Sustainable Development: Valuing the Future for the Environment and Equity

Odin Knudsen and Pasquale L. Scandizzo

Abstract We argue that sustainable development is at its essence the destruction and creation of expansion options under the shroud of uncertainty. Without this options approach, the future is undervalued because of uncertainty and the opportunity to stage investment. As a result of this undervaluation, protecting the environment, in particular combatting climate change, is underinvested in favor of short-term returns. We extend this argument to income and wealth distribution arguing that public policy should favor longer-term investment and suppress returns on shorter-term capital. That is, policy should favor the future. In making this argument, we extend the analysis of the influential book by Piketty to real options analysis of investment under uncertainty. We find that taking into account the term structure of investment is more important than the average rate of return on capital in income and wealth distribution. Valuing the future not only benefits the environment but also results in a more equitable income distribution. Both are at the heart of sustainable development.

1 Introduction

To some, sustainable development means a process of growth where future generations are not made any worst off than present generations (Mitlin 1992). In the strictest interpretation of sustainable development, the substitution between various forms of capital are not permitted. This strict approach to development implies that the stock of man-made capital increases at least at the rate of population growth while natural capital remains preserved. In the weaker form of sustainable development, substitution between various forms of capital is permitted.

O. Knudsen (✉)
Real Options International, Bethesda, MD, USA
e-mail: oknudsen@realoptionsinternational.com

P.L. Scandizzo
FUET, Tor Vergata Economics Foundation, Rome, Italy
e-mail: scandizzo@economia.uniroma2.it

L. Paganetto (ed.), *Stagnation Versus Growth in Europe*,
DOI 10.1007/978-3-319-26952-8_12

Diverging from this approach to sustainable development based on the substitution of capital, we argue in this paper that sustainable development constitutes growth that does not destroy the *options* of future generations but in fact creates new options termed expansion options. This approach to sustainable development bridges between these stricter and weaker interpretations by stating that the *options* of future generations should not be compromised by the actions of present generations. In the stricter interpretation of sustainable development, this options approach would imply that the consumption of natural capital could reduce the options of future generations and therefore should be limited. In the weaker interpretation, sustainable development would imply that growth is permitted even if options are destroyed as long as options of equal or greater value are created.

This process of the destruction and creation of options has profound implications to how the future is value and to income and wealth inequality. We argue that if sustainable development is not approached from this options approach the future will be undervalued in investments and that inequality will become more perverse. Eventually both outcomes lead not only to environmental deterioration but also to social and political instability.

2 Linking the Future with Inequality

Two paramount issues are dominating the political debate in the first decades of the twentieth-first century: the state of the environment and particularly climate change and the inequality of wealth and income. Both issues could lead to economic instability deeper into the century: climate change through destabilizing agricultural production, extreme weather events and uncontrollable migration—events that we are already experiencing; wealth inequality through destabilizing democracies and leading to social upheavals.

In an earlier paper (Knudsen/Scandizzo 2014), we argued that the discount rate was too high once uncertainty and the ability to stage investments were taken into account. We showed that the optimum discount rate should favor the future, that is, be low, and even negative depending on the degree of uncertainty. Also that it was not just the level of the discount rate but its term distribution that was important. We argued that the return on investment should favor more the future for both economic and environmental reasons.

Similarly, Piketty (2013) published a monumental historical and theoretical book on capital and growth. The link between our earlier paper on valuing the future and Piketty's work on inequality is the rate of return on capital. Piketty argued empirically and through simple models that the rate of return on capital was too high with the consequence that capital and wealth distribution were becoming more unequal and threatened to become even more askew in the future. His argument was built upon a simple relationship between the capital output ratio, the rate of return on capital and the growth rate of output. In particular he argued that when the rate of return on capital is higher than the growth rate of output, and the gap between the two increases, capital and wealth distribution also becomes increasingly unequal. To counter this tendency, he argued for a wealth tax.

In this paper, we will deepen this link between our earlier work and Piketty's thesis by arguing that, in a dynamic economy with expansion options and the capacity to stage investment, the rate of discount should be lower than the return to capital within a stage. The resulting lower rate of discount if applied through policy will result in a more equal distribution of income and wealth. In other words, viewing investment choices through the lens of creation and destruction of expansion options will drive investments toward greater economic and environmental benefits and more equal income and wealth distribution in the future. Such an outcome is the essence of sustainable development.

3 Valuing the Future

One of the most perplexing issues of economics is how to value the future. That valuation is critical to the most important decisions facing investors and governments—how to allocate capital between different investments in combatting and adapting to longer term environmental consequences such as climate change.

What separates future outcomes from the present in this allocation is the rate of return on capital or the discount rate for future net returns. Once a rate is chosen, then capital can be allocated between different possible investments until the marginal return on capital is driven down to the discount rate. Likewise, the discount rate reflects the value of consumption today versus that in the future. Higher discount rates imply that present consumption is valued more than future consumption. Investments that yield immediate benefits tend to get priority as the rate of return rises.

There is wide disagreement among economists on what discount rate to use in evaluating projects. Some argue that the average return on capital is the most appropriate, usually around 7 % as investment today could compensate future generations by this return compounded. Others argue that the anticipated growth rate of an economy, currently about 2–3 % in industrial countries is the appropriate discount rate. Some defend the use of the riskless rate of interest, usually specified as the long-term return on top rated government bonds, currently between 1 and 2 %. Finally some like Stern (2006) argue for an ethical rate of discount, nearer 1 %. A survey of economists conducted by Weitzman (2007) yielded the crowd opinion of a mean discount of 4 %.

The choice of discount rates has minor impact when deciding between investments with short-term payoffs say within 5 years but for investments for longer term outcomes like combatting climate change it is critical as the outcomes are likely 50 to a 100 years off. Selection of a 7 % rate of discount means that to avoid 1000 € damage 50 years from now we should only be willing to invest (sacrifice consumption) today of about 33 €. While using a 1 % would up the investment to 608 €.

The choice of the appropriate rate of discount for evaluating investments to mitigate or adapt to climate change has triggered a wide debate in the literature—almost dividing in parallel to that in politics—between more liberal economists who lean to a lower discount rate that would demand action now to combat climate change to more conservative economists who see no need to choose a lower discount rate than the return on capital and question if action today is justified. Heated debate has lead to doubt on one of the most publicized reports on climate change where Stern (2006) argued that actions today were economic justified even though the damage from climate change may even be decades to a century away. His critics countered that he had put the veil of formal economics over a politically driven conclusion. By choosing a low discount rate, he had assured his conclusion under the guise of adhering to rigorous economics. A higher discount rate would find that action today to combat climate change would be limited or even not justified under current estimates of damage in the future.

Reinforcing the debate for waiting to combat climate change has been real option theory where the option to wait plays an essential role. Under uncertainty, the option to wait demands a higher return to investment before action is triggered. Also higher uncertainty raises the value of the option to wait forcing greater time to pass for more information to accumulate before action is triggered. The combination of a conventional rate of discount and the option to wait would force a delay in action to mitigate and to adapt to climate change. These factors operating in parallel along with the low but finite probability that the science on climate change may be wrong have played strongly into the hands of those who argue that governments should devote few resources and take only limited action against climate change.

3.1 The Appropriate Discount Rate for Climate Change Investments

Much of the debate on the appropriate discount rate for evaluating investments to combat climate change centers on the parameters of the Ramsey equation, which derived from a growth model that optimize intergenerational utility. While this Ramsey Growth model is constrained by very specific assumption (a single decision-maker for example), it is considered the "logical" relationship between the rate of return of capital, the time preference of consumption, the elasticity of consumption and the growth rate of consumption. In other words, the Ramsey equation binds a relationship between the return on capital and macroeconomic parameters such as the savings rate, putting to test whether the variables are "reasonable" or consistent with observation.

