Novel Information Processing for Image De-noising Based on Sparse Basis

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Abstract. Image de-noising is one of the important information processing technologies and a fundamental image processing step for improving the overall quality of medical images. Conventional de-noising methods, however, tend to over-suppress high-frequency details. To overcome this problem, in this paper we present a novel compressive sensing (CS) based noise removing algorithm using proposed sparse basis on CDF9/7 wavelet transform. The measurement matrix is applied to the transform coefficients of the noisy image for compressive sampling. The orthogonal matching pursuit (OMP) and Basis Pursuit (BP) are applied to reconstruct image from noisy sparse image. In the reconstruction process, the proposed threshold with Bayeshrink thresholding strategies is used. Experimental results demonstrate that the proposed method removes noise much better than existing state-of-the-art methods in the sense image quality evaluation indexes.

Keywords: ATVD · BP · CS · OMP · Sparse

1 Introduction

From the compressive sensing (CS) theory, we know that a sparse signal can be reconstructed from far fewer samples than the samples required by Nyquist rate. That is why CS has been widely used in medical image and remote sensing. In a CS-based image processing system, de-noising is a classical problem to be improved in the quality of images using a reduced amount of data. In recent years, many researchers have worked on advanced methods for image de-noising, including total variation image regularization [6], texture preserving variational de-noising using adaptive fidelity (ATVD), which is a modified version of TVD algorithm [4], basis pursuit de-noising [3]. In addition to mentioned above methods for image de-noising, we shall next focus on compressive sensing as a new method for image de-noising. This paper is to develop a novel image de-noising algorithm based on compressive sensing, which is faster, simpler and also can

S. Arik et al. (Eds.): ICONIP 2015, Part III, LNCS 9491, pp. 443–451, 2015.
DOI: 10.1007/978-3-319-26555-1_50

keep strong edge preservation. In this novel work, we have applied proposed sparse basis on CDF9/7 wavelet transform and a convex optimization technique for reconstruction of image is used, such as which is also called Basis Pursuit (BP) [3] and greedy pursuit such as Orthogonal Matching Pursuit (OMP) algorithm [9]. Even though after algorithm processing, there is still some noise owing to random can be observed. Hence, we take our proposed threshold just before applied inverse CDF9/7 wavelet transform and make a filter operation. To find the best de-noised image, we used TVD algorithm [6] and TV with adaptive fidelity (ATVD) [4]. For making a good comparison, we have used sparse basis DWT and DCT and used image quality assessment scheme [5] to assessed images de-noised.

This paper is organized as follows. Section 2 gives a review of the compressive sensing (CS) problem statement. Section 3 describes sparse image representation by wavelet lifting scheme. Section 4 describes compressive image de-noising method with an algorithm. Experimental results are reported in Sect. 5. The paper completed with a brief conclusion.

2 About Compressed Sensing

The concept of compressive sensing (CS) is to acquire significant data directly without sampling the signal. Thus, it is shown that if the signal is 'sparse' or compressible, then the acquired data is sufficient to reconstruct the original signal from sparse signal with a high probability [1]. Sparsity is mainly defined by the appropriate basis such as DCT or WT and others transformation for that signal. According to compressive sensing theory, CS can be further described as below:

First let following the notations of [1] and we have $f = f_1, ..., f_N$ be N real-valued samples of a signal, which can be represented by the transform coefficients, x. That is,

$$f = \Psi x = \sum_{i=1}^{N} x_i \psi_i \tag{1}$$

where $\Psi = [\psi_1, \psi_2, ..., \psi_N]$ is an $N \times N$ transform basis matrix, which determines the domain in which the signal is sparse and $x = [x_1, x_2, ..., x_N]$ is an N-dimension vector of coefficients with $x_i =< x, \psi_i >$. We assume that x is S-sparse, meaning that there are only significant elements in x with $S \ll N$.

Suppose a general linear measurement process computes inner product with $M < N$, between f and a collection of vectors, ϕ_j, giving $y_j =< f, \phi_j >$; $j = 1......M$. If Φ denotes the $M \times N$ matrix with ϕ_j as row vectors, then the measurements $y = [y_1, y_2, ..., y_M]$ are given by:

$$y = \Phi f = \Phi \Psi x = \Theta x \tag{2}$$

where y is M-dimensional observation vector, Φ is the $M \times N$ random measurement matrix, and $\Theta = \Phi \Psi$ is called sensing matrix. For reconstruction ability x is S-sparse, if Θ satisfies the Restricted Isometric Property (RIP) [2].

The signal reconstruction is an ill-conditioned problem involves using y to reconstruct the N-length signal, x, that is S-sparse, given Φ and Ψ. Many researchers have developed a number of different algorithms to solve these three CS reconstruction problems. In this paper we have used Basis Pursuit (BP) algorithm [3] to solve the l_1 norm and gradient-based algorithms as OMP algorithm [9] to solve the l_2 norm for proposed image de-noising applications.

