

Modeling and Analysis of the Hydraulic Servo Drive System

Piotr Woś and Ryszard Dindorf

Abstract In the hydraulic servo drive appear structural nonlinearities which cause that designing nonlinear control of the position and power system is hampered. In the article a mathematical model of the servo drive hydraulic control was described. It is useful for the synthesis algorithms in the simulation model. The calculation diagram of the hydraulic servo drive model consisting of the double-acting cylinder with one-sided piston rod and directional control valve was presented. There were presented characteristics of: displacement, velocity, acceleration and pressures as well as the displacement of spool valve at mass load. An algorithm of control the nonlinear object was adapted by using the linearization method of the model process. Simulation examinations will serve for developing the control algorithm which will enable the compensation influences of disruptions such as: friction and changeable load powers mass.

Keywords Hydraulic servo system · Nonlinear dynamic model · Input–output feedback linearization

1 Introduction

Control of the hydraulic servo drive has already been an object of examinations in different centers of education and research for many years [1, 2]. Nonlinear dynamic characteristics of the hydraulic actuator as well as servo valve are caused by large inertia of the movement, friction forces, deformations and springiness of mechanical elements, compressibility of working fluid, and characteristic flows

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[3, 4]. Changes in dynamic parameters of hydraulic servo drive are associated with load and velocity of the movement, maladjustment of the control structure, and influence of many other factors associated with characteristics of working fluid and exploitation parameters. They all have a significant influence on reducing the resistance of control system. Frequent maladjustment of the control structure results from large forces or load moments of the servo drive hydraulic system. Moreover, requirements concerning high accuracy of control positional and velocity in wide scope cannot be fulfilled, because these conditions are often changeable in time depending on the external load. Turbulent character of the fluid flows in valves and appearance of structural nonlinearities, such as saturation pressure or rate of fluid flow, zone of overlap caused by positive windows overlap of the control slider as well as hysteresis caused by magnetizing the armature of control slider. They all cause that the process of physical phenomena in unknown states in all hydraulic systems can be described only in the nonlinear way. Model of the hydraulic servo drive should include dynamic characteristics of the hydraulic actuator or hydraulic engine, flow characteristics of the servo valve or proportional valve, characteristics of electromechanical converters, compressibility (capacity) hydraulic in distinguished servo drive areas, friction forces appearing in elements of the system, efficiency of hydraulic elements, and other factors like the accuracy of carrying slide steam, accuracy of the filtration as well as characteristics of the working fluid [5]. We should pay attention to nonlinear static characteristics of the control valve. Proportional control valves can have positive or negative overlap [6]. In amplifiers, valve sliders are applicable with the value of overlap to 5 % nominal jump, which takes about 0.5 mm up to 1.0 mm. Edges of the spool valve and conet cooperating with it are carried out with the lower tolerance from $\pm 2.5 \mu\text{m}$ [6, 7]. It allows for keeping the nonlinear scope in the vicinity of zero for about $\pm 3 \%$ of jump. In this range, the movement of slider may change the rate of strengthening the valve to 200 % of its value appearing at normal opening the valve. Such large changes of control parameters may lead to the unstable work of servo drive, e.g., during positioning of the hydraulic actuator or hydraulic engine. Slight leakages that appear in the valve are caused by inaccuracies of making the spool and valve body, which corresponds to the negative overlap. Such a situation also appears, when the control system is unable to hold a slider in the position corresponding to turning off the hydraulic actuator from the power supply. Also, the work instability can be caused by pollutants, which block the flow of the valve. It causes the delays of valve action as at positive overlap. Disadvantageous feature is also appearing of losses caused by leakages, and they cause the movement of the piston at the zero control signal. The value of control signal must then change even in the steady state, at the lack of spool valve movement. Also, undervalued cause of the non-linearity of hydraulic servo valves is friction between the spool valve and the sleeve valve. The threshold of insensitivity causes that to the coil of valve must be given the minimal intensity of current in order to trigger corresponding slider movement and flow of working fluid. The friction force as well as action of the well-proportioned electromagnet introduces the hysteresis into static characteristics of the control valve and electromechanical converter [8–10].

