

Chapter 2

Resistors

In the previous chapter, we learned that all transducers for sensors and biosensors generate voltage. In this regard, it is important that we begin our study with a basic understanding of electronics and circuitry.

2.1 Electric Circuit

Figure 2.1 shows an *electric circuit* that includes a battery (power source), a lamp (load), wires (conductors), and a switch to turn the circuit on or off.

Benjamin Franklin (the father of electronics) thought that positive charges move from the positive to the negative terminal of a battery, which he defined as electric current. Figure 2.2 is a redrawing of Fig. 2.1 that illustrates this concept. We will use this type of circuit diagram throughout this textbook.

Franklin's definition was apparently an error, as Joseph Thompson later found that the moving charges in an electric circuit were not positive charges but free electrons, which move in the opposite direction. As of today, however, we are still pretending that there are positive charges that move from the positive to the negative side of a power source, or from the *high voltage* to the *ground*, which works perfectly okay as long as we stick to this convention.

2.2 Current and Voltage

As defined previously, *electric current*, or simply *current*, is the flow of hypothetical positive charges from a region of positive net charge to a region of relative negative charge, which is the opposite of the flow of electrons from a negative to a positive charge. To measure current, we need to define a unit for electrical charge. A coulomb (C) is used for such a unit, which is equal to the charge of 6.25×10^{18} electrons or comparable positive charges. Current is the rate at which positive charges or electrons flow through a given point in a circuit. Current is given in unit amperes (A):

Fig. 2.1 A simple electric circuit

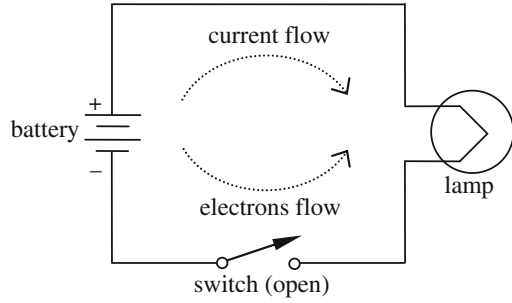
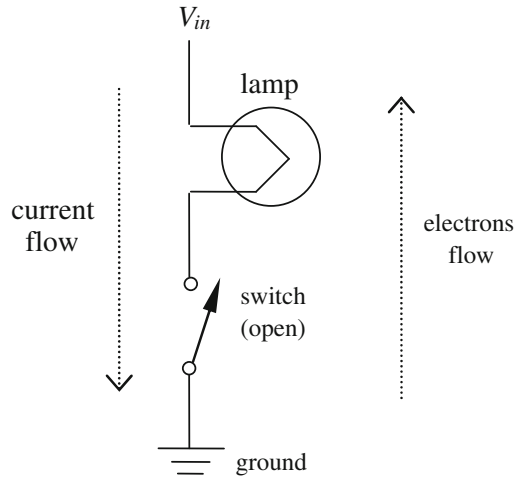


Fig. 2.2 Redrawing of Fig. 2.1. The battery power source is divided into high voltage (V_{in}) and ground



$$\text{amperes(A)} = \text{coulombs (C)}/\text{seconds (s)} \tag{2.1}$$

Another important term in basic circuitry is *electrical voltage*, or simply *voltage*. Voltage is an electromotive force (EMF) that moves positive charges or electrons from one point to another. Voltage at a given point in a circuit is always measured against the ground, which is usually the negative terminal of a power supply. The voltage of a battery, therefore, is the voltage at its positive terminal (high voltage) measured against its negative terminal (ground). A battery has an excess of electrons at negative terminal and a deficiency of electrons at the other.

When the movement of one coulomb of positive charges or electrons between two points (or the given point of a circuit and the ground) generates one joule of work, the voltage between these two points is defined as one volt (V), which is the unit of voltage:

$$\text{volts(V)} = \text{joules (J)}/\text{coulombs(C)} \tag{2.2}$$

The voltages of typical AA or AAA batteries are 1.5 V and that of your car is 12 V.

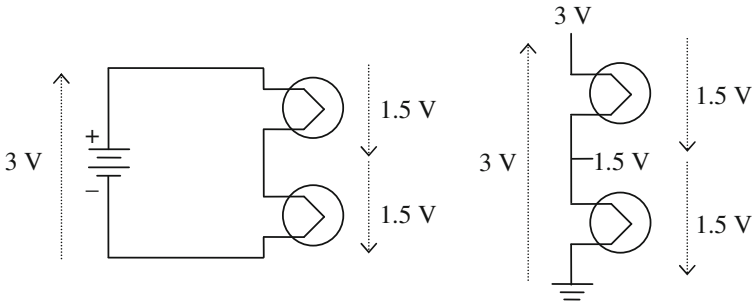


Fig. 2.3 Voltage rise and voltage drop

Figure 2.3 shows a circuit with two identical lamps connected in series with 3 V battery. The voltage of the battery is measured at its positive terminal (V_{in}) with reference to its negative terminal (ground). This particular voltage is referred to as a *voltage rise*, as it generates necessary voltage to the circuit. This voltage is dropped upon traveling the two lamps. As the two lamps are identical, equal amount of *voltage drop* should occur for these lamps, which is 1.5 V each. Therefore, the voltage at the point in between the two lamps (with reference to the ground) should be 1.5 V. In this way, the sum of voltage drops in a given circuit becomes the same as the sum of voltage rise.

2.3 Resistance and Ohm's Law

If a copper wire is connected across the 3 V battery, huge current will flow. If a piece of rubber is connected, however, almost no current will flow. The ratio of voltage over current should tell you how resistive your wire is in delivering electric charges with a given voltage. This ratio is called *resistance* (R). The unit of resistance is the *ohm* (Ω):

$$\text{ohms } (\Omega) = \text{volts(V)}/\text{amperes(A)} \quad (2.3)$$

More generally,

$$R = \frac{V}{I} \text{ or } V = IR \quad (2.4)$$

which is known as *Ohm's law*. Obviously the resistance of conductors is very low, while that of insulators is very high. *Conductors* are the substances that have many free electrons, like metals ($R \rightarrow 0$). The best metallic conductors are gold, silver, and copper among common materials. As gold and silver are very expensive, copper is usually the one most frequently used in wiring electric circuits. *Insulators* are the substances that have fewer free electrons ($R \rightarrow \infty$). The best insulators are glass and rubber among common materials.

Resistors are the passive devices that resist current flow, and are typically made out of materials that fall somewhere in between the properties of conductors and insulators. Resistors are often connected in series or in parallel, which will be discussed in the next sections.