3 Proposed Sparse Representation of Image

Currently, multi-resolution pyramid decomposition and synthesis algorithm, namely the Mallat algorithm, is the most commonly used in wavelet research area. Cohen-Daubechies-Feauveau 9/7 (CDF 9/7) Wavelet Transform (WT) [8] is a lifting scheme based wavelet transform that can reduce the computational complexity. The principle of lifting scheme is described by considering an input image x fed in parallel into a $\widehat{h}$ (low pass filter) and $\widehat{g}$ (high pass filter). The outputs of the two filters are then sub sampled by 2 ($\downarrow$ 2) to obtain low-pass subband y_L and high-pass subband y_H as shown in Fig. 1. The original signal can be reconstructed by synthesis filters h (low pass) and g (high pass), which take the up-sampled by 2 ($\uparrow$ 2) for y_L and y_H as inputs. An analysis and synthesis system has the perfect reconstruction property if and only if $x' = x$.

The mathematical representations of y_L and y_H can be defined as

$$\begin{cases} y_L(n) = \sum_{i=o}^{N_L-1} \widehat{h}(i)x(2n-i), \\ y_H(n) = \sum_{i=o}^{N_H-1} \widehat{g}(i)x(2n-i) \end{cases} \tag{3}$$

where N_L and N_H are the lengths of $\widehat{h}$ and $\widehat{g}$ respectively.

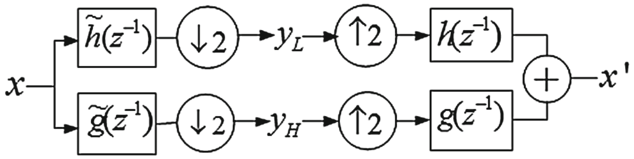


Fig. 1. Discrete wavelet transform (or subband transform) analysis and synthesis system [8].

This paper develops multi-layer lifting scheme WT with sparse basis, which decomposes an image x into 4 parts for each layer: $LL1$, $HL1$, $LH1$,and $HH1$ and each layer multiply by sparse matrix. This concept is also applied to the second and third level decomposition based on the principle of multi-resolution analysis. For example the $LL1$ subband is decomposed into four smaller subbands: $LL2$, $LH2$, $HL2$, and $HH2$. The three layer subbands are sparse so that

Orthogonal Matching Pursuit (OMP) algorithm [9] or Basis Pursuit (BP)[3] can be adopted to rebuild these parts directly.

4 Proposed Image De-noising Method Based on Compressive Sensing

For image de-noising application, we propose another novel image de-noising based on compressive sensing framework. The objective of novel compressive sensing de-noising process is to estimate the original image x with dimension $N \times N$ pixels by discarding the corrupted together with three popular different noises: Gaussian Noise, Poisson noise, and impulse (salt & pepper) noise n from the function f:

$$f = x + n \tag{4}$$

The real value sample of signal f which can be represented by transform coefficients x. That is

$$f = \Psi x = \sum_{i=1}^{N} (x+n)_i \psi_i \tag{5}$$

where $\Psi = [\psi_1, \psi_2, ..., \psi_N]$ is the transform basis matrix using by sparsity CDF 9/7 wavelet transform and $s = [(x+n)_1, (x+n)_2, ..., (x+n)_N]$ is an N-vector of coefficients with $x_i =< x, \psi_i >$ and there are only with $S \ll N$ significant elements in x. Most natural signals can be transformed to sparse domain by several conventional transforms.

We sample from f by mixing matrix or measurement matrix Φ that is stable and incoherence with matrix transform Ψ:

$$y = \Phi f = \Phi \Psi x = \Theta x = \Theta(x+n) \tag{6}$$

where Θ is the compressive sensing matrix. We need to reconstruct the original signal from this observation. It is known that sparsity is a fundamental principle in fidelity reconstruction, and noise is not sparse in the standard domain. Hence, we can reconstruct the exact signal due to sparsity. To remove noise, we develop a novel CS image de-noising Algorithm 1 and its flow chart is shown in Fig. 2.

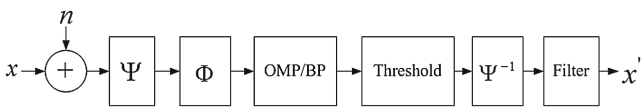


Fig. 2. Proposed image de-noising framework based on compressive sensing.

