Rik Pinxten

MULTIMATHEMACY: Anthropology and Mathematics Education

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Dedicated to the children for whom education through schooling did not (yet) yield equal opportunities in life

Foreword

The book of nature is written in the language of mathematics, so told us Galileo in his famous treatise *The Assayer* (1623). What Multimathemacy makes absolutely clear is how specific, i.e., Eurocentric this statement is. A moment's reflection leads to the following observations. Firstly, it does not concern humans, it only concerns nature, it is therefore an outsider's view and, *in extremis*, a God's eye point of view. Secondly, it assumes that it can be expressed in written form, more specifically, a book. And a single book at that. Thirdly, it takes for granted that there will be just the one language and hence just the one mathematics (or should I use the capital 'M', as Alan Bishop suggests). It has of course taken centuries of development, mathematical, scientific, and cultural, to arrive at this bold statement and it has taken almost four centuries to question it, to unravel its presuppositions, and to suggest alternatives. This book is such an alternative.

The focus of *Multimathemacy* is on mathematical education. There is a nice, helpful and seminal metaphor in terms of a city, that summarizes quite neatly the whole enterprise of the book:

> The city shows many buildings: one impressive skyscraper, a few semi-tall buildings and a huge amount of huts and small dwellings. The skyscraper is the building of AM (Academic Mathematics) with its own logical structure, its neatly designed separate rooms and a staff looking after the maintenance and the eventual enlargement or rehabs of the building. The staff are the mathematicians. The other taller buildings are the substantial mathematics corpuses of Chinese, Indian and other traditions. All of them developed one or the other special branch, working on their own particular intuitions and often applying their knowledge in architecture, irrigation systems and so on. And then one finds a huge amount of small dwellings in the city, harboring particular local knowledge in mathematics, as exemplified in building, sacred doings, tapestry or pottery making and the like (EM or ethnomathematics)

No doubt, above the entrance of the skyscraper, Galileo's words are etched into stone. The educational drama has been and still is that the book of nature written in that specific language, so cherished in the tall building, has become a standard for the whole village. And that must lead to problems, to dropouts and to *mathophobia*. What is needed are forms of mathematical education that take into account this local knowledge and these local practices. It does lead to a quite challenging and detailed view on mathematical education. It takes the form of a framework, called FORMA, wherein all kinds of activities, involving mathematics one way or another, can be 'inserted', depending on the specific cultural background of the pupils.

FORMA is shorthand for 'Frame of Reference for Mathematical Activities' and it brings together the following activities: (a) counting, (b) locating, (c) measuring, (d) designing (shape, size, scale, proportion and other geometric concepts), (e) playing as a tool for exploration, (f) explaining through underlying structures and rules), (g) moving, especially dancing and rhythmic moves and ceremonial actions, (h) generalizing by comparing, (i) logically operating, (j) exchanging and market activities, (k) making music, and (l) story telling. (a) up to and including (f) have been proposed before, most notably by Alan Bishop, but the six remaining activities, (g) up to and including (l), are new. One of the strong features of *Multimathemacy* is that for each of these cases, plenty of illustrative material is presented. Thereby FORMA is not merely a theoretical proposal, an abstract framework or something similar, but also a concrete vision that can be applied almost immediately in specific learning settings.

Let me briefly return to the metaphor. Although it is not the intention of this book to deal with the skyscraper itself, nevertheless—and I am now writing as a philosopher of mathematics—if FORMA were to become accepted, then surely this must have repercussions on the inhabitants of AM (Academic Mathematics), as the skyscraper is called. Probably the strongest effect will be the realization that AM requires itself a specific cultural setting, in the very same way that any other form of mathematical activity requires such a background. This raises all kinds of questions concerning the so often claimed universality of mathematical knowledge and thereby its necessity. How did this universality arise? What does AM look like without this feature? Can there be different AM cultures? Could AM have developed along different routes? Is, in other words, AM contingent? At present there are a number of philosophers and mathematicians who take these questions seriously and one can only hope that philosophers, anthropologists, mathematicians and educators (and whoever else who is interested) will find common meeting grounds to address these issues all over the city as it were and not merely on the roof or, more likely, in the cellars of the skyscraper (often called 'foundations' by the philosophers).

This is not going to be an easy task as one might expect huge resistance to these novel ideas. And it will be huge as even sympathizers sometimes do not seem to escape its seduction as the case of Hans Freudenthal illustrates. Innovative as his ideas were on mathematical education (as this book shows), nevertheless he did hold the belief that a universal language, including mathematics, was possible to communicate with whatever alien life forms. He effectively though partially developed this language in 1960 and named it *Lincos* (*Lingua Cosmica*). Dare one surmise that it was too hard to realize that the skyscraper was just a building next to others in the city and therefore to bear this idea's weight the gaze went upwards into the cosmos to dream away? But then one is reminded of the words of Errett Bishop, the mathematical constructivist, perhaps a bit harsh but no less to the point: “If God has mathematics of his own that needs to be done, let him do it himself.” Probably writing it down in Galileo’s book of nature.

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Preface

How do you start yet another book? On mathematics education? I can say that the figures are tragic: poor people around the world get little education, let alone a good schooling in mathematics and the sciences, which might help them to escape from poverty. Hence, generations of poor people continue to get born, live and die without the promise of a decent life. Since all democratic states in the world (in theory all member states of the UNO, although the latter is not really a synonym of 'democratic state') subscribe to the Universal Declaration of Human Rights, it would follow that the human race officially engages itself to offer a good education to its youth. The facts belie this engagement. Mathematics education is believed to be a pillar of any conceptualization of what could be termed 'a good education'. Hence, offering suggestions for the improvement of mathematics education is a good thing to do. That could have been one opening for a book like this.

Another one is to state yet another truth: I am an ignorant learned person, who has been teaching at good Western universities throughout his life. Yet, the concepts, theories and problems of science I do not know anything about (more than the layperson, or even that is saying too much) are innumerable. Moreover, I am not an exception in this: it is safe to say that all my colleagues share this condition with me, and the amount of knowledge we are lacking is growing every minute. Academic lore has it that the last person, who 'knew everything' about the natural sciences alone, died near the midst of the nineteenth century. One net result of this condition is that the authority of learned knowledge dwindled over the years: today the media are more likely to ask a sportsman or a movie star for their opinion on issues of religion, culture, morality or politics, and even the cost of scientific research today than colleagues of mine. With the corporatization of academia, we witness today that CEOs are hired as chancellors of universities, because they pretend to know how to run a business, and a university is considered more and more to be yet another branch of business (Sahlins 2008). In our 'learned unknowing', we researchers still do want to have a say in what the next generation might best learn in schools, because schooling seems to help or at the very least not hinder lots of people in their career opportunities. And from there, it enhances their chances to live a decent life.

I choose the second entry for this book: I am ignorant, and yet I know something. I am convinced that I am rather able in posing a problem. I have been reading and writing a lot about human creativity, as it unfolds and can be recognized in such human endeavours as thinking, artistic activities, religious and life stance stories and procedures and even in political or community practices. I have been doing fieldwork (as an anthropologist by vocation) while being trained in philosophy, and hence, I have been struggling with problems of meaning, contact, communication and interaction, with identity and with existential issues. That was and is a good schooling, I think: you learn to think, rethink, negotiate and change the problem you consider relevant. It may sound as a truism to most, or maybe even as blasphemy to some, but when I think I can pose problems, then that is largely due to this practice of intercultural negotiation and interaction. Strangely enough, very few people are teaching that at the learned schools, but it comes along in some disciplines (like anthropology or psychotherapy training) when the students are pushed outside and into the field and have to meet with real people in order to 'do science' about them. Because of this peculiar extracurricular learning I benefited from in my contacts with other cultural subjects, I now dare to write the book that is in front of you.

Indeed, this book is not written by a mathematician, but rather by an anthropologist/philosopher. Hence, the lack of knowledge in pure or 'academic' mathematics is obvious from the start. In the words of the famous and uniquely creative mathematician Hardy (1967), I am not concerned with pure or creative mathematical knowledge, but rather or at the most with what he calls 'trivial mathematics' (Hardy 1967, sec. 28). Trivial mathematics deals with elementary geometry, elementary number theory and such, which can be shown to be 'useful' in business calculation, in orientation in the real world, or in a more general way allows people to become educated in mathematical skills that have a beneficial impact in jobs, in daily life and in sustainable ways of life in the present-day predicament. The pure mathematician is not busy with that sort of issue (which is perfectly alright and in agreement with Hardy's well-made point), but mathematics as a subdomain of knowledge has impact and potential here, and that is what I want to highlight. So, even though I am not and cannot be concerned with problems within mathematics as a discipline, I take the particular focus on the constraints, the estrangement, the attraction and the wonder that I can describe when pupils from different cultural groups come into contact with mathematics in schooling and are given to understand that this formal way of thinking is extremely important for their chances in life and for the future of the world in general. The mathematician is, fairly and unavoidably, as stupid or ignorant as me in a million other questions and concepts outside his or her own specialization. Because these colleagues are raised in the same sort of tunnel view on the discipline as a whole, which is supposed to be a building block of science in general, of which nobody has had an overview for the past five generations or so, it is permitted that somebody—even an outsider like me—poses some questions on the primary level of mathematics education. The perspective I advocate is not entirely new: since the 1960s of the past century, the ‘underachieving’ pupils have been the focus of some attention, and since the 1980s, the sociocultural approach has gained at least some status in mathematics education circles (Atweh et al. 2010). Although studies in this realm have not been systematic, and almost never placed mathematical thinking and learning squarely with all other empirical perspectives on knowledge (see below), it should be granted that the detached view of ‘pure mathematics’ is growing less dominant in mathematics education than it has been in the past.

Moreover, I want to claim an important role for ‘trivial mathematics’, to use the phrase of Hardy once more. It is well known indeed that many mathematicians will side with Hardy in believing that mathematics is in some way ‘above’ reality, or might address another reality than the one laypeople have access to. Platonism in the profession seems to help safeguard that belief: there is supposed to be a layer of reality that is beyond the common empirical one, and that layer is the playground of the pure mathematician. Recent research on mathematical literacy and common sense, however, seems to undermine such convictions. Not only has the importance of ‘much of the mathematics taught in schools to individual pupils rapidly decreased’ through the use of PCs and other devices (Gellert et al. 2010, p. 58), but international researches point to the growing need for mathematical literacy understood as the ability to behave mathematically. The emphasis here is on behaving, rather than on ‘pure thinking’. Again, Gellert et al. (2010, pp. 59–60): ‘This ability is to be developed by experiencing mathematical modes of thinking, such as searching for patterns, classifying, formalizing and symbolizing, seeking implications of premises, testing conjectures, arguing, thinking propositionally, and creating proofs and all this at increasingly higher levels of mathematical abstraction’. The relevance of ‘trivial mathematics’ in education is firmly substantiated through research, it seems, when these authors conclude that ‘Giving the pupils the opportunity of experiencing the process of applying mathematics is certainly an essential contribution to developing methodological insights into the process of mathematical modelling’ (idem: 60). For them, it is not just a bonus, or a curiosity, but ‘essential’ learning.

Today, being knowledgeable in mathematics (or in certain branches of the ‘trivial part’ of it) is a severe selection criterion for higher studies, for better jobs and potentially for more cloud in the globalizing world. How could that work, when we are all so ignorant about the totality of knowledge, or even about a larger picture for which to educate? What sort of amazing trust or blindness do we manifestly show when we leave the task of perfecting and enhancing that strange system of education in the hands of those big players today (Greer 2012), who ‘believe’ that the principles of the free market will solve all long-term problems when we just leave the big choices to the free market and have them manage the road that leads to the right horizon? I am not going to solve that question, but given the turn we are taking today in these matters, I at the very least want to take the opportunity to say what I understand on the ways formal reasoning (and especially mathematics education at the elementary level) takes shape in the obviously culturally diverse universal classroom which emerges. The excuse to dive into this sea as an outsider to the club is that I have an 'educated guess' on the causes of failure in the classrooms, precisely because I am a bit more acquainted than specialists in the discipline of mathematics with contexts of learning, different cultural backgrounds and the contextual constraints of our own scientific learning processes.

One further warning before I start. I take for granted two philosophical postulates in my search. I invite the reader to go along with me in accepting them.

In my view, action is the more generic category in the relationships between human beings and the world. Language, and within that texts as such, is a particular type of action, constrained by specific and particularistic rules and traditions which differ from other types of action (such as physical actions, perception, thinking, maybe rhythm). The forceful and rather imposing Western tradition of education through instruction (in schooling primarily) shows a marked preferential use of linguistic action, and even of verbal instruction, in mathematics education. My plea is to look at this consciously and alter it where possible, because this focus and rather excluding perspective in mathematics education is alienating, rather than emancipating (as schooling officially promises to be) for many groups and traditions.

In the second place, I am convinced that sophistication and abstraction in thinking is a high value and an intrinsic aspect of what is called education worldwide. However, the roads to sophisticated thinking, abstraction and formal thinking are many and diverse. In that perspective, I warn against the implicit and taken-for-granted ideological use of value-laden concepts such as 'universal' and 'universalism'. Yes, abstraction and sophisticated thinking will be found to be a high value in all education of the human species. But the premises, the choices for pathways and for particular ends and goals will most probably be differing across cultural traditions. My plea is to respect these divergences in the curricula and in the learning strategies for mathematics education. This is captured in the notion of multimathemacy.

To make that notion clear from the start, it is useful to work with a visual metaphor:

Mathematical knowledge (like any other knowledge, but that is beyond the scope of the present project) can be represented with the visual metaphor of a city. The city shows many buildings: one impressive skyscraper, a few semi-tall buildings and a huge amount of huts and small dwellings. The skyscraper is the building of academic mathematics (AM) with its own logical structure, its neatly designed separate rooms and a staff looking after the maintenance and the eventual enlargement or rehabs of the building. The staffs are the mathematicians. The other taller buildings are the substantial mathematics corpuses of Chinese, Indian and other traditions. All of them developed one or the other special branch, working on their own particular intuitions and often applying their knowledge in architecture, irrigation systems and so on. And then, one finds a huge amount of small dwellings in the city, harbouring particular local knowledge in mathematics, as exemplified in building, sacred doings, tapestry or pottery making and the like (EM or ethnomathematics). All of these buildings in the city work with particular linguistic, religious and social settings, and all of them are also local in the sense that their intuitions, their choices for this or that line of reasoning and learning, their values and their expectations in terms of usefulness, elegance or rightness (truth, inalterability) are not necessarily duplicated in the other buildings. In the eyes and minds of the inhabitants of the AM skyscraper, mathematics education consists for all city dwellers in learning what the skyscraper people decree it to be. On the other hand, mathematics education, according to the view of multimathemacy, has to take the complete range of this diversity into consideration, since it instantiates the many versions of background knowledge and capacities and attitudes the pupils carry with them when being touched by the products and programmes of AM. It is my conviction and expectation that in education it will be wise, enhancing emancipation and generally beneficial for all humans to use educational curricula and learning strategies (in schools or otherwise) when starting from the particular context of each dwelling and try to find a way towards the rooms and structures of one's own and any other building in the city that is supposed to be relevant, interesting and beneficial for one's own world. The latter include, among other buildings, that of AM.

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Part I
My Horizon

Chapter 1
Introduction

1 What Ifs

Mathematics education is one important branch of the ways the learning of formal thinking is developed—or otherwise guided—in the education of children. In the particular historical context we are rapidly drawn into on a worldwide scale—which is broadly indicated as 'globalisation'—knowledge, creativity and other cognitive abilities in formal thinking are becoming more and more prominent for survival in the emerging complex societal and economic landscape.

We have ample data now suggesting that dropout or 'low performance' in mathematics classes, from the first grades on, are increasing rather than diminishing. Especially, the recurrent PISA assessments show that progress in terms of better performance across the board is not the rule (PISA 2010). At the same time, the shift in economic and political power in the world since roughly a generation yields a growing lack of highly qualified graduates (with sufficiently sophisticated mathematical training) to substitute for the now retiring engineers and natural science personnel in industry, as well as in higher education in the West. There are several reasons for this development: the brain drain from Asian countries towards western universities and industrial corporations of the past century has virtually stopped, if not changed direction. The economy in these former brain drain countries is booming and keeps the brains more and more inside the countries (especially in the so-called BRIC countries), but at the same time the dropout rate from mathematics education in the West has not been altered in any systematic way. I will mostly concentrate on a possible remedy for this last issue in this book, and only secondarily deal with the situation in other parts of the world. It is the former one where I see change might be possible soon, provided there is a genuine interest in the matter and on condition that a thorough and open debate about the theme can be engaged in. The primary focus of the book is this 'crisis' in math education. I make use of particular learning theories (especially those to be found within present-day cultural psychology) and of diverse ethnographic studies. But these two

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disciplines are only used as sources for the study of the problem within mathematics education. Finally, a broad discussion on rights, personhood and even democracy is needed. The latter is, of course, to be found in yet other disciplines. In the writing of this book I decided to present my views and choices on this third issue in an appendix at the end. This will allow me to focus fully on the question of math education, and present the deep philosophical-political choices as a background against which the more general and indeed 'global' issues can be checked.

