal interactions through their negotiations in all groups were audiotaped and transcribed.

After the coding procedure and the identification of verbal interactions, a variable named *group activity* was defined to measure the total contributions or actions occurred during the interaction process. Ordinarily, the *group activity* is measured by the number of utterances brought up in the discussion and it has been correlated with high performance (Kempa & Ayob, 1991, 1995; Stamovlasis et al., 2006). It has been acknowledged however that not only the number of contributions enriches the discourse and enhances the probability of an ultimately successful outcome, but the multiplicity and variety of utterances as well. To this end, within the present methodology *information entropy* (H_S) is proposed as a measure of the group activity, because it has two advantages. First, H_S is a theoretically suitable measure to reflect the degree of *novelty* in terms of new categories and/or new patterns. Second, it is a concept of complexity theory and thus it can be co-examined along with other nonlinear measures (see next section).

Coding Procedure: The Key of Inquiry

The first and crucial step in a discourse analysis is the coding procedure. Spoken conversations produce utterances which can be coded according to the theoretical framework of interest and create a series of events/categories unfolding in time. It should be emphasized that the coding procedure does not necessarily implement predetermined categories; the categories/codes could be inductively emerged from the coding procedure, e.g., see qualitative approaches (Denzin & Lincoln, 2005). Thus, the coding procedure is not different from a typical one followed in traditional inquiries. ODA focuses on the ensued symbolic sequences and analyzes them accordingly. The categorical time series analysis could be applied with various

Table 13.1 Example categorization systems

Coding system I		Coding system II	
Cognitive interactions		Interpersonal interactions	
R=	Reflection on the problem	Y=	Expressing approval
E=	Explanation with a physical law	N=	Expressing disagreement
H=	Hypothesis	D=	Expressing doubt
A=	Argument	A=	Asking for approval
T=	Thesis	H=	Asking for help
S=	Skeptical	G=	Providing guidance or help
C=	Recall a physical law		

methodological approaches of data collection and coding procedures, depending on the research questions or hypotheses posted. The coded data resemble the following stings of symbols:

AABBBDCAABAEAABBBEAABAEAABBBABABABEED. . ..

where A, B, D, or E are coded utterances. An utterance is defined as a simple, complete or incomplete phrase or a chain of phrases, which possess recognizable and interpretable elements of communication. When the interaction process involves written messages, e.g., in e-mail communications, the interacting agents have the opportunity to express much more ideas and greater variety of utterances can be recorded, and the coding scheme becomes richer. Note that the content of the coded utterances is not related to dynamics; it is the pattern structure that is associated with the dynamical characteristics. Also in ODA discrete event sequences are recorded regardless of the length of time required to complete the event or the time that elapses between the events. Time length is an interesting feature, which deserves a special focus in the time series analysis; however, it is not examined in this chapter.

A discourse could be coded with category systems of various forms simultaneously and analyzed accordingly with ODA. In Table 13.1, examples of coding systems are presented. Coding systems I and II include codes of two types of interactions, cognitive and social-interpersonal interactions, respectively. Another simple coding scheme might assign a symbol to each participant, so that the turn-taking pattern can be followed and recorded. Different coding schemes facilitate different hypothesis testing of theoretical or practical interest. For instance, it might be desirable to compare the level of cognitive activity to the level of social interaction process evolving simultaneously through a given discourse. Another interesting example of coding scheme might include categories characterizing language function, such as “informative,” “evaluative,” and “affectional,” (e.g., Kumpulainen & Mutanen, 1999). In the latter case the application of ODA may serve testing hypotheses concerning the evolution of discourse at linguistic level and determining potential relationships between language functions and discourse outcomes.

Coding procedures are fundamental parts of the analysis since the results concern the theoretical premise that is behind the coding scheme and drive the

Table 13.2 Coding scheme with cognitive utterances and participants' psychometric measures

Digit	Description	Evaluation
First digit	Actor's level of a psychometric variable (e.g., *M*-capacity)	1 = Low
		2 = Intermediate
		3 = High
Second digit	Cognitive utterance (nominal scale)	1 = Reflection on the problem
		2 = Explanation with a physical law
		3 = Hypothesis
		4 = Argument
		5 = Thesis
		6 = Skeptical
		7 = Recall a physical law
Third digit	Correctness (ordinal scale)	1 = Incorrect
		2 = Partially correct
		3 = Correct

hypotheses posted. It is also possible to include multiple categorical variables in ODA (Pincus, 2001; Pincus, Fox, Perez, Turner, & McGee, 2008). A complex coding that includes multiple categorical variables applied to an educational setting is depicted in Table 13.2. A three-digit code for each utterance includes the following: the first for the speaker, the second for the type of cognitive category, and the third digit evaluates the content correctness at an ordinal scale. Moreover, in a multiple coding scheme, certain individual difference might also be coded and their role in the peer interaction process could be examined.

Finally, after a set of mutually exclusive and exhaustive categories have been derived their reliability should be established by two or more raters. Cohen's Kappa statistic could be used for measuring inter-rater reliability. Typically, values above 0.65 are considered satisfactory.

Symbolic Dynamic Analysis with Orbital Decomposition Method

ODA is based on symbolic dynamics and it is specially designed for the analysis of time series with data measured at the nominal level (Guastello, 2000; Guastello, Hyde, & Odak, 1998). The basic idea of ODA originates from a methodological approach involving calculations of entropy with scale variables, applied primarily to a physical system when characterizing an experimental strange attractor with periodic orbits (Lathrop & Kostelich, 1989). In these systems periodic orbits presuppose basins of attraction, and thus if more basins exist, the more chaotic the motion becomes (Newhouse, Ruell, & Takens, 1978). Analogously, in an interaction process of verbal exchange, the concept *proximal recurrences* of a repeated pattern plays the role of neighboring orbits and thus the greater the variety of these orbits, the more unpredictable the conversation flow will be, and more nonlinear or even chaotic the dynamic character of the evolving process might be. This analogy between periodic orbits and pattern recurrence in a categorical time series allows the application of similar concepts and formulas to the latter and the description of the process under investigation via quantitative means.

The primary form of a symbolic sequence under investigation is a string of symbols: e.g., AABEDBDEAABAEAABBBEA, where A, B, D, and E are the codes for the events occurring during the course of time. Patterns are combination of at least two symbols with varying length. A single symbol is not considered as pattern; however it is included in the analysis. The first two steps of the procedure involve two calculations: a likelihood χ^2 and φ^2 test for a string sequence or pattern of responses with varying length (C), and topological entropy (H_{T}). For $C = 1$ a single utterance (e.g., A) is considered as the unit of analysis, while if $C = 2$, two utterances in the row (e.g., AB or DB) are taken together as the unit of analysis. The calculations include all string lengths starting with $C = 1$ and continuing with $C = 2$, $C = 3$, $C = 4$, etc.

For each increasing string length, a likelihood χ^2 test provides the statistical significance; this is to exclude the pattern that occurred by chance. For a given string length C (e.g., $C = 3$, A-B-D) and N_c strings of length C in the data, the expected frequency of the string is

$$F_{\mathrm{exp}} = P_{\mathrm{A}} P_{\mathrm{B}} P_{\mathrm{D}} N_c \tag{13.2}$$

where P_{A}, P_{B}, and P_{D} are the probabilities for A, B, and D, respectively. The corresponding likelihood χ^2 is given by the formula

$$\chi^2 = 2\sum \left[F_{\mathrm{obs}} \ln\left(\frac{F_{\mathrm{obs}}}{F_{\mathrm{exp}}} \right) \right] \tag{13.3}$$

Note that for $C = 1$ equal probability is considered for the null hypothesis, while for $C \geq 2$ the H_0 is that the odds of the string are equal to the a posteriori combinatorial probabilities of the states. The φ^2 test provides a correction to the χ^2. Moreover, φ^2 test is a measure analogous to the proportion of variance accounted for this string length and it is given by the equation

$$\varphi^2 = \frac{\chi^2}{N_c} \tag{13.4}$$

χ^2 and φ^2 are used to determine the optimal length at which the analysis of the symbolic sequence should be carried out. The optimum length corresponds to the maximum φ^2.

Topological entropy (H_{T}) describes the deterministic *nonrandom complexity* for the time series and it is the upper bound for the metric entropy, which is equal to the positive Lyapunov exponent (Lathrop & Kostelich, 1989). The latter is a measure of the chaoticity of the dynamical process. The calculation of H_T is based on the diagonal entries or trace of a hypothetical transition matrix at each string length ($\boldsymbol{M}^C$). Each cell entry is binary and indicates whether a particular symbolic sequence is followed in time by any other symbolic sequence. The trace tr$\boldsymbol{M}^C$ of this matrix represents instances in which a pattern is followed by itself in a consecutive period of time. This is the *proximal recurrence* (Guastello et al., 1998). The topological entropy (H_T) measure based on the trace of the matrix $\boldsymbol{M}^C$ is given by the equation

$$H_T = \lim_{c \to \infty} (1/C) \log_2 \mathrm{tr}\left(\boldsymbol{M}^C\right) \tag{13.5}$$

The trace tr$\boldsymbol{M}^C$ is the sum of the diagonal elements, which consists of 0 and 1 s. *Proximal recurrences* become less likely for longer patterns based on simple combinatorial probabilities of single utterances; thus, as C increases, H_T is expected to decrease and eventually drops to zero. The longest optimal string length for analyzing the discourse corresponds to the string length C, when the tr$\boldsymbol{M}^C$ becomes zero at $C + 1$, and at which, under optimum conditions, the value of φ^2 is maximized.

As the string length approaches infinity, assuming that the dynamics of the system is described by the transition matrix $\boldsymbol{M}^C$, H_T approaches the base-2 logarithm of the maximum eigenvalue of the matrix, which is the Lyapunov exponent, a measure of the chaoticity of the dynamical process described by the matrix $\boldsymbol{M}^C$ (Lathrop & Kostelich, 1989), and it also reflects its complexity that is not due to chance. The Lyapunov dimensionality then is calculated by the equation

$$D_L = e^{H_T} \tag{13.6}$$

The second entropy measure is the *information entropy* or *Shannon entropy* (H_S). For a set of categories with unequal odds of occurrence it is defined by the following equation

$$H_S = \sum_{i=1}^{r} p_i [\ln(1/p_i)] \tag{13.7}$$

where p_i is the probability associated with each ($i = 1$ to r) categorical outcome (Shannon, 1948; Shannon & Weaver, 1949). Shannon entropy is not related to dynamics; however it is a measure of complexity since it reflects the information content needed to describe the system. H_S has been proposed as a measure of the degree of novelty present within a categorical time series (Attneave, 1959). It indicates the degree to which a categorical time series contains relatively rare patterns, that is, those with low probabilities of occurrence. Topological entropy, on the other hand, does not reflect this degree of novelty because it relies on proximal recurrences.

Having found the optimal string length using the procedure described above, then the calculation of *fractal dimension* can be carried out using the iPL distribution graph (Eq. 13.1). If S is the magnitude of the recurrent pattern and f is the frequency at which each particular pattern occurs, the slope of the $1/f$ curve (Eq. 13.1) can be used as an estimate of fractal dimension.

Nonlinear Hypotheses for Discourse Analysis

The nonlinear analysis applied to categorical time series might be driven by various types of research questions and hypotheses. The identification of repeated patterns of different size, proximally or distantly, is a key feature to be sought. ODA can provide this information along with frequency distributions. By applying ODA to verbal interactions in learning-in-groups settings, the structure within turn-taking patterns can be examined, and a characterization of discourse sessions could be achieved, based on the amount of structure within the discourse patterns. Such macrostructure of a given discourse might be a qualitative emergent property that could be used as a classification criterion. The emerged macrostructure, along with its nonlinear quantitative measures, such as exponents and entropy of a given discourse, might also be correlated to the outcomes and the effectives of the interaction process.

Assigning codes for each person's name initials can accommodate the investigation of whether the interacting group is balanced as far as each member's contribution in the turn-taking patterns. In multiple coding schemes individual differences might be included so that hypotheses regarding their role in the interaction process might be investigated. For example, in Table 13.2, the coding scheme where each member's personal code is replaced with a code representing levels of a psychometric variable, e.g., *M*-capacity, facilitates testing the hypothesis that students with high information processing capacity demonstrate influential contributions to the peer interaction process.

Moreover, two fundamental interrelated theoretical hypotheses can be tested: (1) Verbal interaction processes in learning-in-group sessions display complex dynamic characteristic of *self-organization*. (2) Learning outcomes from group-member interactions are *emergent* phenomena from nonlinear dynamical processes.

Methodologically, the two measures, Lyapunov dimensionality (a measure of turbulence) and fractal dimension (a measure of complexity), are the means of demonstrating when the signature of nonlinearity and complexity is present in a group interaction process. Shannon's entropy is not a dynamic measure per se; however it indicates whether the discourses encompass novel patters regarding spoken utterances. Information entropy might be related to other nonlinear characteristics and is a valuable tool for evaluating complex patterns.

An interesting endeavor is the investigation of the conditions under which dynamical characteristics are present and how these might be associated with effectiveness and learning outcomes. Special cases, such as brain storming situations with emerging phenomena, attract special attention and are potential candidates for the application of ODA. In this inquiry, an additional hypothesis posted was that the *information entropy* of the resulting symbolic sequences, which reflects the group *activity* in each session, is correlated with the group performance.

Discourse Analysis with Orbital Decomposition

The application of ODA to symbolic time series and the related calculations can be carried out with ORBDE software (Peressini & Guastello, 2010). The provided tables and results are explained in the following paragraphs.

Table 13.3 presents the ODA results for a students' discourse, where the time series comprised of cognitive type interactions. All the relevant statistical indices were calculated for $C = 1$ to $C = 5$. The tr$\boldsymbol{M}^C$ becomes zero at $C = 5$, and between $C = 3$ and $C = 4$ strings, the former was chosen as the optimum string length for analysis based on the greater φ^2 value. The anomaly of φ^2 values greater than 1.0 has been described as resulting from a violation of the assumption of a 2×2 matrix, which however does not affect comparison, and the value of φ^2 reflects the proportion of variance accounted for this string length (Guastello et al., 1998). At $C = 3$ the entropy measure is $H_S = 4.137$ and Lyapunov exponent $D_L = 1.390$, indicating a nonlinear complex process. The measures D_L and H_S might be used to compare two categorization systems. For instance, an interpersonal interaction coding scheme that results to values $H_S = 3.252$ and $D_L = 1.134$ shows lower degree of novelty and less turbulence or chaos at the level of social interactions.

The most frequently recurring patterns are listed in Table 13.4. The first and second columns show the most repeated patterns (e.g., HES, SAS and RAC), while in the next columns the expected frequency along with the observed probability is given. In the last column the contribution of each triplet to the total information entropy value is calculated. The findings suggest that certain patterns or structure dominate in the evolution of the discourse; that is, triplets of utterance that express skepticism or doubt on preceding propositions or combine reflection with argumentation appeared more frequently and they might have a decisive contribution to

Table 13.3 Complexity and entropy indicators from orbital decomposition analysis of cognitive type interactions

C	tr$\boldsymbol{M}^C$	H_T	D_L	χ^2	df	N*	φ^2	H_S
1	4	2.00	7.389	68.85	8	114	0.604	1.895
2	7	1.404	4.070	67.937	22	113	0.601	3.324
3	2	0.333	1.390	153.725	27	112	1.373	4.137
4	1	0	1.000	123.534	14	111	1.112	4.483
5	0	–						

String length (C), number of proximal recurrences (trace of binary matrix C), topological *entropy* (H_T), Lyapunov dimensionality (D_L), Shannon entropy (H_S), χ^2, φ^2, and number of strings for $C = 1$–4

Table 13.4 Primary strings of cognitive utterances identified at $C = 3$

Code ($C = 3$)	Utterances' pattern	Frequency	Expected frequency	P_{obs}	Shannon $p \log (1/p)$
HES	Hypothesis-explanation-skeptical	4	0.542	0.036	0.119
SAS	Skeptical-argument-skeptical	4	1.703	0.036	0.119
RAC	Reflection-argument-recall	4	0.210	0.036	0.119
ESR	Explanation-skeptical-reflection	3	0.348	0.027	0.097
STS	Skeptical-thesis-skeptical	3	0.310	0.027	0.097
CAS	Recall-argument-skeptical	3	0.745	0.027	0.097
EAE	Explanation-argument-explanation	3	0.426	0.027	0.097
AEA	Argument-explanation-argument	3	0.585	0.027	0.097
ESH	Explanation-skeptical-hypothesis	3	0.542	0.027	0.097
ACH	Argument-recall-hypothesis	3	0.326	0.027	0.097
HSE	Hypothesis-skeptical-explanation	3	0.542	0.027	0.097

the final outcomes. Table 13.5 depicts the patterns of multiple coding; it suggests that crucial contributions of decisive utterances with correct contents were made by members of high information processing capacity (*M*c), who are essentially undertaking the role of facilitator. Analogously, a number of similar hypotheses regarding the effects of other individual differences in collaborative groups could also be tested.

One of the main concerns expressed in the hypotheses frequently posted is the relation (if any) between group performance and effective dynamic patterns unfolding in the discourse. In the current study, group performance is measured by the group test, which accounted for the correct answers received after negotiation by the group members, and it reflects the amount of learning resulted from the collaborative session. On the other hand, *group activity*, which traditionally is operationalized by the number or frequencies of utterances, is a prerequisite for high-level outcomes. A relation between group performance and group activity was sought by implementing *information entropy* (H_S) as a measure for the latter. H_S proved to be a suitable index to characterize discourse based on certain categorization scheme, since it reflects the degree of novelty of utterance patterns in regard to category/code scheme of the choice. Successful sessions, that is, those of high group-performance, appeared to display higher information entropy, compared to the unsuccessful ones. Even though causality between H_S and group performance cannot be directly established, a probabilistic relation might be derived from empirical data analyzed by means of ordered logistic regression. Figure 13.1 depicts the proposed relation showing the probability of attaining low and high group performance level (effective and ineffective sessions) as a function of *information entropy* H_S (calculated values) encompassed in the utterance patterns of the discourse. The probability of attaining a successful session increases as the information entropy increases, while the probability of attaining an unsuccessful session decreases as the information entropy increases.

Table 13.5 Patterns of multiple coding

Code		Multiple pattern
HES	333 223 363	High Mc-Hypothesis-correct, Int.Mc-Explanation-correct, High Mc-Skeptical-correct
	333 222 362	High Mc-Hypothesis-correct, Int.Mc-Explanation-partially cor., High Mc-Skeptical-correct
	232 223 363	Int.Mc-Hypothesis-correct, Int.Mc-Explanation-partially cor., High Mc-Skeptical-correct
	333 121 263	High Mc-Hypothesis-correct, Low Mc-Explanation-incorrect, High Mc-Skeptical-correct
SAS	262 342 363	Int. Mc-Skeptical- partially cor., High Mc-Argument-partially cor., High Mc-Skeptical-correct
	262 342 363	Int.Mc- Skeptical- partially cor., High Mc-Argument-partially cor., High Mc-Skeptical-correct
	362 141 363	Int.Mc-Skeptical-partially cor., Low Mc-Argument-incorrect, High Mc-Skeptical-correct
	262 342 162	Int.Mc-Skeptical-partially cor., High Mc-Argument-partially cor., Low Mc-Skeptical-incorrect
RAC	313 243 171	High Mc-Reflection-correct, Int.Mc-Argument-correct, Low Mc, Recall-incorrect
	313 243 172	High Mc-Reflection-correct, Int.Mc-Argument-correct, Low Mc, Recall- partially cor.
	313 243 373	High Mc-Reflection-correct, Int.Mc-Argument-correct, High Mc -Recall-correct
	313 243 373	High Mc-Reflection-correct, Int.Mc-Argument-correct, High Mc -Recall-correct
ESR	323 263 313	High Mc-Explanation-correct, Int.Mc-Skeptical-correct, High Mc-Reflection-correct
	323 262 313	High Mc-Explanation-correct, Int.Mc-Skeptical-par.correct, High Mc-Reflection-correct
	121 263 313	Low Mc-Explanation-incorrect, Int.Mc-Skeptical-correct, High Mc-Reflection-correct
HSE	333 263 131	High Mc-Hypothesis-correct, Int.Mc-Skeptical-correct, Low Mc-Explanation-incorrect
	233 363 233	Int.Mc-Hypothesis-correct, High Mc-Skeptical-correct, Int.Mc-Explanation-correct
	333 263 333	High Mc-Hypothesis-correct, Int.Mc-Skeptical-correct, High Mc-Explanation-incorrect

Processes at the Edge of Chaos

One of the main hypotheses is whether the propagation of verbal interactions or the time series of verbal turn-taking patterns conforms to *inverse power law*. It was found that some of the analyzed symbolic sequences followed the iPL distribution. The iPL for one session is demonstrated in Fig. 13.2 showing the log of the

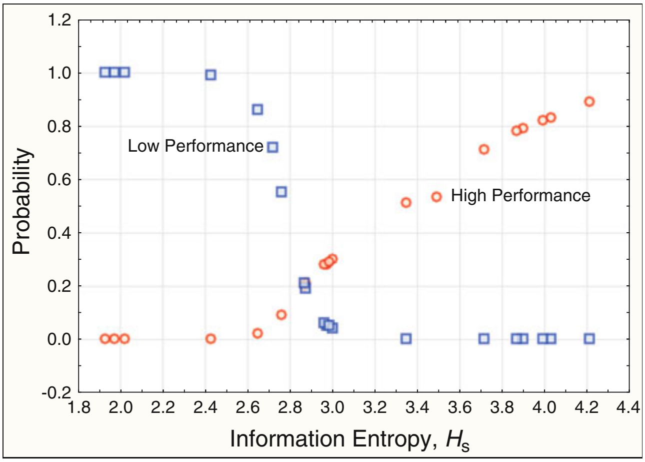

Fig. 13.1 Plot of the probability of attaining low and high group performance as a function of *information entropy* (H_S) encompassed in the dynamical utterance patterns [calculated values using a logistic function]

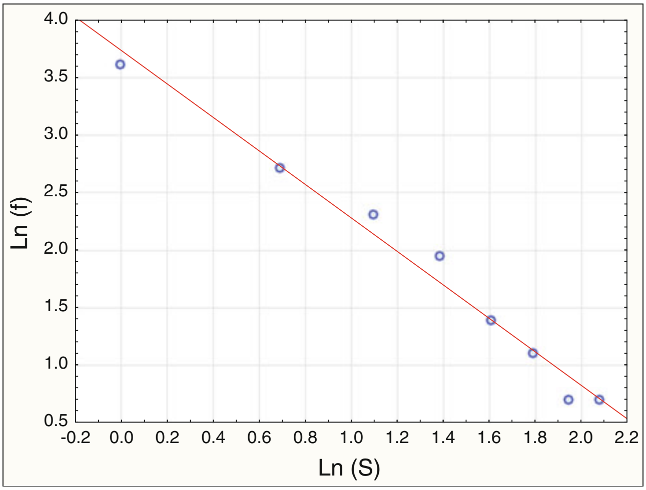

Fig. 13.2 A log-log scale plot of the number of different patterns that occur at various frequencies. The *fitted line* ($R^2 = 0.97$) suggests an iPL distribution with $\beta = 1.46$ ($t = -17.7$, $p < 0.001$)

frequency [ln(f)] at which each of these recurrence phenomena occur as a function of the log of the number of recurrences [ln(S)] for a given pattern at the optimum string length. The distribution has a negative slope which is the *fractal dimension*. The fitted line ($R^2 = .97$; $F = 313.30$; $p < 0.001$) provides the value of $\beta = -1.46$, with 95 % CI [-1.68; -1.26], which is within the typical range ($1 < b < 2$) for EOC processes (Bak, 1996; Kauffman;, 1995). The presence of an iPL denotes that the system is working within the dynamic regime, being at the *EOC* (Waldrop, 1992), a state characterized by both complexity and coherence, and even though the categorical time series is unpredictable on a moment-by-moment basis, it could be somewhat predictable on a global level (fractal distribution). This finding, along with the other nonlinear indices, supports the central hypothesis on the *emergence* of learning outcomes and the creative nature of interactive processes.

Discussion and Overview

This chapter presented the ODA, a novel method for studying dynamical properties of patterns in categorical time series. ODA is based on symbolic dynamics and it was used to identify patterns of interactions in discourses taking place within collaborative group sessions. Symbolic dynamics is an area of mathematics that studies series of entities or categories forming regularities or patterns unfolding in space or time, whereas they can be further examined for structures of higher order. The identification of regularities and hierarchical structures within symbolic sequences is an analogous endeavor to cryptographic analysis, where meaningful patterns of symbols are sought, and it is motivated by similar philosophy as the Turing's computational machine (Hodges, 2012). The main question that challenges this inquiry in a discourse analysis is if, at the optimum unit of analysis (string length), there are certain combinations of utterances, events, or multiple patterns of them, which are the more prevailing or the more creative contributions to the process under investigation.

Research has shown that discourse verbal interactions are not randomly organized in time (Pincus & Guastello, 2005). They possess dynamical structures of nonlinear character with varying dimensionality, order, or entropy. Typical mathematical tools, such as Markov chains used in symbolic dynamic analysis, cannot identify emerging and recurring patterns of utterances. Moreover, the various discourse analysis techniques, which have been applied to psychological and educational research for testing specific hypotheses, have not been grounded on any mathematical formalism or coherent theoretical premise. ODA is filling this gap in the literature of methodology by providing a general philosophy to measuring dynamical properties and unfolding patterns in time series measured at the nominal level. It provides quantitative indices of patterning, information, complexity, entropy, or chaos that can characterize the systems generating these series. The ODA method originates from an orbital decomposition method applied to chaotic time series; however it does not require the presence of chaos per se, but it can distinguish systems of sufficient complexity and quantify them based on measures comparable to chaotic indices, such as topological entropy and Lyapunov dimensionality. D_L is a dynamic measure and it is informative for the degree of turbulence or chaos in the categorical time series; higher values of D_L denote higher degree of complex patterning over the course of conversation that is not due to chance. In addition, *information entropy*, H_S, which increases with longer strings and richer combinations, reflects the decree of novelty characterizing the time series. D_L and H_S at the optimum string length correspond to the most probable structure conveying the dynamical characteristics and the information content, and are used for comparisons and further analyses.

The present study demonstrates that group interactions in cooperative learning settings can be studied effectively with ODA. Methodologically, it challenged the traditional approaches, which due to epistemological fallacies ignore what is between the input and output. It shed light into the "black box" by implementing the proper methodological tools and revealed the determinative role of dynamics, while it opens a new area of investigation for education research. The method could be extended to discourses of various topics in science and other disciplines as well. ODA is an appropriate mean of analysis for any relevant to education processes, such as attention, reading, studying, or playing. Moreover, it is applicable to any time series of qualitative attributes, actions, or events taking place within the school system, such as class attendance, accomplishments, episodes of decent or antisocial behavior (e.g., bulling), to name a few.

Returning to group-interactions inquiry, it must be said that groups are not always functioning as nonlinear systems and discourses do not always display emergent patterns. In the experiments presented in this work, group settings were designed so that activated members were involved in a free interaction process. Discourses, under certain circumstances, show special features of nonlinearity, nonrandom complexity, and novelty as measured by information entropy, which are associated with group performance and productivity. These cases are more likely to occur within unstructured settings where the discourse is allowed to evolve spontaneously without preexisting scenario. The findings support a central hypothesis that the learning outcomes from interactive groups *emerge* from nonlinear dynamical processes. This is in line with theoretical premises and empirical evidences from chaos and complexity research. The identification of iPL and fractal dimensionality supports the hypothesis that in certain cases discourse in a group interaction process could be functioning at the EOC, indicating creative processes and emergent phenomena. The connection between creativity and nonlinear processes has been elaborated in a special issue of *Nonlinear Dynamics Psychology, and Life Sciences* (issue 2, April, 2011). At the individual level, and focusing on the interactive mental resources in task executions, it was pointed out that the effective cognitive processes, those which lead to learning outcomes, are nonlinear dynamical processes. On the contrary, there are linear processes, such as "raw learning" procedures and algorithmic problem solving, which are not associated with learning and creativity (Stamovlasis, 2010, 2011, see also Chap. 9 in his volume). At a theoretical level, any mental process and inductive-type complex problem-solving procedure, where the solution is not hidden in the initial conditions, but is generated via an iterative and recursive process, conform to nonlinear dynamical processes. These are the processes that produce *information* (Nicolis, 1991). That is, these are the creative processes. In this chapter, the central notion of emerging learning or creativity through nonlinear dynamical processes has been extended to processes at social level, referring to a constellation of individuals/students who interact with each other as a coherent unity. It is of paramount significance that irrespective to the unit of analysis, at individual or collective level, the same principles can be demonstrated, and this is the advantage of the theory of complex dynamical systems.

The above findings have also important implications for education. At the group level, a productive interaction process differs from a traditional instructional session, which is based on behavioristic "transfer of knowledge" assumptions. The latter represents a linear process having a predetermined scenario, in which "learning" (if any) is considered to occur as a passive reception of emitted information. In those cases, active involvement is not taking place and the participating minds do not contribute to construction of meaning in the classroom. On the contrary, within interactive groups, learning outcomes *emerge* through an iterative and recursive process. The nonlinear perspective for the *situated learning theory* suggests that collaborative construction of knowledge requires an "activated" group involved in a dynamical interplay. The term activated implies strong interventions and contribution to the evolving session. Given that the outcome is not nested in any of the member's initial repertoire, it has to be created through the interaction process. Thus, creativity is associated with *emergence*, and this is the fundamental element that nonlinear dynamics offers to educational theory and practice. Learning-in-group approaches should encourage and train novice for active participation, in a way that nonlinearity is induced in the interactive process. An evolving discourse—a categorical time series portrayed by ODA—following a trajectory which possesses low-dimensional chaos and operating at the EOC, is more likely to be creative process.

References

Appleton, K. (1997). Analysis and description of students' learning during science classes using a constructivistic-based model. *Journal of Research in Science Teaching, 34*, 303–318.

Arrow, H., McGrath, J. E., & Berdahl, J. L. (2000). *Small groups as complex systems. Formation, coordination, development, and adaptation*. Thousand Oaks, CA: Sage.

Attneave, R. (1959). *Applications of information theory to psychology: A summary of basic concepts, methods, and results*. New York: Holt, Rinehart, & Winston.

Bak, P. (1996). *How nature works: The science of self-organized criticality*. New York: Springer-Verlag.

Bales, R. R. (1999). *Social interaction systems: Theory and measurement*. New Brunswick, NJ: Transaction Publishing.

Bandura, A. (1977). Self-efficiency: Towards a unified theory of behavioral change. *Psychological Review, 84*, 191–215.

Barnes, D., & Todd, R. (1995). *Communication and learning revisited, making meaning through talk*. Portsmouth, NH: Boynton/Cook.

Bowen, C. W. (2000). A quantitative literature review of cooperative learning effects on high school and college chemistry achievements. *Journal of Chemical Education, 77*, 118–121.

Broder, A., Kumar, R., Maghoul, F., Raghavan, P., Rajagopalan, S., Stata, R., et al. (2000). Graph structure in the Web. *Computer Networks, 33*, 309–320.

Denzin, N., & Lincoln, Y. (2005). *Qualitative research*. New Delhi: Sage.

Guastello, S. J. (1998). Self-organization in leadership emergence. *Nonlinear Dynamics in Psychology and Life Sciences, 2*, 303–316.
Guastello, S. J. (2000). Symbolic dynamic patterns of written exchanges: Hierarchical structures in an electronic problem-solving group. *Nonlinear Dynamics, Psychology, and Life Sciences, 4*, 169–189.
Guastello, S. J. (2005). Statistical distributions and self-organizing phenomena: What conclusions should be drawn? *Nonlinear Dynamics, Psychology, and Life Sciences, 9*, 463–478.
Guastello, S. J. (2009). Group dynamics: Adaptation, coordination, and leadership emergence. In S. J. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: Theory of nonlinear dynamics* (pp. 402–433). New York: Cambridge University Press.
Guastello, S. J. (2011). Orbital decomposition: Identification of dynamical patterns in categorical data. In S. Guastello & R. Gregson (Eds.), *Nonlinear dynamics systems analysis for the behavioral sciences using real data* (pp. 499–516). New York: CRC Press.
Guastello, S. J., & Bond, R. W., Jr. (2007). The emergence of leadership in coordination-intensive groups. *Nonlinear Dynamics, Psychology, and Life Sciences, 11*, 91–117.
Guastello, S. J., Hyde, T., & Odak, M. (1998). Symbolic dynamic patterns of verbal exchange in a creative problem solving group. *Nonlinear Dynamics, Psychology, and Life Sciences, 2*, 35–58.
Guastello, S. J., Koopmans, M., & Pincus, D. (Eds.). (2009). *Chaos and complexity in psychology: Theory of nonlinear dynamics*. New York: Cambridge University Press.
Hall, R. H., Rocklin, T. R., Dansereau, D. R., Skaggs, L. P., O'Donnell, A. M., Lambiotte, J. G., et al. (1988). The role of individual differences in the cooperative learning of technical material. *Journal of Educational Psychology, 80*, 172–178.
Hodges, A. (2012). *Alan Turing: The Enigma*. London: Princeton University Press.
Horn, E. M., Collier, W. C., Oxford, J. A., Bond, C. R., Jr., & Dansereau, D. R. (1998). Individual differences in dyadic cooperative learning. *Journal of Educational Psychology, 90*, 153–161.
Johnson, D. W., & Johnson, R. (1991). *Joining together: Group theory and group skills* (4th ed.). Englewood Cliffs, NJ: Prentice Hall.
Johnson, D. W., Johnson, R., & Maruyama, C. (1983). Interdependence and interpersonal attraction among heterogeneous and homogeneous individuals: A theoretical formulation and a meta-analysis of the research. *Review of Educational Research, 53*, 5–54.
Johnson, D. W., Johnson, R., Ortiz, A., & Stanne, M. (1991). Impact of positive goal and resource interdependence on achievement, interaction, and attitudes. *Journal of General Psychology, 118*, 341–347.
Johnson, D. W., Johnson, R., Stanne, M., & Garibaldi, A. (1990). Impact of group processing on achievement in cooperative groups. *Journal of Social Psychology, 130*, 507–516.
Kauffman, S. A. (1995). *At home in the universe*. New York: Oxford University Press.
Kelso, J. A. S. (1995). *Dynamic patterns: The self-organization of brain and behavior*. Cambridge, MA: MIT Press.
Kelso, J. A. S., & Engstrøm, D. A. (2006). *The complementary nature*. Cambridge, MA: MIT Press.
Kempa, R. R., & Ayob, A. (1991). Learning interactions in group work in science. *International Journal of Science Education, 13*, 341–354.
Kempa, R. R., & Ayob, A. (1995). Learning from group work in science. *International Journal of Science Education, 17*, 743–754.
Koopmans, M. (2014a). Nonlinear change and the black box problem in educational research. *Nonlinear Dynamics, Psychology and Life Sciences, 18*, 5–22.
Koopmans, M. (2014b). Change, self-organization, and the search for causality in educational research and practice. *Complicity: An International Journal of Complexity in Education, 11*, 20–39.
Kumpulainen, K., & Mutanen, M. (1999). The situated dynamics of peer group interaction: An introduction to an analytic framework. *Learning and Instruction, 9*, 449–474.
Lathrop, D. P., & Kostelich, E. J. (1989). Characterization of an experimental strange attractor by periodic orbits. *Physics Review, 40*, 4028–4031.

Lawson, A. E. (1978). Development and validation of the classroom test of formal reasoning. *Journal of Research in Science Teaching, 15*, 11–24.

Lazarowitz, R., & Hertz-Lazarowitz, R. (1998). Cooperative learning in science curriculum. In B. J. Rraser & K. G. Tobin (Eds.), *International handbook of science education* (pp. 449–469). Dordrecht, The Netherlands: Kluwer Academic.

Lazarowitz, R., Hertz-Lazarowitz, R., & Baird, J. (1994). Learning science in cooperative settings: Academic achievement and affective outcomes. *Journal of Research in Science Teaching, 31*, 1121–1131.

Lemke, J. L. (1998). Multiplying meaning: Visual and verbal semiotics in scientific text. In J. R. Martin & R. Veel (Eds.), *Reading science* (pp. 87–113). London: Routledge.

Lemke, J. L. (1999). Multimedia literacy demands of the scientific curriculum. *Linguistics and Education, 10*, 1–25.

Lynch, M., & Woolgar, S. (Eds.). (1990). *Representation in scientific practice*. Cambridge, MA: MIT Press.

Mercer, N. (1996). The quality of talk in children's collaborative activity in the classroom. *Learning and Instruction, 6*, 359–377.

Newhouse, R., Ruell, D., & Takens, F. (1978). Occurrence of strange attractors: An axiom near quasi-periodic flows on T^M, $M \geq 3$. *Communication in Mathematical Physics, 64*, 35–41.

Nicolis, S. J. (1991). *Chaos and information processing*. Singapore: World Scientific.

Nicolis, G., & Nicolis, C. (2007). *Foundations of complex systems*. Singapore: World Scientific.

Nicolis, G., & Prigogine, I. (1989). *Exploring complexity*. New York: Freeman.

Noddings, N. (1989). Theoretical and practical concerns about small groups in mathematics. *Elementary School Journal, 89*, 607–623.

Nowak, A., Szamrej, J., & Latané, B. (1990). From private attitude to public opinion: A dynamic theory of social impact. *Psychological Review, 97*, 362–376.

O'Donnell, A. M., Dansereau, D. R., & Rocklin, T. R. (1991). Individual differences in cooperative learning of concrete procedures. *Learning and Individual Differences, 3*, 149–162.

Peressini, A. F., & Guastello, S. J. (2010). *Orbital decomposition: A short user's guide to ORBDE v2.4*. Retrieved May 2014, from http://www.societyforchaostheory.org/resources/#4g

Piaget, J. (1973). *To understand is to invent: The future of education*. New York: Free Press.

