

# The Algorithm of Discovery: Making Discoveries on Demand

**Boris Stilman**

**Abstract** According to our hypothesis the Algorithm of Discovery should be an evolutionary component of the Primary Language of the human brain (as introduced by J. von Neumann in 1957). In our research we identified two such components, Linguistic Geometry (LG), and the Algorithm of Discovery. We suggested that both components are mental realities “hard-wired” in the human brain. LG is a formal model of human reasoning about armed conflict, an evolutionary product of millions of years of human warfare. In this paper we focus on discovering the Algorithm of Discovery, the foundation of all the discoveries throughout the history of humanity. This Algorithm is based on multiple thought experiments, which manifest themselves and are controlled by the mental visual streams. This paper reports results of our investigation of the major components of the Algorithm of Discovery with special emphasis on constructing a series of models and mosaic reasoning. Those approaches are demonstrated briefly on discoveries of the No-Search Approach in LG, the structure of DNA, and the theory of Special Relativity.

**Keywords** Linguistic geometry • Primary language • Artificial intelligence • Algorithm of discovery • Game theory • Mosaic reasoning

What if discoveries are produced routinely as an output of computer programs? What a leap this would mean for humanity? Approaching this time of making discoveries on demand is the purpose of our efforts.

More than 50 years passed since J. von Neumann hypothesized existence of the Primary Language [43]. Unfortunately, the nature of this language is still unknown. Our hypothesis is that the Primary Language is a collection of major algorithms crucial for survival and development of humanity, the underlying “invisible”

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B. Stilman (✉)

University of Colorado Denver, Denver, CO, USA

e-mail: Boris.Stilman@UCDenver.edu; boris@stilman-strategies.com

B. Stilman

STILMAN Advanced Strategies, Denver, CO, USA

foundation of all the modern languages and sciences. We suggested that one of the components of the Primary Language is Linguistic Geometry (LG), a type of game theory [1–4, 11, 12, 14–27, 36–41] that allows us to solve classes of adversarial games of practical scale and complexity. It is ideally suited for problems that can be represented as abstract board games, for example, military decision aids, intelligent control of unmanned vehicles, simulation-based acquisition, high-level sensor fusion, robotic manufacturing and more. The advantage of LG is that it provides extraordinarily fast and scalable algorithms to find the best strategies for concurrent multi-agent systems. Also, unlike other gaming approaches, the LG algorithms permit modeling a truly intelligent enemy. LG is applicable to the non-zero-sum games and to the games with incomplete information (i.e., imperfect sensors, weather, enemy deception, etc.).

We suggested [28] that every human brain “speaks” the LG language, though, only well trained commanders and, especially, advanced strategists are able to utilize it to full capacity. Most importantly, they are able to translate from the LG language, i.e., from the Primary Language, into the natural languages to describe strategies in the spoken language terms.

## 1 Towards Ordinary Discoveries

In our research on revealing other components of the Primary Language, besides LG, we assumed that they look like LG in some respects. Our contention is that the hypothetical Algorithm of Discovery must be one of such components. In a number of papers, we have been developing a hypothesis that there is a universal Algorithm of Discovery driving all the innovations and, certainly, the advances in all sciences [29–35]. All the human discoveries from mastering fire more than a million years ago to understanding the structure of our Solar System to inventing airplane to revealing the structure of DNA to mastering nuclear power utilized this algorithm. The Algorithm of Discovery should be a major ancient item “recorded” in the Primary Language due to its key role in the development of humanity. This line of research involved investigating past discoveries and experiences of construction of various new algorithms, beginning from those, which we were personally involved in [14–39, 42–44].

Thought experiments allow us, by pure reflection, to draw conclusions about the laws of nature [5]. For example, Galileo before even starting dropping stones from the Tower in Pisa used pure imaginative reasoning to conclude that two bodies of different masses fall at the same speed. The Albert Einstein’s thought experiments that inspired his ideas of the special and general relativity are known even better [6, 9, 13]. The efficiency and the very possibility of thought experiments show that our mind incorporates animated models of the reality, e.g., laws of physics, mathematics, human activities, etc. Scientists managed to decode some of the human mental images by visualizing their traces on the cortex [5]. It was shown that when we imagine a shape “in the mind’s eye”, the activity in the visual areas of the brain