In order to discuss the mathematics educational theme it is necessary, I claim, to come clear about what we in the West conceive as education, and especially education through schooling. To make that point in a somewhat unorthodox but possibly revealing way I start this book with a list of "what ifs". They are meant as an experiment in thinking, rather than a thorough scientific analysis. The reader is invited to think out of the box, together with me. The long-term failure to turn the performance figures around justifies this kind of approach. Moreover, as an anthropologist with philosophical training, I want to invite the reader to dare travel along exotic and unknown paths, so that cultural identity and difference can be recognized and dealt with more prominently than is usually the case in the realm of mathematics education at the elementary level.

2 What If 1

What if the vehicles for learning formal thinking vary between groups and traditions, and are to a large extent culture specific? I understand by vehicles the concepts, terms, designs and material objects, but also the lay or naïve strategies and curricula, which are used in learning formal reasoning in different cultures. The very notion of 'Multimathemacy' (Pinxten and François 2011) grants that there are many sorts of skills, technologies, and knowledge traditions, which coexist and have their own, possibly complementary advantages in different contexts. My aim is to identify, codify, and use these cultural traditions as starting points for educational processes. I assume that even though these traditions are culturally situated, the cross-cultural interactions characteristic of our global society create new, complex and diverse situations that can benefit from access to the full range of traditions in teaching and learning mathematics, logic, and thinking in general.

3 What If 2

What if we look upon the present world as the result of many centuries of relatively slow and varied interaction and communication between cultural groups around the globe? Not surprisingly, the result of these interchanges shows differences, but also convergences: I follow the analysis of a global historian (Stuurman 2011) on this point. In Stuurman's view one witnesses the 'emergence of humanity' (in the sense of recognizing other populations/cultures progressively as more human and hence similar to ourselves) over the past 3000 years. When introducing ways of enhancing formal thinking in this context, one cannot consider whether one should do so or not, meanwhile leaving 'the other' untouched. Rather, contact and mutual influence is a fact. Whether or not 'the other' is approached is not a problem anymore: all humans beings are continuously touched by globalisation processes. The quality of the interactions and negotiations one engages in, is what matters today. One can act in the frame of a colonial attitude, i.e. with a presupposed supremacy on one party over the other. Or one can choose for a mutually respectful and humanistic way on the other hand. I choose for the latter approach. This choice will become clear in many parts of the book. The general discussion on this choice, beyond the mere domain of mathematics education, is sketched in the Appendix. The option I stand by implies that globalisation and global agreements do not necessarily mean making education in general—and schooling in particular—ever more uniform, entailing the abolition of all ways of formal thinking except for one (i.e., the western tradition).

4 What If 3

What if I recognize differences in linguistic, cultural and social backgrounds in pupils? Then I can/should argue that what I call education in *formal thinking* in humans probably is a *pluriform* phenomenon.

I am convinced with anthropologists that formal thinking (and within it 'mathematical' thinking) takes a variety of forms throughout a world of many cultures. Thinking styles, premises, preferences for logical operators vary to a substantial degree, although some level of classification logic is most probably universal (Conklin 1971; but see Restivo (2013) for a perspective that socially grounds even classification logic, and Restivo (1992) for a discussion of the variety in mathematical traditions across major civilizations). Hence, any context-free view of formal thinking, such as the maturational view of Piaget (1972), or the behaviourist view of Skinner (1982) is likely to be inadequate in presupposing that the local western format is the only, or the real or even the only right form of logic, of mathematics or of formal thinking in general. Universals are not excluded, but universals a priori (which were supposedly firmly established in western history) are blind to diversity and hence tend to be euro-centric. As in other realms of thought it is high time to allow room for diversity and for comparative study of different formats [see a similar idea on the concept of justice[1] in Sen (2008)].

[1] I am not convinced that the transcendentalist approach of Rawls (Theory of Justice 1970) is the right track: that is, I am opposed on anthropological grounds to decreeing what would be the most consistent and hence universal content of justice. Instead, I choose a perspective like that of Sen (o.c.), which sees a plurality of possible notions and gradual changes within and through practice.

5 What If 4

What if, on the basis of What Ifs 1–3, one adopts insightful teaching and learning processes in their plurality within standard mathematics education? I claim that then students of all classes and backgrounds will learn formal thinking on equal terms, and with comparable rates of success measured by testing and graduation rates.

Curricula and teaching programs should start from the out-of-school knowledge and worldview of the local culture (Pinxten et al. 1983; Bishop 1988; Skovsmose 1994; Vithal and Skovsmose 1997; Powell and Frankenstein 1997; Pinxten and François 2007; Alro and Skovsmose 2002; Mesquita et al. 2011), rather than solely from the preconceptions of western mathematicians.

Different *learning theories* were developed over the years. My choice is to combine socio-cultural approaches (Vygotsky 1962; Cole 1996), cognitive anthropological studies in math education programs, and the sociology of mathematics (Restivo 1992). My perspective supports implementing a multi-perspectival view on formal learning in math classrooms under the banner of 'multimathemacy' (Pinxten and François 2011).

Recapitulating, I see the following structure in the problem definition as I developed it so far:

What Ifs 1 and 2 are the broader contextual presuppositions I have on the question of what formal thinking amounts to in the mixed world of cultures, religions, languages and learning styles that we inhabit. Beyond the heavily euro-centric focus on formal thinking we have been cherishing for centuries, it is time to recognize the cultural basis of learning and thinking. When doing so, two avenues of approach on education (and schooling) are open: one can reject all non-western traditions as irrelevant, underdeveloped or otherwise "wrong" from the point of view of the manifest success of the European scientific development of the past three centuries. Without denying that success, I claim that its shortcomings are becoming apparent today in the dropout rates in western schools, and in the at best modest success rates of development programs abroad. Hence, one can grant the success of (European-originated) science and formal thinking, but also see its incompleteness and possible contextual constraints, and allow for a plurality of foci, stemming from different cultural, linguistic and religious insights. When 'applying' the first and the second view in education (What Ifs 3 and 4) this yields the following policies: the first view will have one argue to eradicate all intuitions, insights, and different ways of formal reasoning and learning strategies in non-western contexts, in order to replace them by the one and only 'right' way, i.e. the way European educational history produced a particular branch of formal reasoning. The second view will have us seek for what Pinxten and François (2011) called elsewhere 'multimathemacy' avenues to mathematics education.

(Footnote 1 continued)

Analogously, mathematics and formal thinking are not what the AM standards assume to be universals, but rather reflections of the vernacular.

Chapter 2
Worldview

If and when I say that out-of-school background knowledge is relevant for school learning, then I have to come clear on what this background knowledge looks like. To that end I return to anthropology. Indeed, not only school education is a culture specific way of transferring knowledge between generations, but the very domain of vernacular or so-called out-of-school knowledge has to be looked into. In the literature this domain is often captured under the label of 'worldview'.

The literature on worldviews leaves one with a feeling of uneasiness. On the one hand some philosophers and scholars in religious studies claim that the progressively scattered field of knowledge of the 20th century points to the importance of worldview as a unifying force for the organization and deployment of scientific research. It would act like a logical or theoretical frame in the project of Enlightenment. On the other hand, explicit use of the term worldview in anthropology is decreasing over the years, to the extent that some announce the end of it (Beine 2010).

1 Anthropological Studies

Worldview has been associated over a long period with the culturally idealistic approaches in anthropology. Starting with Boas, anthropologists have been working in the perspective of the 'psychic unity of humanity', meaning that all human beings would share the same basic categories of thought, but also of sentiment and volition. For example, time, space, causality and such would be universally shared, although particular shapes and contents of them could be distinguished in diverse cultural traditions. In a general sense, this perspective is based on a philosophical program of mentalism: researchers took for granted that 'in the human mind' or 'in immaterial culture' situated in the mind, such categories can be found. Kearney (1984) states, in a rare overview of research on this issue, that this emphasis on the

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This section is based to a large extent on my entry 'Worldview' in the Ency of Social Sciences (Pinxten 2015).

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'mental' across cultural borders is probably due to the close links between linguistics and anthropology in the American case: language structures were presupposed to exist in the mind, and hence by extension all other structures and processes which organize the experiences of an outside world would be situated there too. The encompassing phenomenon, which would synthesize concepts, views and expectations about the way the world is, is the worldview of a particular group: a worldview is the shared 'software' of a cultural group, which processes input from the senses, stores its information and serves as the basis for action for the community (after Kearney 1984: Chap. 2).

Most certainly, Lévi-Strauss reinforced the mentalism in anthropological studies by starting almost exclusively from a linguistic perspective with his structuralist anthropology (Lévi-Strauss 1958). In his theory, the deep structures, which matter most for the study of any domain of culture in any part of the world, are to be found in the human mind. Their universal form is uncontested in structuralism until the '80s of the past century. In the view of Lévi-Strauss sense data, concepts, principles and systems of thought and action are structured logically, much like the data in a computer. Neither in his study of thinking and knowing in non-western cultures, nor in his systematic comparative analyses of myths Lévi-Strauss questions the mentalist presupposition, neither does he doubt the motor function (or the generative force) of the deep structures in the human mind. Thus, in the famous 'La pensée sauvage' (1962) the primacy of classification logic in the human 'hardware' is an unquestioned point of departure, which allows the author to sketch human thinking worldwide as the universal mapping and structuring of inputs from external reality into highly comparable worldviews. The differences he finds are interpreted by means of structural nuances, which are reduced to the workings of the very same universal ironclad logic in the mind. In his numerous studies of mythology a similar intellectual move can be perceived: in these very elaborate analyses Lévi-Strauss (1980) tries to show how the same inborn and hence universal logic is at work in the structuring of human imagination throughout the world. In that sense, the structuralist school of Lévi-Strauss illustrates very strongly what Kearney criticizes as a mentalist view on culture. Although Lévi-Strauss hardly ever uses the term 'worldview' to indicate the ensuing encompassing structures he finds (or builds) across cultures, his use of deep structures in the mind of humanity seems to amount to the same effect.

In the period of the '60s till the '80s of the past century a similar, but more differentiated sub-discipline emerged in American anthropology, known as ethnoscience. It grew out of folk taxonomies—a range of sub-disciplines from ethnobotany to ethnozoology—and was later generalised to cognitive anthropology. In an overview article by two prolific scholars in this branch of anthropology (and linguistics) Werner and Schoepfle (1987) explain that this approach aims at mapping in a systematic way what amounts to be the knowledge of different cultural traditions. In practice, ethnographers trigger terms and meanings from a set of informants while presupposing that all human beings actually work in their mind with a preconceived (or inborn) classification logic. Doing field work then amounts to 'filling in' by means of the informants the formal structures of classification about different domains of reality which are in themselves unquestioned, because all human minds are believed to work in a rather exclusive way within the frame of universal classification logic. Of course, different informants will be competent in different reality domains. Hence, particular taxonomies will be filled in more by some, and less by others. The ethnographer is, in Werner's view, the one who can construct the ideal knowledge system, which is the sum of all competent data and classification thinking around in one particular cultural group. Werner calls this the 'synthetic informant model' or SIM: it holds the knowledge of the 'omniscient informant'. No one member of a community has the total classification system of knowledge, all terms and all concepts, which are formulated in the model. Only the virtual 'omniscient informant', built up as a product of research with many particular informants, adding on bits and pieces along the way, will constitute the overview of knowledge available in the community. The anthropologist thus constructs a sort of superstructure of worldview items which is supposed to represent all partial worldview items in real and particular members of the community. The SIM is warranted, according to the researchers, because it synthesizes in an encompassing model what is present in a scattered way throughout individual cultural subjects. This reasoning is intriguing, since it carries to its limit the a prioris I indicated in Lévi-Strauss's approach. Based on the presupposition of the universality of taxonomic thinking as the preferential or even unique way of mapping reality, the 'omniscient informant' of SIM thus becomes something like the necessary and "natural" master discourse for a community. Although not one person in any particular community will actually incorporate the model, the anthropologist pretends that the construct as SIM would be warranted and truthful, because it would be the most encompassing instantiation of the deep logic at work. It is clear that this approach carries the philosophical idealism of former scholars to an extreme: actual flaws and patchwork patterns in empirical work does not hinder the scholar, because the deep structure logic which the researcher projects into human worldviews guarantees the validity of the virtual model. It is very probable that all cultures build classifications (Conklin 1971). But it is improbable that the preference for the same logical operators is universal, or that the contents are uniform across cultures.

Over the years critical assessments have been developed, yielding more varied and nuanced descriptions and models. On the one hand, different logics have been recognized in non-western cultures: e.g., inference next to classification logic. Thorough experimental work has been done (e.g., Cole 1996). On the other hand, knowledge and learning have been looked at much more as contextualized processes. This made for perspectives on worldviews as differentiated and dynamic ways of production and use of knowledge, rather than fixed or inborn software structures (Lave 1990). Hence, not the worldview as such is the focus of attention, but rather the processes of acquisition, production, use and change of data, models and folk theories. Along the way, the term 'worldview' silently disappeared from the anthropological literature in the past years. A quick survey of textbooks teaches us that almost none of them mention the concept in their index. A rare exception is Bonvillain (2006), who uses the term exclusively in the context of struggles by minorities to claim their rights in a context of marginalization: '(Worldview is the) Culture-based, often ethnocentric, way that people see the world and other peoples.' (Bonvillain 2006: 35).

However, a second approach should be mentioned. It is the materialistic view on culture (in the line of M. Harris and a small group of Marxists) elaborated by Kearney. In his book on 'World View' Kearney (1984) explicitly turns against the philosophically idealist approach focused on in the previous sections of the present chapter. Although he dwells on historical roots most of the time (Boas and others) and is limited in his scope to work before 1980, he makes a deep critique of the mentalism in most of the old work, which is largely valid for the broader cognitive anthropological studies of a later date as well.

Kearney refers succinctly to the work of the Russian psychologist Vygotsky. The so-called socio-historical approach of this early Marxist psychologist was an example of situating thinking, and hence worldview, in socio-historical contexts. With a school of researchers (Luria, Leontiev and many others: see Wertsch 1985) he developed an approach on the cognitive world, which emphasized social functions as well as the practical workings of concepts, models and worldview. Kearney picks this up and develops a materialistic theory on worldview which recognizes static aspects (like classification systems), but at the same time looks at subjects in their manifold interrelatedness: they are related to their self, and to others. On top of that they are, through learning processes, related to concepts and pragmatic procedures which are transferred by groups or communities: notions of dealing with time, space, causality and so on. Kearney emphasizes that the ecological environment has impact on worldviews as well: f. ex. cosmological processes induce views about time, while the daily experience of the geographical environment will serve as foundation for a 'carpentered space' (in the western worldview: Campbell 1989) or else a 'habitat life-space' in some oral traditions (Pinxten et al. 1983; Ingold 2004).

What Kearney did not know, apparently, is that the Vygotsky school became very influential in the West, basically by the rediscovery, translation and subsequent introduction in the Anglo-Saxon world of his writings and of a lot of the publications of his collaborators by scholars such as J. Wertsch and M. Cole. Not only did a series of books get translated and discussed, but at least two English language journals were launched (*Mind, Culture and Activity* and *Culture and Psychology*). Moreover, a whole network of groups and institutional programs was developed in the past three decades, stretching from Russia over Europe to the USA and Japan. So, although Kearney's proposal to work out a materialistic perspective on worldview, with links to the early Marxist psychology of Vygotsky, was not picked up in anthropology, a strong group of psychologists now work out a materialistic view, combining cognition, culture and an action perspective in thorough and cross-culturally oriented research. However, the notion of worldview, still so central in Kearney's book, is not used anymore in this line of 'cultural psychology' (which is the actual title of Cole 1996). Rather, the focus is on types of activity, types of contexts, mathematical skills, time (e.g., Vaalsiner 2006) and so on.

2 Worldview and Sphere

A special case of recent date is the work of Tim Ingold. This influential and prolific anthropologist combines an evolutionary approach with a deep interest in an environmental focus on culture and learning. The old work of ecologists such as von Uexkühl, von Bertalanffy and others studied social and cultural patterns and forms in the ecological context of the subjects. The term 'worldview' is seldom used, but such concepts as 'Umwelt' or ecosystem seem to be conceptually similar or overlapping with what was understood by worldview in the social sciences and philosophy. That these authors came from biological sciences may account for the fact that not many scholars have them brought into contact with the worldview perspective of the humanities. *The Perception of the Environment* by Ingold (2004) is, in my view, the one great exception of recent date in that respect.

Ingold did thorough ethnographic research with the Saami people in Finland. In this major book of his he expands his scope to encompass all hunter-gatherer cultures. A central issue he introduces has to do with cosmology, or the broader notion of 'worldview' (without using the term in any central way). Ingold explains how hunter-gatherer peoples live within nature, whereas at least western peasant cultures share a worldview of humans living vis-à-vis nature. The latter have an objectifying view: humans look upon all other creatures as if from an outsider position. Ingold examines how the globalisation discourse of today's social sciences can only be developed and even understood in this latter worldview: the whole world is constructed as the context which exists outside of human beings, and hence the natural environment is seen as a globe by humans, who distance themselves from it by conceiving it as a whole existing independent and outside of the observer. In the worldview of hunter-gatherers the environment is seen as a sphere, that is to say an environment, which is like a habitat, with all creatures (including humans) living within the whole. The consequences of these different stands on nature or environment are tremendous: in the hunter-gatherer culture humans see themselves on an equal level with animals and other natural phenomena, as is shown in the 'discourse' between animals and humans in such cultures (e.g., illustrated in the rock paintings and drawings Ingold discusses). In the western view on reality as a globe, humans see the environment as the objective other, which is somehow estranged from human beings and can hence be manipulated by them. My reading of this new line of research is that, by linking the cultural and social anthropological models (again) with the broader ecological perspective, Ingold in fact opens up a new line of thinking on worldview.

