Chapter 5
Wheatstone Bridge

In Tasks 2 and 3 of previous chapter's laboratory (diode and transistor temperature sensors), we measured V_{ab} across the 1 Ω resistor to evaluate the current. We did not measure V_a and V_b separately and make a subtraction, as both are on a scale of several volts (measured in 0–20 V scale), while their difference is only in millivolts (measured in 0–200 mV scale). If you have a DMM, you can connect one of its leads to the position *a* and the other to *b* to measure this tiny voltage drop. This is not really possible if you plan to develop a stand-alone sensing device.

In fact, this type of voltage measurement is known as a differential measurement, a common metric particularly in biosensor applications. Many biosensors measure a tiny difference in voltage, current or resistance in comparison with that of a blank or a negative control. If you try to make measurements for the target and the blank separately and evaluate their tiny difference by subtraction, you will need a sensor that has extremely high accuracy and sensitivity, perhaps with six or more significant digits.

There is a simple solution for it: a *Wheatstone bridge*. In the past, it has been used primarily for strain gauge applications, but more recently for a bio-application in the form of a cantilever biosensor. In reality, a Wheatstone bridge can be used for any differential measurement for both physical and biosensors.

5.1 Wheatstone Bridge

A Wheatstone bridge is an electrical circuit used to measure a very small change in resistance, such as a 10 Ω decrease for a 10 kΩ resistive load. This small change is not readily detectable by a typical DMM in the 0–20 kΩ range. The Wheatstone bridge consists of four resistors arranged in a diamond configuration. An input DC voltage, or excitation voltage, is applied between the top and bottom of the diamond, and the output voltage is measured across the middle. When the output voltage is zero, the bridge is said to be balanced. One (or more) of the legs of the bridge may consist of a resistive transducer, such as a thermistor or a strain gauge (often as R_4 in Fig. 5.1). The other legs of the bridge are simply completion resistors with resistance equal to that of a selected resistive transducer. As the resistance of one of the legs changes, the previously balanced bridge becomes unbalanced; this

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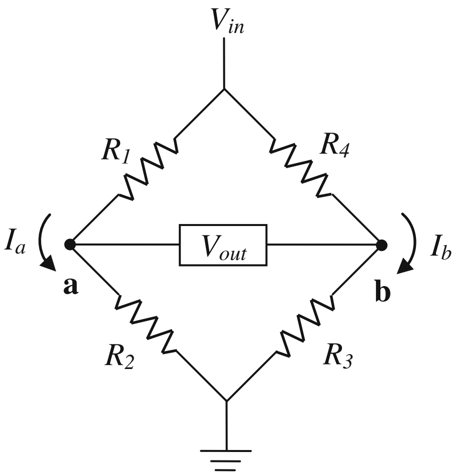
Fig. 5.1 A wheatstone bridge



can occur when a temperature or strain from a resistive transducer changes, for example. The unbalance in the bridge causes a voltage to appear across the middle of the bridge. This induced voltage may be measured with a voltmeter, or the resistor in the opposite leg to the changed resistor may be adjusted to rebalance the bridge. In either case, the change in resistance that caused the induced voltage may be measured and converted to obtain the engineering units of temperature or strain.

The Wheatstone bridge circuit in Fig. 5.1 contains four resistors arranged in a diamond configuration. A voltage, V_{in}, is supplied across the vertical diagonal of the diamond. The voltage, V_{out}, appears across the pair of terminals connected along the horizontal diagonal.

$$V_{\text{out}} = V_a - V_b = I_a R_2 - I_b R_3 \tag{5.1}$$

and,

$$I_a = \frac{V_{\text{in}}}{R_1 + R_2} \quad \text{and} \quad I_b = \frac{V_{\text{in}}}{R_3 + R_4} \tag{5.2}$$

therefore,

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \tag{5.3}$$

When the voltages V_a and V_b are equal, V_{out} will be zero; at this point, the bridge is said to be balanced. In addition, the value of R_4 can be found from:

$$R_1 R_3 = R_2 R_4 \quad \text{or} \quad R_4 = \frac{R_1 R_3}{R_2} \tag{5.4}$$

Assume that the bridge is initially balanced. R_4 is your resistive transducer. The resistance of R_4 is slightly changed. This change can be measured in two different ways:

- Measure V_{out} to calculate the new value of R_4, using Eq. 5.3.
- Use a variable R_1 and adjust it until $V_{out} = 0$. Use Eq. 5.4 with the adjusted R_1 value to calculate the new value of R_4.

