# Financing the Reduction of Emissions from Deforestation: A Differential Game Approach

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**Abstract.** This paper analyzes and compares two versions of a mechanism that aims at mitigating climate change through REDD (Reduced Emissions from Deforestation and Forest Degradation). In this mechanism industrialised countries compensate countries with rainforests if they reduce their deforestation, because it is more cost efficient than restricting carbon emissions from domestic production. The initial question is, which funding possibility yields the best environmental results and is most beneficial for the involved parties. For this purpose, differential games are developed, in which industrialized countries and countries with rainforests denote the two players. Solutions are obtained by applying Pontryagin's Maximum Principle and the concept of Nash and Stackelberg Equilibria. Due to the model assumptions, analytical solutions can be found. It turns out that both versions of the mechanism can be a valuable contribution in the battle against climate change. Moreover, most advantages and disadvantages of the two variants turn out to be robust w.r.t. parameter changes and small modifications of the model.

**Keywords:** REDD · Climate policy · Differential game · Optimal control

## 1 Introduction

In search of promising strategies to combat climate change, REDD is one of the most debated proposals. The basic idea is to pay money to forest owners so that they do not cut down their forest and hence avoid greenhouse gas emissions [4].[1] Rainforests are important $CO_2$-sinks and deforestation causes approximately 20 % of global $CO_2$-emissions [10]. Therefore, the preservation of rainforests within the scope of a REDD-mechanism can play a vital role in the battle against climate change. Already in the Bali Roadmap 2007 the fundamental decision for the implementation of a REDD mechanism was written down,

[1] The currently discussed REDD+ mechanism operates at a national level and does not directly compensate individual forest owners.

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but up to now no agreement on the financing could be achieved. The discussion there focuses on the choice between a market-based and a fund-based approach. In both scenarios, the forest owners receive money if their deforestation is below a certain reference rate. The reference rate should describe how much they would have deforested in absence of a REDD mechanism. In a market-based scenario, forest owners can generate certificates if their deforestation is below the reference rate. That means that reduced deforestation is converted into reduced carbon emissions and these certificates can be sold on an international certificate market. Buyers who have a reduction obligation, imposed by the Kyoto Protocol or a succeeding agreement, can use these certificates towards their emissions reductions compliance targets. That means, if the price of the certificate is lower than the domestic carbon avoidance costs, the buyer can comply with her reduction target in a cheaper way. A fund-based solution implies that a fund is implemented into which everybody, in practice mainly industrial countries though, can pay money. The thus arising sum will be distributed among rainforest countries, according to their reduced deforestation. Donors with reduction obligations are not allowed to count the emission reduction they financed as their own reduction. The strongest advocate of a market-based solution is the Coalition for Rainforest Nations. They argue that only in a market-solution with certificate trading, industrial countries have a monetary incentive to invest into the preservation of the rainforest and thus only in this approach sufficient money can be raised. On the other hand, the largest rainforest country Brasil and the insular state Tuvalu belong to the most vehement opponents of a market-based approach. Their main point is that a market-solution only helps industrial countries to cheaply comply with their reduction obligations but that it does not lead to additional emission reductions [4]. However, both reasonings fall short, as REDD cannot be analysed independently from the negotiations for new reduction obligations in the scope of a successive treaty of the Kyoto Protocol. It can be assumed that industrial countries will be willing to accept more stringent reduction targets if they are able to fulfill them relatively cheaply with the help of certificate trading as part of REDD. Which effect prevails, the increased willingness to transfer money to the South if emission certificates are thereby generated and to accept low emission caps in a market-based approach, or the additional reductions beyond the obligations in a fund-based solution is the starting point for this paper. To the best of our knowledge, this is the first mathematical paper that focuses on this specific topic.

## 2 The Model

### 2.1 The Baseline Scenario

To analyse this issue, two agents will be considered that interact in a finite period $[0, T]$. The first agent will be called *north* and represents industrial countries that do not own forest. The second one, *south*, represents developing countries with rainforests.

