Chapter 12
Propagation of a Short Subterahertz Pulse in a Plasma Channel in Air Created by Intense UV Femtosecond Laser Pulse

A.V. Bogatskaya, A.M. Popov and E.A. Volkova

Abstract The evolution of the electron energy distribution function in the plasma channel created in air by the third harmonic of the Ti:Sa-laser pulse of femtosecond duration is studied. It is shown that strong nonequilibrium of such a plasma leads to the possibility to use the channel for guiding and amplification of few-cycle electromagnetic pulses in subterahertz frequency range at the time of relaxation of the energy spectrum in air determined by the vibrational excitation of the nitrogen molecules. The refractive index and the gain factor as a function of time, electron concentration and frequency of the amplifying radiation are obtained. The propagation of few-cycle radio-frequency pulses through the amplifying medium is analyzed.

12.1 Introduction

An important feature of the plasma structure appearing in the field of an ultrashort laser pulse is its strong nonequilibrium. Such nonequilibrium can be used for a number of applications, in particular, for generation of XUV attosecond pulses and discussed by Agostini and Di Mauro [1] and Krausz and Ivanov [12]. The energy spectrum of photoelectrons appearing in multiphoton ionization of the gas under the conditions where the pulse duration is comparable or smaller than the average time interval between the electron—atomic collisions consists of a number of peaks corresponding to the absorption of a certain number of photons. Such an electron

A.V. Bogatskaya · A.M. Popov (✉)
Department of Physics, Lomonosov Moscow State University, Moscow 119991, Russia
e-mail: alexander.m.popov@gmail.com

A.V. Bogatskaya · A.M. Popov · E.A. Volkova
D.V. Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow 119991, Russia

A.V. Bogatskaya · A.M. Popov
P.N. Lebedev Physical Institute, RAS, Moscow 119991, Russia

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energy distribution function (EEDF) is characterized by the energy intervals with the inverse population. It was demonstrated by Bunkin et al. [7], that such a situation can be used to amplify electromagnetic radiation in a plasma.

The possibility to use the plasma channel created by a high intensity ultrashort pulse of a KrF excimer laser ($\hbar\Omega = 5\,\mathrm{eV}$) in xenon for the amplification of radio-frequency pulses was analyzed by Bogatskaya and Popov [4]. In this paper time dependences of the gain factor with various frequencies ω of the amplyfied radio-frequency radiation in the xenon plasma channel were obtained. The possibilty to amplify the subterahertz radiation in different gases was studied by Bogatskaya et al. [5]. It was demonstrated that the xenon plasma has some advantages as the amplifying medium in comparison with other rare and molecular gases.

In this paper we discuss the possibility of using of the plasma channel created in the atmospheric air as a waveguide for the transportation and amplification of the radio-frequency radiation. The evolution of the electron energy spectrum in the relaxing plasma created by the femtosecond laser pulse is examined using the Boltzmann kinetic equation and the refactive index as well as the gain factor of electromagnetic radiation in the plasma channel are calculated as a function of time and electronic concentration in dependence of frequency in subterahertz band. It is found that for definite range of the laser frequencies there exists also a rather short time interval when such a relaxing air plasma can be also used as an amplifying and guiding medium for radio-frequency ultrashort pulses. The propagation of such pulses through the amplifying medium is studied in the frames of slow varying amplitude approximation.

It should be mentioned that mechanism of the amplification of electromagnetic radiation in the plasma channel discussed in this paper is close from physical point of view to the effect of the negative absolute conductivity in the low temperature gas-discharge plasma predicted by Rokhlenko [16] and Shizgal and McMahon [17], experimentally detected by Warman et al. [18], and discussed in detail in reviews of Aleksandrov and Napartovich [3] and Dyatko [10].