Curiously enough, the Wheatstone bridge was not invented by Charles Wheatstone (1802–1875), but by Hunter Christie. However, Wheatstone was responsible for popularizing the arrangement of four resistors, a battery and a galvanometer. He gave Hunter Christie full credit for the Wheatstone bridge in his 1843 Bakerian Lecture. Wheatstone called the circuit a "*differential resistance measurer*." In regards to Wheatstone's diamond-pattern, it has been suggested that a set of blue willow pattern China, decorated with cross-hatching on an arched bridge, suggested the shape to him.

5.2 Strain Gauge

The oldest yet still popular application of a Wheatstone bridge would be a *strain transducer*, also known as a *strain gauge*. Obviously *strain* is measured with a strain gauge. Strain (ε) is a measure of a body's deformation, and its definition for either compression or tension is:

$$\varepsilon = \frac{\Delta l}{l_0} \tag{5.5}$$

where
Δl change in length
l_0 original length

Figure 5.2 shows a typical setup of a *strain gauge*, which is attached to a body (Fig. 5.3). As the body elongates horizontally, the physical width of a metal coil decreases and the resistance changes accordingly. This resistance change is very small and generally requires a circuit layout known as a *Wheatstone bridge*. The strain gauge is widely used in civil and mechanical engineering applications, but it is also used for an electronic balance.

Strain gauge can also be used to indirectly measure stress (σ), because stress is linearly proportional to strain for a limited range of strain.

$$\sigma = E\varepsilon \tag{5.6}$$

where E = *elastic modulus* (for compressive or tensile strain)

The elastic modulus E is generally known in literature for many types of materials.

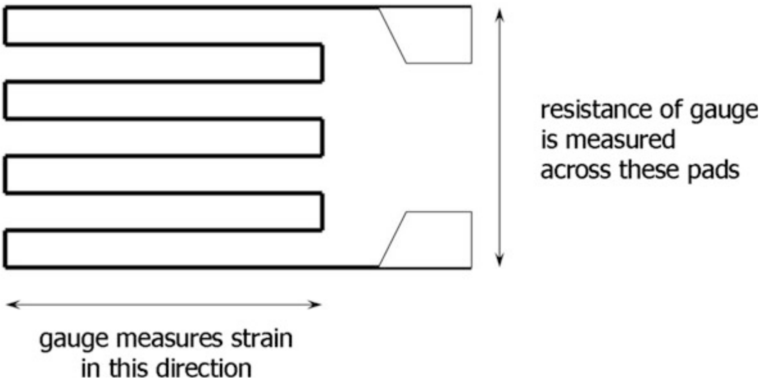


Fig. 5.2 A strain gauge

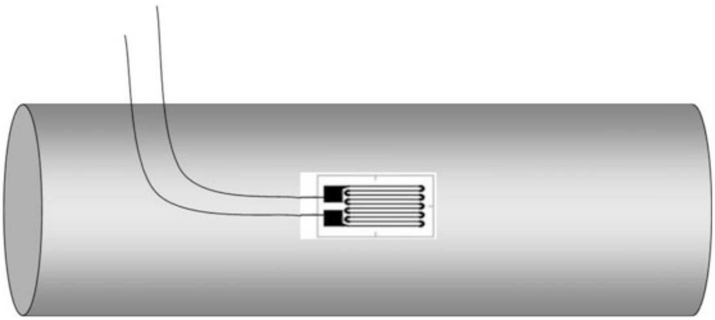


Fig. 5.3 A strain gauge attached to a metal specimen

5.3 Cantilever Biosensor

Recently, strain gauges have been applied to monitor biological reactions, which are often referred to as cantilever bending biosensors, or just *cantilever biosensors*. The working principle of a cantilever biosensor is shown in Fig. 5.4. Upon downward bending of the cantilever, the resistor on top of the cantilever is elongated, which results in an increase in its resistivity. This change in resistivity is read out as an electrical signal via a Wheatstone bridge configuration.