2.4 Resistors in Series, or Voltage Divider

Figure 2.4 shows a circuit with two different resistors connected in series. There is only one path for current to pass through the circuit, meaning that the current flowing through two different resistors should be the same. The input voltage, V_{in} , is the voltage rise. V_1 is the voltage drop across R_1 , and V_2 is across R_2 . As the sum of the voltage rise should be the same as the sum of the voltage drop,

$$V_{in} = V_1 + V_2 \quad (2.5)$$

As explained earlier, there is only one path for the current flow to pass through. The currents flowing through R_1 and R_2 are identical. Let us designate that current as I . Using Ohm's law:

$$I = \frac{V_1}{R_1} \text{ (current through } R_1) = \frac{V_2}{R_2} \text{ (current through } R_2) \quad (2.6)$$

Equation 2.6 can be expressed for V_1 and V_2 :

$$V_1 = I \cdot R_1 \text{ and } V_2 = I \cdot R_2 \quad (2.7)$$

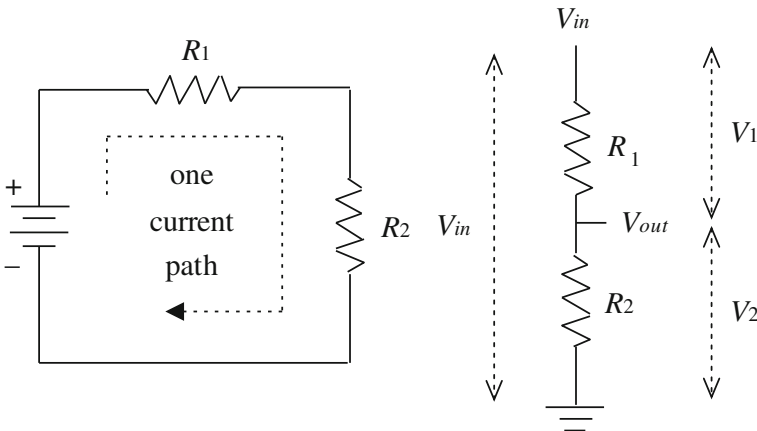


Fig. 2.4 Resistors in series, or voltage divider

Plugging Eq. 2.7 into Eq. 2.5 gives:

$$V_{\text{in}} = I \cdot R_1 + I \cdot R_2 = I(R_1 + R_2) \quad (2.8)$$

For the entire circuit, using the equivalent total resistance R_T and Ohm's law:

$$V_{\text{in}} = I \cdot R_T \quad (2.9)$$

Comparing Eqs. 2.8 and 2.9 gives:

$$R_T = R_1 + R_2 \quad (2.10)$$

Equation 2.10 indicates that the total resistance R_T is equal to the sum of the two resistors. In general, the total resistance of the resistors in series can be expressed as:

$$R_T = R_1 + R_2 + \cdots + R_n \quad (2.11)$$

This particular circuit is also known as a *voltage divider*, as the voltage at the point in between the two resistors (with reference to the ground), V_{out} , is a fraction of the input voltage V_{in} , depending on the ratio of the two resistors R_1 and R_2 . In other words, the voltage divider “divides” the input voltage using two (or more) resistors connected in series.

The output voltage V_{out} is essentially the same as V_2 . From Eq. 2.7:

$$V_{\text{out}} = V_2 = I \cdot R_2 \quad (2.12)$$

To express V_{out} in terms of V_{in} , R_1 , and R_2 , let us use Eqs. 2.9 and 2.10:

$$I = \frac{V_{\text{in}}}{R_T} = \frac{V_{\text{in}}}{R_1 + R_2} \quad (2.13)$$

Plugging Eqs. 2.13 to 2.12:

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2} \quad (2.14)$$

This is the voltage divider relationship, without the need for measuring the current I in evaluating its voltage output V_{out} .

Question 2.1

Calculate V_{out} of a voltage divider for:

- (a) $R_1 = 10 \Omega$ and $R_2 = 5 \Omega$
- (b) $R_1 = 1 \text{ k}\Omega$ and $R_2 = 1 \Omega$

2.5 Potentiometer, or Pot

The voltage divider divides the input voltage at the fixed ratio set by the two resistors. A *potentiometer*, or simply a *pot*, is a variable voltage divider made as a single piece. It is essentially a single resistor with an additional, third terminal. This third terminal is made to mechanically slide through the resistor so that the length of the top and bottom resistors can be altered. Figure 2.5 shows a graphical representation of a pot. Note that the arrow indicates the third terminal, or slider, and its direction does not indicate the flow of current. Pots are commonly used in many different applications, including the volume control of a radio or stereo system, and as position transducers such as a joystick (Fig. 2.6).

Fig. 2.5 A potentiometer or a pot

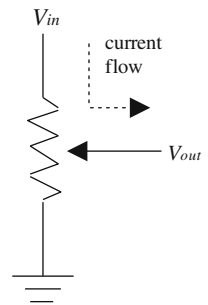
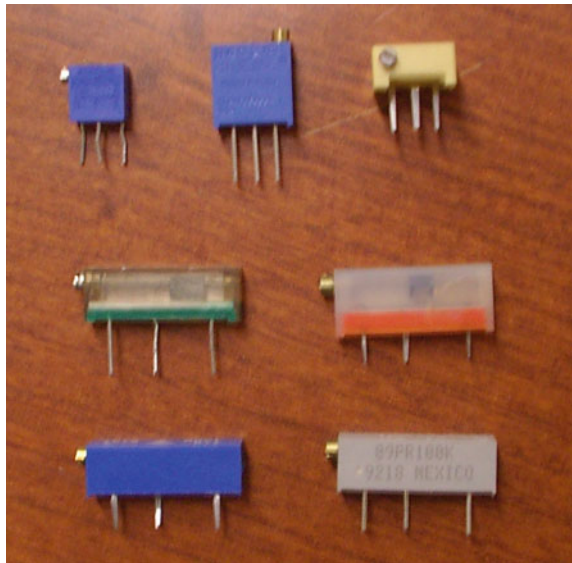


Fig. 2.6 Various single-turn and multi-turn potentiometers for infrequent adjustments, typically mounted directly on PCBs



2.6 Resistors in Parallel, or Current Divider

Figure 2.7 shows a circuit with two different resistors connected in parallel. There are two paths for current to pass through the circuit, meaning that the current is divided into two different resistors. Therefore:

$$I_{in} = I_1 + I_2 \tag{2.15}$$

The voltage drops across the two different resistors should, however, be the same (V_{in}).

$$I_1 = \frac{V_{in}}{R_1} \text{ and } I_2 = \frac{V_{in}}{R_2} \tag{2.16}$$

Plugging Eq. 2.16 into Eq. 2.15:

$$I_{in} = \frac{V_{in}}{R_1} + \frac{V_{in}}{R_2} = V_{in} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \tag{2.17}$$

For the entire circuit, using the equivalent total resistance R_T and Ohm's law:

$$I_{in} = \frac{V_{in}}{R_T} \tag{2.18}$$

Comparing Eqs. 2.17 and 2.18 gives:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or } R_T = \frac{R_1 \cdot R_2}{R_1 + R_2} \tag{2.19}$$

In general, the total resistance of the resistors in parallel can be expressed as:

