Chapter 4
Mathematics Education and Culture: Learning Theories

Let me refer once more to the What Ifs at the beginning of this book. In my view on the history of knowledge in the West, it so happened that western thinkers took for granted that context-independent knowledge is the ideal, and that education should primarily be training by schooling. It then follows that this education by schooling should have this ideal built in. The presupposed universality of knowledge is here understood to be intimately linked to its independence from contextual constraints, both in its applicability and in its aim at genuine truth. As a consequence of this, context-independent knowledge became a highly valued goal in education. Finally, in the pedagogical literature this kind of truth came to prevail over the value of searching strategies through insightful, but often diverse steps in the educational contexts, to the best of my understanding. Cultural, social and other 'external' aspects are eliminated from the educational processes, yielding the sort of world-wide standard of schooled knowledge I find in the universal assessment strategies of PISA, supported by the OECD. This result of historical development in mathematics education through schooling is in conflict with several What Ifs of my perspective.

In the light of all this, it is important to note that this presumably dominant view is not the only one around and that different theories of learning exist. Especially one tradition will be highlighted here, since it is compatible with the What Ifs of my view. I refer to what is generally known as the socio-historical view on learning, and was later renamed the socio-cultural learning theory.

1 Socio-Cultural Learning Theory

In the early days of the Soviet revolution (ca. 1917–1930) L. Vygotsky was working on his alternative learning theory, called the socio-historic theory. In the competitive theories of those days I distinguish between two main currents:

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behaviourism in the Anglo-Saxon world saw learning basically as a process inside the head of the learner. The teacher could manipulate the stimuli (S), which were fed into the so-called black box (the brains) of the learner and check on the impact of the processing of data at the other end, where he looks at the responses (R). While S and R could be measured and manipulated in the educational context, the learning itself was taking place inside the head of the learner and could not be studied scientifically according to this theory. The second theory, which was initiated around that time, and with which Vygotsky had regular discussions, is that of Piaget. In his genetic psychology Piaget saw the learner as a biological being, which developed or 'matured' over the years. Again external inputs are important and can be controlled or adjusted, but the developing processes of a biological nature are of primary importance, and learning can be reduced to a double process of accommodation and assimilation (respectively adapting to the environment and taking in aspects from the latter: Piaget 1972).

Vygotsky was the first to understand learning in a broader socio-cultural or socio-historical frame. In a sense, one can say that for him and his school, learning takes place in the total field of interaction between a learner and his or her environment. In the Marxist tradition, in which Vygotsky was working, his approach is called 'socio-historical', with an intrinsic lack of interest in cultural differences. Vygotsky knew the psychologists of his time (Piaget, Thorndike and others in the West, for example) and objected to 'single factor' views on development (it is all maturation, or it is all stimuli, etc.) in favour of more complex models and theories (Wertsch 1985).

When the theories of the so-called Vygotsky-school were rediscovered by western psychologists in the '60s, translations of major texts in English became available. With the translation of Vygotsky (1962, and the volume of articles edited by Cole et al. 1978) the perspective of this Russian school started a new life. Gradually, the focus was slightly shifted, or rather broadened to be dubbed 'the socio-cultural theory of learning'. This is obvious from the two major professional journals in English which promote research in that perspective today: *Culture and Psychology*, and *Mind, Culture, and Activity.* The same can be said of major psychology books in the Anglo-Saxon group (Vander Veer, Vaalsiner, Alvarez, and many others), which carry the notion of culture in its title (most prominently Cole's 1996 seminal synthesis).

I will restrict the references to learning theory to this particular school, because it is unique in offering ample room for cultural difference and hence might be adequate within the framework sketched by the What Ifs of the present book.