12.2 Kinetics of Photoelectrons in a Plasma Channel Produced in Air by the Ultrashort UV Laser Pulse

To analyze the properties and evolution of the plasma channel created by a high intensity laser pulse of femtosecond duration, it is significant to take into account that the channel appears only due to the multiphoton ionization of molecules. In this case, the avalanche ionization of the gas molecules can be neglected. Moreover, for pulses with the duration of $\tau_p \sim 100\,\mathrm{fs}$, elastic collisions of electrons with molecules of the medium during the pulse can also be neglected. Indeed, the characteristic time of collisions of electrons with nitrogen or oxygen molecules in air at atmospheric pressure and room temperature ($T \approx 0.03\,\text{eV}$) can be estimated as $T_c \approx 1/N\sigma v$, where $N \approx 2.5 \times 10^{19}\,\text{cm}^{-3}$ is the density of the particles, $\sigma \sim 10^{-15}\,\text{cm}^2$ is the elastic collision cross section, and $v \sim 10^8\,\text{cm/s}$ is the velocity of electrons appearing in the photoionization process. Under these conditions one derives $T_c \sim 4 \times 10^{-13}\,\text{s}$. This time exceeds the duration of the laser pulse. This means that the energy spectrum of photoelectrons by the end of the laser pulse is determined only by the photoionization of molecules of the gas and can be obtained from the solution of the problem of the ionization of a single atom or molecule in a strong laser field. The evolution of the spectrum caused by elastic, inelastic and electron-electron collisions, which is described by the Boltzmann kinetic equation, takes place in the postpulse regime. For this reason, under the conditions of interest, the problem of the ionization of the gas by laser radiation can be considered independently from the problem of the evolution of the spectrum of photoelectrons. The solution of the former problem is used as the initial condition for the latter one.

For the intensity range $I \leq 10^{13}\,\text{W/cm}^2$ the ionization probability of O_2 molecules is a cubic function of the radiation intensity I for the third harmonic of the Ti:Sa laser: $w_i \sim I^3$. For the N_2 molecules we have four-photon ionization in this intensity range: $w_i \sim I^4$. For the moderate fields with the laser intensity of the third harmonic of the Ti:Sa laser $\sim 10^{11}$–10^{12} W/cm^2 in accordance with the perturbation theory the probability of the three-photon ionization is significantly larger than the four-photon ionization probability. So plasma channel is formed mainly by the three-photon ionization of O_2 molecules. Also in such fields the AC Stark shift of both bound levels and the continuum boundary can be neglected. Hence, the position of the first peak in the spectrum of photoelectrons corresponds to the energy $\varepsilon_0 = 3\hbar\Omega - I_i$, where $I_i \approx 12.08\,\text{eV}$ is the ionization potential of the oxygen molecule, and Ω is the frequency of the laser radiation. The data collected by Delone and Krainov [9] and Couairon and Mysyrowicz [8] lead to the estimation of the degree of ionization in air as $\alpha = N_e/N \approx 10^{-7} \div 10^{-6}$ for the above mentioned intensity range and the laser pulse duration $\tau_p \sim 100$ fs. Here N_e is the electron density.

Analyzing the evolution of the energy spectrum, we assume that the plasma channel with a given degree of ionization and strongly nonequilibrium electron energy distribution function is formed at the initial (zero) instant of time. The electron energy distribution function (EEDF) is approximated by the Gaussian

$$n(\varepsilon, t = 0) = \frac{1}{\Delta\varepsilon\sqrt{\pi\varepsilon}} \exp\left(-\frac{(\varepsilon - \varepsilon_0)^2}{(\Delta\varepsilon)^2}\right). \tag{12.1}$$

The width of the peak is determined by the pulse duration and for $\tau_p \sim 50-100$ fs can be estimated as $\Delta\varepsilon \approx 0.2\,\text{eV}$. For the above mentioned intensity range above-threshold ionization peaks can be neglected.

This electron energy distribution function is normalized as

$$\int_0^\infty n(\varepsilon, t = 0)\sqrt{\varepsilon}\,\mathrm{d}\varepsilon = 1. \tag{12.2}$$

The quantity $n(\varepsilon, t)\sqrt{\varepsilon}$ is the probability density of the existence of the electron with the energy ε.