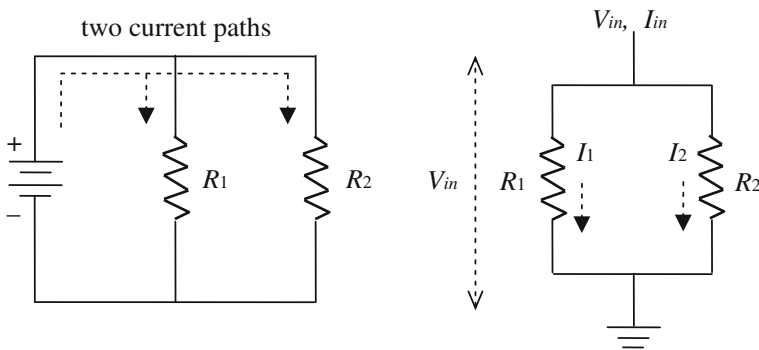


Fig. 2.7 Resistors in parallel or current divider

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \text{ or } R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} \quad (2.20)$$

This particular circuit is also known as a *current divider*, as the current is divided into two paths, I_1 and I_2 . Let us express I_1 in terms of I_{in} , R_1 , and R_2 (i.e. without using V_{in}). Plugging Eq. 2.19 into Eq. 2.18 gives:

$$V_{in} = I_{in}R_T = I_{in} \frac{R_1 \cdot R_2}{R_1 + R_2} \quad (2.21)$$

Plugging Eq. 2.21 into the first equation of Eq. 2.16:

$$I_1 = \frac{V_{in}}{R_1} = I_{in} \frac{R_1 \cdot R_2}{R_1 + R_2} \frac{1}{R_1} = I_{in} \frac{R_2}{R_1 + R_2} \quad (2.22)$$

Similarly,

$$I_2 = I_{in} \frac{R_1}{R_1 + R_2} \quad (2.23)$$

Equations 2.22 and 2.23 are the current divider relationships. The current flowing through branch 1 is affected by the second resistor, and that through branch 2 by the first resistor. This indicates that more current flows through a smaller resistor, while less current flows through a larger resistor.

Question 2.2

Calculate I_1 and I_2 of a current divider for:

- (a) $R_1 = 10 \Omega$ and $R_2 = 5 \Omega$
- (b) $R_1 = 1 \text{ k}\Omega$ and $R_2 = 1 \Omega$.

Does less current flow through the resistor with higher resistance?

2.7 Reading Resistor Values

We have just learned some basic fundamentals of electric circuitry, primarily Ohm's law. The resistor is a cornerstone in understanding Ohm's law, and also one of the most frequently used components in building circuits. A resistor is used to convert electric current into voltage signals. The resistor is also the main component in a voltage divider circuit. Before we begin our lab exercises, we need to learn some fundamental basics, including how to read resistor values.

Figure 2.8 shows a photograph of a resistor with four different colored bands around it. You will notice that the left-end color band is very close to the lead while the right band is further away from the other lead. If not, you have to flip it around.

Fig. 2.8 Resistors



Table 2.1 Resistor color code

	1st band	2nd band	3rd band	4th band
	1st digit	2nd digit	Multiplier	Tolerance (%)
Silver	–	–	10^{-2}	± 10
Gold	–	–	10^{-1}	± 5
Black	0	0	10^0	–
Brown	1	1	10^1	± 1
Red	2	2	10^2	± 2
Orange	3	3	10^3	–
Yellow	4	4	10^4	–
Green	5	5	10^5	± 0.5
Blue	6	6	10^6	± 0.25
Violet	7	7	10^7	± 0.1
Gray	8	8	10^8	–
White	9	9	10^9	–

In some cases, there may exist some distance between the third and fourth bands. (Note that this direction does not matter in actual circuit building, as resistors have no polarity.) Assume that the sequence of the colored bands is: Brown—Black—Red—Silver.

Table 2.1 shows a resistor color code, for identifying the color code. The first two bands correspond to two significant digits: in this case, brown for 1 and black for 0. The third band represents a multiplier; in this case, red for 10^2 . Therefore, we can come up with the formula, $10 \times 10^2 = 1,000 \Omega$ or 1 k Ω . The fourth band represents *tolerance*, i.e., a maximum error; in this case, silver for 10 %. As 10 % of 1,000 Ω is 100 Ω , the actual resistor value would be somewhere between 900 Ω and 1,100 Ω . As the tolerance band is typically either silver (10 %) or gold (5 %), only two significant digits are necessary to show the resistor size.

If the tolerance is less than or equal to 1 %, three significant digits are necessary. In this case, a 5-band color code becomes necessary with the first three corresponding to significant digits, the fourth the multiplier, and the fifth the tolerance. In some cases, a 6-band color code is possible, where the sixth represents the *temperature coefficient*. Four colors are used for the temperature coefficient band; brown (100 ppm), red (50 ppm), orange (15 ppm), and yellow (25 ppm). This coefficient represents the fractional decrease of resistor value per 1 °C rise from room temperature (25 °C). If the temperature coefficient of the above 1,000 Ω resistor is 100 ppm (brown), $1,000 \Omega \times 100 \times 10^{-6} = 0.1 \Omega$. If the temperature rises from 25 to 55 °C, then it follows that $1,000 - (0.1 \times (55-25)) = 997 \Omega$.

Question 2.3

- (a) Identify the resistor values and tolerances of the following resistors.
 Green—Brown—Red—Gold
 Orange—Black—Black—Orange—Brown—Brown
- (b) For a resistor of exactly 30.0 kΩ at 25 °C with a 100 ppm temperature coefficient, calculate its resistance at 37 °C.

2.8 Breadboards

Circuits are typically soldered into a printed circuit board (PCB) (Fig. 2.9), the wiring layout of which is chemically written on a board. This is a common practice in many commercial electrical devices, but not at all appropriate in testing and/or developing a new circuit (prototyping).

Breadboards make this prototyping really easy (Fig. 2.10); all you have to do is insert the leads into the tiny holes to construct a circuit and power it up by connecting the breadboard to the power supply. You can change the circuit layout anytime by simply removing, adding, and relocating components. You can also measure the voltage difference between any two points using a voltage meter

Fig. 2.9 A computer “mother board” is a good example of a PCB



(commonly known as a *voltmeter*), or more practically, with a digital multimeter (DMM).

The early prototyping was made on a plain wooden plate that looked like a breadboard, on which bread is cooled and/or sliced after baking. The current prototyping breadboard, however, no longer look like a real breadboard.

2.9 Laboratory Task 1: Resistors in Series

In this task, you will need the following:

- Breadboard
- Three-output low-capacity AC-to-DC power supply (+5, +12 and -12 V)
- Standard wire (wire gauge 22)
- Wire cutter/stripper
- Resistors (1 k Ω)

Let us learn about breadboards with a practical example: resistors in series (i.e., a *voltage divider*). Figure 2.11 shows the circuit diagram.

