Dynamical Analysis of Neural Networks with Time-Varying Delays Using the LMI Approach

Shanmugam Lakshmanan[1(✉)], C.P. Lim[1], Asim Bhatti[1], David Gao[2], and Saeid Nahavandi[1]

[1] Center for Intelligent Systems Research, Geelong Waurn Ponds Campus, Deakin University, Geelong, Australia
lakshmanan.shanmugam@deakin.edu.au
[2] School of Applied and Biomedical Sciences, Federation University, Ballarat, Australia

Abstract. This study is concerned with the delay-range-dependent stability analysis for neural networks with time-varying delay and Markovian jumping parameters. The time-varying delay is assumed to lie in an interval of lower and upper bounds. The Markovian jumping parameters are introduced in delayed neural networks, which are modeled in a continuous-time along with finite-state Markov chain. Moreover, the sufficient condition is derived in terms of linear matrix inequalities based on appropriate Lyapunov-Krasovskii functionals and stochastic stability theory, which guarantees the globally asymptotic stable condition in the mean square. Finally, a numerical example is provided to validate the effectiveness of the proposed conditions.

Keywords: Neural networks · Interval time-varying delay · Stability · Linear matrix inequality

1 Introduction

Neural networks (NNs) constitute an important research topic in field of science and technology because of their extensive applications to various domains such as signal processing, parallel computing, and optimization problems [1,2]. Recently, studies of NNs along with time-delays have become more popular, which make NN models more complicated and interesting. Time-delays are encountered in neural processing and signal transmission, which can destabilize the whole networks, create oscillatory behaviours, and even cause chaos. Therefore, the analysis of NN models with time-delays plays a vital role in directly applying the NN models in real world problems. Indeed, many researchers have conducted dynamical analysis of NN models with time-delays (see, e.g., [3–6]).

Stability analysis plays a significant role in analysing time-delay systems. In the literature, many researchers have conducted stability analysis of time-delay systems using the Lyapunov-Krasowskii methodology with linear matrix inequalities (LMIs) [7,8]. Due to the effects of time-delay, the stability criteria can be

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classified into two types, i.e., delay-independent stability and delay-dependent stability. The delay-dependent stability criterion is less conservative as compared with the delay-independent one in the case of small time-delays. As such, many researchers have been investigated the delay-dependent stability criterion related to a variety of problems (ref [4–8]). As an example, the authors in [4], discussed the delay-dependent condition for cellular NNs with constant time-delays. Further, the authors in [5–8] argued that delay is varying with respect to time, and derived the stability conditions for NNs with time-varying delays. Based on the proposed results, many researchers have studied the stability criteria of time-varying delays that lie between 0 and their upper bounds, i.e., $0 \leq \tau(t) \leq h$. In practice, a time-delay typically exists in an interval. In other words, a time-delay varies in an interval for which the lower bound is not restricted to 0. For this particular reason, the stability criteria pertaining to the time-delay range has great significance for delayed NN models (see e.g., [9–12]). In [9], the authors initially investigated the stability problem of NNs based on interval time-varying delays by constructing an appropriate Lyapunov-Krasovskii function (LKF) and utilizing the free weight matrix approach. They showed that the proposed results were less conservative as compared with the existing results of NNs with interval time-varying delays.

Recently, NNs with Markovian jumping parameters have been investigated widely due to their random changes of structure. A NN has finite modes, and it may jump from one to another at different times. It has been pointed out in [13] that jumping between different NNs modes can be governed by a Markovian chain. Therefore, many researchers have been investigated NNs with Markovian jumping parameters (see, e.g.,[13–15]). As an example, the authors in [14] investigated the delayed uncertain Hopfield NN models with Markovian parameters, and the problem of state estimation was studied in [15] for jumping recurrent NN models with discrete and distributed delays. However, to the best of the authors' knowledge, the lower bound of time-varying delay is not restricted to 0 in this paper. In addition, not many results pertaining to stability analysis of NNs with Markovian jumping parameters by using convex combination techniques based on the delay interval have been established, which has motivated the present study.