The temporal evolution of the initial spectrum (12.1) was analyzed using the kinetic Boltzmann equation for the EEDF in the two-term approximation. We also assumed that the radio-frequency field amplifying in the plasma is weak enough and does not contribute to the Boltzmann equation. Under above assumptions the kinetic equation was written in a form:

$$\frac{\partial n(\varepsilon, t)}{\partial t}\sqrt{\varepsilon} = Q_{ee}(n) + Q^*(n) + \sum_i \frac{2m}{M_i}\frac{\partial}{\partial \varepsilon}\left(v_{tr}^{(i)}(\varepsilon)\varepsilon^{3/2}\left(n(\varepsilon, t) + T\frac{\partial n(\varepsilon, t)}{\partial \varepsilon}\right)\right). \tag{12.3}$$

The details can be found in the review of Ginzburg and Gurevich [11], and monograph of Raizer [15].

Equation (12.3) has the form of the diffusion equation in the energy space. Here, T is the gas temperature (below, we take $T \approx 0.03\,\mathrm{eV}$), m is the mass of the electron, M_i (i = 1, 2) are the masses of the nitrogen and oxygen molecules respectively, and $v_{tr}^{(i)} = N_i \sigma_{tr}^{(i)}(\varepsilon)\sqrt{2\varepsilon/m}$ is the partial transport frequency, where $\sigma_{tr}^{(i)}(\varepsilon)$ is the transport scattering cross section for N_2 (i = 1) and O_2 (i = 2) molecules, $N_1 = 0.79 \times N$ and $N_2 = 0.21 \times N$ are the concentrations of N_2 and O_2 molecules in the air, $Q_{ee}(n)$ is the integral of electron-electron collisions, $Q^*(n)$ is the integral of inelastic collisions. These integrals are described in detail in the review of Ginzburg and Gurevich [11].

Equation (12.3) with the initial condition (12.1) was solved numerically using an explicit scheme in the energy range $\varepsilon = 0{-}5\,\mathrm{eV}$. The elastic and necessary inelastic cross sections for N_2 and O_2 molecules were taken from data obtained by Phelps [14] and Phelps and Pitchford [13]. The total transport cross section for the electrons in air is presented at Fig. 12.1. The most important for our consideration is the existence of the energy interval $\varepsilon = 1.5 \div 2.3\,\mathrm{eV}$ with the positive derivative $\mathrm{d}\sigma_{tr}/\mathrm{d}\varepsilon > 0$.

Among a lot of inelastic collisions of electrons with nitrogen and oxygen molecules the excitation of vibrational levels of the ground electronic state $N_2(X^1\Sigma^+)$ is of most importance. These cross sections are high enough in the energy range ~2–4 eV and contribute significantly to the temporal evolution of the EEDF discussed below.

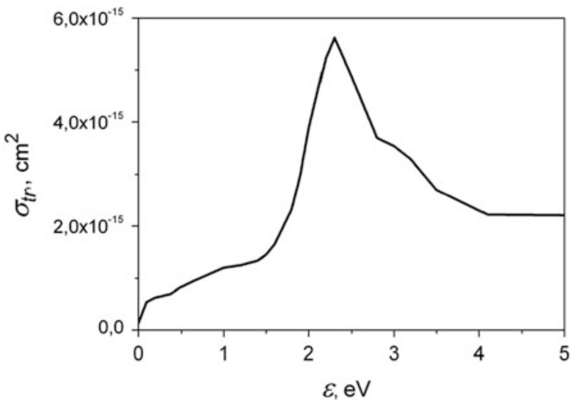


Fig. 12.1 Transport cross section for the electrons in air

The obtained from (12.3) EEDF makes it possible to calculate the temporal dependence of the electrodynamic properties of the plasma channel created by laser pulse. For example, the expression for the complex conductivity $\sigma(\omega) = \sigma'(\omega) + i\sigma''(\omega)$ at the frequency ω can be written in the form (for details see the reviews of Ginzburg and Gurevich [11] and Bunkin et al. [7]):

$$\sigma(\omega) = \frac{2}{3}\frac{e^2 N_e}{m}\int\limits_0^\infty \frac{\varepsilon^{3/2}(v_{tr}(\varepsilon) + i\omega)}{\omega^2 + v_{tr}^2(\varepsilon)}\left(-\frac{\partial n(\varepsilon, t)}{\partial \varepsilon}\right)\mathrm{d}\varepsilon. \tag{12.4}$$