Many bioreceptors can be used with cantilever biosensors, including antibodies, enzymes, and nucleic acids (Fig. 5.5). For diagnostics, each measuring cantilever is paired with an inert reference cantilever. The reference cantilever is used to filter out thermal and chemical interactions between the surrounding media and the measuring cantilevers, since the reference cantilever is not affected by the molecular reaction on the measuring cantilevers.

The first applications of cantilever sensors for biological systems were reported in 1996 with a single cantilever. The first biosensing experiments with cantilever

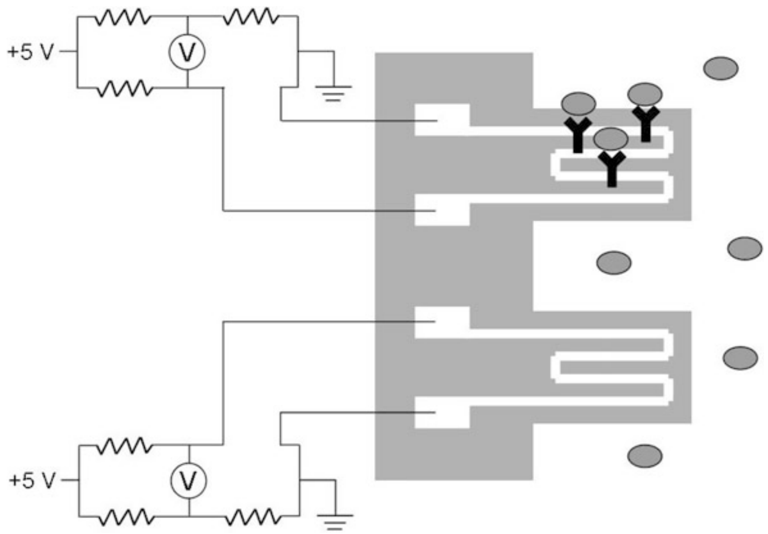


Fig. 5.4 A cantilever biosensor. The antibodies (*Y-shaped*) immobilized on the *top* cantilever capture target molecules, causing it to bend

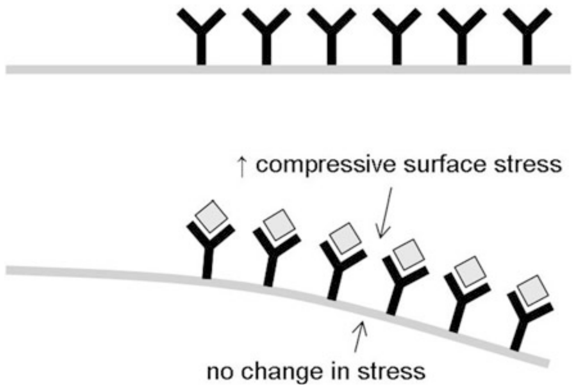


Fig. 5.5 Cantilever bending upon antibody-antigen binding. Antigen binding to the antibodies increases compressive surface stress (i.e., surface tension) on the *top* of the cantilever, while there is no such change on the other side, leading to cantilever bending downwards. If a tensile surface stress is generated, cantilever will bend upwards

arrays were demonstrated in 2000, showing the proof-of-principle for DNA detection and the ability to identify single-base mismatches between sensing and target DNA oligonucleotides.

These cantilever biosensors can be integrated into a lab-on-a-chip device, which also requires a Wheatstone bridge to read out small changes in resistance.