Pincus, D. (2001). A framework and methodology for the study of non-linear, self-organizing family dynamics. *Nonlinear Dynamics, Psychology, and Life Sciences, 5*(2), 139–174.

Pincus, D., Fox, K. M., Perez, K. A., Turner, J. S., & McGee, A. R. (2008). Nonlinear dynamics of individual and interpersonal conflict in an experimental group. *Small Group Research, 39*(2), 150–178.

Pincus, D., & Guastello, S. J. (2005). Nonlinear dynamics and interpersonal correlates of verbal turn-taking patterns in group therapy. *Small Group Research, 36*, 635–677.

Prigogine, I., & Stengers, I. (1984). *Order out of chaos: Man's new dialog with nature*. New York: Bantam.

Schroeder, M. (1991). *Fractals, chaos, power laws*. New York: Freeman.

Shachar, H., & Fischer, S. (2004). Cooperative learning and the achievement of motivation and perceptions of students in 11th grade chemistry classes. *Learning and Instruction, 14*, 69–87.

Shannon, C. E. (1948). The mathematical theory of communication. *Bell System Technical Journal, 27*, 379–423.

Shannon, C. E., & Weaver, W. (1949). *The mathematical theory of communication*. Urbana, IL: University of Illinois Press.

Stamovlasis, D. (2006). The nonlinear dynamical hypothesis in science education problem solving: A catastrophe theory approach. *Nonlinear Dynamics, Psychology and Life Sciences, 10*(1), 37–70.

Stamovlasis, D. (2010). Methodological and epistemological issues on linear regression applied to psychometric variables in problem solving: Rethinking variance. *Chemistry Education, Research and Practice, 11*, 59–68.

Stamovlasis, D. (2011). Nonlinear dynamics and Neo-Piagetian theories in problem solving: Perspectives on a new epistemology and theory development. *Nonlinear Dynamics, Psychology and Life Sciences, 15*, 145–173.

Stamovlasis, D., Dimos, A., & Tsaparlis, G. (2006). A study of group-interaction processes in learning lower-secondary physics. *Journal of Research in Science Teaching, 43*(6), 556–576.

Taggar, S. (2001). Group composition, synergy, and group performance. *Journal of Creative Behavior, 35*, 261–285.

Teasley, S. (1995). The role of talk in children's peer collaborations. *Developmental Psychology, 31*, 207–220.

Vallacher, R. R., & Nowak, A. (2006). Coherence in human experience and psychological science. In P. Van Lange (Ed.), *Bridging social psychology: The benefits of transdisciplinary approaches* (pp. 77–82). Hillsdale, NJ: Lawrence Erlbaum.

Vallacher, R. R., & Nowak, A. (2007). Dynamical social psychology: On complexity and coordination in interpersonal relations. In M. Uhl-Bien & R. Marion (Eds.), *Complexity and leadership, Volume 1: Conceptual foundations* (pp. 49–81). Charlotte, NC: Information Age Publishers.

Vallacher, R. R., & Nowak, A. (2009). The dynamics of human experience: Fundamentals of dynamical social psychology. In S. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: Theory of nonlinear dynamical systems* (pp. 370–401). Boston, MA: Cambridge University Press.

Vallacher, R. R., Van Geert, P., & Nowak, A. (2015). The intrinsic dynamics of psychological process. *Current Directions in Psychological Science, 24*(1), 58–64.

Vygotsky, L. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Waldrop, M. M. (1992). *Complexity: The emerging science at the edge of chaos*. New York: Simon & Schuster.

Webb, N. M. (1989). Peer interaction and learning in small groups. *International Journal of Educational Research, 13*, 21–39.

Webb, N. M. (1991). Task related verbal interaction and mathematics learning in small groups. *Journal of Research in Mathematics Education, 22*, 366–389.

Webb, N. M., Troper, J. D., & Fall, R. (1995). Constructive activity and learning in collaborative small groups. *Journal of Educational Psychology, 87*(3), 406–423.

West, B. J., & Deering, B. (1995). *The lure of modern science: Fractal thinking. Studies of nonlinear phenomena in life sciences* (Vol. 3). New Jersey: World Scientific.

Wheelan, S. A. (2005). *The handbook of group research and practice*. London: Sage.

Zady, M., Portes, O., & Ochs, D. (2002). Examining classroom interactions related to differences in students' science achievements. *Science Education, 87*, 40–63.

Chapter 14
Investigating the Long Memory Process in Daily High School Attendance Data

Matthijs Koopmans

Complex dynamical systems research is motivated by a desire to understand how systems maintain stability over the longer term, and how they transform themselves. To that end, the early cybernetic literature has maintained that the role of time needs to be considered when trying to establish a causal connection between outcomes and input conditions (Ashby, 1957; Wiener, 1961). While the causal attribution of outcomes to changing input conditions is part and parcel of many educational studies, there have been few attempts to deliberately model time when establishing this causality (Koopmans, 2014a). The description of large samples of sequentially organized data through time series analysis is quite common in many other disciplines, such as cardiology (heart rates), meteorology (temperature, precipitation), and econometrics (mortgage rates, interest rates), and in fact, time series can be found on an almost daily basis in newspapers such as the *New York Times* and the *Wall Street Journal*.

Time series are useful whenever it needs to be estimated whether the passage of time influences the causal mechanisms that predispose systems to behave in a certain way. They have been used to study phenomena as diverse as irregular heartbeat (Peng et al., 1993), blood cell perfusion in rat brains (Eke et al., 2000), seasonal variability in the teen pregnancy rates in the state of Texas (Hamilton, Pollock, Mitchell, Vincenzi, & West, 1997), and much more. In spite of the fact that the conceptual foundations of this approach for education have been lucidly laid out quite a long time ago (Glass, 1972), the use of time series in education has not received as much attention as one might expect given the time dependency of many of the processes of interest to the discipline: students learn over time, achievement gaps get narrowed over time, and teachers manage time when they plan and execute their lessons. This lack of attention to the time aspect reflects a tendency, particularly

M. Koopmans (✉)
Mercy College, School of Education, Mercy Hall, Rm. SE 31, 555 Broadway, Dobbs Ferry, NY 10522, USA
e-mail: mkoopmans@mercy.edu

© Springer International Publishing Switzerland 2016
M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_14

in applied research circles, to build studies around the prediction of educational outcomes at the group level, rather than the underlying dynamics of educational processes at the individual level (Koopmans, 2014a).

Cross-sectional methodologies rely on the tacit assumption that measurement results obtained over a large sample of cases can be generalized across a large time spectrum, as defined by the scope of the conclusions drawn from those measurements. Does a "snapshot" standardized test result characterize stable achievement levels over the scope of, say, an entire school year? The assumption that you can generalize from cross-sectionally obtained group averages to the entire time spectrum for individual group members is known in the literature as the *ergodic assumption* (Molenaar, 2004; Chap. 8). If this assumption cannot be taken for granted, its verification becomes an empirical issue, requiring a detailed analysis of individual cases. While Molenaar originally made this argument in the context of psychological research, similar argument can be made for education, which, in many ways relies on the same measurement practices for statistical inference, i.e., the measurement of behavioral constructs across groups of individuals (Kerlinger, 1977). An interesting question to contemplate for educational researchers is what we can learn about systemic behavior and transformation thereof from the detailed and statistically rigorous analysis of the contribution of time to behavior in individual cases. Such understanding cannot be easily obtained through conventional linear statistical techniques, which typically rely on the aggregation of the findings across individuals for statistical inference (Neter, Wasserman, & Kutner, 1985).

The underlying assumption when using time-invariant measures is that the systems under study are stable. This assumption of stability has also pervaded the early dynamical literature that traditionally assumed that systems were in principle in a state of equilibrium, except for the instability that accompanies a transformation process (e.g., Lewin, 1947). The more recent literature on dynamical processes has challenged this assumption and proposed that healthy systems may often be in a state of disequilibrium (Bak, 1996; Goldstein, 1988), resulting in an openness to transformation (Stadnitski, 2012b), whereas this proclivity may be absent in systems that are stable in the sense that their behavior is highly predictable based on past occurrences.

Moreover, systems may appear stable for long periods of time while the endogenous process brings those systems to a critical state called *self-organized criticality* (Bak, 1996). The prototypical example of self-organized criticality is the sand pile model, which states that a continued supply of sand to a pile on a flat surface causes occasional avalanches that reorganize the pile, ostensibly to reduce the friction between the grains that result from the accumulation (Jensen, 1998). The state of friction in a system where change is imminent is called self-organized criticality or being "at the edge of chaos," and it is seen as an indicator of systemic complexity (Waldrop, 1992). One implication of the idea of self-organized criticality is that there is a continuous relationship between the small ordinary events that define the endogenous process in the system and the large cataclysmic events that produce transformation in the system, requiring a single analytical framework capturing both aspects.

One important characteristic of self-organized criticality in systems is self-similarity, also referred to as fractality, $1/f^{\beta}$noise or pink noise. A well-known example of self-similarity is the coastline of Norway, which on a small scale replicates patterns that are also observed on a large scale (Feder, 1988), although the scale at which they replicate is not constant. This independence of the patterns observed on the scale at which they are observed is called *scale invariance*. When measurements are conducted over time, patterns of variability can similarly replicate themselves. Such self-similarity occurs when the same variability patterns are observed within an undetermined variety of different time frames, suggesting an alternating but unpredictable pattern of stability/instability.

Complexity and nonlinear dynamical system theories provide a rich array of transformative scenarios, such as bifurcation and period doubling, sensitivity to initial conditions, hysteresis, second-order change, coupled oscillators, and change through self-organized criticality (Koopmans, 2009), and the search for empirical manifestations of those scenarios requires the detailed analysis of sequentially ordered observations in almost all instances. While time series is a common statistical technique, its fine-tuning to specifically address transformative hypotheses put forward in the dynamical literature is a relatively recent development. Two aspects that have generated particular analytical interest are the use of time series analysis to detect sensitivity to initial conditions and chaos (Kantz & Schreiber, 2004; Kaplan & Glass, 1995; Sprott, 2003), and the measurement of self-organized criticality, fractality, and long memory processes (Beran, 1994). The analysis presented here focuses on the latter of these two applications.

School Attendance as a Dynamical Process

Few educators would dispute that attending school is critical to successful educational outcomes, as it is a prerequisite to exposure to classroom instruction and the learning opportunities it provides. In addition, school attendance is also a mediating variable in the system of causal relationships that includes parental support, student academic engagement, instructional effectiveness, and academic attainment (Astone & McLanahan, 1991; Balfanz & Byrnes, 2012; Kemple, Segeritz, & Stephenson, 2013; Kemple & Snipes, 2000; Roby, 2003). In spite of its apparent importance, the analysis of school attendance has taken the backseat to outcomes such as academic achievement, high school dropout, and college persistence behavior, and to the extent that attendance data get reported, it is reported in aggregated form, averaging daily attendance rates over weekly, monthly, or yearly periods (see, e.g., National Center of Education Statistics, 2008), requiring us to assume that those rates are stable over time. Reporting attendance aggregated across the time spectrum results in significant information loss. A time-sensitive view of attendance may help reveal how existing attendance rates impact future attendance over the immediate and longer term, whether there are cyclical patterns to this impact, and what the timing might be of the response of attendance rates to external events or conditions.

An opportunity presented itself to conduct a statistically rigorous analysis of the dynamical processes that may be manifest in educational time series when the New York City Department of Education started recording and publishing the daily attendance rates of all of its schools in 2004, and continued to do so up to the day of this writing. The resulting data sets provide highly detailed information to estimate about how attendance behavior is affected by the progression of time, how attendance patterns differ from one school to the next, and to what extent transformative scenarios such as the ones mentioned above play out over these attendance trajectories.

Most teachers and school administrators are probably well aware of the ebbs and flows in the daily attendance in their classrooms and school buildings. In formal research, these fluctuations get obfuscated by the aggregations that are seen as necessary to summarize the data meaningfully. Hence, the findings of this research do not connect effectively to local knowledge in the schools about daily attendance (Koopmans, 2015). A related point is that applied research in education tends to prefer the cross-sectional estimation of complex cause-and-effect relationships instead of the estimation of the endogenous process through which those relationships are generated (Sulis, 2012). As a result, the literature provides little guidance about what to expect with regard to the short-range dependencies in daily attendance rates, nor the correlations between observations over longer time periods. The work discussed here aims to address this gap.

Using Time Series Analysis to Uncover Dynamical Patterns

The purpose of the analysis presented here is to uncover the dynamical patterns in daily attendance rates, and illustrate why the estimation of those patterns may yield relevant insights into attendance behavior at the school level. Data were obtained from a total of seven schools and some data preparation was done to make the information suitable for a time series analysis. Since such analyses do not permit missing values, a nearest-neighbor imputation was conducted in instances when daily attendance was not recorded on three or fewer subsequent occasions in a given week. If more than 3 days were missing from a given week, that week was removed in its entirety from the series. Similarly, the summer and winter recess was not considered and the last session before and first session after recess were connected as neighbors to ensure the integrity of the dynamics of the temporal ordering of the information.

The two sections that follow will first describe how the estimation of short-range error dependencies (autocorrelation) proceeds in a conventional autoregressive integrated moving average (ARIMA) analysis (Box & Jenkins, 1970; Cryer & Chan, 2008). It is then shown how long-range patterns can be estimated through an extension of this framework called autoregressive fractionally integrated moving average (ARFIMA), a method introduced by Granger and Joyeux (1980) and Hosking (1981) to model the slowly decaying autocorrelations that characterize the long-term memory process. A third section describes the use of power spectral analysis, a procedure used to convert time series plots into plots that show the periodicity of the data. This analytical procedure can be used to detect long-term fractal patterns (Delignières, Torre, & Lemoine, 2005; Wagenmakers, Farrell, & Ratcliff, 2004).

The capability of the combined ARIMA/ARFIMA approach to address both short- and long-range error dependencies within a single analytical framework (Wagenmakers et al., 2004) makes the approach particularly attractive to analyze daily attendance rates, and sets it apart from many other approaches to the detection of fractality, such as power spectral density (PSD) analysis (Eke et al., 2000), de-trended fluctuation (DFA) analysis (Peng et al., 1993), and rescaled range (R/S) analysis (Hurst, 1965), none of which is particularly well suited to differentiate short-range and long-range processes. Delignières et al. (2005) provide a lucid overview of these and related approaches.

Like ARFIMA, Thornton and Gilden's (2005) spectral likelihood classification is designed to distinguish short-term from the long-term processes, but it approaches the issue as an "either/or" proposition; that is, the short-term model and the long-term model compete to provide the best fit to the data. As a result, this approach does not enable the investigator to examine the contribution of long-range processes to the variability in the trajectory *over and above* the contribution of the short-range ones. Thornton and Gilden rightly argue that such as assessment is unlikely to be of great theoretical interest when first-order dependencies (i.e., correlations between neighboring values on the trajectory) are at issue, but in the context of the analysis of daily attendance patterns in high schools, the question is pertinent whether the long-range modeling component needs to be supplemented by seasonal estimators, i.e., short-range features that are of substantive importance to the field such as the days of the school week. Our knowledge about seasonal fluctuations in attendance may facilitate planning at the classroom, school building, and policy level, and may help us better understand the interplay between exogenous (e.g., parents, SES) and endogenous influences (i.e., school attendance rates in the near and distant past). Such estimation may also enhance our understanding about the extent to which the prediction of variability in daily attendance trajectories is relatively straightforward and to what extent it requires dealing with the complexities in the system's behavior. The ARFIMA approach is better equipped to make these distinctions than approaches based on power spectra. However, the particular strength of power spectral analyses compared to ARFIMA is that the former procedure does not require any assumptions with regard to the distribution of observations across the spectrum. Specifically, it can reliably estimate fractality regardless of whether the original time series is stationary or not, whereas ARFIMA requires stationary data (Stadnitski, 2012b; Wagenmakers et al., 2004), i.e., data whose statistical properties are constant across the entire time spectrum.

Short-Range Dependencies

In this section, I'd like to discuss the estimation of short-range dependencies, a statistical procedure that has been part and parcel of conducting time series analysis for many decades now. Let us start with an example. Figure 14.1a shows the daily

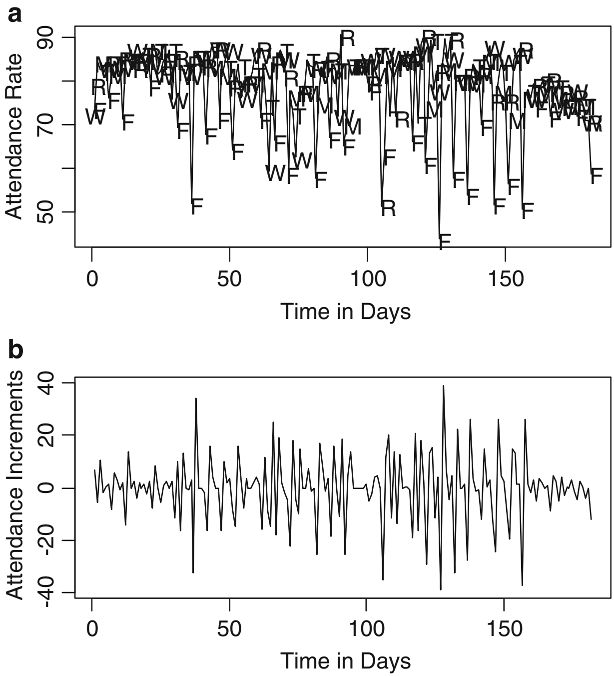

Fig. 14.1 (**a**) Daily attendance 2009–2010 in School 1 with the days of the week marked ("R" represents Thursday, $N = 183$); (**b**) first difference $Y_t - Y_{t-1}$ of the attendance rates in School 1 (attendance increments)

attendance for the entire academic year 2009–2010 in one New York City high school (School 1), marking the days of the week ("R" represents Thursday). The trajectory displays a somewhat drooping appearance with many outlying observations falling way below what would be the average of the series. It can also be seen that there is an overrepresentation of Fs (Fridays) among those low-lying observations. The implication of this pattern would be that average daily attendance rates aggregated across the time spectrum systematically overestimate attendance on Friday and underestimate attendance on the other days of the week.

Figure 14.1b shows a trajectory of attendance increments, or first differences $(Y_t - Y_{t-1})$ in that same school. It can be seen in that figure that in the course of the school year, the differences between given observations and their immediately preceding neighbors become larger, resulting in increased variability, which is to say that the trajectory shows heteroscedasticity across the time spectrum. This trend would go unheeded if traditional central tendency and variability measures are used to characterize these data, leaving us unaware of the increased turbulence in daily attendance as the year progresses.

These examples illustrate very clearly why measures of central tendency and variability are insufficient to characterize the distribution of daily attendance data, as these measures ignore the skewness and the cycles in the first example, and they ignore the increased variability over time in the second one. These examples also indicate why conducting an ordinary least squares regression of daily attendance rates on time yields a biased estimate of their relationship. The observations are not independent, as shown in the first example, and the assumption of homoscedasticity is violated in the second example. The failure of these traditional estimates to handle characteristics that are typical of time-dependent data is part of what motivates ARIMA, which is designed to distinguish two types of error dependency: *the autoregressive process* (AR), and the *moving average process* (MA).

The AR model predicts the value of Y_t as a linear combination of its own past values, plus an error term that is presumed to be an independent identically distributed random variable. The MA model predicts Y_t in terms of accumulated error disturbances, also called innovations. Appendix 1 explicates the ARMA models formally. The investigator can control the number of lags that are used in this prediction for each of these two modeling components. To ensure an unbiased estimation of AR and MA processes, it is essential to verify the stationarity assumption, i.e., the constancy of statistical properties of the data across the entire trajectory. In case of non-stationarity, the first difference of the time series $(Y_t - Y_{t-1})$ is typically used for the estimation. A process that requires such differencing to estimate the ARMA components is called an integrated ARMA or ARIMA process (Cryer & Chan, 2008).

There is a variety of ways to test for the stationarity of a time series. The most well known is the augmented Dickey-Fuller (ADF) test (Fuller, 1996), which regresses the first difference of an observed time series on lag 1 or the original series, and on the past k lags of the first difference of the series. It is then tested whether the beta coefficient in the regression model associated with the lag 1 observation is different from zero, using the parameters for the past k lags as covariates. Rejection of the null hypothesis confirms stationarity of the series (Cryer & Chan, 2008). Thus, using conventional notation, ARIMA (p, d, q) defines the number of AR parameters p and the number of MA parameters q included in the estimation process. The parameter d refers to the order of differencing required, i.e., $d = 0$ for the stationary process, and $d = 1$ for the use of the first difference of a non-stationary process.

Figure 14.2a, b shows simulated examples of a stationary and a non-stationary time series for a sample of 180 observations. Figure 14.2c shows the first difference of the trajectory in Fig. 14.2b, which results in stationarity. In the series shown in Fig. 14.2a, c, it can be seen that the patterns of variability look pretty similar across the series and that the mean of zero appropriately characterizes its central tendency. This is clearly not the case for the trajectory shown in Fig. 14.2b, which characterizes non-stationarity. This latter simulation shows what is known as a *random walk* or *Brownian motion*, an unstable system with strongly correlated observations. The results of the ADF test on these three trajectories are as follows: ADF $= -4.95$, $p < 0.01$; ADF $= -1.43$, $p > 0.01$; and ADF $= -5.95$, $p < 0.01$ for the series in Fig. 14.2a, b, and c, respectively, using $k = 4$ as the lag order. These results confirm the properties that these simulations were set out to show.

When estimating short-range effects across the time spectrum, it is often productive to inspect to the autocorrelation function plots to detect the patterns of

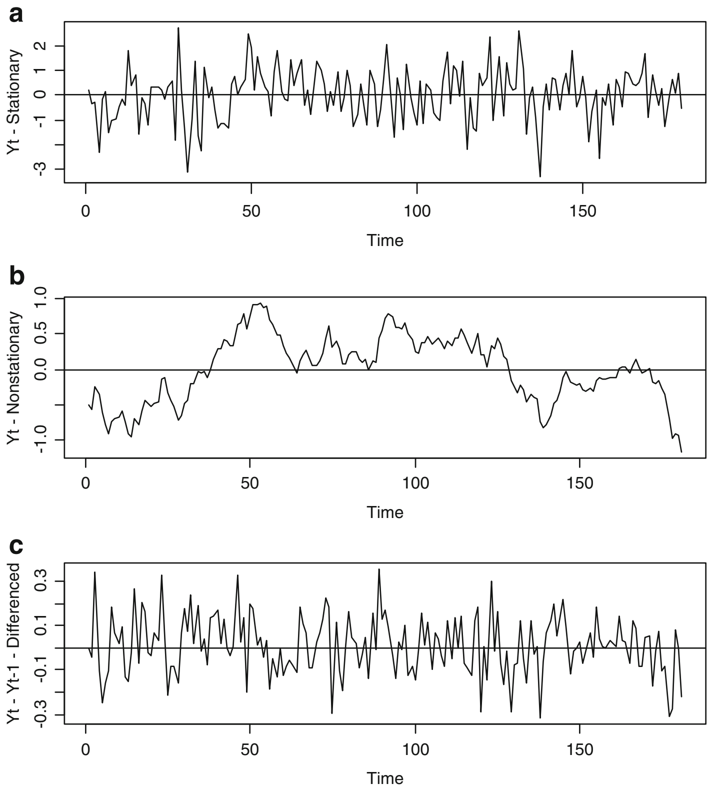

Fig. 14.2 (**a**) Simulation of a stationary time series ($N = 180$, ADF $= -4.95$, $p < 0.01$); (**b**) a non-stationary time series ($N = 180$, ADF $= -1.43$, $p > 0.01$); and (**c**) the first difference $Y_t - Y_{t-1}$ of the series in (**b**) produces stationarity (ADF $= -5.95$, $p < 0.01$)

dependency residing in the data. The use of these plots is illustrated in Fig. 14.3. Three simulated trajectories ($N = 180$) are shown in the left panels and the corresponding ACF plots are shown on the right. Figure 14.3a shows a simulated trajectory without error dependencies (white noise). In this situation, knowing the trajectory does not improve our ability to predict subsequent observations. The ACF plot corresponding to this situation is shown on the right. The spikes in the plot indicate the size of the autocorrelations at the lags indicated on the abscissa. The dotted lines indicate the 95 % confidence interval. The plot shows that none of the autocorrelations up to lag $k = 30$ are different from zero. The trajectory in Fig. 14.3b shows the clustering of neighboring observations that comes with autocorrelation, giving the trajectory in its entirety less of a random appearance than the one shown in Fig. 14.3a. An autoregressive process was generated using an AR (1) model with $\varphi = 0.70$, also with 180 observations. The ACF plot shows what a positive AR (1) process typically looks like. The correlations at the first few lags are significantly different from zero, but they rapidly recede to non-significance as the lag order

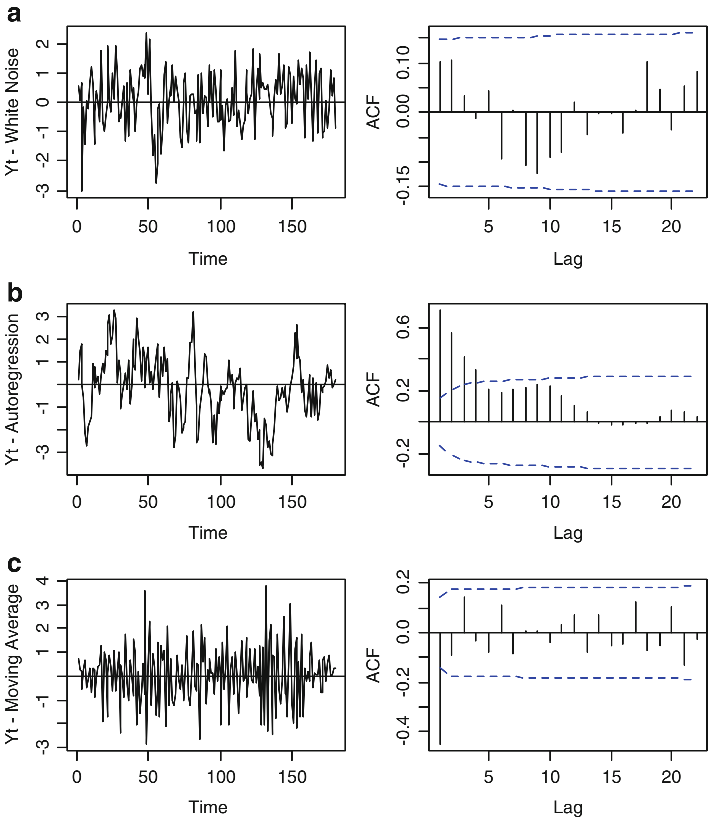

Fig. 14.3 Three simulated time series (*left* panels) with corresponding ACF plots (*right* panels). (**a**) White noise; (**b**) autoregression ($\varphi = 0.70$); (**c**) moving average ($\theta = 0.70$). $N = 180$

increases. Figure 14.3c illustrates an MA (1) scenario at $\theta = 0.70$. A different clustering pattern can be observed in this latter series where consecutive observations tend to alternate across the mean of zero, as is also indicated by the negative autocorrelation shown in ACF plot for the first lag. Note also that, typical of the moving average process, after the first spike, the autocorrelations immediately recede to non-significance at subsequent lag values. The examples presented here can be extended to include AR and MA processes at negative parameter values, multiple AR (p) or MA (q) parameter values, and combined ARMA (p, q) estimates (see, e.g., Box & Jenkins, 1970; Cryer & Chan, 2008; Shumway & Stoffer, 2011).

Seasonal ARMA Processes

One of the advantages of the ARIMA/ARFIMA approach is that the number and size of the lags included in the predictive models are fully up to the investigator, and there may be substantive reasons to model predictions around particular lag sizes, such as cycles denoting the days of the week or months in a year. For the analysis of school attendance in particular, the 5 days of the week are of particular interest to estimate whether daily attendance rates have a seasonal cycle. Consequently, over and above the estimation of the impact of immediately neighboring values (i.e., attendance on the previous day or 2 days), as illustrated above, we would like to estimate the impact of last week's attendance rate. Does knowing the attendance rate on a given day of the week improve our prediction of attendance on that same day the following week? The trajectory shown in Fig. 14.1a illustrates the relevance of this estimation. Appendix 1 shows the formal modeling features of the seasonal ARMA model.

An empirical example of the weekly cycles is shown in Fig. 14.4a, which shows the daily attendance trajectory for School 2 in the 2009–2010 school year, as well as the ACF plot. While the cyclical dependencies may be difficult to detect in the time series, the ACF plot brings them out very clearly as a pronounced spike at the fifth lag. This ACF plot also points to the absence of short-range dependencies at other lag values. Figure 14.4b shows the residuals of the ARIMA model that successfully models the seasonal dependency at five lags ($\varphi_1 = 0.90$, $\theta_1 = -0.71$, and $\theta_2 = 0.16$). The trajectory on the left suggests randomness, and the ACF plot confirms that there are no remaining short-range dependencies in the data. The extreme values shown in Fig. 14.4a were modeled using an intervention analysis framework (Cryer & Chan, 2008, see Koopmans, 2011 for further details about that aspect of the analysis).

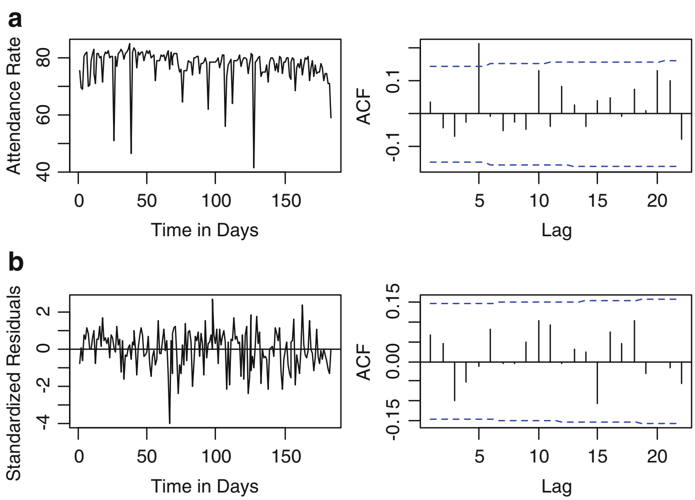

Fig. 14.4 (**a**) Trajectory of daily attendance rates in School 2 (*left* panel) with the corresponding ACF plot at the *right*; (**b**) residuals of a successful ARIMA model with corresponding ACF plot. $N = 183$

Long-Range Dependencies

The estimation of long-range dependencies helps determine whether there is evidence of self-organized criticality in the trajectories. Self-organized criticality would indicate that, as in the sand pile experiments discussed above, there are instances of critical instability and a repeating tension-release process in the face of continued input. In the data discussed here, perhaps long episodes of required attendance behavior create the need for incidental release, with a state of self-organized criticality immediately preceding this release. In time-sensitive measurements, indicators of self-organized criticality are the presence of self-similar patterns, and strong autocorrelations over a wide time spectrum. An important part of the data for long-range dependencies therefore is the detection of these two patterns.

Self-similarity refers to the replication of certain patterns at various scales, i.e., patterns within patterns. These patterns do not replicate in a strictly deterministic way. Rather, it is their *general impression* that remains the same (Beran, 1994). Figure 14.5 shows an example of self-similarity in the daily attendance rates in one school (School 3). The first panel (Fig. 14.5a) shows the daily attendance rate in that school over a 7-year period, from the fall of 2004 through the spring of 2011. Figure 14.5b shows those rates for one school year (2007), and Fig. 14.5c shows the rates for the fall of 2007. Figure 14.5d shows the rates for a 22-day period within the fall of 2007. Comparison of these four trajectories suggests self-similarity in the following three ways. There appears to be a slight downward trend in Fig. 14.5a that replicates itself at the smaller grid levels of Fig. 14.5b, c, and d. In addition, there

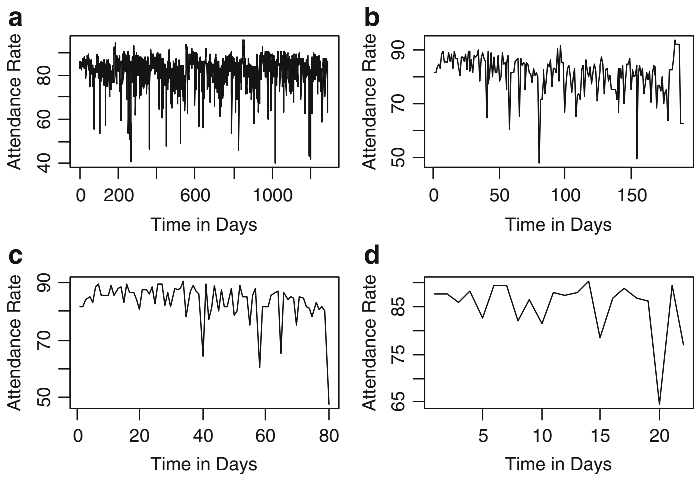

Fig. 14.5 Evidence of self-similarity in the attendance trajectory in School 3. (**a**) Attendance rates over a 7-year period (2004–2011, $N = 1290$); (**b**) rates in the same school over a 1-year period (2007, $N = 190$); (**c**) rates over the fall of 2007 only ($N = 80$); (**d**) rates over 22 days in the fall of that year

are pronounced dips that surface more toward the end of the series. Furthermore, at each level of description, variability appears to increase as the series progresses.

While this self-similar pattern is striking, not all features of the three trajectories replicate across scales: the last few observations toward the end of the trajectories show different variability patterns, and the lower dips do not necessarily occur at the same relative position of the time window. In the face of these conflicting signs, further statistical modeling is needed to empirically confirm the impression that daily attendance trajectories are indeed self-similar. As with the estimation of sensitivity to initial conditions, a large number of data points is needed to estimate a process hypothesized to replicate itself over and over in a scale-invariant manner.

The conventional ARIMA model described above is highly suitable to estimate such short-range dependencies, and a successfully fitted ARIMA model results in randomly distributed residuals. However, ARIMA models are not well suited for the detection and estimation of long-memory effects. The ARFIMA model is specifically designed to analyze the long-term fractional process that indicates self-similarity, by estimating the significance of the parameter d (the differencing parameter) over and above that of the autoregressive and moving average parameters. The use of ARFIMA to estimate long-range processes presumes a stationary trajectory, however. In case of non-stationarity, the investigator has the choice of analyzing the first (or second) difference, of the series, or resorting to different estimation methods altogether to detect fractality (Stadnitski, 2012a).

You may recall that in the short-term ARIMA (p, d, q) model, the parameter d is fixed to be zero if the trajectory is stationary, or $d = 1$ if it is non-stationary, in which case the first difference $Y_t - Y_{t-1}$ is analyzed. The *fractional* part of the ARFIMA (p, d, q) process refers to the fact that the detection of self-similarity through modeling of the long-range processes involves estimating fractions of d falling between $d = 0$ and $d = 1$. Dealing with the stationary case, ARFIMA also presumes that the differencing parameter d ranges from -0.5 to 0.5, with a $d = 0$ indicating no error dependency (white noise). A positive differencing parameter indicates a long-range positive autocorrelation pattern, also known as *persistence*. Conversely, a negative differencing parameter indicates a long-range negative autocorrelation pattern, referred to as *anti-persistence* (Beran, 1994; Stadnitski, 2012a; 2012b). Appendix 2 explicates ARFIMA formally.

A simulation with 1200 data points is shown in Fig. 14.6a, b to illustrate, respectively, short-range dependency in a simulated autoregression ($\varphi = 0.5$) and long-range dependency in a simulated fractal process ($d = 0.35$). The panels on the left show the relative stability of the autoregressive process in Fig. 14.6a compared to the more turbulent manifestation in Fig. 14.6b. The panels at the right of the figure show the characteristics of the corresponding ACF plots. The spikes indicating the size of the autocorrelations quickly recede to non-significance as the lag size increases in Fig. 14.6a, while in the plot in Fig. 14.6b the recession to non-significance proceeds very slowly, indicating persistence.