Fig. 2.10 A breadboard

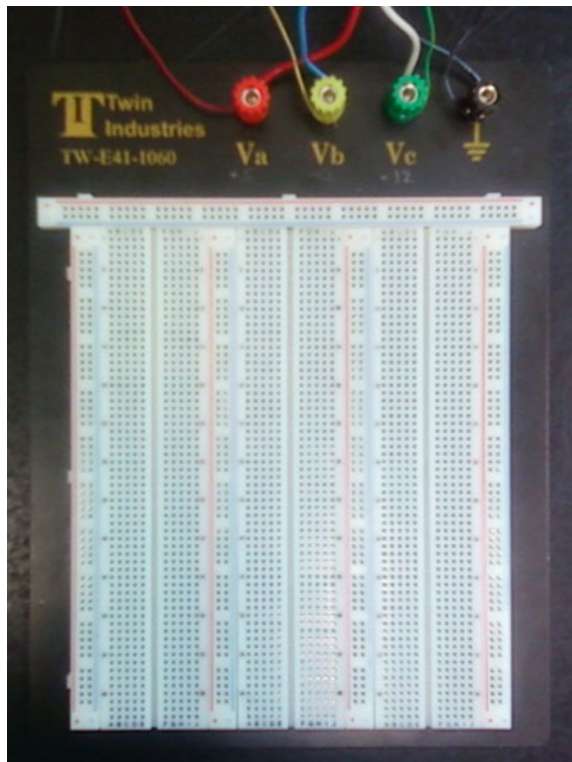


Fig. 2.11 Circuit diagram for Task 1

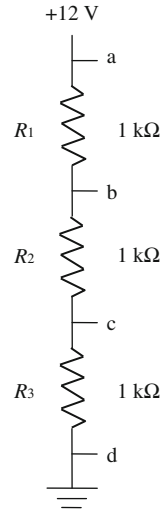
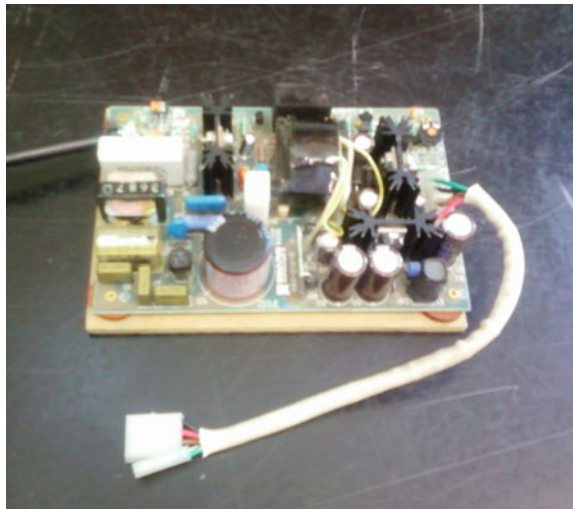
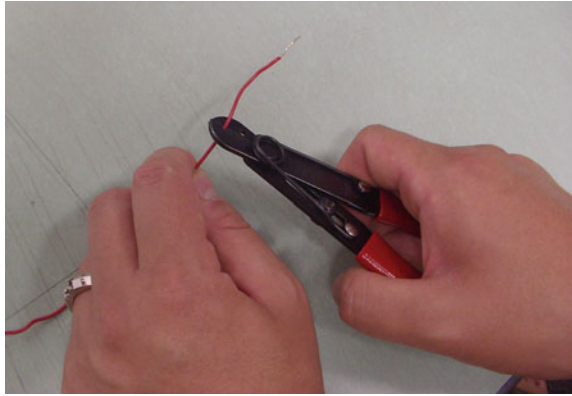


Fig. 2.12 Three-output low-capacity power supply



You need a power supply that generates +12 V, three 1 kΩ resistors, a couple of wires, and a breadboard. These items can generally be purchased from an online vendor; use any search engine, and look for the term “breadboard” or “resistor” and you will find appropriate vendors in your area.

First, hook up a power supply to a breadboard (Fig. 2.12). The previous Fig. 2.10 shows a typical breadboard with power connectors. At the top are *binding posts* which allow you to hook up power to the breadboard tie points. You may use batteries as a power source, but we recommend you use an AC-to-DC power supply that generates +5, +12 and −12 V (three outputs). Again, this kind of power supply

Fig. 2.13 Wire stripping

can be purchased from online vendors as mentioned above. Note that a high-capacity power supply is not necessary; choose one whose current capacity is less than 2 A (it usually costs less than \$10 in the U.S.) (Fig. 2.12).

Your power supply may come with a connector for plugging into the breadboard. If not, you can simply connect it using standard wire. Most breadboards accept wire gauge sizes from 19 to 29, but 22 is probably the most common. Strip off about a quarter inch or half a centimeter of the insulation at each end of the wire. Figure 2.13 shows how to strip off a wire using a wire cutter/stripper. Note that the same job can be done with a nipper, but we recommend using a wire cutter/stripper).

Figure 2.14 also demonstrates how the wire can be fixed to a binding post on a breadboard. Note that other connecting means are also available, such as alligator clips, mount clips, and banana plugs (Fig. 2.15).

Let us connect wires from the power supply to the breadboard. You will need to connect three different DC voltages (+5, +12 and -12 V) and a ground (GND; 0 V; a reference point) that is shared by the above three voltages. Do not connect the power supply to the AC power outlet yet.

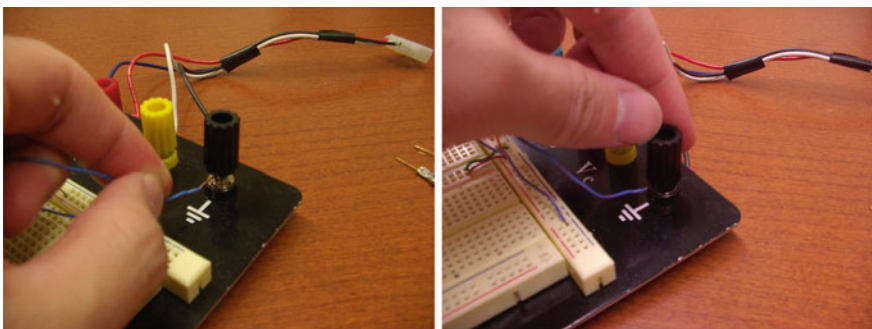
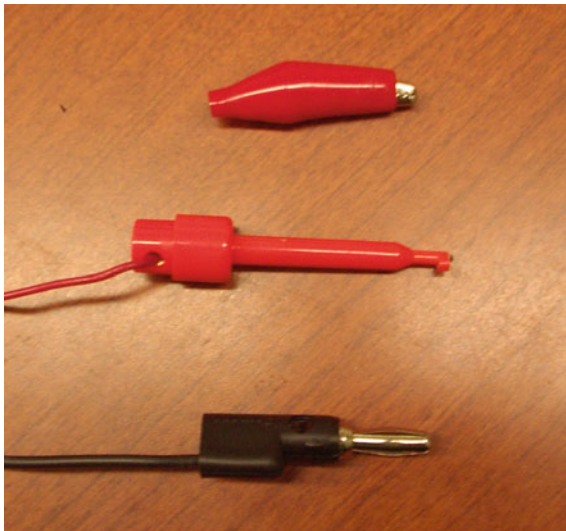
**Fig. 2.14** Wire mounting to a binding post (of a breadboard)