Main concepts of this approach, which have special relevance for my focus, are:

– The recognition of the complexity of learning processes: this issue was mentioned very briefly in a former paragraph. Vygotsky's consistent plea against 'single–factor theories of development (theories that posit one major force of development and a single set of explanatory principles) was aimed primarily at biological reductionism and mechanistic behaviourism.' (Wertsch 1985: 21). The consequence of this position is that learning cannot be reduced to mere

biological processes or mere mechanistic procedures. For Vygotsky learning is something that is part of the complex of learner + socio-historic context. In educational terms this implies that changing elements in the context will have impact on the learning process and on what is/can be learned. But also, the personal characteristics of the subject—e.g., its being mature or not—will be relevant for the learning process. Specifically, mediation and mediators have great importance in development, according to Vygotsky. Mediators in learning are primarily signs and tools, which do or do not form part of the environment of the learner: books, all sorts of artefacts, language and communication styles, institutional settings and so on;

– Higher mental functions have great importance in Vygotsky's view: in order to understand the child 'one must first understand the social relations in which the individual exists' (Wertsch 1985: 58). In Vygotsky's own words: 'we could say that humans' psychological nature represents the aggregate of internalized social relations that have become functions for the individual and forms the individual's structure.' (Vygotsky in: Wertsch: idem). So, the focus on the individual as source of learning is rejected in favour of the social group or origin.

  Especially for the higher mental functions this focus is crucial: mediators such as language, habits, values and so on are the landscape or the pool out of which ideas, insights and choices emerge, and by means of which they are formed and phrased. Without this broad and complex social (and cultural) context no mental functioning of any degree, which we would call human, will emerge. On the other hand, a voluntary and conscious organization of this contextual material in education is of utter importance for the development of especially higher mental functions in Vygotsky's approach.

– The primary educational concept attached to the former paragraph is that of 'zone of proximal development'. In Vygotsky's social theory of learning and development the types and the quality of interaction between the learner and her environment is of primary importance. Put differently, the abilities and the capabilities of the pupil is one part of the interactional complex, and the nature and qualities of what is offered 'from outside of the learner' is a second and equally important part. Within this double structure of interaction, the typical focus of Soviet scholars was on 'how the child can become "what he not yet is"…' in Wertsch's phrasing of the Vygotsky perspective, whereas most of the western developmental psychologies focused on the research of how the child became what he presently is. Piaget focused on the detection of the stage of biological maturation at each phase, and how this could be understood in view of the stages that came before. BehaviourIsm explores how the intricate manipulation of S and R yield the S-R-relationship that is witnessed at any one moment of life. The focus of Vygotsky is on what could become or even what could be made or guided such that the person of tomorrow will be different from the one we see today. Although this is certainly a perspective that is in line with Marxism,

it need not be restricted to that philosophy of human beings and of society: Rousseau and other Enlightenment thinkers held similar ideas (see Chap. 12).

Vygotsky captures the educational potential of this alternative view on the child by introducing the notion of 'zone of proximal development'. A lot has been written on this notion. I will only mention it briefly, in order to use it in the culture-sensitive perspective of multimathemacy later on. The first element of this notion is that of stages of development: a person at one particular stage A is understood to be at a certain point in a possible development. But, in the focus on 'what a person is not yet', this entails that this person is next to or in the near proximity of…stage B. As educators we need to know this, take this into account and then actively use that knowledge in the educational process. On the one hand it is useless to follow a concatenation of stages and progressively more difficult steps in a mechanistic way, for example because the curriculum has it stipulated, or because the theory tells us that is the path to follow. Or, with reference to mathematics, because the mathematicians of the AM view are convinced that this is the intrinsic path for development in mathematical thought. Practically, exclusively curriculum-steered mathematics education is not the right way, according to this approach, because chances are that the particular level, or cognitive structuring of the pupil is disregarded by doing so. And hence a high level of dropout will ensue.

On the other hand, it matters a lot in this perspective what the environment offers at any moment for the particular learner concerned: continuous checking of the appropriateness of that offer for the particular child is of great importance, since proximity between child and teacher's worldview or knowledge is relevant here. If the teacher follows the orthodoxy that seems fit for his own cognitive setup, he may offer material, questions, images or concepts in the classroom that cannot be connected, recognized or otherwise familiarized by the child because the proximity is lacking. Any offer in the curriculum or the teaching will add to feelings of alienation, and be appreciated as 'not speaking to my world' or 'too abstract' or something along these lines. This does not mean that the teacher should only focus on what is familiar to or recognized by the pupil. On the contrary: what is offered should be just one or two steps beyond what is already acquired or known. The zone of proximal development indeed refers to the distance between the actual development level of a child and the potential level, reached with the help of adults and the mediators they use (school books, concepts, end the like: Wertsch 1985: 67). This refers primarily to the nature of higher mental functions, which encompass anything we would call culture (or social being) today.