Inspired by the above account, we aim to analyze the delay dependent stability criteria for NN models with interval time-varying delays by constructing suitable LKF and utilizing the free-weighting matrix approach, convex combination technique in this study. The sufficient condition is derived in terms of LMIs [16] for the considered problem with Markovian jumping parameters. The obtained formulae can be determined by using the Matlab LMI control toolbox. A numerical example is provided to illustrate the effectiveness of the proposed results.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ represent the n-dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively. For a given matrix, A^{-1} and A^T, denote its inverse and transpose, $X \geq Y$ (similarly, $X > Y$), where X and Y are symmetric matrices, i.e., $X - Y$ is positive semi-definite (similarly, positive definite). $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. diag$\{\cdots\}$ stands for a block diagonal matrix. The notation $*$ always denotes the symmetric block in a symmetric matrix. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ indicates a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions and $\mathcal{E}$ stands for the mathematical expectation. $L^2_{\mathcal{F}_0}([-h_2, 0], \mathbb{R}^n)$ denotes the family of all bounded $\mathcal{F}_0$- measurable, $C([-h_2, 0], \mathbb{R}^n)$ -valued random variables $\xi = \{\xi(\theta) : -h_2 \leq \theta \leq 0\}$ such that $\int_{-h_2}^{0} |\mathcal{E}(s)|^2 ds < \infty$.

2 Problem Description and Preliminaries

Consider the following NN model with time-varying delays:

$$\dot{y}_i(t) = -a_i y_i(t) + \sum_{j=1}^{n} b_{ij} g_j(y_j(t)) + \sum_{j=1}^{n} c_{ij} g_j(y_j(t - \tau(t))) + I_i,\ i = 1, \ldots, n \tag{1}$$

where n denotes the number of neurons in the NN, $y_i(t)$ denotes the state of the ith neuron at time t. $g_j(y_j(t))$ is the activation function of the jth neuron at time t. Parameters b_{ij} and c_{ij} represent, respectively, the connection weights and the delayed connection weights, from the jth neuron to the i neuron. I_i is the external bias on the ith neuron, $a_i > 0$ denotes the rate with which the ith neuron resets its potential to the resting state in isolation when it is disconnected from the network and external inputs. The time-varying delay $\tau(t)$ satisfies the following conditions.

$$0 \leq h_1 \leq \tau(t) \leq h_2, \quad \dot{\tau}(t) \leq \mu, \tag{2}$$

where h_1, h_2, and μ are constants. The NN model defined in (1) can be expressed in the matrix-vector form as follows.

$$\dot{y}(t) = -Ay(t) + Bg(y(t)) + Cg(y(t - \tau(t))) + I, \tag{3}$$

where $y(\cdot) = [y_1(\cdot), y_2(\cdot), \ldots, y_n(\cdot)]^T \in \mathbb{R}^n$, $A = diag\{a_1, \ldots, a_n\} > 0$, $B = (b_{ij})_{n\times n}$, $C = (c_{ij})_{n\times n}$, $I = [I_1, \ldots, I_n]$ and $g(y(\cdot)) = [g_1(y_1(\cdot)), \ldots, g_n(y_n(\cdot))]^T$.

Assumption 1: $g_i(\cdot)$ in (1) satisfies

$$l_i^- \leq \frac{g_i(x_1) - g_i(x_2)}{x_1 - x_2} \leq l_i^+,\ \ \forall x_1, x_2 \in \mathbb{R},\ x_1 \neq x_2,\ i = 1, ..., n, \tag{4}$$

where l_i^-, l_i^+ are known constants.

Taking the Markov jumping parameters into account, the delayed NN model defined in (3) becomes

$$\dot{y}(t) = -A(\eta(t))y(t) + B(\eta(t))g(y(t)) + C(\eta(t))g(y(t - \tau(t))) + I \tag{5}$$

where $\eta(t)$ $(t \geq 0)$ is a right-continuous Markov chain on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ taking values in a finite state space $\mathcal{S} = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{\imath\jmath})_{N\times N}$ and transition probability from the $\imath^{th}$ mode at t to the $\jmath^{th}$ mode, at $t + \Delta t$ $(\imath, \jmath \in \mathcal{S})$

$$P\{\eta(t + \Delta t) = \jmath | \eta(t) = \imath\} = \begin{cases} \gamma_{\imath\jmath}\Delta t + o(\Delta t), & \imath \neq \jmath, \\ 1 + \gamma_{\imath\imath}\Delta t + o(\Delta t), & \imath = \jmath, \end{cases}$$

where $\Delta t > 0$ and $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$, $\gamma_{\imath\jmath} \geq 0$ is the transition rate from $\imath$ to $\jmath$, if $\imath \neq \jmath$; while $\gamma_{\imath\imath} = -\sum_{\jmath=1,\ \jmath\neq\imath}^{N} \gamma_{\imath\jmath}$. If we shift the equilibrium point y^* in (5) to the origin by letting $x(t) = y(t) - y^*$, system (5) can be transformed into:

$$\dot{x}(t) = -A_\imath x(t) + B_\imath f(x(t)) + C_\imath f(x(t - \tau(t))), \tag{6}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T$ is the state vector of the transformed system, and $f_i(x(t)) = g_i(x_i(t) + y_i^*) - g_i(y_i^*)$, $i = 1, 2, ..., n$.
From Assumption 1, $f_i(x(t))$ satisfies

$$l_i^- \leq \frac{f_i(x_1) - f_i(x_2)}{x_1 - x_2} \leq l_i^+, \ \ \forall x_1, x_2 \in \mathbb{R},\ x_1 \neq x_2,\ i = 1, ..., n. \tag{7}$$

Let $x(t, \phi)$ be the state trajectories of system (6) with the initial condition $\phi \in L^2_{\mathcal{F}_0}([-h_2, 0], \mathbb{R}^n)$. It can be seen that system (6) admits a trivial solution $x(t, 0) \equiv 0$ corresponding to the initial condition $\phi = 0$.

3 Main Results

For convenience, the following notations are used:

$$L_1 = \mathrm{diag}\{l_1^-, l_2^-, \cdots, l_n^-\}, \quad L_2 = \mathrm{diag}\{l_1^+, l_2^+, \cdots, l_n^+\} \ \text{ and}$$
$$\xi^T(t) = \Big[x^T(t)\ \ x^T(t-h_1)\ \ x^T(t-\tau(t))\ \ x^T(t-h_2)\ \ f^T(x(t))\ \ f^T(x(t-\tau(t))\ \ \dot{x}^T(t)\Big].$$

We derive a range-dependent time-delay stability condition for delayed NNs (6) with Markovian jumping parameters in the following theorem.

Theorem 1. *Given scalars $h_2 > h_1 \geq 0$ and $\mu \geq 0$, the delayed NN model in (6) is globally asymptotically stable in the mean square if symmetric matrices $P_\imath > 0$, $Q_l > 0, R_1 > 0, R_2 > 0$ $(l = 1, 2, 3)$, positive diagonal matrices $W, \Delta, \Gamma_1, \Gamma_2$ and real matrices $N_a, M_a, X_a, Y_a, Z_a (a = 1, 2)$ of appropriate dimensions exist, such that the following LMIs hold:*

$$\begin{bmatrix} \Xi^\imath & \sqrt{h_1}\ \bar{N} & \sqrt{h_2 - h_1}\ \bar{Y} \\ * & -R_1 & 0 \\ * & * & -R_2 \end{bmatrix} < 0, \tag{8}$$

$$\begin{bmatrix} \Xi^\imath & \sqrt{h_1}\ \bar{N} & \sqrt{h_2 - h_1}\ \bar{X} \\ * & -R_1 & 0 \\ * & * & -R_2 \end{bmatrix} < 0 \tag{9}$$

where $\Xi^\imath_{7\times 7}$ with entries:

$$\Xi_{1,1} = -Z_1 A_\imath - A_\imath^T Z_1^T + Q_1 + Q_2 + Q_3 + N_1 + N_1^T - 2L_1 \Gamma_1 L_2 + \sum_{\jmath=1}^{N} \gamma_{\imath\jmath} P_\jmath,$$

$\Xi_{1,2} = -N_1 + N_2^T,\ \Xi_{1,5} = Z_1 B_\imath + \Gamma_1(L_1 + L_2),\ \Xi_{1,6} = Z_1 C_\imath,$
$\Xi_{1,7} = P - L_1^T W + L_2^T \Delta - A_\imath^T Z_2^T - Z_1,\quad \Xi_{2,2} = -Q_1 - N_2 - N_2^T + X_1 + X_1^T,$
$\Xi_{2,3} = -X_1 + X_2^T,\quad \Xi_{3,3} = -(1-\mu)Q_3 - X_2 - X_2^T + Y_1 + Y_1^T - 2L_1\Gamma_2 L_2,$
$\Xi_{3,4} = -Y_1 + Y_2^T,\ \Xi_{3,6} = \Gamma_2(L_1 + L_2),\quad \Xi_{4,4} = -Q_2 - Y_2 - Y_2^T,$
$\Xi_{5,5} = -2\Gamma_1,\ \Xi_{5,7} = W - \Delta + B_\imath^T Z_2^T,\quad \Xi_{6,6} = -2\Gamma_2,\quad \Xi_{6,7} = C_\imath^T Z_2^T,$
$\Xi_{7,7} = h_1 R_1 + (h_2 - h_1)R_2 - Z_2 - Z_2^T,\ \bar{N} = [N_1^T\ \ N_2^T\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T,$
$\bar{X} = [0\ \ X_1^T\ \ X_2^T\ \ 0\ \ 0\ \ 0\ \ 0]^T,\quad \bar{Y} = [0\ \ 0\ \ Y_1^T\ \ Y_2^T\ \ 0\ \ 0\ \ 0]^T.$