Figure 14.7a shows the mean-centered attendance trajectory in School 2 (see Fig. 14.5a for the original trajectory for this school), as well as the ACF plots at

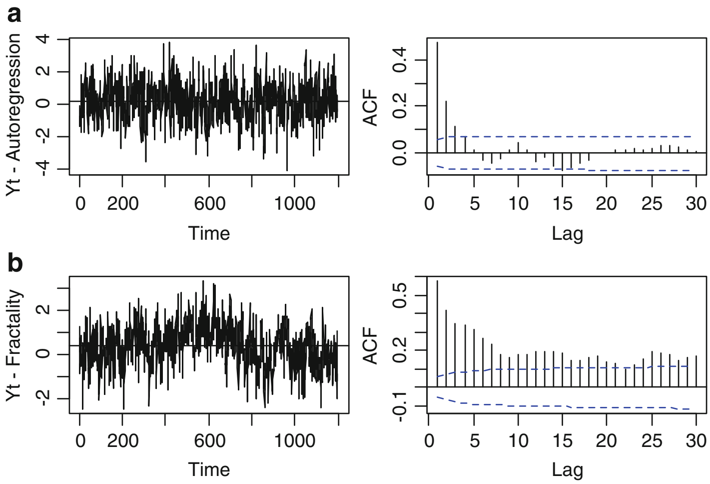

Fig. 14.6 Time series plot (*left* panels) and ACF plot (*right* panels) of (**a**) simulated autoregression ($N = 1200$, $d = 0$; $\varphi = 0.5$) and (**b**) simulated fractality ($N = 1200$, $d = 0.35$)

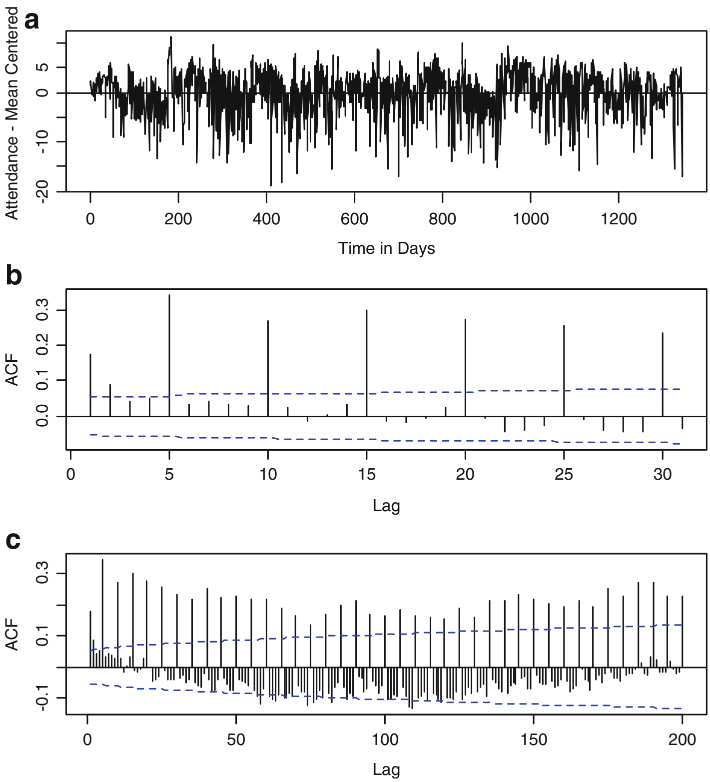

Fig. 14.7 Empirical data with short- and long-range dependencies (School 3); (**a**) attendance rates (mean centered); (**b**) ACF at 31 lags; (**c**) ACF at 200 lags

31 lags and at 200 lags (Fig. 14.7b and c, respectively). The short-term picture in Fig. 14.7b shows a rapidly decaying autocorrelations at the first few lags, as well as a seasonal cycle at the fifth lag that looks quite persistent. The longer term picture shown in Fig. 14.7c shows the persistence of the seasonal dependency as well as some evidence of nonseasonal persistence. Koopmans (2015) describes in greater detail the ARFIMA modeling process through which it was determined that the long-range dependencies made a statistically significant contribution to the variability in the data, even after modeling the short-range and seasonal processes in the trajectory. This analytical process yielded a differencing parameter of $d = 0.13$, indicating some degree of persistence over and above the short-range and seasonal dependencies.

In addition to the differencing parameter d, several other parameters are often used to characterize the dynamical process in time series data. One is the Hurst exponent H, named after Harold E. Hurst, who developed the measure to characterize the scaling dimension in such natural phenomena as water discharges, tree rings, temperature, and precipitation. Hurst originally defined the parameter in terms of the range R and standard deviation S of the measurement trajectories within given time periods to assess how the observed variability depends on the time ranges over which the measurements are taken. A linear correlation between the time range and measurement variability indicates long-range dependencies (Feder, 1988; Mandelbrot, 1997). Within the ARFIMA framework, the estimation of H is based on the differencing parameter d as $H = d + 0.5$. So then the interpretation of the differencing parameter provided above translates into an interpretation of the Hurst exponent as follows: $H = 0.5$ indicates white noise, $H > 0.5$ indicates persistence, and $H < 0.5$ indicates anti-persistence. Hence, the scaling component for School 3 equals $H = 0.63$, again indicating some persistence.

Power Spectral Density

To analyze fractal patterns in time series data, it is common practice to generate power spectra to assist with the detection of self-similarity. The conversion of a time series to a power spectrum involves a mathematical operation called Fourier transform (Shumway & Stoffer, 2011), which re-expresses the trajectory of observed measurements over time as a power versus frequency relationship as follows (Mandelbrot & van Ness, 1968):

$$S(f) \propto 1/f^{\beta}$$

In this function, f represents the frequency and $S(f)$ is the squared amplitude corresponding to that frequency (Delignières et al., 2005). The amplitude, or power, represents the magnitude of the variability in the cycles of dependency between observations at different lag values. The frequency in the spectrum is a *relative frequency*, which expresses the periodicity of the dependencies as $f = \frac{j}{n}$

with $j = 0, 1, 2, \ldots, (n - 1)/2$. Here, j represents the number of cycles and n the number of time points in the series (Shumway & Stoffer, 2011). Thus, the relative frequency ranges from $\frac{1}{n}$ to $\frac{1}{2}$ after the Fourier transform is carried out. Few iterations j represent the long-term process and many iterations represent the short-term (Eke et al., 2000), and the power or amplitude expresses the strength of the dependency between the observations that constitute the cycle.

A power spectrum is produced by log-transforming the relative frequency as well as the power of this function. A power spectral density plot is said to display a *power law* if the relationship of the log power to the log relative frequency is linear with a negative slope. Such a relationship would indicate that if one, for instance, doubles the frequency, the power diminishes by the same rate regardless of the frequency values chosen on the abscissa of the density plot. This feature indicates the *scale invariance* that is one of the signature characteristics of self-similarity (Eke et al., 2000). Generating power spectra is therefore of theoretical as well as diagnostic interest in such cases. The parameter β in the power function above is used to estimate the slope in this plot, assuming that the relationship is linear. This latter proviso is an important reminder that a careful inspection of the power spectrum is required to determine whether this assumption is actually met. For a lucid discussion of the interpretation of linear and nonlinear patterns in power spectra, see Wagenmakers et al. (2004).

A major advantage of power spectral density analysis over ARFIMA is its capability of distinguishing fractality in stationary as well as non-stationary trajectories, typically referred to as fractal Gaussian noise (fGn) and fractal Brownian motion (fBm). As you may recall, ARFIMA requires stationarity in the data, and in the absence thereof, differencing is used to make the data stationary. Some researchers have argued that such a transformation effectively removes intrinsically interesting features from the data, resulting in information loss (Granger & Joyeux, 1980). Comparison of Fig. 14.2b and c illustrates this point. The differencing accomplished in Fig. 14.2c removes many interesting particularities from the data trajectory, such as the lack of consistency of the behavior of the data from one time period to the next in the original series. This feature, which could have major substantive interest in the analyses at hand, completely disappears in the differenced transformation shown in Fig. 14.2c.

The estimation of the Hurst coefficient H, on the basis of which the presence of long-term dependencies is decided, requires a distinction between fGn and fBm processes. Eke et al. (2000) describe how power spectral density analysis can be used to that end. The criteria for deciding whether a given time series belongs to the fBm or the fGn family are described as follows: if the slope of the power spectrum based on an observed time series equals $-1 < \hat{\beta} < 0.38$, fGn should be assumed when estimating H. If $1.04 < \hat{\beta} < 3$, fBm should be assumed. If $0.38 < \hat{\beta} < 1.04$, the process is said to be unclassifiable in terms of fGn vs. fBn. In the fGn case, the theoretical relationship between the Hurst exponent and the

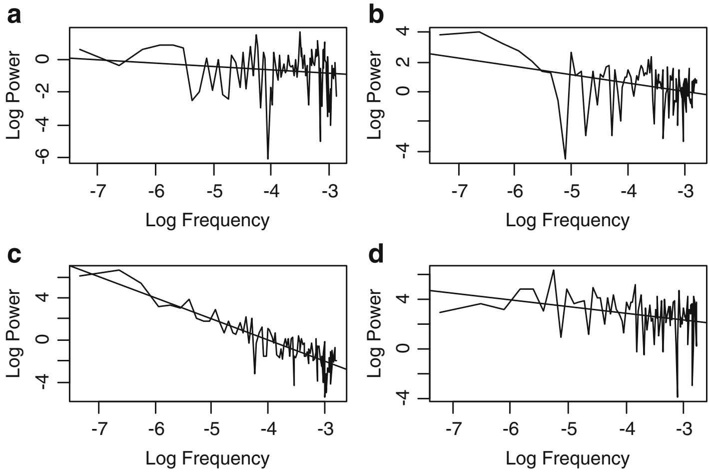

Fig. 14.8 Power spectra for (**a**) simulated white noise ($\beta = -0.21$), (**b**) simulated pink noise ($\beta = -0.57$), (**c**) simulated Brownian motion ($\beta = -2.0$), and (**d**) empirically observed daily attendance rates in School 3 ($\beta = -0.55$). The slopes for the simulated pink noise and Brownian motion spectra and for the attendance rates for School 3 are different from zero ($p < 0.05$)

power exponent β is $H = (\beta + 1)/2$; the power exponent can also be expressed as twice the differencing parameter estimated in ARFIMA, i.e., $\beta = 2d$. As indicated above, the Hurst exponent can be defined as $H = d + 0.5$. In the fBm case, $H = (\beta - 1)/2$. In both fGn and fBm processes, an H of 0.5 marks the boundary between persistence and anti-persistence (Stadnitski, 2012a, 2012b).

Figure 14.8a–c shows, respectively, what these power spectra would look like for simulated trajectories of white noise (no memory), pink noise (long-range memory), and Brownian motion (infinite memory). The ARFIMA simulation routine was used to generate white noise at $d = 0$ ($H = 0.5$) and pink noise at $d = 0.35$ ($H = 0.85$). Brownian motion was generated using phytools (Revell, 2012) with $\beta = 2.0$. In all three cases, the series were set to be 1500 observations long with a random normal distribution. The fourth panel (Fig. 14.8d) shows the power spectrum for School 3 ($N = 1290$ observations). The similarity between the power spectra for School 3 and the simulated pink noise in Fig. 14.8b is clearly discernible here, as are the differences between those two spectra on the one hand, and the white noise and Brownian motion spectra on the other. These differences can be appreciated both in terms of the steepness of the slopes and in terms of the amount of variability left around the fitted lines. As expected, the power function for white noise is flat; the slopes for the pink noise spectra fall well within Eke et al.' range for fGn, while the power spectrum for Brownian motion shows steeper slope with a narrow range of variances throughout indicating infinite memory (continued autocorrelation) in the entire trajectory.

Discussion

Few would argue that time plays a role in daily classroom and school-building activities, and the analysis of what the contingencies are that affect behavior in those contexts is a highly relevant undertaking as it might tell us what the underlying processes are of the transformations that constitute learning (Vygotsky, 1978). However, in educational science, our models of causal attribution tend to be cross-sectional, as we examine whether our instruction, leadership, policy, and other interventions impact the educational outcomes of our student population. These models are incomplete if the endogenous process is overlooked (Koopmans, 2014b). We need to know the behavior of interest over a larger time spectrum in order to understand the system's propensity toward transformation or toward maintaining the status quo. Knowing these propensities is important to qualify our causal attributions about educational effectiveness. Finding no relationship between interventions and outcomes may indicate that the system is resistant to change regardless of the (perceived) merits of the intervention in question. Likewise, it is possible that observed changes are not sustained in the long run in a highly flexible system as it deals with ever-changing adaptive requirements without sustaining the innovations whose effectiveness was demonstrated. We therefore need to acquire more knowledge about the internal systemic processes and how they behave over time because they reveal the system's predisposition toward change. Dynamical theories such as chaos theory and the theory of self-organized criticality are particularly concerned with such systemic propensities.

This chapter addresses two interrelated issues. The first one is that when the phenomena we study potentially have a temporal dimension, as may educational variables do, the contribution of this time dimension to the variability in one's observations needs to be investigated in a fair amount of detail to provide some meaningful answers about how endogenous processes contribute to the transformative process in education. Researchers may counter that longitudinal approaches such as survival analysis, repeated measures analysis of variance, and growth modeling can address this concern. However, these approaches differ from the ones described here in that traditional longitudinal techniques do not provide the degree of detail and resolution in the data that is required to estimate dynamical processes such as cyclical trends, or processes pointing to complexity such as self-organized criticality and sensitive dependence on initial conditions. The circumstances under which the time series approaches described are capable of capturing such complexity are a point of some contention in the dynamical literature. Eke et al. (2000) tested the reliability of fractality estimates using a time series of 2^{17} ($N = 131{,}072$), a length that is unlikely to have any meaningful empirical referents in education. Many researchers have proceeded with series of 2^9 or 2^{10} deemed sufficient for that purpose (Delignières et al., 2005; Stadnitski, 2012a). The challenge for nonlinear time series, in education as well as elsewhere, is the resource intensiveness of collecting information at this level of detail.

There is also a general point to be made about the cross-sectional use of central tendency and variability measures to address questions of educational effectiveness. The use of these measures presumes that the characteristics of interest are stable over time and that the time factor therefore does not have to be measured (the ergodic assumption, Molenaar, 2004). Given the dynamical nature of educational processes, it does not seem likely that the ergodic assumption holds very often; yet there are very few examples of the type fine-grained analyses that are needed to examine quantitatively the influence of time on the variability in our observations. This chapter illustrates one way of addressing this issue. Obviously, daily attendance rates are not the only variable of interest in the educational context. Important work to address the influence of time on educational outcomes and implementation variables includes several of the chapters included in this volume (Garner & Russell, Chap. 16; Pennings & Mainhard, Chap. 12; van Vondel, Steenbeek, van Dijk, & van Geert, Chap. 11), although the estimation of fractality is not the focus of that work.

The second concern addressed in this chapter is the fact that we know very little about how time contributes to school-level daily attendance rates in particular. The availability of a data repository covering more than a decade's worth of data by now has provided a unique opportunity to investigate the applicability of nonlinear time series in education, and learn more about how such attendance rates behave over the longer term. The analysis presented here indicates that in addition to the first-order autoregressive and moving average parameters that enhance the reliability of our descriptions, effective models incorporate seasonal estimators. Here, these estimators indicate that the 5-day weekly cycle exercises considerable influence over the patterns of variability found in the attendance trajectories. Practitioners may have been able to tell us about the seasonality of the daily attendance in their school buildings, but the formal research on high school attendance has traditionally had remarkably little to say about those patterns.

Of particular interest in the context of complexity research are the patterns in daily attendance that go over and above the seasonal influences noted above. The estimation of fractality or self-organized criticality is of interest because it points to complexity in the system as it adjusts to changing circumstances (Beran, 1994; Stadnitski, 2012b). This may be the case for schools as well, where schools showing fractality may have greater susceptibility to those influences whereas schools whose attendance trajectories do not show fractality may be more immune to them (Koopmans, 2015). Another aspect that is of importance to this discussion is the presence of many extreme observations that are likely to be tied to specific contingencies, such as snow days, upcoming vacations, and the irregularities associated with the end of the school year (Koopmans, 2011). Figures 14.1a, 14.4a, and 14.5a in this chapter illustrate the prominence of these observations. Irregularities of this kind are highly influential to the attendance trajectories, but they can usually be explained in terms of specific external contingencies, whereas the cyclical and long-range dependencies are often not as easy to account for. Particularly in those cases where schools show evidence of self-organized criticality or fractal patterns in the trajectories, the development of strong theories to explain those patterns becomes a pertinent issue, requiring investigator to collect additional information about putative causal influences such as parental support, teacher quality, and school responsiveness to student absences.

To develop strong causal theories about attendance behavior, in other words, it is necessary to triangulate the quantitative characteristics of school-level daily attendance trajectories with data from other sources to find out more about the factors that produce irregularity in attendance behavior as well as what the determinants are of self-organized criticality in the schools. Are small high schools to be more likely or less likely to display self-organized criticality? Are schools serving predominantly students from poor families more or less likely to show such patterns? It is up to empirical research to address these questions to help us understand better why given attendance rates are what they are, as well as to theory to articulate the putative causal mechanisms.

In this context, it is also relevant to contemplate what it means to say that there is self-organized criticality in the attendance trajectory for a given high school. Figure 14.5 in this chapter illustrates what it looks like in one school. The patterns shown there seem to suggest a fatigue dynamic, where initial cycles of high attendance/low variability are followed by higher variability and then lower peaks. This pattern appears to replicate in this trajectory in a scale-invariant manner, which is to say that it occurs over large time frames (e.g., a 7-year period), but also in much smaller time frames residing within those larger ones. The value of attendance research from a complexity perspective is that, contrary to the seasonal cycles that are easy to discern for school-building practitioners, these self-similar patterns are much harder to detect let along confirm, while they nonetheless have important implications for policy.

To estimate fractality, Wagenmakers et al. (2004) recommend a competitive modeling approach, along the lines of a stepwise multiple regression, where the statistical goodness-of-fit models including all the short-term estimators of interest are compared to a model including all of those as well as a differencing parameter estimate. Koopmans (2015) shows the applicability of this modeling strategy to daily school attendance trajectories. The literature advises caution when concluding self-organized criticality based on evidence of persistence in time series data, because the possibility remains that the appearance of persistence may in fact mimic a pattern of short-range dependencies (note that d is not lag specific in the general ARFIMA formulation shown in Appendix 2). Therefore, a careful inspection of the plotted trajectories and ACF plots is always indicated, as well as the triangulation of the statistical evidence from ARFIMA with other sources of information that may provide a more substantive description of the dynamics underlying long-range dependencies to help develop a strong causal theory to explain the results of time series analyses.

In closing, I'd like to stress the merits of single-case designs to enhance our understanding of educational processes, and the attendance data presented here are meant to illustrate that point as well. We can learn from studying the particularities of bounded individual systems and investigate in great detail the processes of self-maintenance and transformation as they play out over a large time spectrum and in the interactions between individual agents within the system (students, teachers, administrators, policy makers) and the larger systemic components (classrooms, school buildings, districts, federal agencies) with which these agents interact in an ongoing dynamical interrelationship. In educational science, a distinction is traditionally made between qualitative research, which tends to focus on the particular and quantitative research, which is oriented toward the analysis of data for purposes of statistical inference. The research presented here argues from a complexity angle for the obsolescence of the idea that quantitative and qualitative research are mutually exclusive empirical strategies. The richness of detail provided by the single case uniquely allows for a rigorous quantitative assessment of the dynamical underpinnings of behavior, as well as revealing its qualitative transformations.

Appendix 1: Short-Range Estimation Using ARIMA

The general model AR can be stated as

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + e_t$$

This model estimates Y_t using p lags. The parameter ϕ estimates the influence of past observations on the series at each given lag.

The MA model estimates Y_t in terms of accumulated error disturbances, also called innovations. Using q lags, this estimation can be written as follows:

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

In this equation θ estimates the impact of each innovation on the series.

AR and MA processes can be captured in a single predictive model. For purposes of clarity, we describe a predictive model that uses one lag only, i.e., $p = 1$ and $q = 1$:

$$Y_t = \phi_1 Y_{t-1} + e_t - \theta_1 e_{t-1}$$

A special case is the *seasonal ARMA process*, which estimates the dependencies in terms of days of the week, months in a year, etc. The analysis presented here focuses on the regularities as a cyclical weekly pattern with 5 days in the school week. The model used to address this question can be formally written as

$$Y_t = \phi_1 Y_{t-5} + e_t - \theta_1 e_{t-5}$$

The autocorrelation function (ACF) at lag k is defined as

$$r_k = \sum_{t=k+1}^{n} \left(Y_t - \overline{Y}\right)\left(Y_{t-k} - \overline{Y}\right) / \sum_{t=1}^{n}\left(Y_t - \overline{Y}\right) \quad \text{for } k = 1, 2, \ldots$$

Appendix 2: Long-Range Estimation Using ARFIMA

Some mathematical reorganization of the terms in the ARIMA model as stated in Appendix 1 is required to describe what the estimation of the long-range influences adds to the models that assess the short-range effects on attendance trajectories.

It is often conventional in time series notation to express ARMA processes in terms of the so-called *lag operator*, or *backshift operator*, which is defined as

$$BY_t = Y_{t-1}$$

In plain English, the backshift operator B shifts observations back one time unit to construct a new series. The next lag over can be written as $BBY_t = Y_{t-2}$, or

$$B^2 Y_t = Y_{t-2}$$

In terms of this operator, the ARIMA process described above is often written as

$$\left(1 + \varphi_1 B + \varphi_2 B^2 + \cdots + \varphi_p B^p\right) Y_t = \left(1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q\right) e_t$$

The left side of the equation represents the autoregression (AR) component; the moving average (MA) component is on the right. The mathematical derivation of this formulation, called the *characteristic equation*, from the equations above can be found in Box and Jenkins (1970), Cryer and Chan (2008), and many other standard time series texts. It is assumed in this model that remaining error is randomly distributed, i.e.,

$$e_t \ (t = 1, 2, \ldots) \sim N\left(0, \sigma^2\right) \text{ IID.}$$

The ARFIMA model separates long-term dependencies from the short-term ones by parameterizing d as a differencing estimate:

$$(1 + \varphi_1 B + \varphi_2 B^2 + \cdots + \varphi_p B^p)(1-B)^d Y_t = (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q) e_t$$

It is assumed here that the trajectory is stationary and that $-0.5 < d < 0.5$ (Beran, 1994; Sowell, 1992; Stadnitski, 2012b).

References

Ashby, W. R. (1957). *An introduction to cybernetics*. London: Chapman & Hall.
Astone, N. M., & McLanahan, S. S. (1991). Family structure, parental practices and high school completion. *American Sociological Review, 56*, 309–320.
Bak, P. (1996). *How nature works: The science of self-organized criticality*. New York: Springer.
Balfanz, R., & Byrnes, V. (2012). *The importance of being in school: A report on absenteeism in the nation's public schools*. Baltimore, MD: Johns Hopkins University Center for Social Organization of Schools.
Beran, J. (1994). *Statistics for long-memory processes*. New York: Chapman & Hall.
Box, G. E. P., & Jenkins, G. M. (1970). *Time series analysis, forecasting and control*. San Francisco, CA: Holden-Day.
Cryer, J., & Chan, C. (2008). *Time series analysis with applications in R*. New York: Springer.
Delignières, D., Torre, K., & Lemoine, L. (2005). Methodological issues in the application of monofractal analyses in psychological and behavioral research. *Nonlinear Dynamics, Psychology, and Life Sciences, 9*, 435–462.
Eke, A., Herman, P., Bassingthwaighte, J. B., Raymond, G. M., Percival, D. B., Cannon, M., et al. (2000). Physiological time series: Distinguishing fractal noises from motions. *Pflügers Archives, 439*, 403–415.
Feder, J. (1988). *Fractals*. New York: Plenum.
Fuller, W. A. (1996). *Introduction to statistical time series* (2nd ed.). New York: Wiley.
Glass, G. V. (1972). Estimating the effects of interventions into a non-stationary time series. *American Educational Research Journal, 9*, 463–477.
Goldstein, J. (1988). A far-from-equilibrium systems approach to resistance to change. *Organizational Dynamics, 17*, 16–26.
Granger, C. W. J., & Joyeux, R. (1980). An introduction to long-range time series models and fractional differencing. *Journal of Time Series Analysis, 1*, 15–30.
Hamilton, P., Pollock, J. E., Mitchell, D. A., Vincenzi, A. E., & West, B. J. (1997). The application of nonlinear dynamics to nursing research. *Nonlinear Dynamics, Psychology, and Life Sciences, 1*, 237–261.
Hosking, J. R. M. (1981). Fractional differencing. *Biometrika, 68*, 165–176.
Hurst, H. E. (1965). *Long-term storage: An experimental study*. London: Constable.
Jensen, H. J. (1998). *Self-organized criticality: Emergent complex behavior in physical and biological systems*. Cambridge: Cambridge University Press.
Kantz, H., & Schreiber, T. (2004). *Nonlinear time series analysis* (2nd ed.). New York: Cambridge University Press.
Kaplan, D., & Glass, L. (1995). *Understanding nonlinear dynamics*. New York: Springer.
Kemple, J. J., Segeritz, M. D., & Stephenson, N. (2013). Building on-track indicators for high school graduation and college readiness: Evidence from New York City. *Journal of Education for Students Placed at Risk, 18*, 7–28.
Kemple, J. J., & Snipes, J. C. (2000). *Career academies: Impacts on students' engagement and performance in high school*. New York: MDRC.
Kerlinger, F. N. (1977). *Foundations of behavioral research* (2nd ed.). London: Holt, Rinehard & Winston.
Koopmans, M. (2009). Epilogue: Psychology at the edge of chaos. In S. J. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: The theory of nonlinear dynamical systems* (pp. 506–526). New York: Cambridge University Press.
Koopmans, M. (2011, April). *Time series in education: The analysis of daily attendance in two high schools*. Paper presented at the annual convention of the American Educational Research Association, New Orleans, LA.

Koopmans, M. (2014a). Nonlinear change and the black box problem in educational research. *Nonlinear Dynamics, Psychology and Life Sciences, 18*, 5–22.

Koopmans, M. (2014b). Change, self-organization, and the search for causality in educational research and practice. *Complicity: An International Journal of Complexity in Education, 11*, 20–39.

Koopmans, M. (2015). A dynamical view of high school attendance: An assessment of the short-term and long-term dependencies in five urban schools. *Nonlinear Dynamics, Psychology and Life Sciences, 19*, 65–80.

Lewin, K. (1947). Frontiers in group dynamics. *Human Relations, 1*, 5–41.

Mandelbrot, B. B., & van Ness, J. W. (1968). Fractional Brownian motions, fractional noises and applications. *SIAM Review, 10*, 422–437.

Mandelbrot, B. B. (1997). *Fractals and scaling in finance: Discontinuity, concentration, risk*. New York, NY: Springer.

Molenaar, P. C. M. (2004). A manifesto on psychology as an idiographic science: Bringing the person back in to scientific psychology, this time forever. *Measurement, 2*, 201–218.

National Center of Education Statistics. (2008). *Average daily attendance in public elementary and secondary schools, by state or jurisdiction: Selected years, 1969–70 through 2005–06*. Retrieved June 30, 2010, from http://nces.ed.gov/programs/digest/d08/tables/dt08_040.asp

Neter, J., Wasserman, W., & Kutner, M. H. (1985). *Applied linear statistical models: Regression, analysis of variance, and experimental designs*. Homewood, IL: Irwin.

Peng, C. K., Mietus, J., Hausdorff, J. M., Havlin, S., Stanley, H. E., & Goldberger, A. L. (1993). Long-range anti-correlations and non-Gaussian behavior of the heartbeat. *Physical Review Letters, 70*, 1343–1346.

Revell, L. J. (2012). phytools: An R package for phylogenetic comparative biology (and other things). *Methods in Ecology and Evolution, 3*, 217–223.

Roby, D. E. (2003). Research on school attendance and student achievement: A study of Ohio schools. *Educational Research Quarterly, 28*, 4–15.

Shumway, R. H., & Stoffer, D. S. (2011). *Time series and its applications* (3rd ed.). New York: Springer.

Sowell, F. (1992). Modeling long-run behavior with the fractional ARFIMA model. *Journal of Monetary Economics, 29*, 277–302.

Sprott, J. C. (2003). *Chaos and time series analysis*. New York: Oxford University Press.

Stadnitski, T. (2012a). Some critical aspects of fractality research. *Nonlinear Dynamics, Psychology, and Life Sciences, 16*, 137–158.

Stadnitski, T. (2012b). Measuring fractality. *Frontiers in Physiology, 3*, 1–13.

Sulis, W. H. (2012). Causal tapestries for psychology and physics. *Nonlinear Dynamics, Psychology and Life Sciences, 16*, 113–136.

Thornton, P. L., & Gilden, D. L. (2005). Provenance of correlations in psychological data. *Psychonomic Bulletin & Review, 12*, 409–441.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wagenmakers, E. J., Farrell, S., & Ratcliff, R. (2004). Estimation and interpretation of $1/f^{\alpha}$ noise in human cognition. *Psychonomic Bulletin & Review, 11*, 579–615.

Waldrop, M. M. (1992). *Complexity: The emerging science at the edge of order and chaos*. New York: Simon & Schuster.

Wiener, N. (1961). *Cybernetics, or control and communication in the animal and the machine* (2nd ed.). Cambridge, MA: MIT Press.

Chapter 15
Educational Systems and the Intergenerational Transmission of Inequality: A Complex Dynamical Systems Perspective

Porfirio Guevara and Emilio Porta

Introduction

It is one of the greatest puzzles of our time. Globalization and technological change has lifted hundreds of millions of people out of poverty and improved the lives of many more that, thanks to the internet and advances in education, have joined the global supply networks everywhere, mainly in developing countries during the last half-century. Paradoxically, this trend has also intensified internal divisions in society, like those based on income distribution. Individuals with better skills have managed to outstrip unskilled workers as the formers' knowledge has facilitated access to well-paid jobs and investment opportunities. Unskilled workers, on the other hand, are more likely to access jobs that compete more directly with automatic processes performing routine-intensive activities that affect their employment and returns opportunities. The numbers are astonishing; worldwide, some 780 million adults and 126 million youngsters still lack the most basic reading and writing skills (UNESCO, 2015). As a result, income or wealth-based inequalities has been reported on the rise everywhere (Piketty, 2014; Ravallion, 2014). This is a reminder that despite the startling technological advance of the modern world in solving many of today's most pressing scientific and engineering challenges, the complexities of the systems in which most human activities are embedded prevent us from taking full control of even our good intentions to provide inclusive social and economic progress for everyone.

Although social and neural scientists are still trying to disentangle the multiple causes of inequality and policy measures are currently subject to an intense debate, the role of educational systems on human capital in an increasingly technologically connected society are at the center of deliberations (Noble et al., 2015; Porta & Laguna, 2007). Educational systems are one of the main sources of skills and

P. Guevara (✉) • E. Porta
INCAE Business School, Alajuela, Costa Rica
e-mail: porfirio.guevara@consultor.incae.edu; eporta@gmail.com

© Springer International Publishing Switzerland 2016
M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_15

productivity available to a country. At an individual level, researchers have long established a strong causal link between advances in school attainment and individual earnings via improvements in productivity, and therefore productivity is crucial to explain how much workers earn.[1] But workers' earnings depend not only on specific issues related to their productivity to perform determined tasks, earnings also depend on the pool of additional workers currently available in a country to accomplish such tasks, for which the underlying educational system is crucial. If, on average, only a reduced fraction of individuals finish the school on time—i.e., there is a sizeable fraction of students that repeats or drops out—then one would expect to see a shallow pool of skilled workers in this country and as a consequence high returns to schooling, which would be one of the main sources of inequality. Moreover, if this process is prolonged over time and it systematically targets specific groups, we just need a dominant positive feedback in the system reinforcing small differences to attain the intergenerational transmission of inequality.

Feedbacks are an essential component of any complex dynamical system and positive—or reinforcing—feedbacks have been extensively identified in educational systems (see Koopmans, 2014). Constituent elements in an educational system change and react over time—usually in nonlinear ways—with the collective patterns they create amplifying original differences. One may naturally think, for example, on the influence that the aggregate characteristics of a community has on individual schooling decisions. Students from low-income families are more likely to repeat or dropout the school, starting out at a disadvantage in the labor market and in that way restricting their earnings and likely, those of their offspring. The educational system rewards disproportionally those who complete the process but additionally penalizes extensively who fail to do so as the reverberations of these outcomes are transmitted through generations. Thus the role of educational systems and their efficiencies must be placed at the center of the debate on the transmission of inequalities and social mobility for the purpose of understanding them and designing possible strategies to address them.[2] Consequently, to approach inequality in a relevant dimension our analytical framework must grasp the dynamics of the whole system and not only the behavior of its individual components. Under this perspective two prominent approaches can be applied in the analysis:

(a) Individual-based interventions, which focus on cognitive skills and learning trajectories attained by students during the instruction process

---

[1] The dominant theory of human capital formation is rooted in economics and owns its relevance to outstanding contributions by Mincer (1958), Becker (1962), and Becker and Chiswick (1964). These celebrated authors established the central role of education to explain earnings' differences and inequalities in society, and their ideas have been subject to mounting empirical scrutiny by authors like Hanushek (2009, 2014), Autor (2014), Ravallion (2014), among others.

[2] By *efficiency* we mean the ability of an educational system to graduate the maximum number of students had children entered school at normal age and advanced one grade each year, without repetition or dropout.

and the influence of school and socioeconomic factors on those trajectories (Noble et al., 2015).[3]

(b) Improving the education production process by enhancing the operational activity of human capital generation at macro level.

The approach we follow in this chapter is of an operational and aggregate nature and thus the second category is the relevant to our analysis. The overall behavior of a complex system cannot be deduced from its constituent elements in isolation—which can be regarded as the emergent property—and therefore the analysis of an educational system can be enriched from a modeling perspective by adopting a macro perspective that accounts for the interactions of its elements. When the relevant unit of study is set to be at aggregate or macro-level, it becomes much simpler to focus on the average performance of the students in a system without losing relevant information for the analysis. Additionally, as the macro-level of any complex system is governed by the laws of physics (Carroll, 2010), once we integrate these laws in our model a certain amount of discipline is imposed in its structure increasing the reliability of simulations over long periods of time since these laws are expected to remain unchanged over time.[4] Thus, complex modeling and simulation grounded on scientific principles offers a sound and reliable methodology to understand the relationship between the structure of an educational system and the behavior driving the intergenerational transmission of inequality, as many of these relationships mainly emerge over long periods of time.

To perform the analysis we present a dynamic, nonlinear system dynamics simulation model for primary education, calibrated for the case of Nicaragua during the period 2000–2010, in a similar fashion to the one described by Guevara, Lopez, Posch, and Zuniga (2014). We also illustrate how the model can be extended to disaggregate population by income/wealth and by their opportunities to finish primary school. We believe this approach will help us understand how educational systems work in reality by making explicit some of the channels and feedbacks that influence the relationship between income/wealth and education across

[3] The individual-based interventions which focus on cognitive skills and learning trajectories attained by students during the instruction process and the influence of school factors on those trajectories have also a great deal of interest in this book. See for instance the analysis of learning trajectories over time and the influence of the classroom interactional context by Steenbeek and van Geert (University of Groningen, Netherlands); or the use of orbital decomposition to study the predictability of learning behaviors and patterns of social interaction in educational settings by Stamovlasis (Aristotle University of Thessaloniki, Greece).

[4] The model complies with the first two laws of physics. The First Law (conservation of the matter) states that the amount of people entering the system must not be different from that that ever goes out, ruling out the possibility that the simulation model creates people artificially due to a human error in the computer code. The Second Law proposes that the entropy of a closed system cannot decrease and time has only one direction (see Guevara, 2014).

generations. Under this perspective, we find insightful to portray (the lack of) equality as a *critical factor* of the human capital process—following Guevara and Posch (2015)—and show how income inequality might impact the overall operational efficiency of the system. Our intention is to draw a methodological line related to the transmission of inequalities from a complex system perspective that can be extended and refined in future studies for the purpose of designing and evaluating policies to tackle income distribution in a country via the efficiency of its educational system. This CDS simulation model thus will allow us to draw alternative causal inferences to those documented in studies using simple correlations as in Hanushek (2009) or Hanushek and Woessmann (2014).

Assessing a complex system's topology using correlational methods is helpful albeit insufficient due to the nature of this study. The interactions we aim to capture in our model are embedded in a complex web of multiple subsystems and variables producing outcomes that feedback to these subsystems and their components. Thus we require information about the multiple components' roles in a system and their mutual and simultaneous interplay which likely go beyond correlational procedures (see Guevara et al., 2014). Another fundamental omission in traditional statistical analysis arises from its static nature. It normally takes a snapshot of the complexities of the human capital process over time and its impact on the transmission of inequalities through generations. In a dynamic context, when skills and opportunities for social mobility are to a great extent determined by the economic or social family background, their effects go beyond the direct impact on the actual individuals perceiving such benefits, as it takes the form of an intergenerational wealth transfer. Therefore we need approaches that explicitly deal with these issues and help us to answer critical questions like:

- How can we model the simultaneous and dynamic interrelationship between the efficiency of educational systems and income distribution?
- What are the consequences of income-based unequal opportunities in education systems dominated by self-reinforcing causal relationships?
- What will happen to the school attainment of current and future generations if such causal relationships are held over the long term?

The current empirical literature does not provide answers to these questions and this study aims to start the debate. The intergenerational transmission of inequality is not less controversial from an academic perspective given its complex nature and multidimensionality. Multiple channels of influence interact via feedback mechanisms making clear-cut conclusions difficult to wage. We argue that inequality and low social mobility are not only bad for those individuals born in disadvantaged households; it is also detrimental for the efficiency of the whole educational system which in turn may have implications for the long-term productivity and social and economic progress of countries. As we show next, Nicaragua presents several characteristics that make the country suitable for the analysis from a complex dynamical systems perspective.

The Case of Nicaragua

It is very much the case in Latin America and other regions in the world that more income inequality is associated with less opportunities for the new generations to advance due to low educational mobility (see Fig. 15.2). Low educational mobility in this context means the family background is determinant and a large fraction of socioeconomic advantages and disadvantages are passed on from parents to children, generating a self-perpetuating behavior in the system. In short, more inequality at any point in time is associated with a greater transfer of educational (and consequently economic) status across generations.

The Nicaraguan educational system is a conspicuous case in this regard as it shows certain regularities in its behavior suggesting that an underlying structure is driving the observed outcomes. For example, (1) income and wealth are unequally distributed among the Nicaraguan population and this condition is fairly stable across time and directly projected in the school population (Table 15.1) that reinforces these results. While some improvement in equality is observed over the past three decades, income inequality still remains large as shown by the Lorenz curves in Fig. 15.1.[5] The closer the Lorenz curve is to the equal-distribution line the better the income distribution in the country. So for the last two years that information is available, 2005 and 2009, the country improved its income distribution (2) Most individuals in the upper quintiles of income start and complete primary education without delay, as represented by high promotion and low dropout and repetition rates, while those in the lower quintiles of income are predominantly underperforming in the same terms. In Table 15.2 children in the first quintile (the poorest) have completion rates below 75 % while the richest show promotion rates of 95 %. Similarly repetition and dropout rates are 3 to 7 times higher in the lower quintiles than in the highest quintiles, respectively. These differences tend to remain stable over time. Likewise, Nicaraguan families with high educational attainment tend to have fewer and better educated children

Table 15.1 Nicaragua 2001: Primary-school enrollment by household wealth

Quintile	Students	Percentage (%)
I (poorest)	231,672	26.13
II	225,540	25.44
III	198,213	22.35
IV	161,575	18.22
V (richest)	69,683	7.86
Total	886,683	100.00

Source: LSMS 2001

[5] The Lorenz curve is often used to represent income distribution and shows the proportion of income or wealth (y %) accrued by the bottom x % of the population. A perfectly equal income distribution would be one in which the bottom x % of society would always have x % of the income and can be depicted by the straight line $y = x$ which is called the "line of *equidistribution*".