In recent years a group of scholars in the West (Europe and the US) have been expanding these ideas. The Swedish psychologist Hedegaard (2012) thus focuses more broadly on the child's social situation, including family and societal settings, to situate learning and even schooling. She makes a plea to start 'researching children in their everyday settings' (Hedegaard 2012: 139) rather than in experimental settings. The 'zone of proximal development' then

becomes a particular and highly relevant slice of that 'whole' of the child's environment. But at the same time, other—less school-defined—settings can be focused on in the research. Hence, the child's background (see next section) and the knowledge gathered and readily available there becomes the general field or larger 'zone' of learning, and taking this into account in curriculum development then looks like a very sensible thing to do.

Lave (2012) goes even a step further and points out that looking at a person's learning process implies attention for the larger political context of the learner. She links this thought to Marx, a source of inspiration for Vygotsky at the time, especially his theses on Feuerbach. In contemporary anthropology she points to a parallel (but not Marxist) position in Ingold's recent environmentalist work: "when a child becomes skilled this is a consequence of his or her involvement in a social matrix that is entwined with the natural world, a world that is not so much mastered as it is revealed through deployment of the skill.' (Ingold 2004: 163). This can be considered, in my interpretation, as a reformulation of the idea of Vygotsky idea of learning, when looked at as the rather symmetrical and cooperative interaction between learner and environment. (see also Ingold's idea of sphere, above on 'worldview')

– When I apply these notions within the educational scope in the context of a variety of cultures, it is clear that we have potentially a strong instrument for a genuinely emancipative mathematics education. In the next section this point is explained in full. In Chap. 12 the Vygotsky approach is worked into the Sen-Nussbaum proposal.

2 Background and Foreground Knowledge

From the previous paragraphs it is clear that the interpretation of the phenomenon of learning has obvious implications for the educators and the educational policy makers. More specifically, the interpretation of the widely spreading trend of dropout from school without decent qualifications (without a diploma or a valid certificate for the 'real world') will be quite different, depending on the theory of learning one adopts. Concretely, many a mathematician and quite a number of mathematics teachers I encountered in the past years would explain the dropout rates by stating that these pupils 'just could not cope with mathematical thinking', or even that they were incapable of formal thinking in general. The reference to the PISA assessment is then used to substantiate these opinions. E.g., with the start of a new school year (September in continental Europe) this sort of discussion appears on a yearly basis in the written press. This illustrates the point made in a general way by D'Ambrosio, the founding father of EM. In a reaction on this type of attitude about detached or context-independent mathematical education, he states with disbelief: 'Indeed, some educators and mathematicians claim that content and methods in mathematics have nothing to do with the political dimension of

education.' (D'Ambrosio 2007: 27). In a very thorough study on the way the arguments run when implementing this still dominant view on AM in mathematics education François (2008) gives a painstaking analysis of the role of political perspectives in the context of Flemish (Belgian) education. Political perspectives encompass issues such as social and gender inequality, differences in worldview, textuality or orality, acquaintance with a school culture (its discipline, evaluation tradition, competitive structure and so on), apart from strictly political elements like the open access of schools or the installation of elite schools with high financial thresholds.

Remarks:

1. Coming back to the notion of 'zone of proximal development' I have to make an important note on the possible meanings of this central concept. Vygotsky speaks about a difference in knowledge, established in the child's mind and the potentials of the environment (including the teacher's knowledge). The child has acquired knowledge through what he called 'independent problem solving' (Vygotsky 1978). One possible reading of the parties involved could be that it is an asymmetric couple: that is to say, one could understand the relationship as an asymmetric or unequal one, with the child going from less to more, thanks to the intervention of a superior other, namely the teacher. This interpretation, with a clear educational impact, need not be the one to adopt. In a critical reading, which is compatible with my view, one can interpret the 'zone of proximal development' as knowledge transmission within a symmetric relationship. This is the reading that one finds in Roth and Radford (2010).