Proof. Choose the following LKF for the delayed NN with Markovian jumping in (6),

$$\begin{aligned} V(x(t), t, \eta(t) = \imath) &= x^T(t) P_\imath x(t) + 2\sum_{j=1}^{n} \Big(w_j \int_0^{x_j(t)} (f_j(s) - l_i^- s) ds + \delta_j \int_0^{x_j(t)} (l_i^+ s - f_j(s)) ds \Big) \\ &+ \int_{t-h_1}^{t} x^T(s) Q_1 x(s) ds + \int_{t-h_2}^{t} x^T(s) Q_2 x(s) ds + \int_{t-\tau(t)}^{t} x^T(s) Q_3 x(s) ds \\ &+ \int_{-h_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta. \end{aligned} \tag{10}$$

Let $V(x(t), t, \eta(t) = \imath, t > 0)$ $\mathbb{L}V(t)$ be the stochastic positive LKF. The weak infinitesimal operator is defined as

$$\begin{aligned} \mathbb{L}V(x(t), t, \eta(t) = \imath) &= \lim_{\Delta t \to 0} \frac{1}{\Delta t}\Big[\mathcal{E}\{ Vx((t+\Delta t), r(t+\Delta t), t+\Delta t) | x(t), \eta(t) = \imath \} \\ &\quad -V(x(t), \eta(t) = \imath, t) \Big] \\ &= \frac{\partial V}{\partial t} + \dot{x}^T(t) \frac{\partial V}{\partial x}\Big|_{\eta(t)=\imath} + \sum_{\jmath=1}^{N} \gamma_{\imath\jmath} V(x(t), t, \imath, \jmath). \end{aligned}$$

Now take $\mathbb{L}V(t)$ along a given trajectory of the delayed NN in (6) as follows

$$\begin{aligned} \mathbb{L}V(x(t), t, \eta(t) = \imath) \le\ & 2x^T(t) P_\imath \dot{x}(t) + \sum_{\jmath=1}^{N} \gamma_{\imath\jmath} x^T(t) P_\jmath x(t) + 2[f(x(t)) - L_1 x(t)]^T W \dot{x}(t) \\ & +2[L_2 x(t) - f(x(t))]^T \Delta \dot{x}(t) + x^T(t) Q_1 x(t) - x^T(t-h_1) Q_1 x(t-h_1) \\ & +x^T(t) Q_2 x(t) - x(t-h_2) Q_2 x(t-h_2) + x^T(t) Q_3 x(t) \\ & -(1-\mu) x^T(t-\tau(t)) Q_3 x(t-\tau(t)) + h_1 \dot{x}(t) R_1 \dot{x}(t) \\ & -\int_{t-h_1}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds + (h_2 - h_1) \dot{x}^T(t) R_2 \dot{x}(t) \\ & -\int_{t-h_2}^{t-h_1} \dot{x}^T(s) R_2 \dot{x}(s) ds. \end{aligned} \tag{11}$$

where $W = \text{diag}\{w_1, ..., w_n\}$ and $\Delta = \text{diag}\{\delta_1, ..., \delta_n\}$. It should be noted that

$$-\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)ds = -\int_{t-\tau(t)}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)ds - \int_{t-h_2}^{t-\tau(t)} \dot{x}^T(s)R_2\dot{x}(s)ds.$$

By Lemma 2.1 in [17], it follows that

$$-\int_{t-h_1}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \leq h_1\xi^T(t)\bar{N}R_1^{-1}\bar{N}^T\xi(t) + 2\xi^T(t)\bar{N}[x(t) - x(t-h_1)] \quad (12)$$

$$\int_{t-\tau(t)}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)ds \leq (\tau(t) - h_1)\xi^T(t)\bar{X}R_2^{-1}\bar{X}^T\xi(t)$$
$$+2\xi^T(t)\bar{X}[x(t-h_1) - x(t-\tau(t))] \quad (13)$$

$$\int_{t-h_2}^{t-\tau(t)} \dot{x}^T(s)R_2\dot{x}(s)ds \leq (h_2 - \tau(t))\xi^T(t)\bar{Y}R_2\bar{Y}^T\xi(t)$$
$$+2\xi^T(t)\bar{Y}[x(t-\tau(t)) - x(t-h_2)]. \quad (14)$$