The modelling of the south is similar to [7]. In the absence of an international agreement, the south faces the following optimisation problem:

$$\begin{aligned} \max_{D(t)} \int_0^T e^{-r_s t}\Big\{\big[\overline{P}-\theta D(t)\big]D(t)+Y\big[F_0-F(t)\big]\Big\}dt + e^{-r_s T}\phi F(T) \\ \text{s.t.} \quad \dot{F}(t)=-D(t), \quad F(0)=F_0. \end{aligned} \tag{1}$$

It is here assumed that deforestation in the south $D(t)$ yields two kinds of income: Firstly, timber can be sold at a price that is linearly decreasing in the amount sold. Then revenues through timber sale at time $t$ amount to $\big[\overline{P}-\theta D(t)\big]D(t)$, where $\overline{P}$ is the maximal market price obtained when $D(t)$ tends towards zero and $\theta$ is a positive parameter that determines the steepness of the demand curve. Secondly, the deforested areas can be used for agricultural production. Here, like in [7], it is assumed that the yield per cultivated land is constant. Thereby agricultural income can be modelled as $Y\big[F_0-F(t)\big]$, where $F_0$ denotes the initial size of the rainforest and $F(t)$ is the size of the rainforest at time $t$.

Let $\overline{D}(t)$ be the function that optimizes problem (1) and serves as a reference rate for the definition of *reduced deforestation*.

The economic utility of the north is modelled in a more schematic way. In absence of an international agreement, the north faces the following optimisation problem:

$$\begin{aligned} \max_{E_n(t)} \int_0^T e^{-r_n t}\Big\{aE_n(t)-bE_n(t)^2-c_n\big[S(t)-\underline{S}\big]^2\Big\}dt - e^{-r_n T}\psi S(T) \\ \text{s.t.} \quad \dot{S}(t)=E_n(t)+\gamma\overline{D}(t), \quad S(0)=S_0. \end{aligned} \tag{2}$$

It is important here to reflect the two-edged role of greenhouse gas emissions for the north: On the one hand, emissions are closely linked to production and thereby economic welfare. On the other hand, excessive emissions of greenhouse gases lead to climate change and all the negative impacts related to it.
Analogously to [1,3] it is assumed that production and greenhouse gas emissions of the north $E_n(t)$ grow proportionally and that the utility derived from production is concave. Pinning down the economic utility of the north as $aE_n(t)-bE_n(t)^2$ fulfills both requirements. The last term in curly brackets in (2) reflects the damage caused by the accumulated stock of greenhouse gases $S(t)$. As in [1–3] it is assumed that the damage caused by a certain concentration of greenhouse gases is convex in the stock. As there has always been $CO_2$ in the atmosphere it is not the existence but the concentration above a threshold $\underline{S}$ that causes damage. The damage function used here reflects those two observations, and $c_n$ weights the damages, in comparison to economic utility.

The dynamic constraint in (2) describes the assumption that the accumulated stock of greenhouse gases in the atmosphere increases linearly in the emissions of the north, $E_n(t)$, and those of the south, $E_s(t)$. In this model, the emissions of the south solely stem from deforestation activities. Therefore, deforestation has to be converted into corresponding greenhouse gas emissions. The most natural way to do so is to assume that a certain area of rainforest on average stores a certain amount of $CO_2$, which will be released after logging. This leads to $E_s(t) = \gamma D(t)$, the emissions of the south $E_s(t)$ are thus proportional to deforestation.

### 2.2 A Market-Based Approach

As in [5], a two-stage game is considered. In the first stage, the north agrees on emission caps. In the second stage, emission certificates are traded, or more specifically, the north will buy emission certificates from the south in order to fulfill its emission caps with less restrictions for domestic production.

As this problem will be solved using backwards induction, it is more intuitive to start the detailed description at the second stage. Let $O_n(t)$ denote the emission cap of the north at time $t$ that results from the first stage. The north can emit more than the cap $O_n(t)$, but the transgression has to be compensated through the purchase of emission certificates $Z_n(t)$, thus $E_n(t) = O_n(t) + Z_n(t)$. Increasing emissions according to (2) result in increasing domestic production, but the corresponding certificates have to be bought at market price $p_z(t)$. To comply with the caps in the cheapest possible way the north faces the following optimisation problem:

$$\max_{Z_n(t)} \int_0^T e^{-r_n t} \Big\{ a\big[O_n(t) + Z_n(t)\big] - b\big[O_n(t) + Z_n(t)\big]^2 - p_z(t) Z_n(t) \Big\} dt.$$

Here, no terms for the damages caused by excessive pollution appear, as the trading of emission certificates only redistributes emissions between traders but does not change the overall sum of them. The total amount of pollution will thus be discerned at the first stage, and the associated damage will be considered by the north then.

