Nonlinear Acoustics

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Contents

Abstract

Early detection and continuous tracking of material micro-damages have been one of the most demanding techniques in industries. Due to the sensitivity of acoustic nonlinearity to micro-damage, the nonlinear ultrasonic technique has been explored as a promising tool for early detection of micro-damages. In this chapter, we breifly introduce the earlier efforts and recent development of the nonlinear acoustics and their applications for nondestructive testing and

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evaluation (NDT& E). Some advanced techniques based on measure of nonlinear acoustics for NDT& E are also introduced as potential and attractive means.

Definition and Historical Background

Nonlinear acoustics is a branch of physics and acoustics dealing with acoustic waves propagation in either fluid or solid media with nonlinear response. Generally, in the case of acoustic waves with sufficient amplitudes in the media, the wave motion should be governed by nonlinear wave equations for the reason that linearization is no longer possible in this situation. In solid materials, grain boundaries, lattice anharmonicity, energy absorption, imperfect interfaces etc. can be the sources for arising nonlinearity in the solid media (Landau and Lifshitz 1970; Campos-Pozuelo et al. 2006; Kim et al. 2006b). Due to material nonlinearity, acoustic waveform propagation in the media can distort. The acoustic nonlinear responses include creating accompanying harmonics, multiplication of waves of different frequencies, and, under resonance conditions, changes in resonance frequencies as a function of drive amplitude (Van Den Abeele et al. 2000a).

The use of nonlinear acoustics in solid media can be significant in nondestructive testing (NDT) for the interrogation of micro-damages in materials at an early stage. In other words, the sensitivity of nonlinear acoustic methods for the detection or evaluation of material damage is far higher than that of linear ones. Thus, acoustic nonlinear responses in solid media can be used as a promising nondestructive testing method in solid media for micro-damage detection. Acoustic nonlinearity is usually characterized by a quantitative parameter that quantifies the amount by which an ultrasonic wave is distorted as the acoustic wave travels through the specimen. More detailed definition of nonlinear parameter for various types of ultrasonic waves in solid media will be introduced later in this chapter. The principle of nonlinear ultrasonic NDT is based on the fact that the level of acoustic nonlinearity in the materials containing damage is more than those in materials with no damage.

For acoustic waves propagation in media with nonlinearity (micro-cracks or damage), various nonlinear phenomena will be generated. Thus, different nonlinear acoustic techniques are developed based on the measure of these phenomena. For acoustic wave of single frequency propagating through a nonlinear medium, new components of higher and lower frequency can be generated in the material. Higher-frequency waves must have frequencies that are integer multipliers of the input wave frequency, which is usually called higher harmonics generation technique (Hikata and Elbaum 1966). Under certain conditions the generated wave frequency can be half of the input wave frequency, which corresponds to sub-harmonics generation technique (Chomas et al. 2002). For two or more acoustic waves with different frequencies propagating in the nonlinear media, the interactions of these waves with material micro-damages can produce nonlinear components with other frequencies. These new components with either a sum or difference of frequencies can be used to evaluate or detect the material nonlinearity (Kuvshinov et al. 2013). Monitoring the resonance frequency shift with the increase of excitation amplitude is another nonlinear acoustic-based technique

(Van Den Abeele et al. 2000b; Muller et al. 2005). Nonlinear wave modulation spectroscopy is also an interesting technique that has been widely used to evaluate material nonlinearity. The technique utilizes a continuous high-frequency probe wave and a low-frequency vibration or pump wave. The pump wave is generally a given resonance mode of the tested sample (Kober and Prevorovsky 2014).

Acoustic nonlinearity can be used for characterization of microstructural evolution and detection of material damage. Initially, micro-damaged materials show progressively enhanced features of nonlinear acoustic response. Classical nonlinear acoustic theory has explained the nonlinear behavior in the material with nonlinearity at the atomic/molecular scale very well. The enhanced nonlinear response arising from the complex compliance of local or volumetric damage dominates the nonlinear sources. However, there are two issues that need to be clarified:

1. The mechanism of acoustic nonlinear response in solid media is far more complicated and not yet well understood. In addition, even for the classic quadratic nonlinearity, the measured acoustic nonlinearity should also be carefully clarified (Qu et al. 2011).
2. For different types of damage in solid media, the corresponding nonlinear mechanisms are also different and the nonlinear behavior of acoustic waves in solid media with cracks or damage may induce coupling phenomena with classical nonlinearity, as well as hysteresis and discrete memory (Van Den Abeele et al. 2000a). Thus, quantitative characterization of material nonlinearity by acoustic waves has not yet been demonstrated.

Earlier Efforts

The use of nonlinear acoustic waves for nondestructive testing is receiving increasing attention for the high sensitivity of the approach (Donskoy et al. 2001). Earlier efforts on the theory and applications of nonlinear acoustic have been reviewed in text books, chapters, as well as review papers. M.F. Hamilton and D. T. Blackstock edited one book about nonlinear acoustic in 1998 (Hamilton and Blackstock 1998). In this book, a systematic introduction of nonlinear acoustics is provided. Considering the rapid development in nonlinear acoustic techniques, some researchers reviewed the advance of nonlinear ultrasonic techniques for nondestructive assessment of micro-damage in solid media (Jhang and Kim 1999; Matlack et al. 2015; Li et al. 2017). Recently, J. Rushchitsky edited a book which provides a coherent treatment of the theory of propagating nonlinear elastic waves in solid media (Rushchitsky 2014). The principles of some typical nonlinear ultrasonic techniques are addressed in this chapter.

