Non-aggregative Assessment of Subjective Well-Being

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Abstract

In this paper, we introduce a new methodology for socio-economic evaluation with ordinal data, which allows to compute synthetic indicators without variable aggregation, overcoming some of the major problems when classical evaluation procedures are employed in an ordinal setting. In the paper, we describe the methodology step by step, discussing its conceptual and analytical structure. For exemplification purposes, we apply the methodology to real data pertaining to subjective well-being in Italy, for year 2010.

1 Introduction

The use of ordinal data is spreading in socio-economic analysis, as issues like evaluating multidimensional poverty, well-being and quality-of-life are gaining importance in applied research and policy-making. Many social surveys ask respondents for self-assessments or subjective judgments, often expressed through binary or ordinal scales. Nowadays, many datasets comprising (also) ordinal variables are available to scholars; main examples at European and Italian level are the EU-SILC survey and the multi-topic survey entitled "Multipurpose Survey about Families Aspects of Daily Life", held by Istat (Italian National Statistical Bureau). Despite the abundance of ordinal data, statistical methodologies capable to effectively

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exploit them in studies pertaining to socio-economic evaluation are missing yet. Often, ordinal scores are treated as or transformed into cardinal figures and standard multivariate procedures are applied. Alternatively, ordinal data are simplified into binary variables and counting procedures are employed [1]. In both cases, the informative potential of ordinal data is not adequately exploited. As a matter of fact, the development of evaluation procedures in multidimensional ordinal settings is still an open, and largely unexplored, research field. Recently, a new methodology has been proposed by the Authors and other colleagues with the aim of overcoming counting and composite indicators approaches [4–6]. Its novelty lies in the use of partial order theory as a tool to compute synthetic indicators without aggregating ordinal variables. In the following, we give a step-by-step description of the methodology and, for exemplification purposes, apply it to data pertaining to subjective well-being in Italy. The paper is organized as follows; Sect. 2 introduces the data and motivates the interest for subjective well-being; Sect. 3 outlines the methodology; Sect. 4 presents the results of the analysis; Sect. 5 concludes.

2 Subjective Well-Being Data

The measurement of well-being is one of the most vivid topics in socio-economic statistics, particularly after the Stiglitz Commission stated a growing role of well-being measures, besides GDP, to assess the wealth of countries. In Italy, an ambitious project devoted to well-being assessment is being led by CNEL (National Committee for Economy and Work) and Istat. Twelve well-being dimensions have been identified; among them, our focus is on subjective well-being. Data used in the paper come from "Multipurpose Survey about Families Aspects of Daily Life" for year 2010.[1] The sample is composed of 48,336 statistical units. For sake of simplicity, records with missing values have been deleted reducing the sample to 40,949 units (see Sect. 4 for a remark on the missing data problem). We have selected four variables, pertaining to the satisfaction degree relative to:

1. personal economic status (variable v_1);
2. personal health status (variable v_2);
3. relationships with relatives (variable v_3);
4. relationships with friends (variable v_4).

All of the variables are recorded on a four-degree scale[2] (1 = "not at all"; 2 = "not much"; 3 = "enough"; 4 = "very"). In addition to well-being scores,

[1] Data are available within a protocol agreement signed by Istat and the University of Florence.

[2] In the original dataset, variables are scored as: 4 = "not at all"; 3 = "not much"; 2 = "enough"; 1 = "very". Codes have been reversed in such a way that increasing scores correspond to increasing satisfaction.

information about gender and the region of residence of each statistical unit in the sample are available.

3 Evaluating Subjective Well-Being

As usual in studies pertaining to well-being, the primary aim is two-fold: (1) identifying people who are not satisfied of their own well-being status and (2) measuring their dissatisfaction degree. The existence of incomparabilities among well-being self-assessments makes these goals more subtle than in the unidimensional case, where individual achievements can be linearly ordered. Multidimensional self-assessments can be ordered only partially and this introduces the role of partial order theory in the evaluation procedure.