sketches the contours of the imagined object; thus, mental images have the analogical nature. It appears that we simulate the laws of nature by physically reflecting the reality in our brain. The human species and even animals would have had difficulty to survive without even minimal “understanding” of the laws of environment. Over the course of evolution and during development of every organism, our nervous system learns to comprehend its environment, i.e., to “literally take it into ourselves” in the form of mental images, which is a small scale reproduction of the laws of nature. Neuropsychologists discovered that “we carry within ourselves a universe of mental objects whose laws imitate those of physics and geometry” [5]. In [28], we suggested that we also carry the laws of the major human relations including the laws of optimal warfighting. The laws of nature and human relations manifest themselves in many different ways. However, the clearest manifestation is in perception and in action. For example, we can say that the sensorimotor system of the human brain “understands kinematics” when it anticipates the trajectories of objects. It is really fascinating that these same “laws continue to be applicable in the absence of any action or perception when we merely imagine a moving object or a trajectory on a map” [5]. This observation, of course, covers actions of all kinds of objects, natural and artificial. Scientists have shown that the time needed to rotate or explore these mental images follows a linear function of the angle or distance traveled as if we really traveled with a constant speed. They concluded that “mental trajectory imitates that of a physical object” [5].

Our main hypothesis is that the Algorithm of Discovery is based not on formal logic but on the so called “visual streams”, i.e., mental imaginary movies which run in our brain [10]. (By the way, LG is highly visual as well.) This is how it may work. Within the brain, the visual streams run consciously and subconsciously and may switch places from time to time (in relation to conscious/subconscious use). We may run several visual streams concurrently, morph them, and even use logic for such morphing, although this use is auxiliary. Then we mentally tag some of the objects shown in the movie and create the so-called symbolic shell around the main visual stream. This shell eventually becomes a standard symbolic algorithm that can be communicated to others employing familiar language, logic, mathematics, etc. I named this approach “visual reasoning”. While the “visual” component (including pattern recognition) is, in general, pretty sophisticated, the reasoning component is relatively simple. Fortunately, the full scale mental visibility is rarely used in discoveries, and, in my opinion, the limited visibility can be simulated with a reasonable effort. The “reasoning” component is certainly within the scope of the modern software development.

Our approach to discovering the Algorithm of Discovery is analogous to an attempt to understand the algorithm of a program while watching its execution. Let us assume that this program’s interface includes color movies on various subjects. In addition to this Algorithm, we are trying to discover the instruction set of the “computer” running this program, i.e., the means of the human brain to running it. With multiple published introspections of great scientists we can recreate clips from various movies, i.e., their imaginary thought experiments. What really helps is the assumption that all those movies were “demonstrated” by the programs running

essentially the same algorithm. With our own past developments in LG, we have additional power of asking questions via morphing our own movies and getting answers by watching those morphed movies until the very end. Unfortunately, we do not have this power with the discoveries of other scientists.

## 2 The Algorithm of Discovery

In this section we briefly summarize the results introduced in [29–33]. The Algorithm of Discovery operates as a series of thought experiments, which interface with the rest of the brain and with external environment via imaginary animated movies (plays), which we named visual streams. These streams may or may not reflect the reality. This interface is constructive, i.e., visual streams could be morphed in the desired direction.

The input to the Algorithm is also a visual stream, which includes several visual instances of the object whose structure has to be understood or whose algorithm of construction has to be developed. Sometimes, the object is dynamic, i.e., its structure is changing in time. Then the input visual stream includes this visual dynamics. As a rule, neither the structure of the object nor the details of the dynamics are present in the stream. It simply replicates (mimics) the natural or imaginary phenomenon. The task of the Algorithm of Discovery is to understand its structure including dynamics and/or develop an algorithm for reconstructing this object including its changes in time. This understanding happens in several stages. Importantly, it always ends up with the process of actual reconstruction of the object employing the construction set developed by the Algorithm on the previous stages. If the Algorithm investigates a natural real life object this imaginary reconstruction may be totally unrelated to the construction (replication) utilized by the nature. Usually, this reconstruction process is artificially developed by the Algorithm of Discovery with the only purpose to reveal the structure of the object. However, if the algorithm of natural replication is the goal of discovery than the Algorithm of Discovery will employ a set of different visual streams to reveal the relevant components utilized by the nature [35].

All the visual streams are divided into classes, Observation, Construction and Validation. They usually follow each other but may be nested hierarchically, with several levels of depth.