5.4 Laboratory Task 1: Wheatstone Bridge

In this task, you will need the following:

- A breadboard, wires, wire cutter/stripper, a power supply, and a DMM.
- Four 1 kΩ resistors.

Figures 5.6 and 5.7 show the circuit layout. Before beginning this task, the exact resistance values for the four resistors should be measured with a DMM. Label the resistors R_1, R_2, R_3, and R_4, respectively. Table 5.1 show the DMM readings.

Theoretical V_{bd} can be calculated from Eq. 5.3, using the experimentally measured V_{in}, R_1, R_2, R_3, and R_4 values

$$V_{bd} = V_{in}\left(\frac{R_2}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right) = (5.04)\left(\frac{989}{988+989} - \frac{988}{988+986}\right) \\ = -0.0013\,\mathrm{V} = -1.3\,\mathrm{mV}. \tag{5.7}$$

The experimental reading is off by +1.3 mV from its theoretical reading, but they are very close to each other.

Question 5.1
You constructed the Wheatstone bridge circuit first and tried to measure the R_1 value by connecting the DMM leads to the points '*a*' and '*b*'. Your DMM reading was 0.75 kΩ rather than 1 kΩ. Why is this happening? Hint: Between points '*a*' and '*b*', there are two branches of resistors, one with R_1 and the other with $R_2 + R_3 + R_4$.

Alternative Task 1: Wheatstone Bridge
Switch the locations of the four resistors and calculate V_{bd}. Compare this with the actual DMM measurement. Try several different combinations until the V_{bd} value (both calculated and measured) becomes close to 0 mV.

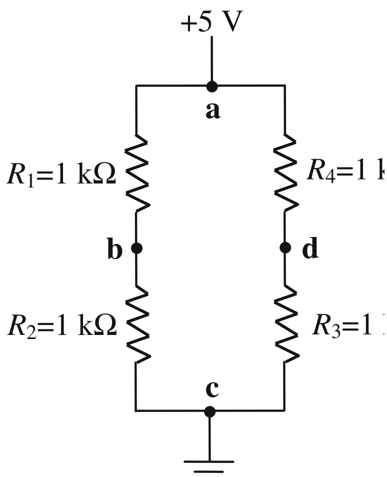


Fig. 5.6 Circuit diagram for Task 1

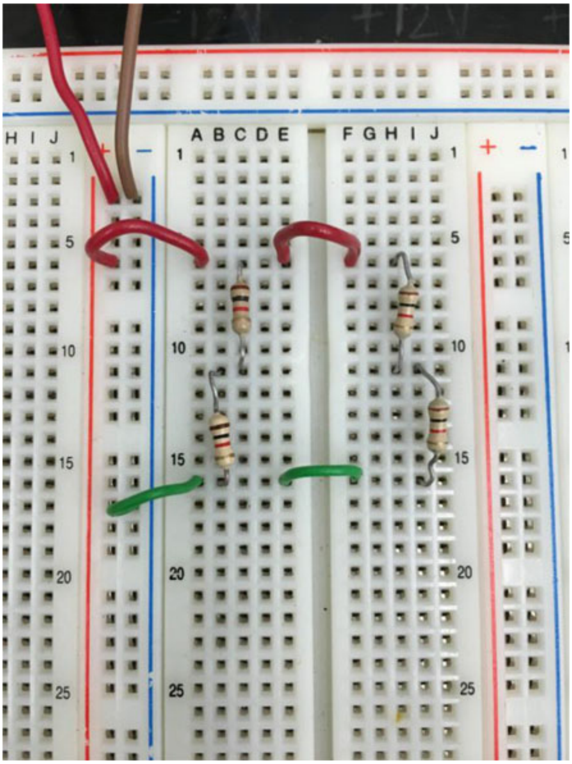


Fig. 5.7 Circuit photo for Task 1

Table 5.1 Experimental data from Task 1

$V_{in} = V_a$	R_1	R_2	R_3	R_4	V_{bd} (experimental)
5.04 V	0.988 kΩ	0.989 kΩ	0.988 kΩ	0.986 kΩ	0.0 mV

5.5 Laboratory Task 2: Wheatstone Bridge for a Thermistor

In this task, you will need the following:

- A breadboard, wires, wire cutter/stripper, a power supply and a DMM.
- 20 kΩ pot and a screw driver.
- 100 kΩ resistor.
- 30 kΩ thermistor.
- 30 kΩ resistor.
- Two 1 kΩ resistors.