If the south emits less than in the baseline scenario, it can sell certificates $Z_s(t) = \gamma(\overline{D}(t) - D(t))$. The south now has to balance the utility derived from deforestation (1) and the income from selling certificates:

$$\begin{aligned} \max_{D(t)} \int_0^T e^{-r_s t} \Big\{ \big[\overline{P} - \theta D(t)\big] D(t) + Y\big[F_0 - F(t)\big] + p_z(t)\gamma\big[\overline{D}(t) - D(t)\big] \Big\} dt \\ + e^{-r_s T} \phi F(T) \\ \text{s.t.} \quad \dot{F}(t) = -D(t), \quad F(0) = F_0. \end{aligned}$$

The equilibrium market price $p_z(t)$ is the price that leads to $Z_n(t) = Z_s(t)$ $\forall t \in [0, T]$. Let $Z_n^*\big[O_n(t), t\big]$, $D^*\big[O_n(t), t\big]$ be the (Nash-Equilibrium-)solutions of this game.

Now the first stage can be considered. At the first stage, the trade-off between economic utility and damage through emissions is optimized. The north faces:

$$\max_{O_n(t)} \int_0^T e^{-r_n t}\Big\{a\Big\{O_n(t) + Z_n^*\big[O_n(t), t\big]\Big\} - b\Big\{O_n(t) + Z_n^*\big[O_n(t), t\big]\Big\}^2 \\ - p_z(t)Z_n^*\big[O_n(t), t\big] - c_n\big[S(t) - \underline{S}\big]^2\Big\}dt - e^{-r_n T}\psi S(T)$$
$$\text{s.t.} \quad \dot{S}(t) = O_n(t) + \gamma\overline{D}(t), \quad S(0) = S_0.$$

The constraint results from the fact that trade only redistributes the emissions between regions but does not directly change overall emissions, thus $E_n(t) + E_s(t) = O_n(t) + Z_n(t) + \gamma\overline{D}(t) - Z_s(t) = O_n(t) + \gamma\overline{D}(t)$.

### 2.3 A Fund-Based Approach

For the modelling of a fund-based approach again two stages will be required. In the first stage, the north agrees on emission caps and assigns a price to a certain area of saved rainforest. As the north (Stackelberg leader) is able to foresee the reaction of the south (follower) it thus also determines how much it is willing to pay into the fund for the preservation of the rainforest. In the second stage, the south optimizes deforestation according to the money in the fund and the assigned price.

Just like in the market-based approach, it is more intuitive to start the detailed description in the second stage, in which the emission caps $O_n(t)$ and the prices offered by the north $p_f(t)$ are already set. The south therefore considers the following problem:

$$\max_{D(t)} \int_0^T e^{-r_s t}\Big\{\big[\overline{P} - \theta D(t)\big]D(t) + Y\big[F_0 - F(t)\big] + p_f(t)\gamma\big[\overline{D}(t) - D(t)\big]\Big\}dt \\ + e^{-r_s T}\phi F(T)$$
$$\text{s.t.} \quad \dot{F}(t) = -D(t), \quad F(0) = F_0.$$

Let $D^*\big[O_n(t), p_f(t), t\big]$ be the optimal path of deforestation.

Now, at the first stage, the north optimally chooses the emission caps and the price offers. As in the market-based scenario, economic interests and the avoidance of damages have to be balanced:

$$\max_{O_n(t), p_f(t)} \int_0^T e^{-r_n t}\Big\{aO_n(t) - bO_n(t)^2 - p_f(t)\gamma\Big\{\overline{D}(t) - D^*\big[O_n(t), p_f(t), t\big]\Big\} \\ - c_n\big[S(t) - \underline{S}\big]^2\Big\}dt - e^{-r_n T}\psi S(T)$$
$$\text{s.t.} \quad \dot{S}(t) = O_n(t) + \gamma D^*\big[O_n(t), p_f(t), t\big], \quad S(0) = S_0.$$

As there is no carbon trading, reduced emissions from reduced deforestation directly lead to decreased overall emissions. Thus it holds that $E_n(t) + E_s(t) = O_n(t) + \gamma D(t)$, which leads to the dynamic constraint above.