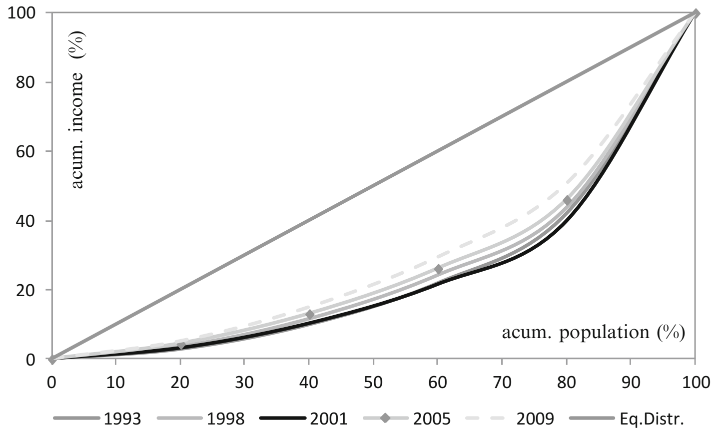

Fig. 15.1 Nicaragua 1993–2009: Lorenz curves. Source: constructed with data from SEDLAC (2015)

Table 15.2 Nicaragua 2001: promotion, repetition, and dropout in primary by household wealth

Quintile	Promotion	Repetition	Dropout
I (poorest)	74.5	9.7	14.9
II	83.7	6.3	9.5
III	87.0	4.2	8.5
IV	92.1	3.2	4.5
V (richest)	94.9	2.8	2.2

Source: Living Standards Measurement Survey (LSMS), 2001 (Porta, Arcia, Macdonald, Radyakin, & Lokshin, 2011)

and these children tend to repeat and dropout less than those in less educated families (World Bank, 2001). Consequently, educational and social mobility is very low in the country, Nicaragua scores very low in mobility (very high position in Fig. 15.2) even respect to other Latin American countries to which it is often compared (Andersen, 2001; SEDLAC, 2015). So Fig. 15.2 shows countries ranked from low to high inequality (left to right): Argentine, Peru, Nicaragua, and Bolivia being the most equal countries, and Brazil, Paraguay, and Honduras being the least. On the other hand, moving along the vertical axis from bottom to top represents a movement from more mobility in educational status across generations to less educational mobility. In countries such as Argentine, Bolivia, and Ecuador, the correlation between parental economic status and the adult outcomes of children is the weakest: Less than 10 % of any educational advantage or disadvantage that a father had had is passed on to a son in adulthood. In contrast, in Honduras, Panama, and Nicaragua, more than 15 % of any advantage or disadvantage is inherited by the next generation. If a father had twice the average of years of education in Bolivia, for example, he would expect his son to end up having only about 8 % above average; in Nicaragua, this would be more than 20 %. In such settings the Nicaraguan poor are

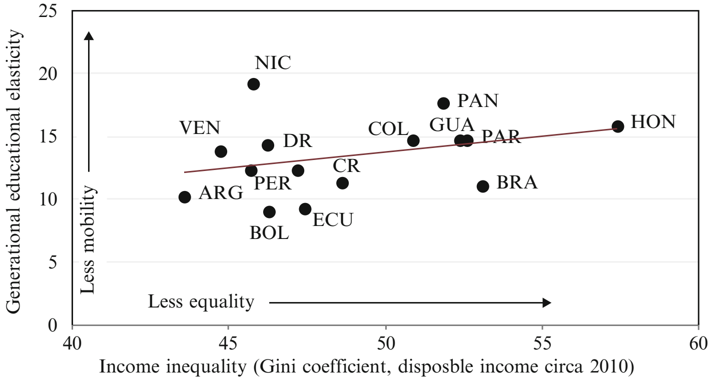

Fig. 15.2 More inequality is associated to less mobility in Latin America. Source: constructed with data from SEDLAC (2015) and World Bank (2015)

more likely to see their children growing up to be the next generation of poorly educated people, and the rich are more likely to see their children at the top rungs of the social ladder. Therefore, the large disparities that exist in the education system of Nicaragua just replicate the inner pattern of income inequality. This result also holds on a global scale and among regions as confirmed by Porta (2011) in more than 80 countries and it is what we would like to capture in our model.

The Model

When attempting to understand the complex dynamic behavior of an educational system we first need a fair understanding of the underlying structure—in term of its stocks, flows and feedbacks—that may influence the system's observed behavior. A *stock* variable is something that can be accumulated, i.e., water in a reservoir or population in a country. It is measured at one specific time and that measurement represents a quantity existing at that point in time (say, persons). A *flow* variable is analogous to the mathematical concept of *rate*, which measures a variable over a period of time and when coupled to a stock, the flow variable is measured in the same units of the stock *per time unit* (say, persons per year). Finally, feedbacks are closed chain of interactions between the elements of a system forming a loop that can be of two classes: positive and negative. Positive feedback loops are self-reinforcing (more population—more births—more population). Negative feedback loops are self-correcting as they counteract change (larger population—more deaths—smaller population).

Stock and flow variables are natural candidates to be included in any educational system structure because *time* is intrinsically embedded in these variables and it is possible to identify and capture the components' mutual influences as well as their direction of influence.

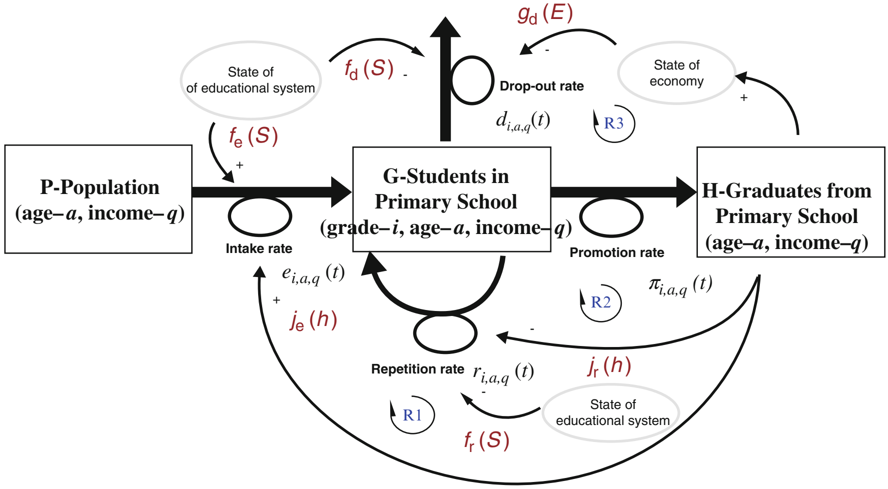

Fig. 15.3 State variables (stocks) are represented by *rectangles* and are disaggregated by age-*a*, income-*q*, and grades-*i*. Flows change the values of these stocks and are represented by *arrows*. The other variables such as the *state of the education system*, *state of the economy*, and *graduates from primary school* (human capital, *h*) are the critical factors that modify these flows (nonlinearly) and generate positive feedbacks such as R1, R2, and R3 (reprinted with permission, see Guevara et al., 2014)

A well-documented feedback in demographic educational modeling and simulation is the *assortative mating* characteristic that suggests that educated families are more likely to send their offspring to school where they can meet peer students, or in its dynamic version—the *role model effect*—as more educated households place education a top priority for the next generations (Behrman & Rosenzweig, 2005; Durlauf, 1998, Morrison, 2008). In the context of a causal loop diagram, we can capture this effect using the population of literates whose effects influence directly the system's transition rates (see reinforcing feedbacks R1 and R2 in Fig. 15.3). High literacy in a country reduces repetition rates because more literate parents persuade and are persuaded by their peers to support children's academic activities and their collective efforts are more effective (Durlauf, 1998; Oreopoulos, Page, & Stevens, 2006). This leads to an improvement in promotion rates leveraging primary graduates which also increases the amount of literates in the population. A second causal loop effect captures the influence of educated population on aggregate economic growth (Dowrick, 2004). A country with a sustained economic growth is more likely to improve households' budgets and support youngsters' education because there will be more enrollment and less dropout. In such setting more students finish primary school *ceteris paribus* (R3 in Fig. 15.3), and the share of persons with complete primary education increases improving human capital in the country. More human capital in turn improves economic growth in the long run when a more educated labor force exploits better economic opportunities in the market and more efficiently, boosting up the country's productivity. Notice that extending the previous feedbacks loops to include income-based differences in a population is straightforward as we only have to disaggregate the same variables included in these loops by wealth or income percentiles. These positive feedbacks spawn the conditions for income-based feedbacks that would induce an intergenerational transmission of inequalities in society.

To grasp how the whole structure works in an educational context one may begin dividing up the entire course of school levels into grades, represented by stock variables through which a population flows via transition rates: intake, repetition, dropout, and promotion. Clearly, these transitions occur from the first to the last grade in school; flow-variables capture these processes via differential equations for intake (e), repetition (r), dropout (d), and promotion (p)—as shown in Fig. 15.3—while feedbacks are the mechanisms driving these dynamical processes. This chapter uses the primary school completion rate (PCR)—which measures the number of graduates from primary school in a given year as a proportion of the total number of children in the population reaching the appropriate age for graduation—as an output indicator to track progress, efficiency, and the dynamics of education systems. The system dynamics model presented here builds upon Guevara et al. (2014) who presented a model of the Nicaraguan educational system originally disaggregated by age and grade only, which we extend by disaggregating all population stocks and their respective inflows and outflows by quintiles of income.[6] An income-disaggregated population thus is more relevant for our purpose because beyond the simple age-grade disaggregation, a richer picture emerges due to the indisputable relationship between each income-group and the educational transition rates: intake, repetition, dropout, and promotion. As we climb up the ladder of income quintiles in the system, from the lowest to the highest, intake rates increase unambiguously while repetition and dropout decrease monotonically.

Three *critical factors* are included in the model: the *state of the education system (S),* the *index of human capital (or the state of adult literacy)* in the country (h), *and* the *state of the economy (E).*[7] To explicitly capture feedbacks and nonlinear relationships in the model, all parameters governing transition rates in the model are specified through the multiplicative interaction of their respective initial values (at time $t = 2{,}000$) and the nonlinear effects that the critical factors exert on each parameter through a number of functions f, g, j, that make explicit the nonlinear impact of variables S, E, and h on enrollment (e), repetition (r), and dropout (d).

---

[6] We calibrated the model using a complete set of quantitative information circa 2000 (most of data used in the model comes from the 2001 LSMS). Before 2001, primary-school data available were not disaggregated by income.

[7] A particularity of these factors is that they cannot be developed or purchased instantaneously; they resemble stocks, which thus must be accumulated over time to reach a particular level. For instance, the *state of adult literacy* in a population cannot be raised immediately; it has to be developed through the transmission of basic learning capabilities on to children, which takes several years. So to explicitly use adult literacy as a critical factor in this model, the flow of primary school graduates is accumulated in a stock.

These critical factors interact nonlinearly with the model components in a closed chain of causal relationships. This is explained in some detail next (Fig. 15.3).

The state of the education system (S) indicates the presence of adequate physical space, supporting personnel, and all related amenities (power, water and toilets, chalkboards, chairs, etc.) that make school activities suitable for students. School infrastructure in this model comprises a stock that increases with newly built classrooms and decreases with those that wear out after a period of 20 years of activity. Classroom requirements are measured considering the actual amount of students in the system and an observed good practice of maintaining an average of 30 students per classroom. "Saturated" classrooms reduce enrolment and increase repetition and dropout.

The *index of human capital* (h) measures the share of graduates from primary school as a proportion of the relevant population. This share has a direct influence on enrollment and repetition. This index also affects dropout indirectly via the state of the economy (R3 in Fig. 15.3). Currently enrolled students in primary education have only three possible directions r, d, or p. While d and p are both exit strategies in this system, the latter is clearly preferred to the former as school graduates are expected to have the skills and experience intended for them. So at the end of the school course, graduate students can be accumulated in a stock we label *human capital*.

The *state of the economy* (E) is used to quantify economic progress through a measure of relative per capita Gross Domestic Product (GDP) in the country. The relative income measure is the per capita GDP at any point in time compared to that recorded in the country in year 2000.[8] GDP grows at 5 % per year on average and this growth rate increases with the education level of the country (Calvacanti, De Abreu, & Veloso, 2013). The intuition behind this formulation is that per capita income and the level of education move in the same direction and this reduces dropout rates as more people can afford education costs (Porta & Laguna, 2007). An increase in primary completion rates raises human capital (h) and more human capital reinforces economic progress at aggregate and individual level (Hanushek, 2009). So when the relative income in the country is low, children are more likely to abandon school as their parents cannot afford the cost of education (Arcia, 2003; Oreopoulos et al., 2006). Countries exhibiting such characteristics would typically exhibit low per capita income and low economic growth.

In this study we aim to understand how coordinated interactions of these critical factors work in a complex dynamic environment like the educational one. Coordination in this setting describes a situation where multiple, interdependent elements interact simultaneously, following their own dynamical processes with limited control by a central authority and with a clear impact on school outcomes. In practical terms, Guevara and Posch (2015) show that coordinated actions that improve infrastructure (state of the educational system, S), economy (state of the economy, E) and literacy (human capital, h) simultaneously are more effective to reach full completion in education.

[8] Real GDP per capita in 2000 was US$1,035 (World Bank, 2015).

However, given that these critical factors follow different accumulation paths (different timing), as we add more critical factors to the system it would take longer for them to line up in the right way to reach a particular configuration, making coordination more difficult. We assess their coordinated impact on the system using the PCR indicator aforementioned over a long period of time (i.e., 2010–2050). When all these properties are merged in a simulation model, the underlying system is expected to bring about features commonly observed in complex systems like tipping points, phase transitions, etc.

Simulations

Baseline Scenario

Under the baseline scenario, the model exploits all assumptions and parameter values used for calibration along with an average economic growth rate of 5 % (see Appendix, Tables 15.3. and 15.4). Table 15.3 presents the initial values of population stocks and Table 15.4 presents some parameter values used for repetition and dropout rates across age and income groups in year 2000.[9] With these specifications, the model generates synthetic data that allows a direct comparison of simulated PCR (continuous line) to corresponding observed time values (dotted line) from 2000 to 2010, when the last empirical result was published (World Bank, 2015) and period 2010–2050 for forecasting and analysis. Figure 15.4 shows that the model closely replicates real data for the case of Nicaragua.

The bump registered by the simulated PCR in Fig. 15.4 during 2003–2005 occurs as a result of the substantial over/under official age student population accumulated in the educational system during the 1990s coupled with decreasing repetition and dropout rates of the mid-2000s. An educational system with such characteristics can even temporarily overshoot the 100 % completion level when these over/under age students are driven out of the system via higher graduation and/or less dropout and repetition (see Guevara & Posch, 2015). We disaggregated completion rates by income quintile, and simulated them for the period 2000–2015. Thus, this illustration shows Nicaragua as a five-tiered education system. As can be reasonably expected, the first and second quintiles (poorest) are also the worst performers, well below the national average (black thick line) with completion rates under 80 % during the period of analysis, while the top two quintiles are well above the 90 % PCR. The same bump is also observed in Fig. 15.5, particularly at the top quintiles. This result comes in the model's simulation as a consequence of top

[9] Of course the entire data set used to calibrate the model is far larger than that and the one provided in the appendix is just for the sake of illustration. For the complete data set used in the calibration process please contact the authors. Similarly for a detailed description of all assumptions (feedbacks and nonlinear relationships) see Guevara et al. (2014).

Table 15.3 Nicaragua: Population values disaggregated by age and income quintile, circa 2000

Population	Income					
Age	Q1	Q2	Q3	Q4	Q5	Total
0	49,058	47,763	41,961	34,207	14,757	187,746
1	47,794	46,532	40,880	33,326	14,377	182,909
2	46,547	45,318	39,814	32,457	14,002	178,138
3	45,324	44,127	38,767	31,604	13,634	173,455
4	44,131	42,966	37,747	30,772	13,275	168,891
5	42,979	41,844	36,762	29,969	12,928	164,482
6	41,878	40,772	35,820	29,201	12,597	160,268
7	40,838	39,759	34,930	28,475	12,284	156,286
8	39,862	38,809	34,095	27,795	11,991	152,552
9	38,945	37,917	33,311	27,156	11,715	149,043
10	38,069	37,064	32,562	26,545	11,451	145,692
11	37,210	36,227	31,827	25,946	11,193	142,403
12	36,343	35,383	31,085	25,341	10,932	139,084
13	35,451	34,515	30,323	24,719	10,664	135,672
14	34,529	33,617	29,534	24,077	10,387	132,144
15	33,581	32,694	28,723	23,416	10,101	128,516
15>	693,281	674,974	592,990	483,413	208,541	2,653,199
Total	1,345,820	1,310,282	1,151,132	938,417	404,827	5,150,480

Table 15.4 Nicaragua: primary education repetition and dropout parameters, circa 2000

Repetition					
Age	Poorest	II	III	IV	Richest
0	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
5 or above	0.15	0.12	0.08	0.05	0.00
Dropout					
Age	Poorest	II	III	IV	Richest
0	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
5 or above	0.05	0.02	0.02	0.00	0.00

income quintiles showing more progress not only in reducing repetition and dropout rates but also in enrolling their children at the official school age. These results are consistent in the country's survey data that show decreasing completion rates in nearly all quintiles after reaching a maximum level, the fifth quintile even overshooting 100 % (LSMS, 2001, 2005, 2009).

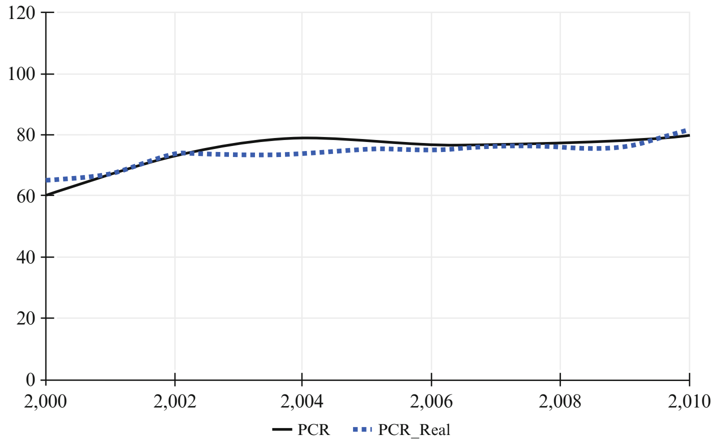

Fig. 15.4 Nicaragua 2000–2010. Simulated (*continuous line*) and observed PCR

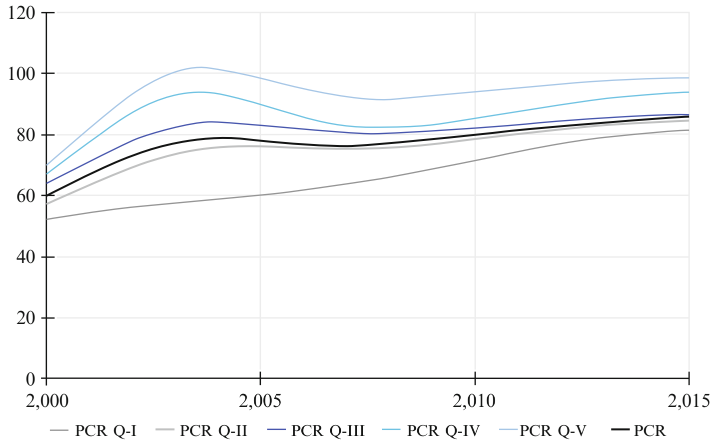

Fig. 15.5 Nicaragua 2000–2015. Simulated primary completion rates by income quintiles

More Scenarios

Note, however, that despite the economic and social differences, students and population in general interact on a more regular basis. Therefore, despite every income quintile being clearly delimited in the Nicaraguan completion rates, these layers are still interdependent as they jointly determine the aggregate amount of literacy in the population which we assume impacts the system's transition rates. The magnitude of these interactions can be better appreciated in results shown by

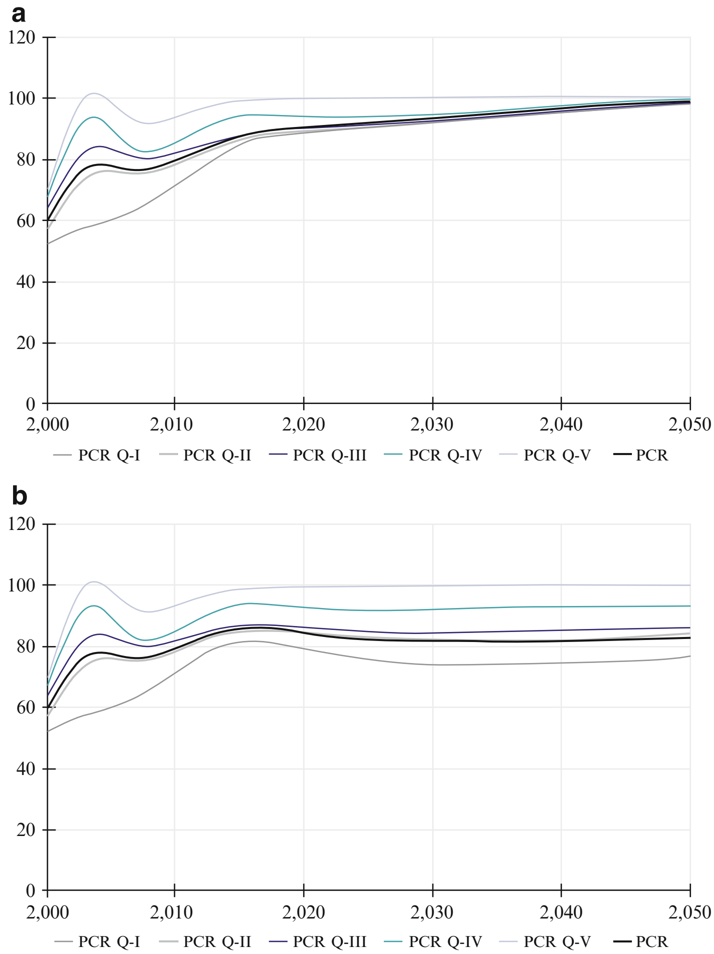

Fig. 15.6 Nicaragua 2000–2050. Simulated completion rates by income quintiles assuming (**a**) economic growth rate of 5 %. (**b**) economic growth rate of 3 %

Fig. 15.6 under two alternative scenarios: one with a strong economic growth and one with a weak economic growth.

In Fig. 15.6a we show that under a strong 5 % economic growth the fifth quintile (the richest) reaches 100 % on its own while the other four quintiles must "wait" until they altogether reach a similar level of completion rate to finally progress toward a maximum completion rate, i.e., the second quintile waits until 2018 for the first quintile to catch up, and similarly the third and fourth wait for the previous ones before advancing in 2020 and 2030, respectively. It also has to do with the fact that the first two quintiles include more than 50 % of the total primary student population and the first four quintiles more than 90 %. On the other hand, assuming a slower economic growth rate of 3 % in Fig. 15.6b,[10] we observe that the first four quintiles primary completion rates do not advance to catch up with the fifth one. Thus, following the patterns generated by the simulations, it is easy to tell that when the overall education system (the black thick line in Figs. 15.5 and 15.6) is below the 80 % threshold and economic growth in the country is not strong, even if one waits for complexities to play out over a long period of time, the system will not eventually converge to 100 % completion level. As GDP per capita is set to be low at the beginning of the simulation for all income quintiles—except the first one—while drop out and repetition rates are very high, income and education will not reinforce each other to fuel completion rates towards its maximum level. A long period of time of robust economic growth would be needed to bring those values of the four lower income quintiles to a level consistent with a full primary completion rate.

Therefore when the whole system has reached a steady-state below the maximum completion rate, policy interventions may be necessary in order to drive it more rapidly toward the higher equilibrium level. The magnitude of these interventions should be adequate to accelerate completion rates—particularly those at the lower income quintiles—up to the point at which the system crosses the lower equilibrium threshold. Once this critical level of 80 % PCR is exceeded, a transition phase occurs via the positive interactions between human capital and economic activity that becomes capable of fueling itself to drive the system to a path of self-sustained momentum until reaching the maximum level. It is at this is point when a society manages to break the clogs-to-clogs cycle at least in primary education.

Discussion

Complex dynamic mechanisms drive many social, economic, and natural processes in modern highly connected societies and the prospects for advancing at the right pace in human development can more likely be accomplished if the impact of past, present, and future events that shape the development paths of countries are identified and understood. But satisfactory answers need consistent models showing alternative paths and the consequences that intertwined factors in human and natural systems may have on the shape and direction of such paths.

In this model, the life opportunities of Nicaraguan children are, at the broadest level, determined by the income, education, and direction they receive from their families which then are reinforced across generations. The stronger and more enriching family environment children receive, the stronger and more enriching family environment they will pass on to the next generation. Using this complex

[10] Here we assume that Nicaragua's population grows at 3 % which means that a 3 % growth in its Gross Domestic Product (GDP) would not change per-capita GDP, which is the ratio of GDP divided by population.

approach we are able to capture several empirical observations about this educational system and project key outcomes into the future.

Complexity modeling and simulation can be regarded as an informed guide for decision-making intelligence providing consistent forecasts when properly designed and constructed. Decision-making intelligence that is timely, relevant, and accurate adds significant value to decision makers when such insights provide consistent information that reduces the uncertainties of future events; this is what a well-designed model should aim to. This in no way is suggesting that models are capable of predicting future events accurately. What it indeed suggests is that in principle all simulated paths should be consistent predictions based on the logic of the model's structure and the impact of the assumed nonlinear relationships. So if the model's logic has been articulated in a consistent way, the model predictions will remain sound, regardless of which particular scenario unfolds, and that provides sound information about the real system. This perspective is likely to lead to a view that the more we learn about the functioning of complex systems using simulation models, the better we will interfere in real-world systems.

As we have already discussed in this chapter, a very useful perspective in demographic education modeling might consider populations as a collection of elements whose combined activities shape the realm of the environment they are embedded in—the behavior of individual components influences the dynamic behavior observed at aggregate level—and the aggregate behavior of that population reciprocally influences individuals' courses of action. We find it particularly interesting to track behavioral patterns generated by segregations stemming from a population whose educational systems—governed by reinforcing feedbacks tend to perpetuate initial conditions—dragging the whole system in poor outcomes due to disadvantaged initial conditions of some segments in the population. When a model's population is disaggregated by income, a much richer collection of behavioral patterns is achieved due to the innate particularities that each income-group possesses regarding enrollment, repetition and dropout. The richest quintiles behave very much like an average developed country with high promotion and low repetition and dropout rates while the first and second quintiles, covering more than 50 % of the population, are more representative of the country reality, showing low completion rates and high repetition and dropout. This, however, has further implications for the educational system as a whole, advancing toward full primary completion at country level becomes increasingly difficult if the poorest quintiles are not brought along with the rich ones, in particular the first quintile. The reason is straightforward and well-recognized in economics; educated people generate positive influence on others to whom they interact with, a concept normally regarded as a positive externality (captured by the model's feedbacks and nonlinear interactions) or in other words the public good nature of equality, in the economic sense of the term. When a sector of the population lacks educational skills such positive externality is interrupted generating a negative effect on their peers (think on the difficulties to transmit ideas efficiently when people lack basic education). All in all, it means that we must turn the impact of these reinforcing feedbacks into an affirmative force that drives high educational accomplishment and better distribution and mobility in society. Even if certain segments of the population—i.e., like the low-income and low-educated—are initially segregated in society, technological advances in communication and transportation make such segments more likely to interact with more educated and affluent ones on a more frequent basis for cultural, social, or economic reasons. So reducing income-based outcomes in educational systems is not just a policy measure to show our solidarity with the most disfavored groups, it is also an effective operational policy required for well-functioning systems. This reasoning thus downplays the premise often argued that inequalities work as an incentive for social mobility implying that at a system level decision makers should not prioritize on policies to level the playing field for all individuals. It is likely that similar results can be obtained with other inequalities like those based on gender, geographical areas, race, etc.

Although the magnitude of intergenerational educational mobility is lower in Nicaragua than in many other countries, the "persistence" pattern derived from reinforcing feedbacks is consistent with low social class mobility in the country and does not differ from the rest of the world. Therefore, we expect that research on the intergenerational transmission of inequality from a complexity system perspective like the one portrayed here can inspire new endeavors to better understand the underpinning of such mechanisms in other countries.

Appendix

The following description of the simulation model is an excerpt from Guevara et al. (2014) reprinted with permission from the journal Nonlinear Dynamics, Psychology, and Life Sciences.

The Simulation Model

The educational model has 3 state variables: Population (P), Population in Primary School (G), and Primary School Graduates (H). These are represented by stocks (rectangles) in Fig. 15.3. P stands for the country's total population, disaggregated into age cohorts and it is the main input to the education system (Eq. 15.1). The arrows in Fig. 15.3 are differential equations that modify the stocks; hence, population increases with births and decreases with deaths. Equation 15.1 shows that the birth rate B, is the product of a constant fractional vector β multiplied by the country's population (i.e., the sum of all age-cohorts). Similarly, death rate D, is the result of a constant ϕ multiplied by the stock of population. In the model, aging $[A(t)]$ represents the transition of the population from one age cohort to the next, after it has remained an average length of time (υ) in that cohort. $P_a(0)$ is the initial population.

$$P_a(t) = \int_{t=t_0}^{T} [B(t) + A_{a-1}(t) - A_a(t) - D_a(t)]\,\mathrm{d}t + P_a(0) \tag{15.1}$$

$$\text{where} \quad B(t) = \beta \sum_a P_a \quad 0 < \beta < 1$$

$$D_a(t) = \phi P_a \quad 0 < \phi < 1$$

$$A_a(t) = {}^{P_a}\!/_{\upsilon} \quad \upsilon = 1,$$

$P_a(t) =$ stock of age-a population, $a = 0, 1, 2, \ldots, 15,$ and Adults (16 or more).

$A_a(t) =$ aging rate, $B(t) =$ birth rate, $D_a(t) =$ death rate,

The second state variable, G, is a matrix broken down by grade and age, encompassing children currently enrolled in school. Equation 15.2 shows that it consists of *6* grades according to the official cycle length in the country. In words, $G_{1,a}(t)$ represents the population of age-a students attending the first grade. Once children enter the school system they may follow three mutually exclusive directions: (1) passing to the next level through promotion ($p_{i,a}(t)$) from grade i to $i+1$ and growing older by 1 year (from a to $a+1$); (2) repeating the year ($r_{i,a}(t)$) just passing to the next age cohort (from a to $a+1$) but remaining in the same grade (i); or (3), dropping-out of the grade i at age a ($d_{i,a}(t)$). Note that in Eq. 15.2 intake [$e_{1,a}(t)$] only occurs in the first grade, denominated by $p_{0,a-1}(t)$, and promotion replaces it as an inflow after the second grade. Thus,

$$G_{i,a} = \int_{t=t_0}^{T} \left[p_{i-1,a-1}(t) + r_{i,a-1}(t) - p_{i,a}(t) - d_{i,a}(t) - r_{i,a}(t)\right]\mathrm{d}t + G_{i,a}(0) \tag{15.2}$$

where $G_{i,a}(t) =$ population in grade $i = 1, 2, \ldots, 6$; age $a = 0, 1, 2, \ldots, 15, 16$ (age 16 and above).

$p_{i,a}(t) =$ promotion grade i at age

$e_{1,a}(t) =$ intake rate grade 1 at age a; $p_{0,a-1}(t) \equiv e_a(t)$

$r_{i,a}(t) =$ repetition grade i at age a

$d_{i,a}(t) =$ dropout grade i at age a

All transition rates are specified as the product of a vector of fractions such as intake ($\alpha_{1,a}$), repetition ($\rho_{i,a}$), dropout ($\delta_{i,a}$), and promotion $\left(\pi_{i,a} \equiv \left(1 - \delta_{i,a} - \rho_{i,a}\right)\right)$ multiplied by the stock of people in the respective grade (in the case of intake, by the population stock, P). In addition, these fractional values change across grades but remain constant within grades $\left(\delta_{i,a}, \rho_{i,a}, \pi_{i,a} = \delta_i, \rho_i, \pi_i\right)$. The corresponding formulations are Eqs. 15.3–15.6.

$$e_a(t) = e(P_a(t), \alpha_{1,a}) = \alpha_{1,a} P_a(t) \tag{15.3}$$

$$d_{i,a}(t) = \mathrm{d}(G_{i,a}(t), \delta_{i,a}) = \delta_i G_{i,a}(t) \tag{15.4}$$

$$r_{i,a}(t) = r\left(G_{i,a}(t), \rho_{i,a}\right) = \rho_i G_{i,a}(t) \tag{15.5}$$

$$p_{i,a}(t) = p(G_{i,a}(t), \pi_{i,a}) = \pi_i G_{i,a}(t) \tag{15.6}$$

$$\alpha_{1,a},\ \delta_{i,a},\ \rho_{i,a},\ \pi_{i,a} \in (0,\ 1)\ \text{for every}\ a$$

The third stock in Fig. 15.3, H, accumulates graduates from primary education as shown in Eq. 15.7. Equation 15.8 describes the construction of an index h of per-capita human capital which is the number of living people who have completed primary school compared to the country's population. This index ranges from 0 to 1 where 0 implies that no adult (i.e., no person aged 16 and above) has completed primary education and 1 means that all adults have at least finished it. Therefore

$$H_{a=16} = \int_{t=t_0}^{T} \left[\sum_a p_{6,a}(t) - D_{a=16}(t)\ \ \mathrm{d}t \right] + H_{a=16}(0) \tag{15.7}$$

$$h_{a=16} = \frac{H_{a=16}}{P_{a=16}}, \qquad \text{where } 0 \le h \le 1 \tag{15.8}$$

Equations 15.1 to 15.8 allow the construction of the two performance indicators: the gross enrollment rate (from Eqs. 15.1 and 15.2) and the primary completion rate (from Eqs. 15.1 and 15.6):

$$\mathrm{GER} = \sum_{i,a} \frac{G_{i,a}(t)}{P_{7-12}(t)} \tag{15.9}$$

$$\mathrm{PCR} = \sum_{a} \frac{p_{i,a}(t)}{P_{12}(t)} \tag{15.10}$$

Model Calibration

To calibrate the model it is necessary having a complete dataset for at least one point in time in which all stock variables are disaggregated by age, income group, and level of education attained. In this model that data point corresponds to year 2000 (LSMS 2001, 2005 and World Bank, 2015) and Table 15.3 shows this data point for the population variable used in the model, disaggregated by age and income. Likewise, Table 15.4 presents average parameter values for repetition and dropout rates across all grades for year 2000.

References

Andersen, L. (2001) *Social mobility in Latin America: links with adolescent schooling*. IADB Research Network Working Paper #R-433.

Arcia, G. (2003). *The incidence of public education spending: impact of the education for all—fast track initiative*. Consulting Report, The World Bank.

Autor, D. (2014). Skills, education, and the rise of earnings inequality among the "other 99 percent". *Science, 344*, 843–850.

Becker, G. S. (1962). Investment in Human Capital. *The Journal of Political Economy, 70*(5), 9–49. part 2.

Becker, G. S., & Chiswick, B. R. (1964). Education and the distribution of earnings. *American Economic Review, 56*(1/2), 358–369.

Behrman, J. R., & Rosenzweig, M. R. (2005). Does increasing women's schooling raise the schooling of the next generation? *American Economic Review, 95*, 1745–1751.

Calvacanti, P., De Abreu, S., & Veloso, F. (2013). On the evolution of total factor productivity in Latin America. *Economic Inquiry, 51*(1), 16–30.

Carroll, S. (2010). *From eternity to here: The quest for the ultimate theory of time*. New York: Penguin Group.

Dowrick, S. (2004). Ideas and education: level or growth effects and their implications for Australia. In: *Growth and Productivity in East Asia*. NBER-East Asia Seminar on Economics, (pp. 9–40). National Bureau of Economic Research, Inc.

Durlauf, S. N. (1998). What should policymakers know about economic complexity? *Washington Quarterly, 21*(1), 157–165.

Guevara, P. (2014). Toward a common structure in demographic educational modeling and simulation: A complex systems approach. *Complicity: An International Journal of Complexity and Education, 11*(2), 86–101.

Guevara, P., Lopez, L., Posch, A., & Zuniga, R. (2014). A dynamic nonlinear model for educational systems: a simulation study for primary education. *Nonlinear Dynamics, Psychology, and Life-Sciences, 18*(1), pp.91–108

Guevara, P., & Posch, A. (2015). Dynamic complexity, entropy, and coordination in educational systems: A simulation of strategic and exogenous interventions. *Systemic Practice and Action Research, 28*(2), pp.179–196.

Hanushek, E. (2009). The economic value of education and cognitive skills. In G. Sykes, B. Schneider, & D. N. Plank (Eds.), *Handbook of education policy research* (pp. 39–56). New York: Routledge.

Hanushek, E., and Woessmann, L. (2014). Institutional structures of the education system and student achievement: A review of cross-country economic research. In: Strietholt et al (ed.) *Educational Policy Evaluation through International Comparative Assessments* Waxman 2014. Retrieved May, 2015 from http://hanushek.stanford.edu/sites/default/files/publications/Hanushek%2BWoessmann%202014%20EduPolEval.pdf.