The visual streams operate in a very simple fashion similar to a child construction set. The Construction stream utilizes a construction set and a mental visual prototype, a model to be referenced during construction. This is similar to a list of models pictured in a manual (or a visual guide) enclosed to every commercial construction set. It appears that all the thought experiments in LG related to construction investigated so far, utilized those manuals. Imagine a child playing a construction set. He needs a manual to construct an object by looking constantly at its picture included in this manual. This model comes from the Observation stream as its output. It is not necessarily a real world model. It is not even a model from the

problem statement. It is created by the Observation stream out of various multiple instances of the real world objects by abstraction, specifically, by “erasing the particulars”. A final version of the object constructed by the Construction stream should be validated by the Validation stream.

The Algorithm of Discovery initiates the Observation stream, which must carefully examine the object. It has to morph the input visual stream and run it several times to observe (mentally) various instances of the object from several directions. Often, for understanding the object, it has to observe the whole class of objects considered analogous. If the object is dynamic (a process) it has to be observed in action. For this purpose, the Observation stream runs the process under different conditions to observe it in different situations. The purpose of all those observations is erasing the particulars to reveal the general relations behind them. A good example of multiple observations of processes is related to the thought experiments with various objects with respect to the inertial reference frames when discovering the theory of Special Relativity [9]. This includes experiments with uniformly moving ships, trains, experiments with ether as well as experiments for catching a beam of light (Sect. 5). Once the relations have been revealed, a construction set and a visual model have to be constructed by the Observation stream. Both are still visual, i.e., specific,—not abstract. However, they should visually represent an abstract concept, usually, a class of objects or processes, whose structure is being investigated. For construction, the Observation stream utilizes the Construction stream with auxiliary purpose (which differs from its prime purpose—see below). Note that the model construction is different from the subsequent reconstruction of the object intended to reveal its structure. This model may differ substantially from the real object or class of objects that are investigated. Its purpose is to serve as a manual to be used for references during reconstruction. Various discoveries may involve a series of models (Sect. 3).

When the model and the construction set are ready, the Algorithm of Discovery initiates the Construction stream with its prime purpose. This purpose is to construct the object (or stage the process) by selecting appropriate construction parts of the set and putting them together. If an object has a sequential nature the construction also takes place sequentially, by repetition of similar steps. If multiple models have been produced the final object construction can also be considered as a model construction. At some point of construction, the parts are tagged symbolically and, in the end, visual reasoning with symbolic tagging turns into a conventional symbolic algorithm to be verified by the subsequent Validation stream.

Models and construction sets may vary significantly for different problems. Construction of the model begins from creation of the construction set and the relations between its components. Both items should be visually convenient for construction. The Algorithm of Discovery may utilize a different model for the same object if the purpose of development is different. Such a different model is produced by a different visual stream.

In many cases the Algorithm of Discovery employs “a slave” to visually perform simple tasks for all types of visual streams. This slave may be employed by the Construction stream to “see” construction parts and put them together. More

precisely, imagine a child playing a simplistic construction set. To avoid offending children, I had named this personality a Ghost. This Ghost has very limited skills, knowledge and, even, limited visibility. The Observation stream may utilize the Ghost to familiarize itself with the optional construction set, to investigate its properties. Next, the Construction stream may use the Ghost to perform the actual construction employing those properties. Eventually, the Validation stream may use the Ghost to verify visually, if properties of the constructed object match those revealed by the Observation stream. In all cases, the Ghost is guided by the Algorithm of Discovery or, more precisely, by the respective visual streams.

As was already discussed, the initial visual model is usually guided by a very specific prototype, where the Observation stream has actually erased the particulars. However, this specificity does not reduce generality in any way. This sounds like a paradox. Essentially, every component of this model carries an abstract class of components behind it. This way visual reasoning about the model drives reasoning about abstract classes, which is turned eventually into the standard formal reasoning. This happens as follows. A visual model drives construction of the formal symbolic model so that the key items in a visual model have tags representing the respective formal model. At first, the formal model is incomplete. At some stage, a running visual stream is accompanied by a comprehensive formal symbolic shell. Running a shell means doing formal derivation, proof, etc. synchronized with a respective visual stream. While the shell and the stream are synchronized, the visual stream drives execution of the shell, not the other way around. For example, a formal proof is driven by animated events within the respective visual stream. The visual streams, usually, run the creation of the visual model, the construction set and the final construction of the object several times. During those runs as a result of persistent tagging the symbolic shell appears. Multiple runs utilize the same visual components but during initial runs the synchronization of the stream and the shell is not tight. Further on, synchronization is tightened by morphing the visual model and/or adjusting symbolic derivation if they initially mismatch. Eventually, the stream and the shell switch their roles. In the end, it appears that the stream becomes the animated set of illustrations, a movie, driven by the running symbolic shell. For example, during the final runs (and only then), the visual streams, presented in [29–34], are driven by the constraints of the abstract board game, the abstract set theory and/or the productions of the controlled grammars. At this point the visual stream and the symbolic shell can be completely separated, and the visual stream can be dropped and even forgotten.