The Partial Order of Well-Being Self-assessments By self-assessments, any statistical unit in the population is assigned a four-component vector $\boldsymbol{p}$, in the following called a *profile*, comprising his/her scores on variables v_1, v_2, v_3 and v_4. In total, there are $4^4 = 256$ different profiles $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_{256}$, together with the (absolute) frequencies $n_1, \ldots, n_{256}$ of statistical units sharing them. Profiles can be partially ordered according to the following natural definition:

Definition 3.1 Profile $\boldsymbol{p}_h$ is more satisfied than, or equally satisfied as, profile $\boldsymbol{p}_k$ (written $\boldsymbol{p}_k \trianglelefteq \boldsymbol{p}_h$) if and only if $p_{ki} \leq p_{hi}$ for each $i = 1, \ldots, 4$, where p_{hi} and p_{ki} are the i-th components of $\boldsymbol{p}_h$ and $\boldsymbol{p}_k$, respectively.

The set P of profiles endowed with the partial order $\trianglelefteq$ gives rise to the *profile poset* $(P, \trianglelefteq)$, which, for notational convenience, will be similarly indicated as P. It has a top element (4444), denoted by $\top$, and a bottom element (1111), denoted by $\perp$, which represent the best and the worse element, respectively.

Setting the Threshold Given P, the open problem is how to extract information pertaining to well-being, out of it. The identification of unsatisfied profiles (just like the identification of poor individuals in customary poverty studies) is a normative act, which cannot be performed only through data analysis. As a purely mathematical structure, P conveys no absolute socio-economic information and cannot suffice to identify unsatisfied profiles. Identification is therefore performed introducing exogenously a *threshold* (here denoted by $\boldsymbol{\tau}$), which in principle is up to experts and policy-makers to select. In the literature about social evaluation, multidimensional thresholds are usually identified based on the selection of cut-offs for each evaluation dimension. In a partial order framework, where emphasis is put on profiles rather than variables, it is more natural to identify the threshold directly in terms of profiles on "the edge of dissatisfaction". This way, one can take into account interactions among achievements on different well-being factors, which are crucial in a multidimensional setting. We must also notice that due to multidimensionality, more than one profile may be on the dissatisfaction edge and thus the threshold $\boldsymbol{\tau}$

may be (and usually is) composed of several elements. As proved in [4], under very general conditions, $\boldsymbol{\tau}$ may be always chosen as an *antichain* of P, that is, as a set of mutually incomparable elements of the profile poset. It is clear that, in real studies, the choice of the threshold is a critical task, affecting all the subsequent results. Therefore preliminary data insights, experts' judgments and any other source of information should be involved in selecting it.

Identification of Unsatisfied Profiles Given the threshold, the next step is to define an *identification function*, denoted by idn(·), that quantifies in [0, 1] to what extent a profile of P may be classified as unsatisfied. Notice that idn(·) does not measure the intensity of dissatisfaction (which will be later assessed in a different way), but the degree of membership to the set of unsatisfied profiles. The methodology is thus fuzzy in spirit, to reflect the classification ambiguities due to multidimensionality and partial ordering. In view of its formal definition, it is natural to impose the following four conditions on idn(·):

1. If, in satisfaction terms, profile $\boldsymbol{p}$ is better than profile $\boldsymbol{q}$, then its degree of membership to the set of unsatisfied profiles must be lower than the degree of $\boldsymbol{q}$, in formulas:

$$\boldsymbol{q} \trianglelefteq \boldsymbol{p} \Rightarrow \mathrm{idn}(\boldsymbol{p}) \leq \mathrm{idn}(\boldsymbol{q}).$$

2. Profiles belonging to the threshold are, by definition, unsatisfied profiles; therefore the identification function must assume value 1 on them:

$$\boldsymbol{p} \in \tau \Rightarrow \mathrm{idn}(\boldsymbol{p}) = 1.$$

   From conditions (1) and (2), it follows that a profile is unambiguously classified as unsatisfied if it belongs to the threshold or if it is worse than an element of the threshold:

$$\mathrm{idn}(\boldsymbol{p}) = 1 \Leftrightarrow \boldsymbol{p} \trianglelefteq \boldsymbol{q}, \;\; \boldsymbol{q} \in \tau.$$

   In poset theoretical terms, the subset of profiles satisfying the above condition is called the *downset* of τ and is denoted by $\tau\downarrow$.
3. A profile $\boldsymbol{p}$ is unambiguously identified as "not unsatisfied" if and only if it is better than *any* profile belonging to the threshold:

$$\mathrm{idn}(\boldsymbol{p}) = 0 \Leftrightarrow \boldsymbol{q} \trianglelefteq \boldsymbol{p}, \;\; \forall \boldsymbol{q} \in \tau.$$

   This condition aims to exclude that a profile which is incomparable with even a single element of the threshold may be scored 0 by the identification function.
4. If P is a linear order, then idn(·) must assume only values 0 or 1. In other words, if no incomparability exists, the identification function must classify profiles as either unsatisfied or not.