The real part of this expression describes the dissipation of the energy of the electromagnetic wave in the plasma. For weakly ionized plasma the absorption coefficient at the frequency ω can be represented in the form:

$$\mu_\omega = \frac{4\pi\sigma'}{c} = \frac{2}{3}\frac{\omega_p^2}{c}\int\limits_0^\infty \frac{\varepsilon^{3/2} v_{tr}(\varepsilon)}{\omega^2 + v_{tr}^2(\varepsilon)}\left(-\frac{\partial n(\varepsilon, t)}{\partial \varepsilon}\right)\mathrm{d}\varepsilon, \tag{12.5}$$

where $\omega_p^2 = 4\pi e^2 N_e/m$ is the squared plasma frequency.

The imaginary part of the expression (12.4) determines the plasma refractive properties. The refractive index of the weakly ionized plasma can be written in a form:

$$n_\omega = 1 - \frac{2\pi\sigma''}{\omega} = 1 - \frac{1}{3}\omega_p^2\int\limits_0^\infty \frac{\varepsilon^{3/2}}{\omega^2 + v_{tr}^2(\varepsilon)}\left(-\frac{\partial n(\varepsilon, t)}{\partial \varepsilon}\right)\mathrm{d}\varepsilon. \tag{12.6}$$

In the case when $\nu_{tr}(\varepsilon) = const$ for any expression for the EEDF we derive well-known formulas (see, for example, the monograph of Raizer [15]) for refractive index and absorption coefficient in the weakly ionized plasma:

$$n_\omega = 1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2 + \nu_{tr}^2}, \quad \mu_\omega = \frac{\omega_p^2 \nu_{tr}}{c(\omega^2 + \nu_{tr}^2)} \tag{12.7}$$

In particular, we note that the refractive index $n_\omega < 1$, i.e. plasma is optically less dense medium than the neutral gas.

In more general case when transport frequency is the function of the electron energy the refractive index and the absorption coefficient depends on the specific form of the electron energy distribution function. Typically EEDF decreases with the energy, i.e. $\partial n/\partial\varepsilon < 0$ and, consequently, both integrals in (12.5) and (12.6) are positive and, hence, $n_\omega < 1$, i.e. plasma remains to be optically less dense medium, and $\mu_\omega > 0$. However, in the process of the photoionization of atoms or molecules by short pulses, energy ranges with the positive derivative, $\partial n/\partial\varepsilon > 0$, appear to exist for the initial instant of time. Such energy intervals make a negative contribution to the integrals in (12.5) and (12.6). In particular, it was demonstrated by Bunkin et al. [7] that the integral in (12.5) can become even negative in the low-frequency range $\omega < \nu_{tr}$ in gases with the pronounced Ramsauer minimum for the EEDF with energy interval with positive derivative, $\partial n/\partial\varepsilon > 0$. In the papers of Bunkin et al. [7] and Bogatskaya et al. [5] it was found that for the plasma with the EEDF similar to (12.1) the amplification of the electromagnetic radiation with $\omega < \nu_{tr}$ will be possible, if the condition

$$\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\varepsilon/\sigma_{tr}(\varepsilon) < 0 \tag{12.8}$$

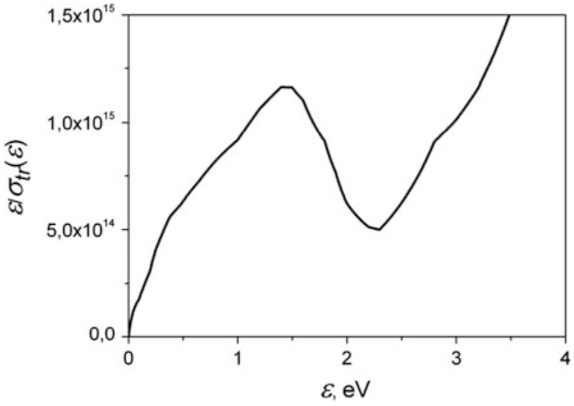
is fulfilled. Typically, the condition $\omega < \nu_{tr}$ is satisfied for the subterahertz frequency range $\omega \leq 10^{12}\,\mathrm{s}^{-1}$. As about change the sign of the integral (12.6) for the EEDF defined by (12.1) it will take place for the gases with

$$\frac{\mathrm{d}}{\mathrm{d}\varepsilon}\varepsilon^{1/4}\Big/\sigma_{tr}(\varepsilon) < 0 \tag{12.9}$$

This inequality is much more soft and is performed for many gases.