2 Dynamic Model of the Hydraulic Servo System

The mathematical model of occurring physical phenomena in the studied drive system was created after assuming the following [7, 8]: volume module of the oil compressibility is fixed in the entire scope of change pressures and temperatures in the hydraulic system, the p_s pressure in the crowded wire of pump is permanent during the system work, hydraulic control valve has zero overlap, output p_t pressure from the control valve into the container is negligibly small toward pressures in the cylinder chambers, leaks of pressures between the pump and control valve are being omitted, temperature and viscosity of the oil are established during the system work, and delay time of the control valve is equal to zero. Calculation diagram of the hydraulic servo drive model, consisting of the double-acting cylinder with one-sided piston rod and directional control valve, is presented in Fig. 1.

Marked parameters presented in the mathematical model of analyzed hydraulic servo drive were compared in Table 1.

Considering such nonlinearities as evolution characteristics of the flow, friction, and stiffness of working fluid in cylinder chambers from the position, such equations were determined in the following form:

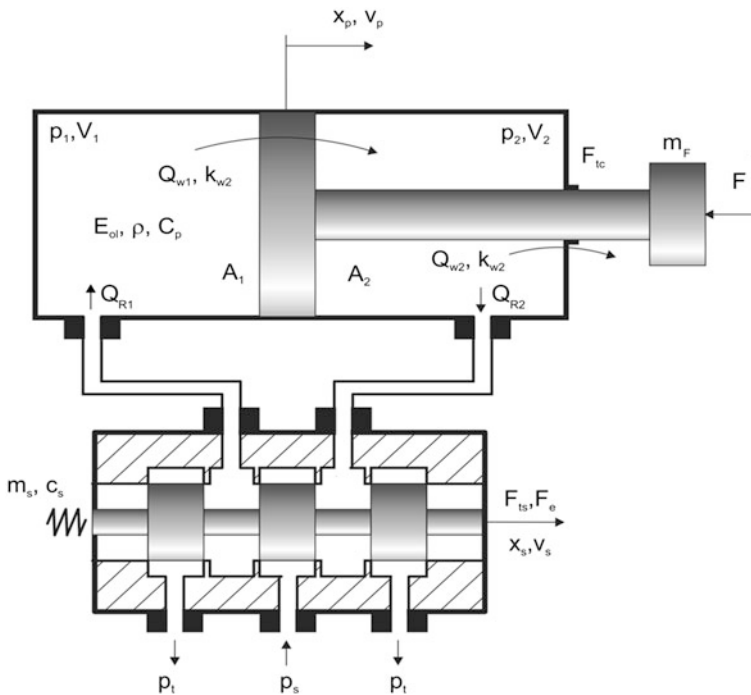


Fig. 1 Scheme of the hydraulic servo model

Table 1 Markings presented in the model

Mark	Term
x_p, x_s	Displacement of the piston, spool valve
v_p, v_s	Velocity of the piston, spool valve
p_1, p_2, p_s, p_t	Pressures
V_1, V_2	Volumes of the individual cylinder chambers
A_1, A_2, A_u	Surfaces of the piston and cross section of seals
m_p, m_s	Total mass on the piston, mass of the spool valve
Q_i	Volumetric flow rate
F, F_e	Load external power, electromagnet power of the valve
F_{tc}, F_{ts}	Friction force in the cylinder and in the valve
$F_{tp}, F_{tl}, F_{tk}, F_{tu}$	Components of the friction forces: adhesion, sticky friction, kinetic, sealing
c_s, c_p	Stiffness of the valve spring and stiffness of the cylinder
f_b, f_s, f_u	Coefficients of sticky friction in the cylinder and valve as well as rates of friction insulating the cylinder
E_{ol}	Reduced substitute module of working fluid
K_q	Coefficient of flow through the valve
K_e	Coefficient of the strengthening in the electromechanical converter of the valve
C_q	Coefficient of resistance flow through the valve
ρ	Oil density
μ	Dynamic rate of oil viscosity
d	Diameter of the spool valve
k_{w1}, k_{w2}	Coefficients of leakages in the cylinder

– equation of the movement of piston

$$\begin{cases} \frac{dx_p(t)}{dt} = v_p(t) \\ \frac{dv_p(t)}{dt} = \frac{1}{m_p} [A_1 p_1(t) - A_2 p_2(t) - F_{tc} - F] \end{cases} \quad (1)$$