Fig. 2.15 *Top* Alligator clip. *Middle* Mount clip. *Bottom* Banana plug



Now, return back to the breadboard. Notice that the tie points come in groups of five, horizontally. These five points are electrically connected to each other, allowing you to connect one wire to four others. There are also *distribution strips* which are usually used for distributing power (+5, +12 or -12 V) and ground (0 V) around the breadboard, since the circuit will likely connect to these in several places. On the breadboard shown in Fig. 2.14, these distribution strips are conveniently marked with red or blue lines, indicating they are also electrically connected throughout the entire line. For example, the distribution strips on top are horizontally connected, while the others are vertically connected.

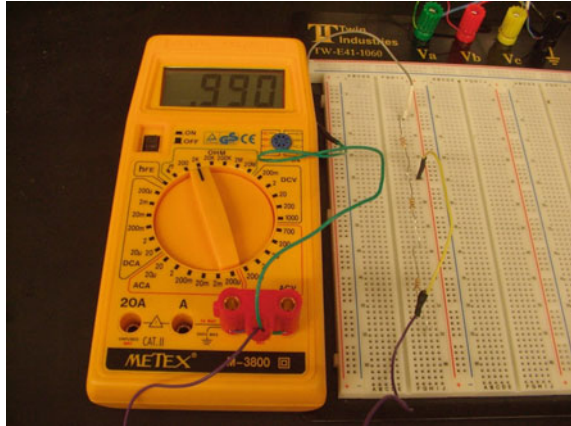
You should have three 1 k Ω resistors. Insert the first lead to one horizontal tie point and the second lead to the other tie point. Then insert one lead of your *digital multimeter (DMM)* to each of the tie points. As the resistor does not have any polarity, you can connect it any way you want.

The DMM can measure electric current, resistance, and voltage. Each measurement comes with a different measurement range. Set up the DMM to measure resistance, with a maximum range of 2 k Ω . It is okay to use the 20 or 200 k Ω range to measure 1 k Ω resistors, but we would not get sufficient significant digits. Record the exact resistance values of all three resistors (Fig. 2.16).

You should not measure the resistance values while the circuit is connected to a power supply. Even if the power supply is not connected to the AC outlet, the power supply itself contains many components, especially resistors, which will affect your resistance readings.

For the next question and subsequent experimental procedures, assume your measurements showed resistance values of 990 Ω for the first resistor (R_1), 1020 Ω for the second (R_2), and 980 Ω for the third (R_3).

Fig. 2.16 Resistance measurements with a DMM



Question 2.4

If the tolerance band is gold, are the above resistors within their acceptable tolerance?

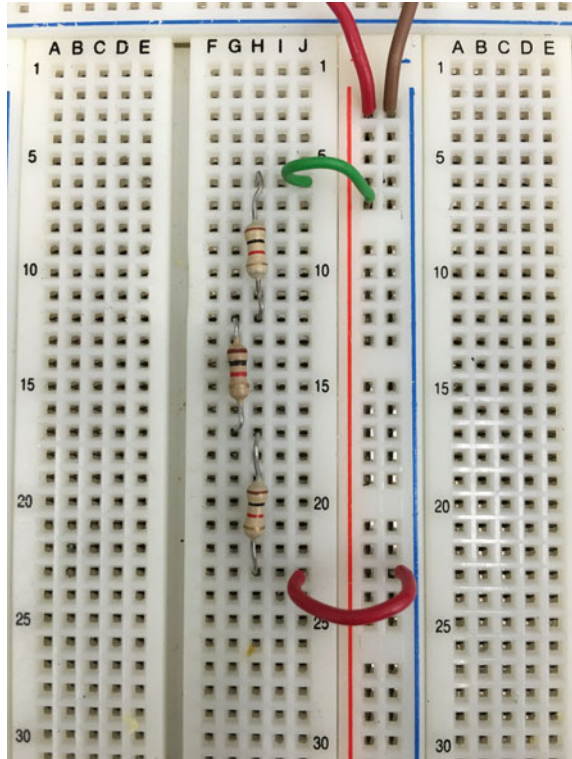
Power one pair of distribution strips with +12 V and GND by connecting wires from the binding posts onto the breadboard. Then connect three 1 k Ω resistors in series as shown below. Use additional wires to connect these series resistors to the DC power source (+12 V and GND).

The setup in Fig. 2.17 is not very ideal. Generally, the wires should be cut as short and as close to the board as possible to prevent possible signal interference and so forth. If you have the luxury of owning a collection of pre-cut and pre-stripped wires, you may choose to use those; however, the above practice is generally acceptable.

The theoretical voltages at four different points (*a*, *b*, *c*, and *d*) in Fig. 2.11 can be calculated. V_a should be measured by inserting the positive lead of your DMM to point *a* and the ground lead to point *d*. Make sure your DMM is set to measure DC voltages, not AC. You can also connect the ground lead to any location within the distribution strip that is connected to GND. Remember that electric voltage is always measured with reference to ground (GND). Set your DMM to voltage measurement with a 20 V range. V_d is 0 V and there is no need to measure it. The experimental measurement should show $V_a = 12.5$ V. (Note that the power supply is supposed to generate 12.0 V, but the actual output will vary depending on the output from your AC outlet). This is your input voltage, V_{in} .

Now calculate the voltage at point *b*. Given the three resistor values previously measured ($R_1 = 990 \Omega$, $R_2 = 1020 \Omega$ and $R_3 = 980 \Omega$), the top resistor is equal to R_1 (990 Ω) and the bottom to $R_2 + R_3$ (1020 $\Omega + 980 \Omega$). From the voltage divider relationship,

Fig. 2.17 Circuit photo of Task 1



$$V_b = V_{in} \frac{R_{bottom}}{R_{top} + R_{bottom}} = (12.5) \frac{1020 + 980}{990 + 1020 + 980} = 8.36 \text{ V} \quad (2.24)$$

We can repeat the same for point *c*. The top resistor is $R_1 + R_2$ ($990 \Omega + 1020 \Omega$) and the bottom is R_3 (980Ω).

$$V_c = V_{in} \frac{R_{bottom}}{R_{top} + R_{bottom}} = (12.5) \frac{980}{990 + 1020 + 980} = 4.10 \text{ V} \quad (2.25)$$

Experimental measurements of voltages at point *b* (with reference to GND) and point *c* (with reference to GND) should not differ very much from these calculations.