Koopmans, M. (2014). Nonlinear change and the black box problem in educational research. *Nonlinear Dynamics, Psychology, and Life Sciences, 18*(1), 5–22.

Living Standards Measurement Surveys (LSMS 2001, 2005, 2009). Nicaragua. The World Bank.

Mincer, J. (1958). Investment in human capital and personal income distribution. *Journal of Political Economy, 66*(4), 281–302.

Morrison, K. (2008). Educational philosophy and the challenge of complexity theory. *Educational Philosophy and Theory, 40*, 19–34.

Noble, K. G., Houston, S. M., Brito, N., Bartsch, H., Kan, E., Kuperman, J. M., et al. (2015). Family Income, parental education and brain structure in children and adolescents. *Nature Neuroscience, 18*(5), 773–778. doi:10.1038/nn.3983.

Oreopoulos, P., Page, M. E., & Stevens, A. H. (2006). The intergenerational effects of compulsory schooling. *Journal of Labor Economics, 24*(4), 729–760.

Piketty, T. (2014). *Capital in the 21stcentury*. Massachusetts: Harvard University Press.

Porta, E. (2011). *A data set for estimating global indicators of education inequality*. Washington, DC: World Bank, Human Development Network.

Porta, E., Arcia, G., Macdonald, K., Radyakin, S., & Lokshin, M. (2011). *Assessing sector performance and inequality in education*. Washington: The World Bank.

Porta E. and Laguna, J.R. (2007). *Educational Equity in Central America: A pending issue for the public agenda*. USAID Working Paper.

Ravallion, M. (2014). Income inequality in the developing world. *Science, 344*, 851–855.

SEDLAC (2015). Socio-Economic Database for Latin America and the Caribbean. CEDLAS and The World Bank, http://sedlac.econo.unlp.edu.ar/eng/. Accessed in May, 2015.

UNESCO (2015). Literacy Data Show a Persistent Gender Gap. http://www.uis.unesco.org/literacy/Pages/literacy-day-2014.aspx. Accessed in August, 2015.

World Bank (2001). Nicaragua Poverty Assessment Challenges and Opportunities for Poverty Reduction. Report No. 20488-NI.

World Bank. (2015). World development indicators. Washington, DC: The World Bank, http://data.worldbank.org/data-catalog. Accessed in May, 2015.

Chapter 16
The Symbolic Dynamics of Visual Attention During Learning: Exploring the Application of Orbital Decomposition

Joanna K. Garner and Daniel M. Russell

Introduction

In educational research, the objects of study include individuals in classrooms and other environments where learning takes place. A wide variety of methodologies have been used to describe and explain learning-related phenomena, but until recently the majority of quantitative analysis techniques have supported the formulation of acontextual, mechanistic linear models. Such an approach has been criticized for overlooking the richness and complexity that exists within and between individuals as they go about their work (Winne, 2015). However, interest in complexity and dynamic systems approaches to learning and educational aspects of human development has begun to arise (Davis & Sumara, 2008; Kaplan, Garner, & Semo, 2015; Kunnen & Bosma, 2000; Stanton & Welsh, 2012). This reflects researchers' increasing willingness to think about behaviors and processes as manifestations of emergent, self-organizing psychological or social systems. Authors on the leading edge of these developments have adapted analytical tools from other fields to align their research methods with their new epistemological and theoretical perspectives, but the number and variety of examples available to the educational researcher remains limited. Therefore, a primary aim of this chapter is to provide a worked example of one particular analytical approach called orbital decomposition (OD; Guastello, Hyde, & Odak, 1998) and to demonstrate how basic psychological processes used in learning can be described using a complexity-informed, dynamic systems perspective.

J.K. Garner, Ph.D. (✉)
The Center for Educational Partnerships, Old Dominion University,
Norfolk, VA 23508, USA
e-mail: jkgarner@odu.edu

D.M. Russell, Ph.D.2
School of Physical Therapy & Athletic Training, Old Dominion University,
Norfolk, VA, USA

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_16

Complex Dynamic Systems

In mathematics and computational scientific fields, a complex dynamic system is a collection of hierarchically structured components that mutually influence one another over time, often in nonlinear ways, within the confines of a numerically specifiable space (Capra, 1983). Phenomena as diverse as the weather, the human brain, and the economy have all been modeled using complex dynamic systems (Juarrero, 2010). When principles of complex dynamic systems are adopted as a conceptual framework for thinking about interpersonal behaviors and human development, far-reaching implications arise because of new assumptions about the nature of the phenomena under consideration and the methods through which they should be studied (Granic & Patterson, 2006; Stanton & Welsh, 2012; Thelen & Smith, 1994).

The transition from a mechanistic to a complex dynamic systems perspective requires movement towards an approach in which phenomena are conceived as emerging from the interaction of many parts which cannot be simply reduced to the behavior of those parts (Stanton & Welsh, 2012). According to this view systems have properties such as adaptive self-organization, meaning that they may exhibit fractal dimensionality, with the structure of smaller parts resembling the structure of larger parts like the branches of a tree. Within the system numerous hierarchically arranged components influence one another in a mutual, reciprocal manner (Granic & Patterson, 2006). Self-organizing systems also have the capacity for relations among components to become reconfigured, sometimes unpredictably, in response to features of the environment. In addition, systems are inseparable from the context in which they evolve, such that even small changes can produce perturbations to the system that consequently emerge as large, nonlinear shifts in behavior patterns (Guastello & Liebovitch, 2009; Prigogine & Stengers, 1984). Such shifts in the system's behavior can appear abruptly, manifesting empirically as an inflection point on a graph, and achieved through internal positive and negative feedback loops that either promote or prevent destabilization (Granic & Patterson, 2006; Prigogine & Stengers, 1984). Stability in both system configuration and behavior can be modeled via a topographical landscape of mathematical probabilities, with valleys and basins representing areas in which the system is likely to arrive and stay for a period of time. Such locations, depicted in quantity and relative capacity to stabilize the system, are termed attractors (Kelso, 1995). Multiple attractors form a landscape that represents the "behavioral repertoire" or an array of potential states of the system at a given point in time (Granic & Lamey, 2002, p. 267).

In this chapter we consider the phenomenon of studying—learning from informational text and graphics—from a complex dynamic systems perspective. As a means of articulating how the differences between linear mechanistic and nonlinear probabilistic systems apply to this topic, imagine that a student is presented with the task of learning about the structure, function, and blood flow patterns of the heart. The student has three informational sheets of paper in front of them, labeled A, B, and C, which contain diagrams and paragraph-length explanations. The task requirements are to be able to draw and label components of the heart and associated patterns of blood flow. A blank sheet of paper is also available for note-taking during the studying period. The student has a limited amount of time to complete the studying task, and must regulate their attention and cognitive processes accordingly.

Based on this description, if we could trace the student's visual attention, we might expect to find a relatively simple, linear trace of task behavior. The student might review the task instructions in order to make sure that they understand the task, and then might review each of the three informational sheets in turn, i.e., look at and read Sheet A followed by Sheet B followed by Sheet C. Another simple, linear option might be for the student to take notes after reading each informational sheet, such that their task progression looks like this: Instructions, Sheet A, Notes, Sheet B, Notes, Sheet C, Notes, Finish. However, the act of learning from text involves more than simply reading the information. Thus, another more segmented option might emerge, involving the activation of comprehension-monitoring processes that drive the student to move back and forth between the sheets of information such that their progression looks like this: Instructions, Sheet A, Sheet B, Sheet A, Sheet B, Sheet C, Sheet B, Sheet A, Sheet B, Sheet A, Sheet C, Sheet A, Finish. In addition, if the student takes notes then a trace of their visual attention might show that notes are visited periodically during the back-and-forth reading activity, leading to a pattern such as Sheet A, Notes, Sheet B, Notes, Sheet A, Sheet B, and Notes. Finally, if we add in goal-oriented performance-monitoring processes on top of comprehension-monitoring processes, the trace of the task progression might include periodic review or looping between informational sheets, task instructions, and notes, hence: Instructions (planning), Sheet A, Notes, Sheet A, Notes, Sheet B, Sheet A, Notes, Sheet B, Notes, Instructions (comprehension monitoring), Sheet B, Sheet C, Notes, Sheet C, Sheet A, Sheet B, Notes, Instructions (performance monitoring), Finish.

Thus, studying may emerge through the interaction of hierarchically organized elements of a cognitive-metacognitive system, and may manifest itself as sequences of visual attention that are contingent on prior ones such as when new information sparks the reader to look back, search for something elsewhere in a text, reread the task instructions, and take notes. Progressions of attention may in fact include repeating sub-loops nested in larger dynamic sequences. If so, then it is appropriate to question the utility of mechanistic models that focus on linear progressions through phases such as planning, execution, and reflection, and instead consider ways in which a dynamic approach might be fruitful. If visual attention is sensitive to context and the initial conditions of the task in ways that cannot be predicted at the outset, or if individuals' trace behaviors can shift midway through the task from being linear to chaotic and back again, then we propose that it may be worthwhile to investigate the phenomena through the application of analytical tools that are associated with complexity and nonlinearity. One example of such an approach is symbolic dynamics.

Symbolic Dynamics

The behavior of a complex dynamic system can be described using mathematical terms that model the relations between successive states or configurations of the system as they occur over time. At its most basic, the mathematical analysis of such a system involves the pursuit and identification of a function, f, that is applied to the system, x, over n number of times (Sundararajan, 2012). However, the time series of data derived from the recorded behavior of a system will often reveal the presence of short-term or transient functions, meaning that one or more functions "that characterizes one part of the time series does not characterize the entire series," (Guastello, Peressini, & Bond, 2011 p. 465). Put simply, repetitive sequences of behavior may occur periodically, and may be interspersed over a time interval. If the behavior is to be characterized from a dynamic systems perspective, techniques are needed to discover these intermittent behavioral sequences (Guastello, 2005; Guastello & Gregson, 2011; Guastello et al., 1998). Symbolic dynamics is one area of mathematics that includes a variety of methods for achieving precisely this goal, thus opening up its application to real-time behavioral data that changes in structure according to ongoing feedback to the system. For example, it may capture changes in the structure of visual attention sequences, when individuals revise an action plan in service of a learning goal.

In contrast to the numerical data that one may typically associate with time series, strategies under the heading of symbolic dynamics have in common the capacity for detecting patterns in *sequences of nominal level data* derived from the observation or simulation of successive states of a system (Guastello, 2008; Marcus & Williams, 2008). Because it is the analysis of shift spaces—the series of discrete states in which a dynamical system exists over time—symbolic dynamics can benefit researchers by reducing the otherwise complicated system dynamics into a data set consisting of a string of symbols such as A, B, C, and D (Tabrizian, 2010). The resulting data series reflects the state of the system at discrete intervals over time, such as the location of an individual's gaze during a learning task. The meaning of the data string can then be interpreted in light of the theoretical framework being applied to the topic of investigation, such as the meaning of particular sequences of visual attention during reading and note-taking.

Strings of nominal data are particularly useful for analyzing systems whose attractor landscape includes configurations that recur periodically over time (Sundararajan, 2012), such as when engaged in a task that requires repeated steps. The application of symbolic dynamic mathematical modeling techniques can reveal the degree to which the system's behavioral traces reflect regularity or randomness, thereby offering potential clues as to the degree to which the series of system's states is in fact a manifestation of the presence of some deeper, underlying structural properties of the system and the context in which the system is embedded. This feature of symbolic dynamics makes it appropriate for studying self-organizing systems in which order arises or *emerges* in a nonlinear and non-predictable way from the dynamic interaction among its components (Crutchfield, 1994).

Orbital Decomposition

Orbital decomposition (OD) refers to a series of computations that identify structure and complexity in dynamical patterns, such as those that might be evident within a time series of behaviors or social interactions over time (Guastello & Gregson, 2011). Consistent with a particular approach to complexity known as chaos theory, OD seeks to quantify the systematicity of recurring sequences in data. It is based on the notion that a time series of events can be chaotic, meaning that although the data series can appear random it can actually be characterized by one or more mathematical functions (Guastello, 2000, 2008).

Initially used by Guastello et al. (1998), OD assumes that within a time series of data are a number of coupled oscillators or orbits that manifest as patterned recurrences in nominal data strings (Pincus et al., 2014). An oscillator produces a periodic change in its state which when coupled to another oscillator can lead to complex changes in state, such as chaos, which have been studied and modeled in mechanical, electrical, and biological systems (Kelso, 1995). The OD analysis decouples the orbits to determine their nature by modeling the contributing oscillators as patterned recurrences in the nominal data. This is done by finding patterns in the nominal data string and determining how frequently they repeat and whether they repeat immediately (Guastello et al., 2011). Beginning with a sequence length (C) of one, this process continues with increasing sequence length up to the longest pattern that is immediately repeated in the data. This analysis therefore identifies oscillations in the data string and quantifies the probability of each occurring. For each C length, up to the maximum detected, additional variables are quantified requiring the researcher to identify the appropriate or optimal C length to consider. This may be determined by finding the longest string length that immediately repeats itself, or by finding the string length for the largest resulting χ^2 value, which indicates the length at which the proportion of expected versus observed frequencies of data string patterns is the largest (Pincus et al., 2014). The description of how each variable is calculated will be provided shortly.

Once the optimal pattern length has been revealed, indices of informational complexity can be calculated. A frequently used measure is Shannon entropy (Shannon, 1948; Shannon & Weaver, 1949), which is calculated using the probabilities of each event in the data series. It provides an indication of the degree to which a data series is random versus the degree to which systematic patterns exist. When single behavioral sequences are repeated continuously, there is a lack of novelty and the Shannon entropy index is low. Conversely, if sequences are never repeated and the novelty is very high, then the index will also be high. Topological entropy, on the other hand, is a measure of content. It can be used to calculate a Lyapunov exponent, which reveals the fractal dimensionality or complexity of the recurring patterns. Shannon entropy and topological entropy are inversely related to one another. The more random the time series, the less complex the content of the information contained within it is likely to be, leading to high Shannon entropy and low topological entropy (Katerndahl & Parchman, 2013; Pincus & Guastello, 2005). In essence, the indices that OD yields reveal the degree to which recurring patterns are incorporated into time series data. In the following section we elaborate on the steps of OD, including the mathematical formulae used to derive its constituent indices.

When adopting a symbolic dynamic approach the first step is to define the different categories of system dynamics. This involves identifying the exclusive categories of system behavior that underpin the phenomena under investigation. Examples might include individuals who are speaking within a social interaction, categories of behaviors exhibited at a particular time and location, or, as in the present study, the different materials to which each participant could orient themselves during a learning episode. After specifying an exhaustive list of categories, the second step is to define the data sampling rate. At this point standard behavioral sampling methods can be applied, such as coding the state of the system at a constant interval (e.g., every second or every 10 s), and/or when a change in state occurs. If the category is recorded at a constant interval of one per second the result is a series of nominal data points representing repeated locations over 24 seconds, such as EEEAEAAAADADADDDADDADDAA.

Deciphering the above symbol sequence is possible if two parameters are known: (1) what the letter symbols represent, and (2) the length of time that each instance of a symbol indicates. In this example, the sequence describes the materials that an individual attends to during 24 s in the context of a learning episode. The letter sequence indicates that an individual began a studying task by looking at the task instructions (designated as material E) for 3 s before switching visual attention to material A for 1 s and then back to the instructions for 1 s before returning to material A for 4 s, etc. In all cases, coding may continue until the end of a given time period or trial. In addition to recording and analyzing the letter sequences such as listed above, the researcher may elect to de-emphasize the relative amount of time spent in each successive state and instead focus on the sequence itself. Doing so emphasizes the patterns of shifting among states or materials, and allows the researcher to investigate questions pertaining to the sequential structure of the time series. If this were to be the case for the previous example, repetitive notation is removed and the data series becomes EAEADADADADADA. Either form of letter sequence (i.e., coded at equal time intervals or only changes in state) can be analyzed using OD.

A computer program which performs OD analysis (ORBDE, v.2.4) was created by Peressini & Guastello (2014) and can be freely downloaded from the Society for Chaos Theory in Psychology & Life Sciences webpage: http://www.societyforchaostheory.org/resources/#4menu. ORBDE requires data files to be in ASCII format with up to 80 characters in a line being read in sequence before the next line of data. These files can be readily created in Microsoft Windows Notepad by representing each category with a single lowercase or uppercase letter (ORBDE is case sensitive) from the Roman alphabet. Up to 52 different mutually exclusive categories can be used and a sequence of approximately 1000 letters can be readily handled by the program. Version 2.4 of ORBDE allows more than one code to be aggregated to an event; however for simplicity we will only be considering the standard analysis of only one code being applied to an event.

The computations of ORBDE begin by finding patterns (C) of length 1 and then continue with increasing sequence lengths. To enhance understanding we provide some example calculations below using the sequence EAEADADADADADA. For each performed pattern the software counts the frequency or number of occurrences (F_{ob}), and determines whether it is proximally (immediately) repeated or not. The observed probability for each pattern (p_i) is computed by dividing the number of times it was performed by the maximum number of times a pattern of that C length could have occurred, N^* (e.g., $p_{\text{A}} = 7/14 = 0.5$, $p_{\text{AE}} = 1/13 = 0.077$). The calculation of expected frequency (F_{ex}) depends on the C length. For $C = 1$, F_{ex} for each pattern is computed by dividing the maximum times a pattern of that length could be performed, N^* (e.g., $N^* = 14$) by the number of different performed patterns (e.g., for A, D, or E, $F_{\text{ex}} = 14/3 = 4.667$). For $C \geq 2$, the F_{ex} is computed by the combined observed probability of each single letter code multiplied by N^*:

$$F_{\text{ex}} = p_i p_i \ldots N^* \tag{16.1}$$

(e.g., for EA, $F_{\text{ex}} = 2/14 \times 7/14 \times 13 = 0.929$). All patterns that were performed only once or not at all are represented by a single F_{ex} that is calculated by subtracting the sum of F_{ex} for all patterns that occurred at least twice from N^* (e.g., $C = 2$ non-repeating patterns, $F_{\text{ex}} = 13 - (0.929 + 0.929 + 2.321 + 2.321) = 6.5$). A partial calculation for Shannon entropy (Shan) is then computed for each pattern (i):

$$\text{Shan} = p_i \left[\ln\left(\frac{1}{p_i} \right) \right] \tag{16.2}$$

Along with a partial calculation for the χ^2 test (SqDev), where ln is the natural logarithm:

$$\text{SqDev} = F_{\text{ob}} \ln\left(\frac{F_{\text{ob}}}{F_{\text{ex}}} \right) \tag{16.3}$$

These partial statistics tables are repeated up to the sequence length, C, that is requested by the researcher. A final statistics table is then provided that summarizes the calculations for the different C sequence lengths where at least one pattern is immediately repeated. Shannon entropy (H_{S}) is calculated from the probabilities for each pattern at a given C length:

$$H_{\text{S}} = \sum_{i=1}^{t} p_i \left[\ln\left(\frac{1}{p_i} \right) \right] \tag{16.4}$$

In contrast, topological entropy H_{T} is computed as the base 2 logarithm of the number of recurring strings or patterns of length C (N_C) in the data series (see Appendix):

$$H_{\mathrm{T}} = \lim_{C\to\infty}\left(\frac{1}{C}\right)\log_2(N_C) \tag{16.5}$$

N_C can be estimated by the trace of a transition matrix ($\mathrm{tr}\mathbf{M}^{\mathrm{C}}$), which represents the number of strings that are immediately repeated (e.g., for $C = 2$, $\mathrm{tr}\mathbf{M}^{\mathrm{C}} = 3$), and can be substituted into Eq. (16.5) to become

$$H_{\mathrm{T}} = \lim_{C\to\infty}\left(\frac{1}{C}\right)\log_2\left(\mathrm{tr}\mathbf{M}^{\mathrm{C}}\right) \tag{16.6}$$

H_{T} is provided for each string length C preceding $\mathrm{tr}\mathbf{M}^{\mathrm{C}} = 0$.

The stability of an oscillator in phase space can be quantified by the largest Lyapunov exponent (i.e., how likely similar orbits will converge or diverge over time). As string length increases to infinity, H_{T} asymptotically approaches the largest Lyapunov exponent. Therefore, the largest Lyapunov exponent equals the largest eigenvalue of $\mathbf{M}^{\mathrm{C}}$, which can be approximated by $\mathrm{tr}\mathbf{M}^{\mathrm{C}}$. Based on the Lyapunov exponent the Lyapunov fractal dimension D_{L} can be quantified as

$$D_{\mathrm{L}} = \mathrm{e}^{H_{\mathrm{T}}} \tag{16.7}$$

As noise can effect these calculations it remains important to use a statistical test to assess whether the findings could be simply explained by chance. To do this a χ^2 test is computed in ORBDE based on the F_{ob} and F_{ex}:

$$\chi^2 = 2\sum\left[F_{\mathrm{ob}}\ln\left(\frac{F_{\mathrm{ob}}}{F_{\mathrm{ex}}}\right)\right] \tag{16.8}$$

Using the degrees of freedom provided in the summary table along with the χ^2 value, statistical significance can be determined by looking up the critical χ^2 value in a standard statistics textbook. In an effort to quantify the proportion of variance accounted for in one categorical variable by another ORBDE computes ϕ^2 according to:

$$\phi^2 = \frac{\chi^2}{N^*} \tag{16.9}$$

This calculation is based on Cramer's V which computes ϕ from an $N \times M$ contingency table, where N are the expected frequencies of category membership and M are the observed frequencies of category membership (Guastello et al., 1998). For a 2×2 contingency table, ϕ is equivalent to the correlation coefficient, but for an $N \times M$ contingency table ϕ provides only an estimate of the strength of the relationship and is not bounded by 1. Hence ϕ^2 can also be larger than 1. While clearly it is not possible to account for more than 100 % of the variance in a variable, ϕ^2 still provides a useful index of the goodness of fit for comparing across similar conditions.

In addition to the indicator of dimensionality derived from the entropy analysis, an additional approach for assessing the presence of self-organization via fractal dynamics and the quantification of the fractal dimension is to determine if a frequency distribution of the observed recurring patterns follows an inverse power-law relationship (IPL). ORBDE identifies the patterns that appear in the time series and the number of recurrences for each pattern (X). Based on these results the number of different patterns with the same recurrence frequency can be quantified (Y). An IPL distribution occurs when Y is an exponential function of X (Fig. 16.3):

$$Y = aX^{-b} \tag{16.10}$$

The intercept (a) and the shape/rate of decay of the curve (b) can be estimated using nonlinear regression. The shape of the IPL curve is of particular interest as it can be interpreted as an estimate of the fractal dimension. The larger the magnitude of b, the greater the complexity, meaning the greater the ratio of the frequency of less recurrent patterns relative to the frequency of more recurrent patterns. Nonlinear regression analysis, which can be performed in SPSS or a similar statistical software program, can also provide a measure of R^2 which indicates how well the data can be modeled as an IPL distribution. To begin with, the data file must contain the Xs (number of recurrences for each pattern) and Ys (number of different patterns with the same recurrence frequency). The dependent variable is Y and the model expression used in SPSS is $a \times X \times \times -\mathrm{b}$. The nonlinear regression analysis estimates parameters a and b based on initial starting values (e.g., 0), and computes R^2 according to the ratio of residual to corrected sums of squares.

Self-Regulated Learning from Text

Self-regulated learning (SRL) is the process of thoughtfully engaging in behaviors that result in the achievement of a learning goal (Boekaerts & Corno, 2005; Pintrich, 2000; Winne & Hadwin, 2008; Zimmerman, 2000, 2008). Engagement includes "self-generated thoughts, feelings, and actions that are planned and cyclically adapted to the attainment of personal goals" (Zimmerman, 2000, p. 14). A SRL episode is a period of time during which an individual is actively and metacognitively engaged in planning, monitoring, executing, and evaluating strategic behaviors in service of a particular academic outcome (Garner, 2009). Tasks are often executed independently without direct guidance from an instructor and may involve reviewing assignment details, reading one or more texts, inspecting diagrams, taking notes, practicing problems, and self-testing. Importantly, the behavioral sequences that give rise to task completion are contingent on one another. The perceived success of each step in the learning process is derived from previous behaviors and the feedback and subjective evaluations generated by the outcome of metacognitively controlled cognitive processes such as reading.

Global task strategies include high-level, metacognitively controlled methods for approaching and executing the task. They refer to decisions to strategically allocate time and attentional resources according to an assessment of the task in relation to the learner's skills. These strategies should influence the way that visual attention is deployed. For example, in order to read words and sentences and interpret diagrams and figures, learners must control their visual attention across space (materials) and time. Skilled reading may therefore appear as a nonlinear progression of attention through the text, especially for readers who are able to use metacognitive self-monitoring processes to work towards carefully specified goals (Bråten & Stromsø, 2011; Pressley & Harris, 2006). Research has shown that successful readers distribute their attention unequally across both text and visual aids (diagrams and figures) depending on their global task strategy and how these choices impact specific cognitive and metacognitive processes (Bråten & Stromsø, 2011; Moreno & Mayer, 2000; Pressley & Harris, 2006; Schnotz & Bannert, 2003). Deviations from linear attentional processes also correlate with domain knowledge; content novices are more likely than experts to distribute attention evenly among sources or ideas because they are not able to use critical ideas to organize and frame the meaning of supporting details (Afflerbach & Cho, 2009; Maggioni & Fox, 2009).

When it comes to reading comprehension, it seems that some deviation from equitable distribution of attentional resources is adaptive. For example, Pressley and Afflerbach (1995) reported that highly effective learners tended to engage in pre-reading behaviors such as previewing and goal setting. During reading, they then spent more time in some parts of the text than others, took notes, returned iteratively to particular sections to clarify or identify information, and connected their reading to their prior understanding of the world. After reading new information, effective readers took time to summarize or reflect on their reading and reread selectively depending on the outcome of a self-evaluation of comprehension quality.

An additional manifestation of a global task strategy is note-taking, "a complex activity that requires comprehension and selection of information and written production processes" (Piolat, Olive, & Kellogg, 2004, p. 291). Note-taking is one of the most frequent behaviors students engage in while working with informational text (DeZure, Kaplan, & Deerman, 2001; Howe, 1970; Kauffman, 2004) and it is an important predictor of the outcomes of a SRL episode when students work with technical information (Bernacki, Byrnes, & Cromley, 2012; Kauffman, 2004). Notes quality has also been shown to indicate the likelihood of success on assessments of learning episodes associated with both textbooks and lectures (Peverly, Vekaria, & Garner, 2014). Some researchers, however, have questioned the universal value of note-taking. One reason for concern is the potential cost of continuously dividing one's attention during a learning episode (Long, 2014; Piolat et al., 2004). The potential cost of switching attention back and forth between materials may depend on the degree to which the student has executive capacity to control their attention (Altemeier, Jones, Abbott, & Berninger, 2006), but also on the specific part of the learning activity in which the student is engaged.

In sum, research pointing to nonlinear and non-equitable distributions of attention during reading, plus equivocal findings on the frequently occurring strategy of note-taking that link note-taking to the distribution of attentional resources, led us to wondering if a dynamic approach might reveal insights into the ways in which global task strategies disrupt or promote patterns of visual attention during learning.

The Present Study

Process-driven theories of self-regulated learning postulate ways in which time-based behavioral sequences are related to performance outcomes (e.g., Winne & Hadwin, 2008; Schnotz & Bannert, 2003). Commonly applied methodological approaches to collecting data about the processes that take place during learning include self-report questionnaires, rubric-driven analyses of think-aloud protocols, and other forms of qualitative data. Each of these approaches has its limitations; self-report can be subject to presentation bias and other sources of error (Dupreyat & Mariné, 2005; Winne & Jameson-Noel, 2002), and think-aloud protocols can be disruptive to ongoing psychological processes (Ariasi & Mason, 2010). Perhaps most problematic is that these methods provide little insight into and lack mechanisms for identifying and quantifying the patterns or *contingencies* that result from the moment-by-moment operation of psychological processes, even though these contingencies and the sequences may reveal just how it is that particular psychological processes operate in concert with one another. Symbolic dynamics offers a means by which these behavioral contingencies can be explored.

In the present study, SRL is conceptualized phenomenologically as studying (Winne & Hadwin, 2008). The primary goal was to explore and characterize the presence of nonrandom patterns of behavior during studying, with a view to considering the viability of the proposition that visual attention sequences during SRL can be modeled as a complex dynamic system. The direction and duration of attentional gaze served as an indicator of the contents of the learner's conscious attention, an accepted indicator of the strategic processing that takes place during the metacognitive control of working memory (Ainsworth, 2006; Just & Carpenter, 1980; Mayer, 2009; Norman & Shallice, 1980; Schnotz & Bannert, 2003). By providing participants with an authentic studying task that included multiple, overlapping sources of information to review, we hoped to trigger cognitive and metacognitive processes including goal setting, comprehension monitoring and self-checking strategies that lead to repeated reading, repeated reference to the task instructions, and note-taking.

A second goal was to apply principles of symbolic dynamics to explain how visual attention sequences in learning shift over time, and in particular to glean information about the structure of repeated sequences using orbital decomposition. We evoke a principle of structure, which is to say that we do not expect participants to behave randomly when faced with the learning task. The consequence of this is that we expect that the derivation of nonrandom patterns of visual attention will be possible, such that structure will be evident. We also propose to uncover contextual sensitivity, which is to say that we expect that the dynamic structure of participants' actions will be sensitive to the informational content provided in each of the source materials. Contextual sensitivity may manifest itself as some materials acting as points of origin for attentional sequences more so than others. This is of importance in relation to principles of complex dynamic systems, which include the existence of attractors—points or states to which the system iteratively returns.

The study focused on the following research questions:

1. What is the structure in the patterns of visual attention to different materials during a learning episode, and how does it vary with global task strategy?
2. Does the dynamic structure of participants' gaze sequences demonstrate characteristics of complexity and self-organization, and do these characteristics vary with global task strategy?

Method

Participants

Twenty-four undergraduate college students ($n = 12$ female, $n = 12$ male; mean age $= 22.14$ years) participated in the study in return for extra credit towards a course grade.

Materials and Procedure

Participants engaged in a self-paced, time-limited learning task in which they were instructed to learn about the structure and pattern of oxygenated and deoxygenated blood flow to and from the human heart (Fig. 16.1). Task materials consisted of three physically separated pages of diagrams of the heart accompanied by informational text. The first page presented the structure of the heart, the second page presented the arrangement of blood flow into and out of the heart and lungs, and the third page included a detailed and annotated graphic with explanation of where oxygenated and deoxygenated blood could be found. Participants were also given a blank sheet of paper on which they could take notes, and a separate page containing written instructions for their learning task. Participants were aware that they would need to learn the information for a test. A timer was visible to participants, who were instructed that they could have up to 10 min to study. After studying was complete and participants had answered several questions about their studying methods, participants were given a posttest.

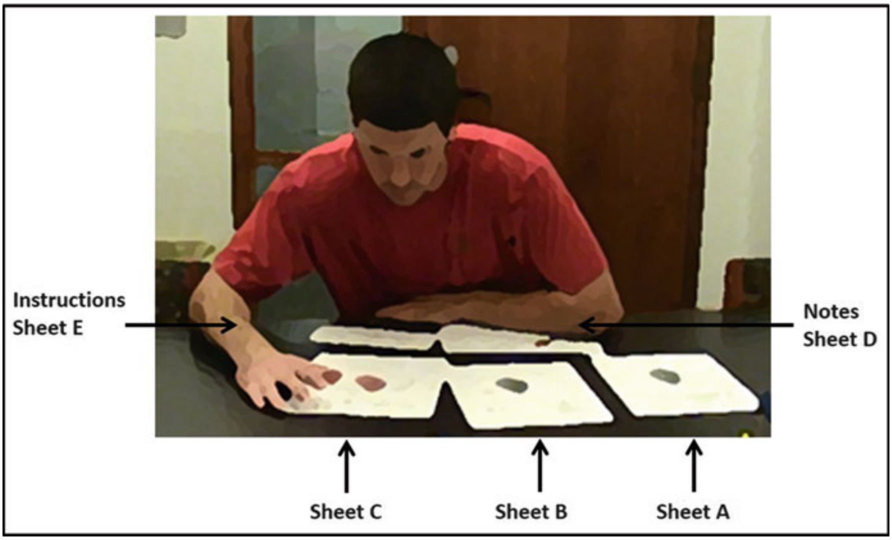

Fig. 16.1 Spatial configuration of task materials

Data Analysis

Studying behavior was recorded to DVD using a video camera. Footage from each participant's learning episode was then coded according to what the participant was looking at during each second of the recording, and how long the gaze lasted before a transition to something else. Coding was exhaustive, meaning that at any given point during the learning episode, visual attention was coded in one of the six categories: sheet 1 (A), sheet 2 (B), sheet 3 (C), notes/note-taking (D), instructions (E) or elsewhere, including a countdown timer (F). In accordance with procedures described by Guastello (2011), the resulting time series was converted to a string of letters, such as AAABBB, which would represent 3 s of looking at sheet 1 (A) followed by 3 s of looking at sheet 2 (B).

These letter sequences were quantified in several different ways. Because the length of this letter string varied among individuals, total sequence length was calculated. This reached a maximum value of 600, reflecting 1 letter per second for up to 600 s. This allowed the calculation of the percentage of time spent fixated on each material (e.g., total time spent on Sheet A = 128, total sequence length = 412 s, percentage of time spent on A = 128/412 = 31.1 %). Because we were interested in transitions of visual attention we calculated the number of transitions from one material to another, with the sum reflecting how many transitions occurred between categories. The mean time between transitions was then computed by dividing the total sequence length by the number of transitions plus one.

The symbol sequence was also turned into a graphical image using custom-written Matlab software (Mathworks, R2010a). Symbol sequence figures (Engbet et al., 1997)

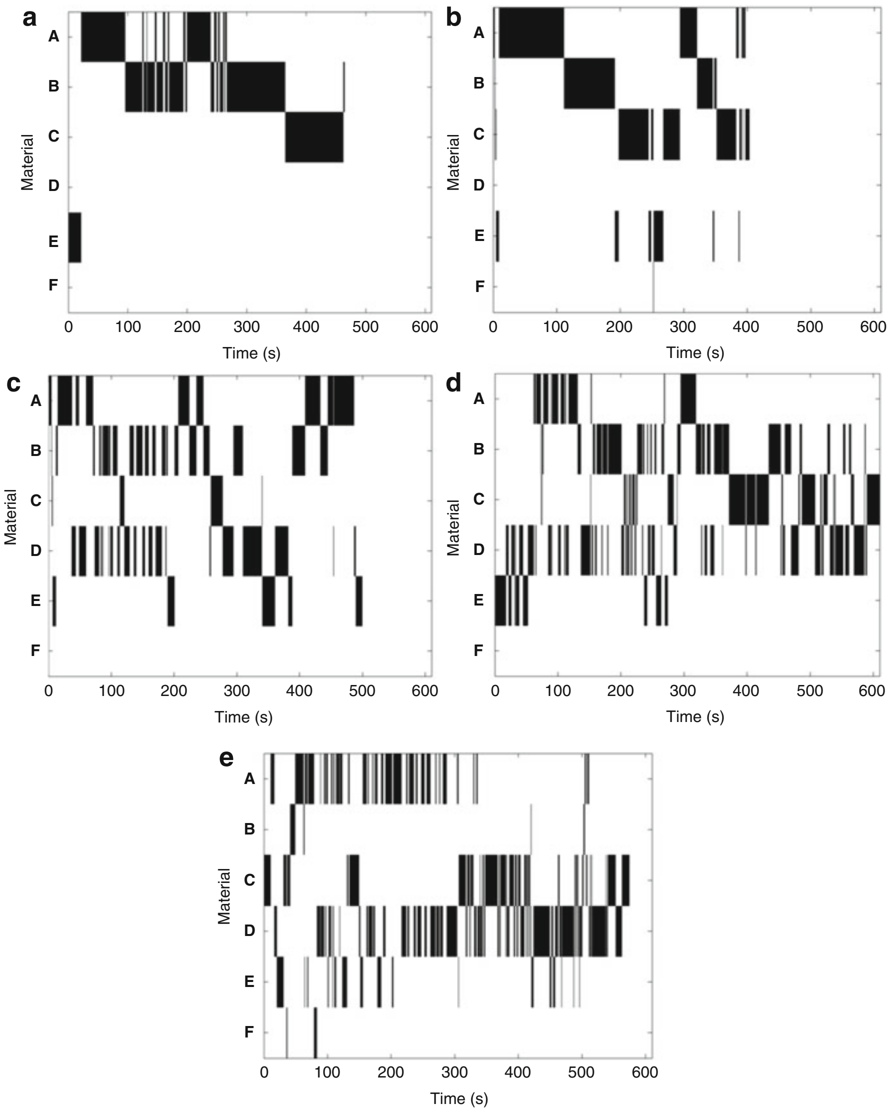

Fig. 16.2 Symbol sequence figures for individuals with Shannon entropy scores at the (**a**) 1st, (**b**) 25th, (**c**) 50th, (**d**) 75th, and (**e**) 99th percentiles

were generated to visually represent fixations and transitions in visual attention across the learning episode (Fig. 16.2). These figures provide an immediately visible depiction of differences in the patterns exhibited by participants.