A stream may schedule other streams by creating almost a “program with procedure calls”. Essentially, it may schedule a sequence of thought experiments to be executed in the future. These experiments will, in their turn, initiate new visual streams. In this case, the purpose, the nature, and the general outcome of those experiments should be known to the stream created this sequence. However, this sequence is different from the list of procedure calls in conventional procedural (or imperative) programming. The algorithms of those “procedures”, i.e., the algorithms to be produced by the respective thought experiments are generally unknown. The experiments are not programmed—they are staged. The actual algorithm should be

developed as a result of execution of such experiment. In a sense, this is similar to the notion of declarative programming when a function is invoked by a problem statement while the function's body does not include an algorithm for solving this problem.

The ability of a visual stream to schedule a sequence of thought experiments permits to create a nested top-down structure of visual streams with several levels of depth. Though, we suspect that the actual depth of the nested programmed experiments never exceeds two or three.

Proximity reasoning as a type of visual reasoning was introduced due to the need for approaching optimum for many discoveries. It is likely that all the technological inventions and discoveries of the laws of nature include "optimal construction" or, at least, have optimization components [13]. Thus, various construction steps performed by the Algorithm of Discovery require optimization, which, certainly, makes construction more difficult. As the appearance of this Algorithm is lost in millennia, for its main purpose, it could not certainly utilize any differential calculus even for the problems where it would be most convenient. For the same reason, it could not utilize any approximations based on the notion of a limit of function. Those components of differential calculus could certainly serve as auxiliary tools. In that sense, in order to reveal the main optimization components, the most interesting problems to be investigated should lack continuity compelling the Algorithm of Discovery to employ explicitly those components. Based on several case studies [34], we suggested that this optimization is performed by the imaginary movement via approaching a location (or area) in the appropriate imaginary space. Having such space and means, the Algorithm employs an agent to catch sight of this location, pave the way, and approach it. Contrary to the function based approach, which is static by its nature, the Algorithm operates with dynamic processes, the visual streams. Some of those streams approach optimum (in a small number of steps); other streams show dynamically wrong directions that do not lead to the optimum and prevent the Algorithm from pursuing those directions. Both types of streams represent proximity reasoning. We suggested that proximity reasoning plays a special role for the Algorithm of Discovery as the main means for optimization. Proximity reasoning is a type of visual reasoning. This implies that the Algorithm should reason about the space where distances are "analogous" to the 3D Euclidian distances. Roughly, when we approach something, the distance must be visually reduced, and this should happen gradually. The space for proximity reasoning should provide means to evaluate visually if the animated images representing various abstract objects approach each other or specific locations [34]. Construction of those spaces is the key component of the Algorithm of Discovery.

### 3 A Series of Visual Models for Discoveries

A discovery, i.e., a development of the final algorithm for the object construction is based usually on constructing a series of models. Each of those models may, in its turn, be based on multiple experiments and may result from multiple uses of the

Observation and Construction streams. Interestingly, those models may represent the same object, though, be totally different. The purpose of these models is to look at the object from different prospective to reveal different properties. The models do not appear at once. Experiments with one model demonstrate the need for the next one. The model construction is based on the wide use of the principle of erasing the particulars. For each model some of the particulars of an object under investigation are erased while other particulars are emphasized. A good example of such multiple models is the discovery of the No-Search Approach [24, 25, 27, 31, 33, 34]. This discovery is based on the four different models that represent the same abstract object, the State Space of the 2D/4A Abstract Board Game (ABG). This ABG is a reformulation of the well-known R. Reti chess endgame [4].