Higher Harmonic Generation

Higher harmonic generation is one of the typical nonlinear phenomena for acoustic waves in nonlinear media, and it is viewed as the classical acoustic nonlinear phenomenon. Physically, the phenomenon of higher harmonic generation is related to nonlinearity in the elastic behavior of material (Goldberg 1956). To date, a number of investigators have applied this nonlinear ultrasonic technique to assess fatigue damage in different materials. The correlation between dislocation density levels within the fatigued material and increase in acoustic nonlinearity has been reported (Nagy 1998; Jhang 2000). Many researches have carried out measurements of acoustic nonlinearity versus material degradation in different kinds of material (Nagy 1998; Jhang 2000; Cantrell and Yost 2001; Kim et al. 2006a; Li et al. 2012a). The trend of acoustic nonlinear response in the specimen with improved material properties by heat treatment was also researched (Li et al. 2013). The second harmonic generation of surface wave was discussed, and experiments on SAW (surface acoustic wave) harmonic generation were first reported in aluminum and steel (Rischbieter 1967; Herrmann et al. 2006a). Vella discussed the nonlinear interaction of two collinear SAWs, where the rigorous theory of thermoelasticity was used to derive exact expressions for the nonlinear force and stress fields (Padmore and Stegeman 1976). Considering that second harmonic generation accompanies more structural information and the distinct advantages of guided wave techniques, nonlinear ultrasonic guided waves draw significant attention from the NDE community for material characterization and micro damage detection. An investigation of second harmonic generation of guided waves in isotropic plates has been first reported by Deng (1999). de Lima and Hamilton (2003) investigated the second harmonics field of elastic wave propagation in an isotropic plate. Srivastava et al. reported the possibility of existence of antisymmetric or symmetric Lamb wave modes at higher harmonics (Srivastava and di Scalea 2009). Higher harmonic generation of various guided waves has been used to evaluate material nonlinearity either in plates or tube-like structures as well as composites (Pruell et al. 2007; Liu et al. 2013; Li et al. 2012).

Nonlinear Wave Modulation Spectroscopy

Nonlinear wave modulation spectroscopy is another nonlinear acoustic technique for evaluation of material nonlinearity. The technique is based on the interactions of two waves with probe frequency and pump frequency manifesting themselves as side-band components in the frequency spectra of the received signal. The nonlinear mechanism has been illustrated through the behavior ensued by the existence of a crack in a sample by Sutin (Solodov 1998). In addition, vibro-acoustic modulation techniques were formerly conceived for the detection of localized defects such as cracks in structural parts. The Nonlinear response of acoustic waves modulation spectroscopy caused by various nonlinearity sources was also reviewed (Van Den Abeele 2000). In cases of localized defects such as cracks, the modulation arises from the nonlinear stiffness that ensues from the interfacial contact. In such cases, the behavior can even be chaotic (Solodov and Korshak 2002). Compared to the higher harmonic generation technique, the modulation technique offers some advantages (Donskoy and Sutin 1998). First, higher harmonic generation requires a homogeneous travelling path to take advantage of the cumulative effect – thus this is difficult to fulfill in the presence of reflecting boundaries and other structural inhomogeneities. Second, high voltages are needed, which frequently add some nonlinear background signal and that may affect the sensitivity of the technique. Recently, nonlinear impact resonance acoustic spectroscopy (NIRAS) was proposed based on the conventional nonlinear wave modulation technique. It has been shown to be highly sensitive to defects, especially to small cracks in materials (Klepa et al. 2012; Eiras et al. 2014; Hilloulin et al. 2014). Wave mixing can also be considered as a special case of nonlinear wave modulation spectroscopy, which, due to the cross-interactions of two acoustic waves at different frequencies, happens mainly in certain mixing zones in the specimen (Croxford et al. 2009; Demcenko et al. 2012).

Shift of Resonance Frequency

Nonlinear resonance techniques monitor the resonance frequency shift and attenuation variations with increasing amplitude of the excitation. The resonance frequency and attenuation are determined for different excitation levels. The material nonlinearity is manifested as a downward resonance peak shift and a decrease of the quality factor (*Q*) – inverse of attenuation – with increasing excitation amplitude. Therefore, from the downward resonance frequency shift, the third-order elastic nonlinear term can be obtained as