Stern in the Review selects very specific values on the Ramsey equation parameters, in particular a 0.1 time preference, meaning that future consumption is nearly equal to present consumption. Then he specifies the long run growth of consumption at 1.3 % and a constant elasticity of consumption to utility. These assumptions yield a discount rate of 1.4 %. Using this discount rate, Stern Review argues that expenditures today are justified to avoid a 1 % perpetual loss in global GDP due to climate change. It proposes that the choice of a "low," ethical discount rate is justified as appropriate when extinction is a possibility.

Nordhaus (2007) in a blistering attack on the Stern Review argues that the low discount rate is inconsistent with reasonable assumptions on the observed rate of return on capital and the savings rate:

> To a first approximation, the *Review*'s assumptions about time discounting and the consumption elasticity would lead to a doubling of the optimal global net savings rate.

Also the ethical argument is suspect according to Nordhaus:

> Global per capita consumption today is around $10,000. According to the *Review*'s assumptions, this will grow at 1.3 % per year, to around $130,000 in two centuries. Using these numbers, how persuasive is the ethical stance that we have a duty to reduce current consumption by a substantial amount to improve the welfare of the rich future generations?

Even more dramatically, the observed return on capital is about 7 %; in a 100 years, $10,000 invested today would yield nearly nine million dollars. Even if the rate of capital is less, it is clear that future generations will have a higher income perhaps sufficient to compensate for the effects of climate change. Combined with the uncertainty of climate change's effects, critics of action today argue that economies should wait to make the substantial sacrifices of income in combatting climate change or investing in adaptation. But a counter argument is that the effects of climate change could be sudden and dramatic. Action today is justified even if the probability is low of catastrophic climate change effects.

Weitzman (2013) in a later article tackles the discounting problem with the prospect of a catastrophic event sometime in the future where consumption could be driven to zero—the economist's way of characterizing mass human extinction. How much would a current generation pay to avoid such a disaster in perhaps the distant future? Weitzman uses a fat-tailed distribution, that is, a probability distribution where the tails have higher probability than say a normal distribution, which is thin tailed. The fat-tail distribution puts a higher probability on extreme, catastrophic events. His conclusion is nonsensical as he quickly points out—an infinite amount. The obvious solution is to limit the upper bound of present consumption loss for avoiding the catastrophic outcome. But such a bound is arbitrary so it does not get us any farther in asking on how much society should sacrifice to avoid a future loss, that is, what should the discount rate be in evaluating current investment.

Pindyck (2013) in a follow up article argues that even a thin tailed probability distribution could result in a similar conclusion to the fat-tail one. He adds that if marginal utility is bounded then the sacrifice today does not need to be infinite. But then again what is that bound?

Pindyck adds another wrinkle to the problem by arguing that there are other possible catastrophes in the future, including nuclear attack, bioterrorism etc. so even if say a 10 % loss of current GDP was justified by climate change avoidance of other catastrophic events could also demand additional sacrifice of income. How then with multiple catastrophic events does society determine the relative weight to give to each? As Pyndck writes:

> If catastrophes—climate or otherwise—would each reduce GDP and consumption by a substantial amount, then they cannot be treated individually. Potential non-climate catastrophes will affect the willingness to pay to avert or reduce a climate catastrophe, and affect the economics of "climate insurance."

In another approach, Weitzman tries an intermediate path by defining a risk-adjusted discount rate. He describes two forms of risk—diversifiable risk and non-diversifiable risk. In the former, risk can be reduced by diversifying a portfolio and in the later it cannot.

The basis of this approach to the discount factor is the CAPM approach to asset management where an asset that has a low or negative correlation with the market is considered more valuable. This hedged investment, one whose returns move less in sync or against the market, should have a lower discount rate than the return to equity (capital). If the return of a climate investment is closely correlated with the private returns of the market, then no hedge is implied and the discount rate is closer to the return on capital. If it is a hedge moving in an opposite direction to the market, then it is closer to the risk free rate. Weitzman in a highly stylized model shows that in such a hedge scenario the discount rate should be less than the market return and move down over time.

Critical to how fast the discount rate should move down over time is the Beta with respect to the returns to the broad market. By assuming that investment today in climate mitigation is essentially a hedge against a future event yields an adjustment of the current discount rate and a declining rate over time. For example if Beta is equal to 0.5 and the return on capital is 7 % and the risk free rate is 1 % then the initial rate of a hedged investment should start with a discount rate of 4 %. In 25 years the rate should be 3 % and in 50, 2.3 %.

While acknowledging that no one can specify the exact Beta to use, Weitzman concludes that, over reasonable ranges, Beta is a real and significant pull downward to the discount rate of climate hedged investments.

3.2 An Alternative Approach to Environmental and Longer Term Investments

The debate over the appropriate discount rate is unresolved but critical. Today one can only agree that it should not be too high—near the return on capital—as this rejects the possibility of a low probability but catastrophic events. But then again it should not be too low—near the risk free rate on bonds—as this would weigh the future consumption nearly the same as today's consumption. It is in the intermediate range of say 4 % that as the economists' survey pointed out consensus exists—the Goldilocks consensus. Whether it should diminish over time depends on how diversifiable is the hedge resulting from investment in climate mitigation or adaptation. Essentially that is the state of the discussion but gets us no closer in evaluating real world investments, especially those that have benefits and costs which extend deep into the future such as investment in climate change mitigation and adaptation. An alternative approach is demanded which steps around the debate on the discount rate and instead appeals to our intuition.

We know that the future is uncertain perhaps deeply uncertain. We also know that investors take actions today that have very uncertain outcomes or may result in possible benefits many years from now. We also know that companies engage in research with uncertain outcomes that may or may not result in development of a product for the market. We also know that society takes actions today to avoid irreversible but uncertain damage in the future. And as argued above, a "reasonable" rate of discount discourages or possible eliminates all these actions.

What perhaps is going on is not that the rate of discount is incorrect, whatever it may be, but that the analysis is incomplete. Most investments are staged or phased in. Land is acquired, a store built in a market, and then more stores are opened if successful. Research is conducted and if promising results are found, a development stage is embarked upon. Countries grow in stages from the movement from agriculture to industry to services. A staged approach is the observed manner by which investment takes place. It is also the prudent approach especially when uncertainty in the future is high.

Furthermore, an investment may be made in a stage where apparently the returns in that stage do not justify the costs unless making that investment captures the option to expand into future returns and that benefit of expansion is weighed in the initial decision to invest. If the future is not viewed solely through the prism of future diminished value, but through the lens of possible future opportunities that can only be potentially realized if the initial investment is made, then apparently unjustified investments could become the norm. If strings of continuing future investment opportunities are envisioned but only can be captured if a string of investments are made, then the "apparent" rate of discount may be less than the rate of discount when these expansion opportunities or options are missed. In fact, it may appear that the corrected rate of discount once future options are taken into account is actually negative. That is the investment should be made as the future is more valuable than the present.

The discount rate applied in any stage is not the issue but how uncertainty and staging investments leads to action now even in the face of "market" interest rates. In other words, for investments in mitigating or adapting to climate change, the issue at hand is not only what is the discount rate applied within a stage but also how does uncertainty and prudently staging investments bring forth future expansion options.