Bogatskaya and Popov [4] demonstrated that the presence of the Ramsauer minimum in the transport cross section of xenon and as a consequence the rapidly increasing range of the $\sigma_{tr}(\varepsilon)$ can be responsible for the appearance of the amplification of electromagnetic radiation in the plasma created by multiphoton ionization by short laser pulse. Both N_2 and O_2 molecules do not characterized by the Ramsauer minimum. Nevertheless, the transport cross section for electron scattering on nitrogen molecule is characterized by large positive value of the derivative $\mathrm{d}\sigma_{tr}/\mathrm{d}\varepsilon$ in the energy range of $\sim$1.5–2.3 eV. As a result, the condition (12.8) is

Fig. 12.2 The value $\varepsilon/\sigma_{tr}(\varepsilon)$ for electrons in air

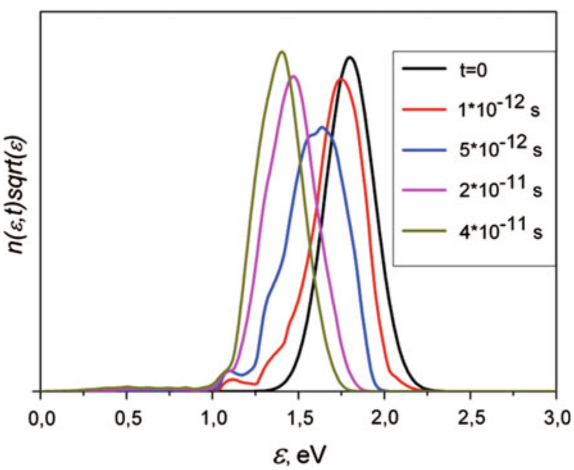


satisfied in this range (see Fig. 12.2). It means that it is also possible to obtain the negative values of the absorption coefficient.

We would like to stress, that for the case under study integral in (12.6) can also become negative. It means, that the refractive index for such strongly nonequlibrium plasma will be greater than unity, $n_\omega > 1$ and plasma will become optically more dense medium than the nonionized gas. Thus, such a plasma channel can be considered as waveguide for both transportation and amplification of microwave radiation.

Results of the numerical simulations for the EEDF evolution in time in air are presented at Figs. 12.3 and 12.4 for two different energy positions of the initial photoelectron peak. As can be seen, for the initial energy of photoelectrons $\varepsilon_0 = 1.8\,\text{eV}$ (this energy value is very close to the ionization of oxygen molecules by the third harmonic of the Ti:Sa laser) the electron energy distribution function is characterized by pronounced maximum, which is gradually shifted toward lower

Fig. 12.3 The EEDF in air at various times after the creation of the plasma channel. Initial peak of photoelectrons is characterized by $\varepsilon_0 = 1.8\,\text{eV}$ and electron concentration $N_e = 10^{13}\ \text{cm}^{-3}$



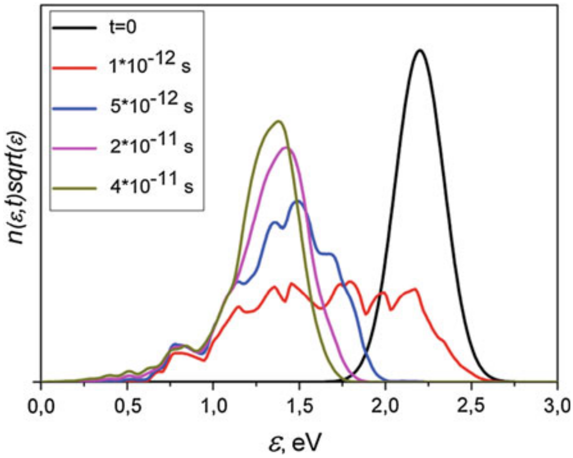


Fig. 12.4 The EEDF in air at various times after the creation of the plasma channel. Initial peak of photoelectrons is characterized by $\varepsilon_0 = 2.2$ eV and electron concentration $N_e = 10^{13}$ cm^{-3}