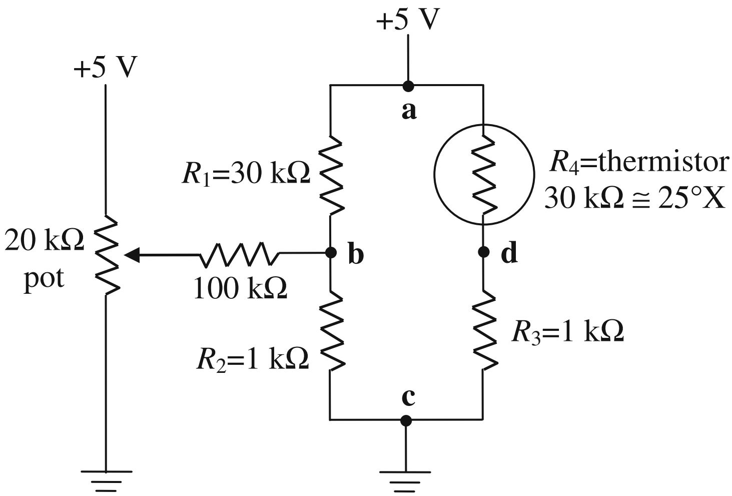


Fig. 5.8 Circuit diagram for Task 2

Figures 5.8 and 5.9 show the circuit layout. The resistance value of typical resistors varies as the environmental temperature changes. A thermistor is a resistor that is much more sensitive to temperature than typical resistors. We will use a thermistor whose resistance is 30 kΩ at 25 °C (room temperature or RT). Before you begin, measure the resistance value of your thermistor with your DMM. If the DMM reading is 32 kΩ, you should change R_1 to 32 kΩ.

The 20 kΩ pot and the 100 kΩ resistor attached in parallel with R_1 act as a "fine-tuner" for R_1. As they are parallel to R_1, the actual resistance between the points *a* and *b* should be slightly smaller than R_1. Adjust the 20 kΩ pot from one end to the other and measure V_{bd} for those two extremes. (The easy way to do this is to set your pot to generate 0 V and measure V_{bd}, then simply flip its direction to generate maximum voltage.) The voltage at one extreme should be positive while that at the other extreme negative. This indicates you are able to balance the bridge. If not, try to swap R_2 and R_3, or use a different resistor for R_1.

Once the bridge is balanced, the overall equivalent resistance on the top left part of the bridge (i.e., the whole part including R_1, the pot, and the 100 kΩ) can be easily calculated from Eq. 5.4 ($R_1R_3 = R_2R_4$), shown in Table 5.2.

Now, hold the thermistor with your fingers to increase the temperature that it is exposed to. Do not touch the pot. The resistance value should decrease, changing the balance of the bridge (in other words, V_{bd} is no longer 0 mV). Given the $R_{1,eq}$ obtained above and experimentally measured V_{bd}, V_{in}, R_2 and R_3, we can calculate the new R_4 with finger heating, also shown in Table 5.2.