In climate related investments to mitigation or adaptation, returns and costs are highly uncertain even when calibrated against the best of the climate models. Also because of feedback effects, unknowns on sequestration of carbon and future emissions, the mitigation needed to abate climate change is uncertain. Likewise the effects and costs of climate change are also uncertain, particularly in any location. For example how high to build a seawall or a dam cannot be determined from past data but must be estimated based on models of possible future events. Both mitigation and adaptation require, for efficiency in allocation of capital, a stage approach that allows possible future investments as more information is retrieved from events and more refined projections. But if initial apparently uneconomic first stage investments are not made then the option to avoid more consequential damage is not realized.

The stage approach is embodied in real option theory where investments under uncertainty are staged, potentially proceeding to later stages as more information is accumulated. In this real options approach, the first stage faces the option to wait, which tends to postpone enactment of the investment but also can benefit from an expansion or exit option in a second stage. These later options can expire—have a finite time for taking advantage of the opportunity—as in the case of irreversible environmental damage or have increasing investment costs due to increasing costs in achieving the same benefits.

In the first case of the expiring option, if action is not taken in a timely manner, so much environmental damage is caused that it cannot be reversed. Such a case could be the destruction of biodiversity or a rainforest such as the Amazon. In the second case of increasing investment costs, the damage is reversible but at higher and higher costs over time. In both cases, we can frame the investment choice as a staged and option laden cost benefit analysis that could be discounted at some conventionally agreed discount rate. The two stages are determined by the fact that investment opportunities are sequentially ordered and second stage investment and related benefits are contingent on the first stage investment being undertaken. In other words, adopting the first stage investment produces an uncertain flow of net benefits, evolving over time according to a stochastic process, and, at the same time, empowers the decision maker to adopt, if and when she decides to do so, a second stage investment. This in turn is expected to produce a second round of uncertain net benefits, which also follow a stochastic process, but are ruled by different parameters. The economic attractiveness of the first stage investment depends on an extended net present value ENPV—the expected stream of benefits and costs along with the option to wait and the expansion or second stage option.[1]

We begin with the simplest case where we have a stage investment choice. In the first stage, an uncertain stream of benefits and costs are assumed to follow a geometric Bernoulli random process, where outcomes in the future become more uncertain the more distant in time. For simplicity, we assume that there is no trend component with net benefits randomly distributed around a mean. In other words, the first stage faces only a random process where the future outcomes are more and more uncertain with time but distributed about a constant mean. This process as applied to climate change would mean that the effects are not getting worse but the possible outcomes are more uncertain the farther out we look into the future.

[1] In the Mathematical Appendix to this paper, the detailed assumptions and proofs are presented. In the body of the paper, we will explain the results in graphs.

By itself, the first stage investment decision would be quite demanding in terms of return. The benefit cost ratio would have to be above one by the value of the option to wait, whose value depends on the uncertainty of the process. In other words, the uncertainty of future outcomes would demand prudence before undertaking a first stage investment. This prudence is characterized by the value of waiting for more information.

If however there is the potential for a second stage investment that can only be realized if the first stage is undertaken, the incentive to do the first stage is enhanced. By initiating the first stage investment, the option to do a later investment is realized. The value of this second stage expansion option depends on uncertainty with the higher the uncertainty the more value is this options everything else equal. The investment decision to proceed with the first stage is thereby influenced by the opportunity to acquire this second stage option.

As scientists have argued in IPCCC reports, weather events will in the future become more variable and severe. We model this crudely that the variance of the process when the second stage is applicable will be greater, that is more severe in the second stage. We therefore have a process of investments that is phased between two stages and where the outcomes are more variable in the second stage.

To illustrate refer to Fig. 1 where we have modeled the investment decision for the first stage based upon the variability of outcomes for a second stage and for different mean second stage benefit cost ratios. We also assume that variability exists in the first stage but is less than in the process of outcomes for the second stage. In figure one, we assume for illustration a standard deviation of 30 % for the first stage process.

Along the vertical axis is the benefit-cost ratio necessary to trigger the first stage investment. Note that when there is no uncertainty in the second stage it takes a benefit-cost ratio of 1.5 (rather than 1) to embark on the first stage investment. This "wedge" of a premium of 50 % represents the value of the option to wait. The benefit-cost ratio must overcome this wedge with superior benefits.

As the uncertainty of the second stage process increases (moving out on the horizontal axis), the second stage expansion option begins to counter the waiting option. Depending on the benefit cost ratio of the second stage, the curves crossover the unity benefit cost ratio for the first stage indicating that a benefit-cost ratio of less than one in the first stage can trigger the first stage investment.

The calculations underlying Fig. 1 are for a discount rate of 4 %. The less than one benefit cost ratio for first stage investment demonstrates that for even higher discount rates, first stage investments can be triggered even if the benefit-cost ratio of the second stage is less than one provided that uncertainty in the second stage is significantly high.

In Fig. 1 as just described, the level of uncertainty in the first stage was not related to that of the second stage process. If we relax that assumption—driving a relationship between the two—where the first stage uncertainty increases (non-linearly) with the second stage process, we get Fig. 2.

In this case, the increases in uncertainty of the second stage drives up the value of both the option to wait and the expansion option, requiring a higher level of

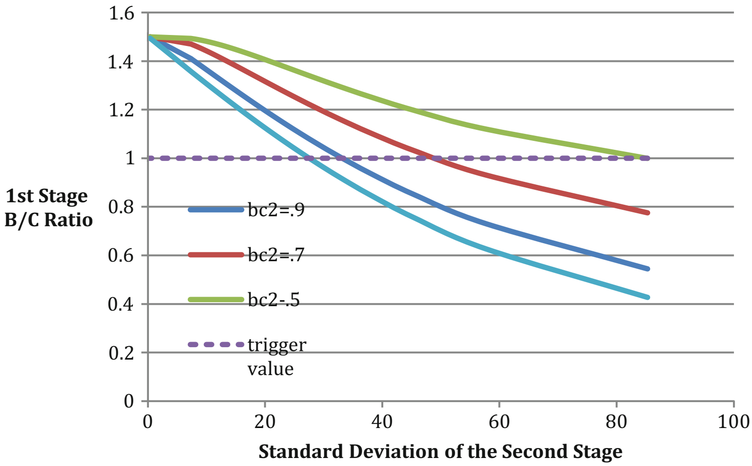

Fig. 1 Effect of second stage uncertainty with option to wait equal investment in stages

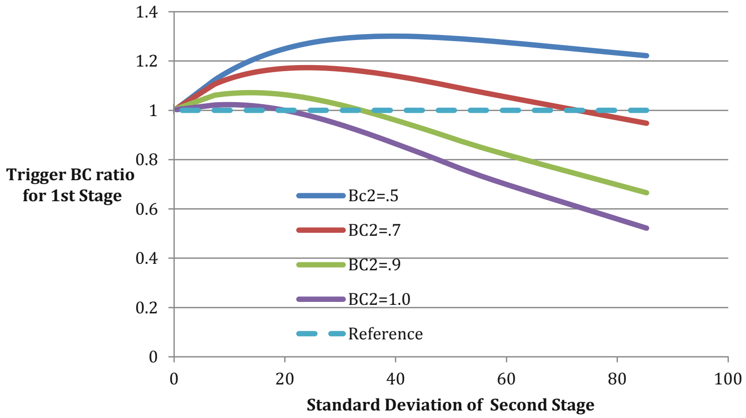

Fig. 2 Trigger for first stage with uncertainty linked

uncertainty in the second stage to drive the benefit cost ratio of the first stage below 1, the certainty trigger value. For benefit cost ratios of near 1 for the second stage, the uncertainty can be relatively low for benefit-cost ratios of the second stage to decline below 1. For low benefit cost ratios in the second stage (B-C = 0.5 in Fig. 1), the uncertainty has to be quite high before the benefit cost ratio is 1 for the first stage. Interestingly even if the benefit cost ratio of the second stage and the first stage is less then one investment in the first stage can be triggered by second stage uncertainty.