– equation of flow through control inter space of the valve
for $x_s(t) > 0$

$$\begin{cases} Q_{R1}(t)_+ = K_q x_s(t) \sqrt{|p_s - p_1(t)|} \\ Q_{R2}(t)_+ = -K_q x_s(t) \sqrt{|p_2(t) - p_t|} \end{cases} \quad (2)$$

for $x_s(t) < 0$

$$\begin{cases} Q_{R1}(t)_- = K_q x_s(t) \sqrt{|p_1(t) - p_t|} \\ Q_{R2}(t)_- = -K_q x_s(t) \sqrt{|p_s - p_2(t)|} \end{cases} \quad (3)$$

- equation of pressures in the cylinder chambers
for $x_s(t) > 0$

$$\begin{cases} \frac{dp_1(t)}{dt} = \frac{E_{ol}}{V_1 + A_1 x_p(t)} (K_q x_s(t) \sqrt{|p_s - p_1(t)|} - A_1 v_p(t) - Q_{w1}(t)) \\ \frac{dp_2(t)}{dt} = \frac{E_{ol}}{V_2 - A_2 x_p(t)} (-K_q x_s(t) \sqrt{|p_2(t) - p_t|} + A_2 v_p(t) + Q_{w1}(t) - Q_{w2}(t)) \end{cases} \quad (4)$$

for $x_s(t) < 0$

$$\begin{cases} \frac{dp_1(t)}{dt} = \frac{E_{ol}}{V_1 - A_1 x_p(t)} (K_q x_s(t) \sqrt{|p_1(t) - p_t|} - A_1 v_p(t) + Q_{w1}(t)) \\ \frac{dp_2(t)}{dt} = \frac{E_{ol}}{V_2 + A_2 x_p(t)} (-K_q x_s(t) \sqrt{|p_s - p_2(t)|} + A_2 v_p(t) - Q_{w1}(t) - Q_{w2}(t)) \end{cases} \quad (5)$$

Flow coefficient K_q through the valve is determined as

$$K_q = C_q \pi d \sqrt{\frac{2}{\rho}} \quad (6)$$

where d is the diameter of the spool valve, C_q is the coefficient of resistance flow through the valve, and ρ is the oil density.

Leakages appearing in the cylinder are proportions to the difference of pressures in the cylinder chambers:

for $x_s(t) > 0$

$$\begin{cases} Q_{w1}(t)_+ = k_{w1}(p_1(t) - p_2(t)) \\ Q_{w2}(t)_+ = k_{w2}p_2(t) \end{cases} \quad (7)$$

for $x_s(t) < 0$

$$\begin{cases} Q_{w1}(t)_- = k_{w1}(p_2(t) - p_1(t)) \\ Q_{w2}(t)_- = k_{w2}p_2(t) \end{cases} \quad (8)$$

where k_{w1}, k_{w2} are the coefficients of leakages in the cylinder equation of the spool valve movement:

$$\begin{cases} \frac{dx_s(t)}{dt} = v_s(t) \\ \frac{dv_s(t)}{dt} = \frac{1}{m_s} (-F_{ts}(t) - c_s x_s(t) + F_e(t)) \end{cases} \quad (9)$$

where

force of the sticky friction is

$$F_{ts}(t) = f_s v_s(t) \quad (10)$$

and the force of the electromagnet spool valve is [5]

$$F_e(t) = K_e(0.3u(t) + 6.5) \quad (11)$$

In the simulated system a friction force appearing in the hydraulic cylinder was taken into account:

$$F_{tc}(t) = F_{tl}(t) + F_{tz}(t) \quad (12)$$

Friction in servo-hydraulic drives is a phenomenon causing standbys of the movement and reducing the efficiency of system as well as at the same time influences on attenuation mechanical oscillation [8]. The fundamental element of Eq. (12) is the sticky friction force, which has a primary importance in the modeling of dynamics drive system:

$$F_{tl} = f_l v_p(t) \quad (13)$$

The coefficient of sticky friction is determined by the relation $f_l = \frac{\mu A}{h}$ where A is the total area of the joint, h is the layer of the oil film (value of float), and μ is the dynamic coefficient of the oil viscosity.

