Chapter 8
Conclusion, Outlook, and Open Problems

In this monograph, we have given a tutorial for studying dynamical systems on networks. By reading our tutorial, you should now have a reasonable understanding of (1) why it is interesting and desirable to study dynamical systems on networks; (2) several of the popular families of problems and models; (3) basic considerations about dynamical systems on networks; (4) the use and range of validity of techniques for analytical approximations such as mean-field theories, pair approximations, and higher-order motif expansions; and (5) time-scale issues and challenges for investigating dynamical systems on time-dependent networks. We have also given pointers to software implementations for direct numerical simulations and for solving the systems of equations that result from the aforementioned approximation methods. As our monograph is a tutorial rather than a literature review, there is a lot that we haven't covered, and we strongly encourage you to scour the literature for interesting problems to study and generalize. The primary purpose of our tutorial is to equip you with the background knowledge to be able to do so successfully. We have also provided numerous references to get you started.

Before saying "goodbye," it is also worth commenting on some of the particularly challenging problems that are available. For example, although one can study problems by purely computational means, we believe that it is desirable (when possible) to try to develop analytical techniques in order to gain insights on these problems. In network science, mathematically rigorous results tend to be rare—see, e.g., [85, 86]—but approximations and heuristic techniques have been employed on many toy problems. A key goal is to find the "next easiest" sets of dynamical systems on networks to study using more general versions of these techniques and to use them to help develop these methods further (and to devise new methods). Additionally, many of the dynamical processes that have been investigated are either some type of percolation or fairly (or even very) closely related to percolation, and it is also important to move beyond these types of models.

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There are many open issues in the study of dynamical systems on time-dependent networks. One needs to examine the balance of time scales for dynamics on networks versus the dynamics of the networks themselves [139, 252], and the effects that this has on the validity and choices of analytical techniques to use is also very important. Most of the studied situations with both dynamics on networks and dynamics of networks tend to be rather unrealistic, so this is an area that is particularly ripe for further study.

Finally, many networks are *multiplex* (i.e., include multiple types of edges) or have other *multilayer* features [28, 173]. The existence of multiple layers on which different dynamical processes can occur and the possibility of both structural and dynamical correlations between layers offers another rich set of opportunities in the study of dynamical systems on networks. The investigation of dynamical systems on multilayer networks is only in its infancy, and this area is also loaded with a rich set of problems [28, 173, 186, 271].

Goodbye.