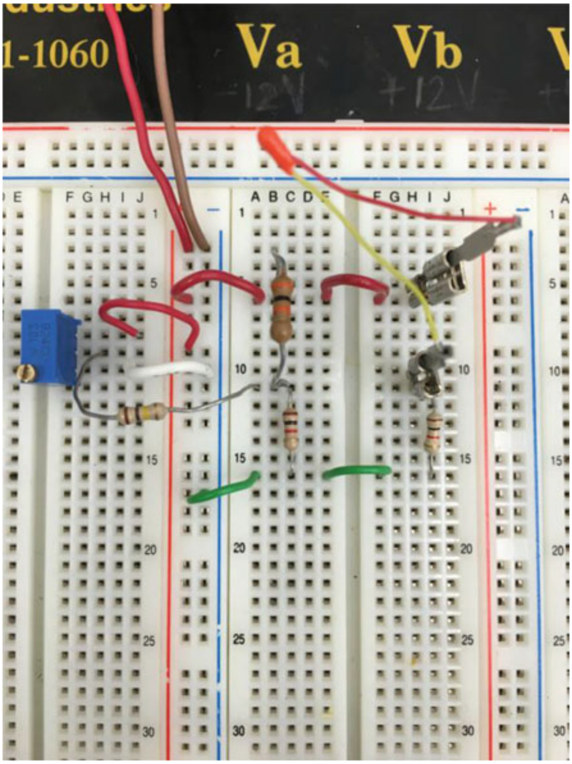


Fig. 5.9 Circuit photo for Task 2

R_4 with finger was calculated as follows:

$$\begin{aligned} V_{\mathrm{bd}} &= V_{\mathrm{in}}\left(\frac{R_2}{R_1+R_2}-\frac{R_3}{R_3+R_4}\right) \\ &= (5.04)\left(\frac{989}{30830+989}-\frac{988}{988+R_{4,\mathrm{finger}}}\right) = -0.0620 \end{aligned} \tag{5.8}$$

Solving for $R_{4,\mathrm{finger}}$ gave 21.8 kΩ. Apparently, the temperature rise caused the resistance of our thermistor to drop by 9.0 kΩ.

Question 5.2
Calculate the current flowing through the two branches when the bridge is balanced (V_{bd} = 0 mV). Hint: Use R_2/V_b and R_3/V_d to calculate the current. V_d (=V_b; why?) can be measured from V_{in}, R_3 and R_4. Are they identical?

Question 5.3
Calculate the current flowing through R_1 and the 100 kΩ resistor, using the values shown in Table 5.2. (Note that the arrow symbol on the pot does not represent the direction of current, but simply a schematic symbol for a pot.) Hint: The sum of these two currents should be the same as the current flowing through R_2.

Table 5.2 Experimental data from Task 2

$V_{in} = V_a$	$R_{1,eq}$	R_2	R_3	R_4 @ RT
5.04 V	unknown	0.989 kΩ	0.988 kΩ	30.8 kΩ

V_{bd}			$R_{1,eq} = R_2R_4/R_3$ @ balanced
@ V_{pot} = 0 V	@ $V_{pot} = V_{in}$ (5.04 V)	@ balanced	= 0.989 × 30.8/0.988 = 30.83 kΩ
−6.5 mV	+36.9 mV	0.0 mV	

V_{bd} w/finger	R_4 w/finger
−62.0 mV	21.8 kΩ

Question 5.4
Calculate R_4 using the above calculation and measurement when V_{bd} = +50 mV.

Alternative Task 2: Wheatstone Bridge for a Resistor
Task 2 can be repeated with regular 1 kΩ resistors for both R_1 and R_4. In this case, a 1 kΩ resistor (R_4) acts as a very insensitive thermistor. Repeat the entire Task 2.

5.6 Laboratory Task 3: Wheatstone Bridge for a Strain Gauge

In this task, you will need the following:

- A breadboard, wires, wire cutter/stripper, a power supply and a DMM.
- 20 kΩ pot and a screw driver.
- 10 kΩ resistor.
- A strain gauge, 120 Ω, with ribbon leads.
- Three 120 Ω resistors.

Figures 5.10 and 5.11 show the circuit layout. Similar to Task 2, you will need to balance the bridge; in other words, you need to make V_{bd} = 0 mV. Once you achieve the zero balance, try to bend the strain gauge (by touching it with your finger) and record V_{bd}. You can back-calculate the new resistance value for the strain gauge ($R_{4,\text{strain}}$) (Table 5.3).