The first model is the so-called Pictorial LG that includes a network of zones, a pictorial representation of several types of local skirmishes. This representation is obtained by “projecting” optional variants of skirmishes from the State Space onto the Abstract Board. Moreover, to make those projections visible, an Abstract Board is mapped into the area of a 2D plain. This mapping permits to easily visualize Pictorial LG as a network of straight lines drawn on a sheet of paper or displayed on a screen. The straight lines represent trajectories of pieces, i.e., the planning routes of mobile entities. This type of representation permits to erase (abstract from) the particulars of movement of various entities such jumps, turns, promotions, etc. Small circles (representing stops) divide a trajectory into sections (the steps). This way, movement through the State Space of the ABG is visualized by the “physical” movement of pieces along the trajectories of the Pictorial LG. Moreover, the first model permits to conduct experiments that investigate visually if a piece moving along trajectory can approach a “dynamic” area on the Abstract Board while this area moves away, e.g., shrinks. However, a conclusion about approaching or non-approaching an area should be considered as local with respect to the ABG because both trajectories and areas are just “projections” of variants of movement and subspaces of the State Space on the Abstract Board. In order to expand from local to global conclusions the first model invokes the second one.

The second model is the so-called Mountain-triangle drawn on a sheet of paper or displayed on a screen. Essentially, it represents the same State Space of the ABG, though, with distinguished Start State at the upper vertex of the triangle. In addition, the Mountain-triangle represents the brute-force search tree of the ABG that grows top down from the upper vertex of the triangle and the terminal states located in the bottom side of it. Certainly, it is a rough representation of the State Space. However, it is convenient for visualizing a tree with top-down direction. It is called “Mountain” to reflect analogy with a climber’s descends and ascends performed by the Ghost when visiting branches of the tree. The second model permits to elevate projection subspaces introduced in the first model into the full State Space. This elevation is based on the expansion of the ABG terminal states (introduced in the 2D/4A problem statement). The expansion experiments consist of the blowing inside the triangle various bubbles rooted in the bottom and directed to the top (“closer” to the Start State). Those bubbles represent various subspaces of the State Space. In order to establish link with their projections the second model invokes



joint experiments with the first one. Those experiments reveal formal description of several bubbles, i.e., the subspaces of the ABG State Space. These descriptions are based on the zones, the components of the Pictorial LG. It appears that those bubbles and their complementary subspaces have complex structure and fill the full State Space. Further experiments demonstrate the need in decomposition of the State Space into multiple well-defined subspaces and their intersections to reveal the complete structure of the bubbles. Basically, we are talking about precise accounting for intersections of multiple bubbles which represent a clearly defined mosaic of tiles (Sect. 4). The second model invokes the third one.

The third model is the so-called State Space Chart. This model is again a representation of the ABG State Space. It is a square drawn on a plain and broken into four quadrants (by the vertical and horizontal lines) and a circle around the center of the square. This third model represents mosaic of eight tiles, four quadrants and four circular segments, the proper subsets of the respective quadrants. These tiles represent important subspaces of the ABG State Space which are described employing zones of the Pictorial LG. It appears that the Start State of the 2D/4A ABG (reflected by a small circle) belongs to the upper left quadrant. The subspaces represented by the circular segments have special value. All the states of those subspaces have a well-defined strategy leading to a specific result of the game, a white win, a black win, or a draw. Thus, the third model together with the first and second models provides means to investigate if there are strategies leading from the Start State to each of the circular segments. Those strategy-candidates are represented visually as lines linking the Start State to the appropriate segment. The investigation is based on four thought experiments that utilize effectively the visual dynamics of the first model. The experiments permit to eliminate two classes of strategy-candidates, the white winning strategies and the so-called Pure draw strategies. The rest of the candidates are preserved for the precise final testing on the fourth model. It should employ minimax search algorithm and choose the only real strategy existing in this problem.

The fourth model is the Solution Tree, the conventional search tree of the 2D/4A ABG. As usual, it grows top-down and employs minimax. However, the legal moves included on the Tree are those prescribed by the strategy-candidates preserved by the experiments with the third model. Specifically, these are classes of the black winning strategies and the so-called Mixed draw strategies. The final construction experiment yields the Tree with the optimal moves only. So, it is not a search tree in conventional sense. The two strategy-candidates being tested provide ultimate forward pruning that leads to constructing the final Solution Tree. Both candidates are described by the visual algorithms utilizing the first model of the Pictorial LG. Thus, every legal move to be included on the Solution Tree is selected employing the strategies generated as the outcome of the first three models. Moreover, it explicitly uses the first model to reflect visually the game state change in the Pictorial LG. The fourth model demonstrates that the only real strategy for this problem is the Mixed draw strategy. In terms of the mosaic reasoning (Sect. 4), application of the strategy-candidates to constructing the Solution Tree is the

iterative application of the transformation matching rule leading to complete Solution Tree mosaic.