energies. While the average electron energy is more then ~ 1.5 eV (see the dependence at Fig. 12.2), it is naturally to expect the positive value of the gain factor. It should be emphasized that for larger energy of the initial photoelectrons ($\varepsilon_0 = 2.2\,\text{eV}$) the temporal evolution of the EEDF is quite different from that was discussed above (see Fig. 12.4). Due to significant value of the cross sections for the vibrational excitation of N_2 molecules by electrons with energies above ~ 2.0 eV the characteristic time of relaxation of the EEDF for $\varepsilon_0 = 2.2\,\text{eV}$ decreases dramatically and photoelectrons are found to be distributed over the energy range of $1.0-2.2\,\text{eV}$ even for the $t = 1$ ps. Later the Gaussian-type EEDF is formed again, but the average energy of photoelectrons for these instants of time is less 1.5 eV, and the positive value of the gain factor can not be achieved.

The electron energy distribution functions obtained in the numerical simulations were used to calculate the gain factor of electromagnetic radiation $(k_\omega = -\mu_\omega)$ in the air plasma for different values of the initial peak position and the frequency of the amplified radiation $\omega = 5 \times 10^{11}\,\text{s}^{-1}$. These data are presented at Fig. 12.5.

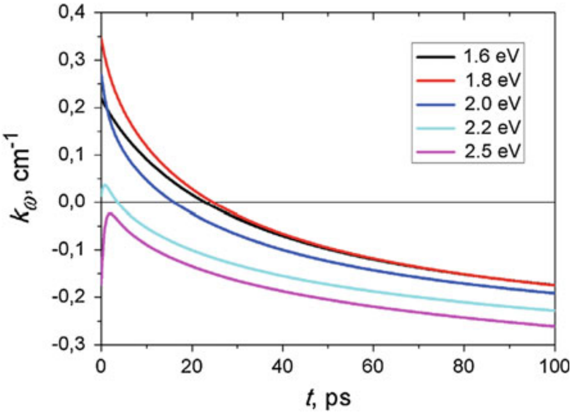


Fig. 12.5 Time dependence of the gain factor of the electromagnetic radiation with frequency 0.5 THz in the atmospheric air plasma channel for different energies of the photoionization peak

The data presented clearly demonstrate that the amplification of the radiation is possible if the energy of photoelectrons is less than ~ 2.25 eV. On the other hand, the energy of the initial photoelectron peak should not be less than 1.5 eV. The maximum value of the gain factor can be obtained for the initial photoelectron peak position $\varepsilon_0 = 1.8\,\text{eV}$.

Such energy of photoelectrons appears to exist for the three-photon ionization by the laser radiation with $\hbar\Omega \approx 4.63\,\text{eV}$ which is very close to the third harmonics of the Ti:Sa laser. Even for such value of ε_0 the gain factor is found to be positive during approximately 25 ps. It means that the plasma channel in air can be used for amplification of only extremely short few-cycled radio-frequency pulses. For example, for $\omega = 5 \times 10^{11}\,\text{s}^{-1}$ it is possible to amplify the pulses of two or three cycle duration. For higher frequencies of amplified radiation the gain factor drops dramatically as the condition $\omega < \nu_{tr}$ is not satisfied already.