To determine the functional form of the identification function, we start by defining $\mathrm{idn}(\cdot)$ on the linear extensions of the profile poset. A *linear extension* ℓ of P is a linear order defined on the set of profiles and obtained turning incomparabilities of P into comparabilities. In a linear extension, all the elements are comparable, particularly the elements of the threshold τ selected in P. Therefore, in any linear extension ℓ, we may find an element $\boldsymbol{\tau}_\ell$ of the threshold that is ranked above any other element of τ. According to condition (4), it is natural to define the identification function[3] on ℓ putting $\mathrm{idn}_\ell(\boldsymbol{p}) = 1$ if $\boldsymbol{p} \trianglelefteq_\ell \boldsymbol{\tau}_\ell$[4] and $\mathrm{idn}_\ell(\boldsymbol{p}) = 0$ otherwise. In other words, identification in linear extensions reduces to the unidimensional problem of classifying profiles as above the threshold or not. We now extend the definition of the identification function from the set of linear extensions to the profile poset. The starting point is a simple but fundamental results of partial order theory that we state without proof [7]:

Theorem 3.1 *Any finite poset P is the intersection of its linear extensions:*

$$P = \bigcap_{\ell \in \Omega(P)} \ell$$

where $\Omega(P)$ is the set of linear extensions of P.

The close connection between P and $\Omega(P)$ suggests to express $\mathrm{idn}(\cdot)$ as a function of the idn_ℓs:

$$\mathrm{idn}(\cdot) = F(\{\mathrm{idn}_\ell(\cdot),\ \ell \in \Omega(P)\}).$$

To specify the functional form of $F(\cdot, \ldots, \cdot)$, we require it (1) to be symmetric (the way linear extensions are listed by is unimportant) and to satisfy the properties of (2) associativity, (3) monotonicity, (4) homogeneity and (5) invariance under translations. Symmetry and associativity are justified since the intersection operator is symmetric and associative; monotonicity assures that $\mathrm{idn}(\boldsymbol{p})$ increases as the number of linear extensions where $\mathrm{idn}_\ell(\boldsymbol{p}) = 1$ increases; homogeneity and invariance under translations assure that $\mathrm{idn}(\cdot)$ changes consistently if the $\mathrm{idn}_\ell(\cdot)$s are rescaled or shifted. By the theorem of Kolmogorov–Nagumo–de Finetti, it then follows that $F(\cdot, \ldots, \cdot)$ has the form of an arithmetic mean,[5] so that:

$$\mathrm{idn}(\boldsymbol{p}) = \frac{1}{|\Omega(P)|} \sum_{\ell \in \Omega(P)} \mathrm{idn}_\ell(\boldsymbol{p}).$$

[3] We denote this identification function by idn_ℓ to remind that it depends upon the linear extension considered.

[4] We denote with $\trianglelefteq_\ell$ the order relation in ℓ.

[5] More precisely, of a weighted arithmetic mean, but in our case there is no reason to assign different weights to different linear extensions.

Since idn_ℓ is either 0 or 1, $\mathrm{idn}(\boldsymbol{p})$ may be alternatively seen as the fraction of linear extensions where $\boldsymbol{p}$ is classified as unsatisfied:

$$\mathrm{idn}(\boldsymbol{p}) = \frac{|\{\ell \in \Omega(P) :\ \mathrm{idn}_\ell(\boldsymbol{p}) = 1\}|}{|\Omega(P)|}.$$

In a sense, the evaluation procedure implements a counting approach, but on linear extensions of P and not directly on well-being variables. This way, it exploits the structure of the underlying partial order, to quantify the degree of membership of a profile to the set of unsatisfied profiles, with *no* variable aggregation. By construction, all of the elements in $\tau \downarrow$ are classified as unsatisfied in any linear extension of P and therefore are scored to 1 by $\mathrm{idn}(\cdot)$, as required by condition (2) above. Similarly, profiles above any element of τ are scored to 0, consistently with condition (3). All of the other profiles in P are classified as unsatisfied in some linear extensions and as not unsatisfied in others and thus are scored in $]0, 1[$ by $\mathrm{idn}(\cdot)$. Once each profile $\boldsymbol{p}$ (and thus any statistical unit sharing it) has been scored by $\mathrm{idn}(\cdot)$, synthetic well-being indicators may be obtained. In particular, we focus on the (fuzzy extension of the) *Head Count Ratio*, which is defined as the arithmetic mean of the identification function over the entire population and which represents the "relative amount" of dissatisfaction in it.