$$\frac{f - f_o}{f_o} \approx \frac{\delta}{2} \cdot \varepsilon^2 \tag{1}$$

where ε is strain, f_o is the resonance frequency in the linear strain range, f is the frequency in the nonlinear stain range, and δ is a measure of material nonlinear parameter. In practice, f_o is the resonance frequency for the lowest excitation level. The technique can be used to provide insight into the ultimate stress in brittle materials such as concrete and the yield stress for ductile materials (Zarembo et al. 1989). However, experimental evidence in polycrystalline solids and rocks revealed a linear dependence of the resonance frequency and attenuation shifts with strain amplitude along with an unexpectedly high third harmonic amplitude (Read 1940; Guyer et al. 1995; Guyer and Johnson 1999; Johnson and Rasolofosaon 1996). These observations do not align with the classical nonlinear behavior and were associated with hysteresis in the stress-strain relationship. Such behavior was found to be the characteristic of materials with defects at the mesoscale level: rocks, concrete, soil, cracked materials, etc., which are collectively termed nonlinear mesoscopic elastic materials, NMEM (Guyer and Johnson 1999). By including hysteresis in modeling resonance experiments (Nazarov et al. 2003), it was demonstrated that the resonance frequency shift is proportional to the strain amplitude ($\Delta\varepsilon$), so that

$$\frac{\Delta f}{f_o} = \alpha_f \cdot \Delta\varepsilon \tag{2}$$

along with a linear decrease of attenuation as

$$\frac{1}{Q} - \frac{1}{Q_o} = \alpha_Q \cdot \Delta\varepsilon \tag{3}$$

where Q is the quality factor (inverse of attenuation) and Q_o is the quality factor obtained in linear strain regime. These dependences (Eqs. 2 and 3) may be of a higher order if the characteristics of the hysteretic function change (Pecorari and Mendelsohn 2014). The parameters α_f and α_Q quantify the extent of hysteresis and are presumed to have the same physical origins (Johnson and Sutin 2005).

Recent Trends

Slow Dynamics

Slow dynamics (SD) is a novel nonlinear acoustic method, which is based on monitoring changes in the thermodynamic state of the material. This may be done, for example, by recording the resonant frequency for a specific resonance mode of an object and obtaining the change in the sound wave speed. One way to monitor SD is to probe the change in resonant frequency with a low-amplitude acoustic wave before and, again, after some disturbance of the thermodynamic equilibrium state. Rheological models were used to simulate the phenomenological behavior of slow dynamics (Bentahar et al. 2006; Favrie et al. 2015; Guyer et al. 1998; Nazarov and Radostin 2015; Ten Gate and Shankl 1996). However, although the nonlinear hysteresis and relaxation can be fairly well represented by these models, the underlying mechanisms may be different in different materials (Nazarov and Radostin 2015). Indeed, the physical origins of slow dynamics are still not very well understood. On the other hand, there seems to exist a strain amplitude threshold while at low strain amplitudes – say below ~10^{-7} the material exhibits nonlinear classical behavior. Beyond this threshold amplitude, hysteresis is activated, which is accompanied by a slow dynamic recovery; that is, the material enters a nonequilibrium or nonclassical regime. Slow dynamic effects appear to be related to the damage features such as micro-cracking. Various resonance-based techniques have been developed to assess the mechanical integrity of various materials (Van Den Abeele et al. 2000b). Slow dynamic effects coexist during dynamic excitation (Johnson and Sutin 2005), so that the measurement of the hysteretic parameters may be affected by slow dynamic effects. Such an effect can be minimized by increasing the time lapse between consecutive acquisitions.

Subharmonic Phased Array Technique

To enhance the selectivity in the nonlinear ultrasonic response induced by micro-cracks, recently the subharmonic phased array technique was developed

(Ohara et al. 2007, 2009; Sugawara et al. 2015). It combines the sensitivity of nonlinear ultrasonics and the high power input of the phased array approach. Ohara et al. (2007) proposed a practical closed-crack imaging apparatus, called the subharmonic phased array for crack evaluation (SPACE) on the basis of subharmonic generation by short bursts and phased array algorithm with frequency filtering.

The experimental configuration of SPACE is shown in Fig. 1; a $LiNbO_3$ single-crystal transmitter with a polyimide wedge was used to generate intense ultrasound and a phased array sensor system was used as a receiver to concentrate on reception. When high-energy ultrasonic excitation is used, scattering of fundamental and subharmonic waves occur at the open and closed parts of the crack, respectively. The scattered waves received by the array sensor are converted to digital signals. Subsequently, they are digitally filtered at fundamental and subharmonic frequencies. After their phase shift following the delay law, they are added. Finally, the root-mean-square value is calculated as intensity at a focal point. This process is repeated over a scan area with incremental steps to create images. The fundamental and subharmonic images obtained can indicate the open and closed parts of cracks, respectively. Researchers used the SPACE technique to evaluate closed fatigue cracks and stress corrosion cracks. The measurement accuracy of SPACE for such micro-cracks is yet to be determined. It has been demonstrated that SPACE is very useful for correcting the underestimation of crack depths.