The authors stipulate that the kind of educational strategy one adopts will allow for a different interpretation of the notion. In examples from a mathematics classroom they explain how the actual interactions between teacher and pupil will fill in the notion of the 'zone' very differently. In Roth and Radford the following situation is described: 24 students sit in a circle on the ground. In the centre of the circle a series of papers are spread out, accompanied by labels such as 'cube', 'ball', and other geometric concepts. On the papers the appropriate geometric figures are depicted. The teacher sits next to the papers, surrounded by the students. She explains what is written on the label cards and hands a bag with 'mystery objects' to a particular student. The student picks out an object, e.g., a cube and enters a dialogue with the teacher. Within a symmetric interpretation of the use of the 'zone of proximal development' the dialogue will be a genuine dialogue, where the teacher invites the student to match his object with the label of 'square and cube' and the accompanying card with square figures.

The dialogue goes back and forth (because of the attitude of the teacher, who acts as a genuine interlocutor rather than as the one who knows or knows more than the student): the teacher asks what label will be adequate, and the student responds by silence. In a second moment, when prompted, the student asks a question about the first question: what does that question mean? So, the student starts negotiating the question. The teacher recognizes the relevance of the negotiation and rephrases

her question, starts pointing to the labels, and so on. The student will finally understand, reformulate the question and solve the problem formulated by the teacher. It is important to see that in this pedagogy a series of aspects are specific and allow for a symmetric perspective on the learning situation:

– everybody sits at the same level, with eye contact,
– the learning process is developing as a continuous turn taking between provider (teacher) and student, where the latter can negotiate the questions, appeal to the teacher to look at them from the student point of view and think along rather than instruct.

2. Secondly, it is good to expand this notion from the rather restricted cognitive version of Vygotsky, who spoke about 'developmental level as determined by independent problem solving' (after Wertsch's translation 1985: 67). This makes the notion more precise, but also more limited. The broader notions of 'background knowledge' and 'foreground knowledge' which were introduced by Skovsmose (2005) cover a larger and more cross-cultural or maybe even trans-cultural area: not only cognitive differences, and eventually linguistic differentiations are taken into account, but the broader field of relevance covered by the studies on worldview (see Part I of this book). Indeed, children bring learning styles, values, rules and habits on authority relations, time management and time notions, and a certain framing of what a task would be, along with them when they enter school. All of these aspects, and possibly more, form the package of what is called 'background knowledge' (BK). From the point of view of the educator it then matters a lot to take all this into account and define what would be the possible 'foreground knowledge' (FK) of each child: what are the potential next steps, and what would be the translation of further perspectives within the frame of the BK of each child? What I call 'political' in this instance is the stand the educator and/or the school will take vis-à-vis this complex of BK + FK for each child: they can disregard BK and hence organize education from the perspective of a standard and so-called culture-independent point of view. In mathematics education this means that the curriculum and the standard learning procedure of AM prevail, and no modifications in terms of BK + FK should be considered. My contention is that large dropout numbers will then be the consequence. Indeed, in my analysis AM has clearly social, political and cultural foundations as well. Hence, it is not 'neutral' in that sense. Using AM as the sole basis of instruction, translated into a uniform curriculum for all in mathematics education will hence yield misunderstandings, alienation and eventual dropout on the part of the children, since they do not share some or a lot of the implicit and taken for granted social and cultural aspects of AM.