Algorithm 1. Proposed Image De-noising Algorithm with CS

1 Introduce noise to the image f.

2 Perform sparse domain CDF 9/7 wavelet transform to signal f into several sparse directions to CDF 9/7 wavelet sub-band $f_{j,l}$, where j is the decomposition level and l the number of direction level at each scale are expressed as

$$\widehat{f}^1 = W^1 f = \left[\widehat{f}^1_{LL}\widehat{f}^1_{LH}\widehat{f}^1_{HL}\widehat{f}^1_{HH}\right]^T$$

$$\widehat{f}^2 = W^2 \widehat{f}^1_{LL} = \left[\widehat{f}^2_{LL}\widehat{f}^2_{LH}\widehat{f}^2_{HL}\widehat{f}^2_{HH}\right]^T$$

$$\widehat{f}^3 = W^3 \widehat{f}^2_{LL} = \left[\widehat{f}^3_{LL}\widehat{f}^3_{LH}\widehat{f}^3_{HL}\widehat{f}^3_{HH}\right]^T$$

where $W^j, j \in 1,2,3$ represents the 2D CDF9/7 wavelet transform matrix of level j.

3 Apply CS scheme to each direction and decomposition level as follows:

$$y = \Phi_{j,l} \Psi x_{j,l}$$

4 Apply $l_1 - norm$ approach BP or OMP to reconstruct the signal x from y using step 3.

5 Apply proposed threshold (T) with the aid of Bayeshrink thresholding strategies via proposed threshold and is given by

$$T(j,l) = \beta \frac{\sigma^2_{j,l}}{\sigma^2_{w,j,l}}$$

$$\sigma^2_{j,l} = \left(\frac{median(|HP_{j.l}|)}{0.6745}\right)^2$$

$$\widehat{\sigma}^2_{w,j,l} = max(\widehat{\sigma}^2_y - \widehat{\sigma}^2, 0)$$

where $\widehat{\sigma}^2_y = \frac{1}{M \times N}\sum_{j,l=1}^{M,N} y^2_{j,l}$. β is the parameter define by threshold. With proposed threshold, a de-noised OMP/BP coefficient $\widehat{x}_{T,j,l}$ is calculated as follows:

$$\widehat{x}_{T,j,l} = \begin{cases} \frac{x_{r,j,l}[x_{r,j,l} - T^{\alpha}]}{|x_{r,j,l}|} & \text{for } |x_{r,j,l}| \geq T(j,l) \\ 0 & \text{for } |x_{r,j,l}| < T(j,l) \end{cases}$$

where $x_{r,j,l}$ is the coefficients value after image reconstruction by OMP/BP. α is smooth signal parameter and we have chosen $\alpha = 20$ and β is 0.3 for proposed threshold.

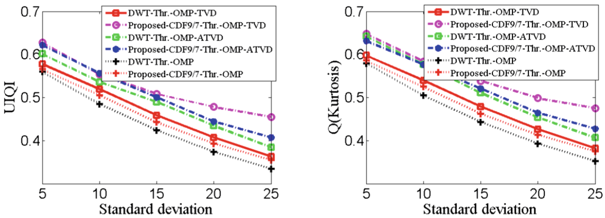
6 After threshold, we apply inverse CDF 9/7 wavelet transform to recovered image.

7 Finally recovered image is applied to filter operation methods (TVD algorithm [6], or ATVD algorithm [4]) to obtain de-noised image x'.

8 Sparse basis DWT or DCT is used to decompose the image into feature and non-feature regions in steps 2 and apply inverse DWT or DCT in step 6 for evaluation of de-noised image.

5 Experimental Results

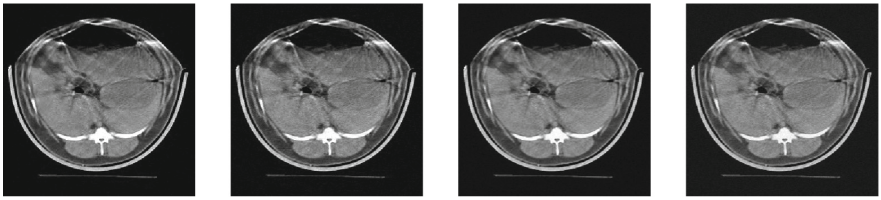
For evaluating the performance of proposed de-noising algorithm on images were selected from open source databases [7]. Consider, the noise is a combination of three popular noises including fixed Poisson noise at $\lambda = 0.9686$, 100 % impulse noise density, and white Gaussian noise with a fixed deviation level $\sigma \in [5, 10, 15, 20, 25]$ in the effect of low-light noise distribution in digital camera. This low-light noise is used as camera sensor matrix and camera performance. For comparison, besides the proposed de-noising frame based on proposed sparse basis CDF 9/7 WT with BP or OMP including TVD/ATVD and other de-noising techniques with sparse basis DWT and DCT by OMP or BP including TVD/ATVD algorithm are also used. This paper compares the evaluation of quality indexes (EIQ) for de-noised images on several measurements $M > N(N = 256)$. It is noted that the more measurements of M, the better will be the quality of de-noised images. Figure 3 shows the quality indexes of de-noised images obtained with different de-noising methods performed on the X-ray images. From these figures, UIQI and Q(Kurtosis) values of proposed sparse basis CDF9/7 WT with Gaussian measurements, OMP and proposed threshold show higher values than others. However, there is some noise inside de-noised image, so de-noising method such as TVD/ATVD has been applied there. The experimental results clearly show that the proposed method based on proposed sparse basis CDF 9/7 WT and proposed threshold by OMP or BP algorithm including TVD, ATVD out-performed all other four proposed methods for all values of the noise deviation in the range [5, 25]. Figure 4 shows the visual comparison of de-noised X-ray images for $\sigma = 10$ and for the necessity of filtering TVD or ATVD for the proposed de-noising algorithm. The comparison is clear with data plotted in Fig. 3(a)–(b), which shows the relationship between the UIQI and Q(Kurtosis) for different σ performed on an X-ray image. Similarly the visual comparison of de-noised brain (MR)-1 image in Fig. 6 for $\sigma = 5$ with data plotted in Fig. 5(a) and (b). The proposed sparse basis CDF 9/7 WT with BP and proposed threshold including TVD/ATVD show higher