As our interest was in the sequential structure of the time series each letter sequence was recoded to include only the transitions between the materials prior to

OD analysis. Orbital decomposition analysis was performed on these recoded strings of letters using ORBDE v1.2 (Peressini & Guastello, 2010). Final dependent variables computed by ORBDE included the trace matrix (tr$\mathbf{M}^{C}$), Shannon entropy (H_S), topological entropy (H_T), Lyapunov dimension (D_L), χ^2, and ϕ^2 at different sequence lengths (C). Additionally, the patterns that were detected for $C \geq 2$ were further analyzed. Unique sequences shared among at least two or more participants were inspected for beginning and second location in order to determine where participants were most and least likely to look next, after beginning a sequence with a particular material. Also calculated were the most frequently occurring unique sequences for each individual participant, and for the sample as a whole. This was done for the raw number and proportions of sequences, so that the number of times that the sequence was carried out could be expressed as a percentage of the number of unique sequences begun by a particular stimulus material. Finally, the number of recurrences for each pattern (X) and the number of different patterns with the same recurrence frequency (Y) were determined for each individual, and were analyzed for evidence of an inverse power-law distribution using nonlinear regression analysis. All nonlinear regression analyses and statistical analyses were conducted using SPSS 20.0.

Results

Research Question 1. What is the structure in the patterns of visual attention to different materials during a learning episode, and how does it vary with global task strategy?

Symbol Sequence Plots

Participants were free to study each of the different materials for as long and as often as they wanted within the total possible study time of 10 min. Figure 16.2a–e graphically reveals the visual attention patterns of four individuals during the studying task. These symbol sequence plots depict time on the x-axis and each location of visual gaze on the y-axis, thus providing a second-by-second account of the location of visual gaze in relation to learning materials, notes, and the task instructions.

It is clear from these figures that each individual had a different gaze pattern and that the complexity of these patterns varied across the participants. For example, the individual in Fig. 16.2a spent time studying the instructions, and then read material A followed by material B. He/she alternated between A and B for the majority of the time, before spending a block of time looking at material C. The participant ended by inspecting material B for a short period. In contrast, the individuals in

Fig. 16.2d and e immediately exhibited variability in visual attention by shifting between the different materials, notes, and instructions.

In addition to the information provided by these figures, quantitative analysis techniques were employed to reveal more about the structure in visual attention as it pertained to global task strategy (note-taking). Time spent looking at each of the materials will be assessed first, before we describe the gaze sequences revealed by the OD analysis.

Time Spent Viewing Materials

Individuals did not spend equal time looking at each material and differed from each other in how they allocated time across materials (Fig. 16.2). Regarding global task strategy, two-thirds of the sample ($n = 16$) elected to take notes (Fig. 16.2c–e) while they were reading the materials. Notes are indicated as material D. We hereafter refer to them as a group of "note-takers." The other one-third of the participants ($n = 8$) completed the task by reading without taking notes, and we hereafter refer to them as a group of "readers".

The overall time on task was not significantly different between the groups (see Table 16.1 for descriptive statistics), but a two-way mixed design ANOVA revealed that the proportion of time spent attending to each material was contingent upon the content of the material and the participant's overall strategy of reading versus reading and note-taking, $F(2.91, 63.90) = 9.06$, $p = 0.000$.

Sidak post hoc tests revealed statistically significant differences with large effect sizes depending on whether participants read or took notes for Sheet A, Notes/

Table 16.1 Descriptive statistics (mean and standard deviation) for the timing variables of visual attention for the reader and note-taker groups, and overall

	Participant group					
Variable	Readers ($n = 8$)		Note-takers ($n = 16$)		All ($n = 24$)	
	M	SD	*M*	SD	*M*	SD
Time (s)						
Total task	355.4	143.1	440.3	161.2	412.0	157.6
Before a transition	11.0	4.1	7.0*	4.0	8.4	4.4
Proportion of time						
Sheet A	43.7	14.3	24.7*	11.0	31.1	15.0
Sheet B	26.5	11.8	21.1	11.4	22.9	11.6
Sheet C	25.0	18.3	18.5	8.1	20.7	12.4
Notes D	0.0	0.0	27.0*	18.8	18.0	19.9
Instructions E	3.4	2.5	7.2*	4.7	5.9	4.4
Elsewhere F	1.3	2.1	1.6	1.6	1.5	1.7
Number of transitions	33.9	15.5	80.4*	44.4	64.9	43.1

*Significant group differences, $p < 0.05$

Sheet D, and Instructions/Sheet E (p's < 0.05). Readers spent proportionally more time inspecting sheet A than did note-takers (Cohen's $d = 1.57$), but note-takers spent proportionally more time inspecting the task instructions ($d = 0.91$). The pattern of group differences remained the same when the absolute time spent viewing the materials was considered. Post hoc comparisons between materials revealed that readers spent significantly more time attending to materials A and B than the notes, instructions, or elsewhere (materials D–F, p's < 0.05), but material C did not differ from any other location. In contrast, note-takers viewed materials A–C and the notes (material D) for approximately the same time (p's > 0.05), but significantly more than the instructions (material E), (p's < 0.05). All materials (A–E) were viewed significantly more than any nonmaterial location (designated F), (p's < 0.05). In sum, differing proportions of time spent viewing each of the materials indicated differences in the salience of each material to the participants, and changes in relative salience were dependent on whether or not notes were taken.

Orbital Decomposition: Origin of Various Gaze Sequences

Sequences were ordered by the material that represented their origin. When the frequencies of sequences initiated at each location were examined, it became apparent that the materials differentially impacted the likelihood that a sequence would originate at that location. Table 16.2 includes the number of unique sequences for each group, the percentage of the total number of unique sequences generated, the aggregated total number of repetitions of the unique sequences, and the mean number of repetitions for sequences initiated at each location per participant. As is apparent from Table 16.2, Sheet A was the most likely point of origin for a repeated gaze sequence for readers, but the notes page was the most likely point of origin for note-takers. In addition, the instructions were more likely to act as a point of origin for the note-takers, who also initiated a substantially higher number of different unique sequences from this location than did the readers.

Table 16.2 Frequencies of sequences that began at each point of origin for readers and note-takers

	Readers ($n = 8$)				Note-takers ($n = 16$)			
Learning material	No.	Percentage	Total no.	Mean	No.	Percentage	Total no.	Mean
Sheet A	22	32 %	282	35.25	44	19 %	680	42.50
Sheet B	16	24 %	219	27.38	48	21 %	615	38.48
Sheet C	21	31 %	144	18	34	15 %	530	33.13
Notes	0	0 %	0	0	68	30 %	1438	89.90
Instructions	5	7 %	21	2.62	26	11 %	155	9.19
Room/timer	4	6 %	12	1.5	10	4 %	29	1.81

Taking notes had a large and significant effect on the number of transitions between materials, $F(1, 22) = 8.11$, $p = 0.01$, $d = 1.23$ (Table 16.1). Note-takers made on average more than twice as many transitions between materials while studying than participants who only read. The time spent looking at a material before switching visual attention elsewhere was also significantly less for note-takers than readers, $F(1, 22) = 5.12$, $p = 0.03$, $d = 0.98$. The symbol sequence figures and quantitative measures about the time viewing the materials revealed significant differences between note-takers and readers in their visual attention of the different materials. Next, we consider if the sequences of viewing the materials demonstrate any patterns and differences between these groups.

Orbital Decomposition: Repeated Sequences

The OD analysis broke the time series of categories (in our case, visual attention to a particular material) into patterns of different length that reoccur immediately or at any point in time. Given that participants could remain focused on the same material over many seconds, each time series was turned into the sequence with which materials were viewed (e.g., AAAABBAAAC became ABAC). Under these circumstances orbital decomposition could be used to inform about patterns in the sequence of materials. For increasing sequence lengths, designated C, all patterns of that length were identified, along with how many times each pattern occurred. Additionally, the number of different patterns that were immediately repeated were counted. This process continued until the longest pattern sequence that immediately reoccurred was determined. The length of this longest pattern was designated as the maximum C length.

In our sample, the maximum C length varied among participants, revealing individual differences in the maximum number of gaze locations that comprised an immediately repeated sequence. Across the sample the minimum C length was 2 and the maximum C length was 32, meaning that at least one individual did not immediately return to any sequence that was longer than 2 locations in length, but at least one individual immediately returned to a sequence that was 32 locations in length. The overall mean C length for the sample was 8.2 (SD = 6.6). The mean length of any immediately repeated gaze sequence was 9.1 locations for note-takers (SD = 7.6) but only 5.1 locations for readers (SD = 3.6); however this difference failed to reach the level of significance ($p = .30$, $d = 0.45$).

As an illustration, Table 16.3 provides a summary of the more frequently occurring patterns of sequence lengths $C = 2$–5 for readers and note-takers. Only patterns which reoccurred at least twice for an individual participant were included in the analysis and only those that occurred at least an average of once per person are displayed in the table. Data for the other less frequent patterns are summarized together. Mean frequency is the number of recurrences of a particular pattern across all members of a group divided by the number of participants in that group. Given the large discrepancy in the number of total recurrences between the groups, the

Table 16.3 Mean frequency of patterns of sequence lengths (C = 2-5) performed by readers and note-takers

Readers ($n = 8$)		Note-takers ($n = 16$)	
Pattern	Mean frequency (% of recurrences)	Pattern	Mean frequency (% of recurrences)
$C = 2$ (252 recurrences of 12 unique patterns)		$C = 2$ (1144 recurrences of 26 unique patterns)	
AB	7.8 (24.6 %)	DA	10.1 (14.2 %)
BC	6.1 (19.4 %)	BD	7.7 (10.8 %)
CA	5.1 (16.3 %)	DB	7.7 (10.8 %)
BA	4.8 (15.1 %)	CD	7.0 (9.8 %)
AC	4.0 (12.7 %)	DC	6.9 (9.6 %)
EC	1.0 (3.2 %)	AE	5.2 (7.3 %)
6 other patterns	2.8 (8.7 %)	BA	3.3 (4.5 %)
		AB	3.1 (4.4 %)
		CE	3.0 (4.2 %)
		BC	2.6 (3.6 %)
		CB	2.1 (3.0 %)
		AD	1.8 (2.4 %)
		DE	1.7 (2.4 %)
		ED	1.6 (2.3 %)
		CA	1.4 (2.0 %)
		EC	1.1 (1.6 %)
		10 other patterns	5.3 (7.3 %)
$C = 3$ (191 recurrences of 19 unique patterns)		$C = 3$ (1009 recurrences of 62 unique patterns)	
ABA	5.9 (24.6 %)	DAD	8.6 (13.7 %)
BAB	4.1 (17.3 %)	ADA	8.4 (13.3 %)
CAC	2.9 (12.04 %)	DBD	6.0 (9.5 %)
ABC	1.9 (7.9 %)	BDB	5.4 (8.5 %)
BAC	1.6 (6.8 %)	DCD	5.3 (8.3 %)
ACE	1.5 (6.3 %)	CDC	5.1 (8.0 %)
BCA	1.0 (4.2 %)	ADB	4.3 (6.8 %)
12 other patterns	5.0 (20.9 %)	BAB	1.5 (2.4 %)
		54 other patterns	18.6 (29.4 %)
$C = 4$ (133 recurrences of 21 unique patterns)		$C = 4$ (742 recurrences of 76 unique patterns)	
ABAB	4.6 (27.8 %)	DADA	7.2 (15.5 %)
CACA	2.3 (13.5 %)	ADAD	7.1 (15.4 %)
ACAC	1.8 (10.5 %)	DBDB	4.6 (9.8 %)
BABA	1.4 (8.3 %)	DCDC	4.1 (8.8 %)
17 other patterns	6.6 (39.9 %)	CDCD	3.9 (8.5 %)
		BDBD	3.7 (8.0 %)
		70 other patterns	15.8 (34.1 %)

(continued)

Table 16.3 (continued)

Readers ($n = 8$)		Note-takers ($n = 16$)	
Pattern	Mean frequency (% of recurrences)	Pattern	Mean frequency (% of recurrences)
$C = 5$ (102 recurrences of 19 unique patterns)		$C = 5$ (544 recurrences of 69 unique patterns)	
BABAB	5.0 (39.2 %)	DADAD	6.7 (19.7 %)
CACAC	1.5 (11.8 %)	BDBDB	3.9 (11.4 %)
ABABA	1.1 (8.8 %)	DBDBD	3.9 (11.4 %)
16 other patterns	5.1 (40.2 %)	CDCDC	3.4 (10.1 %)
		DCDCD	3.4 (9.9 %)
		ADADA	1.7 (5.0 %)
		63 other patterns	11.1 (32.5 %)

Only patterns with a frequency of at least once per participant are displayed

percentage of the total recurrences is also provided for each pattern. The percentage provides an indication of the strength of attraction for that particular pattern compared with other patterns performed by the participants of that group.

The overall number of recurrences decreased as sequence length increased. However, the total number of unique patterns increased with longer sequence length up to $C = 4$, after which they decreased for $C = 5$ and beyond. What is clear in comparing the two groups is that taking note-taking led to many more unique patterns (greater flexibility) and total recurrences for C lengths 2, 3, 4, and 5.

The most common patterns of visual attention across materials also differed with global task strategy. For note-takers, visual attention to the notes (material D) was involved in a large percentage of sequence recurrences (63.6 % of $C = 2$; 86.1 % of $C = 3$; 73.7 % of $C = 4$; 83.4 % of $C = 5$). The five most frequently occurring patterns at each of the different sequence lengths ($C = 2$–5) all involved notes (material D) and an instructional material (A–C). Note-takers generated the majority of their transitions between an instructional material and the notes. This meant that note-takers made proportionally less sequences (19.3 % of $C = 2$) switching directly between instructional materials (e.g., AB, AC, BA, BC, CA, or CB), compared with readers who generated 89.7 % of $C = 2$ recurrences between the materials A, B, and C.

Another difference in the use of materials between the groups was in the proportion of $C = 2$ recurring patterns that included the task instructions, which suggests a link between the global task strategy of note-taking and metacognitive monitoring processes. Note-takers looked at the instructions in 21.1 % of $C = 2$ recurrences, while readers only viewed the instructions in 6.3 % of the total recurrences. No differences in the utilization of instructions were observed for higher sequence lengths. A final difference between the groups was in the proportion of patterns that involved more than two materials. Of the seven most frequent

$C = 3$ patterns by readers (mean frequency ≥ 1), four involved three different materials (i.e., ABC, BAC, ACE, BCA) amounting to 25.2 % of the total recurrences. In contrast, only one of the most frequent sequences performed by the note-takers included three different materials (i.e., ADB) totaling only 6.8 % of the total recurrences. For C lengths of 4 and 5 none of the most frequent patterns included more than two different materials. These findings indicate that participants mostly went back and forth between two materials. However, readers executed more visual attention sequences that switched between three materials than did note-takers.

Research Question 2. Does the dynamic structure of participants' gaze sequences demonstrate characteristics of complexity and self-organization, and do these characteristics vary with global task strategy?

Orbital Decomposition: Overall Measures of Complexity

OD computes several variables that provide indices to different aspects of complexity, including Shannon entropy, topological entropy, and dimensionality, based on the maximum Lyapunov exponent. These values are quantified for each sequence length. For comparison between the groups we selected $C = 2$, because many longer sequences were made up of this length pattern (Table 16.3) and 23 out of 24 participants produced a $C = 2$ pattern that was immediately repeated, which was a necessary pattern for the calculation of some indices.

Shannon entropy quantifies the probability of the patterns occurring. Lower values indicate fewer patterns being performed many times, while higher values indicate more different patterns, in other words greater novelty. Note-takers were found to have more novelty in their visual attention sequences than readers (Table 16.4). This finding is in line with the analysis of the number of unique patterns performed, which revealed that note-takers performed many more unique repeated sequences than readers.

While Shannon entropy quantifies the repetition of sequences it ignores the temporal structure of these sequences, i.e., whether they immediately reoccur or not. In contrast, topological entropy and dimensionality measures were estimated based on the immediate recurrence of a pattern. The number of different sequences that were immediately repeated (trace of the transition matrix) for $C = 2$ was significantly greater for note-takers than readers (Table 16.4). Since topological entropy has been interpreted as indicating turbulence, this finding shows that note-takers' overall sequence of visual attention was structurally more complex than that exhibited by readers. When the dimensionality of the visual attention sequence was estimated according to the maximum Lyapunov exponent based on topological entropy, for $C = 2$ the dimensionality for all participants was nonlinear ($D_L > 1$). In addition, the dimensionality was significantly greater for the note-takers than for the readers (Table 16.4). Simply stated, note-takers performed more unique repeated sequences (i.e., orbits) and repeated these patterns more often during the studying episode, suggesting both more structure and more complexity in their visual

Table 16.4 Group descriptive statistics (mean and standard deviation) of the final statistics obtained from orbital decomposition at the repeated sequence length of 2

	Readers ($n = 7$)		Note-takers ($n = 16$)	
Indicator	*M*	SD	*M*	SD
Shannon entropy	1.85	0.46	2.34**	0.34
Trace matrix	3.71	1.50	6.38*	2.68
Topological entropy	0.89	0.31	1.25*	0.38
Lyapunov dimension	2.54	0.75	3.73*	1.20
χ^2	31.64	18.78	60.48	40.36
ϕ^2	0.82	0.23	0.70*	0.21

*$p < 0.05$, **$p < 0.01$

attention patterns. These results indicate that the more complex visual attention sequences performed by note-takers were not simply due to greater stochastic noise, but rather arose deterministically from a richer attractor landscape.

Inverse Power-Law: Characteristics of a Self-Organizing System

Self-organizing systems that display fractal properties have frequency distributions that follow an IPL function, where there are many more small compared with large parts. This is in contrast with a normal frequency distribution, where the highest frequency would be for the medium-size components. In our case, the number of different patterns served as the system parts. These data were summed for each number of pattern reoccurrences and the resulting frequency distribution was plotted. The resulting distributions followed an IPL function and were not Gaussian.

Figure 16.3 shows an example of this phenomenon for a single participant, where the vast majority of patterns only repeat once and very few patterns reoccur up to 19 times. For this participant, nonlinear regression analysis determined the following function, $Y = 169.83 \times X^{-2.96}$, $R^2 = 0.99$. The shape of the IPL function ($b = -2.96$) provides information about the ratio of patterns that repeat few times to those that repeat many times, and is a measure of the system's fractal dimensionality (D_F). A D_F equal to -2.96 indicates a fractal process. This finding was consistent across all subjects, with an average $D_F = -3.11$ indicating strong fractal dimensionality in the distribution of different repeating visual gaze sequences. Integrity of the IPL function can be quantified by the variance accounted for, with a high R^2 value revealing high structural integrity (Table 16.5). The overall mean R^2 across participants was 0.94, supporting the conclusion that visual gaze sequences displayed characteristics of a self-organizing system. Thus, the fractal

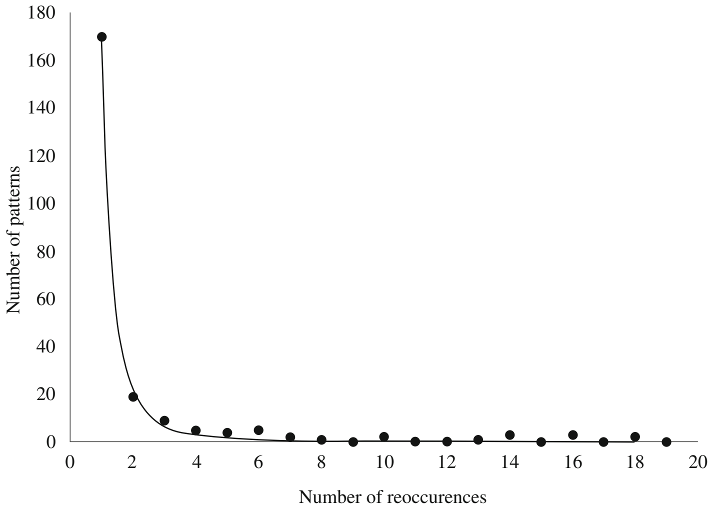

Fig. 16.3 Illustration from a single participant of the inverse power-law relationship between the number of reoccurrences of patterns and the number of unique patterns that were repeated

Table 16.5 Group and overall descriptive statistics (mean and standard deviation) for the inverse power-law analysis on repeated sequence lengths

	Readers ($n = 8$)		Note-takers ($n = 16$)		All ($n = 24$)	
Function statistics	*M*	SD	*M*	SD	*M*	SD
b	−4.08	5.31	−2.62	0.71	−3.11	3.07
R^2	0.86	0.23	0.98*	0.04	0.94	0.13
Highest repeated pattern frequency	9.50	5.66	13.5	8.20	12.17	7.57

$^*p < 0.05$

structure of the pattern of reoccurring gaze sequences during a SRL episode is congruent with known properties of self-organizing systems.

While all participants demonstrated significant IPL relationships for the frequency of repetitions of sequences, differences were observed between the groups according to global task strategy. On average, the readers had larger D_F. However, this was not significant ($p > 0.05$), likely due to the large variability between subjects (Table 16.5). In contrast, note-takers still had a large average D_F but exhibited much reduced variability. A Mann-Whitney U test revealed significant differences between the two group's R^2 fit indices, with note-takers exhibiting significantly greater R^2 values ($p < 0.05$) and therefore greater structural integrity in visual attention patterns than the readers. In sum, although both groups exhibited visual gaze sequences that closely resembled the properties of self-organizing systems, greater flexibility and adaptability in the sequence of studying the materials were found for note-takers.

Discussion

To our knowledge, this study is the first of its kind to use symbolic dynamics to investigate the nature of gaze sequences during a self-regulated learning episode. Drawing on principles of complexity science and dynamic systems theory, we used orbital decomposition (OD) to methodologically frame the visual inspection of learning materials as an example of learners' exploration of an attractor landscape, and we proposed that participants' tactical decisions would impact their behavior within it. Our two research questions concerned (1) the presence and nature of patterned sequences in relation to global task strategies such as note-taking, and (2) the degree to which these patterns revealed evidence that the cognitive and metacognitive processes responsible for directing attentional guidance during learning can exhibit the features of a complex, dynamic, and self-organizing system.

The OD analysis explicated gaze sequences and allowed for the quantification of the degree to which these sequences were repeated. It revealed that regardless of whether or not a note-taking strategy was used, visual attention sequences were most likely to consist of iterative repetitions representing movement between just two locations. None of the participants exhibited a linear behavioral trace of looking at each material without returning to the same material. Sequences longer than two locations were detected but these were primarily comprised of iterative repetitions of two-location sequences (e.g., ADAD) rather than sequences that represented sequential attention to all of the learning materials (e.g., ABDC). This was particularly evident for the note-taking group, for whom notes comprised a significant attentional attractor.

The process of checking for congruence and completeness between informational sources is known as intertextual comparison, and its use has been associated with improved learning outcomes (Stahl, Hynd, Britton, McNish, & Bosquet, 1996; Kobayashi, 2009). In our study readers, more than note-takers, executed intertextual visual sequences. The note-takers' apparent reluctance to make intertextual comparisons has implications for modeling the process of how learners create coherent representations of meaning from multiple sources of information, and what happens when they read and take notes to support comprehension. Studying from informational texts requires two interrelated processes, the first being the derivation of meaning within and across sources, and the second being the integration of information with prior knowledge (Holsanova, Holmberg, & Holmqvist, 2009; Kintsch, 1991; Schnotz & Bannert, 2003). The process of selecting, organizing, and integrating information is postulated to end with the formation of a mental model, an abstract internalized representation of concepts and their relations that supports future performance (Glenberg, Meyer, & Lindem, 1987; Johnson-Laird, 2013; Leopold & Leutner, 2012; Mayer, 2001, 2005; Schnotz & Bannert, 2003; Van Meter & Firetto, 2013). When creating a mental model by inspecting multiple texts, the learner must read and compare the different sources.

Unfortunately, research on students' learning strategies has also shown that intertextual comparisons are rare, unlikely to be generated spontaneously, and very unlikely to manifest in written form through the content of notes (Hagen, Braasch, & Bråten, 2014; Stahl et al., 1996). Our findings corroborate this; direct unmitigated intertextual comparisons were extremely unlikely if the individual was taking notes. In our study, the information contained in each informational sheet was designed to complement and build on the contents of the other sheets. However, although it was entirely feasible for participants to produce sequences such as ABD (A followed by B followed by Notes) or ADBD (A followed by Notes followed by B followed by Notes), these sequences almost never appeared if the individual took notes. Readers, on the other hand, were far more likely to demonstrate intertextual sequences such as AB or ABAB.

Note-taking is often promoted as a useful supplement to careful reading that can greatly improve comprehension and is commonly included in lists of activities associated with effective text-based self-regulated learning (Kauffman, 2004). According to Kierwa (1987), note-taking can exert a meaningful impact on learning by promoting encoding in working memory, and by providing external storage that relieves cognitive resources that are otherwise heavily taxed during the comprehension process. However, studies have shown that students' notes are often verbatim rather than elaborative, or highly deficient in content, which can lead to students studying from incomplete or incorrect information (Britt & Sommer, 2004; Hagen et al., 2014; Kierwa, DuBois, Christensen, Kim, & Lindberg, 1989). In one study, simple rereading of text was even found to be more beneficial than note-taking (Kierwa et al., 1989), while another applied study reported that students' performance in a lecture class was actually superior when note-taking was not permitted (Piolat et al., 2004). Additionally, students with cognitive and attentional disabilities are often overwhelmed by the task of taking notes while listening or reading new information (Kauffman, 2004).

Our findings suggest that note-taking and reading are quite different tasks from the perspective of the patterns of visual gaze associated with them. For participants in our study who took notes, the notes page was a salient part of the learning process and it strongly impacted the sequence and tempo of visual attention and gaze sequences. The notes page became a temporal as well as spatial attentional attractor during learning—as a group, note-takers on average devoted about one-quarter of the 10-min studying time attending to their notes. Together, these findings highlight the substantial effect that note-taking can have on the way that visual attention sequences are executed during studying.

Self-monitoring refers to attempts to reflect upon and react to judgments of progress in relation to a set task or goal (Greene & Azevedo, 2009; Pintrich, 2000; Winne & Hadwin, 2008). Strategies for self-monitoring include pausing to compare information or to check on the accuracy of notes (Azevedo, Guthrie, & Seibert, 2004; Kauffman, Zhao, & Yang, 2011; Zimmerman & Paulsen, 1995). In our study, a second manifestation of the way in which global task strategy became linked to visual attention patterns came from the analysis of the proportions of time that individuals spent viewing particular learning materials and the task instruction, which we suspect links to attempts to self-monitor comprehension and progress through the task. In the context of the present study, we used reference to the instructions as an indicator that the learner was engaged in self-monitoring, presumably due to the realization that one portion of the task had been accomplished, or the desire to check that learning was aligned with the performance outcomes listed, or both. The analysis of the proportions of time spent viewing each material showed that on average, participants who engaged in note-taking spent twice as long viewing the task instructions than participants who did not take notes. We speculate that this may have been due to a variety of factors relating to self-monitoring; note-takers may have been less sure of the task at hand, may have felt the need to use the instructions to guide their note-taking, or may have been using the task instructions to gauge the completeness of their learning behaviors.

In sum, we found that the dynamics of visual attention were not linear and progressive. Instead, they were patterned and repetitive in nature. Global task strategy, particularly note-taking, led to significantly higher rates of attentional shifting throughout the learning episode, with proportionately less time spent alternating back and forth between information sheets and more time alternating attention between material and the notes page. Note-taking was also associated with increased use of the task instructions compared to the groups who read but did not take notes. To our knowledge, these findings are novel among studies on SRL and learning from text.

Our study also sought to determine the degree to which visual attention during learning could be understood as a manifestation of a complex, dynamic system. The second research question asked to what degree novelty and dimensionality might be found in the attentional sequences, and to what extent such indices of self-organization might vary according to global task strategy choices such as note-taking. We found that the complexity and organization of visual attention sequences varied with global task strategy; whereas readers tended to initiate sequences from the first informational sheet, note-takers initiated sequences from the notes page that they were creating. Note-takers initiated a greater number of repeated novel attentional sequences overall, as indicated by higher Shannon entropy values, with a significant proportion of them originating from the notes page.

The greater complexity found in the note-taking group was not due to greater noise, but from deterministic processes. The topological entropy was significantly higher for this group, indicating greater turbulence and a higher fractal dimensionality based on the largest Lyapunov exponent (Table 16.4). Although the fractal dimension computed as the steepness of the curve (b) in the IPL analysis did not reach significance between groups, the significantly higher R^2 values (Table 16.5) demonstrated that the frequency distributions for the note-takers could be better modeled as an IPL function than that of the readers (Fig. 16.3). This is a hallmark of self-organizing systems.

Note-taking was associated with greater variability in attentional sequences, with the notes page acting as an attractor or anchor point of origin for a large number of different patterns of attention. In addition, the note-taking group spent more time viewing the instructions and used the instructions as the origin of significantly greater numbers of unique, repeated attentional sequences. This is an example of note-takers' exhibiting more novelty in attentional sequences—they were less rigid than readers, exhibited a richer range of patterns, and were not "stuck" in a small number of high-frequency repetition patterns. Metaphorically speaking, their attractor landscape was more varied, and they moved more freely among the valleys. It seems that note-taking may foster psychological processes that have greater structure and flexibility than reading alone.

Limitations

While OD exhaustively detects patterns in the data and computes several complexity metrics, like any other analysis technique it has its limitations. First, it is critical for the researchers to define appropriate categories for coding and to record the categories with an appropriate time resolution or sequence of categories. These factors require knowledge about the area of investigation and potentially some trial and error. Recent developments in the ORBDE software allow several categories to be coded at the same time allowing more complex assessments of patterns in behavior or learning (Peressini & Guastello, 2014). Such a capability raises questions about how to classify behaviors or utterances, and upholds rather than relieves the researcher's incumbent responsibility to proceed with caution in this regard.

Second, ORBDE exhaustively determines patterns in the data, but only reports the number of occurrences of each pattern in the total data length or whether a pattern is immediately repeated once or not at all. It does not indicate how many times a sequence immediately repeats itself, even though such information may be helpful for understanding some dynamic aspects of a system. A third issue is that ORBDE computes complexity metrics for any C length that has at least one pattern that immediately repeats. Therefore, an issue faced by the researcher is the determination of the appropriate C length from which to consider the complexity metrics. It has been recommended that the optimal C length is the length of the longest sequence that is immediately repeated (Guastello, Koopmans, & Pincus, 2011). In our opinion, this potentially places undue focus on a single pattern. Guastello et al. preempt this concern by stating that it is also important to consider statistical techniques, including χ^2, and the value of Shannon entropy, which both provide information about structure in the data series. In the current study our goal was to compare readers and note-takers on different metrics computed by ORBDE. For statistical comparison it made more sense to compare values for the same sequence length; otherwise the metrics would have been heavily dependent on the C length used. Fourth, it should be noted that selecting maximum C length tends to lead to a low trace of the matrix, which in turn approaches a topological entropy value close to 0 and a D_L close to 1.

Finally, a substantial conceptual limitation for our study is the lack of prior work on reading and SRL that has articulated component processes as being dynamically interrelated and constituent parts of a complex dynamic system. Therefore, although we can describe the patterns of visual attention that were evident in the data, basic questions remain. For example, it is far from clear whether it is advantageous to exhibit greater or lesser complexity or flexibility when engaged in a SRL episode. Similarly, our proposal that self-monitoring processes might trigger particular sequences of visual attention is speculative, as is our suspicion that visual sequences that originate from either notes or one information sheet in particular might be meaningfully construed (and therefore perhaps, in future research, manipulated) as attractors in a landscape. However, in spite of these concerns we believe that careful use of OD promises to reveal dynamical structure in learning behaviors that cannot be readily quantified with an interval or ratio scale and therefore have heretofore been neglected.

Future Directions

The inclusion of complex dynamic systems into educational research requires the development of a particular epistemological posture about the nature of the phenomena being studied. A general step-by-step approach for adopting such a perspective has been articulated by Stanton and Welsh (2012). In regard to theory, they call for the researcher to conceptualize the phenomena of interest as a multicomponent, dynamic system of mutually influential variables, rather than as a collection of independent, dependent, and moderator variables that interact according to mechanistic, linear relationships. In our view, regarding the problem of how to investigate learning from text, this requires an acknowledgement of the potential for simultaneous and reciprocal influences among groups of cognitive, metacognitive, and affective constructs, and a far more dynamic progression through SRL phases than has typically been conceptualized.

A consequential second and more empirically focused step is for researchers to identify a "collective variable" of interest, an "observable phenomenon that captures the interrelatedness of diverse system elements" (Lunkenheimer & Dishion, 2009 p. 290 in Stanton & Welsh, 2012). The collective variable, analogous to a dependent variable, must be quantifiable through observation, linked to a construct of interest, and able to account for the ever-changing interaction between the system and the context. The collective variable in the present study was a measurement of visual gaze patterns, which we designated as a proxy measure for visual attention and the target of active processes in working memory. Visual gaze patterns were predicted to be sensitive to the contingencies among behaviors needed to execute the learning process as well as choices of strategy and task conditions.

Third, the researcher must characterize attractor states, which are conceptual and empirical locations where the system may settle. The possible potential locations or states in which the system may be found are designated as the landscape or plane of attractors. In the present study the attractors were the repeated gaze patterns and the attractor plane was comprised of all possible sequences or locations that the individual could visit during their learning episode. Our findings reveal marked differences in how the attractor plane was navigated by readers and note-takers. In this way, we accomplished an additional step, which Stanton and Welsh describe as the process of identifying the "dynamic trajectory of the collective variable" (p. 21). Using OD we were able to determine how each learner's cognitive-metacognitive system moved around the attractor plane. Through symbol sequence plots and the summary presented in Table 16.2, we were able to construct a representation of the dynamic "signature" of systems' behavior over time.

In a final step, Stanton and Welsh challenged researchers to identify and then intervene in the control parameters of the system. Control parameters, like independent variables, are features that can be manipulated in order to impact an outcome. A naturally occurring control parameter emerged in the case of our study; individuals' decisions to take notes or simply read the material resulted in significant differences in the collective variable of visual gaze patterns.

In sum, by adopting a complex dynamic systems approach we found that global task strategy propagated very different patterns of task execution behaviors during learning from text. Simply stated from the perspective of task execution, reading and note-taking appear to be very different from one another. Greater intertextual comparisons were made by readers, for whom the primary information sheet served as an attentional anchor. Note-takers relied heavily on a notes page and the task instructions to structure their attentional sequences, and made many more (and more flexible) shifts in visual attention throughout the learning episode. What is not clear, and should be the subject of further research, is how these differences may relate to individual differences in content knowledge, strategic knowledge, reading comprehension, and processing capacity. Also unknown is how such differences might be associated with performance outcomes, and whether global task strategy might exhibit a nonlinear effect on such an outcome depending on other aspects of each learner's metacognitive system.

Learner's systems were "found" in some locations within the attractor plane with much greater frequency than in others. This suggests that as SRL episodes progress, learners may variously become entrained or entrenched in attractors. Whether such entrenchment is adaptive or maladaptive, and when and how interventions might be timed to interrupt or perturb the learner's SRL system to promote its movement to a different attractor or its stability in service of a particular learning sub-goal, are also intriguing questions for future research. To answer this and other questions we challenge SRL theorists to increase their consideration of behavioral contingencies within and across learning episodes, and to incorporate their findings in ways that might add complexity-based ideas to currently dominant models that emphasize a trajectory through distinct phases of learning. Integrating the two perspectives together may yield helpful insights, such as understanding the conditions that occur just prior to an individual's decision to shift strategies and take notes, pause to judge the quality of their learning, or conclude a learning episode.

Appendix: Calculation of Number of Recurring String Patterns (N_C)

N_C is estimated by first creating a separate transition matrix $\mathbf{M}^{\mathrm{C}}$ for each C sequence length with the allowable strings on each axis. Each cell then contains 1 if the row string is immediately followed by the column string or 0 if it does not:

		AD	DA	EA	AE
$\mathbf{M}^{\mathrm{C}}=$	AD	**1**	0	0	1
	DA	0	**1**	0	0
	EA	0	1	**1**	0
	AE	1	0	0	**0**

The diagonal of the matrix (bolded in the example above) indicates if a string is immediately followed by itself (1) or not (0). Rather than using the determinant of this matrix, which could be very large and computationally intensive, the trace of $\mathbf{M}^{\mathrm{C}}$ (tr$\mathbf{M}^{\mathrm{C}}$) can be easily computed by summing the 1s from the diagonal (Lathrop & Kostelich, 1989). Hence, tr$\mathbf{M}^{\mathrm{C}}$ represents the number of strings that are immediately repeated (e.g., for $C = 2$, tr$\mathbf{M}^{\mathrm{C}} = 3$) and is used to compute H_{T}.

References

Afflerbach, P., & Cho, B. (2009). Identifying and describing constructively responsive comprehension strategies in new and traditional forms of reading. In S. Israel & G. Duffy (Eds.), *Handbook of reading comprehension research* (pp. 69–90). Mahwah, NJ: Erlbaum Associates.

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*, 183–198.

Altemeier, L., Jones, J., Abbott, R. D., & Berninger, V. W. (2006). Executive functions in becoming readers and reading writers: Note taking and report writing in third and fifth graders. *Developmental Neuropsychology, 29*, 161–173.

Ariasi, N., & Mason, L. (2010). Uncovering the effect of text structure in learning from a science text: An eye-tracking study. *Instructional Science, 39*, 581–601.

Azevedo, R., Guthrie, J., & Seibert, D. (2004). The role of self-regulated learning in fostering students' conceptual understanding of complex systems with hypermedia. *Journal of Educational Computing Research, 30*, 87–111.

Bernacki, M. L., Byrnes, J. P., & Cromley, J. G. (2012). The effects of achievement goals and self-regulated learning behaviors on reading comprehension in technology-enhanced learning environments. *Contemporary Educational Psychology, 37*, 148–161.

Boekaerts, M., & Corno, L. (2005). Self-regulation in the classroom: A perspective on assessment and intervention. *Applied Psychology: An International Review, 54*, 199–231.

Bråten, I., & Stromsø, H. I. (2011). Measuring strategic processing when students read multiple texts. *Metacognition and Learning, 2*, 111–130.

Britt, M. A., & Sommer, J. (2004). Facilitating textual integration with macro-structure focusing tasks. *Reading Psychology, 25*, 313–339.