I give two examples to illustrate this point:

1. During my observations of learning styles in some oral traditions (with Navajo Indian studies, but also with some immigrant groups in Belgium) I was struck by the rather obvious fact that learning is not initiated in the child through

instruction. No parent instructs her child to do things this or that way. Rather, children from birth on, are put in an erect position (on a cradle or otherwise) such that eye contact is always possible. Hence, the child sees what the mother or father are doing. When the child turns toddler and infant the same procedure is followed: the child is present and is encouraged to look at what the adult does. Then 'of a sudden' the child will start imitating: it starts a small loom for sash belts and weaves 'like it has seen doing' the others. Or it will herd sheep and goat through canyons by imitating the use of environmental data (sun, rocks, canyons, etc.) the way they were used by elders. I literally never heard an adult instruct a child. When the children started school, teachers complained to me that they were 'silent': they did not ask questions, nor were they attracted by competition. Parallel observations were reported by e.g., Farrer (1991). When she went through a long cycle of participant observational research with a Mescalero Apache informant, she asked all sorts of questions, never receiving an answer. Finally, her informant shouted at her: 'Pay attention!'. This testimony of her refers to the same emphasis on looking and imitating in order to learn, rather than instruction or verbal transfer of knowledge. Education is 'stealing with your eyes' and 'paying attention', rather than being instructed verbally on what to think or do.

Similar things were observed by collaborators who worked on Turkish immigrant children, coming from a rural area of Turkey with basically poor schooling. They also were silent in the classroom, except for pupil-to-pupil interventions in the classroom. That is, when the teacher explained something they did not grasp, one of the Turkish children would switch to what we called the 'pupil code' and translated or interpreted for the colleagues what could be the intention of the teacher. This occurred regularly, and the teacher got quite annoyed by what she took for a lack of discipline. The researchers were able to show that children in such mixed classes (at primary level) used several codes in the classroom, in order to cope with this 'other-cultural' setting:

- the teacher code, with the teacher as the one who has and communicates knowledge,
- the pupils also practiced the child code, meaning they voiced their unrest or tiredness by being noisy, just like any other pupil, yielding mild protest,
- and they practiced the pupil's code, meaning they added information for a small group of their likes with what I call similar BK, in order to help understand what the teacher's code is all about (De Munter and Soenen 1997).

When such different codes are actually at work within a mixed classroom, it is good to recognize them and eventually to integrate them in a creative and productive way in the teaching practice. In fact, we learned from the school ethnographies that we did that the usual burnout of teachers in this kind of schools was overcome by integrating the different codes within the classroom practices and train teachers to cope in this multifaceted way with diversity within the class.

2. a different example stems from the same type of field experiences. In working with children and adults in Navajo Nation, USA I was often surprised by the procedure I had to follow in seeking collaboration from the local people. I would approach a person and ask her or him to work with me on a theme (mostly on spatial knowledge as expressed in the language). Typically, the person addressed would enter a longer or shorter period of silence and, when admitting to collaborate after a while, would ask for coffee. When the work started the person would go for it until it was pretty much finished, regardless of the hours of time spent on the job. When colleagues would stop the work at around 5 p.m., or would leave for the weekend, Navajo would object that the job was not finished. The idea that work, thinking or whatever be subordinated to a fixed, context-independent time frame (the 9 to 5 job) was experienced as counterintuitive. It proved to be utterly estranged from the cultural habits of the informants. Moreover, the structuring of problems and events in the typical script format, which is so central to our literary tradition,—the so-called plot format—was often causing uneasiness and alienation. That is to say, in the western settings we are used to work with a hidden and obvious plot structure: an introductory phase of action, with the presentation of protagonists and small actions is always the first phase. This is then followed by the 'building up' of the plot towards a moment of action and heightened tension, and ending in the resolution of conflict in a happy ending/the elimination of all the bad guys. This is utterly strange for the story telling tradition I was working in. Things happened, and protagonists came and went in the story, and of a sudden (to my mind) the event ended. The very idea of the structure of the plot, with a distinct beginning, middle and end, was absent. On the few occasions where Navajo people made their own filmed report (e.g., about a ceremony) this was very striking: even then, in the medium where plot is seen as intrinsic within the western tradition, it was impossible for the westerner to recognize that familiar structure. Rather, here again, the task seems the crucial structuring element: a task involves spending time, using paraphernalia, doing and saying things. But the tasks dictate what time is spent, not the other way around; and the structure of the event is experienced as made up by actors, circumstances and happenings rather than by a preconceived textual plot or storyline (Hymes 1981; Pinxten 1995).