Measuring Dissatisfaction Intensity Two profiles may share the same identification degree, but still represent conditions of different dissatisfaction severity. Consider, for example, profile (4144), which belongs to the threshold, and profile (1111), which is the bottom of P, both scored 1 by the identification function. To obtain a more complete picture of subjective well-being, it is therefore of interest to separately assess the dissatisfaction intensity of a profile $\boldsymbol{p}$, which in the following will be called the *gap*[6] of $\boldsymbol{p}$. To this goal, we:

1. Introduce a metric $d(\cdot, \cdot)$ on linear orders, to measure the distance between a profile and the threshold in each linear extension ℓ of P.
2. Given a linear extension ℓ, for any profile $\boldsymbol{p}$ classified as unsatisfied in it, its distance $d(\boldsymbol{p}, \tau\ |\ell)$ to the threshold is computed. This distance is then scaled to $[0, 1]$, dividing it by the maximum distance to the threshold achievable in ℓ, that is, by $d(\bot, \tau\ |\ell)$. The rescaled distance is denoted by $\hat{d}(\boldsymbol{p}, \tau\ |\ell)$.
3. Similarly to the identification step, the gap $g(\boldsymbol{p})$ of profile $\boldsymbol{p}$ is obtained averaging distances $\hat{d}(\boldsymbol{p}, \tau\ |\ell)$ over the set of linear extensions $\Omega(P)$.

Many different metrics may be defined on a linear extension. Here we simply define it as the absolute value of the difference between the rank of a profile and the rank of the highest ranked element of the threshold. Formally, let $r(\boldsymbol{p}|\ell)$ be the rank

[6]The terminology is taken by the practice of poverty measurement.

of profile $\boldsymbol{p}$ in linear extension ℓ and let

$$r(\tau\,|\ell) = \max_{\boldsymbol{q}\in\tau}(r(\boldsymbol{q}\,|\ell))$$

be the rank of the highest ranked element of the threshold in ℓ. Then the distance between a profile and the threshold is simply $d(\boldsymbol{p}, \tau; \ell) = |r(\boldsymbol{p}|\ell) - r(\tau\,|\ell)|$ and, since $d(\perp, \tau; \ell) = r(\tau\,|\ell) - 1$, we also have

$$\hat{d}(\boldsymbol{p}, \tau; \ell) = \frac{|r(\boldsymbol{p}|\ell) - r(\tau\,|\ell)|}{r(\tau\,|\ell) - 1}.$$

Finally we put

$$g(\boldsymbol{p}) = \frac{1}{|\Omega(P)|}\sum_{\ell\in\Omega(P)} \hat{d}(\boldsymbol{p}, \tau\,|\ell).$$

Some comments on the gap function are in order. First, it is computed only for profiles $\boldsymbol{p}$ such that $\mathrm{idn}(\boldsymbol{p}) > 0$. Secondly, it is anti-monotonic, since clearly if $\boldsymbol{q} \neq \boldsymbol{p}$ and $\boldsymbol{q} \trianglelefteq \boldsymbol{p}$, then $d(\boldsymbol{p}, \tau\,|\ell) < d(\boldsymbol{q}, \tau\,|\ell)$ in each linear extension ℓ and thus $g(\boldsymbol{p}) < g(\boldsymbol{q})$. Thirdly, the gap function achieves its maximum value 1 on the bottom element of P (in our case, on profile (1111)). In general, it attains strictly positive values even on the elements of the threshold, achieving value 0 if and only if the threshold is composed of a single profile. This fact, due to the existence of incomparabilities among elements of the threshold, reveals how subtle multidimensional evaluation may be, compared to the unidimensional case. Once the gap function is computed, it may be averaged on the entire population, to obtain the overall *Gap* indicator which complements the Head Count Ratio previously introduced.