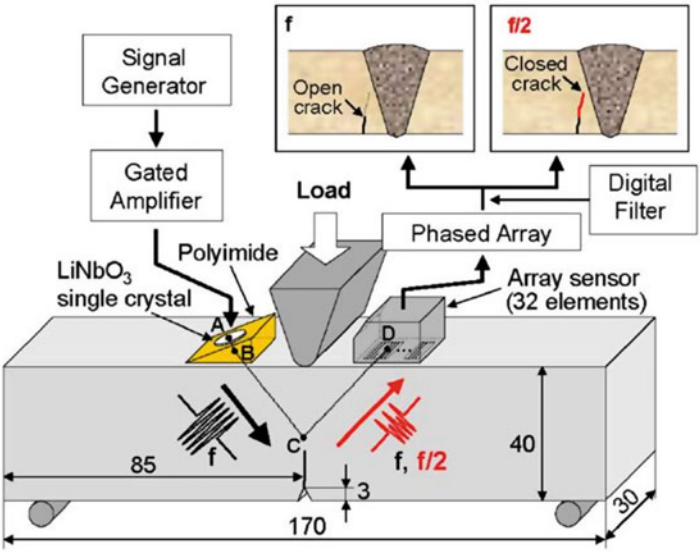


Fig. 1 Experimental configuration of SPACE (From Ohara et al. (2007))

Nonlinear Laser Ultrasonic Technique

Traditionally, contact methods based on piezoelectric transducers and/or capacitive probes have been used to monitor the generation of harmonic energy in a material substrate. These approaches suffer from a number of problems, however, that place restrictions or limits on measurement capabilities. There are often special requirements for specimen preparation, for example, that may require optically flat and parallel surfaces, or requirements on specimen conductivity. In addition, measurements are typically limited to a single measurement location, due to the need for hard-bonding of transducers or restrictions imposed by the placement of the probes. The spatial resolution capabilities of a typical measurement are also very coarse in nature, and involve an averaged or integrated signal over the entire receiver area, which can impact the quality of the measurements. An alternative method for detecting and characterizing the harmonic ultrasonic field in a material involves the use of laser interferometry techniques (Scruby and Drain 1990; Jia and de Billy 1992; Moreau 1995; Hurley and Fortunko 1997; Stratoudaki et al. 2011). Laser interferometry has long been used for dynamic motion measurements and it offers several advantages for making nonlinear ultrasonic measurements. Because coherent light is used as the probe, measurements can be made in a noncontact, remote, and nonintrusive manner. High spatial resolutions are also possible (1–10 microns) without reductions in measurement sensitivity. Interferometric measurements are also directly related to the optical wavelength used, which provides an absolute measure of the ultrasonic displacement levels. They also have a truly broadband frequency response, which is difficult to achieve with piezoelectric probes. And, finally, by raster-scanning the laser beam position with respect to the material surface, a high-resolution image of the harmonic (and fundamental) ultrasonic displacement field(s) can be created.

Recently, laser techniques for generation of nonlinear SAW pulses were developed resulting in the observation of strong nonlinear effects, such as the formation of shock fronts and drastic changes of the pulse shape and duration (Jia and de Billy 1992). It was demonstrated that a nonlinear compression as well as an extension of a wave pulse may take place depending on the nonlinear acoustic parameters (Hess et al. 2014; Liu et al. 2013; Liu et al. 2014). A noncontact testing with laser-generated ultrasonic transmitter is a very attractive technique in surface wave inspection. It can be used for online inspection and structural health monitoring, where contact methods with embedded sparse sensors cannot be applied such as are the cases of high temperature applications.

Nonlinear Ultrasonic NDT

In this chapter, we view the second harmonic generation as the typical example to illustrate the nonlinear ultrasonic NDT methods. Thus, the physical meaning of nonlinear ultrasonic technique in this section is the measure of second harmonic waves for nondestructive testing.

Nonlinear Ultrasonic Bulk Waves NDT

To detect material nonlinearities, a single frequency ultrasonic wave is launched into the specimen, and the signal responses from the ultrasonic wave as a function of propagation distance are received. The ultrasonic wave is distorted due to material nonlinearity, and consequently, higher harmonics are generated (Li et al. 2012). Thus, the received signal is composed of not only the fundamental frequency wave but also second or higher harmonic frequency waves. The measurement of harmonic generation for microstructural characterization is typically aimed at determining the value of the nonlinear acoustic parameter. The phenomenon of second harmonic generation is related to nonlinearity in the elastic behavior of material when the relationship between the one dimensional stress and strain is nonlinear (Cantrell and Yost 2001):

$$\sigma = E\varepsilon\left(1 - \frac{1}{2}\beta\varepsilon\right). \tag{4}$$

Substituting Eq. (4) into the equation of motion of a solid element

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial\sigma}{\partial x}, \tag{5}$$

yields the nonlinear wave equation as follows:

$$\rho\frac{\partial^2 u}{\partial t^2} = E\frac{\partial^2 u}{\partial x^2} - E\beta\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}, \tag{6}$$

where E is Young's modulus, β is the nonlinear coefficient, u is the displacement, x is a coordinate value in the wave propagation direction, σ is the longitudinal stress, and ρ is the mass density, respectively.