Here again, my contention is that it proves important to know, respect and take into account this way of going about with and in reality, and not superimpose a local, western approach or format as the obvious, the best or even the only conceivable way of doing in building up knowledge and transferring a view on reality.

These two examples may suffice to make the point. Other aspects will be highlighted later in the book. It will be clear that schooling and education through instruction are highly specific ways of transferring knowledge. I claim that we have to be conscious of this and question the presumed supremacy or dominance of the schooling format.

3 Socio-Cultural Learning Theory and Mathematics Education

Some of the cross-cultural psychological and the anthropological studies on mathematics and mathematics education has been set up within the framework of socio-cultural perspectives on learning, and on instruction (mainly through schooling). References to the Vygotskian perspective are sometimes explicit and clear, sometimes not. For the purpose of this book, I lump together those most relevant studies in the field, which are focused on mathematics education, disregarding whether or not they work with the framework of the Vygotsky-inspired learning theory.

– Modern mathematics in a traditional culture: M. Cole did fieldwork with the Kpelle in Liberia during the 60s and 70s of the past century. A quarter of a century later Cole (1996) produced a seminal work, which thought through a series of questions on psychology in a comparative scope and within the Vygotsky-perspective. However, the first ethnographic work was not deeply inspired by Vygotsky at all.

In the first reports of the Kpelle research Cole was struggling with the western psychological presuppositions. He thus relates that, in line with his training as an experimental psychologist, he had set up a 'scientific' experimental setting in the middle of the bush, among the Kpelle. He remarks along the way that the Kpelle did not have schools at that point, and were not acquainted with the detached, context-free use of knowledge which is so characteristic of western schooling. Evidently, texts were absent too in the education of the young Kpelle. A net result of the experiment showed that Kpelle did not master classification logic, and thus seemed to illustrate the idea of underdevelopment (which was obvious for the Peace Corps and scientific workers at the time) pretty nicely. However, Cole was becoming critical of his own mindset and asked himself and his collaborators the question: was it the case that the Kpelle did not know classifications, or was the experimental setup and the questioning in themselves so utterly foreign to them that they did not produce any adequate or relevant answer? So, he questioned his own position and the implications of his way of approaching the non-westerner (Cole et al. 1971; Gay and Cole 1967) instead of being satisfied with the taking for granted of his own implicit colonial attitude.

One of the reasons for this shift in mind was that he saw people dong calculations in the markets, and apply proportional thinking quite adequately while making costumes in the street. So he concluded that there probably was no lack of knowledge with them, but rather a lack of interaction and communication between the researcher and the Kpelle. He then started looking at the Kpelle language and found quite interesting things which corroborated the observations in the field and questioned the appropriateness of an experimental setting some more.

When asking broadly about appropriate naming of phenomena the researchers kept stumbling on the category of '*sen*', which translates as 'thing'. They were able

to identify a chart of '*sen*', which was repeatedly used in numbers of sentences. Through substitution of labels in sentences, the workings of the charts became apparent. Obviously, the chart and the ensuing classifications of phenomena were 'odd' or 'not logical' according to western views, but they were systematic and had the function of ordering phenomena in the Kpelle worldview. So, instead of identifying the classification or lack thereof according to western criteria, Cole and his co-workers started taking into account the Kpelle criteria of relevance. What does this mean in actual practice? 'Consider for a moment how rare a straight line, a perfect circle, … are in nature' (Cole et al. 1971: 144). In the western approach to the external world we will nevertheless ascribe such features to the 'things' we distinguish in the world: the sun is circular, the wall of a house and a road are produced as straight lines, etc. Not so in Kpelle country. Hence, the relevance of circle and straight line is extremely limited, in contrast to that of curbs, paths, winding roads or passes in the woods, and so on.