Alternative Task 1: Resistors in Series

Change the three resistors on the breadboard to $R_1 = 3 \text{ k}\Omega$, $R_2 = 8 \text{ k}\Omega$ and $R_3 = 1 \text{ k}\Omega$ (the actual resistance values will vary), and repeat Task 1.

- Identify color bands for all three resistors.
- Measure the actual resistor values using a DMM.
- Measure V_{in} ($= V_a$).

- Calculate V_b and V_c using the voltage divider relationship.
- Experimentally measure V_b and V_c and compare with the calculations.

Repeat the whole experiment with $R_1 = R_2 = R_3 = 100 \text{ k}\Omega$ (the actual resistances will vary).

2.10 Laboratory Task 2: Resistors in Parallel

Use the same materials and equipment used for Task 1. Let us construct a circuit that contains resistors in parallel (i.e., *current divider*) (Figs. 2.18 and 2.19).

Note that R_1 and R_2 are connected to the same row of tie points. If we are using identical resistors from Task 1 ($R_1 = 990 \text{ }\Omega$, $R_2 = 1020 \text{ }\Omega$ and $R_3 = 980 \text{ }\Omega$), the total resistance of the two resistors in parallel, i.e., between the points a and c , is:

$$R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{990 \cdot 1020}{990 + 1020} = 502 \text{ }\Omega. \quad (2.26)$$

Compare this value with the actual resistance reading of your DMM between points a and c . Remember that the power from the resistor circuit should be disconnected before making any resistance measurements, as the power supply circuit can function as a resistor even if it is not connected to the AC outlet.

Using this “total” resistance, we can calculate the voltage output at point c using the voltage divider relationship:

$$V_c = V_{\text{in}} \frac{R_{\text{bottom}}}{R_{\text{top}} + R_{\text{bottom}}} = (12.5) \frac{980}{502 + 980} = 8.27 \text{ V}. \quad (2.27)$$

Fig. 2.18 Circuit diagram for Task 2

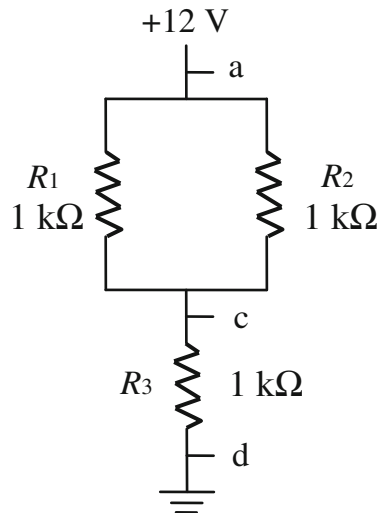
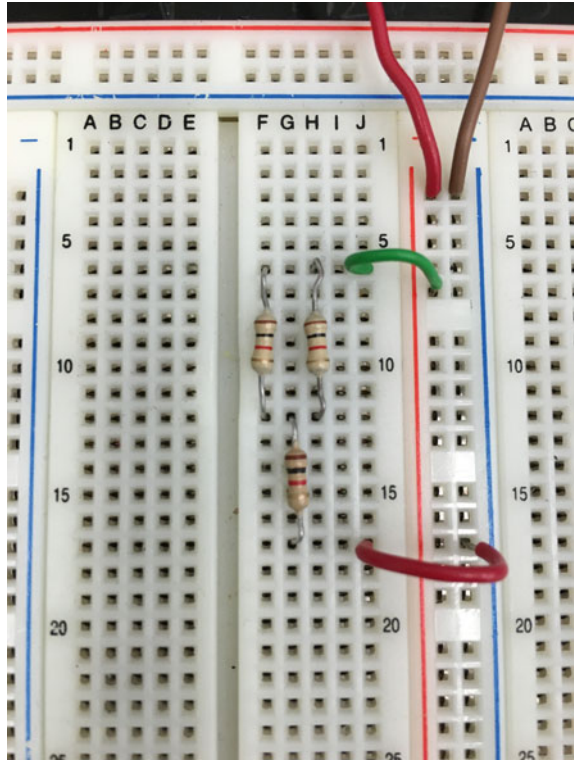


Fig. 2.19 Circuit photo of Task 2



Compare this value with the actual voltage measurement at point *c* (again, with reference to GND) using a DMM.

Alternative Task 2: Resistors in Parallel

Repeat Task 2 with $R_1 = 3 \text{ k}\Omega$, $R_2 = 8 \text{ k}\Omega$ and $R_3 = 1 \text{ k}\Omega$, and $R_1 = R_2 = R_3 = 100 \text{ k}\Omega$.

2.11 Laboratory Task 3: “Droop”

Repeat Tasks 1 and 2 with $R_1 = R_2 = R_3 = 10 \text{ M}\Omega$ (Fig. 2.20). The actual resistance measurements show $R_1 = 9.9 \text{ M}\Omega$, $R_2 = 10.1 \text{ M}\Omega$ and $R_3 = 10.2 \text{ M}\Omega$. $V_{\text{in}} = V_a = 12.5 \text{ V}$. For Task 1:

$$\begin{aligned}
 V_b &= V_{\text{in}} \frac{R_{\text{bottom}}}{R_{\text{top}} + R_{\text{bottom}}} = (12.5) \frac{10.1 + 10.2}{9.9 + 10.1 + 10.2} = 8.40 \text{ V and} \\
 V_c &= V_{\text{in}} \frac{R_{\text{bottom}}}{R_{\text{top}} + R_{\text{bottom}}} = (12.5) \frac{10.2}{9.9 + 10.1 + 10.2} = 4.22 \text{ V,}
 \end{aligned}
 \tag{2.28}$$

and for Task 2:

$$V_c = V_{in} \frac{R_{bottom}}{R_{top} + R_{bottom}} = (12.5) \frac{10.2}{\frac{9.9 \cdot 10.1}{9.9 + 10.1} + 10.2} = 8.39 \text{ V.} \quad (2.29)$$

However, the experimental measurements for Task 1 show:

$$V_b = 5.05 \text{ V}, \quad V_c = 2.52 \text{ V}$$

and for Task 2:

$$V_c = 6.28 \text{ V}$$

These measurements are significantly lower than our calculations. This phenomenon is known as *voltage droop*, or simply *droop*.

Even though a DMM has its own power supply (typically a 9 V battery), a small portion of electric current flowing through our circuit is bypassed through the DMM to make the actual measurement. A large resistor, typically around 10 MΩ, is installed in a DMM. The resistor sizes of our main circuit are substantially lower than 10 MΩ, so that almost all of the electric current flows through our main circuit, while only a very small portion of current flows through the DMM. However, if the main circuit consists mostly of large resistors (i.e., high loads), relatively large currents will flow through the DMM, resulting in a voltage loss within the main circuit.