Capra, F. (1983). *The turning point*. Toronto, ON: Bantam.

Crutchfield, J. P. (1994). Is anything ever new? Considering emergence. In G. Cowan, D. Pines, & D. Meltzner (Eds.), *The science of complexity XIX* (pp. 1–15). Reading, MA: Addison-Wesley.

Davis, B., & Sumara, D. (2008). Complexity as a theory of education. *Transnational Curriculum Inquiry, 5*, 33–44.

DeZure, D., Kaplan, M., & Deerman, M. A. (2001). *Research on student notetaking: Implications for faculty and graduate student instructors*. Center for Research on Learning and Teaching. University of Michigan, No. 16. Retrieved from http://www.crlt.umich.edu/sites/default/files/resource_files/CRLT_no16.pdf

Dupreyat, C., & Mariné, C. (2005). Implicit theories of intelligence, goal orientation, cognitive engagement, and achievement: A test of Dweck's model with returning to school adults. *Contemporary Educational Psychology, 30*, 43–59.

Engbet, R., Shiek, M., Scheffczyk, C., Kurths, J., Krampe, R., Kliegl, R., et al. (1997). Symbolic dynamics of physiological synchronization: Examples from bimanual movements and cardio-respiratory synchronization. *Nonlinear Analysis, 30*, 973–984.

Garner, J. K. (2009). Conceptualizing the relations between executive functions and self-regulated learning. *Journal of Psychology, 143*, 405–426.

Glenberg, A. M., Meyer, M., & Lindem, K. (1987). Mental models contribute to foregrounding during text comprehension. *Journal of Memory and Language, 26*, 69–83.

Granic, J., & Lamey, A. V. (2002). Combining dynamic systems and multivariate analyses to compare the mother-child interactions of externalizing subtypes. *Journal of Abnormal Child Psychology, 30*, 265–283.

Granic, I., & Patterson, G. R. (2006). Toward a comprehensive model of antisocial development: A dynamic systems approach. *Psychological Review, 113*, 101–131.

Greene, J., & Azevedo, R. (2009). A macro-level analysis of SRL processes and their relations to the acquisition of sophisticated mental models. *Contemporary Educational Psychology, 34*, 18–29.

Guastello, S. J. (2000). Symbolic dynamic patterns of written exchanges. *Nonlinear Dynamics Psychology & Life Sciences, 4*, 169–187.

Guastello, S. J. (2005). Nonlinear models for the social sciences. In S. A. Whelan (Ed.), *The handbook of group research and practice*. Thousand Oaks, CA: Sage.

Guastello, S. J. (2008). Chaos and conflict: Recognizing patterns. *E:CO, 10*, 1–9.

Guastello, S. J. (2011). Orbital decomposition: Identification of dynamical patterns in categorical data. In S. J. Guastello, & R. A. M. Gregson (Eds.), *Nonlinear dynamical systems analysis for the behavioral sciences using real data* (pp. 499-516). Boca Raton, FL: CRC Press.
Guastello, S. J., & Gregson, R. A. M. (2011). *Nonlinear dynamical systems analysis for the behavioral sciences using real data*. Boca Raton, FL: CRC Press.

Guastello, S. J., Hyde, T., & Odak, M. (1998). Symbolic dynamics patterns of verbal exchange in a creative problem solving group. *Nonlinear Dynamics, Psychology, and Life Sciences, 2*, 35–58.

Guastello, S. J., Koopmans, M., & Pincus, D. (Eds.). (2011). *Chaos and complexity in psychology: The theory of nonlinear dynamical systems*. Cambridge, MA: Cambridge University Press.

Guastello, S. J., & Liebovitch, L. S. (2009). Introduction to nonlinear dynamics and complexity. In S. J. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: The theory of nonlinear dynamical systems* (pp. 1–40). New York: Cambridge University Press.

Guastello, S. J., Peressini, A. F., & Bond, R. W. (2011). Orbital decomposition for ill-behaved event sequences: Transients and superordinate structures. *Nonlinear Dynamics, Psychology, and Life Sciences, 15*(4), 465–476.

Hagen, A. M., Braasch, J. L. G., & Bråten, I. (2014). Relationships between spontaneous note-taking, self-reported strategies and comprehension when reading multiple texts in different task conditions. *Journal of Research in Reading, 37*, S141–S157.

Holsanova, J., Holmberg, N., & Holmqvist, K. (2009). Reading information graphics: The role of spatial contiguity and dual attentional guidance. *Applied Cognitive Psychology, 23*, 1215–1226.

Howe, M. J. (1970). Notetaking strategy, review and long-term retention of verbal information. *Journal of Educational Research, 63*, 285.

Johnson-Laird, P. N. (2013). Mental models and cognitive change. *Journal of Cognitive Psychology, 25*, 131–138.

Juarrero, A. (2010). Intentions as complex dynamical attractors. In J. H. Aguilar & A. A. Buckareff (Eds.), *Causing human actions: New perspectives on the causal theory of action*. Cambridge, MA: MIT Press.

Just, M. A., & Carpenter, P. A. (1980). A theory of reading: From eye fixations to comprehension. *Psychological Review, 87*, 329–354.

Kaplan, A., Garner, J. K., & Semo, S. (2015). *Teacher role-identity and motivation as a dynamic system*. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL.

Katerndahl, D., & Parchman, M. (2013). Effects of family presence on the content and dynamics of the clinical encounter among diabetic patients. *Journal of Evaluation in Clinical Practice, 19*, 1067–1072.

Kauffman, D. F. (2004). Self-regulated learning in web-based environments: Instructional tools designed to facilitate cognitive strategy use, metacognitive processing, and motivational beliefs. *Journal of Educational Computing Research, 30*, 139–161.

Kauffman, D. F., Zhao, R., & Yang, Y.-S. (2011). Effects of online note taking formats and self-monitoring prompts on learning from online text: Using technology to enhance self-regulated learning. *Contemporary Educational Psychology, 36*, 313–322.

Kelso, S. (1995). *Dynamic patterns: The self-organization of brain and behavior*. London: MIT Press.

Kierwa, K. A. (1987). Note taking and review: The research and its implications. *Journal of Instructional Science, 16*, 233–249.

Kierwa, K. A., DuBois, N., Christensen, M., Kim, S., & Lindberg, N. (1989). A more equitable account of the note-taking functions in learning from lecture and from text. *Instructional Science, 18*, 217–232.

Kintsch, W. (1991). A theory of discourse comprehension: Implications for a tutor for word algebra problems. In M. Carretera, M. Pope, R. Simmons, & J. I. Pozo (Eds.), *Learning and instruction: European research in an international context* (Vol. 3, pp. 235–353). Oxford, UK: Pergamon Press.

Kobayashi, K. (2009). The influence of topic knowledge, external strategy use, and college experience on students' comprehension of controversial texts. *Learning and Individual Differences, 19*, 130–134.

Kunnen, E. S., & Bosma, H. A. (2000). Development of meaning making: A dynamic systems approach. *New Ideas in Psychology, 18*, 57–82.

Lathrop, D., & Kostelich, E. J. (1989). Characterization of an experimental strange attractor by periodic orbits. *Physical Review A, 40*, 4028–4031. Retrieved from http://www.robots.ox.ac.uk/~parg/mlrg/papers/periodic_orbit.pdf.

Leopold, C., & Leutner, D. (2012). Science text comprehension: Drawing, main idea selection, and summarizing as learning strategies. *Learning and Instruction, 22*, 16–26.

Long, G. M. (2014). Positive effects of restricting student note-taking in a capstone psychology course: Reducing the demands of divided attention in the classroom. *Teaching of Psychology, 41*, 340–344.

Lunkenheimer, E. S., & Dishion, T. J. (2009). Developmental psychopathology: Maladaptive and adaptive attractors in children's close relationships. In S. J. Guastello, M. Koopmans, & D. Pincus (Eds.), *Chaos and complexity in psychology: The theory of nonlinear dynamical systems* (pp. 282–306). New York: Cambridge University Press.

Maggioni, L., & Fox, E. (2009). *Adolescents' reading of multiple history texts: An interdisciplinary exploration of the relation between domain-specific reading and epistemic beliefs*. Paper presented at the Annual Meeting of the American Educational Research Association, San Diego, CA.

Marcus, B., & Williams, S. (2008). Symbolic dynamics. *Scholarpedia*. Retrieved from http://www.scholarpedia.org/article/Symbolic_dynamics

Mathworks. (Version R2010a). Natick, MA: Mathworks.

Mayer, R. E. (2001). *Multimedia learning*. Cambridge, NY: Cambridge University Press.

Mayer, R. E. (2005). Cognitive theory of multimedia learning. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (pp. 31–48). Cambridge, NY: Cambridge University Press.

Mayer, R. E. (2009). *Multimedia learning* (2nd ed.). Cambridge, NY: Cambridge University Press.

Moreno, R., & Mayer, R. E. (2000). A coherence effect in multimedia learning: The case for minimizing irrelevant sounds in the design of multimedia instructional messages. *Journal of Educational Psychology, 92*, 117–125.

Norman, D. A., & Shallice, T. (1980). *Attention to action: Willed and automatic control of behavior*. University of California San Diego CHIP Report 99.

Peressini, A. F., & Guastello, S. J. (2010). *Orbital decomposition: A short user's guide to ORBDE v1.2*. http://www.societyforchaostheory.org/tutorials/

Peressini, A. F., & Guastello, S. J. (2014). *Orbital decomposition V2.4*. Retrieved from http://www.societyforchaostheory.org/tutorials/#4

Peverly, S., Vekaria, P. C., & Garner, J. K. (2014). Both handwriting speed and selective attention are important to lecture note-taking. *Reading and Writing, 27*, 1–30.

Pincus, D., Eberle, K., Walder, C. S., Kemp, A. S., Lenjav, M., & Sandman, C. A. (2014). The role of self-injury in behavioral flexibility and resilience. *Nonlinear Dynamics, Psychology, and Life Sciences, 18*(3), 277–296.

Pincus, D., & Guastello, S. J. (2005). Nonlinear dynamics and interpersonal correlated of verbal turn-taking patterns in a group therapy session. *Small Group Research, 36*, 635–677.

Pintrich, P. R. (2000). The role of goal orientation in self-regulated learning. In M. Boekaerts, P. R. Pintrich, & M. Zeidner (Eds.), *The handbook of self-regulation* (pp. 451–502). Burlington, MA: Elsevier.

Piolat, A., Olive, T., & Kellogg, R. T. (2004). Cognitive effort during note taking. *Applied Cognitive Psychology, 19*, 291–312.

Pressley, M., & Afflerbach, P. (1995). *Verbal protocols of reading: The nature of constructively responsive reading*. Hillsdale, NJ: Erlbaum.

Pressley, M., & Harris, K. R. (2006). Cognitive strategy instruction: From basic research to classroom instructions. In P. A. Alexander & P. H. Winne (Eds.), *Handbook of educational psychology* (2nd ed., pp. 265–287). Mahwah, NJ: Erlbaum.

Prigogine, I., & Stengers, I. (1984). *Order out of chaos: Man's new dialogue with nature*. New York: Bentam.

Schnotz, W., & Bannert, M. (2003). Construction and interference in learning from multiple representations. *Learning and Instruction, 25*, 279–294.

Shannon, C. (1948). A mathematical theory of communication. *Bell System Technology Journal, 27*, 379–423.

Shannon, C., & Weaver, W. (1949). *The mathematical theory of communication*. New York: University of Illinois Press.

Stahl, S. A., Hynd, C., Britton, B. K., McNish, M. M., & Bosquet, D. (1996). What happens when students read multiple source documents in history? *Reading Research Quarterly, 31*, 430–456.

Stanton, M., & Welsh, R. (2012). Systemic thinking in couple and family psychology research and practice. *Couple and Family Psychology: Research and Practice, 1*, 14–30.

Sundararajan, A. (2012). *Shifting and moving: An introduction to dynamical systems*. Retrieved from http://math.uchicago.edu/~may/REU2012/REUPapers/Sundararajan.pdf

Tabrizian, M. (2010). *Shifts as dynamical systems*. Retrieved from http://math.berkely.edu/
Thelen, E., & Smith, L. B. (1994). *A dynamic systems approach to the development of cognition and action*. Cambridge, MA: Bradford.
Van Meter, P., & Firetto, C. (2013). Cognitive model of drawing construction: Learning through the construction of drawings. In G. Schraw, M. T. McCrudden, & D. Robinson (Eds.), *Learning through visual displays* (pp. 247–280). Charlotte, NC: Information Age Publishers.
Winne, P. H. (2015). *Comments, Vice President's forum: diversity vis-à-vis individual differences: Pondering the implications for theory and research*. Annual meeting of the American Educational Research Association, Chicago, IL.
Winne, P. H., & Hadwin, A. F. (2008). Studying as self-regulated learning. In D. J. Hacker, J. Dunlosky, & A. C. Graesser (Eds.), *Metacognition in educational theory and practice* (pp. 277–304). Mahwah, NJ: Erlbaum.
Winne, P. H., & Jameson-Noel, D. L. (2002). Exploring students' calibration of self-reports about study tactics and achievement. *Contemporary Educational Psychology, 27*, 551–572.
Zimmerman, B. J. (2000). Attaining self-regulated learning: A social-cognitive perspective. In M. Boekaerts, P. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation* (pp. 13–39). San Diego, CA: Academic Press.
Zimmerman, B. J. (2008). Investigating self-regulation and motivation: Historical background, methodological developments, and future prospects. *American Educational Research Journal, 45*, 166–183.
Zimmerman, B. J., & Paulsen, A. S. (1995). Self-monitoring during collegiate studying: An invaluable tool for academic self-regulation. *New Directions for Teaching and Learning, 63*, 13–27.

Chapter 17
A Catastrophe Model for Motivation and Emotions: Highlighting the Synergistic Role of Performance-Approach and Performance-Avoidance Goal Orientations

Georgios Sideridis and Dimitrios Stamovlasis

Introduction

In educational psychology, it is widely acknowledged that learning outcomes presuppose motivation, in that motivation is acting as the "source of energy" required for effective self-regulation and achievement (Atkinson, 1964). Motivation has been considered in implicit theories of intelligence and was implemented as a construct to understand and explain school performance (Gonida, Kiosseoglou, & Leondari, 2006; Gonida, Voulala, & Kiosseoglou, 2009). In addition, a link between motivation and performance with concomitant effects on emotions has been reported (Pekrun, Elliot, & Maier, 2006). Several researchers have attempted to unravel the relationship between anxiety and performance or motivated behavior and performance (e.g., Elliot & Harachiewicz, 1996; Hardy & Parfitt, 1991) with few endeavors examining the relationship between motivation and arousal (Elliot & McGregor, 2001; Pekrun et al., 2006, 2009). Shedding light onto the motivation-performance relationship, different researchers suggested various roles for arousal, considering it in the role of an antecedent (Elliot & Church, 1997), a consequence (Elliot & Thrash, 2002), a mediating (Cury, Da Fonséca, Rufo, Peres, & Sarrazin, 2003), or a moderating variable (Barron & Harackiewicz, 2001). All related studies have fostered linear methodologies and they have failed to converge convincingly, suggesting a specific model with motivation and emotion involved in self-regulation

G. Sideridis (✉)
Harvard Medical School, Clinical Research Center, Boston Children's Hospital, 300 Longwood Avenue, MA 02115, Boston
e-mail: georgios.sideridis@childrens.harvard.edu

D. Stamovlasis
Aristotle University of Thessaloniki, Department of Philosophy and Education, 54 124 Thessaloniki, Greecee
e-mail: stadi@edlit.auth.gr

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2_17

and performance. Recently, a catastrophe theory model has been proposed for explaining variability in students' arousal under achievement situations (Stamovlasis & Sideridis, 2014). In this work, nonlinear dynamics and self-organization theory were combined to explain instabilities in arousal level in educational settings. In the present chapter, further evidence is provided for nonlinear dynamic effects in emotional sates as a function of achievement goal orientations (Elliot & Harackiewicz, 1996).

Next, a short description of achievement goal theory including an explanation as to how motivated behavior, in the form of goal orientations, co-varies with arousal is provided. Furthermore, a rationale was developed for employing catastrophe theory to elaborate on the type of relationship between motivated behavior and arousal.

Achievement Goal Theory

Goal theory was developed with the pioneer work of Dweck, Nichols, Ames, and other theorists. The premise of goal theory was based on early observations that some children "who show impairment in the face of difficulty are initially equal in ability to those who seek challenge and show persistence" (Dweck & Leggett, 1988, p. 256). The above remark helped Dweck (1986) describe motivation using two classes of goals: the goals that aim to validate one's ability, termed *performance*, using normative evaluative standards of what success is, and the goals that aim to aid understanding and learning of a skill in the absence of normative evaluations but rather the employment of intraindividual standards of what success is, termed *mastery* goals. The early works from that dichotomization pointed to the fact that performance goals are *maladaptive* and mastery goals are *adaptive* for achievement-related gains (Dweck & Leggett, 1988). For newer conceptualizations see Grant and Dweck (2003) and Elliot and Murayama (2008).

Later, Elliot and Harackiewicz (1996) proposed that performance-oriented individuals may approach a task to prove their competence, worth, and likeability, but they may also target at avoiding negative end states (performance and/or affective). This thesis resulted in the dichotomization of performance goals into *approach* and *avoidance* forms pointing to the need to revise achievement goal theory (Harackiewicz, Barron, Pintrich, Elliot, & Thrash, 2002), in a way that would clearly identify the positive effects of performance-approach goals (particularly with regard to academic achievement). The main hypothesis from that dichotomization was that performance-approach goals are linked to positive achievement outcomes, because the positive valence associated with approaching success would likely act as a "promoter" of regulation (even when individuals are highly challenged). On the contrary, for performance-avoidance goals the expectation was that the focus on failure (even being on avoiding failure) would be associated with agitation-related emotions when one fails to attain personal standards of success, with important links to one's functioning, and efficacy during an achievement situation. The differentiation between the two performance goals on affective grounds is the main purpose of this study. A secondary purpose is to explain their differences using a nonlinear perspective particularly given the fact that linear approaches have had little success in explaining substantial amounts of variance in academic and nonacademic outcomes due to goal orientations (Harackiewicz et al., 2002; Sideridis, 2003). Linearity has been questioned a long time ago, as it graphically has been stated: "There is no apparent reason, intuitive or otherwise, as to why human behavior should be more linear than the behavior of other things, living or nonliving" (Brown, 1995, p. 1). The latter is relevant to epistemological issues that are raised by the new paradigm of nonlinear dynamics (see also Stamovlasis, 2010, 2011).

Catastrophe Theory and Goal Orientations

Goal orientations, as psychological constructs, describe human behavior under achievement situations or in other words they describe the fundamental function of *self-regulation* (Carver, 2006; Elliot, 1999). From earlier times, performance goals associated with one's need to prove his/her basic worth, competence, and likeability were identified as being linked to a potentially disastrous functioning, in terms of self-regulation without however being considered in their dichotomized forms (Dykman, 1998). Performance goals, approach, and avoidance, in their conceptualization, include a distinct set of affective criteria that are largely responsible for their regulation properties (Elliot, 1999). In other words, the affective response associated with each type of goal saliently defines the goal and explains the regulation that is the outcome of that affective response. For example, a person who is motivated by performance-approach goals is certainly stressed during an achievement situation when challenged up to a point, more so than a person who adopts mastery goals. Their affective response is different due to foci. Individuals adopting performance-approach goals heavily value normative evaluations. Thus, low performance may be synonymous to failure. However, the "approach" focus drives the person to strive and "approach" success in an achievement situation. This is why in such a strong association between performance-approach goals there is need for achievement (Elliot, 1999, 2005; Elliot & Church, 1997; Elliot et al., 2011) and actual performance (Harackiewicz et al., 2002). On the contrary, for individuals who adopt performance-avoidance goals the tension associated with failure is far greater compared to those who target at approaching success. That is, the tension from the agony to avoid failure is so strong that cognitive functioning is impaired (Dweck & Leggett, 1988). Regardless of type of task (cognitive or physical) the resources associated with effective regulation are gradually withdrawn in these cases and overtaken by an emotional over-response, which is usually immense and difficult to regulate. Covington (1992) has nicely described performance-avoidance individuals as persons who do not have a skill, performance, involvement, or motivation deficit but rather experience an amazing overflow of negative emotions. They further added that *"the tension that persists during test taking itself appears to cause a massive failure to recall what was originally learned."* Thus, performance-avoidance goals have been largely held responsible for the maladaptiveness of

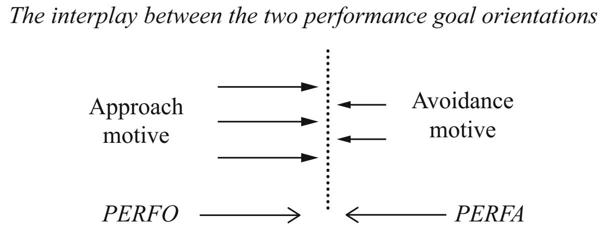

Fig. 17.1 Conceptualization of the *approach—avoidance* dynamics of achievement goals

performance goals, when these goals included both approach and avoidance terms (Dweck, 1986). The main objective of this study is to test the hypothesis that emotional dysregulation could be explained by performance-avoidance goals, however, with the presence and the synergetic effect of performance-approach goals. Now how all these could be related to chaos and catastrophe theories?

Given that the two forms of performance goals represent two opposite modes of motivation one may seek a proper theory and modeling method to empirically explain the data. If one attempts to predict the operation of these two opponent processes using a linear additive equation model, one may find difficulties to explain the outcomes for reasons related to methodological and epistemological issues. These complications arise from the incompatibility of linear modeling with the nature of the processes being investigated (Stamovlasis, 2010). An intriguing idea, but workable as well, is to visualize the interaction of *approach* and *avoidance* motives as *force field dynamics* (Fig. 17.1). This scheme assumes the concomitant operation of opponent processes and it is quite realistic since the synergy of two "complementary opposites" is ubiquitous in nature and human experience (Kelso & Engstrøm, 2006).

In the social and behavioral science literature, the force field dynamics has been an effective modeling approach to explain social behavior (Tesser, 1980; van der Maas & Hopkins, 1998; Van der Maas, Molenaar & van der Pligt, 2003), cognitive phenomena (Stamovlasis, 2006, 2011; Stamovlasis & Tsaparlis, 2012), overload and fatigue (Guastello, Boeh, Schumaker, & Schimmels, 2012; Guastello et al., 2013), to mention a few.

Force field dynamics cannot be described merely as a linear sum because the interplay of opponent processes, acting upon the self-regulation mechanism, can result in surprises and unexpected outcomes in emotional states or arousal. A suitable modeling methodology is offered by catastrophe theory and particularly by implementing the cusp model (Guastello, 2002; Thom, 1975). The cusp model posits that the change in behavior can be described by two control variables, the *asymmetry* and *bifurcation* associated within the mathematical Eq. 17.1,

$$\frac{\delta f(y)}{\delta y} = y^3 - by - \alpha \tag{17.1}$$

in which a response variable y, e.g., arousal, is predicted by the bifurcation variable b and the asymmetry variable a. The change in behavior y is depicted on

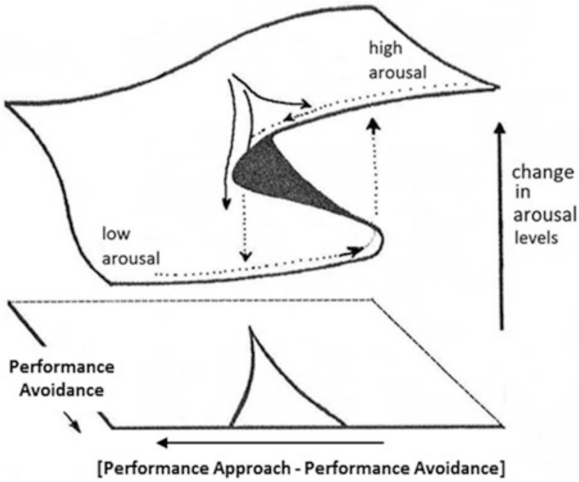

Fig. 17.2 A three-dimensional representation of the cusp catastrophe response surface describing the relationship between achievement goals (performance approach and performance avoidance) and arousal (heart rate per minute)

a three-dimensional surface (Fig. 17.2). When the bifurcation variable has low values, change in behavior will be smooth and predictable being linearly correlated with the asymmetry; when the bifurcation variable takes on high values the behavior is predicted to become discontinuous (Arnol'd, 1992). In this work the *approach* and *avoidance* motives are considered as acting or contributing to the control variables. Specifically, this study attempted to elucidate the role of *performance-avoidance goals*, which in theory fit the role of a bifurcation factor.

The applicability of catastrophe theory in educational psychology and specifically in the area of motivation and goal orientations is also demonstrated by Guastello (1987, 2002); Sideridis and Stamovlasis (2014); Sideridis, Stamovlasis, and Antoniou (2015); and Sideridis, Antoniou, Stamovlasis, and Morgan (2013). It is worth mentioning here that the perspectives for a nonlinear motivation theory are also reflected to the empirical endeavors that include time-series analyses of motivational flows demonstrating scale-free and chaotic properties (Guastello, Johnson, & Rieke, 1999; Navarro & Arrieta, 2010; Navarro, Arrieta, & Balén, 2007; Navarro, Curioso, Gomes, Arrieta, & Cortés, 2013).

Importance of the Present Study and the Related Hypotheses

In the present endeavor, as in many studies where the nonlinear methodology is fostered, the central aim is the theory development and the elucidation of findings that appear to provide limited or controversial evidences regarding roles and/or relations among variables. Thus, it is important here to employ catastrophe theory in order to unravel the affective basis that differentiates performance-approach from performance-avoidance goals and mastery goals for the following reasons: (a) the relationship between goal orientations and affect has been largely understudied with limited exceptions (e.g., Pekrun et al., 2006); (b) goal orientation theory considers affect as a by-product of the goal-performance relationship but the authors and recent evidence suggest that this speculation does not hold (e.g., Grant & Dweck, 2003; Pekrun et al., 2006); (c) the relationship between the approach forms of mastery and performance goals needs to be further elucidated (i.e., if they are really different); (d) the presence of nonlinear relationships between goals and affect needs to be further tested, particularly given the fact that linear models have not been particularly explanatory (Sideridis, 2003); and (e) a theoretical account of the relationship between affect and goal orientations can be further formed using the premises of catastrophe theory and nonlinear dynamics—if supported by the data (Stamovlasis & Sideridis, 2014). This exploration further helps refine our conception of the form and function of goal orientations (see Grant & Dweck, 2003) and eventually intervene on their deleterious effects (if there are such effects).

Ergo, the purpose of this study was to resolve an empirical but of great theoretical importance issue, concerning the relationship between goal orientation forms and a potential dysregulation process under achievement situations. It was expected that goal orientations would be responsible for individual differences in emotional regulation and would be accountable for those differences as a function of their foci (i.e., approach avoidance, mastery, or performance). The following two hypotheses were posited in this research: (a) mastery-approach goals and performance-approach goals are linked to self-regulation control, but together do not explain emotional dysregulation phenomena measured by arousal; (b) performance goals, approach, and avoidance synergetically explain emotional dysregulation, via the cusp catastrophe with positive paths linking performance-avoidance goals to the catastrophic event.

Method

Participants, Procedures, and Measures

The participants ($N = 70$, 9 males and 61 females) were first-year college students from a public university majoring in psychology, and they were selected because they were offering in-class presentation as part of a course requirement. Students' participation was voluntary. They were informed that the purpose of the study was to examine how motives relate to performance and were asked to wear a heart rate-monitoring device. Students were assured of the confidentiality of their participation and were informed that they could withdraw from the study at any time.

A few students who did not feel comfortable in wearing the device were excluded from further consideration. Furthermore, three participants were excluded because the monitoring device failed to provide valid estimates. The participants wore the heart rate-monitoring device approximately 15 min prior to the presentation until after the end of it. Prior to wearing the device, the students had to complete measures of goal orientation, modified to be specific for the task at hand (i.e., in-class presentation). Limitations of the present investigation might originate from peculiarities of the sample chosen, which comprised predominantly female university students.

Goal Orientations

Students' goal orientations were assessed using a modified version of the Elliot and McGregor's (2001) 2×2 scales, without the mastery-avoidance construct. The major modification was the attempt to make the items context specific. Sample items were the following: (a) Do you enjoy to offer presentations because it is an important learning experience (mastery approach)? (b) How important is it to you to do better than your classmates in that presentation (performance-approach goals)? (c) Is it important to you to avoid performing worse than the other students in that presentation (performance-avoidance goals)? Cronbach's alpha coefficients were .88 for mastery approach, .79 for performance approach, and .82 for performance avoidance, which were acceptable.

Arousal

Participant' heart rate per minute (HRPM) was assessed using a commercial heart rate-monitoring device (Polar 610i). There was ample evidence regarding the reliability and validity of the device for the assessment of the cardiac response (Durant et al., 1993; Godsen, Carroll, & Stone, 1991; Treiber, Musante, Hartdagan, Davis, Levy, & Strong, 1989; Wajciechowski, Gayle, Andrews, & Dintiman, 1991). The device stored the data in ASCII format. Data were collected using 10-s intervals; thus, there were 6 data points per minute. An example of the patterns of HRPM for a participating student is shown in Fig. 17.3.

Data Analysis and Results

Initially, an auxiliary linear analysis via multilevel random coefficient modeling (MRCM) was performed in order to elucidate some indicative differences between the various goal orientations (Bryk & Raudenbush, 1992). After running the model without any predictor variable (unconditional), the variabilities between and within persons were estimated at 65.5 % and 34.5 %, respectively

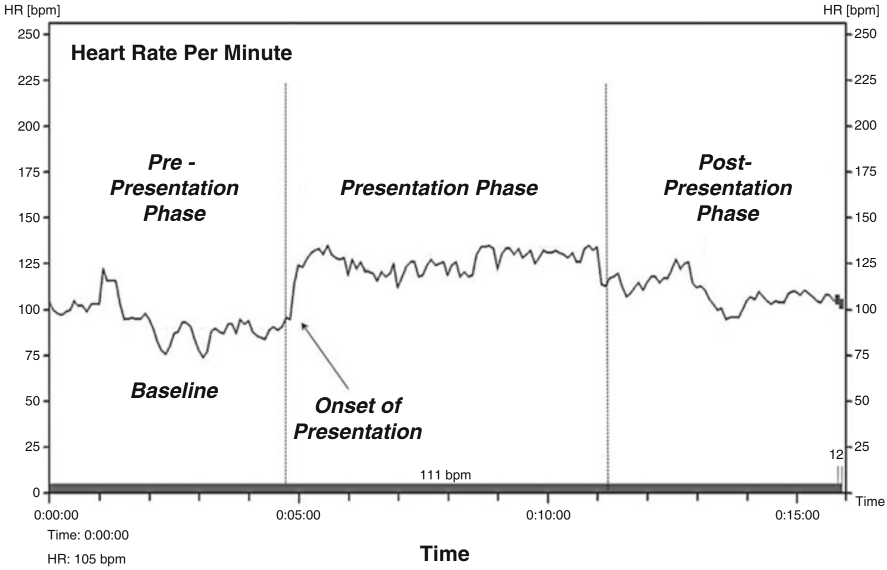

Fig. 17.3 Data on heart rate per minute (HRPM) for a participant of the study. The three phases of HRPM are displayed in the *dotted lines*

(Raudenbush & Bryk, 2002), while it was established that there were ample levels of variability due to condition (baseline versus presentation). Given that this slope would subsequently comprise a dependent variable that would be modeled as a function of goal orientations, it was imperative that variability due to condition was present (Nezlek, 2001). These findings further substantiate the fact that goal orientation differences have a strong affective basis. Moreover, it was found that the increase in arousal was significantly more salient for individuals adopting performance-avoidance goals compared to those who adopted either performance-approach or mastery-approach goals, while no difference was observed between mastery-approach and performance-approach goals at any comparison suggesting equivalence in functioning with regard to affect (see Appendix). The above provided an indication that performance-avoidance goals hold a protagonist role. The expectation, however, to obtain an explanatory conceptual model involving achievement goals failed by means of linear methods. The next step was the application of a nonlinear framework, which encompasses the analysis of HRPM and the application of the cusp catastrophe.

For analyzing the time series HRPM data in a nonlinear fashion, the methodological approach presented in Stamovlasis and Sideridis (2014) was adopted. In this method, which is different from the typical analysis of a single case presented by Koopmans (this volume, 2015), the time series was partitioned in 12 epochs and then the cusp catastrophe model with difference equations and polynomial regression techniques was implemented using cross-sectional data. The dependent variable, heart rate per minute, was defined as the difference in HRPM between two

Table 17.1 Cusp model for students' arousal levels (heart rates per minute): multiple regression slopes, standard errors, *t*-tests, and model fit

Model	Adj R^2	*b*	*S.E.*	*t*	Model *F*
Cusp	0.58				33.7****
z_1^3		−0.003	0.010	−2.52*	
$z_1 x$ *PERFA*		−0.194	0.051	−3.80***	
(PERFO-PERFA)		−0.338	0.126	−2.67**	

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$, **** $p < 0.0001$

time points z_1 and z_2, that is, the difference between two epochs corresponding to periods before (baseline) and during or at the end of the presentation. Heuristically the epochs 4 and 9 were selected although alternative points provided almost equivalent estimates (see Stamovlasis & Sideridis, 2014) for details.

The modeling equation (Guastello, 2011) for the cusp model is

$$\Delta z = z_2 - z_1 = b_1 + b_2 {z_1}^3 + b_3(\mathrm{PERFA})z_1 + b_4[\mathrm{PERFO} - \mathrm{PERFA}] \qquad (17.2)$$

with PERFO = performance approach and PERFA = performance avoidance. Based on the above model, as performance-avoidance levels increase, the response variable, HRPM, bifurcates into two behavioral modes, which are two behavioral *attractors* representing high and low levels of arousal, respectively. Evidence in favor of the cusp model was found pointed to the explanation of significant amounts of variance in the dependent variable over the competing linear models. Similar cusp models involving mastery goals were not supported, suggesting that mastery goals may not be directly involved in the (dys)regulation process.

The cusp analysis results are summarized in Table 17.1. Findings suggested excellent model fit as the total amount of variance explained by the cusp model was equal to 59 %. The cusp model was overall statistically significant [adjusted $R^2 = 0.59$ $F(3, 66) = 33.7$, $p < 0.0001$] having the cubic term [$t(66) = -2.52$, $p < 0.05$], the bifurcation, PERFA [$t(64) = -3.80$, $p < 0.001$], and the asymmetry parameter (PERFO-PERFA) [$t(66) = -2.67$, $p < 0.01$] significantly different from zero. Interestingly, the signs of the coefficients b_3 and b_4 were negative suggesting that when high scores in performance avoidance (the bifurcation variable) were coupled with high values in the asymmetry factor (i.e., the difference between performance-approach and performance-avoidance goals) with the latter dominating that relationship (i.e., performance approach < performance avoidance), then HRPM pulses were high. The opposite was true when high performance-avoidance scores were coupled with a domination of performance-approach goals in the asymmetry factor (i.e., when performance approach > performance avoidance). These findings suggest that the relationship between the asymmetry and heart rate per minute is linear up to a point when performance-avoidance scores increased greatly and enters a stage of uncertainty; there sudden jumps between attractors of low and high arousal occur, while the behavior resembles as being in and out of control phase. The linear competitor models explaining less than 10% of the variance were found inferior to the proposed cusp, the superiority of which is not based merely on its statistical significance, but mainly on its theoretical interpretation and explanatory value (see Sect. “Discussion”).

Discussion

This study provided the empirical evidences for the relationship between goal orientations and arousal, which adds to establishing connections between psychological constructs and emotional regulation (or dysregulation) under achievement circumstances. The finding is supportive of the basic premises of achievement goal theory with regard to the role of the goals on affect and extends its boundaries to the nonlinear regime by providing better explanatory models.

A finding, however in linear terms, suggests that performance-approach individuals may have elevated levels of anxiety prior to a stressful event, compared to mastery-approach individuals but not during the presentation condition. That is, the two approach forms of goals are not differentiated as earlier works on their conceptualization and function suggest (Midgley, Kaplan, & Middleton, 2001). Earlier differences between mastery- and performance-approach goals had been observed with regard to achievement (Harackiewicz et al., 2002), and anxiety (Elliot & McGregor, 1999). However, to our knowledge the present work is the only one that relates goal orientations with the emotional response as measured by heart rate per minute (Sideridis, Antoniou, & Simos, 2013). By taking into account the contribution of performance-approach/avoidance goals to the cusp structure, earlier conceptualizations of approach goal orientations (mastery and performance) with regard to emotional regulation (Dweck & Leggett, 1988) might be reconsidered.

The emotional effect was evident by the significant change in the physiological response between the baseline and presentation condition (Fig. 17.3). Linear coefficients indicated that this increase was significantly more salient for individuals adopting performance-avoidance goals compared to those who adopted either performance-approach or mastery-approach goals. The finding is consistent with the relevant literature, where the performance-avoidance goals have been characterized as *maladaptive* in that the emotional overflow experienced by individuals significantly disturbs effective self-regulation (Darnon, Butera, Mugny, Quiamzade, & Hulleman, 2009). However, the nonlinear analysis suggested that the phenomenon under investigation does not only consist of emotional overflow, but the arousal system enters a chaotic phase unpredictably, which might further grow towards dysregulation and failure.

The “behavior” of performance-avoidance goals was consistent and in accord with expectations of revised goal theory (Harackiewicz et al., 2002). Individuals adopting performance-avoidance goals started with elevated anxiety at baseline and had significantly elevated physiological response during the presentation compared to both baseline levels (at the within-person level) but also the other goal orientations (at the between-person level). Thus, performance-avoidance goals were somehow responsible for the negative affective response during this stressful event. This finding agrees with earlier works for which the negative links of performance-avoidance goals with anxiety and stress were established (Elliot & McGregor, 1999). These links however were not elucidated or explained within a comprehensive explanatory model.