The construction of a series of visual models including the switch procedure from one model to another is a major component of the Algorithm of Discovery. Those series were identified in all the discoveries we investigated so far including discoveries in LG, in revealing the structure of DNA and in the theory of Special Relativity.

## 4 Mosaic Reasoning for Discovering Objects

Mosaic reasoning as a type of visual reasoning was introduced due to the analogy of the Construction stream operation with assembling a mosaic picture of small colorful tiles. Another, maybe, even more transparent analogy is known as a jigsaw puzzle when a picture is drawn on a sheet of paper and then this paper is cut into small pieces, mixed up, to be assembled later into the original picture. As Sir Thompson [42] pointed "... the progress of science is a little like making a jig-saw puzzle. One makes collections of pieces which certainly fit together, though at first it is not clear where each group should come in the picture as a whole, and if at first one makes a mistake in placing it, this can be corrected later without dismantling the whole group". Both analogies, the pictorial mosaic and the jigsaw puzzle, represent well the key feature of the Algorithm of Discovery construction set. However, we prefer the former because the jigsaw puzzle looks more like an assignment in reassembling a construct, a picture, which has already been created and, certainly, well known. In that sense, a tile mosaic is created from scratch, including choosing or even creating necessary tiles. In addition, a jigsaw puzzle is reassembled out of pieces based on random cuts. On the contrary, in pictorial mosaic, in many cases, every tile should have unique properties; it should be shaped and colored to match its neighbors precisely. A similar specificity is related to a group of adjacent tiles, the aggregate.

In the following sections we will utilize discoveries of the structure of DNA and Special Relativity to demonstrate mosaic reasoning for objects and processes, respectively.

For many discoveries, the components of the construction set should be developed with absolute precision, in the way that every part should be placed to its unique position matching its neighbors. We will use the same name, the tiles, for those construction parts. If precision is violated the final mosaic will be ruined and the discovery will not happen. Though a group of tiles, an aggregate, may be configured properly, its correct placement in the mosaic may be unclear and requires further investigation. Moreover, a tile itself may have complex structure which may require tailoring after placement in the mosaic. In some cases, a tile is a network of rigid nodes with soft, stretchable links.

Mosaic reasoning may stretch through the observation, construction, and validation steps of the Algorithm of Discovery operating with tiles and aggregates of

tiles. Overall, mosaic reasoning requires tedious analysis of the proper tiles and their matching rules. Investigation of the matching rules is the essential task of the Observation stream. Multiplicity of those rules and their specificity with respect to the classes of construction tiles make the actual construction very complex. Selecting a wrong tile, wrong tailoring, choosing a wrong place, or incompatible neighbors may ruin the whole mosaic. The matching rules are the necessary constraints that control the right placement of the tiles. Missing one of them, usually, leads to the wrong outcome because the Algorithm of Discovery is pointed in the wrong direction.

Some of the matching rules impact mosaic locally while other rules provide global constraints. The global matching rules include the requirement of the top-down analysis and construction, the global complementarity rule, certain statistical rules, the transformation rules, etc. For many if not all natural objects and processes, their structure is not reducible to a combination of the components. Large groups of tiles, i.e., large aggregates, may obey the rules which are hardly reducible to the rules guiding placement of singular tiles. This matching rule must be understood globally first, implemented in the mosaic skeleton construction, and, only then, reduced to the placement of the specific tiles. An example of the global matching rule for the discovery of the structure of DNA is the choice of the helical structure of the DNA molecule including the number of strands [35, 44]. The rule of the global complementarity means that placement of one aggregate may determine precisely the adjacent aggregate. In case of DNA, one strand of the helix with the sequence of the base tiles attached to it determines the unique complementary second strand with the corresponding sequence of the base tiles. The global statistical rules related to the whole mosaic may reflect the relationship between major structural components, the large aggregates. If understood and taken into account by the Observation stream, they may focus the Construction stream and lead to a quick discovery. In the case of DNA, the so-called Chargaff rules reflect the structural relationship between the base tiles of the complementary strands of the double helix [35, 44]. Yet another class of global matching rules is called transformation rules. This is an algorithm for reconstructing an aggregate out of another aggregate and placing this aggregate in the proper location. Applied sequentially, such a rule permits to turn an aggregate, the so-called generator, into the set of adjacent aggregates. This way the whole mosaic could be constructed. For example, the whole mosaic of the DNA molecule could be constructed if the generator and the singular transformation are defined. Over the course of four experiments, the double helix generator was constructed. It includes a pair of nucleotides with sugar-phosphate backbone and purine-pyrimidine base. The transformation is a combination of translation and rotation. Interestingly, this type of construction may be utilized by the Algorithm of Discovery as a convenient procedure to reveal the structure of an object, e.g., the DNA molecule, while the nature may have used a totally different algorithm for producing the same object.