Since the amplitude of the second harmonic wave is much lower than that of the fundamental wave, a perturbation method can be used to solve this nonlinear equation. The displacement u is assumed to be of the form

$$u = u_1 + u_2. \tag{7}$$

Using the perturbation method, the following two equations are obtained:

$$\rho\frac{\partial^2 u_1}{\partial t^2} = E\frac{\partial^2 u_1}{\partial x^2}, \tag{8}$$

and

$$\rho\frac{\partial^2 u_2}{\partial t^2} = E\frac{\partial^2 u_2}{\partial x^2} - E\beta\frac{\partial u_1}{\partial x}\frac{\partial^2 u_1}{\partial x^2}. \tag{9}$$

By choosing u_1 as a sinusoidal wave of a single frequency

$$u_1 = A_1 \sin(kx - wt), \tag{10}$$

the second-order solution can be obtained as

$$u_2 = A_2 \cos 2(kx - wt), \tag{11}$$

where $A_2 = A_1^2 k^2 \beta x / 8$, A_1 is the amplitude of the fundamental wave and A_2 is the amplitude of the second harmonic, while k is the wave number. The acoustic nonlinear parameter β is related to the amplitude of second harmonic, which is normalized by the square of the fundamental wave amplitude

$$\beta = \frac{8A_2}{{A_1}^2 k^2 x}. \tag{12}$$

In experiments, the quantity $\widehat{\beta}$ is measured:

$$\widehat{\beta} = \frac{A_2}{A_1^2} \propto \beta. \tag{13}$$

Thus, the material nonlinearity can be evaluated by detecting the fundamental and the second harmonic amplitudes of an ultrasonic test in a specimen.

An example for the nonlinear ultrasonic bulk wave NDT is provided in Fig. 2. A 5 MHz piezoelectric transducer (PZT) is employed to generate a signal with center frequency of 5 MHz. An attenuator and an amplifier are connected to the transmitting transducer and to the receiving transducer, respectively. The center frequency of the receiver is set at 10 MHz to obtain the corresponding second harmonic frequency component. Both transducers are carefully placed on each side of the specimen with holders designed to ensure uniform coupling conditions. As shown in Fig. 3, a Hanning window is imposed on the steady-state part of the signal, and signals are digitally processed by using the fast Fourier transform (FFT) to obtain amplitudes A_1 at the fundamental frequency and A_2 at the second harmonic frequency, respectively.

Nonlinear Ultrasonic Guided Waves NDT

With the high sensitivity of the nonlinear ultrasonic approach and the advantages of guided wave techniques illustrated above, the nonlinear ultrasonic guided waves have drawn significant attentions for material characterization and micro-damage detection. Compared with bulk waves, second harmonic fields of guided waves are much more complex because of dispersion and their multimode nature. In general, the effect of second harmonic generation can be small and easily overlooked due to the dispersive nature of guided waves. Consequently, proper mode tuning with

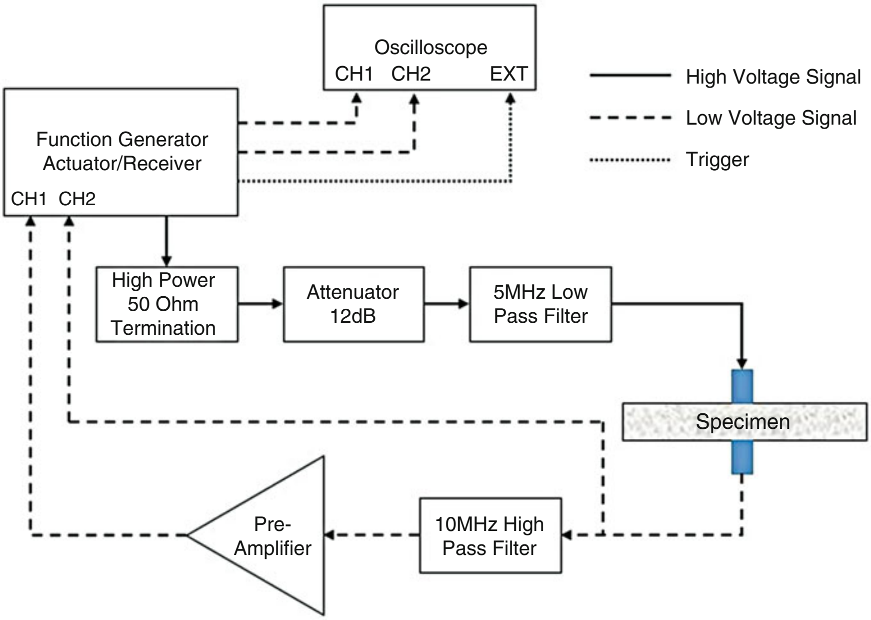
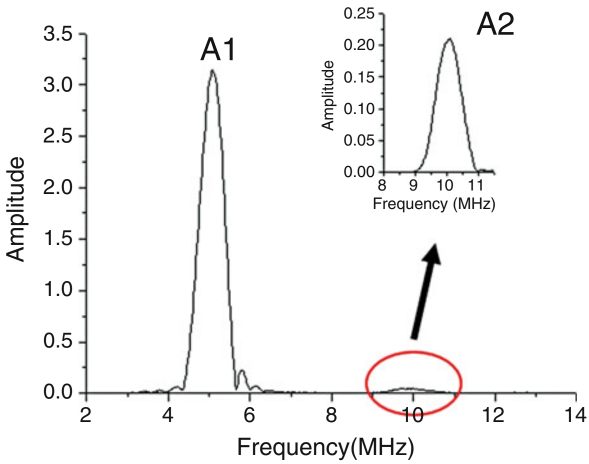


Fig. 2 Experimental setup for second harmonic generation of bulk waves

Fig. 3 Typical measured primary and second harmonic wave amplitudes



physically based features is highly demanded to enhance the efficiency of nonlinear guided wave generation and reception.