3 Applications

Ingold did link his approach to culture with natural sciences, especially with biology. On the other hand, in the work cited as well as elsewhere (Ingold 2010) he shows a keen interest in the production and use of artefacts, including what is generally referred to as art. In that sense he allows for applied anthropology to be linked with worldview. In a very specialized field, outside of anthropology, this interdisciplinary link between the worldview focus and applied anthropology is booming in recent research (of the first and second decade of this century). In what is now known as ethnomathematics—linked in name and by means of some researchers in that field with the cognitive anthropological approach, but mainly emanating from mathematics education circles—the relevance of worldview is keenly discussed. The founding father of this sub-discipline, the Brazilian mathematician D'Ambrosio, (e.g., 1985) has made multiple references to the living conditions, the cultural practices and beliefs, the social aspects of schooling and of learning in general, and how all of these have impact on the success or failure of mathematics education.

A group of sociologists and anthropologists carried out research in the same perspective. One of the influential scholars in this regard is the Danish mathematician Skovsmose, who successfully launched the concepts of 'background knowledge' (BK) and 'foreground knowledge' (FK), (Skovsmose 1994). BK refers to the out-of-school knowledge a child brings to school from her culture: concepts and models about reality, linguistic categories, learning styles and strategies to build and use knowledge. FK then encompasses all those concepts and skills the teacher can add on to the BK of each child to introduce it by means of insightful and relevant (to the child) steps to a further level of formal thought. Here again the notion of 'worldview' as such is hardly ever used in the literature. But to my mind the combination of BK and FK seems to add up to the content of what researchers in the humanities and the social sciences understood by worldview (Pinxten and François 2011). The whole engagement with ethnomathematics is to understand what formal thought (and mathematical thinking as an integral part of it) amounts to in different traditions of thinking and learning. From this knowledge the mathematics educator will then try to develop learning procedures and curriculum material, that will allow to counter the serious dropout rates of minority groups and lower social groups.

4 Religion, Ideology and Worldview

In the past decades the concept of worldview is abundantly and almost exclusively used in religious contexts. Not surprisingly it then has also a normative ring.

Already in the Marxist perspective on worldview, which was introduced by Kearney (1984), action and action strategies were an integral part of the concept (his Chap. 2). That focus broke away from the purely objectifying 'idealist' view of structuralists and cognitive anthropologists. In a broader philosophical view we find worldview linked with the old German notion of *Weltanschauung*, with a decidedly normative aspect: it refers to the way the world is known and is (to be) interacted with. Most scholars refer to Wilhelm von Humboldt (1767–1835) as the originator of this notion. Today, philosophers and scholars in religious studies continue in this line of research. Thus, the well-known scholar in comparative study of religion Ninian Smart gives it a central place in his study on human beliefs (Smart 2000), while other Christian scholars diminish the role of the cognitive or knowledge dimension of worldview in order to underline the prominence of religious and existential dimensions. Thus states: worldview is ‘a commitment, a fundamental orientation of the heart, that can be expressed as a story or a set of presuppositions (assumptions which may be true, partially true, or entirely false) which we hold (consciously or subconsciously, consistently or inconsistently) about the construction of reality, and that proved the foundation on which we live and move and have our being” (Sire 2004: 15–16).

The conservative philosopher of science and of religion, MacGrath (2004) approvingly cites MacIntyre in his disbelief about the project of Enlightenment and calls for a reuse of a worldview notion, that includes knowledge but places it firmly in a broader, not only rational but clearly religious (Christian) foundation.

Whatever the chances for success may be for such a re-emergence of a religious worldview notion, the general criticism on a detached and purely cognitive perspective on worldview and knowledge has been picked up by numerous religious scholars outside of anthropology.

The Belgian philosopher and logician, Leo Apostel, started an interesting research group in the past decades. He was trained by the philosopher of rhetorics Chaim Perelman and by the logician Rudolph Carnap. From the latter’s Vienna Circle work he borrowed the ideal of a “unified science”. Apostel worked extensively with Jean Piaget, adopting the focus on logically coherent development of knowledge from him. Finally, he was close to the research group of Nobel Laureate Ilya Prigogine, who combined ontological interests with thermodynamics. Apostel embraced some of the criticism on Enlightenment thinking, and started out to work on an ethically responsible approach to science (parallel to I. Stengers, M. Serres, B. Latour and others today in the French tradition). This critical position did not reject rationality or a strong scientific approach to knowledge, but looked at science as a form of contextualized thinking and acting. Apostel engaged in extensive collaborative research with scholars from different disciplines, and also took care to involve researchers from various life stance and religious positions. In the course of this work he developed the idea that scientific knowledge is too much scattered and hence vulnerable to particular and even particularistic interests. At the same time the group around Apostel recognized that science is produced by human beings (taking in the criticism by Kuhn (1962) and sociologists of science) and that the latter were in need of a synthetic and self-critical worldview to found their intuitions and heuristics. He founded a research centre at the Free University of Brussels (CLEA, *Centre Leo Apostel*) which is entirely devoted to the study of worldviews in philosophy and science. He thus merged the ‘unified science’ ideal of the Vienna Circle with the pluralistic and interdisciplinary approach he explored in the ’70s and ’80s of the past century. In one of his last publications (Apostel and Vanderveken 1991) the project is outlined as a major research program for the whole of science, recognizing at the same time that science is a human endeavour and thus in need of more than only rationalistic principles. In a sense, the criticism on Enlightenment thinking by conservative thinkers and religious scholars (such as MacGrath) is taken up as a fair critique. It is answered by the exploratory research in worldviews: a new, scientifically screened and tested worldview is needed, according the group, in order to overcome the failures and inconsistencies of the old Enlightenment project. The centre was successful in attracting research grants, producing PhDs and publishing worldwide. It certainly acquired a high status by organizing a series of path breaking symposia, followed by an intriguing book series with a first class academic publisher, under the inviting title 'Einstein meets Magritte'. The series title in itself highlights the programmatic perspective of the Worldview group in Brussels: both science and art, both cognition and vision. Over the years a plethora of great names has become attached to the initiatives of this group, all of them engaging themselves with the worldview notion in one way or another (e.g., see Aerts et al. 2005).

Whether the program will be successful in the end is an open question. It is relevant here to mention the initiative since it focuses squarely and uniquely on worldview, linked to scientific research. In their perspective science needs a unifying worldview today. This worldview will certainly be cognitive, but does entail ontological, religious-moral and political stands as well. Because scientific rigour is combined with societal engagement it presents an intriguing positive alternative to the merely ideological worldview concepts of the religions. In that sense the Brussels worldview program is linked with the older anthropological perspective (explicit use is made of anthropological material, but also of the contemporary sociology and ethnography of science with Latour and others in Apostel and Vanderveken 1991). On the otherhand the ecological and evolutionary thinkers (e.g., von Bertalanffy) as well as genetic psychologists (e.g., Piaget) are recognized as foundational for the contemporary project. A question that remains is to what extent the group is aware and critical about the transcendental implications of former philosophies of science (such as Kant and others) and is able to break away from that deep-seated pretention and move to a genuine comparative and possibly pluralistic perspective on worldview. The latter focus is explored at the very least in some of the publications (see Aerts et al. 2005, 2011; Note et al. 2009; Vanderbeken et al. 2011).

5 The Notion of Sphere

As mentioned earlier, the British anthropologist Tim Ingold developed an explicitly ecological perspective on thinking and learning. He speaks (e.g., Ingold 2004) of environment instead of world, nature and such. Older philosophical works have proposed a series of terms, some of which were adopted for a while, and have been dropped later: Russell used the term 'external world' (Russell 1918) and others that of 'nature'. The most successful label may well be that of 'Umwelt' (von Uexküll 1926) with its clearly biological or ecological ring. It was often used in its German version in Anglosaxon literature, like the works of general system theory and ecological theory. It might well be closest to Ingold's notion of environment: a (member of a) species lives in, adapts to and transforms the natural surroundings to some extent. That is to say, the creature does not live on its own, as an isolated phenomenon, but is best studied in the complex context in which it survives. Ingold (2004) seeks to distinguish between two clearly distinct cultural types of environment. Put differently, human communities think and manipulate the surroundings in at the least two very different ways, according to Ingold. On the one hand there is the way which is taken for granted by western traditions: humans think themselves somewhat detached from the rest of nature and in a way objectify all other creatures and phenomena, as if they belong to a reality that is somewhat distinct from humans. In religious studies and philosophy this way of thinking nature is called the 'God's Eye View': westerners position themselves mentally in an outside viewers position, which can only be found in the religious imagination as that of God, who created nature and humans. In a sense, the mental setup of the western cultural subject is to imagine oneself looking over God's shoulder upon everything, thus turning it all into objects (Pinxten 2010). In the perception of environment this results in thinking and speaking of reality in terms of globe and global: you can only speak that way when taking this objectifying perspective and distance yourself from the phenomena of nature according to Ingold (2004). On the other hand, Ingold found in his detailed study of hunter-gatherer communities that their imaging and conceptualizing of natural phenomena is fundamentally different. These cultural subjects always see themselves mentally as part of an encompassing network of phenomena, forces, and processes. They think of nature as a habitat, of which all phenomena are integral parts. Ingold calls this intuitive model of nature a sphere. You are not distancing yourself from nature in that mental setup, but you are within it, part of it. The notion of sphere is opposed to that of globe in this particular sense.

In my own fieldwork with Navajo Indians I had experiences, which seem to substantiate Ingold's idea of sphere: when I made a mud scale model of what earth and sky would look like in my understanding of them from field notes (a shallow basket of the earth topped by a similar upside down structure representing the sky: Pinxten et al. 1983) my informants showed embarrassment. During later interviews with them the scale model had disappeared, and people told me: 'you can go only so far up in the sky'. In other words, there is no total view of a globe, no outsider's position, which is thinkable. That would be a counterintuitive way of thinking and speaking. Hence I developed a notion of 'action habitat' to describe the intuitive model of nature held by them. It points to what Ingold understands by sphere: human beings are in a network of phenomena and forces, together with all other phenomena and that is what we mean by nature or reality.

6 Back to Mathematics Education

What is the relevance of this section on worldview for mathematics education?

In the list of What Ifs my number 3 speaks about these issues in a very general sense. The hypothesis I can produce from that What If reads: knowing the background knowledge of the particular cultural and linguistic groups in the class room is important because the results of that research can be used in order to develop appropriate culture-sensitive mathematics education. Again the culture-sensitivity pertains both to the curriculum materials and to the learning procedures.

The discussion on worldview makes clear in a particular way what is meant here. When developing curriculum material it is obvious from such studies that western mathematics education works under the implicit assumption of the universality of the notions of world, environment and space, which are those of Academic Mathematics. When considering set theory, for example, it is taken for granted that the learner (as an instance of the community of human beings in general) reasons about reality in terms of things (set, element), part-whole relations between things, and so on. The child coming from e.g. a Navajo Indian or a Cherokee cultural background is immediately confronted with a way of thinking and speaking about the world, which is fundamentally different from the process or event cosmos it is living in and reasoning about. The intuitive clash between these two worldviews is not explicitly addressed in regular mathematics curricula, because a basic difference at that ontological level is not expected by the AM mathematician. Literature on the Chinese tradition (Needham 1965, vol. III) and on other traditions (Ascher 1998) point to these fundamentally different worldviews, linked with the structural diversity at the linguistic level. The dropout of children at an early stage of education can possibly be linked to the fact that it is left to the children to solve this 'clash of ontologies' while being a pupil in the mathematics class. That is a yeoman's job, and blindness for it is not a good pedagogical principle.

At the deepest level of understanding the very notion of sphere (in the sense of the network of phenomena one lives with and in) against that of globe (in the sense of the 'external world' vis-à-vis which a knower positions herself) obviously implies another worldview. But it also impacts on the strategies and procedures for thinking and learning which are available to the learner. Objectifying reality in the globe-view entails a mental setup where the knower sees herself as detached from the rest of reality. The very experience of a habitat world, which implies the in-relationship and the necessary interrelatedness of everything, is absent. Or better still, it is counter-intuitive. Even ecologists in western culture cannot adopt that mental setup: they want to include context in their decisions and concepts, but then continue to speak about the whole, as if they have an outsider's position. Their point would then come down to defending a more responsible view, when including the context in their global vision. In the sphere perspective, the knower sees herself necessarily within nature, interrelated with and impacting on everything else. Thus, western subjects have a long tradition of believing that 'thoughts are free' and hence have no moral sanctions attached to them. It is primarily action and speech, which are liable to have impact and hence fall under moral rules. In the sphere view, however, thinking, speaking, acting (and ritual) are all considered to be sides of the same coin, and all of these human activities intrinsically impact on reality (McNeley 1981). Such a view can only be understood within the scope of the sphere or habitat view on nature, with humans as fully part and parcel of the sphere.

In mathematics education these different intuitive worldviews entail different types of background or out-of-school knowledge. It is my contention that AM carries the objectifying or God's eye View as intrinsic background knowledge. And that knowledge is not shared or acquired by the pupil coming from a sphere-cultural view. Hence, the latter will likely get in trouble when trying to build up mathematical knowledge through steps of insight, since the background knowledge she falls back on does not connect properly with the out-of-school view of AM. This is not the whole story, which explains dropout, but I propose it is one avenue to investigate in order to understand systematic dropout of several cultural groups.

7 Conclusions

Worldview or out-of-school knowledge or background knowledge are used as largely overlapping notions in the present work. What I mean to say is not that they are synonyms, but rather that the domains they are supposed to cover are pretty much the same. Rather, one term is used in a particular discipline (e.g., worldview in religious studies and in religious denominations), and another one in mathematical education studies (background and foreground: Skovsmose's work).

In the present work I will not go into the nuanced differences, but only consider the basic point on the relevance of this type of knowledge for planning and implementing mathematics education in general. This point is important, because to my experience the rationalistic view on education of formal thinking often seems to disregard the impact of the type of knowledge that the child brings to school. Hence, dropout will be partially linked to this lack of attention.

Part II
The General Context of Education

Chapter 3
Education in a Post-industrial World

1 Postmodern Society

In the wake of globalization the OECD started the so-called PISA assessments in mathematics. When one takes a look at the test material a clearly Eurocentric perspective on 'mathematical problems' can be detected: typical questions turn around the amount of km^2 of an oil drip, or the comparative sales numbers of CDs, or the amount of electricity produced by a wind tower. All of these, and many others, are taken from the post-industrial world of experience of urban, and mostly western populations. On the basis of the results on these tests, OECD gives a ranking of best performances in mathematics on a worldwide scale. It is clearly the aim of the educational program of OECD to promote a more uniform and exclusively western perspective on mathematics education. For one thing, the aims of the OECD as such are to promote 'economic cooperation', meaning market capitalism: all member states of the organization are capitalist countries to begin with. Education appears to be subordinated to the economic logic of this body: OECD has consistently promoted a global assessment of school results (by means of a sort of world exam system), because a worldwide market is believed to benefit from uniform schooling in a series of competences, which would be the same across the globe. Most probably the implantation of production units and markets anywhere would fare better in a short time span with such policy. At the same time, the emphasis on entrepreneurial competences in students and teachers has been growing in these countries over the past decade.

Very few criticisms on the market ideology of OECD's educational views can be found in the literature. I see two main types of criticism that should be voiced here:

(a) today Euro-centrism in education is framed in neoliberal terms:
One critique focuses on the way this global streamlining of education runs against the idea of '*Bildung*' (general humanistic education). The original idea of '*Bildung*' in the 19th century German context of von Humboldt and others focused on a humanistic full development of personal capacities of the student,

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with a supplementary emphasis on critical thinking, as a basis for innovation in science and culture. In several countries of Europe the old value of '*Bildung*' through education is part of the basic rights of each person, often even laid down in the Constitution (e.g., in Germany). Especially the narrowing down of education (through schooling) in our time to the training of young people to become economic agents in a market system is then seen as an offence against these basic rights.

Over the years the educational program aiming at the development of human capacities to their full and optimal extent, was taken up in a different and somewhat revolutionary gist by Freire (2005). Illustrating his political stand, it is good to remember that he was removed from his academic position by the military in Brazil, returning after their reign as the Minister of Education for a while. Freire is best known for his severe criticism of the school system as a way to enhance uniformity in thinking and learning, and hence to disregard a person's individual qualities in favour of uniformity and discipline. I appreciate Freire's basically humanistic perspective on education as yet another version of the old Humanism and Enlightenment ideals.

In recent publications Nussbaum is the one who has most strongly encouraged a counter-current. Both in her 'Not for profit' (Nussbaum 2012) and in 'Creating capabilities' (Nussbaum 2011) she develops ideas which come closer to the old idea of '*Bildung*' than to the market and management model that is so typical of global postmodern society in the view of OECD. With the economist-philosopher A. Sen she has been developing the capabilities approach for decades. The basic idea is that all humans have a series of capabilities, which should be able to grow and develop in such a way that each individual human being can grow to become a full person. The first and major reference here is the set of capabilities of each individual, and the assessment should be of the community or society, which enables or hinders the trajectory from capability to full person in each case. It is clear that neither the market nor the state is the first criterion of the assessor, but rather the individual set of capabilities. Nussbaum (2012) makes a strong plea for such a new humanistic perspective on education because the soft and locally varying sources from the humanities have been recognized to guarantee the growth and protection of democratic mentalities in overall more just political and economic systems so far.[1] These criteria for a good education fall outside of the scope of the OECD concerns.