Further, we add the following zero equation with any chosen matrices of Z_1 and Z_2

$$2[x^T(t)Z_1 + \dot{x}^T(t)Z_2][-A_\imath x(t) + B_\imath f(x(t)) + C_\imath f(x(t-\tau(t)) - \dot{x}(t)] = 0. (15)$$

Noting that for positive diagonal matrices Γ_1, Γ_2 and Assumption 1, one has

$$-f^T(x(t))\Gamma_1 f(x(t)) + 2x^T(t)\Gamma_1(L_1 + L_2)f(x(t)) - 2x^T(t)L_1\Gamma_1 L_2 x(t) \geq 0. \quad (16)$$

$$-f^T(x(t-\tau(t)))\Gamma_2 f(x(t-\tau(t))) + 2x^T(t-\tau(t))\Gamma_2(L_1 + L_2)f(x(t-\tau(t)))$$
$$-2x^T(t-\tau(t))L_1\Gamma_2 L_2 x(t-\tau(t)) \geq 0. \quad (17)$$

Substituting (12)–(14) into (11) and adding (15)–(17) into (11), yields

$$\mathbb{L}V(x(t), t, \eta(t) = \imath) \leq \xi^T(t)\Big[\Xi^\imath_{\tau(t)}\Big]\xi(t) \quad (18)$$

where $\Xi_{\tau(t)} = \Xi + h_1\bar{N}R_1^{-1}\bar{N}^T + (\tau(t) - h_1)\bar{X}R_2^{-1}\bar{X}^T + (h_2 - \tau(t))\bar{Y}R_2^{-1}\bar{Y}^T$. Taking the mathematical expectation $\mathbb{E}$ on both sides of (18) and from LMIs (8)–(9), we can obtain

$$\mathbb{E}\Big\{\mathbb{L}V(x(t), t, \eta(t) = \imath)\Big\} \leq \mathbb{E}\Big\{\xi^T(t)\Big[\Xi^\imath_{\tau(t)}\Big]\xi(t)\Big\} \leq -\lambda\{\mathbb{E}\{|x(t, \phi, \imath_0)\|^2\}\},$$

where $\lambda = \lambda_{min}\Big(-\Xi^\imath_{\tau(t)}\Big)$. This implies that system (8) is globally asymptotically stable in the mean square. Notice that $(\tau(t)-h_1)\bar{X}R_2^{-1}\bar{X}^T + (h_2-\tau(t))\bar{Y}R_2^{-1}\bar{Y}^T$ is a convex combination of matrices $\bar{X}R_2^{-1}\bar{X}^T$ and $\bar{Y}R_2^{-1}\bar{Y}^T$ on $\tau(t) \in [h_1,\ h_2]$. Therefore, by following the convex analysis approach, $\Xi_{\tau(t)} < 0$ if and only if

$$\Xi^\imath_{\tau(t)}\Big|_{\tau(t)=h_1} < 0, \quad (19)$$

$$\Xi^\imath_{\tau(t)}\Big|_{\tau(t)=h_2} < 0. \quad (20)$$

Using Schur complement, (19)–(20) are equivalent to (8)–(9), respectively. This completes the proof.

Remark 1. A range-dependent time-delay stability criterion has been proposed for a delayed NN model with Marakovian jumping parameters. The sufficient condition has more information of the lower and upper bounds of time-varying delays. Moreover, we have introduced few free weight matrices, and expressed the derived sufficient condition in two LMIs (8) and (9) by using the convex combination technique, which is based on $\tau(t) \in [h_1, h_2]$. Here, it should be mentioned that the restrictive condition of $\mu \leq 1$ has been removed in Theorm 3.1, and we can easily derive the corresponding results in the non-differentiable case of time-varying delays when $Q_3 = 0$ in LKF (10).