Aiming on an explicated answer to the above research question, this study examined the achievement goals within a nonlinear perspective and presented a cusp catastrophe model for changes in arousal levels under an oral presentation condition. The findings showed that the difference of performance-approach minus performance-avoidance goals contributed to the asymmetry factor, which has a linear relationship with arousal, while performance-avoidance goals acted as the bifurcation factor. The interpretation of the model suggests that there is a cutoff value in performance-avoidance goals beyond which the linear relationship with the arousal terminates and the system enters a nonlinear phase. This is the area, where bimodality is observed and the outcome is ambiguous since the response could move between two different levels of arousal, low and high, respectively. These moves are discontinuities in the mathematical sense as behavior moves between the two attractors (upper and lower).

The findings agree with the recognition of a synergetic effect for both psychological constructs. When individuals are motivated by performance goals they have the concurrent operation of an approach and an avoidance motive. The difference between them can play an important role and this is reflected on the magnitude of the asymmetry factor. A cusp structure discloses a *hysteresis effect* existing in the response variable; that is, participants with the same values on performance goals could be found in the upper or the lower mode of the response surface. At the individual level, given the sensitivity of the parameters and the dynamics of the system, an extended interpretation suggests that small differences in the valence of motivation or random fluctuation may cause arousal to oscillate between the two attractors, causing systems' dysregulation through emotional instability (see Fig. 17.4). This portrays the *maladaptive* function of the performance-avoidance motive demonstrated within the nonlinear dynamics and complexity framework.

We maintain that the application of the nonlinear model added significant findings to the literature. First, the amount of variance explained was surprisingly large for this line of research as goal orientations usually have accounted for less than 10 % of the variance in academic and nonacademic variables (Hulleman, Schrager, Bodmann, & Harackiewicz, 2010). Explaining almost 60 % of the total variability in heart rate is certainly of paramount importance. Second, the cusp model showed that both linear relations and sudden shifts in emotional states are expected and elucidated the role of performance-avoidance goals. This finding has implications for the theory of goal orientations and the appropriate analytical means to model their relationship, particularly with regard to emotional regulation. Certainly, the nonlinear dynamics perspective provided the means to verify theoretical assumptions with regard to the synergistic role of performance-approach and performance-avoidance goals (Covington, 1984).

It is important to emphasize also that the cusp model implies an underlying nonlinear dynamical process. The observed bifurcation in fact showed a different

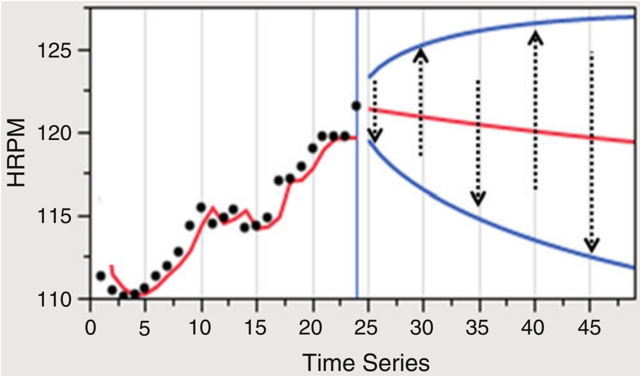

Fig. 17.4 Data for a participating student from the start of a heart rate-monitoring (HRM) device (reflecting the waiting period) until the onset of an in-class presentation. The *vertical line* indicates the onset of a chaotic stage in which behavior/arousal can oscillate between two attractors, due to sensitivity of the parameters and the dynamics of the system. This chaotic epoch of emotional instability may result in systems' dysregulation and/or self-regulation failure

ontology that characterizes complex adaptive systems and thus the epistemological shift is the additional merit of this investigation (Nicolis & Nicolis, 2007). The nonlinear dynamics has provided a comprehensive explanatory model for the function of performance-avoidance goals in the self-regulation process. The above provided a strong link between *achievement goal theory* and *self-organization theory*, introducing the former into the meta-theoretical framework of nonlinear and complexity sciences (Stamovlasis & Sideridis, 2014).

Future research is encouraged to examine the presence of interactions between goal orientations and other motivational/affective variables in order to unravel the complex relationship between goals and self-regulation. Also the role of nonlinear dynamics and complexity theory in this regulation could be further investigated given that chaotic events interfere with competence (White, 1959) and may be linked to anxiety and depression (Skar, 2004). It will be stimulating to further reveal self-organization evidence via catastrophe modeling of motivation goals in conjunction with cognitive variables, contributing therefore to unified theoretical frames.

Appendix

Specification and results from multilevel random coefficient model (MRCM) predict arousal from goal orientations. This model tests also the alternative hypotheses (comparing to cusp model) that goal orientations have significantly different intercepts and slopes in their HRPM compared to grand mean estimates (Table 17.2):

Table 17.2 Coefficients of MRCM, standard deviations, *t*-ratio, *p*-value, and degrees of freedom

Variables	Coefficient	*S.E.*	*t*-ratio	*p*	*df*
For intercept Π_0					
Intercept β_{00}	117.392	0.586	200.38	0.001**	2,296
Mastery-approach β_{01}	−1.467	0.595	−2.464	0.001**	2,296
Performance-approach β_{02}	−0.108	0.814	−0.132	0.895	2,296
Performance-avoidance β_{03}	4.010	0.823	4.873	0.001**	2,296
For slope of phase Π_1					
Intercept β_{10}	10.468	0.828	12.634	0.001**	2,296
Mastery-approach β_{11}	−1.290	0.842	−1.533	0.125	2,296
Performance-approach β_{12}	−1.256	1.151	−1.091	0.276	2,296
Performance-avoidance β_{13}	3.213	1.164	2.760	0.006*	2,296

Note: Coefficients reflect fixed effects of the model. S.E. = Standard error of measurement
*$p < .05$, **It is below $p < .001$ but was rounded to .001 in order to avoid the fact that a probability equal to zero does not exist

$$Y_{ti} = \pi_{0i} + \pi_{1i}(\text{Phase}) + +e_{ti} \quad (\text{Level 1})$$

$$\begin{aligned}\pi_{0i} = \beta_{00} + \beta_{01}(\text{Mastery} - \text{approach}_i) + \beta_{02}(\text{Performance} - \text{approach}_i) \\ + \beta_{03}(\text{Performance} - \text{avoidance}_i) + r_0 \quad (\text{Level 2})\end{aligned}$$

$$\begin{aligned}\pi_{1i} = \beta_{10} + \beta_{11}(\text{Mastery} - \text{approach}_i) + \beta_{12}(\text{Performance} - \text{approach}_i) \\ + \beta_{13}(\text{Performance} - \text{avoidance}_i) + r_1 \quad (\text{Level 2})\end{aligned}$$

References

Arnol'd, V. I. (1992). *Catastrophe theory* (3rd ed.). Berlin: Springer.

Atkinson, J. (1964). *An introduction to motivation*. Princeton, NJ: Van Nostrant Reinholdt Company.

Barron, K. E., & Harackiewicz, J. M. (2001). Achievement goals and optimal motivation: Testing multiple goal models. *Journal of Educational Psychology, 80*, 706–722.

Brown, C. (1995). *Chaos and catastrophe theories*. London: Sage.

Bryk, A., & Raudenbush, S. W. (1992). *Hierarchical linear models for social and behavioral research: Applications and data analysis methods*. Newbury Park, CA: Sage.

Carver, C. S. (2006). Approach, avoidance, and the self-regulation of affect and action. *Motivation and Emotions, 30*, 105–110.

Covington, M. V. (1992). Goal theory, motivation and school achievement: An integrative review. *Annual Review of Psychology, 51*, 171–200.

Covington, M. V. (1984). The motive for self-worth. In R. Ames & C. Ames (Eds.), *Research on motivation in education* (Vol. 1). New York: Academic.

Cury, F., Da Fonséca, D., Rufo, M., Peres, C., & Sarrazin, P. (2003). The trichotomous model and investment in learning to prepare for a sport test: A mediational analysis. *British Journal of Educational Psychology, 73*, 529–543.

Darnon, C., Butera, F., Mugny, G., Quiamzade, A., & Hulleman, C. S. (2009). "Too complex for me!" Why do performance-approach and performance-avoidance goals predict exam performance? *European Journal of Psychology of Education, 4*, 423–434.

Durant, R. H., Baranowski, T., Davis, H., Rhodes, T., Thompson, W. O., Greaves, K. A., et al. (1993). Reliability and variability of indicators of heart-rate monitoring in children. *Medicine and Science in Sports and Exercise, 25*, 389–395.

Dweck, C. S. (1986). Motivational processes affecting learning. *American Psychologist, 41*, 1040–1048.

Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review, 95*, 256–273.

Dykman, B. M. (1998). Integrating cognitive and motivational factors in depression: Initial tests of a goal-orientation approach. *Journal of Personality and Social Psychology, 74*, 139–158.

Elliot, A. J. (1999). Approach and avoidance motivation and achievement goals. *Educational Psychologist, 34*, 169–189.

Elliot, A. J. (2005). A conceptual history of the achievement goal construct. In A. J. Elliot & C. S. Dweck (Eds.), *Handbook of competence and motivation* (pp. 52–72). New York, NY: Guilford.

Elliot, A. J., & Harackiewicz, J. M. (1996). Approach and avoidance achievement goals and intrinsic motivation: A mediational analysis. *Journal of Personality and Social Psychology, 70*, 461–475.

Elliot, A. J., & Church, M. (1997). A hierarchical model of approach and avoidance achievement motivation. *Journal of Personality and Social Psychology, 72*, 218–232.

Elliot, A. J., & McGregor, H. A. (1999). Test anxiety and the hierarchical model of approach and avoidance achievement motivation. *Journal of personality and social psychology, 76*(4), 628–644.

Elliot, A. J., & McGregor, H. A. (2001). A 2×2 achievement goal framework. *Journal of Personality and Social Psychology, 80*, 501–519.

Elliot, A. J., & Thrash, T. (2002). Approach-avoidance motivation in personality: Approach and avoidance temperament and goals. *Journal of Personality and Social Psychology, 82*, 804–818.

Elliot, A. J., & Murayama, K. (2008). On the measurement of achievement goals: critique, illustration, and application. *Journal of Educational Psychology, 100*, 613–628.

Elliot, A. J., Murayama, K., & Pekrun, R. (2011). A 3 x 2 achievement goal model. *Journal of Educational Psychology, 103*, 632–648.

Godsen, R., Carroll, T., & Stone, S. (1991). How well does the polar vantage xl heart rate monitor estimate actual heart rate? *Medical Sciences in Sports and Exercise, 23*, 14.

Gonida, E. N., Voulala, K., & Kiosseoglou, G. (2009). Students' achievement goal orientations and their behavioral and emotional engagement: Co-examining the role of perceived school goal structures and parent goals during adolescence. *Learning and Individual Differences, 19* (1), 53–60.

Gonida, E., Kiosseoglou, G., & Leondari, A. (2006). Implicit theories of intelligence, perceived academic competence and school achievement: Developmental differences and educational implications. *The American Journal of Psychology, 119*(2), 223–238.

Grant, H., & Dweck, C. S. (2003). Clarifying achievement goals and their impact. *Journal of Personality and Social Psychology, 85*, 541–553.

Guastello, S. J. (1987). A butterfly catastrophe model of motivation in organization: Academic performance. *Journal of Applied Psychology, 72*, 165–182.

Guastello, S. J. (2002). *Managing emergent phenomena: Non-linear dynamics in work organizations*. Mahwah, NJ: Lawrence.

Guastello, S. J. (2011). Discontinuities and catastrophes with polynomial regression. In S. Guastello & R. Gregson (Eds.), *Nonlinear dynamics systems analysis for the behavioral sciences using real data* (pp. 252–180). New York: CRC Press.

Guastello, S. J., Johnson, E. A., & Rieke, M. L. (1999). Nonlinear dynamics of motivational flow. *Nonlinear Dynamics, Psychology, and Life Sciences, 3*, 259–274.

Guastello, S. J., Boeh, H., Schumaker, C., & Schimmels, M. (2012). Cusp catastrophe models for cognitive workload and fatigue. *Theoretical Issues in Ergonomic Sciences, 13*, 586–602.

Guastello, S. J., Boeh, H., Gorin, H., Huschen, S., Peters, N., Fabisch, M., et al. (2013). Cusp catastrophe models for cognitive workload and fatigue: A comparison of seven task types. *Nonlinear Dynamics, Psychology and Life Science, 17*, 23–47.

Harackiewicz, J. M., Barron, K. E., Pintrich, P. R., Elliot, A. J., & Thrash, T. M. (2002). Revision of achievement goal theory: Necessary and illuminating. *Journal of Educational Psychology, 94*, 638–645.

Hardy, L., & Parfitt, G. (1991). A catastrophe model of anxiety and performance. *British Journal of Psychology, 82*, 163–178.

Hulleman, C. S., Schrager, S. M., Bodmann, S. M., & Harackiewicz, J. M. (2010). A meta-analytic review of achievement goal measures: Different labels for the same constructs or different constructs with similar labels? *Psychological Bulletin, 136*, 422–449.

Kelso, J. A., & Engstrøm, D. A. (2006). *The complementary nature*. Cambridge: MIT Press.

Koopmans, M. (2015). A dynamical view of high school attendance: An assessment of the short-term and long-term dependencies in five urban schools. *Nonlinear Dynamics, Psychology, and Life Sciences, 19*, 65–80.

Midgley, C., Kaplan, A., & Middleton, M. (2001). Performance-approach goals: Good for what, for whom, under what circumstances, and at what cost? *Journal of Educational Psychology, 93*, 77–86.

Navarro, J., & Arrieta, C. (2010). Chaos in human behavior: The case of work motivation. *The Spanish Journal of Psychology, 13*, 224–256.

Navarro, J., Arrieta, C., & Balén, C. (2007). An approach to the study of dynamics of work motivation using the diary method. *Nonlinear Dynamics, Psychology, and Life Sciences, 11*, 473–498.

Navarro, J., Curioso, F., Gomes, D., Arrieta, C., & Cortés, M. (2013). Fluctuations in work motivation: Tasks do not matter! *Nonlinear Dynamics, Psychology, and Life Sciences, 17*, 3–22.

Nezlek, J. B. (2001). Multilevel random coefficient analyses of event and interval contingent data in social and personality psychology research. *Personality and Social Psychology Bulletin, 27*, 771–785.

Nicolis, G., & Nicolis, C. (2007). *Foundations of complex systems*. Singapore: World Scientific.

Pekrun, R., Elliot, A. J., & Maier, M. A. (2006). Achievement goals and discrete achievement emotions: A theoretical model and prospective test. *Journal of Educational Psychology, 98*, 583–597.

Pekrun, R., Elliot, A. J., & Maier, M. A. (2009). Achievement goals and achievement emotions: Testing a model of their joint relations with academic performance. *Journal of Educational Psychology, 101*, 115–135.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models*. Newbury Park, CA: Sage.

Sideridis, G. D. (2003). On the origins of helpless behavior in students with learning disabilities: Avoidance motivation? *International Journal of Educational Research, 39*, 497–517.

Sideridis, G. D., & Stamovlasis, D. (2014). The role of goal orientations in explaining academic cheating in students with learning disabilities: An application of the cusp catastrophe. *Ethics and Behavior, 24*, 444–466.

Sideridis, G. D., Antoniou, F., & Simos, P. (2013). The physiological effects of goal orientations on the reading performance of students with dyslexia: A Pilot Study. *Procedia, Social and Behavioral Sciences, 96*, 1545–1551.

Sideridis, G. D., Antoniou, F., Stamovlasis, D., & Morgan, P. (2013). The relationship between victimization at school and achievement: The cusp catastrophe model for reading performance. *Behavioral Disorders, 38*(4), 228–242.

Sideridis, G. D., Stamovlasis, D., & Antoniou, F. (2015). Reading achievement, mastery, and performance goal structures among students with learning disabilities: A nonlinear perspective. *Journal of learning disabilities* (accepted for publication, DOI: 10.1177/00222194 15576524).

Skar, P. (2004). Chaos and self-organization: Emergent patterns at critical life transitions. *Journal of Analytical Psychology, 49*, 243–262.

Stamovlasis, D. (2006). The nonlinear dynamical hypothesis in science education problem solving: A catastrophe theory approach. *Nonlinear Dynamics, Psychology and Life Science, 10*, 37–70.

Stamovlasis, D. (2010). Methodological and epistemological issues on linear regression applied to psychometric variables in problem solving: Rethinking variance. *Chemistry Education Research and Practice, 11*, 59–68.

Stamovlasis, D. (2011). Nonlinear dynamics and neo-piagetian theories in problem solving: Perspectives on a new epistemology and theory development. *Nonlinear Dynamics, Psychology and Life Science, 15*, 145–173.

Stamovlasis, D., & Sideridis, G. (2014). Ought approach - ought avoidance: Nonlinear effects under achievement situations. *Nonlinear Dynamics, Psychology and Life Science, 18*(1), 67–90.

Stamovlasis, D., & Tsaparlis, G. (2012). Applying catastrophe theory to an information-processing model of problem solving in science education. *Science Education, 96*(3), 392–410.

Tesser, A. (1980). When individual dispositions and social pressure conflict: A catastrophe. *Human Relations, 33*, 393–407.

Thom, R. (1975). *Structural stability and morphogenesis*. Reading, MA: W.A. Benjamin.

Treiber, F. A., Musante, L., Hartdagan, S., Davis, H., Levy, M., & Strong, W. B. (1989). Validation of a heart rate monitor with children in laboratory and field settings. *Medical Science in Sports and Exercice, 21*, 338–342.

van der Maas, H. L. J., & Hopkins, B. (1998). Developmental transitions: So what's new? *British Journal of Developmental Psychology, 16*, 1–13.

van der Maas, H. L. J., Molenaar, P. C. M., & van der Pligt, J. (2003). Sudden transitions in attitudes. *Sociological Methods and Research, 23*, 125–152.

Wajciechowski, J. A., Gayle, R. C., Andrews, R. L., & Dintiman, G. B. (1991). The accuracy of radio telemetry heart rate monitoring during exercise. *Clinical Kinesiology, 45*, 9–12.

White, R. (1959). Motivation reconsidered: The concept of competence. *Psychological Review, 66*, 297–333.

Glossary

$1/f^{\beta}$ Noise See Pink Noise.

Anti-persistence The presence of long-range negative autocorrelations in a time series.

ARFIMA Autoregressive fractionally integrated moving average, a statistical approach to the analysis of time series data that estimates correlations over the long term of the trajectory.

Attractor Represents the stable state of a system operating in a dynamical equilibrium. A frequently recurring and stable state a system adopts or an area of state space that a system tends to inhabit.

Attractor Landscape A state space with several attractors a system could occupy.

Autocorrelation The correlation between repeated observations within subjects over time.

Autopoiesis A defining characteristic of living systems. Literally it means self-making, self-creating, or self-generating, and it characterizes the system in terms of processes rather than structures.

Average Speed A network-level measure of all the shortest paths between agents in a network; indicates how quickly information could flow in the network.

Betweenness Centrality Agents who stand between two or more agents or groups such that they can influence communication flows between those groups.

Bifurcation Point The two divergent paths in the cusp surface are joined at the bifurcation point at which the behavior is ambiguous, and beyond this point the system enters the bifurcation set, the area where discontinuous changes take place.

Bifurcation Set The area on the control plane where discontinuous changes take place.

Bifurcation Variable The control variable, beyond a threshold value of which a bifurcation effect is taking place.

Bifurcation The phenomenon where the trajectory of a system's behavior evolving in time splits into two divergent paths or branches.

Bimodality Refers to the probability distribution of the dependent variable, where two distinctly different modes exist or two simultaneously present states.

© Springer International Publishing Switzerland 2016

M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2

Brownian Motion A time series in which individual observations in close proximity are highly correlated, while the trajectory shows a high degree of volatility overall.

Causal Dynamics This dynamics is about the complex causal dynamics within a unity of a whole of a reciprocal causal relation. This dynamics describes the potential self-amplifying loop effect, which is thriving on the full generative power of interaction.

Causal Complexity Links complexity with the complex causal dynamics and the corresponding potential nonlinear effects, taking place within loop-like networks of ever-evolving entities in many dimensions.

Clogs-to-Clogs Cycle The clog, a shoe with a thick wooden sole, was worn by manual workers in the north of England. The implication is that the effort required to raise wealth from poverty is often not continued to the third generation, and that the success is therefore not sustained. A similar idea is found in other languages: in Italian "from stables to stars to stables"; in Japanese "the third generation ruins the house"; in Chinese "wealth does not survive three generations," and in Spanish "he who doesn't have it, makes it and he who has it, waste it."

Collective Variable An empirical variable that provides a nonexhaustive description of the behavior of the system in a particular context.

Collinearity A condition in a network in which vectors are equal to, or multiples of, other vectors.

Complementarity A concept in interpersonal theory that defines the interplay between people in interaction; how the interpersonal behaviors of both participants fit together, mutually adjust to each other, and how this dynamically changes during interactions.

Complex Dynamic Systems Theory The theory that describes how open or closed systems develop through complex processes in which the behavior of the system in its entirety cannot be reduced to the behavior of individual elements of that system.

Complexify An inquiry process that embraces ambiguity and complexity, explores patterned emergence, re-organization, and complex dynamics, and engages in the "why's" of thinking.

Complexity Research The term refers to the approaches used to investigate complex living, social systems in ways that try to avoid the lenses of mechanism, reductionism, and positivism in social science.

Complexity Systems' behavior in terms of the following properties: (1) simple components relative to the whole system, (2) feedbacks, and (3) recursive interactions among components. All these characteristics mean in general that the behavior of the system in its entirety cannot be reduced to that of its constituent members.

Constraints The various physical and mathematical limitations or restrictions operating on whatever emerges during emergence. Examples include the actual containing vessels holding reagents together in cases of chemical emergence, or the mathematical constraints of circle packing dynamics underlying the hexagonal shape of Benard convection cells.

Content (of Interactions) The content of interactions indicates what (dyadic) states occur most frequently in the interactions.

Context It refers to spatial, temporal, or formal (conceptual–informational) sets or webs of interrelationships.

Coupled Oscillators Systems with periodic orbits that are linked.

Degeneracy A situation where some simulated random networks in a Monte Carlo process are almost void of ties or almost full of ties, thereby producing an average that is not an appropriate estimate to use a baseline comparison with an observed network.

Delay Convention Refers to the way the dynamical system moves from one stable area to another. In this convention, it is assumed that the system remains in the old equilibrium zone until the last possible point before it passes to the new equilibrium area. The state of the system is determined by the local minimum of potentials.

Deterministic A signal that is causative and nonrandom.

Differencing Parameter (*d*) The parameter d estimates the degree of long-term dependency in a time series that is assumed to be stationary.

Divergence Deviation from a linear relationship between the response predictors demonstrated by two diverging response gradients-deviating paths toward the upper or the lower part of the surface.

Dynamical Minimalism A principle stating that complex dynamic behaviors can be produced by simple rules and/or a few interacting variables.

Dynamical System A system that changes over time. At any given time, the system is in a particular state, and follows an evolution rule that describes how it changes states over time. Generally, a continuous dynamical system will be described by differential equations.

Edge of Chaos See self-organized criticality.

Edges Ties that link agents in a network, represented as a line or arrow between agents.

Emergence The coming into being of radically novel structure, patterns, organization, dynamics, and even laws of behavior in complex nonlinear systems when the conditions are right.

Emergent Behavior Aggregate-level attributes of a system that arise from the interactions of its components that cannot be explained by the individual behavior or the sum of these components.

Emergent Phenomena The outcomes of processes of emergence, these outcomes being characterized as unpredictable, nondeductible, and uncomputable from and irreducible to the substrates already existing in the system along.

Endogenous Processes The behavior of a system as impacted by its own past behavior.

Entropy A concept originating from classical thermodynamics that expresses a measure of disorder in a system (see also Information Entropy).

Epistemology (Batesonian), Epistemological Errors Traditionally, epistemology refers to the study of how knowledge is constructed as a formal function of philosophy. However, Bateson used "epistemology" to refer to the way individuals (and social groups) constructed knowledge and meaning as personal epistemologies. "Epistemological errors" occurred when personal interpretations conflicted with reality, such as errors that occur when confusing the map for the territory.

Ergodicity The equivalence of data structures underlying observed in between-subjects and within-subject distributions.

Exogenous Processes The behavior of a system as impacted by influences external to the system.

Externality An unintended effect (positive or negative) that an activity has on others' well-being for which there is no (positive or negative) compensation for the person causing that effect.

Far from Equilibrium A state of ongoing turbulence in a system and lack of proximity at most temporal instances to the attractor(s) in the system's behavior.

Feedbacks (Positive/Negative) A feedback is a closed chain of causal interactions between interrelated variables. In a positive feedback, a change in two or more linked variables produces a response in the same direction: more (less) of one variable leads to more (less) of the other variables; while in the negative feedback the response is in the opposite direction.

Fourier Transform The conversion of a time series to a periodogram, which re-expresses the trajectory of observed measurements over time in terms of the variability accounted for by periodic cycles at varying lag values.

Fractal Dimension The dimension of phase space a system occupies that can be a non-whole number. It is a measure of a system's complexity and it can be measured by the slope of the iPL distribution graph. If S is the magnitude of the recurrent pattern and f is the frequency at which each particular pattern occurs, the slope of the $1/f$ curve can be used as an estimate of fractal dimension.

Fractal Distribution See inverse power law distribution.

Fractional Differencing A method of estimating long-term dependencies in a stationary time series.

Generative Change Shows change as a complex, causal, generative, potential nonlinear process. A process that takes place within the structure of a dynamic unity of the whole of a reciprocal causal relation.

Generative Complexity Generative complexity links complexity with causal complexity. Shows complexity as encompassing causal, generative processes of change that generate complex emergent effects over time, potential nonlinear, thriving on the full generative power of interaction.

Generativity Generativity is a complex, general, dynamic capability of an entity, like a learner, linked to a complex state of being and doing: that is, of knowing how to go on. This may be an individual or a collective state of being and doing, to be linked to an individual and collective capability.

Gini Coefficient A measurement of the income distribution of a country's residents is based on the Lorenz curve (see definition). This number ranges between 0 and 1 with 0 representing perfect equality and 1 representing perfect inequality.

Grid Measures Measures that can be derived from State Space Grid analysis to study the content and structure of interactions.

Homophily Similarity between or among individuals. Homophilous relationships are more likely to form bonds than are heterophilous relationships (network analysis).

Hub Centrality The degree to which an agent has many out-degree links to agents who have many in-degree links, or the degree to which agents communicate to people who are in the know (e.g., a school principal); standardize to a 0-1 scale.

Hurst Exponent (*H*) An estimator of long-range autocorrelations, or dependencies, in time series data.

Hysteresis Is the effect, where cases with the same values of the two controls, asymmetry and bifurcation, can be found in both distributional modes, that is, they can exhibit two types of behavior corresponding to both behavioral attractors. For a dynamical system, hysteresis effect denotes memory for the path through the phase space of the system. Also hysteresis could refer to the time lag between input and output in a process.

Inaccessibility The region on the response surface existing in between the two behavioral modes. This area is inaccessible in the sense that the corresponding behavior is unlikely. The points within this area are pulled toward to either attractor.

Indeterministic A system that is causative, but its complexity means that its behavior cannot be completely predicted.

Information Entropy (Shannon Entropy, H_S) A variable of a dynamical system, associated with the information needed to describe the system and thus it is related to system's complexity.

Interaction Trajectory The chronological representation of a dyadic interaction in a State Space Grid.

Interpersonal Theory Theory that describes communication processes between people in terms of a two-dimensional model called the Interpersonal Circle.

Inverse Power Law (iPL) A statistical distribution, which mathematically is expressed by the equation $S(f) \propto f^{-\beta}$, where S is the size of an event (or object) and f is the frequency of the event (or object).

Lag Distance between two observations in a time series, expressed in terms of the number of time intervals.

Lineality It refers to sequential sets of relations or processes [Opposite: Recursivity].

Linear Causality Cause and effect as a relationship in which cause precedes effect, and changes in outcomes are proportional to changes in input conditions.

Linearity Mathematical relations that can be plotted on a graph as a straight line.

Log Frequency The logarithm of the relative frequency.

Log Power The logarithm of the amplitude of the cycles in a time series.

Lorenz Curve Plots the proportion of total income (y axis) that is cumulatively earned by the bottom x% of the population. When using quintiles, each quintile has 20 % of the population, sorted in ascending order by the relevant income variable. The line at 45° represents perfect equality of incomes.

Lyapunov Exponent The rate of divergence of nearby trajectories in state space; it is a measure of the chaoticity of a dynamical process resembling random variability.

Maxwell Convention Refers to the way the dynamical system moves from one stable area to another. In this convention, it is assumed that the system immediately jumps to a new equilibrium area, while the state of the system is determined by the global minimum of the potential function.

Metapatterns A term coined by Gregory Bateson and further elaborated upon by Tyler Volk. Refers to patterns of patterns or patterns that span multiple contexts while maintaining a general functional quality and/or meaning.

Meta-stability See Self-organized criticality.

Monte Carlo Procedure A procedure for predicting the likely range of trajectories of a given activity by projecting that trajectory forward in time from different initial conditions. Initial conditions are typically selected based on a statistical probability.

Network Cliques Groups of agents in a network who communicate among themselves more than they communicate with agents outside the group.

Network Density The number of actual links in a network divided by the total possible links, standardized 0-1 with 1 being highest possible density.

Nondeterministic A system that has a random component (measurement error) in its output.

Nonlinear Dynamical Systems That branch of mathematics having to do with equations/functions that can represent evolutionary change in a system. It includes such constructs as phase space, attractors, bifurcation, technical chaos, and so forth.

Nonlinear Regression Analysis A statistical approach for estimating the parameters of a nonlinear function.

Nonlinearity It refers to mathematical relations that do not appear as a straight line when plotted on a graph.

Orbital Decomposition (OD) A series of computations to identify the systematicity of recurring sequences in a string of data.

Path Dependence Refers to the idea that current and future states, actions, or decisions depend on the sequence of states, actions, or decisions that preceded them.

Patterned Recurrence A sequence that repeats itself.

Periodogram Spectral density plot.

Permutation, Dekker A recent algorithm for performing permutations that is more robust against skewness, network collinearity, and network autocorrelation that are other forms of permutation.

Permutation, Y A traditional algorithm for performing permutations in a matrix.

Persistence The presence of long-range positive autocorrelations in a time series.

Philosophical Frameworks Cohesive frames of thinking based on the work of a particular philosopher, set of philosophers, or school or schools of philosophy. Such frameworks provide a solid foundation for cohesive and consistent work within a particular paradigm.

Pink Noise A pattern of correlated errors over the long range of a time series that contracts slowly toward statistical nonsignificance as the lag size increases.

Power Law A linear relationship between the log power and the log frequency in a power spectrum.

Power Spectral Density The expression of a time series in terms of cycles of dependency between observations at given lag sizes and the magnitude of the variability explained by those cycles, both log-transformed.

Power Spectrum The log-amplitude of the cycles in a time series as a function of the log-relative frequency (see also Inverse Power Law).

Practices and Discourses The practices and discourses of a paradigm are those ways of doing specific kinds of functions and the ways of talking about specific topics within that paradigm.

Proximal Recurrence An immediately repeated sequence.

Quadratic Assignment Procedure A statistical method for regressing one matrix onto another.

Radical Novelty A characteristic of emergent phenomena indicating such interrelated properties as unpredictability, nondeducibility, irreducibility, and uncomputability.

Randomness The lack of predictability of a signal in a nondeterministic time series (see also White Noise).

Reciprocity A two-way (shared) relationship between or among individuals.

Recurrence The exact repetition of particular observed values in a trajectory.

Recursive Causality An iterative relationship between cause and effect.

Recursivity It refers to processes that cycle back through sets of relations that are modified in some way after each cycle [Opposite: Lineality].

Regularity The predictability of a signal or time series.

Relationship The meaning of "relationship" is based on Bateson's idea that everything exists in relationship. Protons exist in relationship to electrons. Hemoglobin exists in relationship to oxygen, blood, and cells. Relationships are functional and dynamic and provide the binary tensions that drive more complex sets of relationships and functions (Kelso & Engström, 2006).

Relative Frequency An expression of the periodicity of the dependencies in a time series in terms of the number of cycles in a series, divided by the total number of observations in that series.

Relative Mobility Measures a person's rank on the income, earnings, or wealth ladder compared to her parents' rank at the same age.

Random Graph A matrix created by random, chance combinations of dyadic relationships. Random graphs do not have patterns of relationship that can be attributed to any systematic social force such as friendships or relationships between leaders and followers; thus they are useful for comparing against actual observed networks to determine if there are any patterns in the observed network that can be attributed to social forces.

Research Scaffold Promote a deeper, more connected research agenda that is contextually relevant and publicly meaningful.

Scale Invariance The nested structure of replicated patterns in a self-similar process.

Self-generating A self-replicating, recursive process of regeneration or renewal in living systems. For example, cells in the organs of our body renew themselves continuously through the process of self-generation.

Self-organization Refers to the systems that evolve in time, operating far-from-equilibrium, and taking on ordered structures without requiring any outside intervention. It is the "order for free" notion.

Self-organized Criticality Critical state in a system resulting from the accumulation of certain input conditions, such that further exposure to those conditions leads to higher-level transformation.

Self-organizing A process whereby a pattern spontaneously emerges from the interaction of lower-level constituent parts in the behavior of a system.

Self-reflective The capacity of learning systems to learn from their previous responses, behaviors, or thinking when navigating their world.

Self-reflexive Constituted by self-reflective responses, this is the characteristic of living systems to be self-reflective.

Self-regulated Learning The process of thoughtful engagement using cognitive, metacognitive, and affective control strategies in service of a learning goal.

Self-regulating Self-maintaining, a property of living systems to maintain identity over time.

Self-similarity Across Scales A characteristic of fractals that patterns are repeated across scales; the parts of a fractal reflect the same structure of the whole. Examples in nature include cauliflower, broccoli, and trees.

Self-similarity The replication of patterns at varying scales, i.e., patterns within patterns. The replication is not strictly deterministic, but rather a matter of general impression.

Self-transcending Constructions A descriptive and neutral term for emergence indicating how the radically novel outcomes, i.e., emergent phenomena, are constructed out of antecedent substrate components. Processes of emergence need to be potent enough that substrates are transformed effectuating radically novel emergent integrations.

Shannon Entropy (H_S) See Information entropy.

Simmelian Ties The degree to which agents belong to 3-way reciprocal relationships, standardize to a 0-1 scale.

Spectral Density Power spectral density (see above) without the log transformations.

Stability Continued proximity of a set of repeated observations to a fixed point attractor.

Star Configurations Patterns of relationships in which several agents are connected by another actor.

State Space Grid (SSG) A graphic representation of a state space. An SSG consists of at least two orthogonal dimensions that describe all the possible states a system can adopt.

State Space A description of all possible states a system can adopt.

State Behavior a system can adopt.

Stationarity Constancy of statistical properties (mean, variance) in an ordered sequence of observations across the entire trajectory.

Structure (of Interactions) Structure represents the degree of variability in interactions, in terms of stability, flexibility, and chaos.

Sudden Jumps Abrupt changes between the behavioral modes occurred even with slight changes in the control variables.

Symbolic Dynamics An area of mathematics that examines the structure in the sequence of nominal data.

Theoretical and Conceptual Framework (see Chapter on Bateson Philosophy) Theoretical and conceptual frameworks are the factual knowledge, principles, and laws that support the particular "theory" or "concept."

Time Scales Complex Dynamic Systems Theory assumes that development takes place on three different time scales that are all interrelated to each other. On the micro-level time-scale development occurs in real-time from second to second. On the meso-level time-scale development occurs in real time from hour to hour or day to day. On the macro-level time-scale development occurs in developmental time from week to week, month to month, or year to year. The exact definition of the time scale depends on the variables that are studied.

Time Series A sequence of temporally ordered observations.

Topological Entropy (H_T) A notion and mathematical concept describing the deterministic nonrandom complexity for the time series.

Trajectory See time series.

Transitivity The tendency for the friends of friends to become one's friends (network analysis).

Transcontextuality Transcontextuality does not refer to just multiple or overlapping contexts, but to thinking in ways that span and incorporate views from multiple contexts. The idea of Bateson's "multiple perspectives" or "double description" is based on information from two or more contexts that are used in forming a unified understanding.

Transdisciplinarity Transdisciplinarity is similar to transcontextuality but refers to the more specific sense of "discipline" as a context of inquiry.

Transversality Theorem Any smooth map may be deformed by an arbitrary small amount into a map that is transverse to a given submanifold. This theorem facilitated the classification of singularities that permitted the identification of seven types of "elementary" catastrophes.

Turbulence A state of high variability with an irregular appearance.

Uncomputability The most recent way of characterizing the radically novel nature of emergent phenomena in the sense that these radical novel characteristics cannot be predicted nor deduced *nor computed* from the nature of the substrates. For example, the computational emergence found in artificial life cannot be computed (predicted) from the dynamics of the substrate on/off cells in cellular automata.

Vector A single row or column in a matrix.

White Noise Uncorrelated errors/random scatter of data points without predictive value.

Worldview "Worldview," in this discussion, is based upon, but expanded from, the notion of world hypotheses. Fundamentally, worldview incorporates these world hypotheses but also extends to the sets of social, cultural, and idiosyncratic belief frameworks that guide one's interpretation of perceptions of the world.

Index

© Springer International Publishing Switzerland 2016
M. Koopmans, D. Stamovlasis (eds.), *Complex Dynamical Systems in Education*, DOI 10.1007/978-3-319-27577-2

C

D

T