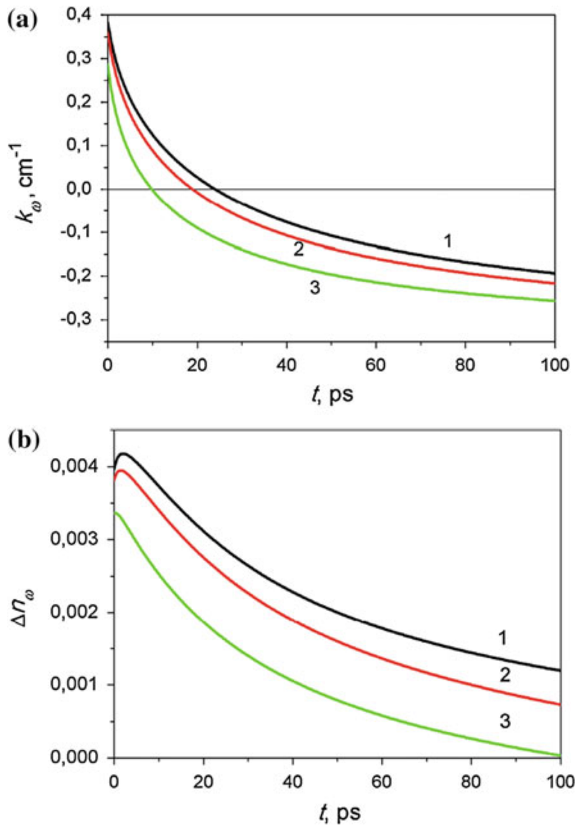


Fig. 12.6 Time dependencies of the gain factor (**a**) and refractive index (**b**) for the RF radiation with frequencies (THz) 0.2 (1), 0.5 (2), 1 (3) in the plasma channel for the initial photoelectron peak position $\varepsilon_0 = 1.8$ eV. Electron concentration $N_e = 10^{13}$ cm^{-3}

The results of simulation for gain factors and refractive indexes $\Delta n_\omega = n_\omega - 1$ for initial peak position at 1.8 eV and for different frequencies of the radio-frequency (RF) field are presented at Fig. 12.6. It can be seen that increasing of the RF frequency results in shortening of the time interval can be used for guiding and amplification of the microwave radiation. On the other hand for microwave field frequencies up to 10^{12} W/cm^2 the time interval for positive value of $\Delta n_\omega = n_\omega - 1$ is several times larger than the interval for positive gain factor.

12.3 Propagation and Amplification of the Radio-Frequency Pulses in the Plasma Waveguide

As it is known, propagation of the electromagnetic radiation in the medium is described by the wave equation:

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \vec{j}}{\partial t}. \tag{12.10}$$

Here $\vec{E}$ is the electric field strength, $\vec{j}$ is the density of the electric current in the plasma.

To analyze the process of microwave pulse propagation qualitatively we use the optical parabolic approximation for the solution of (12.10) (for details see the monograph of Akhmanov and Nikitin [2]). According to this approximation for the pulse propagation along z-direction E should be represented as

$$\vec{E}(\vec{r}, t) = \vec{E}_0(\rho, z, t)\cdot \exp(i(kz - \omega t)) \tag{12.11}$$

Here E_0 is the envelope of the radio-frequency pulse, and $k = \omega/c$ is the wave number.

If we neglect the temporal dispersion, the expression for $\vec{j}$ can be written in a form $\vec{j} = \sigma\vec{E}$. Here σ is the conductivity determined by expression (12.4). In this paper we assume that the radio-frequency pulse is linearly polarized and its intensity is weak enough and do not contribute to the temporal evolution of the EEDF in the plasma channel. As the electronic density in the plasma channel is low enough, the refractive index at the frequency $\omega = 5 \times 10^{11}$ s^{-1} is close to unity and it is possible to assume that the radio-frequency pulse propagates in the channel with the speed of light. After some approximations one can obtain the folowing equation for the E_0:

$$ik\left(\frac{\partial E_0}{\partial z} + \frac{1}{c}\frac{\partial E_0}{\partial t}\right) = -\frac{1}{2}\nabla_\perp^2 E_0 - \Delta n_\omega(t - z/c)k^2 E_0 + \frac{i}{2}k_\omega(t - z/c)kE_0 \tag{12.12}$$

The first term in the right part in (12.12) stands for the diffraction divergence of the electromagnetic field, the second one describes the plasma focusing (defocusing) features and the third term represents the absorption (amplification) process. Actually, the amplification duration τ corresponds to the amplification distance of about $c \cdot \tau \sim 1$ cm. So the laser pulse creates the air plasma channel characterized by amplifying «trail» (see Fig. 12.7). If we launch the laser pulse and the few-cycled RF pulse just one after another simultaneously, the last one will continually locate in the amplifying zone of the laser pulse.