Dry friction force F_{tz} and factor of the sticky friction f_l were appointed stimulating the system of constant tension $u(t)$ and measuring the pressure $p_1(t)$ as well as $p_2(t)$ in the cylinder chambers of in the equilibrium of velocity v_p what allows for calculating the total force of friction. During the movement of the piston rod with the total constant velocity, friction force F_{tc} is expressed as

$$F_{tc} = f_l \cdot v_p + F_{tz} \quad (14)$$

Value of the friction force F_{tz} appearing in the system is determined by Eq. (15) according to the Stribeck model [8]:

$$F_{tz}(t) = (F_{tk} + F_{tp}) \operatorname{sgn}(v_p(t)) + F_{tu} \quad (15)$$

where $F_{tk} = f_{tk} F_N$ is the kinetic friction force, f_{tk} is the coefficient of the kinetic friction, and F_N is the normal force.

Friction appearing in sealing is determined in following relation (16) [8]:

$$F_{tu} = \frac{f_u A_u}{2} (p_1(t) + p_2(t)) \quad (16)$$

where f_u is the coefficient of the friction, $A_u = \pi dl$ is the surface of the sealing, and d and l are the diameter of the piston and length of the sealing.

3 Designing the Control System—Linearization with Feedback

In examining and designing hydraulic servo drive, computer simulation of the model played an important role. Natural way to adapt algorithms of control the nonlinear object is to use the classic method of linearization model process around the point of work and use the linearization of model algorithm. Applying the linearization toward the nonlinear system, the algebraic transformation is explored which eliminates nonlinearities of the object. Therefore, it is necessary to apply the so-called input–output feedback linearization [9]. Nonlinearities are being eliminated (entirely or partly) from the object so that after closing the system was linear.

In effect of the linearization carried out according to the scheme [7], obtained function of coupling the valve for displacement of the spool valve $x_s(t)$ is as follows:

$$x_s(t) = f(p_1, p_2, x_p, v_p, v) \quad (17)$$

for $x_s(t) > 0$

$$\begin{aligned} x_s(t) = & \left[a_0 x_p(t) + a_1 v_p(t) + a_2 \frac{(p_1(t)A_1 - p_2(t)A_2)}{m} \right. \\ & \left. + v(t)_p \left(\frac{A_1 A_2 E_{ol}(V_2(t) + V_1(t))}{m_p (V_1(t) + x_p(t)A_1) \cdot (-V_2(t) + x_p(t)A_2)} \right) - v(t) \right] \cdot \alpha_+(t) \\ \alpha_+(t) = & \frac{m_p (V_1(t) + x_p(t)A_1) \cdot (-V_2(t) + x_p(t)A_2)}{E_{ol} K_q (A_1 \sqrt{|p_s - p_1(t)|} (V_2(t) - x_p(t)A_2) + A_2 \sqrt{|p_2(t) - p_l|} (V_1(t) + x_p(t)A_1))} \end{aligned} \quad (18)$$

for $x_s(t) < 0$

$$\begin{aligned} x_s(t) = & \left[a_0 x_p(t) + a_1 v_p(t) + a_2 \frac{(p_1(t)A_1 - p_2(t)A_2)}{m} \right. \\ & \left. + v_p(t) \left(\frac{A_1 A_2 E_{ol}(V_2(t) + V_1(t))}{m_p (V_1(t) + x_p(t)A_1) \cdot (-V_2(t) + x_p(t)A_2)} \right) - v(t) \right] \cdot \alpha_-(t) \\ \alpha_-(t) = & \frac{m_p (V_1(t) + x_p(t)A_1) \cdot (-V_2(t) + x_p(t)A_2)}{E_{ol} K_q (A_1 \sqrt{|p_1(t) - p_l|} (V_2(t) - x_p(t)A_2) + A_2 \sqrt{|p_s - p_2(t)|} (V_1(t) + x_p(t)A_1))} \end{aligned} \quad (19)$$

Table 2 Simulation parameters

P_s	8 MPa	m_s	0.28 kg
A	$1.256 \times 10^{-3} \text{ m}^2$	E_{ol}	172 MPa
κ	0.69	d	6×10^{-3}
ρ	900 kg/m ³	m_p	20–125 kg
C_d	0.6	c_s	1500 N/m