**Table 1.** Parameter values used for the analysis

| $T$ | $\overline{P}$ | $\theta$ | $F_0$ | $Y$ | $r_n$ | $r_s$ | $\gamma$ | $\phi$ | $\psi$ | $\underline{S}$ | $S_0$ | $a$ | $b$ | $c_n$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 120 | 0.1 | $3.9 \cdot$ e9 | 11 | 0.05 | 0.05 | 6.66 | 10 | 10 | $2.2 \cdot$ e6 | $3 \cdot$ e6 | 300 | 0.05 | $5 \cdot$ e−7 |

## 3 Results

For the presentation of results parameters were chosen as specified in Table 1. Parameters either represent a real price or quantity or are chosen to fulfill the following criterion. In the baseline scenario, north and south should emit 27,600 and 6,900 million tonnes of $CO_2$, respectively. This criterion results from [8,10]. The remaining freedom in the choice of parameters was used to ensure interpretable behaviour of both players in the two REDD scenarios.

All optimisation problems from Sect. 2 can be solved analytically using the Matlab toolbox *Symbolic*. However, the display of each of the closed-form formulae would require more than 25,000 characters and is therefore omitted, while Fig. 1 reveals the most important information.

The upper left plot in Fig. 1 shows that the introduction of a market-based as well as a fund-based REDD mechanism leads to a significant decline in deforestation. In the market-scenario, deforestation decreases on average by 30 %. A fund leads to approximately 25 % less deforestation. It is in line with theory that a market-based REDD mechanism leads to a sharper decline in deforestation, because the north has more incentive to buy certificates than to donate money.

In the upper left plot in Fig. 1, it can be seen that actual emissions of the north do not decrease in any of the REDD scenarios relative to the baseline. In the market-based approach, the north agrees on lower emission caps than in the fund or the baseline scenario. However, the north purchases large quantities of certificates from the south, with the result that it actually emits more in the market-based approach than in the fund or baseline scenario.

The remaining question, which of these contrary effects prevails, is answered in the lower plot in Fig. 1. It shows that both REDD mechanisms can lead to a decline in global total emissions. In detail, the fund-based approach is able to reduce emissions more effectively than the market-based approach. The additional reductions beyond the obligations in the fund-based solution is thus more substantial than the fact that more emissions through deforestation are avoided in the market based approach.

These findings seem to be very stable with respect to the choice of parameters. Sensitivity analyses have been carried out for all parameters. Regardless of the considered combination of parameters, market-based REDD leads to less deforestation while fund-based REDD results in less total global emissions. In a longer version of this paper [9] it is shown that these key findings are also robust w.r.t. small modifications of the model.

However, the chances that an agreement becomes implemented rather depend on the benefit that the involved parties derive from it than on the benefit for the environment. It can be shown that both regions benefit from the introduction