R_4 with strain was calculated from the following:

$$\begin{aligned} V_{bd} &= V_{in}\left(\frac{R_2}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right) \\ &= (5.04)\left(\frac{119}{117+119} - \frac{120}{120+R_{4,\text{strain}}}\right) = -0.0404 \end{aligned} \tag{5.9}$$

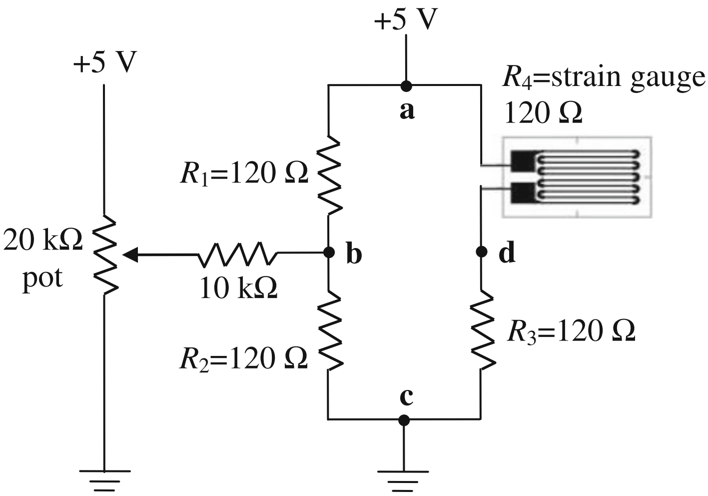


Fig. 5.10 Circuit diagram for Task 3

Fig. 5.11 Circuit photo for Task 3

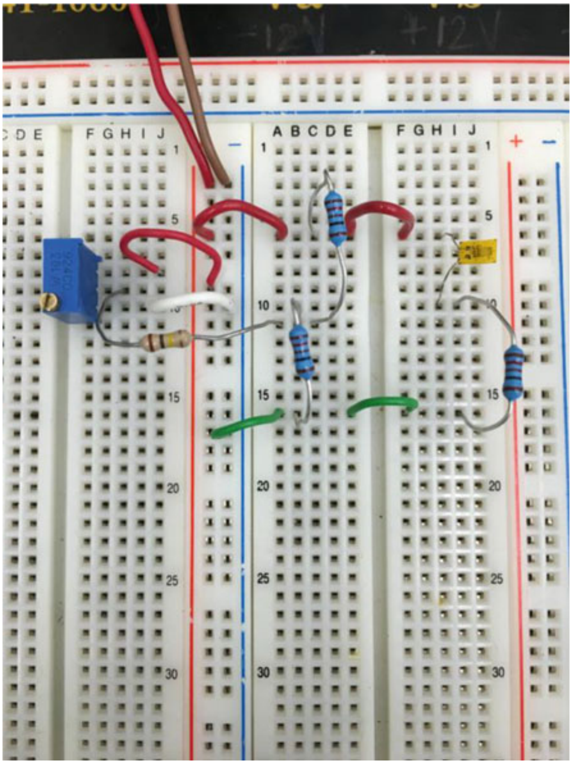


Table 5.3 Experimental data from Task 3

$V_{in} = V_a$	$R_{1,eq}$	R_2	R_3	R_4 w/no strain	$R_{1,eq} = R_2R_4/R_3$ @ balanced
5.04 V	unknown	119 Ω	120 Ω	118 Ω	= 119 × 118/120 = 117 Ω

V_{bd} w/strain	R_4 w/strain
–40.4 mV	114 Ω

Resistance decreased by 4 Ω (118 Ω → 114 Ω) with strain, indicating the strain gauge was compressed (shorter length of wires). If the resistance increased, the strain gauge would be stretched (longer length of wires).

Question 5.5
Calculate R_4 using the above calculation and measurement when V_{bd} = +5 mV.

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