This two-stage analysis can be extended to multiple stages as demonstrated in the mathematical appendix with uncertainty in later stages, effectively operating counter to the discount rate generating an effective discount rate which has uncertainty embedded in it. This uncertainty can be shown to drive the discount rate to very low levels and even cause a cross-over point where the effective discount rate is such that the future becomes more valuable than the present. In other words, with high levels of uncertainty in the future and phased investment, governments should be willing to invest now to acquire future options to tackle climate change. Such a point of view is consistent with common sense, when disaster is possible in a highly uncertain future and where a staged approach to investment can be implemented.

We have demonstrate that perhaps the critical factor in initiating investments today is not just the discount rate, but the uncertainty of the potential benefits of future investments. While uncertainty in a one-stage investment decision plays a delaying role for more information before commitment, the presence of a second stage with higher levels of uncertainty can drive this first stage investment. The critical factor then in deciding to invest today moves beyond consideration mainly of the discount rate, but also must focus on the variability of benefits in the future and how expansion opportunities can be captured by acting now. Strategically phasing investment is not just the common sense way to proceed with the challenge of confronting longer term environmental issues such climate change that are critical to sustainable growth, but is consistent with real option theory.

But critical to this approach is that the investor, whether in government or the private sector, is able to view the future as opportunity to expand to realize uncertain gains or avoid uncertain costs. If the future is viewed as a static replication of the present then the immediate and short-term rate of return will dominate and growth will not be sustainable or will be low. Dynamic, sustainable economies are signified by entrepreneurs and government that view the future as a string of uncertain opportunities, whose potential value override the short-term return on capital. Thus, capital should be allocated among investments according to the equality of marginal return to marginal cost of capital, both extended by the value of future options. In such an economy and with this insight by the public and private sector, ex ante it would appear that investment is too high and too extended to the long term. It would appear that investors are willing to accept a rate of return lower than the immediate short-term investor opportunity.

The issue is how to motivate such a view of the future among investors. How does government help create a dynamic economy where future uncertain value motivates investment? Risk aversion has little to do with it as we have made no assumptions about the appetite for risk—the key is to create expansion opportunities (options) and at the same time to avoid having investment stuck in the short term returns.

Public policy and incentives must be oriented to research and development, the creative engine behind expansion options. It must also discourage short-term investment through tax policy. Such tax policy already exists through much reduced capital gains taxes in the United States but usually the time period for capital gains to be realized are too short (1 year). Also policy must focus on instruments that are tied to the future. For example carbon markets where investors can buy carbon credits that can have long term rising value as the effects of climate change demand more stringent reductions in carbon emissions. Government can also cause disruptive investments that tear apart static markets with innovations such as in automobiles or telecommunications. The key for government policy is to create dynamism both through pushing returns to the future and by creating new, but uncertain investment opportunities in the future. In its own public investment analysis, government must adopt an imaginative approach which uses real options and creatively values options for expansion in the future putting more value on these options the more uncertain are returns (or avoidance of costs) in the future. Whatever the mechanism what remains true is that the short term return on capital is too high weighing too heavily present returns and not the future where expansion options reside.

4 Inequality and the Rate of Return on Capital

In the previous section, we have argued that the rate of return on capital cannot be high in the short term as it discourages needed investment for the long term, especially when economies are faced by the effects of uncertain but long term environmental consequences such as those driven by climate change. We have supplemented that truism with an additional factor—that in an economy with expansion options the optimal rate of return to maximizing net benefits is lower than the rate of return on capital. How much lower depends on the value of these expansion options, which in turn depends in part on uncertainty. It follows that public policy should encourage an economy where expansion options are abundant and encourage an entrepreneur class motivated to seek these opportunities. But in many economies such conditions do not exist, resulting in a rate of return on capital that is too high, that is above the optimal effective return on capital.

As demonstrated by Piketty, the consequences of this higher rate of return on capital can be increased inequality if the return is higher than the growth rate of the economy and this gap increases over time because growth is falling more than the rate of return. Piketty's argument is that in this case, capital income will grow faster than wage income with a consequent worsening of income distribution. While he predicts a lowering of growth rate, which would converge to a level of about 1 % per year, closer to the long term historical rate, he does not identify any specific mechanism which would determine growth. However, one possible determinant of a lower rate of growth in the future may reside in the deterioration of the environment induced by the present generation tendency to undervalue the future.

One way to explain such a link between Piketty's argument and the undervaluation of the longer term growth options described in the first part of this paper, is a simple economic model.

Start from the Harrod–Domar model, where demand is defined by the Keynesian multiplier:

$$Y = \frac{dK}{s}$$

where Y expected (demand side) income, dK autonomous investment and s marginal propensity to consume.

Supply Q is assumed to be proportional (through the capital output ratio k) to installed capacity K:

$$kQ = K$$

In general, demand will grow at the rate dI/sY where $I = dK$, and supply at the rate $\frac{dQ}{Q} = \frac{dK}{K}$ and equilibrium (Y = Q) will be achieved only under the so called warranted rate of growth $g = \frac{dK}{K} = \frac{s}{k}$. This equilibrium can be achieved by chance (but it would be unstable) or, in a stable way, if one of the two parameters, i.e., either the saving rate or the capital output ratio can adjust. The Kaldor–Pasinetti solution consists in using the (functional) income distribution as a way to change the marginal propensity to save, while Solow's solution uses the flexibility of the capital output ratio in a neoclassical production function.

Piketty's case can be seen as a yet another way to explain how a stable g can be achieved. The inverse of the capital output ratio 1/k, in fact, can be interpreted as $Q/K = (P + W)/K = g/s = 1/k$, where P and W are respectively income from profits and from wages. Thus, in order to achieve equilibrium, return on capital, $\rho = \frac{g}{s} - \frac{W}{K} \rightarrow \rho = \frac{g}{s} - \frac{(1-\alpha)}{k} = \frac{\alpha}{k}$, so that capital's income share must be $\alpha = 1 - \left(\frac{g}{s} - \rho\right)k$ and must adjust downward the higher is the (warranted) growth rate, the lower the propensity to save and the lower the rate of return to capital. In Piketty's analysis this rate is assumed to be determined according to Ramsey neoclassical formulation as: $\rho = \phi + \eta g^*$, where ϕ is the pure rate of time preference, η is a measure of the convexity of the representative agent utility function, and $g*$ is the expected growth rate, which would coincide with Solow "optimal" rate if equated to the marginal productivity of capital. This rate will be overestimated if the rate of discount used to select investment projects does not consider the value of future expansion options. If $g^* = g$ and $k = k^*$ to achieve equality between the warranted and the expected rate, this will imply $g^* = \frac{s}{(1-s\eta)}\left[\phi + \frac{(1-\alpha)}{k^*}\right]$ but, with the environmental effects of project choices neglected, by systematically disregarding the project expansion /growth options, effective growth will fall short of the predicted (and the optimal one). As a consequence, realized growth g^{**} will be less than expected: $g^{**} < g^*$ i.e., $g^{**} = g^* - e$, where e is the reduction in growth as a consequence of the systematic over-estimate of growth, so that realized growth will be: $g^{**} = \frac{s}{(1-s\eta)}\left[\phi + \frac{(1-\alpha)}{k^*}\right] - e = \frac{s}{(1-s\eta)}\left[\phi + \frac{(1-\alpha)}{k^{**}}\right]$, where k^{**} is the capital output ratio which ensures equilibrium at the realized rate of growth. Because effective growth is lowered by the use of a larger than optimal discount rate, project selection will be less efficient, capital accumulation will be lower, but the capital output ratio (and the marginal productivity of capital in a production function setting) will be higher. As a consequence, the rate of return to capital will be higher seemingly validating the discount rate used. Thus, through an overvalued discount rate, an undervaluation of the future will reverberate in a lower realized growth rate and essentially create a vicious circle of increasing inequality, deteriorating environment and decreasing growth. Note that this undervaluation e will be greater the higher is uncertainty as future expansion options will be more valuable.