Based on earlier investigations (Deng 1999; de Lima and Hamilton 2003; Bermes et al. 2007; Srivastava and di Scalea 2009; Li et al. 2012; Li and Cho 2016), it was found that the second harmonic amplitude grows linearly with the propagation distance at the internal resonant conditions of the second harmonic guided wave mode and the primary guided wave mode. If the wave mode chosen satisfies these two conditions, the second harmonic amplitude will be cumulative. A series of double-frequency wave components will also be generated by the driving sources of nonlinearity. In practice, interest is focused on the second harmonic generation with the cumulative effect since the cumulative second harmonic plays a dominant role in the second harmonics field after a certain propagating distance.

The nonlinear parameter for symmetric Lamb wave modes can be represented by in-plate displacement on the surface as

$$\beta_{L.s} = \frac{A_2}{A_1^2} \frac{8}{k_l^2 x} \frac{\cosh^2(ph)}{\cosh(2ph)} \left(1 - \frac{k^2 + q^2}{2k^2}\right), \tag{14}$$

where A_2 and A_1 are the in-plate displacement amplitude of the second harmonic mode and the primary wave mode with symmetric feature. The nonlinear parameter for anti-symmetric modes can also be derived with the same procedure as

$$\beta_{L.a} = \frac{A_2}{A_1^2} \frac{8}{k_l^2 x} \frac{\sinh^2(ph)}{\cosh(2ph)} \left(1 - \frac{k^2 + q^2}{2k^2}\right). \tag{15}$$

Using the same method, the nonlinear parameters of the Lamb wave can be represented by the out-of-displacement on the surface as

$$\beta_{L.s} = \frac{A_2}{A_1^2} \frac{i8}{k_l^2 x} \frac{p}{k} \frac{\sinh^2(ph)}{\sinh(2ph)} \left(1 - \frac{2k^2}{k^2 + q^2}\right), \tag{16}$$

$$\beta_{L.a} = \frac{A_2}{A_1^2} \frac{i8}{k_l^2 x} \frac{p}{k} \frac{\cosh^2(ph)}{\sinh(2ph)} \left(1 - \frac{2k^2}{k^2 + q^2}\right), \tag{17}$$

where $\beta_{L.s}$ and $\beta_{L.a}$ are the nonlinear parameters for the symmetric and anti-symmetric Lamb modes, respectively. It is important to note that the cumulative second harmonic filed is symmetric even if the primary wave mode is antisymmetric.

The formulas of the nonlinear parameter for the Lamb wave mode show that the acoustic nonlinear parameter for Lamb waves is a function of frequency, material properties, and geometric information of the waveguide. The nonlinear features of Lamb modes can be affected by the mode type, frequency of the incident signal, material properties, and the geometric information of the waveguide.

The second harmonic modes of guided waves display a cumulative effect under the conditions of phase matching and nonzero power transfer from the fundamental wave mode to the second harmonic wave mode. The cumulative effect of the second harmonic amplitude is of significant advantage for detection in experimental work to ensure measurement of the nonlinear effect with sufficient signal-to-noise ratio. In earlier investigation, phase-matching Lamb wave modes were chosen to evaluate material nonlinearities (Pruell et al. 2007; Deng and Pei 2007; Li and Cho 2014). The concept of phase matching is based on the choice of the Lamb wave modes whose phase velocity equals that of the double frequency guided wave mode. The dispersion curves of Lamb wave propagation in the specimens used in this investigation were calculated numerically, as shown in Fig. 4.

Figure 5 shows the experimental setup used to generate and detect nonlinear guided waves. A high power termination was connected to the actuator to generate a tone burst signal of 20 cycles at a central frequency of 3.85 MHz. The generated sinusoidal signal then passes through the 6 dB attenuator, which was set to purify the signal to produce a high signal-to-noise ratio. A narrow-band contact piezoelectric transducer, whose nominal frequency is 2.25 MHz, was used to excite a longitudinal wave, and a receiver with central frequency of 5 MHz was set for the detection of the second harmonic wave centered at 5 MHz. The angle of the wedge is calculated using Snell's law. High vacuum