Computational Aspects The number of linear extensions of a poset like that involved in the present paper is too huge to list them and perform exact calculations of the identification and the gap functions. In practice, one extracts a sample of linear extensions and computes approximate results on it. The most effective algorithm for (quasi) uniform sampling of linear extensions is the Bubley–Dyer algorithm [2]. For the purposes of this paper, the algorithm has been implemented through a C routine, which is part of an R [8] package for poset calculations, under development by the Authors [3]. The computations required the extraction of 10^{10} linear extensions and took approximately 7.5 h on a 1.9 GB Intel Core 2 Duo CPU E8400 3.00 GHz $\times$ 2, with Linux Ubuntu 12.04 64 bit.

4 Application to Subjective Well-Being Data

To show the evaluation methodology in action, we now apply it to the data presented in Sect. 2. First of all, a threshold has been selected, namely the antichain $\tau = \{(1144), (1211), (3111)\}$. More emphasis has been given to dissatisfaction relative to economics (first component of the profiles) and health (second component of the profiles), than to dissatisfaction pertaining to relationships with friends and relatives. However, it is the combination of scores that matters in identifying unsatisfied profiles. Consider, for example, the first element of the threshold: no matter how good relationships with friends and relatives are, if an individual reports heavy economic and health problems, the corresponding profile will be scored 1 by the identification function. Similarly, if the health status is slightly better, but relational problems arise, then the profile is again scored to 1 by the evaluation function (second element of the threshold). Analogously, for the economic dimension. The choice of the threshold requires in fact judgments on the "global meaning" of the profiles.[7] Compensations among dimensions may exist, but this may depend upon the achievement levels in complex ways. Our choices could be argued indeed, but what is relevant here is to consider the flexibility of the approach, which allows to tune the threshold according to the aims and the contexts. Chosen the threshold, the identification function has been computed. The result is reported in Fig. 1. As it may be seen, its values range from 0 to 1 and some "levels" may be identified. Some profiles are almost unsatisfied, other are "just a little" unsatisfied and so on. This shows how the proposed procedure is successful in revealing the nuances of subjective well-being, overcoming rigid black or white classifications. A similar computation has been performed to get the gap function. Table 1 reports the results at regional and national level, also split by males and females. The Head Count Ratio ranges from about 7 % to almost 25 % and Gap ranges from about 8 % to about 16 %, revealing heavy interregional differences. Regions clearly separate in three main groups, below, around or above the national levels for both indicators. Broadly speaking, this distinction reflects the North–South axis, which is a typical feature of the Italian socio-economic setting, where southern regions are generally in worse socio-economic situations than the northern ones. However there is some remarkable shuffling among territorial areas and some regions from the South (Molise and Basilicata) turn out to score similarly to regions from the Centre and *vice versa*, as in the case of Umbria. The position of Trentino-Alto Adige is remarkable and confirms that this region is an outlier in the Italian context, due to its prerogatives and autonomy as a region under special statute and thanks to the efficiency of its administrative system. A closer look to Table 1 reveals also

[7] The choice of the threshold requires exogenous judgments and assumptions by social scientists and/or policy-makers. It must be noted, however, that the methodology allows for such exogenous information to be introduced in the analysis in a neat and consistent way. One could also add to the analysis judgments on the different relevance of well-being dimensions. Partial order theory, in fact, provides the tools to handle this information in a formal and effective way. We cannot give the details here, but some hints can be found in [6].

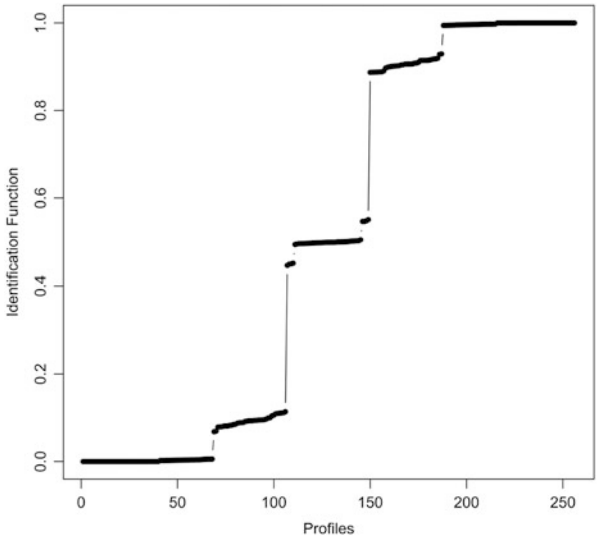

Fig. 1 Identification function (profiles ordered by increasing values of the identification function)