In an economy with expansion options, we have thus found that the rate of return needs to be adjusted downward by a factor closely related to the value of these options. The key to growth is not only a lower rate of return on capital but the continuing string of expansion opportunities. While the resulting growth rate may or may not be higher than the effective rate of return on capital when adjusted for future expansion options, in a dynamic economy with many opportunities for expansion, growth will be higher. Since such a dynamic economy demands a lower return on capital, the spread between return on capital and growth should be less and therefore inequality be less.

5 Conclusions

We have argued that sustainable growth is critically linked to appropriately valuing the future and to reversing the tendency to increase inequality as demonstrated by Piketty. To achieve these elements of sustainable growth a lower rate of return on capital is required, along with investors motivated by expansion opportunities. An economy characterized by the dynamic creation of expansion opportunities results in an effectively optimal discount rate lower than the rate of return on capital. While such an economy would see investment expand to this lower discount rate, it is also likely that growth would be higher. While it cannot be shown that this growth would exceed the rate of return on capital and thus result in lower inequality, we can argue that the return on capital would be more closely aligned with the growth rate. Ideally if growth still exceed the options adjusted return on capital, public policy should intervene to bring about more equality or at least halt the slide into increasing inequality.

Mathematical Appendix

We assume that the effects of mitigation and adaptation policies may be decomposed into two distinct and independent components, both following a geometric Brownian motion:

$$\begin{array}{c} dy_i = \alpha_i y_i dt + \sigma_i y_i dZ_i \\ i = 1, 2 \end{array} \tag{1}$$

where dy_i is the stochastic increase in value created by investment in each period of each stage, dZ_i is random variable with mean $EdZ_i = 0$ and variance $E(dZ_i)^2 = dt$. The α_i represents the drift or trend term and σ_i the (instantaneous) standard deviation of y_i. The α_i represents the general trend in growth that may take place. It will be greater than zero for technological change or any other condition that may improve the prospect of growth and zero or negative otherwise. The σ_i is the standard deviation parameter measuring the instant variability of this growth as the economy is subject to various random shocks with the uncertainty of outcomes when viewed from the present becoming more uncertain the further in the future. With reference to climate change, the process underlying the first stage can be taken to represent the current phase, dominated by the primary effects of C02 accumulation, and thus by the benefits that would ensue from appropriate **mitigation policies**. The process underlying the second stage, on the other hand, upon appropriate investment is undertaken, would enable benefits to be released from both **imitation** and **adaptation** investment policies.

Undertaking investment I_1 immediately would produce an expected stream y_1 of benefits from mitigation and also secure the option to adopt investment I_2, which would in turn give access to a stream of benefits y_2 from stage 2, characterized by further mitigation as well as adaptation. Expected benefits for stage 1 (mitigation) are thus assumed to be the consequence of a known investment level I_1 for the same stage. Expected benefits for stage 2 (adaptation & mitigation), on the other hand, are also assumed to be the consequence of given investment costs, which are also known with certainty, but increase with the time of adoption at a fixed rate g, i.e., $I_2 = I_0 e^{gt}$. Thus, in each stage, the effect of a known level of investment is stochastic and its expected impact per time period equals y_i, but the second stage effect per unit of investment varies with uncertainty because the size of the investment required to produce a given level of benefits increases over time.

At the status quo, the decision maker is holding an option—the option to commit the first stage (mitigation) investment costs. Once this first stage is exercised, a new options is created to proceed to further stages of mitigation and adaptation. More specifically, we may represent the decision problem at each of the two stages as follows. In stage 2, since the option has been obtained by entering stage 1 at time τ_1, the decision maker seeks to solve the problem to gain the greatest possible expected value of her payoff:

$$V(y_2) = \sup E_y \left[\left(\int_{\tau_2}^{\infty} \left(e^{-\rho(s-\tau_2)} y_s - I_0 e^{g(s-\tau_2)} \right) ds \right) \right] \tag{2}$$

where g is a positive rate of growth and τ_2 the (stochastic) time at which the option to adapt will be exercised. Expression (4a, 4b) indicates that the costs of investing in the second stage adaptation will be higher the farther in time is the moment at which they will be incurred.

For stage 1, on the other hand, the decision maker solves the problem to gain the greatest possible expected value of her payoff, conditioned to the future solution of stage 2:

$$V(y_1) = \sup E_y \left[e^{-\rho\tau_1} \left(\int_{\tau_1}^{\infty} e^{-\rho(s-\tau_1)} y_s ds - I_1 + V(y_2) \right) \right] \tag{3a}$$

where I_1 denotes first stage investment costs and τ_1 is the (stochastic) time at which the first stage mitigation option is exercised.

Assuming that the dynamics of the risk contained in the cash flow, dz, can be replicated by existing assets, both options in the two stages can be evaluated by applying contingent claim evaluation. As Dixit and Pindyck (1994, pp. 122–123) show, this evaluation problem has a state dependent solution, contingent on whether the value of the stochastic variable (the cash flow y_t) is above or below a critical threshold of investment adoption ($y_{ip}, i = 1, 2$)[2]:

$$V(y_2) = \frac{y_2}{\delta_2} e^{-\delta_2\tau_1} - I_0 e^{(g-\delta_2)\tau_1} \text{ if } y_2 \geq y_{2p} \tag{3b}$$

$$V(y_2) = \left(\frac{y_2}{y_{2p}} \right)^{\beta_2} \left[\frac{y_{2p}}{\delta_2} e^{-\delta_2\tau_1} - I_0 e^{(g-\delta_2)\tau_1} \right] \text{ if } y_2 < y_{2p} \tag{3c}$$

and

$$V(y_1) = \frac{y_1}{\delta_1} - I_1 + V(y_2) \text{ if } y_1 \geq y_{1p} \tag{4a}$$

$$V(y_1) = \left(\frac{y_1}{y_{1p}} \right)^{\beta_1} \left[\frac{y_{1p}}{\delta_1} - I_1 + V(y_2) \right] \text{ if } y_1 < y_{1p} \tag{4b}$$

In (3a) and (4a, 4b) β_i (i = 1, 2) is the positive root of the characteristic equation (Dixit and Pindyck 1994, p):

$$r - \beta_i \alpha_i - \frac{\beta_i}{2} (\beta_i - 1) {\sigma_i}^2 = 0 \tag{5}$$

[2] Assuming dy_t/y_t normally distributed implies y_t log normally distributed. Given this assumption $E(y_t) = ye^t$, that discounted at rate r gives $\int_{\Omega} \int_0^{\infty} y_t e^{-rs} ds d\omega = \int_0^{\infty} y e^{-(r-\alpha)s} ds = y/(r-\alpha)$ with $r < \rho$. See Dixit and Pindyck (1994, p. 71).

r being an appropriate rate of discount that represents the cost of delaying the investment.[3]

Note that the instantaneous variance of the process ${\sigma_i}^2$ increases monotonically as β_i decreases, it is equal to $r - 2\alpha_i$ at $\beta_i = 2$, and it goes to infinity as $\beta_i \to 1$. We characterize the area for $1 < \beta_i \leq 2$, with variance approaching infinity as β_i approaches 1, as a situation of "deep uncertainty".