It can be seen from (12.12) that in the case $n_\omega > 1$ the plasma channel will partly suppress the diffration divergence of the RF radiation. If the condition

$$(n_\omega - 1)k^2R^2 > 1 \tag{12.13}$$

(here R is the plasma channel radius) is satisfied the channel looks like the waveguide and can transport the radiation without divergence. For $\omega = 5 \times 10^{11}\,\mathrm{s}^{-1}$ and $k = \omega/c \approx 16.7\,\mathrm{cm}^{-1}$ and $\Delta n_\omega \sim 0.0035$ (see Fig. 12.6) the guiding regime of propagation will be realized for $R > 1$ cm.

For the guiding regime of the RF pulse propagation corresponding to the compensation of the divergence term in (12.12) by focusing term the (12.12) can be rewritten in a form

$$\frac{\partial E_0(z,\tau)}{\partial z} = \frac{1}{2}k_\omega(\tau)E_0(z,\tau), \tag{12.14}$$

where the new retardation time is introduced: $\tau = t - z/c$. From (12.14) one obtains:

$$E_0(z,t) = \Phi(t - z/c)\exp\left(\frac{1}{2}k_\omega(t - z/c)z\right). \tag{12.15}$$

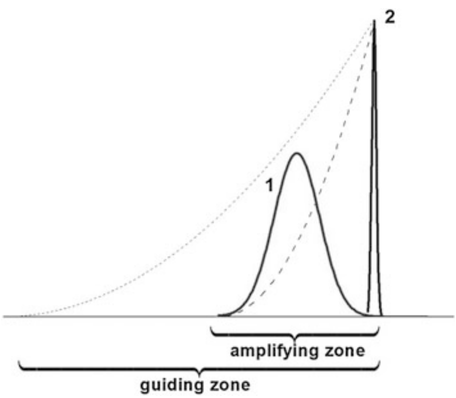


Fig. 12.7 Spatial structure of radio (*1*) and laser (*2*) pulses for a given instant of time. *Dash curves* are the spatial profiles of the gain factor and refractive index

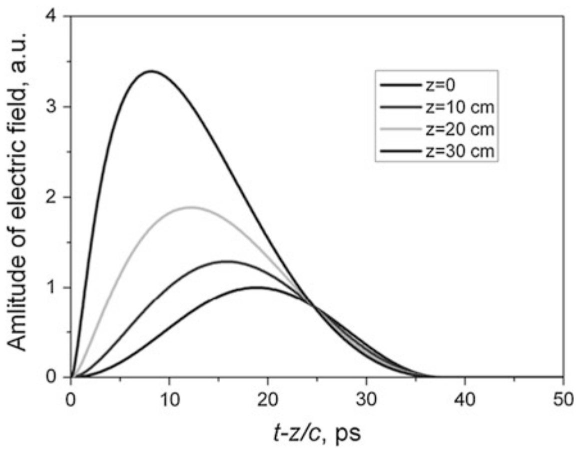


Fig. 12.8 Time dependence of the electric field strength in the amplifying pulse for different propagation lengths

Here Φ is the initial envelope of the RF pulse. We assume that it has the Gaussian form and temporal duration of three optical cycles. Figure 12.8 shows that the significant increase of the radio-frequency pulse amplitude can be obtained during its propagation despite the short time of amplification.

To conclude the discussion of radio frequency pulse guiding and amplification in a plasma channel we would like to stress that (12.12) and (12.14) are obtained in the optical parabolic approximation. It was demonstrated by Brabec and Krausz [6] that this approximation can be applied for the propagation of the pulse of few duration cycles only qualitatively.

12.4 Conclusions

In this paper it has been shown that a plasma channel created in the atmospheric air by the third harmonic of the Ti:Sa laser can be used for amplification and guiding of few-cycle electromagnetic pulses in subterahertz frequency range. Despite the short time duration of the positive gain factor there is an opportunity to reach significant amplification by the simultaneous launching of the laser and few-cycle RF pulses with approximately the same propagation velocity.

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