(b) secondly, and independently from the choice in favour of or away from a humanistic ideal of education, it proves doubtful that this narrowing down is efficient. The dropout rates within western countries are substantial. With the international growth economy balance tilting towards the BRIC countries today, the brain drain from these countries has stopped to a large extent, causing a rather dramatic shortage of highly educated youngsters, especially in mathematics, the natural sciences and engineering. My point is that the choice, which is forgotten or looked over is that for an opening of horizon, rather than

[1]Chapter 12 discusses at length this approach by Nussbaum and Sen.

the narrowing of the tunnel. Washington et al. (2013) illustrate how the attempt in President G.W. Bush's program along similar lines as the OECD perspective (the National Mathematics Advisory Panel, or NMAP) was meant to lead students more quickly to the high-tech market in the USA. However, this did not result in a lower dropout rate at all. On top of that, the focus on algebraic capabilities only narrows down the understanding of mathematics to procedural thinking, disregarding any other grounding: for example, geometry and arithmetic have more of a link with daily experience and can hence be useful for insightful learning, whereas algebra does not. The growing importance of algebra in schooling tends to diminish the role of geometry and arithmetic in favour of less insightful procedural thinking. The authors state that the choices made here are counterproductive for the lower classes, which are most likely to be victims of dropout. So, the result of the choice for the market-economy perspective instead of the broad humanistic education does not benefit the lower social groups and hence does not seem to solve the problem it aims to address. In that sense, it is rather inefficient and inadequate for the problem at hand, apart from the particularly narrow and instrumentalistic perspective on humankind it embraces. In capitalist countries the OECD and the NMAP perspectives are known by now, but beyond them 'bodies such as the World Bank, the IMF and UN agencies/are/increasingly insisting that aid and loan packages be tied to the use of education for competitiveness in the global economy' (Thomas 2010: 98). The trend is clear, and it has nothing to do with intrinsic humanistic ideals about education or wellbeing.

But why should we change the institutional system we have put to work over so many generations? Is this a purely ideological choice, and at what cost, I could ask myself? Recent critical studies by Mesquita et al. (2011) have looked at the question from the perspective of the less fortunate groups in our society. This comes down to disadvantage for those who are dependent on the welfare system, which itself is under fire today. The general conclusion on what these authors term 'the asphalt children' is far from promising for future society, by any standard: 'The welfare state is designed to maintain the new world order…and keeping the marginal peoples invisible.' (Mesquita et al. 2011: 4). It should be clear that 'the new world order' refers to the political perspective of President Bush Jr. and the neoconservatives after the 9/11 attacks. The assessments of the OECD (the PISA investigations) in fact corroborate this statement: over the years the situation of the lower social groups does not ameliorate. Belgium offers a typical illustration of this point: in the consecutive researches of PISA a segment of the students were (very) high performers in mathematical skills, but another large segment consistently landed in the group of bad performers. It proves to be the case that the immigrant population and the socially lower groups constitute that second segment: dropouts and youth who leave school at the age of over twenty without any diploma run over 20 % in the cities now. This is just one case, but there is little reason to believe that the UK (with its big cities), France or the USA would show a better and more equal distribution of good performance.

2 Mathematics Education in the Postmodern/ Post-industrial World

I am picking up the point on mathematics education once more, and situate choices in this postmodern context. What difference would it make whether you hold this or another kind of notion of mathematics education? In what way can we consciously choose here, and what are the options? I grant that these are huge questions. However, it is possible to reflect on them and say some sensible things.

Pais (2011) wrote a lengthy analysis of the debate that is recently developing on the status of ethnomathematics (EM) and its relationships to what is called standard or regular or 'Academic Mathematics' (AM). Some of the criticisms on alternative approaches to mathematics education, like EM, start from an essentialist view on mathematics, Pais claims. For example, when Adam et al. (2003) develop the idea that a variety of indigenous groups in the world (and the lower social groups I mentioned before) may benefit from an EM perspective in education they work under the following explicit presupposition: 'EM is not a philosophy, much less a 'pedagogic philosophy'. Rather it is a lens through which mathematics itself can be viewed.' (Adam et al. 2003: 329). Hence, in the context of mathematics education it may be beneficial for the students '(t)hat mathematics may be imbued with an ethnomathematical perspective…' (idem: 330). Concretely, this would entail to take the child's cultural knowledge in the out-of-school world (his BK or Background Knowledge, in the terms of Skovsmose) seriously and aim for 'an integration of the mathematical concepts and practices originating in the learner's culture with those of conventional, formal Academic Mathematics.' (idem: 332).

This proposal is rather fiercely attacked in the discussion by Rowlands and Carson (2002, 2004). They suggest that one should stop with postcolonial criticism and address the value of approaches to formal thinking instead. That point might seem fair: the postcolonial critique in itself does not necessarily touch upon the value of mathematics as such, nor of any other cognitive product for that matter. However, this does not pertain to the educational approaches, I would claim. But further, and more substantially, Rowlands and Carson dig deep into the matter by stating that the wish that all systems of thought 'are equal in value and in dignity' is a beautiful wish. However, our tradition of mathematics, they continue, gives 'more extended, more refined, and more efficacious cognition.' (Rowlands and Carson 2004: 332). Even more straightforwardly they state: 'the reason we are attempting to 'privilege' modern, abstract, formalized mathematics is precisely because it is an unusual, stunning advance over the mathematical systems characteristic of any of our ancient traditional cultures.' (idem: 337).

Pais (2011) states he is offended by this kind of position. He rightly claims that the argument about the superiority of AM is not substantial: AM did not grow in the void, independent of contexts. When this would be claimed nevertheless (as it seems to be the case), then a view on mathematics as internal logic lays at the foundation. In a sense this is a déjà vu-argument: a lot of what is called science was naively seen to be of a reality in its own, untouched by economic and political contexts and interests. I am not going into this, but it suffices to say that ever since Kuhn (1962) and Feyerabend (1973) such a naïve position is generally questioned. Furthermore, the main point is missed by criticism such as Rowlands and Carson's (2002): whatever the value of this or that cognitive or intellectual tradition, the focus should be on education. This is in fact what Adam et al. (2003), and of course Barton (2006) and others have been trying to focus on. Pais (2011) makes the point that mathematical thinking, with its particular tradition of logical consistency and so-called context-independent reasoning, in and through schooling establishes and perpetuates power relationships. Modestly, one can say that 'mathematics empowers people not so much because it provides some kind of knowledge or competence … but because it gives people a value' (Pais 2011 after Skovsmose 2005: 217). Indeed, when a student qualifies for the mathematics classes she will be able to choose for a branch of higher education courses, whereas failure in mathematics will entail the barring from such courses. At the same time, precisely these courses will yield more prestige and more comfortable professional positions in future life. This is the point already made by Bourdieu in his famous books on the way schooling continues and often creates selection and hence exclusion from status and wealth (especially Bourdieu 1981). Moreover, the selection system of schools works along the paths of testing and examining particular competences (and not other ones) within a fixed background, precisely as I mentioned for the OECD assessment doctrine. Hence, the net result of the actual production cycle of knowledge through mathematics education, using AM exclusively, shows 'how school mathematics constructs a set of learning standards that are more closely related to the administration of children than with an agenda of mathematics learning' Pais has it (Pais 2011: 218). It is not neutral in any way, and the naïve position of the 'believers' such as Rowlands and Carson then lands in implicit political choices that have nothing to do with the nature of mathematical thinking, and everything with the way education in mathematics is organized. The discussion on such implicit and explicit choices is exactly the one I want to focus on.

In this post-industrial/postmodern world we need all expertise we can get in knowledge and skills. Innovation and creativity are of the utmost importance. It is not at all clear that the competences—including mathematical skills and the accompanying curricula—of yesterday should still be the core of what schools should offer to the youth at large today. I do not say that all historical successes should be thrown in the wastebasket. Rather, I claim that it is time to look carefully at the strength and value of the older curricula for the present state of the world. Indeed, in my view mathematics is, just like any other part of knowledge, contextual—social or cultural—in its developments, its exploration strategies and its implicit or intuitive presuppositions. Hence, it is very likely that it would be wise to take into account these contextual elements in the educational procedures and the curriculum material in the learning contexts. The latter would mostly be schools, but of course not exclusively so. Perhaps, the out-of-school contexts may become more important in the postmodern/post-industrial world than before.

In order to make my point in a rather unexpected way, I draw on the distinctions in types of mathematics education, proposed by Freudenthal (1985). In his typical straightforward way Freudenthal sums up four different perspectives on mathematics education, linked to various ways of understanding mathematics as a branch of knowledge:

- the mechanistic view: in this perspective human beings are often portrayed as very much alike to computers. Hence, the type of reasoning or problem solving of computer is the maxim of education: students should be mechanically trained very much in the way one programs a computer. Freudenthal refers to Skinner and other believers in programmed instruction here.
- the structuralist view: because of the conceptualization of mathematics as a logically consistent system of thinking, untouched by experience, mathematics education is conceived as a logically coherent and non-empirical package of knowledge transfer. The Bourbaki group is probably the best-known exponent of this approach. According to Freudenthal this perspective is disastrous for education, since it starts from the view on mathematics of the advanced mathematician, and disregards the trajectory of the average child to progress along the different roads of the children's BKs to gain more mathematical knowledge.
- the empirical view: this is the opposite of the former view, in that it stresses almost exclusively the empirical reality of the child, not minding the need for theory construction through education. A lot of the practice of ethnomathematics might be stuck here: one can teach about lay practices in weaving, designing and so on without taking the next step, i.e. towards abstraction. Nevertheless, in a European dominated history with a Cartesian angle of rationalism, the empirical emphasis was a therapeutic turn (Davies and Hersh 1981).

This argument is elaborated on in an extensive way in Raju (2007), in his analysis of the 'theologification' of mathematics in the first and second 'Math Wars' (from 1000 to 1600 C.E.). This religious restyling of mathematics entailed that proof and anti-empirism became dominant in mathematics and that the empirical turn of Indian and Arab traditions was therefore refused. The distinction made between types of argumentation in Perelman and Tyteca (1957) cautiously makes the same point for mathematics: Perelman argued that only theology works with convictions and logical deductions on the basis of them; all other human ways of reasoning go by way of persuasion, which invites opponents to 'think along' with the speaker and which is pro-empirical knowledge. Perelman and Tyteca mention (in a footnote) that mathematics resembles the sciences here and should break away from theological frames.

- the realistic view (inFreudenthal's terms): in mathematical education the curriculum and the teaching procedures should start with the reality the way it is experienced by the child, and then go to 'mathematize' those experiences. I will come back throughout the book on this point: what could it mean to mathematize a world of experience? I do not discuss this notion in a theoretical way, but try to offer diverse examples from different cultures around the world.

In a parallel way Mesquita's severe critique on the marginalization of the street children in big urban areas today leads her to the following suggestion: 'As we are a complement of each other, our mathematics is a complement of the mathematics of the other.' (Mesquita et al. 2011). In other words, mathematics education should grant room for other insights, concepts and traditions than the AM views, and integrate the lot in larger perspectives. In actual fact, the primary status of the rule of logical consistency seems to be given up by doing that. Mesquita's analysis and comments are an instance, for me, of what Skovsmose and Valero call 'the breaking of political neutrality' (Skovsmose and Valero 2010). The authors develop a synthetic view on the field of mathematics education and political positions. They distinguish between three types of relationship between mathematics education and democracy:

(a) many researchers hold that there is an 'intrinsic resonance' between both, based on the nature of mathematics. The latter is pure and not contaminated by societal influences. In that view mathematical learning and thinking is an exclusively cognitive phenomenon (it is all in the mind, one could say) and hence training of the mind by mathematical reasoning will automatically benefit democracy.
(b) Some researchers have been documenting the discriminating and even 'destructive power of mathematics' (Skovsmose and Valero 2010: 41). The latter becomes clear when one sees that mathematics, also through its authority of truth and its investment in ironclad proofs, is often used to disqualify lay ethical and political judgment as 'uncertain', inapt for proof, because of the lack of 'numbers' to make the point. With the growing impact of science and technology in our globalizing world, this runs counter to democracy. There is an 'intrinsic dissonance' between both.
(c) The authors choose a third position: they advocate a 'critical relationship' between democracy and mathematics. Mathematics and mathematics education should be seen as neutral or detached from 'real life', politics and so on. Instead, the critical insights of teachers, parents and students about the use of mathematics in society and economics, and the conscious and justifiable ways of teaching and of examining mathematical knowledge and knowledge transfer should be addressed openly and critically by all involved. Thus, the authors stretch Freudenthal's view on realism, I think, in order to include the real knower in her natural (societal) context.

In view of Freudenthal's list and looking at the scenery of mathematics education in front of me with the eyes of the anthropologist, who learned how people survive in different environments by developing varying solutions in continuous series of adaptation and modification, I think Freudenthal's fourth view is the most promising one. What is missing in this picture is an adequate theory of learning, which could cope sufficiently with this complex of 'unity in diversity'. The good news is that such theory of learning exists. My next chapter will focus on it.

Chapter 4
Mathematics Education and Culture: Learning Theories

Let me refer once more to the What Ifs at the beginning of this book. In my view on the history of knowledge in the West, it so happened that western thinkers took for granted that context-independent knowledge is the ideal, and that education should primarily be training by schooling. It then follows that this education by schooling should have this ideal built in. The presupposed universality of knowledge is here understood to be intimately linked to its independence from contextual constraints, both in its applicability and in its aim at genuine truth. As a consequence of this, context-independent knowledge became a highly valued goal in education. Finally, in the pedagogical literature this kind of truth came to prevail over the value of searching strategies through insightful, but often diverse steps in the educational contexts, to the best of my understanding. Cultural, social and other 'external' aspects are eliminated from the educational processes, yielding the sort of world-wide standard of schooled knowledge I find in the universal assessment strategies of PISA, supported by the OECD. This result of historical development in mathematics education through schooling is in conflict with several What Ifs of my perspective.

In the light of all this, it is important to note that this presumably dominant view is not the only one around and that different theories of learning exist. Especially one tradition will be highlighted here, since it is compatible with the What Ifs of my view. I refer to what is generally known as the socio-historical view on learning, and was later renamed the socio-cultural learning theory.

1 Socio-Cultural Learning Theory

In the early days of the Soviet revolution (ca. 1917–1930) L. Vygotsky was working on his alternative learning theory, called the socio-historic theory. In the competitive theories of those days I distinguish between two main currents:

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behaviourism in the Anglo-Saxon world saw learning basically as a process inside the head of the learner. The teacher could manipulate the stimuli (S), which were fed into the so-called black box (the brains) of the learner and check on the impact of the processing of data at the other end, where he looks at the responses (R). While S and R could be measured and manipulated in the educational context, the learning itself was taking place inside the head of the learner and could not be studied scientifically according to this theory. The second theory, which was initiated around that time, and with which Vygotsky had regular discussions, is that of Piaget. In his genetic psychology Piaget saw the learner as a biological being, which developed or 'matured' over the years. Again external inputs are important and can be controlled or adjusted, but the developing processes of a biological nature are of primary importance, and learning can be reduced to a double process of accommodation and assimilation (respectively adapting to the environment and taking in aspects from the latter: Piaget 1972).

Vygotsky was the first to understand learning in a broader socio-cultural or socio-historical frame. In a sense, one can say that for him and his school, learning takes place in the total field of interaction between a learner and his or her environment. In the Marxist tradition, in which Vygotsky was working, his approach is called 'socio-historical', with an intrinsic lack of interest in cultural differences. Vygotsky knew the psychologists of his time (Piaget, Thorndike and others in the West, for example) and objected to 'single factor' views on development (it is all maturation, or it is all stimuli, etc.) in favour of more complex models and theories (Wertsch 1985).

When the theories of the so-called Vygotsky-school were rediscovered by western psychologists in the '60s, translations of major texts in English became available. With the translation of Vygotsky (1962, and the volume of articles edited by Cole et al. 1978) the perspective of this Russian school started a new life. Gradually, the focus was slightly shifted, or rather broadened to be dubbed 'the socio-cultural theory of learning'. This is obvious from the two major professional journals in English which promote research in that perspective today: *Culture and Psychology*, and *Mind, Culture, and Activity.* The same can be said of major psychology books in the Anglo-Saxon group (Vander Veer, Vaalsiner, Alvarez, and many others), which carry the notion of culture in its title (most prominently Cole's 1996 seminal synthesis).

I will restrict the references to learning theory to this particular school, because it is unique in offering ample room for cultural difference and hence might be adequate within the framework sketched by the What Ifs of the present book.