In order to find the values of the thresholds y_{ip}, $i = 1, 2$ We first evaluate the value of the second stage option $V(y_2)$, which can be expressed (Dixit and Pindyck 1994 p. 122) as: $A_2 {y_2}^{\beta_2}$, by using the optimal stopping conditions (in this case they indicate the optimal stopping of the process of waiting before committing to the new phase). These conditions imply that optimal switching from waiting to adoption occurs the first time the value hits the boundary of the continuation region. This requires in turn that the following conditions are satisfied at the switching point y_{2p}:

Value Matching:

$$A_2 {y_2}^{\beta_2} = \frac{y_2}{\delta_2} - I_0 e^{g\tau_2} \tag{6}$$

Smooth Pasting:

$$\beta_2 A_2 {y_2}^{\beta_2 - 1} = \frac{1}{\delta_2} \tag{7}$$

In (6) and (7), as we have already specified, y_2 is the expected value of the value flow from the decision to invest in adaptation, I_2 is the investment outlay of the second stage, $\delta_2 = r - \alpha_2$, where r is the risk free interest rate (or any other appropriate rate of discount), and A_2 and β_2 two parameters that can be determined, respectively, solving the system (6) and (7) and applying Ito's lemma (Dixit and Pindyck, op. cit. p. 4). In expression (6), the LHS represents the option to undertake a subsequent phase of adjustment to climate change, which is non zero, and dominates the NPV on the RHS in the so called continuation region, i.e., for the values of expected benefits y_2, for which it is not worth undertaking the project. Once the adaptation phase is undertaken, on the other hand, the option is no more alive, and its value is zero. We may capture this behavior by defining the so called extended NPV or NPVE as:

$$NPVE(2) = NPV(2) - A_2 y_2^{\beta_2} \tag{8}$$

where $NPV(2) = \frac{y_2}{\delta_2} - I_2$ is the expected net present value of entering the adaptation phase. Note that for the adaptation phase, with a positive value of the option to wait,

[3] According to CAPM, the opportunity cost of capital of any investment can be determined as: $r = \rho + \vartheta r_M$, where ρ is the risk free interest rate, and ϑ the regression coefficient of the rate of return of the investment considered and the average market return r_M.

the NPVE may only be lower than NPV. Moreover, if the value of the option to wait prevails over the expected net present value, the NPVE will be negative, indicating that it would be better to defer the investment.

Proposition 1 For a two stage strategy, entry in the first stage is more attractive the higher the uncertainty of the second stage.

Proof Substituting (6) into (5) and solving for y_2, we obtain the value of the optimum switching point y_{2p}, i.e., the minimum present value of expected income necessary to prompt the entry into the development phase:

$$\frac{y_{2p}}{\delta_2} = \frac{\beta_2}{\beta_2 - 1} I_2 \quad (9)$$

and, by substituting into the option expression $A_2 {y_2}^{\beta_2}$, we obtain the expression for the constant:

$$A_2 = \frac{1}{\delta_2 \beta_2} \left[\frac{\beta_2}{\beta_2 - 1} I_2 \delta_2 \right]^{1-\beta_2} \quad (10)$$

Equation (9) states that at the optimum entry point, the discounted value of y_{2p} must exceed the investment costs by a factor of $\beta_2/(1-\beta_2)$ or the uncertainty 'wedge'. From the characteristic Eq. (5), it can be shown that β_2 is negatively related to σ_2: as uncertainty increases the 'wedge' also becomes larger, requiring a larger value of y_2 before the development phase is undertaken.

For the entry in the first stage, on the other hand, we have to consider the value matching condition given by the equality between the option to adopt the strategy under consideration and the difference between the option to go into the second stage and the investment in the first phase:

$$A_1 y_1^{\beta_1} = \frac{y_1}{\delta_1} - I_1 + A_2 y_2^{\beta_2} \quad (11)$$

In other words, the value of the option to enter the first stage (mainly mitigation) will equal expected net income from this stage minus the investment costs needed to enter plus the value of the option to enter the second stage (mainly adaptation).

The smooth pasting condition correspondent to the value matching in (11) is the following:

$$\beta_1 A_1 y^{\beta_1 - 1} = \frac{1}{\delta_1} \quad (12)$$

and, by substituting (10) and (12) into (11), we obtain the expression for the entry point y_{1p} of the mitigation phase:

$$\frac{V_{1p}}{\delta_1} = \frac{\beta_1}{\beta_1 - 1}\left[I_1 - \frac{1}{\delta_2\beta_2}\left[\frac{\beta_2}{\beta_2 - 1}I_2\delta_2\right]^{1-\beta_2} y_2^{\beta_2}\right] \tag{13}$$

Since $\beta_2 \geq 1$, the value of the option to proceed to the second phase effectively consists of two components: (i) a benefit component, given by the value of the stochastic variable yielding the benefits of phase two should the corresponding option be exercised and, (ii) a cost component, given by the cost of exercising the option. A condition of deep uncertainty may be characterized by uncertainty on the value of β_2. Note, however, that while this value may range from one to infinity, the corresponding range of expression (13) is:

$$\begin{aligned} &\lim_{\beta_2 \to 1} \frac{y_{1p}}{\delta_1} = \frac{\beta_1}{\beta_1 - 1}\left(I_1 - \frac{y_2}{\delta_2}\right) \text{ and} \\ &\lim_{\beta_2 \to \propto} \frac{y_{1p}}{\delta_1} = \frac{\beta_1}{\beta_1 - 1} I_1 \end{aligned} \tag{14}$$

Proposition 2 The larger the investment costs anticipated for the second stage, and the greater their rate of increase over time, the more attractive will be immediate entry in the first stage.

Proof For $y_2 < y_{2p}$, we can interpret the result in (13) by writing the entry condition as:

$$\frac{y_{1p}}{\delta_1} = \frac{\beta_1}{\beta_1 - 1}\left[I_1 - Ee^{-r\tau_2}A_2 y_{2p}^{\beta_2}\right] = \frac{\beta_1}{\beta_1 - 1}\left[I_1 - Ee^{-r\tau_2}\frac{I_2}{\beta_2 - 1}\right] \tag{15}$$

where $Ee^{-r\tau_2}$ discounts at the present the option value at the expected time to entry into the adaptation stage. For any value of y_2, the following equality holds:

$$A_2 y_2^{\beta_2} = Ee^{-r\tau_2}A_2 y_{2p}^{\beta_2} = Ee^{-r\tau_2}\left(y_{2p} - I_2\right) = Ee^{-r\tau_2}\frac{I_2}{\beta_2 - 1} \tag{16}$$

i.e., the value of the adaptation option at any time equals its expected discounted value at exercise time. From this equality, solving for the expected discount factor, we obtain:

$$Ee^{-r\tau_2} = \left(\frac{y_2}{y_{2p}}\right)^{\beta_2} \tag{17}$$

Because $A_2 y_{2p}^{\beta_2} = \frac{y_{2p}}{\beta_2\delta_2}$ by the smooth pasting condition and $y_{2p} = \frac{\beta_2\delta_2}{\beta_2 - 1}I_0 e^{gt}$, substituting into (13) we find:

$$\frac{y_{1p}}{\delta_1} = \frac{\beta_1}{\beta_1 - 1}\left[I_1 - \left(\frac{\beta_2\delta_2}{\beta_2 - 1}I_2\right)^{1-\beta_2}\frac{y_2^{\beta_2}}{\delta_2\beta_2}\right] = \frac{\beta_1}{\beta_1 - 1}\left[I_1 - \left(\frac{v_2(\beta_2 - 1)}{\beta_2}\right)^{\beta_2}\left(\frac{\beta_2\delta_2 I_2}{\beta_2 - 1}\right)\right] \tag{18}$$

where $v_2 = y_2/\delta_2 I_2$ is the benefit cost ratio of stage two investment, i.e., the expected net present value of net benefits from entering the adaptation phase divided by the correspondent value of investment costs. Expression (18) shows that, for any given value of the benefit cost ratio and of uncertainty, the entry point is *lower* the *higher* the exercise price (i.e., the investment costs) of the adaptation option. If the benefit—cost ratio of the action to undertake in the long run is greater than one, in particular, the perspective of larger investment to undertake in the future will also reduce the immediate entry point and may even cause it to become negative.

Because $A_2 y_2^{\beta_2} = Ee^{-r\tau_2}\frac{y_{2p}}{\delta_2\beta_2}$, and we can write (16) as follows:

$$\frac{y_{1p}}{\delta_1} = \frac{\beta_1}{\beta_1 - 1}\left[I_1 - \frac{Ee^{(g-r)\tau_2}I_0}{(\beta_2 - 1)}\right] \tag{19}$$

Proposition 3 The higher the uncertainty associated with the second stage, the more attractive will be immediate entry in the first stage.

Proof From (16) and (17), we derive:

$$A_2 y_2^{\beta_2} = Ee^{-r\tau_2}\frac{I_2}{\beta_2 - 1} = \left(\frac{y_2}{y_{2p}}\right)^{\beta_2}\frac{I_2}{\beta_2 - 1} \tag{20}$$

Differentiating with respect to $\beta_2(\sigma_2)$, we obtain:

$$\frac{\partial\left(A_2 y_2^{\beta_2}\right)}{\partial\beta_2} = \left(\frac{y_2}{y_{2p}}\right)^{\beta_2}\frac{I_2}{(\beta_2 - 1)^2}\left[\left(\frac{y_2}{y_{2p}}\right)\log\left(\frac{y_2}{y_{2p}}\right) - 1\right] \tag{21}$$

This derivative is less than zero for $\frac{y_2}{y_{2p}} < 1$, i.e., for all values of second stage pay off that all below the threshold to exercise the second stage investment option. As a consequence, a decrease in the value of β_2, corresponding to an increase in the variance σ_2 and in the second stage volatility will increase the value of the adaptation option in Eq. (18), reducing, in turn, the threshold value of the first stage (mitigation option) y_{1p}. Note that, even though, an increase in uncertainty tends to increase the waiting time before the second stage option is exercised, thus reducing its value in proportion to the discount factor $Ee^{-r\tau_2}$, this effect is always overcome by the increase in the expected value of the payoff : $\frac{I_2}{\beta_2 - 1}$ at the exercise point.

Proposition 4 If the cost of second stage investment increases over time at a rate higher than the discount rate, an increase in the second stage uncertainty will make more attractive to invest earlier in both mitigation and adaptation.

Proof The second term in square parenthesis on the right hand side of (19) and (20) is the expected net present value from implementing the second stage adaptation/mitigation strategy, as it is given by the product of the expected discount factor $E e^{-r\tau_2}$ (which equals 1 at the time of the exercise of the second stage option) by the net benefit $\frac{y_{2p}}{\delta_2} - I_2 = \frac{I_0 e^{gt_d}}{\beta_2 - 1}$. Thus, if $g > r$, the higher the rate g at which the second stage investment costs increase with time, the lower the return required to adopt the first stage investment in mitigation. Note that two different effects will characterize the second stage option: on one hand, a higher basic investment required in the second stage will imply a lower threshold cost for adaptation, thus making less attractive this prospect. In expressions (19), this effect manifests itself as a lengthening of the expected time of exercise of the adaptation option, which reduces the expected benefit of acquiring it through first stage mitigation. On the other hand, if delaying adaptation makes its costs grow over time at a rate greater than the discount rate, i.e., $g > r$, the expected benefits from acquiring the adaptation option will increase with a reduction in the investment in mitigation and this effect will tend to dominate, the greater the difference $g - r$. As we have seen above, the expected discount factor depends on the benefit cost ratio and, for any given level of this ratio, investment costs simply measure project size. If the extended NPV from the long term, and more highly uncertain adaptation stage, exceeds the value of the investment in the first stage, in particular, expression (19) indicates that it may be worth entering the project even with zero or negative net benefit prospects in the first stage.

Proposition 5 The higher the uncertainty associated with the second stage, the greater will be the extended net present value (ENPV) of engaging in (first stage) mitigation.

Proof The ENPV can be defined as the expected NPV plus the options created and minus the options destroyed. In our case, the value of the option created is given by (20), while the value of the option that would be destroyed by immediate investment in mitigation is:

$$A_1 y_1^{\beta_1} = Ee^{-r\tau_1}\left(\frac{y_{1p}}{\delta_1} - I_1\right) = Ee^{-r\tau_1}\left\{\frac{1}{\beta_1 - 1}\left[I_1 - \frac{\beta_1 E e^{(g-r)\tau_2} I_0}{(\beta_2 - 1)}\right]\right\} \tag{22}$$

Thus, the ENPV of the first stage is:

$$ENPV_1 = \frac{y_1}{\delta_1} - I_1 - Ee^{-r\tau_1}\left\{\frac{1}{\beta_1 - 1}\left[I_1 - \frac{\beta_1 E e^{-r\tau_2} I_2}{(\beta_2 - 1)}\right]\right\} + Ee^{-r\tau_2}\frac{I_2}{\beta_2 - 1} \tag{23}$$

We have already seen that the value of the second stage option increases with uncertainty. This directly creates option value in the ENPV and reduces the option value destroyed by first stage entry, since it makes the value of the first stage waiting option (the term in parenthesis) decline with an increase in uncertainty.

Proposition 6 A sufficient condition for the existence of a finite threshold of adoption at any one time in the case of an infinite sequence of options is that uncertainty is sufficiently large.

Proof Assuming an infinite sequence of options, from Eqs. (18) and (19), by using mathematical induction, we can write the recursive form:

$$\frac{y_{tp}}{\delta_t} = \frac{\beta_t}{\beta_t - 1}\left[I_t - \left(\frac{Ee^{-r\tau_{t+1}}}{\beta_{t+1} - 1}\right)\left(\frac{y_{t+1.p}}{\delta_{t+1}\beta_{t+1}}\right)\right] \tag{24}$$

where in analogy to the notation used so far, t denotes the stage of the process and the suffix τ_t the time of entry in the option of stage t. Expression (24) suggests that in the case of a sequence of options, consideration of the next development option should, in some sense, "suffice" to calculate the entry point at each stage.