Calculate the actual voltage at point *c* with resistors in series. Figure 2.20 below shows that a DMM is trying to measure the voltage at point *c*. The “top” part resistance is:

$$R_1 + R_2 = 9.9 + 10.1 = 20.0 \text{ M}\Omega. \quad (2.30)$$

Fig. 2.20 Circuit diagram for Task 3

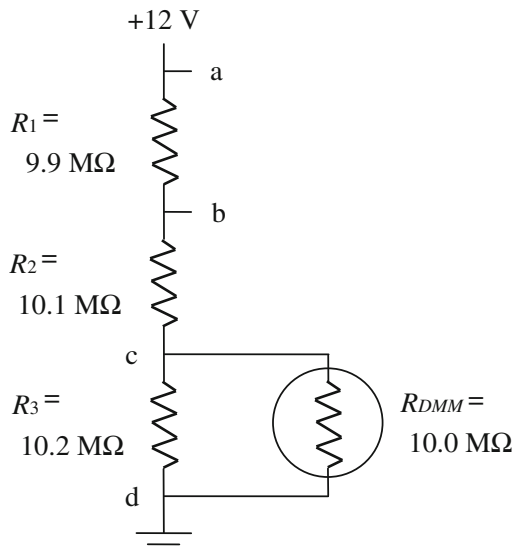
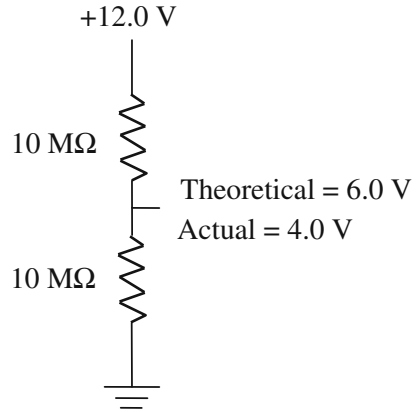


Fig. 2.21 Calculating DMM resistance



The “bottom” part can be treated as resistors in parallel,

$$\frac{R_3 \cdot R_{\text{DMM}}}{R_3 + R_{\text{DMM}}} = \frac{10.2 \cdot 10.0}{10.2 + 10.0} = 5.05 \text{ M}\Omega. \quad (2.31)$$

The voltage divider relationship yields:

$$V_c = V_{\text{in}} \frac{R_{\text{bottom}}}{R_{\text{top}} + R_{\text{bottom}}} = (12.5) \frac{5.05}{20.0 + 5.05} = 2.52 \text{ V}. \quad (2.32)$$

Obviously it is possible to back-calculate the resistor value inside of a DMM with experimental data for V_c .

Question 2.5

Repeat the above calculation (with a 10 MΩ DMM connected) for V_b of Task 1 with 10 MΩ resistors and V_c of Task 2 with 10 MΩ resistors.

Question 2.6

Back-calculate the value of the resistor inside of the DMM in Fig. 2.21, given the following experimental measurements.

2.12 Laboratory Task 4: Potentiometer (Pot)

A pot is basically a variable voltage divider. The circuit layout shown in Figs. 2.22 and 2.23 includes a 20 kΩ pot, where a small adjustment shaft can be found on its top or side, and three terminals (usually labeled 1, 2 and 3) can be found at the bottom. Typically terminal 1 is connected to GND and terminal 3 to the high potential. In reality, terminals 1 and 3 can be swapped as there is no polarity in pots. Within a pot, terminal 2 is connected to a slider whose position can be adjusted with an adjustment shaft on the top. Based on the actual location of a slider, the top and

Fig. 2.22 Circuit diagram for Task 4

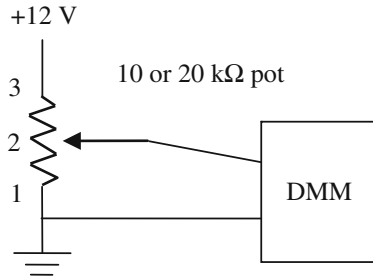
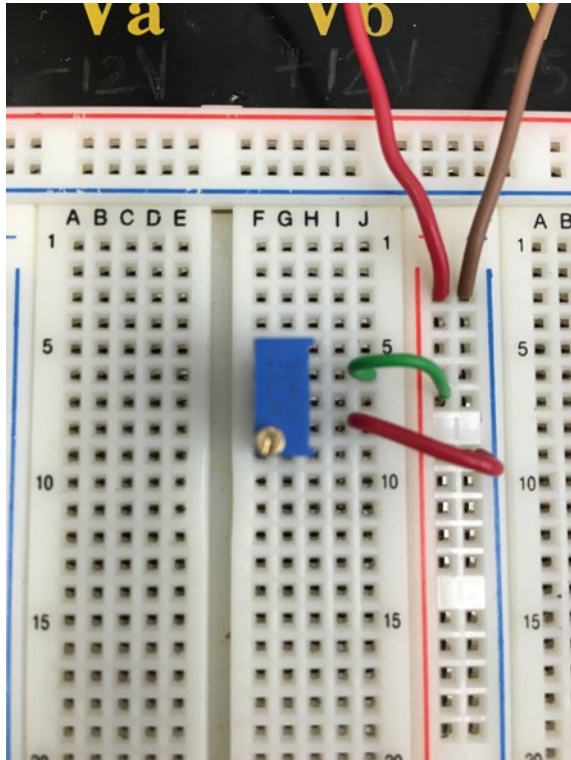


Fig. 2.23 Photo of a pot circuit to be used in Task 4



bottom resistances are varied such that the output voltage from terminal 2 can be varied as well; thus it is a variable voltage divider. If only terminals 1 and 3 are used, it will simply work as a 20 kΩ resistor. If only terminals 1 and 2 (or 2 and 3) are used, it will become a variable resistor.

As the output voltage of a voltage divider simply depends on the ratio of top and bottom resistances, the “size” of a pot (i.e., whether it is 1 or 20 kΩ pot) does not really affect the output voltage. However, it does affect the current applied to the given circuit. Pots are very useful in adjusting input voltages for a given circuit, and will be used throughout this textbook.

In a single-turn pot, a single 360° turn of a shaft would make the slider move from one end to the other. With +12 V input voltage, the output from terminal 2

would vary from 0 to 12 V with a single 360° turn. In a multi-turn pot, $10\text{--}20 \times 360^\circ$ turns may be required to make the slider move from one end to the other. We will use a multi-turn pot for more subtle control of output voltage.

In this task, you will need the following:

- A breadboard, wires, wire cutter/stripper, a power supply and a DMM.
- 10 or 20 k Ω pot.
- A small, flathead screw driver.

Here is the circuit diagram:

- As the pot itself does not have any polarity, switching pins 1 and 3 from top to bottom position does not make any difference. You should measure voltage output from pin 2 of your pot (with reference to ground).
- Read the output voltage from your pot. A voltage close to 12 V indicates the slider (terminal 2) is located very close to terminal 3. A voltage close to 0 V indicates the slider is located very close to terminal 1. Turn the adjustment shaft using an appropriate screw driver to make the output voltage 0 V. This is usually achieved by turning the shaft clockwise. Once you reach the end point, you will hear a clicking sound, indicating you have gone too far. Record the output voltage.
- Make a full (360°) counterclockwise turn of the adjustment shaft. This action should decrease the output voltage a little bit. Record the output voltage.
- Continue this until you reach the maximum voltage ($V_{in} \approx 12$ V).
- Record your data.
- Plot the output voltage against the number of turns. Perform linear regression and obtain the equation and R^2 value. You will notice that the output voltage is linearly proportional to the number of turns (Fig. 2.24).