(a) (b)

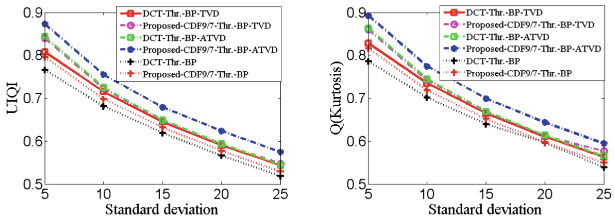
Fig. 3. Plots of image quality indexes for X-ray image versus noise standard deviations (σ) in the range [5, 25] for six different proposed de-noising approach.

quality of de-noised brain (MR)-1 image as shown in Fig. 6(a) and (c) as compared with sparse basis DCT for Fig. 6(b) and (d) in the range $\sigma = [5, 25]$. So, the TVD and ATVD is more important for this proposed de-noising algorithm. We also observes from Fig. 7 the computational complexity for novel de-noising framework based on CS.



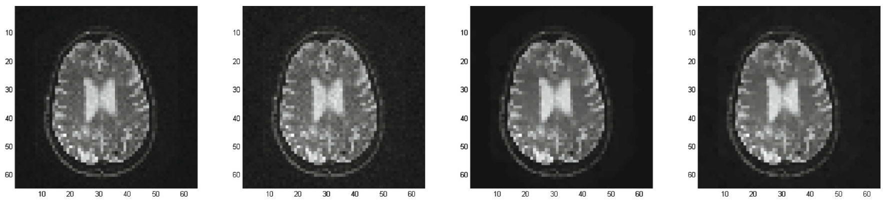
(a) (b) (c) (d)

Fig. 4. De-noising of X-ray image (a) Sparse CDF9/7 WT with BP algorithm and Thr., (b) DWT with BP and Thr., (e) Sparse CDF 9/7 WT with BP, Thr., and TVD, (f) DWT with BP, Thr., and TVD.



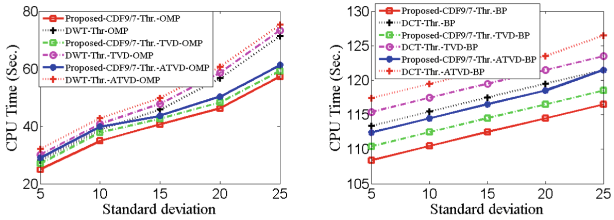
(a) (b)

Fig. 5. Plots of image quality indexes versus noise standard deviations (σ) in the range [5, 25] for six different proposed de-noising approach for brain (MR)-1 image.



(a) (b) (c) (d)

Fig. 6. De-noising of brain (MR)-1 image with (a) CDF9/7 WT with BP and Thr., (b) DCT with BP and Thr., (c) CDF 9/7 WT with BP, Thr., and ATVD, (d) DCT with BP, Thr., and ATVD.



(a) (b)

Fig. 7. Comparison of computational complexity for novel de-noising scheme for (a) X-ray, (b) brain (MR)-1 image.

6 Conclusion

In this paper, we proposed sparse domain CDF 9/7 wavelet transform for image compression based on CS. We also proposed an image de-noising algorithm based on compressive sensing framework that addresses the simultaneous removal of undesired noise components and preservation of high-frequency details in images. Experimental results demonstrate that the proposed sparse basis CDF 9/7 wavelet transform and proposed threshold are more efficient and effective for removing noise than sparse DWT and DCT in the image reconstruction algorithm OMP and BP including TVD and ADTV of proposed approach in terms of proposed image quality indexes and others like UIQI.

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