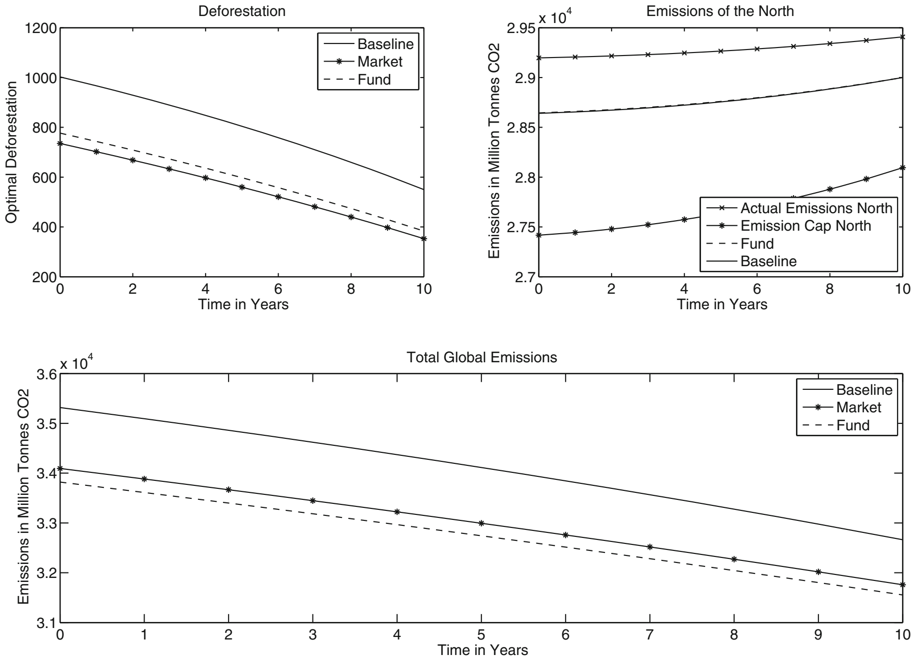


**Fig. 1.** Deforestation (u.l.), emissions of the north (u.r.), total global emissions

of any REDD mechanism. For the south, the welfare gains are larger in the case of market-based REDD. This reproduces the fact that most rainforest countries favour financing of REDD through carbon trading [4]. For the north, fund-based REDD results in a higher welfare gain. The EU officially does not prefer any of the two mechanisms. However, Norway and Germany already started paying into a REDD-like fund [6]. The reason for this result might be that in a fund-based approach the north, as the donor, can set the price of reduced deforestation whereas in a market scenario the price results from supply and demand.

This finding, however, is not independent of the choice of parameters. That means, a fund only results in more profit gain for the north as long as environmental awareness $c_n$ is below a threshold. For $c_n = 10^{-6}$ and above, market-based REDD becomes more profitable. For the south it is the other way round. If the north's environmental awareness is relatively high ($c_n = 5 \cdot 10^{-6}$), the south changes its preference towards a fund.

## 4 Conclusion

The starting point of this paper is to analyse the main distinguishing features of market-based and fund-based REDD. For that purpose, a model that reproduces the main ideas of both REDD mechanisms is developed and the results are analyzed. It is shown that the introduction of any of the two mechanisms leads to less deforestation and less global emissions, and can thereby be a valuable contribution in the battle against climate change. As widely believed in the real discussion, also the model shows that market-based REDD can reduce deforestation more effectively and industrialized countries are willing to pay higher compensation payments. However, also as assumed in public discussion, the therewith avoided emissions are partly compensated by increasing emissions of the industrialized countries. In fund-based REDD, industrialized countries hardly increase their emissions, because it is not possible to use the avoided emissions from avoided deforestation towards their own emissions-reduction compliance targets. This effect is strong enough to offset the upside of the market-based approach, and total global emissions are lower in the fund-based approach than in the market-scenario. Therefore, given our current understanding, we would advocate a fund-based mechanism. However, much more research can be done to put suggestions of this sort on a sound scientific basis. A first future extension of the model could be to add the damage caused by climate change to the utility function of the south. This significantly raises the complexity of the model but might yield further interesting results.

## References

1. Andrés-Domenech, P., Martín-Herrán, G., Zaccour, G.: An empirical differential game for sustainable forest management (2008)
2. Benchekroun, H., Long, N.V.: On the multiplicity of efficiency-inducing tax rules. Econ. Lett. **76**, 331–336 (2002)
3. Breton, M., Zaccour, G., Zahaf, M.: A differential game of joint implementation of environmental projects. Automatica **41**, 1737–1749 (2005)
4. Dooley, K., Leal, I., Ozinga, S.: An overview of selected REDD proposals (2008). http://www.fern.org/sites/fern.org/files/media/documents/document_4314_4315.pdf
5. Helm, C.: International emissions trading with endogenous allowance choices. J. Public Econ. **87**, 2737–2747 (2003)
6. Internet Platform of the Amazon Fund. http://www.amazonfund.gov.br
7. Fredj, K., Martín-Herrán, G., Zaccour, G.: Slowing deforestation pace through subsidies: a differential game. Automatica **40**, 301–309 (2008)
8. Olivier, J. G. J., Janssens-Maenhout, G., Muntean, M., Peters. J. A.H.W.: Trends in global CO2 emissions: 2013 Report. PBL Netherlands Environmental Assessment Agency (2013)
9. Riesner, B.: Financing the reduction of emissions from deforestation: a differential game approach. Master thesis, TU Wien (2015)
10. Stern, N.: The Economics of Climate Change: The Stern Review. Cambridge University Press, Cambridge (2007)