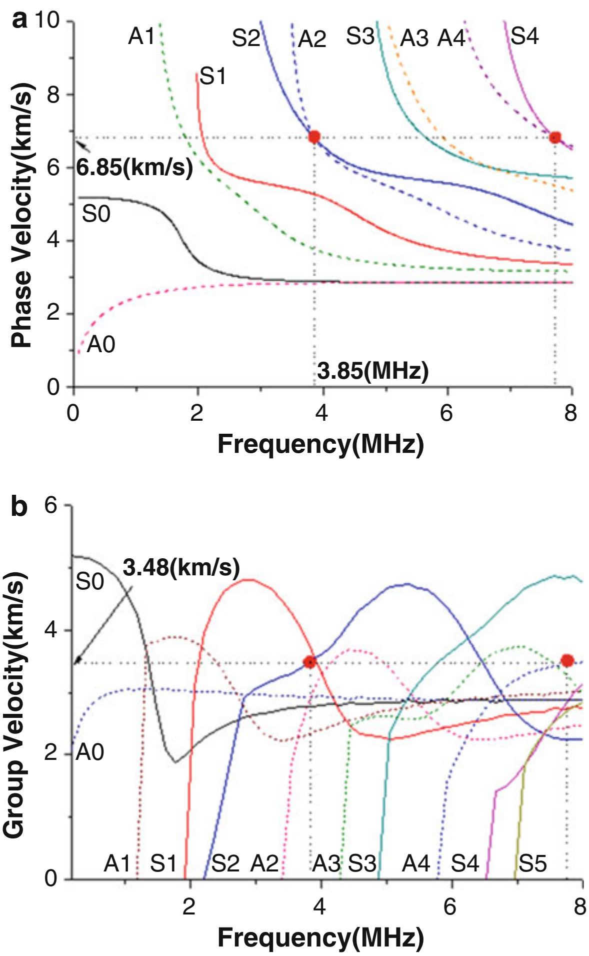


Fig. 4 Phase velocity (**a**) and group velocity (**b**) dispersion curves for guided waves propagation in a stainless steel plate with 1.35 mm thickness

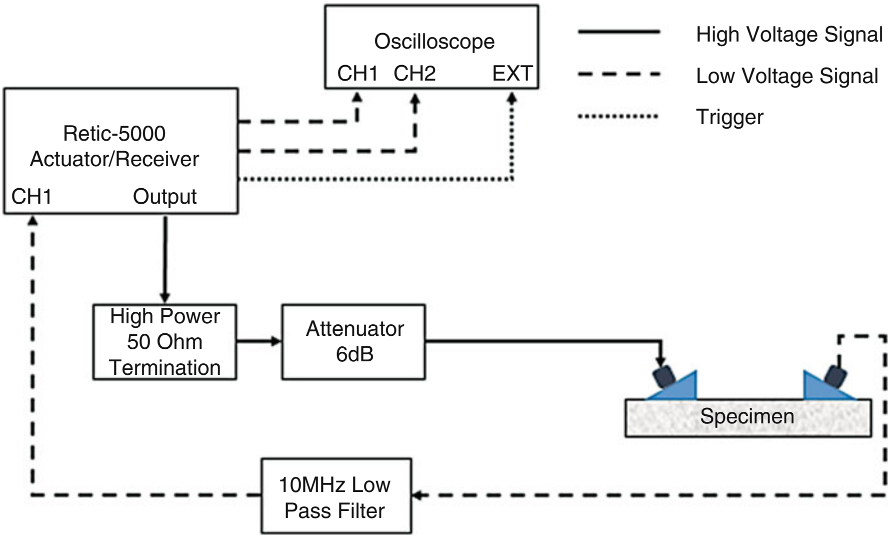


Fig. 5 Experimental setup for nonlinear ultrasonic measurements

grease was used to acoustically couple the transducer and the wedge as well as the wedge and the specimen. The initial distance between the two wedges was set at 40 mm. The obtained time domain data recorded in the oscilloscope is processed using the fast Fourier transform after the signal passed through the 10 MHz low-pass filter.

Group velocity and frequency of the propagating signal are checked to identify the Lamb wave mode. Figure 6a shows the two typical waveforms of the received signal in time domain under different propagation distances in the undamaged specimen. The group velocity of the experimental signal is calculated as 3.32 km/s. The fundamental frequency of the signal is 3.85 MHz. Comparing the value with that shown in the group velocity dispersion curve in Fig. 6b, it can be shown that the propagating signal phase matches S2 Lamb wave mode.

A typical waveform of a received signal in the time domain resulting from the propagation in an undamaged specimen is shown in Fig. 7. Sixteen cycles of a sinusoidal signal (tone burst) are generated by a high power actuator and are modulated with a Hanning window. The received time-domain signal is processed in the frequency domain with the fast Fourier transform (FFT) to obtain its frequency spectrum; the existence of a second harmonic wave in the undamaged specimen demonstrates that there is nonlinearity in the specimen or measurement instruments. The plot in Fig. 8 shows the average data with error bars from the three measurement sets. The increase in the relative nonlinear parameter values with propagation distance shows the cumulative effect of the second harmonic generation.