Main concepts of this approach, which have special relevance for my focus, are:

– The recognition of the complexity of learning processes: this issue was mentioned very briefly in a former paragraph. Vygotsky's consistent plea against 'single–factor theories of development (theories that posit one major force of development and a single set of explanatory principles) was aimed primarily at biological reductionism and mechanistic behaviourism.' (Wertsch 1985: 21). The consequence of this position is that learning cannot be reduced to mere biological processes or mere mechanistic procedures. For Vygotsky learning is something that is part of the complex of learner + socio-historic context. In educational terms this implies that changing elements in the context will have impact on the learning process and on what is/can be learned. But also, the personal characteristics of the subject—e.g., its being mature or not—will be relevant for the learning process. Specifically, mediation and mediators have great importance in development, according to Vygotsky. Mediators in learning are primarily signs and tools, which do or do not form part of the environment of the learner: books, all sorts of artefacts, language and communication styles, institutional settings and so on;

- Higher mental functions have great importance in Vygotsky's view: in order to understand the child 'one must first understand the social relations in which the individual exists' (Wertsch 1985: 58). In Vygotsky's own words: 'we could say that humans' psychological nature represents the aggregate of internalized social relations that have become functions for the individual and forms the individual's structure.' (Vygotsky in: Wertsch: idem). So, the focus on the individual as source of learning is rejected in favour of the social group or origin.

  Especially for the higher mental functions this focus is crucial: mediators such as language, habits, values and so on are the landscape or the pool out of which ideas, insights and choices emerge, and by means of which they are formed and phrased. Without this broad and complex social (and cultural) context no mental functioning of any degree, which we would call human, will emerge. On the other hand, a voluntary and conscious organization of this contextual material in education is of utter importance for the development of especially higher mental functions in Vygotsky's approach.
- The primary educational concept attached to the former paragraph is that of 'zone of proximal development'. In Vygotsky's social theory of learning and development the types and the quality of interaction between the learner and her environment is of primary importance. Put differently, the abilities and the capabilities of the pupil is one part of the interactional complex, and the nature and qualities of what is offered 'from outside of the learner' is a second and equally important part. Within this double structure of interaction, the typical focus of Soviet scholars was on 'how the child can become "what he not yet is"…' in Wertsch's phrasing of the Vygotsky perspective, whereas most of the western developmental psychologies focused on the research of how the child became what he presently is. Piaget focused on the detection of the stage of biological maturation at each phase, and how this could be understood in view of the stages that came before. BehaviourIsm explores how the intricate manipulation of S and R yield the S-R-relationship that is witnessed at any one moment of life. The focus of Vygotsky is on what could become or even what could be made or guided such that the person of tomorrow will be different from the one we see today. Although this is certainly a perspective that is in line with Marxism,

it need not be restricted to that philosophy of human beings and of society: Rousseau and other Enlightenment thinkers held similar ideas (see Chap. 12).

Vygotsky captures the educational potential of this alternative view on the child by introducing the notion of 'zone of proximal development'. A lot has been written on this notion. I will only mention it briefly, in order to use it in the culture-sensitive perspective of multimathemacy later on. The first element of this notion is that of stages of development: a person at one particular stage A is understood to be at a certain point in a possible development. But, in the focus on 'what a person is not yet', this entails that this person is next to or in the near proximity of…stage B. As educators we need to know this, take this into account and then actively use that knowledge in the educational process. On the one hand it is useless to follow a concatenation of stages and progressively more difficult steps in a mechanistic way, for example because the curriculum has it stipulated, or because the theory tells us that is the path to follow. Or, with reference to mathematics, because the mathematicians of the AM view are convinced that this is the intrinsic path for development in mathematical thought. Practically, exclusively curriculum-steered mathematics education is not the right way, according to this approach, because chances are that the particular level, or cognitive structuring of the pupil is disregarded by doing so. And hence a high level of dropout will ensue.

On the other hand, it matters a lot in this perspective what the environment offers at any moment for the particular learner concerned: continuous checking of the appropriateness of that offer for the particular child is of great importance, since proximity between child and teacher's worldview or knowledge is relevant here. If the teacher follows the orthodoxy that seems fit for his own cognitive setup, he may offer material, questions, images or concepts in the classroom that cannot be connected, recognized or otherwise familiarized by the child because the proximity is lacking. Any offer in the curriculum or the teaching will add to feelings of alienation, and be appreciated as 'not speaking to my world' or 'too abstract' or something along these lines. This does not mean that the teacher should only focus on what is familiar to or recognized by the pupil. On the contrary: what is offered should be just one or two steps beyond what is already acquired or known. The zone of proximal development indeed refers to the distance between the actual development level of a child and the potential level, reached with the help of adults and the mediators they use (school books, concepts, end the like: Wertsch 1985: 67). This refers primarily to the nature of higher mental functions, which encompass anything we would call culture (or social being) today.

In recent years a group of scholars in the West (Europe and the US) have been expanding these ideas. The Swedish psychologist Hedegaard (2012) thus focuses more broadly on the child's social situation, including family and societal settings, to situate learning and even schooling. She makes a plea to start 'researching children in their everyday settings' (Hedegaard 2012: 139) rather than in experimental settings. The 'zone of proximal development' then becomes a particular and highly relevant slice of that 'whole' of the child's environment. But at the same time, other—less school-defined—settings can be focused on in the research. Hence, the child's background (see next section) and the knowledge gathered and readily available there becomes the general field or larger 'zone' of learning, and taking this into account in curriculum development then looks like a very sensible thing to do.

Lave (2012) goes even a step further and points out that looking at a person's learning process implies attention for the larger political context of the learner. She links this thought to Marx, a source of inspiration for Vygotsky at the time, especially his theses on Feuerbach. In contemporary anthropology she points to a parallel (but not Marxist) position in Ingold's recent environmentalist work: "when a child becomes skilled this is a consequence of his or her involvement in a social matrix that is entwined with the natural world, a world that is not so much mastered as it is revealed through deployment of the skill.' (Ingold 2004: 163). This can be considered, in my interpretation, as a reformulation of the idea of Vygotsky idea of learning, when looked at as the rather symmetrical and cooperative interaction between learner and environment. (see also Ingold's idea of sphere, above on 'worldview')

– When I apply these notions within the educational scope in the context of a variety of cultures, it is clear that we have potentially a strong instrument for a genuinely emancipative mathematics education. In the next section this point is explained in full. In Chap. 12 the Vygotsky approach is worked into the Sen-Nussbaum proposal.

2 Background and Foreground Knowledge

From the previous paragraphs it is clear that the interpretation of the phenomenon of learning has obvious implications for the educators and the educational policy makers. More specifically, the interpretation of the widely spreading trend of dropout from school without decent qualifications (without a diploma or a valid certificate for the 'real world') will be quite different, depending on the theory of learning one adopts. Concretely, many a mathematician and quite a number of mathematics teachers I encountered in the past years would explain the dropout rates by stating that these pupils 'just could not cope with mathematical thinking', or even that they were incapable of formal thinking in general. The reference to the PISA assessment is then used to substantiate these opinions. E.g., with the start of a new school year (September in continental Europe) this sort of discussion appears on a yearly basis in the written press. This illustrates the point made in a general way by D'Ambrosio, the founding father of EM. In a reaction on this type of attitude about detached or context-independent mathematical education, he states with disbelief: 'Indeed, some educators and mathematicians claim that content and methods in mathematics have nothing to do with the political dimension of education.' (D'Ambrosio 2007: 27). In a very thorough study on the way the arguments run when implementing this still dominant view on AM in mathematics education François (2008) gives a painstaking analysis of the role of political perspectives in the context of Flemish (Belgian) education. Political perspectives encompass issues such as social and gender inequality, differences in worldview, textuality or orality, acquaintance with a school culture (its discipline, evaluation tradition, competitive structure and so on), apart from strictly political elements like the open access of schools or the installation of elite schools with high financial thresholds.

Remarks:

1. Coming back to the notion of 'zone of proximal development' I have to make an important note on the possible meanings of this central concept. Vygotsky speaks about a difference in knowledge, established in the child's mind and the potentials of the environment (including the teacher's knowledge). The child has acquired knowledge through what he called 'independent problem solving' (Vygotsky 1978). One possible reading of the parties involved could be that it is an asymmetric couple: that is to say, one could understand the relationship as an asymmetric or unequal one, with the child going from less to more, thanks to the intervention of a superior other, namely the teacher. This interpretation, with a clear educational impact, need not be the one to adopt. In a critical reading, which is compatible with my view, one can interpret the 'zone of proximal development' as knowledge transmission within a symmetric relationship. This is the reading that one finds in Roth and Radford (2010).

The authors stipulate that the kind of educational strategy one adopts will allow for a different interpretation of the notion. In examples from a mathematics classroom they explain how the actual interactions between teacher and pupil will fill in the notion of the 'zone' very differently. In Roth and Radford the following situation is described: 24 students sit in a circle on the ground. In the centre of the circle a series of papers are spread out, accompanied by labels such as 'cube', 'ball', and other geometric concepts. On the papers the appropriate geometric figures are depicted. The teacher sits next to the papers, surrounded by the students. She explains what is written on the label cards and hands a bag with 'mystery objects' to a particular student. The student picks out an object, e.g., a cube and enters a dialogue with the teacher. Within a symmetric interpretation of the use of the 'zone of proximal development' the dialogue will be a genuine dialogue, where the teacher invites the student to match his object with the label of 'square and cube' and the accompanying card with square figures.

The dialogue goes back and forth (because of the attitude of the teacher, who acts as a genuine interlocutor rather than as the one who knows or knows more than the student): the teacher asks what label will be adequate, and the student responds by silence. In a second moment, when prompted, the student asks a question about the first question: what does that question mean? So, the student starts negotiating the question. The teacher recognizes the relevance of the negotiation and rephrases her question, starts pointing to the labels, and so on. The student will finally understand, reformulate the question and solve the problem formulated by the teacher. It is important to see that in this pedagogy a series of aspects are specific and allow for a symmetric perspective on the learning situation:

- everybody sits at the same level, with eye contact,
- the learning process is developing as a continuous turn taking between provider (teacher) and student, where the latter can negotiate the questions, appeal to the teacher to look at them from the student point of view and think along rather than instruct.

2. Secondly, it is good to expand this notion from the rather restricted cognitive version of Vygotsky, who spoke about 'developmental level as determined by independent problem solving' (after Wertsch's translation 1985: 67). This makes the notion more precise, but also more limited. The broader notions of 'background knowledge' and 'foreground knowledge' which were introduced by Skovsmose (2005) cover a larger and more cross-cultural or maybe even trans-cultural area: not only cognitive differences, and eventually linguistic differentiations are taken into account, but the broader field of relevance covered by the studies on worldview (see Part I of this book). Indeed, children bring learning styles, values, rules and habits on authority relations, time management and time notions, and a certain framing of what a task would be, along with them when they enter school. All of these aspects, and possibly more, form the package of what is called 'background knowledge' (BK). From the point of view of the educator it then matters a lot to take all this into account and define what would be the possible 'foreground knowledge' (FK) of each child: what are the potential next steps, and what would be the translation of further perspectives within the frame of the BK of each child? What I call 'political' in this instance is the stand the educator and/or the school will take vis-à-vis this complex of BK + FK for each child: they can disregard BK and hence organize education from the perspective of a standard and so-called culture-independent point of view. In mathematics education this means that the curriculum and the standard learning procedure of AM prevail, and no modifications in terms of BK + FK should be considered. My contention is that large dropout numbers will then be the consequence. Indeed, in my analysis AM has clearly social, political and cultural foundations as well. Hence, it is not 'neutral' in that sense. Using AM as the sole basis of instruction, translated into a uniform curriculum for all in mathematics education will hence yield misunderstandings, alienation and eventual dropout on the part of the children, since they do not share some or a lot of the implicit and taken for granted social and cultural aspects of AM.

I give two examples to illustrate this point:

1. During my observations of learning styles in some oral traditions (with Navajo Indian studies, but also with some immigrant groups in Belgium) I was struck by the rather obvious fact that learning is not initiated in the child through

instruction. No parent instructs her child to do things this or that way. Rather, children from birth on, are put in an erect position (on a cradle or otherwise) such that eye contact is always possible. Hence, the child sees what the mother or father are doing. When the child turns toddler and infant the same procedure is followed: the child is present and is encouraged to look at what the adult does. Then 'of a sudden' the child will start imitating: it starts a small loom for sash belts and weaves 'like it has seen doing' the others. Or it will herd sheep and goat through canyons by imitating the use of environmental data (sun, rocks, canyons, etc.) the way they were used by elders. I literally never heard an adult instruct a child. When the children started school, teachers complained to me that they were 'silent': they did not ask questions, nor were they attracted by competition. Parallel observations were reported by e.g., Farrer (1991). When she went through a long cycle of participant observational research with a Mescalero Apache informant, she asked all sorts of questions, never receiving an answer. Finally, her informant shouted at her: 'Pay attention!'. This testimony of her refers to the same emphasis on looking and imitating in order to learn, rather than instruction or verbal transfer of knowledge. Education is 'stealing with your eyes' and 'paying attention', rather than being instructed verbally on what to think or do.

Similar things were observed by collaborators who worked on Turkish immigrant children, coming from a rural area of Turkey with basically poor schooling. They also were silent in the classroom, except for pupil-to-pupil interventions in the classroom. That is, when the teacher explained something they did not grasp, one of the Turkish children would switch to what we called the 'pupil code' and translated or interpreted for the colleagues what could be the intention of the teacher. This occurred regularly, and the teacher got quite annoyed by what she took for a lack of discipline. The researchers were able to show that children in such mixed classes (at primary level) used several codes in the classroom, in order to cope with this 'other-cultural' setting:

- the teacher code, with the teacher as the one who has and communicates knowledge,
- the pupils also practiced the child code, meaning they voiced their unrest or tiredness by being noisy, just like any other pupil, yielding mild protest,
- and they practiced the pupil's code, meaning they added information for a small group of their likes with what I call similar BK, in order to help understand what the teacher's code is all about (De Munter and Soenen 1997).

When such different codes are actually at work within a mixed classroom, it is good to recognize them and eventually to integrate them in a creative and productive way in the teaching practice. In fact, we learned from the school ethnographies that we did that the usual burnout of teachers in this kind of schools was overcome by integrating the different codes within the classroom practices and train teachers to cope in this multifaceted way with diversity within the class.

2. a different example stems from the same type of field experiences. In working with children and adults in Navajo Nation, USA I was often surprised by the procedure I had to follow in seeking collaboration from the local people. I would approach a person and ask her or him to work with me on a theme (mostly on spatial knowledge as expressed in the language). Typically, the person addressed would enter a longer or shorter period of silence and, when admitting to collaborate after a while, would ask for coffee. When the work started the person would go for it until it was pretty much finished, regardless of the hours of time spent on the job. When colleagues would stop the work at around 5 p.m., or would leave for the weekend, Navajo would object that the job was not finished. The idea that work, thinking or whatever be subordinated to a fixed, context-independent time frame (the 9 to 5 job) was experienced as counterintuitive. It proved to be utterly estranged from the cultural habits of the informants. Moreover, the structuring of problems and events in the typical script format, which is so central to our literary tradition,—the so-called plot format—was often causing uneasiness and alienation. That is to say, in the western settings we are used to work with a hidden and obvious plot structure: an introductory phase of action, with the presentation of protagonists and small actions is always the first phase. This is then followed by the 'building up' of the plot towards a moment of action and heightened tension, and ending in the resolution of conflict in a happy ending/the elimination of all the bad guys. This is utterly strange for the story telling tradition I was working in. Things happened, and protagonists came and went in the story, and of a sudden (to my mind) the event ended. The very idea of the structure of the plot, with a distinct beginning, middle and end, was absent. On the few occasions where Navajo people made their own filmed report (e.g., about a ceremony) this was very striking: even then, in the medium where plot is seen as intrinsic within the western tradition, it was impossible for the westerner to recognize that familiar structure. Rather, here again, the task seems the crucial structuring element: a task involves spending time, using paraphernalia, doing and saying things. But the tasks dictate what time is spent, not the other way around; and the structure of the event is experienced as made up by actors, circumstances and happenings rather than by a preconceived textual plot or storyline (Hymes 1981; Pinxten 1995).

Here again, my contention is that it proves important to know, respect and take into account this way of going about with and in reality, and not superimpose a local, western approach or format as the obvious, the best or even the only conceivable way of doing in building up knowledge and transferring a view on reality.

These two examples may suffice to make the point. Other aspects will be highlighted later in the book. It will be clear that schooling and education through instruction are highly specific ways of transferring knowledge. I claim that we have to be conscious of this and question the presumed supremacy or dominance of the schooling format.

3 Socio-Cultural Learning Theory and Mathematics Education

Some of the cross-cultural psychological and the anthropological studies on mathematics and mathematics education has been set up within the framework of socio-cultural perspectives on learning, and on instruction (mainly through schooling). References to the Vygotskian perspective are sometimes explicit and clear, sometimes not. For the purpose of this book, I lump together those most relevant studies in the field, which are focused on mathematics education, disregarding whether or not they work with the framework of the Vygotsky-inspired learning theory.

– Modern mathematics in a traditional culture: M. Cole did fieldwork with the Kpelle in Liberia during the 60s and 70s of the past century. A quarter of a century later Cole (1996) produced a seminal work, which thought through a series of questions on psychology in a comparative scope and within the Vygotsky-perspective. However, the first ethnographic work was not deeply inspired by Vygotsky at all.

In the first reports of the Kpelle research Cole was struggling with the western psychological presuppositions. He thus relates that, in line with his training as an experimental psychologist, he had set up a ‘scientific’ experimental setting in the middle of the bush, among the Kpelle. He remarks along the way that the Kpelle did not have schools at that point, and were not acquainted with the detached, context-free use of knowledge which is so characteristic of western schooling. Evidently, texts were absent too in the education of the young Kpelle. A net result of the experiment showed that Kpelle did not master classification logic, and thus seemed to illustrate the idea of underdevelopment (which was obvious for the Peace Corps and scientific workers at the time) pretty nicely. However, Cole was becoming critical of his own mindset and asked himself and his collaborators the question: was it the case that the Kpelle did not know classifications, or was the experimental setup and the questioning in themselves so utterly foreign to them that they did not produce any adequate or relevant answer? So, he questioned his own position and the implications of his way of approaching the non-westerner (Cole et al. 1971; Gay and Cole 1967) instead of being satisfied with the taking for granted of his own implicit colonial attitude.