Furthermore, the Kpelle language frames human experiences in ways that differ from Indo-European languages. According to the ethnographic work of Cole classification and discrimination of 'things' or phenomena in the Kpelle universe is based primarily on colour, rather than form or number of things (Cole et al. 1971). In quite different research contexts other colleagues had hinted at similar differences at a basic level in the way diverging cultural traditions and languages would think and express the universe: e.g., when speaking about the emphasis on process and event in Hebrew language and in some non-western traditions of knowledge the sociolinguist Fishman (1979) would point out that westerners would subconsciously and inadvertently opt for a 'thingification' of the world of experiences of other traditions. Thus, in education, about other cultures he witnessed several examples of this sort: when presenting Mexican culture in the classroom the taco and the sombrero would be shown, and when thinking about Chinese philosophy the two 'halves' yin and yang would be pointed to (rather than the dual perpetual dynamics).

– situated learning and generalisation: In the wake of the translations of Vygotsky's work in the Anglosaxon world scholars like Lave, Rogoff and others started elaborating on this other view on learning. Learning is gradually seen as an interactive process between learners and their environments. The social and cultural networks, the links between the learner's BK and the categories and styles of thinking in the environment as well as communication modes are considered to be relevant in the learning processes, and hence need to be taken into account when looking at education. One step further, one can plead for the broader founding of institutionalised education in social and cultural contexts.

A lot of these issues can be found in the research model, which speaks about 'situated learning'. Especially Jean Lave (e.g., Lave and Wenger 1991) should be mentioned in this realm. In this group of researchers some members focused on mathematical education. Jurow (2004) gives an overview of the foci found in this line of research: 'situated learning …(is) based on the assumption that people learn

through gradual participation in the socially and culturally organized practices of a community.' (Jurow 2004: 281).

When conceptualizing the development of formal thinking in terms of situated learning, the focus turns to such 'organized practices' as the speech habits and formats, the materials, the school context and the context of the pupil's homes, all of which impact on and actually shape the participation of children in the mathematical activities. One is reminded of the 'realistic' option Freudenthal argued for (see above). Looking into these dimensions of the situated learning complex for mathematics classes, Jurow analyzes what generalizing (or abstraction) could mean in the minds of the children participating in the math classes.

Generalizing as a necessary cognitive step in learning formal thinking implies that children learn to 'move away from' the situations in which a particular insight had emerged, or in which a specific concept was formed. In this the particular perspective of situated learning, this step of moving away is instigated by offering or seeking to recognize similar or comparative aspects of situations in new experiences and practices. This approach is not new, of course it draws on work of Vygotskyan like Davydov (1990), but also American scholars like Greeno (1998) and Lerman (2000). In Jurow the following distinct processes of interaction and communication are investigated in empirical settings of mathematics classes:

- *Linking*: 'the process of creating and applying classification systems' (Jurow 2004: 287). That is to say, in different situations aspects are recognized as sufficiently alike or similar to be put together as belonging to one class. With this step towards abstracting children learn to estimate the likeness of phenomena vis-à-vis each other. In the example of the observational setting of Jurow this step was recognized in children estimating the capacity of different ponds with guppies (fish). In terms of the size of the ponds estimates were then made on the possible 'overpopulation' of the ponds involved in one or the other class of ponds.

  Thus, through linking one can describe what people do, how they behave in practice in different situations.
- *Orienting* is the next step: once similarities and differences are clearly expressed in the classification act of linking, the learner orients herself in view of the classification made. In the example of the growth of the guppy population, this involves the mapping in a graph of the evolution in growth: a line that goes up in a curve or in a straight line, for instance.

  When orienting oneself, one moves away one more step from the concrete and draws a line as the representation of an evolution being watched.
- *Evaluating*. A final step in the generalization process is evaluation: the two situations and the representations are compared. In the example studied, the actual evolution of the guppy populations (in two stages of evolution) are looked at again, and their representation in the different graphs (straight line versus curve) are compared with each other: does the mapping make sense? Does either of the lines catch a relevant or important aspect of the two situations, so that this abstract representation of the actual situations is useful? What is learned through

such experiences, moreover, is that *conjecture* is a valid and indeed powerful way of reasoning about concrete situations, while distancing oneself meanwhile from the situations. What the learner does is to fall back on 'what if' questioning, as part of generalization, regardless of the concrete or particular situation one is in. It is an act of imagination which can be engaged in whatever the real life situation one is involved in.

These are a few lines of research in the socio-cultural learning theory, applied to mathematics and mathematics education.