4 Numerical Example

A numerical example is presented to illustrate the potential benefits and effectiveness of the developed method for delayed NNs with Markovian jumping parameters. Consider a three-order delayed NN of (6) with mode $\imath = 2$ and the following parameters

$$A_1 = \begin{bmatrix} 1.8 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1.3 & 0 \\ 0 & 0 & 1.8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.4 & 1.5 & 0.1 \\ 0.56 & 0 & -1.4 \\ 0.1 & 1 & 1.2 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} -0.5 & 1.2 & 0 \\ -0.5 & 0 & 1 \\ 0.45 & 1.25 & 0.3 \end{bmatrix}. \quad C_1 = \begin{bmatrix} 1.5 & 1 & 0.8 \\ 0 & 1.5 & 0 \\ 0.25 & 1.2 & 0.5 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1.8 & 0 & 0.24 \\ 0 & 0.8 & 0 \\ 0.54 & 1.2 & 0.9 \end{bmatrix}.$$

In this example, the activation function is assumed to satisfy Assumption 1 with $l_1^- = l_2^- = l_3^- = 0$, $l_1^+ = 0.2$, $l_2^+ = 0.2$ and $l_3^+ = 0.2$. Then the transition probability matrix is assumed to be $\Gamma = \begin{bmatrix} -7 & 7 \\ 6 & -6 \end{bmatrix}$. Let $h_1 = 0.5, h_2 = 1, \mu = 1.1$. Using the Matlab LMI control toolbox to solve LMIs (8)–(9) in Theorem 1, we obtain the following feasible matrices:

$$P_1 = \begin{bmatrix} 6.1996 & -1.5709 & -3.7824 \\ -1.5709 & 5.2842 & -0.2011 \\ -3.7824 & -0.2011 & 7.4683 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 6.0627 & -1.5764 & -3.7717 \\ -1.5764 & 5.1513 & -0.0142 \\ -3.7717 & -0.0142 & 7.1369 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 3.5372 & -0.9294 & -1.6140 \\ -0.9294 & 2.7700 & 0.0282 \\ -1.6140 & 0.0282 & 3.5646 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 3.7535 & -1.0086 & -1.6783 \\ -1.0086 & 2.8251 & -0.0018 \\ -1.6783 & -0.0018 & 3.7860 \end{bmatrix}.$$
$$Q_3 = \begin{bmatrix} 1.3690 & -0.4091 & -0.7585 \\ -0.4091 & 0.8596 & 0.0282 \\ -0.7585 & 0.0282 & 1.3981 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 2.5604 & -0.5296 & -1.3084 \\ -0.5296 & 3.3156 & 0.0930 \\ -1.3084 & 0.0930 & 3.4785 \end{bmatrix},$$
$$R_2 = \begin{bmatrix} 2.6831 & -0.5575 & -1.3459 \\ -0.5575 & 3.3690 & 0.1050 \\ -1.3459 & 0.1050 & 3.5392 \end{bmatrix}, \quad W = \mathrm{diag}\{6.1985,\ 6.7736,\ 6.3091\},$$
$$\Delta = \mathrm{diag}\{6.6897,\ 7.0455,\ 7.8224\}.$$

Therefore, the proposed NN model with time-varying delays is globally asymptotically stable. In addition, Fig. 1(a) shows the convergence of the state trajectories of the delayed NN model to the zero equilibrium point with different initial conditions. The response of the Markovian jumping modes are shown in Fig. 1(b).

5 Conclusions

In this paper, we have studied the range-dependent time-delay stability criteria for delayed NN models with Markovian jumping parameters. Based on suitable

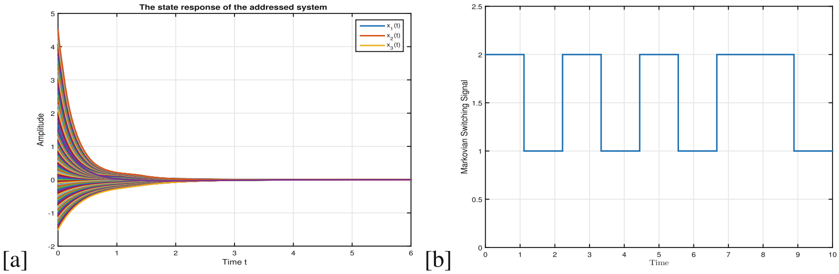

Fig. 1. (a). State trajectories of delayed NNs with different initial conditions. (b). The response of Markovian jumping signal when mode $\imath = 2$.

LKF, integral inequalities, LMI framework, and convex combination technique, the conditions for delay-dependent stability criteria are derived. From the numerical example, it is evident that the proposed method is effective, and is able to provide less conservative results.

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