One of the reasons for this shift in mind was that he saw people dong calculations in the markets, and apply proportional thinking quite adequately while making costumes in the street. So he concluded that there probably was no lack of knowledge with them, but rather a lack of interaction and communication between the researcher and the Kpelle. He then started looking at the Kpelle language and found quite interesting things which corroborated the observations in the field and questioned the appropriateness of an experimental setting some more.

When asking broadly about appropriate naming of phenomena the researchers kept stumbling on the category of ‘*sen*’, which translates as ‘thing’. They were able to identify a chart of '*sen*', which was repeatedly used in numbers of sentences. Through substitution of labels in sentences, the workings of the charts became apparent. Obviously, the chart and the ensuing classifications of phenomena were 'odd' or 'not logical' according to western views, but they were systematic and had the function of ordering phenomena in the Kpelle worldview. So, instead of identifying the classification or lack thereof according to western criteria, Cole and his co-workers started taking into account the Kpelle criteria of relevance. What does this mean in actual practice? 'Consider for a moment how rare a straight line, a perfect circle, … are in nature' (Cole et al. 1971: 144). In the western approach to the external world we will nevertheless ascribe such features to the 'things' we distinguish in the world: the sun is circular, the wall of a house and a road are produced as straight lines, etc. Not so in Kpelle country. Hence, the relevance of circle and straight line is extremely limited, in contrast to that of curbs, paths, winding roads or passes in the woods, and so on.

Furthermore, the Kpelle language frames human experiences in ways that differ from Indo-European languages. According to the ethnographic work of Cole classification and discrimination of 'things' or phenomena in the Kpelle universe is based primarily on colour, rather than form or number of things (Cole et al. 1971). In quite different research contexts other colleagues had hinted at similar differences at a basic level in the way diverging cultural traditions and languages would think and express the universe: e.g., when speaking about the emphasis on process and event in Hebrew language and in some non-western traditions of knowledge the sociolinguist Fishman (1979) would point out that westerners would subconsciously and inadvertently opt for a 'thingification' of the world of experiences of other traditions. Thus, in education, about other cultures he witnessed several examples of this sort: when presenting Mexican culture in the classroom the taco and the sombrero would be shown, and when thinking about Chinese philosophy the two 'halves' yin and yang would be pointed to (rather than the dual perpetual dynamics).

– situated learning and generalisation: In the wake of the translations of Vygotsky's work in the Anglosaxon world scholars like Lave, Rogoff and others started elaborating on this other view on learning. Learning is gradually seen as an interactive process between learners and their environments. The social and cultural networks, the links between the learner's BK and the categories and styles of thinking in the environment as well as communication modes are considered to be relevant in the learning processes, and hence need to be taken into account when looking at education. One step further, one can plead for the broader founding of institutionalised education in social and cultural contexts.

A lot of these issues can be found in the research model, which speaks about 'situated learning'. Especially Jean Lave (e.g., Lave and Wenger 1991) should be mentioned in this realm. In this group of researchers some members focused on mathematical education. Jurow (2004) gives an overview of the foci found in this line of research: 'situated learning …(is) based on the assumption that people learn through gradual participation in the socially and culturally organized practices of a community.' (Jurow 2004: 281).

When conceptualizing the development of formal thinking in terms of situated learning, the focus turns to such 'organized practices' as the speech habits and formats, the materials, the school context and the context of the pupil's homes, all of which impact on and actually shape the participation of children in the mathematical activities. One is reminded of the 'realistic' option Freudenthal argued for (see above). Looking into these dimensions of the situated learning complex for mathematics classes, Jurow analyzes what generalizing (or abstraction) could mean in the minds of the children participating in the math classes.

Generalizing as a necessary cognitive step in learning formal thinking implies that children learn to 'move away from' the situations in which a particular insight had emerged, or in which a specific concept was formed. In this the particular perspective of situated learning, this step of moving away is instigated by offering or seeking to recognize similar or comparative aspects of situations in new experiences and practices. This approach is not new, of course it draws on work of Vygotskyan like Davydov (1990), but also American scholars like Greeno (1998) and Lerman (2000). In Jurow the following distinct processes of interaction and communication are investigated in empirical settings of mathematics classes:

- *Linking*: 'the process of creating and applying classification systems' (Jurow 2004: 287). That is to say, in different situations aspects are recognized as sufficiently alike or similar to be put together as belonging to one class. With this step towards abstracting children learn to estimate the likeness of phenomena vis-à-vis each other. In the example of the observational setting of Jurow this step was recognized in children estimating the capacity of different ponds with guppies (fish). In terms of the size of the ponds estimates were then made on the possible 'overpopulation' of the ponds involved in one or the other class of ponds.
  Thus, through linking one can describe what people do, how they behave in practice in different situations.
- *Orienting* is the next step: once similarities and differences are clearly expressed in the classification act of linking, the learner orients herself in view of the classification made. In the example of the growth of the guppy population, this involves the mapping in a graph of the evolution in growth: a line that goes up in a curve or in a straight line, for instance.
  When orienting oneself, one moves away one more step from the concrete and draws a line as the representation of an evolution being watched.
- *Evaluating*. A final step in the generalization process is evaluation: the two situations and the representations are compared. In the example studied, the actual evolution of the guppy populations (in two stages of evolution) are looked at again, and their representation in the different graphs (straight line versus curve) are compared with each other: does the mapping make sense? Does either of the lines catch a relevant or important aspect of the two situations, so that this abstract representation of the actual situations is useful? What is learned through

> such experiences, moreover, is that *conjecture* is a valid and indeed powerful way of reasoning about concrete situations, while distancing oneself meanwhile from the situations. What the learner does is to fall back on 'what if' questioning, as part of generalization, regardless of the concrete or particular situation one is in. It is an act of imagination which can be engaged in whatever the real life situation one is involved in.

These are a few lines of research in the socio-cultural learning theory, applied to mathematics and mathematics education.

Part III
Epistemological Questions

Chapter 5
Foundational Questions?

When reading recent literature about the foundational questions in mathematics, one gets rather disillusioned. Or else, one could get a feeling of liberation. Disillusion may be your share when you still believe that without a firm foundation no unity in mathematical knowledge will be reached, and hence the 'skyscraper of Academic Mathematics' will remain a fiction for a very long time, if not forever. Relativism will set in and any overview of the field, which has the authority to allow for uniform curricula, as well as development programs, will become obsolete (e.g., Tymoczko 2000). On the other hand, some scholars will feel liberated: granting the enormous benefits of large branches in AM and at the same time emphasizing the political contexts in which any type of knowledge lives and thrives, these scholars will point to the great opportunities that are available to local educators and emancipation brokers now (e.g., Mukhopadhyay and Roth 2013).

1 Whitehead and Dingler

It is in this dual frame of mind that I was reminded of Whitehead's beautiful work on the polyvalent ways formal thinking can be envisioned, at the very least for geometry. In a remarkable book (Whitehead 1953) this deep thinker on mathematics and knowledge in general proposes to adopt what was later called a multi-perspectival point of view (Campbell 1989).

Whitehead grants that historically the geometric notion of point has been considered to be the only genuine 'primitive' in (Euclidean) geometry. The point defined as the geometrical primitive without extension, is the basic constituent of the line, which can be defined as a set of points. And so on. The 'primitive' point is the building block of the line, which can then serve as the basic constituent of the plane, which in turn will be the primary constituent of the volume. At each level of complexity particular properties and specific operations can further be defined. Whitehead makes the remarkable move to shift perspective in this reasoning. He proposes to pick any notion of this range and call it the 'primitive' for geometric reasoning.

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Hence, when taking the line as the primitive notion, the point can easily be defined by means of the intersection of two lines. But also the volume can be seen as the primitive: the ‘touching of volumes’ or their overlapping or the projection of one on the other is what will result in the notions of line, plane or even point. The latter are then the theoretically more complex notions of which the volume is the primary constituent or the so-called ‘primitive’.

Whitehead explored this line of reasoning in view of its relevance for our, western (and eventually scientific) model of the material world. I am intrigued by his approach because it allows me to tinker with the worldview or the background knowledge, which stayed mostly implicit in mathematics education. My point is that this culturally entrenched background knowledge obviously does not have any absolute or objective status, i.e. grounded in reality itself so to speak. Instead, its presumably absolute status was historically just taken for granted. Whether the language structure of the Indo-European languages has a determining role in this historical status of this particular and local (‘western’) worldview has been discussed repeatedly: the works of B.L. Whorf are most notable in that respect (Whorf 1958). However, when investigating the spatial knowledge of a very different tradition (with a ‘verb language’) it gradually dawned on me that the acceptance of ‘point’ as logical primitive was not obvious at all (Pinxten et al. 1983). And subsequently the work of Whitehead allowed me to open up the discussion at the deep level of intuitive spatial reasoning.

Navajo and the Athapaskan languages as instances of ‘verb languages’, as well as the way such oral traditions conceived of learning and thinking, had me go back to yet another intriguing scholar on the fundaments of geometric thought, namely Hugo Dingler. In the wake of the intuitionism school of thought in mathematics foundation studies, Dingler concerned himself with geometry. In a little volume Dingler (1933) rethinks the purely theoretical and somewhat transcendental status of Euclid’s geometry. His point is quite simple and at the same tremendously refreshing: he demonstrates how Euclid’s axiomatic theory can best be understood as a process of abstracting the empirical findings and the concepts and proto-models. Geometry (and mathematics in general) can best be understood in the context in which the human knower or knowledge builder is living, and most of all acting and reacting. Dingler makes the point that Euclid took the empirical knowledge and the semi-theoretical notions of his predecessors and put them in a more coherent and more context-free phrasing, ending up with a thoroughly abstract and logically tight theory, which we came to know as the axiomatic geometry of Euclid.

The relevance of Dingler’s interpretation of Euclid’s geometry for mathematics educators and anthropologists who focus on mathematics education in non-western cultures is not adequately appreciated in the literature, I claim. Oral traditions do not use texts, obviously. Let alone they would be oriented to the authority of texts. It took us, western scholars, several centuries to really take this in: until recently myths were studied as if they were texts, with a mother version, with the authority of at least a basic text, which has foundational status (Hymes 1981). In contrast with all this, we now know that myths in oral cultures are interactions or performances, rather than texts. The story teller presents a certain amount of themes to the audience and actively explores the themes together with the audience: sometimes hearers become speakers in the event, while themes shift and change according to the tastes of the audiences and the circumstances of the performance. In a word, myths are performances of a group rather than a more or less unaltered text (Hymes 1981). Since story telling is a major means of learning for an oral community, and members of such a tradition transfer their lore by means of this dialogical way of learning, it is clear that action and interaction are of the essence in oral culture. Orthodoxy or textual transfers are weird notions in that sort of traditions. For me it follows that Dingler's approach to the oral culture of Euclid in Ancient Greece is likely to be more appropriate to understand how geometry thinking grew back then than the later 'text as authority' view which is the dominant view on Euclid's geometry in the literate culture of the West during the past few centuries.

If this analysis of orality holds water, it then follows that mathematics educators in a mixed or other-cultural setting should be interested in the impact orality as such has on the pedagogical avenues chosen by the educators. My suggestion is that it is likely that we may profit from taking the performance character of story telling into account when devising curriculum material and strategies for learning.

I now want to combine the brilliant opening of Whitehead with the action-centred approach of Dingler, and see where it would bring me in another perspective on mathematics education. From there I will delineate a cause to look at the impact on the latter in different cultural settings. Obviously, both Whitehead and Dingler focus on geometry and do not speak about other branches of mathematics, let alone about set theory as a foundational sub-discipline. My concern is with the learning of elementary mathematical notions and skills at the onset of schooling. Hence, I acknowledge but disregard the logical problems of higher mathematics in its attempts to develop proofs for the consistency of mathematics as a whole. The latter tradition of mathematics (with proof theory, set theory and the like) is enormously important, to be sure, but its relevance for the learning of insights at the elementary level is not at all proven to my mind. As stated in the beginning of this book: I want to make a plea for 'trivial mathematics' as a means for education. Hence I boldly adopt the stand that we can disregard the (fairly recent) discussion of philosophy of mathematics for the purposes at hand and concentrate exclusively on counting, measuring, proportional thinking, drawing and the like in order to see where and how they yield important and insightful material for the first steps in mathematics education. I will come back on this choice later on. For now, I develop the point for elementary geometry as instigated in the works of Whitehead and Dingler.

Given Whitehead's plurality of perspectives on the 'primitive' notion from which to start geometry and applying it in teaching, I take the following stand: it can be argued that different perspectives or different premises are allowed, and hence the choice of one or the other is historically contingent. Put differently, there is not one uniquely necessary point of departure in formal thinking such as geometry. This position is, of course, in line with the ethnographic findings on spatial thinking (e.g., Pinxten et al. 1983). What Dingler adds to this way of reasoning, in my combination of both these studies, is that human action and interaction can be taken as the basis or source of the empirical foundation for formal thinking. Put differently, humans explore the world by means of physical actions, complemented by exploration through the senses (seen as forms of action by me). Finally, the most distant or 'detached' way of interacting with the environment is the purely theoretical building of knowledge. That would amount to a contemporary interpretation of Dingler's early insights (and of the whole school of intuitionists in Erlangen and in Amsterdam: Beth, Brouwer and other scholars in that tradition). If I take that view, the anthropologist in me can then argue that in the culturally diverse world of anthropology a range of actual types, formats and traditions of acting and of mentally processing spatial and other mathematical characteristics can be empirically distinguished. For the sake of economy of research phenomena like language use, perception and thinking are seen as ever so many different modes of action in my perspective, materialized in equally many different particular shades and forms. If I apply this double 'unpacking' of the naturalization of formal thinking when developing ideas on mathematical education, I can justifiably defend a program of multimathemacy (Pinxten and François 2011). However, this last step needs more explanation.

The steps I proposed so far are the following:

- people are socialized in particular traditions of speaking, perceiving, thinking and acting; these cultural differentiations should be taken into account when devising education,
- in a general sense action (and interaction) is the generic category when studying humans: perception, bodily actions, thinking and knowing, linguistic processes and social interaction are forms or particular domains of action. More specifically, mathematical thinking should hence be conceived in this same broad perspective on action, linking operations and forms in mathematics—through whatever series of detachment—with bodily notions and empirical actions (Dingler),
- modelling spatial (and other mathematical) aspects of reality can be gone into from different perspectives (Whitehead). Hence, any orthodoxy in curriculum is better avoided, and the emphasis should be to allow for a variety of many different trajectories, both conceptually and in learning procedures.

The next step is then to put all this together in a synthetic proposal, in view of education in mathematical knowledge and skills. The questions we should be addressing in mathematics education will then become:

- what mental processes, action forms and notions can we discern in the many cultural traditions we know about in anthropology? How can we inspire education (in schools and outside of them) by means of this knowledge?
- finally, and most importantly, how can I take this diversity in worldviews into account as 'background knowledge' or out-of-school knowledge when developing curricula and teaching procedures for mathematics education? This is,

basically, what the term multimathemacy wants to promote: recognize the polyvalence and plurality of formats, processes and notions which make up the out-of-school knowledge in the many cultural groups we know about, and use these as pedagogical starting points in developing educational programs and procedures.

I hope to have made clear that Whitehead and Dingler (amongst a small group of other philosophers) allow us to 'think outside of the box' of received knowledge here. Some fellow travellers in mathematical education circles might benefit from their company. There is no need to go into the debate on foundations of mathematical knowledge again (see Tymocko 2000), but it is good to know that some philosophers offered inspiration for the rather unorthodox position of multimathemacy.

Chapter 6
Language and Thought

For centuries the relationships between language and thought, or alternatively between language and culture, have been the subject of fine scholarship. In linguistics, anthropology, psychology and philosophy great minds have been busying themselves with this complex. To be sufficiently consistent with the previous parts of this book, I will focus on language and culture. By choosing these labels I deliberately place both phenomena of language and thought in their cultural context.

The prolific anthropological linguist Dell Hymes reorganized thinking about these issues in an early publication (Hymes 1970). In his view anthropology uses linguistics in a number of ways. It is important to treat the levels of relevance of the linguistic factor in detail. Beyond that, the very question of the possible relationships between language, action and thought will be gone into to some extent.

(a) At the bottom level, there is *the practice in and knowledge of the language* of the subjects one studies. It is obvious that ignorance of the language of the pupils will not help in the teaching process. Languages represent and interact with reality in a variety of ways. In mathematics education this variety can be crucial for the success of the learning processes. The many ethnographic accounts we have carry all sorts of data about counting, designing, modelling and so on in the thousands of cultures we have been studying. Representative overviews on formal thinking in the cultures of the world do not exist as yet, but there are relatively good and easy instruments to engage in comparative research: Ember et al. (2010), Ascher (1991), Powell and Frankenstein (1997), to mention just a few. Apart from these partial overviews, there are innumerable ethnographic reports that deal extensively or in a subsidiary way with 'mathematical' knowledge in other traditions. The Human Relations Area Files bibliographic system allows one to have an overview easily (cf. Ember, Ember and Pelegrine, o.c.).