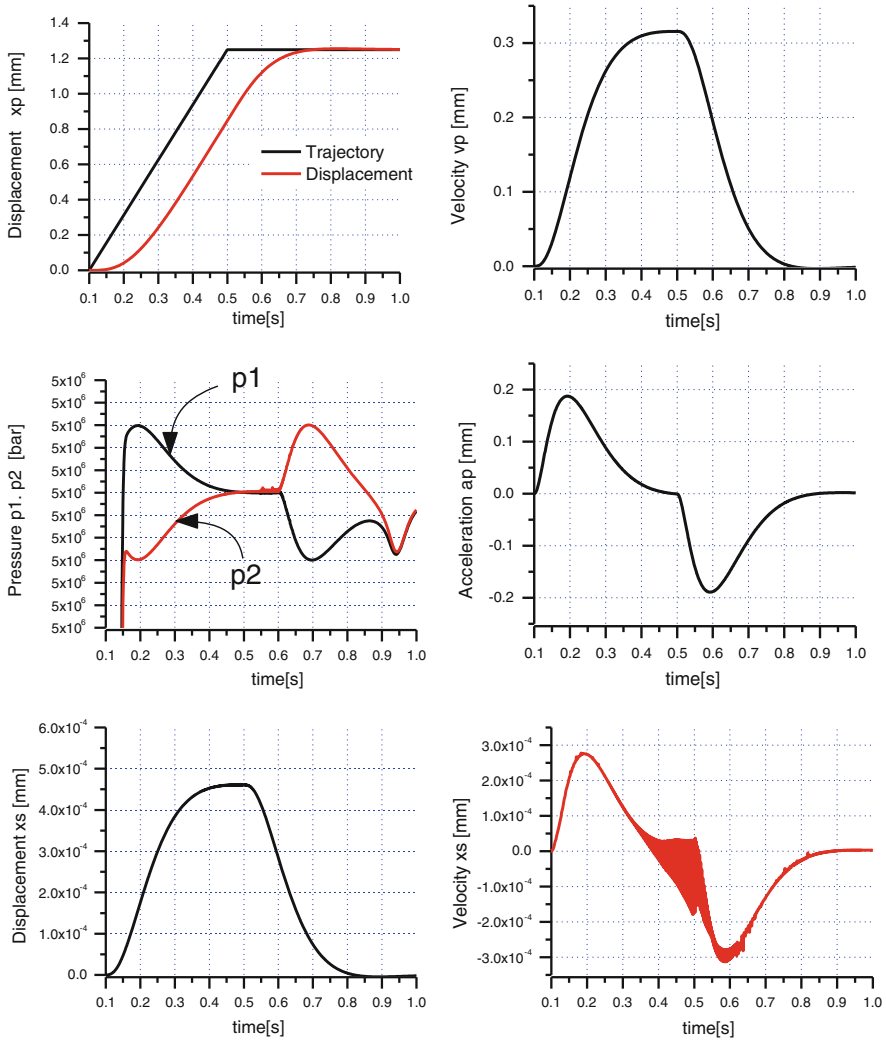


Fig. 2 Dynamic characteristics of the model

where $v = k_s(x_p^{\text{ref}} - x_p)$, x_p^{ref} is the set signal, and a_0, a_1, a_2 are the parameters of the model [8].

For the simulation purposes, constant values of parameters were implemented, see Table 2.

In Fig. 2, the chosen dynamic characteristics such as displacement $x_p(t)$, velocity $v_p(t)$, acceleration $a_p(t)$, pressures in individual chambers $p_1(t)$ and $p_2(t)$ as well as displacement the spool valve $x_s(t)$, and velocity $v_s(t)$ at mass load 100 kg received as a result of the simulation model are presented.

4 Conclusions

The considered solution regards algorithms of the control hydraulic servo drive for which characteristics do not change in time. The accepted theoretical description does not change during the system work. In such assumption, we may accept the sufficient theoretical description only once and select parameters of the adjuster in the course of enforcing the movement drive. Unfortunately, the disadvantage of presented solution is the large sensitivity to mistakes appearing in the description of the controlled object. A lack of the system resistance to interferences appears in case of the parametric model uncertainty [2]. Additionally, all variables of the state must be available for analysis. Since the values of many parameters are not possible to be appointed by direct measurements, they must be appointed as a result of the parametric identification of created mathematical model of the examined object. It is a difficult and laborious process, often loaded by a large dose of the uncertainty. As a result, to assure the right regulation during the work, we must adopt parameters of control process.

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