Table 1 Head Count Ratio and Gap at regional and national level

Regions	ID	Head count ratio			Gap		
		Total	Males	Females	Total	Males	Females
Piemonte - Valle d'Aosta	1	0.16	0.16	0.17	0.13	0.12	0.13
Lombardia	2	0.14	0.13	0.16	0.11	0.11	0.12
Trentino-Alto Adige	3	0.08	0.07	0.08	0.08	0.08	0.08
Veneto	4	0.15	0.14	0.16	0.12	0.11	0.13
Friuli Venezia Giulia	5	0.14	0.13	0.15	0.12	0.10	0.13
Liguria	6	0.14	0.12	0.15	0.11	0.10	0.12
Emilia Romagna	7	0.14	0.13	0.16	0.11	0.11	0.12
Toscana	8	0.15	0.14	0.16	0.12	0.11	0.13
Umbria	9	0.20	0.17	0.23	0.15	0.13	0.17
Marche	10	0.17	0.15	0.18	0.12	0.11	0.13
Lazio	11	0.19	0.17	0.22	0.13	0.12	0.15
Abruzzo	12	0.20	0.17	0.22	0.13	0.12	0.15
Molise	13	0.18	0.18	0.18	0.12	0.12	0.12
Campania	14	0.24	0.22	0.26	0.15	0.14	0.16
Puglia	15	0.24	0.21	0.26	0.16	0.14	0.17
Basilicata	16	0.20	0.16	0.23	0.13	0.11	0.14
Calabria	17	0.22	0.19	0.25	0.15	0.13	0.16
Sicilia	18	0.23	0.21	0.24	0.15	0.15	0.16
Sardegna	19	0.22	0.20	0.24	0.15	0.13	0.17
Italy		0.18	0.16	0.20	0.13	0.12	0.14

a strong correlation between the Head Count Ratio and the Gap. As the average level of dissatisfaction increases, the distance between the "unsatisfied" and the others becomes larger, giving evidence of a social polarization process, particularly affecting southern regions. Focusing on males and females separately reveals other interesting features in the territorial pattern of subjective well-being. Regional Head Count Ratios and Gaps are systematically higher for females than for males, revealing a kind of "gender polarization" across the country. Female subjective well-being differentiates regions more neatly than male scores at the extent that regions form quite separated clusters, enforcing the evidence of strong variations in the structure of subjective well-being along the North–South axis. Again, Trentino-Alto Adige is an exception, in that the difference between males and females is very small.

Remark on Missing Data As stated, records with missing data have been excluded by the analysis. Given the aim of the paper (to present the essentials of a new evaluation methodology), this is an acceptable choice. Indeed, an interesting feature of the methodology is that missing data could be handled quite easily. Each statistical unit in the population is assigned to an element of the profile poset and his/her well-being equals the value of the identification function on that element. When some components of a profile are missing, the statistical unit can only be associated to a subset of the profile poset, comprising the profiles compatible with the available information. Consequently, a range of possible well-being scores may be associated to the statistical unit. Similarly, a range of variation for the overall well-being score could be also derived. Due to the limited space available, here we cannot pursue this analysis further.

5 Conclusion

In this paper, we have introduced and applied to real data a new methodology for multidimensional evaluation with ordinal data, that overcomes the limitation of approaches based on counting or on scaling of ordinal variables. The proposed methodology exploits results from partial order theory and produces synthetic indicators with no variable aggregation.[8] The approach is still under development, particularly to give it sound mathematical foundations, to tune it towards real applications and to overcome the computational limitations due to sampling from the set of linear extensions. Future and broader applications to real data will determine whether the methodology is valuable. The issue of well-being evaluation

[8]It is of interest to notice that in standard multivariate approaches, aggregation often exploits interdependencies among variables. Unfortunately, in quality-of-life studies, it turns out that interdependencies may be quite weak. Our approach, which is multidimensional in nature, overcomes this issue by addressing the evaluation problem as a problem of multidimensional comparison.

is gaining importance day by day, for both scholars and policy-makers. We hope to be contributing to address the problem in a more effective way.

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