An example from work in my own research group can illustrate this point. In the report on a thorough study about the dropout and success in mathematics classes of Turkish immigrant children of 7–8 years of age in Ghent, Belgium, Huvenne (1994) starts by pointing out that the Turkish language has some features which should be

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addressed explicitly by any teacher coming from another cultural background (i.e., Flemish). She mentions two examples to illustrate the problem:

- in Turkish the plural is constructed differently from most European languages: in the European languages one uses a noun in the plural and adapts the verb accordingly, f.ex,: ‘Two apple **are** falling’. But also: ‘Apples **are** falling.’ In Turkish (which is given in literal translation here, rather than Turkish), the plural is only indicated when a number is added, but not when it is not qualified explicitly by numbers. So, the sentence ‘a juniper is an evergreen’, would read in the unqualified plural: ‘juniper **is** evergreen’. In the company of a number, it would read: ‘five junipers **are** evergreens’. When pupils are not made aware of the difference in plural forms in English (or another European language like Dutch/Flemish) and Turkish (an Altaic language), this may backfire in math classes.
- similarly, prefixes do not obtain in Turkish. The consequence of this is that the position of things or phenomena in space gets expressed by means of adding a suffix. Like in: ‘table **the**’ instead of ‘on the table’, or ‘box **the**’ instead of ‘in the box’.

These simple examples point to the relevance of knowing enough of the language of students to teach adequately in a mixed mathematics class. Similar examples can be given for many languages, to be sure. The remedy here is: knowing the language and anticipating the difficulty. In a second stage the difficulty can be solved by explicit treatment of the difference together with the pupils.

(b) The *use of metaphors* is a second level of using linguistics in anthropology (Hymes 1970): in mathematics education I think primarily of the use of limbs (fingers, arms, etc.) in counting, but also of spatial forms of the environment in the development of geometric thinking. Some examples will illustrate this point. In the standard work on metaphors of the past decades, Lakoff and Johnson (1980) give a superb overview of the ways metaphors structure our background knowledge.

In his intriguing research on Rwandese mathematics, Huylebrouck (1997) explains why the duodecimal system rather than the decimal one, is the basis of at least some of the counting traditions in Black Africa. On the basis of finger parts and positions, and of combinations thereof numbers from 1 to 12 are formed and communicated by using the fingers of one or of both hands, in different spatial positions. But, the author continues, this is only part of the story. Beyond often used quantities (say from 1 up to 100) large quantities are expressed through the use of images about other big phenomena. For example, in Kinyarwanda the following concepts and notations obtain:

10.000: ‘inzovu’ which translates as ‘an elephant’
20.000: ‘inzovu ebyilli’ which translates as ‘two elephants’ (after Huylebrouck, n.d., based on data from several African studies scholars).

In my own fieldwork I had ample occasion to see how metaphors worked for Navajo Indians in the USA. Even young kids are able to find their way (with a herd) in winding canyons that stretch out for tens of miles. The particular form of a rock is often described by means of words for animals, but also analogies with human postures: Standing Rock, Eagle Rock, Snake Rock, and so on are ever so many markers for the boy or girl who is herding sheep and goats in this vast and rough territory. The washes and springs have names, which recall particular features of animals, humans or plants, and so on (Pinxten et al. 1983; Pinxten and François 2011). Children learn to orient themselves in this environment by means of such markers and their particular form, position or inclination.

Barton (2008) gives several examples, which illustrate this point. A quite different instance concerns the way spatial positions are defined in different languages and cultures. In the Cartesian tradition of the westerner a position is defined by means of the combination of distances on orthogonal dimensions, which cross each other in one point. This system is known, for two and three dimensions, as the Cartesian coordination system. Implicit in this way of reasoning and measuring is that the focal point or crossing point is in fact the position of ego, of the individual who positions anything in the world vis-à-vis his or her own point of origin (indicated as O for each of the dimensions, with O as the only point of origin for all orthogonal dimensions). This is even more the case in the so-called polar coordinate system of Newton and Bernouilli (Barton 2008), where the unique reference point is the ego, from whom a dimension starts out into space.

In Maori and Tahiti language and thinking, the position of something is defined with reference to both the speaker (ego) and the addressee. Compare the Cartesian positioning (I) and the Maori-Tahiti positioning (II) shown in Figs. 1 and 2.

The very notion of 'position' is similar, but different in both cases. In the Cartesian positioning system a third and person-independent frame is adopted; in the Maori-Tahitian case the frame is the sum of two personal perspectives.

(c) *linguistic structure and thinking* (Hymes 1970):
A famous research program in linguistics and anthropology is that on universalism versus relativism, otherwise known as the debate on linguistic relativism. The point of the debate is that different languages and language families in the world tend to coexist with different worldviews. When languages are different from one another at a (deep) structural level, the question arises whether the language structure does (co)determine the thinking and knowledge processes of the corresponding cultural communities. Until the '80s of the past century two main and opposing positions could be discerned on this issue:

- universalists claim that at the deep structural level universal categorization is the rule. That claim is often accompanied by a presupposition that deep structures would be innate and hence universal across languages. Especially through Noam Chomsky's TGG (Transformational Generative Grammar) analysis this line of thinking became very influential (Chomsky 1968; Schaff 1977). However, a severe critique developed over the years,

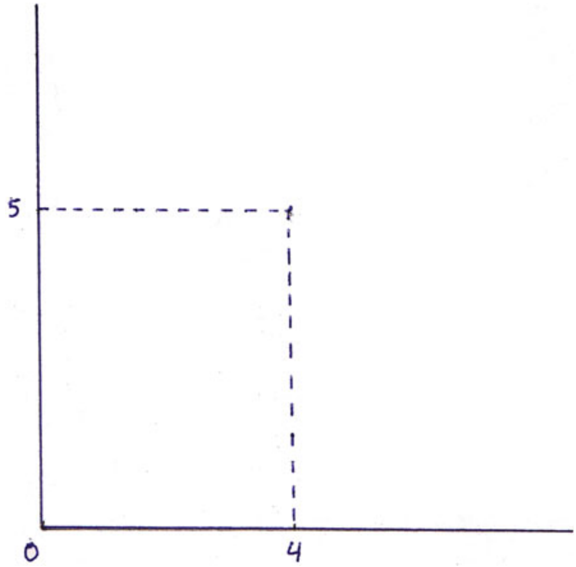

Fig. 1 Cartesian positioning (*4* horizontal, *5* vertical)

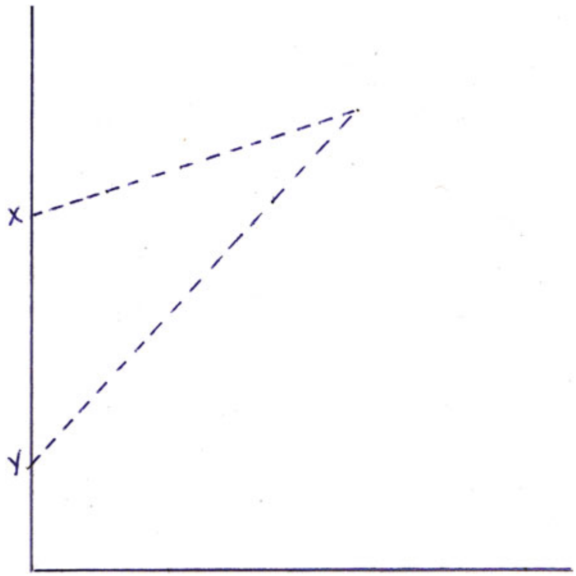

Fig. 2 Maori-Haitian positioning (75° of viewer x, 45° of viewer y)

claiming that Chomskian linguistics was too Eurocentric and started from an empirical basis that was harmfully limited and particular: linguistic anthropologists (Hymes 1970), but also linguists working with Asian and African languages objected that the presupposed universality of Chomsky's grammatical categories did not hold in Japanese, Bantu and so on.

- the opposite group of researchers was named the ‘linguistic relativists’: the scholars in this group have two different kinds of roots. On the European continent there is a tradition, which goes back to the 18th century and has a communitarian or nationalistic ring to it. For example, the 19th century German scholar Herder and others developed the idea of culture-specific languages at the time of emergence of the nation state in Germany (Gipper in Pinxten, ed.). On the other hand, anthropologists and linguists working on a variety of languages and cultures in the USA and elsewhere made strong proposals about deep differences between languages and ways of thinking in (classical) Chinese, in black African traditions and most of all in Native American traditions. Two major linguists in the USA thus were linked together in what became known as the Sapir-Whorf hypothesis. Both of them worked with Native Americans, quite independently from the European tradition. They found that some languages were basically verb-languages, and suggested that the processual view on nature the speakers adopted might be triggered by this deep linguistic structure. For example, the dynamic space-time worldview of the Hopi matches with the grammatical structure of a verb language (Whorf 1956). On the other hand the ‘objectification’ of phenomena in western ways of representing the world can only be understood in a language, which has verb and noun categories.

A heavy debate developed over the question how the Sapir-Whorf hypothesis could be refined and made into one or more pointed and empirically testable hypotheses. Furthermore, factual and comparative studies over the years yielded a more complex and differentiated picture still: some studies showed that some elementary categorization is likely to be universal to a large extent. For example, colour differentiations in languages vary from two to eleven ‘basic colour terms’, but this variation is not random at all: first light/dark shades only, then blues, reds and browns in the three colours category and so on (Berlin and Kay 1969). In classification studies it seems well established now that plants and animals are classified by means of a pretty uniform sort of class logic, which is most likely to be the foundation of natural science classifications as well (Atran 1993). For the purpose of this book, it is not necessary to go into this here. I can suffice by stating that over the years the following position seems to have emerged: the either/or dichotomy of relativism versus universalism is left behind in favour of a nuanced set of positions:

(a) for those aspects of reality which are blatantly important for survival of human beings, universal categorization in the worldviews seems to obtain, regardless of the linguistic deep structures. Scholars have proposed to speak here of the ‘natural kind’ status of these aspects (the philosopher Quine 1969), or of their high degree of ‘entitavity’ (Campbell 1989). That is to say, it is of great importance for survival that humans distinguish between edible and poisonous plants, or that they distinguish between water and fire. These phenomena and their discrimination have then a high status of ‘natural kindness’ or a high degree of entitativity. Although different languages describe these phenomena in different terms and linguistic categories, the universal presence of the categories can be shown.

(b) For those myriad phenomena where the survival value is less clear, or indirect at best, the width of differentiation grows and relativism seems more appropriate as an approach. In actual fact, all more abstract or less empirically testable phenomena belong in this second group. Hence, the variety of forms to speak about such non-empirical phenomena as supernatural forces or beings, ghosts and so on is next to endless across the world.

So, where universalist theory might have been focusing on the 'basic' and vital phenomena in terms of survival, the relativists have a good point when looking at the worldviews in general, including innumerable other aspects of reality as well. Accepting this double position makes it possible to sensibly use the Whorfian hypothesis for the study of the manifest variety of worldview items, but does not preclude the narrowing down of the range of variation and the increase of overlap on the more vital aspects.

The most telling example I can come up with is, not surprisingly, from Native American work on these issues. At the same time it is good to remind readers that the range of empirical data is vast, and is still growing. Famous other examples can be mentioned in work of older China specialists like Granet (1934) and Needham (1956).

But for me the most outstanding example comes from the Ataphaskan languages. Working with the Navajo Indians and focusing on their spatial knowledge, I got more and more intrigued by their language. Navajo (like the other Apache languages, but also Cherokee, classical Chinese and a set of other languages in the world) is a so-called verb-language. That is to say, there is no genuine noun category. There is no verb 'to be' in any real sense of the term in Navajo, and a set of verbs which translate as 'to move, to go, continue subsisting' and the like take the central place of the verb 'to be' in the Indo-European language family. Apart from that, there is a whole set of not less than nine verb stems, the so-called classificatory verb stems, which define a sub-domain of ontology. They all point to different forms of manipulation, of handling and treating aspects of actions or phenomena in reality. They come closest to the domain of the Indo-European verb 'to be'. I explain this a bit in detail to make the point in a stronger way (see especially Garrison 1974).

For example, when speaking about a phenomenon that stretches out 'in horizontal position, touching the background it is lying upon and with absolute dominance of the lateral or the depth dimension' (Pinxten et al. 1983: 94) one uses the verb (third person singular) *silà*: *tooh silà* (water stretching out… like the rivers San Juan or Rio Grande), but also *hoodzoh silà* (a borderline, i.e. a series of milestones running through the desert).

However, when the water is spread out in a thin surface, like in water spilled from a fallen glass, you should use a different classificatory verb stem: *to sikaad* (water spreading out in an shallow pool). The same verb is used to say: '…*Lukachukaidoo sénikàni goyaa nàhoodeeskaad*', meaning 'from Lukachukai the

Round Rock mesa stretches out widely,' indicating a flat mesa-like surface in the landscape. Finally, when the water is to be held in an open container by humans, like water in a glass, one speaks of *si'à*. And that same verb form is used to indicate the holding or grabbing of a stone or another bulky object.

So, the verbstem—*là* refers to a way of handling (water, rocks, humans, whatever) when the phenomenon manifests itself as a stretched, stream-like feature. The verb stem *'à* is used when a/the same phenomenon persists or continues in a spatial form which is to be handled like a solid or robust thing. It could be water (when in a container), or whatever, requiring such manipulation by an agent (humans or winds or any other agents). When the same phenomenon has to be handled yet differently, like a flowing instance or a thin leaflike thing, the stem—*kaad* is in order. What is emphasized, I claim, is the ways of moving, or the persisting as process and the implications of that for the ways of handling, manipulating phenomena in a material or a virtual/imaginary way.

In yet other words, the Navajo does not think in a world consisting of objects (rendered in a grammatical noun category) and operations on them (expressed in verbs), but rather nature is construed as an event world. An accompanying—and probably co-determining—phenomenon of this worldview is the deep grammatical structure of a language that can be termed a verb-language, rather than a language which distinguishes fundamentally between noun and verb categories (or noun phrase and verb phrase categories, in the Chomskian version). A correlate of this can be detected in the fact that such verb-languages like Navajo, do not know or use the time categorisation of past/present/future that is familiar from Indo-European languages. In the case of Navajo, this means that over twenty-five aspects are used to express all possible differentiations of actions and action changes, including what westerners would identify as 'temporal' features (Pinxten 1995).

When taking into account these findings in my subsequent endeavour to develop curriculum material for a bicultural program in geometry teaching, my Navajo collaborators and myself engaged in delicate and time consuming processes of communication in order to fully take into account these deep elements of out-of-school knowledge. For example, a group of teachers from the Rough Rock bicultural school (Rough Rock, Arizona) took it upon themselves to define some technical terms. They sat together in isolation and decided to assign certain terms to be used primarily in a geometric sense.

For example, the verb stem *k'ee* becomes a key semantic unit which indicates the abstract notion of 'line' in a geometry class. It means 'stepping' or 'placing one foot after the other' in everyday language use.

Likewise, the verb *alnii'* 'defines a …division of some bigger unit in two' (Pinxten et al. 1983: 81). In everyday life *shinii'* (first person singular) means 'the middle of my body'. Thus, in a more abstract sense, the verb was reserved by the teachers to be used as a technical term for the geometric notion of centre (Pinxten et al. 1987). In actual fact, we reached only a short list of geometric terms, and the classical geometric notions (square, rectangle, etc.) proved to lie beyond the reach of these Navajo primary school teachers. It would take a more thorough project, involving a wider and possibly more representative sample of teachers from all over the reservation, to finalize that task.

However restricted and somewhat amateurish this example may appear, it illustrates the point of how language structure and thought co-vary and impact deeply on mathematical thinking. In my perspective it indicates equally how the mathematics curriculum can be rethought to genuinely take into account the culture specific background knowledge of the pupils in the mathematics classroom. In that sense, it points at the very least to a potential for what was called earlier a multi-mathemacy approach.

(d) *linguistic theory and anthropological modelling and theory* (Hymes 1970): especially French structuralism was a forceful example of this relationship. Lévi-Strauss (1958, 1980) developed an encompassing approach for all cultural phenomena, including language, art, social and religious activities and products. To come to this theory he started from the relatively strong linguistic theories of his time and expanded them to cover the broader panorama. According to that view the (simpler) structures of language should be recognized at any other level of complexity: myths, marriage patterns, conceptualizations and any other phenomena in cultural traditions are presupposed to be organized along the same structural principles, in Lévi-Strauss's theory. Hence, he felt warranted to use the models and explanations from linguistics and expand their domain of use to any broader domain of culture and society.

A somewhat similar presupposition is present in Barton's remarkable book on the nature of mathematics and mathematical learning processes. The book goes under the title 'The language of mathematics' (Barton 2006). Barton makes the point that mathematics is essentially communication. Not only is any and every concept communicated in one or the other vernacular, but the act of communicating is essential for what mathematics really is: so, for arithmetic, 'It is the expression of the quantity sense, as a number system, that constitutes mathematics.' (Barton 2006: 71). In the first place Barton thus makes the point that mathematics is not innate. It is made by human beings and communities, and hence is likely to vary across cultures. In the second place, mathematics has to do with 'choice': 'The claim being made in this book is that humans select (often unconsciously) which pattern to abstract using many criteria, and not all (not even most) of the criteria used are mathematical' (Barton 2006: 82).