# turns	0	1	2	3	4	5	6	7	8	9	10	11
V	0.09	0.71	1.26	1.9	2.49	3.11	3.65	4.21	4.71	5.29	5.82	6.33
# turns	12	13	14	15	16	17	18	19	20	21	22	23
V	6.88	7.41	8.01	8.56	9.08	9.58	10.1	10.5	11	11.4	11.8	12.3

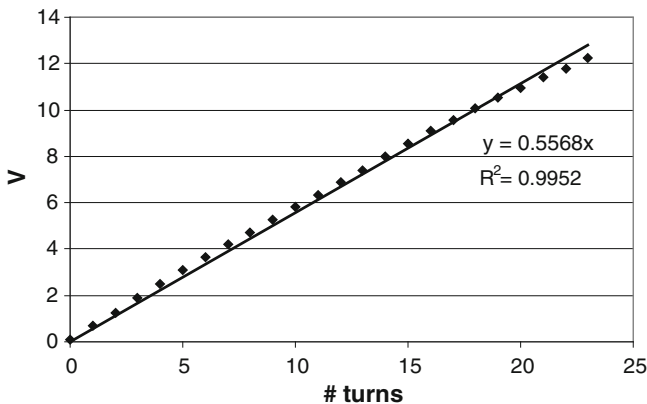


Fig. 2.24 Experimental data for Task 4

2.13 Further Study: Thévenin's Theorem

You may have found it a bit difficult to solve Question 2.6 earlier, back-calculating the inner resistance of a DMM. For this kind of complicated circuit calculation, you may find *Thévenin's theorem* useful. Thévenin's theorem states that a portion of an electric circuit can be simplified as a single voltage source (V_{Th} —Thévenin equivalent voltage or simply *Thévenin voltage*) and a single resistor (R_{Th} —Thévenin equivalent resistance or simply *Thévenin resistance*).

Your voltage output to the DMM comes from point *c*. You are trying to simplify the circuit that is circled with a dotted line in Fig. 2.25. Let's first assume that this is your only circuit; i.e. your DMM is disconnected. With the DMM disconnected, we know that the voltage output at point *c* should be 4.0 V from the voltage divider principle. This (4.0 V) is your V_{Th} .

Figure 2.26 shows we now have a new voltage source at point *c*, you want to relocate the voltage source to this hypothetical point. This relocation implies that the original voltage source of +12 V should be eliminated, while the high voltage and ground should be connected to each other. From this new voltage source, you can look back at the circuit and find there are two branches, one with two 10 M Ω resistors and the other with one 10 M Ω resistor. This is equivalent to parallel resistors with 20 and 10 M Ω resistors. The total resistance can be calculated as: $(20 \times 10)/(20 + 10) = 6.67$ M Ω . This is your R_{Th} .

Hence the components circled with a dotted line in Fig. 2.25 are now converted into an open circuit consisting of a single voltage source ($V_{Th} = 4.0$ V) and a single resistor ($R_{Th} = 6.67$ M Ω). If you connect your DMM (with unknown inner resistance x) to this open circuit, you can use the voltage divider principle to evaluate x . If your actual voltage reading at point *c* is 2.4 V,

Fig. 2.25 Circuit used for Question 2.6

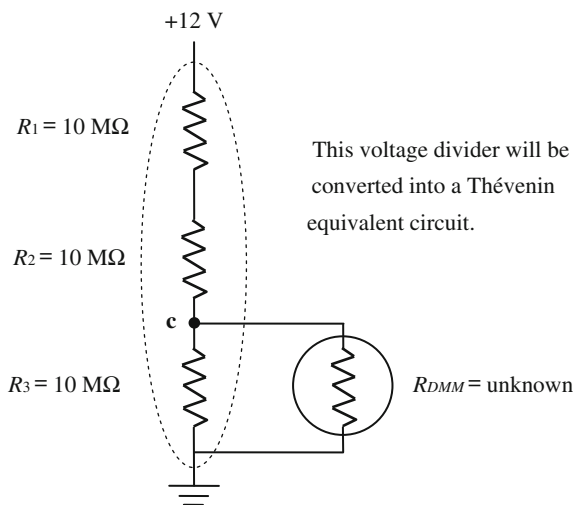


Fig. 2.26 Thévenin circuit for Question 2.6

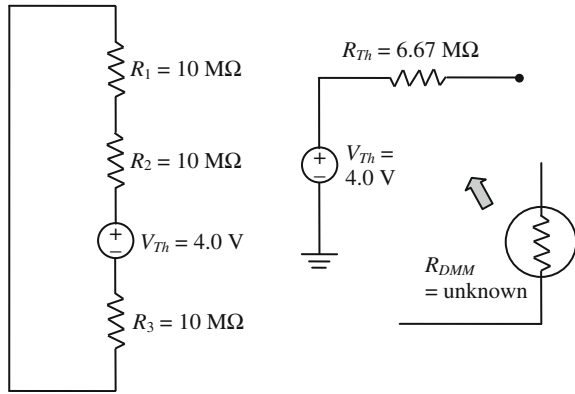
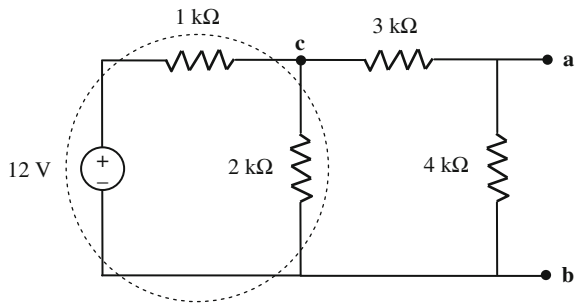


Fig. 2.27 Circuit used for Question 2.7



$$V_{out} = V_{in} \frac{R_{bottom}}{R_{top} + R_{bottom}}$$

$$2.4 = (4.0) \frac{x}{6.67 + x} \tag{2.29}$$

$$x = 10$$

You can also calculate the actual voltage reading at point *c* with the known inner resistance of the DMM (10 MΩ): $(4.0) \times [10/(6.67 + 10)] = 2.4$ V (Fig. 2.26).

Question 2.7

Determine V_{ab} in Fig. 2.27 using Thévenin’s theorem. Hint: Make the first Thévenin equivalent circuit (short dotted line) at point *c*. Then the circuit becomes a voltage divider problem with three resistors (R_{Th} , 3 and 4 kΩ) with the input voltage V_{Th} .

References and Further Readings

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