The local matching rules include the local complementarity rule, the interchangeability rule, etc. The local complementarity means, roughly, that a protrusion on one tile corresponds to the cavity on the complementary adjacent tile. For the

DNA molecule this is usually a hydrogen bond of a base tile (a protrusion) that corresponds to a negatively charged atom of the adjacent tile (a cavity). The local complementarity often expresses itself in the requirement of various kinds of symmetry within the pairs of matching construction tiles. The whole class of the local matching rules is based on interchangeability. In simple terms, if two aggregates that include several tiles are not identical but interchangeable, their internal structure may be unimportant. There are several levels of interchangeability. Two aggregates could be essentially the same, i.e., their skeletons coincide. Importantly, those skeletons must include nodes which serve as the attaching points of the aggregates to the rest of the mosaic. The notion of an internal skeleton depends on the problem domain and is specific for different types of mosaic. For example, two different aggregates for the DNA mosaic may have identical ring structures but the atoms and respective bonds that do not belong to those structures may be different. Another lower level of interchangeability of the aggregates does not require their skeletons to coincide. The only requirement is that the attaching points of those aggregates are identical. In all cases interchangeability means that the stream can take one aggregate off the mosaic and replace it with another. This will certainly change the picture but the whole structure will stand. We named those aggregates plug-ins. It appears that plug-ins played crucial role in the discovery of the structure of DNA because such a plug-in was the key component of the helical generator, a purine-pyrimidine base [35, 44].

Besides mosaic structural components that include tiles, aggregates, global and local matching rules, there is an unstructured component that we named a mosaic environment. Such environment may impact the structure of tiles, aggregates, application of matching rules, and the whole mosaic while being relatively unstructured itself. In case of DNA, this was the water content whose lack or abundance could seriously impact the structure of the whole mosaic.

## 5 Mosaic Reasoning for Discovering Processes

A different type of mosaic, the mosaic of processes, was constructed by Einstein while discovering his theory of Special Relativity [6]. In reality, this was not a construction from scratch—it was a reconstruction of the Galileo-Newton mosaic into new one, the Einstein mosaic. Both mosaics consist of moving tiles, the inertial frames, i.e., those frames moving along straight lines with constant velocities with respect to each other. Contrary to the static mosaic of objects considered above, the inertial frames mosaics represent processes developing in time. Moreover, various entities like human beings or water waves could be moving within those frames. Essentially, these are processes of processes. Mathematically, all those frames should be considered as those in the 4D space with time as the fourth dimension. For the Galileo-Newton mosaic this is a 4D Euclidian space, while for the Einstein mosaic this is a Minkowski space. Note that none of those mathematical constructs were actually used by Einstein for his discovery [6]. Visualization of the 4D spaces

is impossible; however, visualizing those mosaics as sets of the 3D processes developing in time supports fully various thought experiments and related visual streams. Typically, one of the frames is chosen as “static” like the one associated to the platform with the Ghost standing on it while the other frame is “moving” and is associated to a train passing by this platform and another Ghost walking inside a car of the moving train. There is no notion of an adjacent tile. However, there is still a notion of the transformation matching rules utilized by the Construction stream for transforming a generator into the whole mosaic of tiles. In those mosaics all the inertial frames are equivalent (or indistinguishable in terms of the laws of Physics), hence, any tile could serve as a generator and a plug-in simultaneously. For the Galileo-Newton mosaic the required transformation is just the Galileo transform while for the Einstein mosaic it is the Lorentz transform.