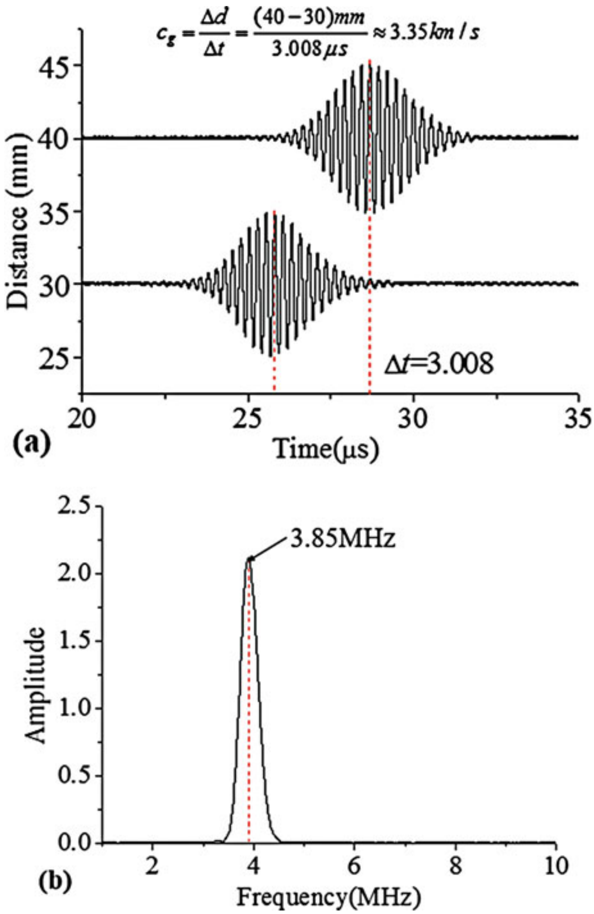


Fig. 6 Group velocity (**a**) and frequency spectrum (**b**) of the experimental signal

Concluding Remarks

Various nonlinear phenomena accompany acoustic wave propagation in solid media with material nonlinearity. Even though the full mechanism of nonlinear response of waves caused by micro-damage is not yet well understood, the use of nonlinear acoustics can be a promising qualitative means for nondestructive testing, since acoustic nonlinearity is a much more sensitive indicator of micro-damage as compared to conventional linear ultrasonic features. The damage-induced material nonlinearity can be represented by the measure of acoustic nonlinearity with appropriate techniques, such as the second harmonic waves, frequency mixing response, as well as the subharmonic generation and nonlinear resonance frequency shift. It is also important to note that nonlinear features of various types of ultrasonic waves are also very different from each other; thus the guideline for proposing nonlinear ultrasonic technique for NDT should also be different.

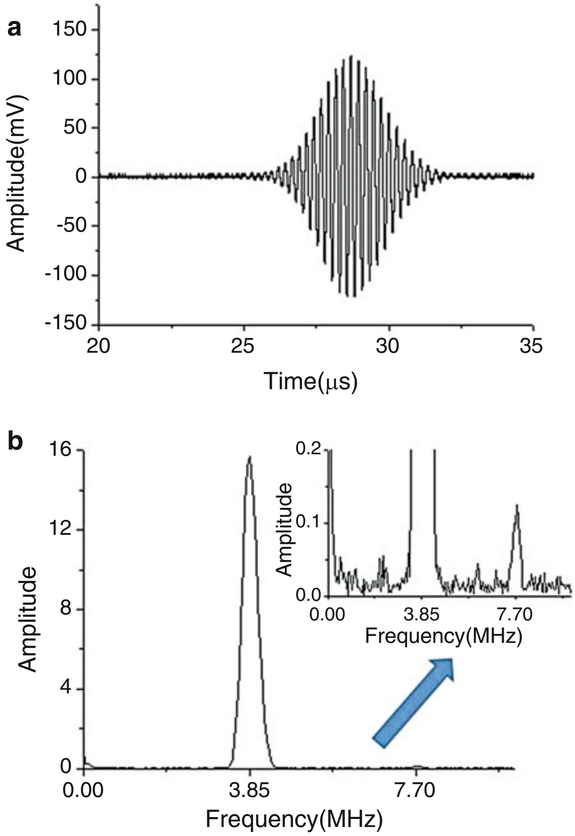


Fig. 7 Typical received signal: time domain signal and Fourier spectra of the fundamental and second harmonic signals

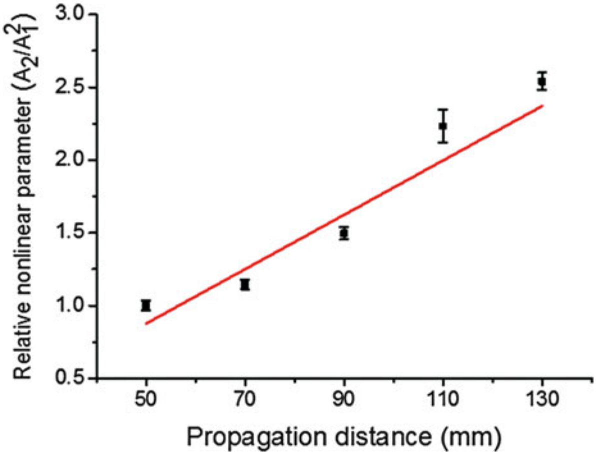


Fig. 8 Relative nonlinear parameter $\overline{\beta}$ versus propagation distances

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