Substituting the explicit value of the option at each stage, expression (24) implies[4]:

$$\begin{aligned}\frac{y_{tp}}{\delta_t} &= \frac{\beta_t}{\beta_t - 1}\left[I_t - \sum_{i=1}^{T-t}(-1)^{i+1}\frac{Ee^{-r\tau_{t+i}}I_{t+i}}{\prod_k^i\left(\beta_{t+k} - 1\right)}\right] \\ &= \beta_t\left[E\sum_{i=0}^{T-t}(-1)^i\exp\left[-\left(r\tau_{t+i} + \sum_{k=0}^{i}\log\left(\beta_{t+k} - 1\right)\right]I_{t+i}\right]\end{aligned} \tag{25}$$

where the alternant signs are due to the fact that in each stage the waiting option is a cost, while the forward (expansion) option is a benefit. In order to prove proposition 6, we investigate the properties of the series in (18) for $T \to \propto$. According to the Leibnitz criterion, a series with alternating signs $\langle a_\tau \rangle, \tau = 1, 2, 3, \ldots .T$ *absolutely converges* if and only if $a_\tau \geq a_{\tau+1}$ (i.e., its terms decline monotonically as the series progresses) and $a_T \to 0$ as $T \to \propto$ (i.e., its terms tend to zero as the series becomes arbitrarily large). These two conditions will be satisfied for the series in (18), provided that

[4] The full expression for T periods is:

$$\frac{y_{1p}}{\delta_1} = \frac{\beta_1}{\beta_1 - 1}\left[I_1 - \left(\frac{\beta_2\delta_2}{\beta_2 - 1}\left(I_2 - \left(\frac{\beta_3\delta_3}{\beta_3 - 1}\left(I_3 - \ldots\ldots\frac{\beta_T\delta_T}{\beta_T - 1}I_T.\right)^{1-\beta_T}\right)\frac{y_T^{\beta_T}}{\delta_T\beta_T}\right)^{1-\beta_{T-1}}\ldots\ldots\ldots\right)^{1-\beta_2}\frac{V_2^{\beta_2}}{\delta_2\beta_2}\right]$$

$$\frac{Ee^{-r\tau_{\tau+i-1}}\left(Ee^{-r(\tau_{\tau+i}-\tau_{\tau+i-1})}I_{t+i}-\left(\beta_{t+i}-1\right)I_{t+i-1}\right)}{\prod_{k}^{i}\left(\beta_{t+k}-1\right)}\leq 0 \tag{26}$$

and

$$\exp\left[-\sum_{k=0}^{i}\left(r\tau_{\tau+k}+\log\left(\beta_{t+k}-1\right)\right)\right]I_{t+i}\rightarrow 0 \text{ as } i\rightarrow\propto \tag{27}$$

Expression (27) implies:

$$\frac{Ee^{-r(\tau_{t+i}-\tau_{t+i-1})}I_{t+i}}{\left(\beta_{t+i}-1\right)}\leq I_{t+i-1} \text{ for. } i=1.2.... \tag{28}$$

Indicating with g_{t+i} the rate of growth of investment between t + i−1 and t + I, condition (28) can be expressed as follows:

$$Ee^{\left(g_{t+1-i}-r\right)(\tau_{t+i}-\tau_{t+i-1})}\leq\beta_{t+i}-1 \tag{29}$$

which in turn implies:

$$r\geq g_{t+1-i}\frac{\log\left(\beta_{t+i}-1\right)}{\tau_{t+i}-\tau_{t+i-1}} \tag{30}$$

Note that the logarithm on the RHS of expression (20) will be zero or negative when forward uncertainty (i.e., uncertainty characterizing the option to be exercised in the future) is sufficiently large or $\beta_{t+i}\leq 2$. In this case, the series in (25) will always converge, if the discount rate is positive and could converge also if it were negative, provided that its absolute value were not too large.

Proposition 7 In the case of a finite sequence of options, both increase in uncertainty and investment costs will increase the value of the future reducing or even reversing the effects of the discount rate. In the case of an infinite sequence, the threshold will depend only on current estimates of uncertainty and investment costs.

Proof Consider the specific value of the limit of the series in expression (25). To determine it in the special case of continuity, note that we can write (25) in the equivalent form:

$$\frac{y_{tp}}{\delta_t} = \frac{\beta_t}{\beta_t - 1}\left[I_t - E\sum_{i=1}^{T-t}\Big(\exp\Big[-\Big(r\tau_{t+i} + \sum_{k=1}^{i}\log\big(\beta_{t+k} - 1\big)\Big]I_{t+i}\right. \\ -\exp\Big[-\Big(r\tau_{t+i-1} + \sum_{k=1}^{i-1}\log\big(\beta_{t+k} - 1\big)\Big]I_{t+i-1}\Big) \tag{31}$$

If the stages of growth are sufficiently close to another (i.e., if the waiting time before exercising each subsequent option is arbitrarily small), expression (31) can be written, for $t = 0$, in the time continuous form:

$$\frac{y_{tp}}{\delta_t} = \frac{\beta(t)}{\beta(t) - 1} I(t)\left[1 - E\int_t^T de^{-\int_0^t (r - g(u) - h(u))du}\right] \tag{32}$$

In (31) $I_t \to I(t) = I(t)e^{g(t)}$, $(\beta_t - 1) \to (\beta(t) - 1) = (\beta(t) - 1)e^{-h(t)}$, where $g(t)$ is the rate of growth of investment and $h(t) = -\log(\beta(t) - 1)$ a measure of the rate of increase of uncertainty so that $\prod_{k=t}^{T}\frac{1}{(\beta_k - 1)} = e^{-\int_t^T \log\left(\beta_{t+k} - 1\right)du} = e^{-\int_t^T h(u)du}$, in the continuum. Solving the integral on the RHS of (31), we find:

$$\frac{y_{tp}}{\delta_t} = \frac{\beta(t)}{\beta(t) - 1} I(t)\left[1 - Ee^{-\int_t^T (r - h(u) - g(u))du}\right] \to \frac{\beta(t)}{\beta(t) - 1} I(t) \text{ as } T \to \propto \tag{33}$$

If $r - h(u) \geq g(u)$

Thus, in general, as shown in the table below, the degree of increase in uncertainty over time will reduce or may even overwhelm the rate of discount in valuing the future. However, in the case of an infinite sequence of options, convergence is insured only if the rate of discount, corrected for the increase in uncertainty, is less than the rate of growth of investment and in this case, only the waiting option matters.

r > g + h		r < g + h		r = g + h	
Infinite sequence	Finite sequence	Infinite sequence	Finite sequence	Infinite sequence	Finite sequence
Only next option matters	Uncertainty and investment growth reduce discounting effect	Only waiting option matters	Discounting is reversed	Only waiting option matters	Only waiting option matters

References

Dixit A, Pindyck RS (1994) Investment under uncertainty. Princeton University Press, Princeton, NJ

Knudsen OK, Scandizzo PL (2014) Do we undervalue the future?: implications for mitigating and adapting to climate change. In: Paper presented at XXVI Villa Mondragone international economic seminar, July 2014

Mitlin D (1992) Sustainable development: a guide to the literature. In: Environment and urbanization, vol 4. IIED, London, pp 111–124

Nordhaus WD (2007) A review of the stern review on the economics of climate. J Econ Lit 45 (3):686–702. doi:10.1257/jel.45.3.686

Piketty T (2013) Capital in the twenty-first century. Harvard University Press, Cambridge, MA

Pindyck RS (2013) Climate change policy: what do the models tell us? J Econ Lit 51(3):860–872

Stern N (2006) Stern review on the economics of climate. HM Treasury, London

Weitzman ML (2007) A review of the stern review on the economics of climate change. J Econ Lit 45(3):703–724, http://www.cepe.ethz.ch/education/EnergyPolicy/Weitzman.pdf

Weitzman ML (2013) Tail-hedge discounting and the social cost of carbon. J Econ Lit 51 (3):873–882