The Galileo-Newton mosaic has been around for several hundred years and was able to explain numerous experiments. There was no need for any reconstruction. Only during a couple of decades, before the Einstein’s discovery in 1905, several questions were raised. Reconstruction of mosaics was preceded by two series of thought experiments. Some of them were pure thought experiments while many others were replayed in real world, though, initially, they were certainly conceived as the thought ones.

The first series has led to revealing the principle of invariance of the laws of physics, the foundation of both mosaics. These laws should be the same in all the inertial reference frames, i.e., for all the tiles. This series could be traced back to the Galileo experiments in the main cabin of a large swimming ship below its deck [8]. Uniform movement of entities inside the ship (also moving uniformly) is indistinguishable from those on the land. However, assuming that the ship is transparent their velocities would look different from the land due to addition of the ship’s velocity (as a vector). While the Galileo’s experiments dealt with mechanical movements the same principle should have covered the laws of electromagnetism by Maxwell-Lorentz. This meant, in particular, that there should not be any distinction in how the induction occurs in both cases, whether the magnet or the conducting coil is in motion. However, according to the classic theory this experiment was interpreted differently for those cases. This meant different laws for different tiles. It was noted by several scientists, including Föppl [7], and emphasized by Einstein [6].

The second series of thought experiments revealed special nature of light or, more precisely, electromagnetic waves. This series could be traced back to the experiments with water and sound waves as traveling disturbances in a medium. As is the case with movements of other entities, velocities of the waves inside the Galileo’s ship are indistinguishable from those over the land (for the sound waves) or near the sea shore (for the water waves). Analogously to other moving entities on the transparent ship their velocities would look different from the land due to addition of the ship’s velocity. According to the Maxwell’s theory, light as well as all types of electromagnetic waves travels at a speed of approximately 186,000 miles per second. This includes AM and FM radio signals, microwaves, visible light, ultraviolet, X-rays, gamma rays, etc. Beginning from Maxwell himself, scientists believed that

these waves propagate as disturbances of the invisible medium called the ether, and their velocity was registered relative to this ether. The ether should have had interesting properties. It should spread through the entire universe and should not affect big bodies like planets and stars as well as the smallest ones like specks of dust. In addition it has to be stiff for the light wave to vibrate at a great speed. Numerous thought experiments were intended to demonstrate that the ether waves are passing by the Ghost at a faster speed if he is moving through the ether towards the light source. Some of those experiments have actually been implemented in real world. These include experiments by Fizeau, Michelson and Morley as well as those contemplated by Einstein himself. The most influential was the thought experiment of the Ghost riding uniformly at the speed of light alongside a light beam and observing “frozen” light. The 13 real life experiments refuted all their thought prototypes by registering no difference to the speed of light. Multiple mental executions and morphing (over the period of ten years) of the riding light experiment led Einstein to conclusion that this was not a real effect—the light would not freeze but would run at the same speed according to the same Maxwell equations as for the Ghost standing on the land.

The visual streams utilized in the two series of thought experiments considered above led conclusively to adoption of the two principles [6]. The first was the rigorous spreading of the principle of relativity to all the physical systems that undergo change meaning that the laws governing this change are the same for all the inertial frames of reference. The second principle stated the constancy of the velocity of light  $c$ , whether the ray is emitted by a stationary or by a moving body.

Adoption of those new principles led in its turn to the construction of the Einstein mosaic of the processes. The first matching rule was the rule of simultaneity which was the algorithm based on the relation between time and signal velocity. The second matching rule was the length measuring rule which was the algorithm for applying the measuring-rod. Those rules utilized by the Construction stream for constructing processes involving rigid bodies moving at the speed close to the speed of light demonstrated visually (within visual stream) and mathematically the effects of time dilation and length contraction. The major matching rule derived from the above principles was the Lorentz transform. It could be visualized as a hyperbolic rotation. Applying it to the generator, an arbitrary inertial frame, permitted to construct the whole Einstein mosaic of processes, the universe of inertial frames.

Our research demonstrated that the Algorithm of Discovery does not search for a solution in the search space. Instead, it constructs the solution out of the construction set employing various tools and guides. The right choices of the construction tiles and the matching rules by the Observation stream permit focusing the Construction stream to produce a desired series of models with a proper mosaic and, eventually, to make a discovery. All the results on the Algorithm of Discovery are still hypothetical and have to be verified by software implementations. The very first implementations have been initiated at the University of Colorado Denver.

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