Finally, Barton draws on Wittgenstein in order to get a grip on what language and communication would amount to. Following upon his philosophy of language games (Wittgenstein 1961) this thinker developed some views on mathematical worlds (in the plural). Language games are the types and formats of communication of an individual in groups, using the codes and rules of each group separately, in different contexts or periods to communicate with other human beings. There is no one unique and encompassing language or even deep structure of language, let alone an innate one, but there are many different language games engaged in by different people at different moments and/or in diverging contexts. Barton summarizes the views on mathematics of Wittgenstein: 'mathematical expressions are rules, not descriptions. Mathematics is neither a description of the world nor a useful science-like theory: it is a system, the statements of which are the rules, which must be used to make meaning within that system.' (Barton o.c.: 127). Of course, Wittgenstein's book on the nature of mathematics (Wittgenstein 1967) already holds many explicit arguments against the transcendentalism of most AM-approaches, and can be interpreted as a plea for an 'anthropological' rather than a logical foundation of mathematical thinking (if the search for foundations can be considered to be sensible still: Bouveresse (1971): Chap. 3).

Putting things in such general terms allows Barton to speak about mathematical worlds, in the plural. With a beautiful visual metaphor he states that mathematics around the world can be conceptualized as a 'braid with different strands' (idem: 124). It then becomes possible to argue, in the line of Restivo (e.g., 1992) and Restivo et al. (1993), that mathematical thinking and culture do have an intricate relationship. In a weak hypothesis it can be said that mathematics is a social and cultural phenomenon; in a strong hypothesis one can claim that the different forms of mathematics that exist throughout the world will be incommensurable (Restivo 1992). The reader will recognize in this line of thinking an attitude that seems to yield my notion of multimathemacy. Before I go into that, I will dwell some more on Barton's mathematics-as-language theory.

1 Evaluation: Language, (Mathematical) Thought and Culture

Looking back at the previous paragraphs I have to point out at present what is relevant in all this for the mathematics teacher. In order to do this in a responsible way I have to narrow down the target audience a bit.

As mentioned before, a majority of humanity is living in an urbanised world of experience in the present era, and this percentage will most probably continue to grow over the next decades. Urbanisation implies cultural (ethnic, religious, gendered) mixing of the population. The baseline of my position then, is that education will benefit from taking this feature into account. Furthermore, I claim that AM generally disregards this point, or shows a degree of blindness for it. Indeed, the well-established attitude (mentioned by Barton 2006, amongst many others) of AM has been to consider mathematics equal to what the European tradition of formal thinking has been producing. Secondly, that attitude led educators to decree that education in that realm is developed in a culture-free way, uniformly defined by mathematicians who were raised in that particular tradition. The PISA assessments by the OECD, which I mentioned before, can be appreciated as a further global step in that particular history. Following the What Ifs in the first chapter of this book, I doubt the universal validity of the educational mathematical products and procedures and think the time is ripe to think about a plurality of educational approaches and curricula. Hence, I feel free to offer alternative and diverse entries here. However, this does not mean that the 'traditional' AM-based curricula and learning procedures would be wrong or useless. Rather, they are considered to be possibly adequate for a particular subset of pupils, coming from a specific cultural background.

Looking into these issues led me to consider in particular the possible impact of linguistic elements on mathematics education. At the most concrete level the impact of the linguistic factor is almost beyond discussion: when I do not know a particular term (e.g., rectangular) I have a handicap in my thinking every time this word is used. Both in the few Turkish examples and in the Navajo material mentioned this point was illustrated. In other words, when a term and concept are missing from the background knowledge of the pupil, it needs to be provided through education. A further point, which is relevant for all school education but is even more typical for mathematics and mathematics education, concerns the quality of the background knowledge concepts and terms. It is one thing to have a hunch or a vague understanding of what a rectangular form would amount to; it is quite another way of reasoning to have a precise insight and to be able to use the appropriate notions in different contexts. I showed that insights can differ across cultures: Barton's examples of Tahiti and Maori positioning are clear instances. At this point the use of metaphors and imprecise concepts from one's cultural background can be mentioned: when a Navajo child estimates a distance by means of the (sun) time it takes to walk along a path through a canyon, taking notice of particular rocks, washes and bushes (Pinxten and François 2011), she develops an adequate piece of knowledge with survival value as little sheep herder. In the words of Bishop (1988), the child uses mathematics (with the small 'm'). However, for the purpose of school education the child is not doing Mathematics (with a capital 'M'). In order to do that, it should reason with the abstract notion of distance, implying such notions as measure, straight line, addition and multiplication, and so on. Most of all, the child should be able to distance itself from the particular context of experience and learn to use notions and procedures in an abstract way, not tainted by specific contexts. While granting this point, I advocate that education will benefit from integrating the particular cultural contexts of experience in the curricula and in the learning processes of the formal education. I make a plea to that end because I think insightful steps should be maximalized. And, obviously, insights are typically installed through contextualized and experience-bound learning.

A major general point in this reasoning is that no educational approach is neutral or 'objective' as such: any proposal will be value-laden and culture specific. The imagination of a Navajo Indian differs from that of an inhabitant of Brussels, Belgium, and that of a Tibetan monk in Nepal is not the same of the dreams and concepts of the Australian Aboriginal. To substantiate this line of thinking I return to the proposal of a hierarchy of 'selectors' developed by the psychologist Campbell (1974).

In his attempts to integrate evolutionary theory (defining the human species in relation to other animal species within the natural selection processes) and the peculiar types of selectors or 'learning processes' he sketches a hierarchy, going from very simple to utterly complex types of learning. At the bottom level, the amoeba shows very elementary selector mechanisms, which makes it move away from an acid environment, for example. In a systematic growth of complexity of selectors covering twelve levels Campbell (1974) thus identifies such well-known selectors as e.g., instinct (in reptiles, but also in higher animals), reflex, imitation, and others, yielding conscious learning and scientific searching at the highest level and exclusively in humans. The higher we climb in the hierarchy, the more complex and the less automatic the selectors will be. The more complex ones are more 'situated', i.e., working in ever more complicated contexts of experience. In the words of Campbell all creatures in the hierarchy are 'family of each other' to some extent, because humans share the lowest selectors with the amoeba or other, rather primitive animals. But since mammals have a more complex setup they will develop and use most or all selectors, whereas the creatures on the lowest steps will not. Drawing on this way of thinking through the complete set of learning mechanisms, developed by Campbell, I now look at systematic or structured learning, i.e. at teaching practices in and outside of the school system. Secondly, I look at learning theories.

It seems obvious that some sort of automatic training, through drill or through reward/punishment procedures, disregards contextual features. For example, techniques aimed at training toddlers to control urinating and defecating before they go to school on a more permanent basis, make use of relatively unconscious or 'animal-like' learning procedures. In a recent initiative of the Flemish Government (so-called 'pedagogical shops', Flemish Parliament 2007) official help in specialized centres is subsidized, using the simple learning theories of behaviourism. Contextual dimensions are almost absent from these programmes: neither religious, nor other cultural parameters are used in these programmes. For the children who have problems at this stage of development, the strict and automatic links between stimuli and responses are more thoroughly trained, and installed by means of more systematic rewarding and punishing processes. However, at the level of language learning and of mathematics teaching, the complexity of the cognitive processes and the involvement of conscious learning is incomparably higher. Of course, this is the old and relatively 'undecidable' problem of the nature-nurture discussion. I take the stand that it is to be expected that at these more complex levels of learning processes the environmental factors become more important. It need not be argued that mathematics education can be considered to be situated within the zone of higher complexity.

Having said all this, it follows that contextual aspects are likely to be both more present and more relevant for mathematics learning processes, than say for the control of the muscles that regulate the exit of excrements from the body. My contention, then, is that all forms of mathematics education will have cultural parameters. That implies that so-called western and AM-designed mathematics education is not culture-neutral. Hence, learning theories that take cultural dimensions into account will be considered adequate and the simpler ones should be considered inadequate most of the time, or for most of the complex tasks of mathematics teaching. More particularly, behaviourist theory of learning should be used sparsely here, and the more encompassing socio-cultural learning theories (Cole 1996; Lave 1995) may be the more appropriate ones. A consequence of this choice will be that learning and hence teaching procedures will be varied and adapted to the cultural background of the pupils. For the discussion in this part of the book, this means that the linguistic structure of a particular culture is to be evaluated as an integral part of the pupil's background knowledge. Hence, the linguistic structure of AM is particular and has to be explicitly dealt with in the mathematics class, certainly when that class counts pupils from other cultural backgrounds and differing language structures.

2 Philosophical Discussion

In Barton's book (Barton 2006) the conceptual field of mathematics and mathematics education is widened in a very interesting way. The author explains how other cultures think and talk about spatial concepts differently, yielding different geometric notions. In his attempt to build an alternative theory about the nature of mathematics, Barton chooses to characterize mathematics as a linguistic and communicational phenomenon. Indeed, the notions of number, proportion and so on are not innate. Rather: '(I)t is the expression of the quantity sense, as a number system, that constitutes mathematics' (Barton 2006: 71). This 'expression' is in language and has to be communicated in order to 'be'. Furthermore, the AM conceptualization of mathematics corresponds with the languages in which it was developed. Concretely, the relationship of capital and mathematics cannot be overlooked here. Going to Asia, the Arab countries or the Pacific area, different mathematical notions and strategies obtain. For example, proofs are an integral part in most of the views on AM, but they might be exclusively western (originating in the ancient Greek tradition: Phalet 1970). I can only agree with these points. From there on, Barton struggles to identify mathematics as an independent, yet tightly linked phenomenon vis-à-vis language:

- most probably language and mathematics developed together in history,
- 'mathematics arises after, not before, human activity, in response to human thinking and communicating about quantity, relationships, and space within particular socio-cultural environments.' (Barton 2006: 143),
- hence, mathematics could have been different than what we know today as AM, and
- mathematics is made 'through communication'.

I agree with three of these points, but object to the final remark: it overstretches the role of communication. Granted that mathematics takes shape IN communication, or while being communicated. A former citation of Barton spoke about 'expression'. However, this is not saying that language or communication are the foundations of thinking, as most philosophers of language would have it. Contrary to this view I would advocate for the conceptual primacy of action and interaction over language and communication.

In fact, a lot of the arguments in Barton's book underscore this critique.

I propose to separate the action/interaction aspect from the linguistic level. In this discussion, I choose the side of action philosophers: action and interaction are the conceptually primary modes of relating with the environment and the other humans. Within that mode one can distinguish a whole range of types of action, going from mere bodily action, over perception as action processes, memory as a complex system of searching, classifying, processing and reclassifying data, and linguistic actions (such as speaking, writing, and so on) as ever so much distinct types of action. In relationships of acting and reacting with both human and non-human agents, these actions take the form of interactions. I will not go into the literature in this perspective here (but see in Pinxten 2010). It suffices to say that the somewhat enigmatic conclusion of Barton that language and mathematics developed together in ancient history, can be granted with different associations by claiming the choice that both can be qualified as different forms of action. Actually, the theory of 'speech acts' developed this line of thinking for language (Searle 1969).

The many examples of 'dynamic worldviews' Barton refers to in the empirical parts of his book take on a different meaning by doing so: Maori, Tahiti or indeed Chinese (yin yang) ways of formal thinking yield sometimes surprising and uneasy translations in his book. Example:

- Pacific navigators practice 'path navigation', where they measure a distance in terms of the amount of action time that is needed to pass from one island to the next: 'The basis of their navigation is to determine where they are on their journey, not their exact position.' (Barton 2008: 34). It would be better to say very explicitly: the journey of moving over the ocean along a path and the exact position at any moment of movement is what interests them, not the position in a fixed 'grid' that is mentally put over the ocean.
- and another one: it is perfectly legitimate to describe any geometric figure in action terms (Barton 2006, Chap. 2, Sect. 1): a square can be seen as the form of a thing or object, or it can be defined by 'squaring', as in 'fly, then stop-and-turn, the fly, then stop-and turn, etc.'. Velocity, the direction of a movement, the type of turn taken, the action performed, etc. can all be constitutive elements of geometric figures. My suggestion is that they should be considered to be the more generic and indeed general identification. The geometry of Euclid would then be a special case of this action-defined version, working with fixed objects and their fixed aspects of form.

When one adopts this perspective, then we do not reason anymore of adjacent, different and possible equally relevant or valid forms of mathematics, and their languages. Rather, the actions with mathematical bearing or relevance are the generic elements, which are 'expressed' (to use Barton's term) in conventional actions of a particular sort by the Pacific navigators, and expressed in a quite different way in the verbal action of mathematical language in AM.

An intriguing by-product of this line of reasoning is that the postcolonial critique by scholars such as Joseph (1992), Powell and Frankenstein (1995) and many others becomes obvious: when a certain culturally dominant society takes the road of empire building and proclaims that its way is the only civilized or genuinely learned way, then mental colonialism is a fact. When this particular domination takes knowledge to be primarily if not exclusively textual as in the schooled tradition of the West, then the reductionist view is taken as a dogma that good thinking is what this short lived dominant group takes it to be, considering any broadening of scope to be dangerous, "unscientific' or indeed heretic. The heavily political discussions over African mathematics (e.g., the Ishonga bone: Huylebrouck, n.d.), as well as over the relative value of Islamic, Chinese or other traditions of learned schools of knowledge are cases, which illustrate this point (Restivo 1992).

Part IV
Multimathemacy and Education

Chapter 7
Multimathemacy and Education. General Principles

1 Looking Back and Forward

Scattered throughout the literature on mathematics education and cultures, social groups and so on, one finds local or particular suggestions for other curricula, and for different approaches to learning procedures in mathematics classes. My point is that we need to sit back, reconsider and make proposals about a more inclusive and at the same time more differentiated pedagogical view on mathematics education. The analysis and the eventual implementation of alternatives should first and foremost start at an early age, say around the age of six or seven, in order to systematically and thoroughly prevent the presently registered dropout in the school career.

My points of departure are listed in the What ifs of the entry to this book. Now it is time to dive in and have a look at the concrete perspectives I can offer in this line of thinking. Of course, some scholars have been travelling already along the road I spotted here. I will recognize their work, and try to build on their insights. I do recognize the ethnographic works of colleague anthropologists who have been interested in the ways of counting, measuring and designing in some of the non-western cultures. In the first phase of ethnomathematics their work was considered to be of central importance in order to develop a critique on AM, and maybe even to come to grips with alternative approaches. As we know, EM was redefined several times (Pinxten and François 2011) and now encompasses IT and engineer's uses of mathematics in the West together with street mathematics and the original ethnographic material. In a sense, it would be easier to define it in a negative way: anything in more or less formal reasoning that AM considers to be imprecise (like the folk forms of measuring and counting) or unorthodox (like the relative disinterest in proof in IT and engineering calculations), and so on. But that posits EM as 'anything but AM', which is at best a poor definition. In a very 'orthodox'

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reasoning along these line the famous mathematician René Thom thus defined rationality (as the backbone of science and of mathematics proper) in such a way that less than 2 % of what passes as natural science research in our times will qualify as scientific, according to Thom. All the rest, including such Nobel laureate branches of research as Quantum Mechanics and most of contemporary cosmology —would be classified by him as 'magical thinking' (Thom 1987). In terms of pure consistency and progression through tight proofs, Thom may have a point, but no single bridge will have been built, nor would a rocket have landed on the moon if we would disregard all the 'magic' this mathematician finds improper or 'not yet scientific' thinking.

My point here is that I claim we have a very similar problem with the prerogatives of AM in view of a broad and relevant mathematics educational program. In other words: do we need the strictest criteria and the highest degree of consistency when teaching mathematics to humankind, given the diversity of FK we have in the classrooms around the world? Or can we look at the different activities children engage in and select the mathematically relevant aspects of these in order to elaborate and make it progressively more abstract in such a way that they bare on a range of problems and contexts? Are we making young people capable of reasoning in a (semi-) mathematical way when picking that line, with more success than when starting out from the correct or orthodox order which reigns inside the house of AM proper? Of course, we will only achieve high quality when the pupil is directed towards mathematical reasoning in the strict sense of the term. But educationally it might be defendable to start from the relatively messy, but operational, world of experience of the child to begin with. Moreover, given many different cultural backgrounds with the children who are drawn into any type of globalizing formats today, the diversity and plurality of FK notions should concern us, because they will become the many points of departure in the pedagogical project I put in front of the reader here.

At first, the reader might object (see Rowlands and Carson 2002) that AM (or 'our' mathematics) is the only high-class mathematics around. Even though at the present point in history this can be argued for, nobody will deny that this tradition is indebted to many others to begin with (India, Islam countries, etc.). Moreover, the recognition of one's powerfulness at a particular point in history does in no way imply that it is intrinsically, or 'by nature' superior to other ways of thinking. But, aside from this discussion, it does not entail at all that the educational approach to continue this learned tradition will necessarily benefit from disregarding its historical and socio-cultural constraints.

The reader might object that multimathemacy, as I advocate it here, will imply a varied and many-headed educational program in order to be implemented. In other words, uniformity in curricula and in teaching and reasoning styles threatens to get lost. In still other words, relativism will become rampant, and then knowledge (and certainly truth) is kicked out as 'collateral damage'. As I argued at some length in the previous chapters not all disasters of the world will descend on us. But yes, uniformity will be loosened and maybe lost to a certain extent. It remains to be seen whether this is a loss or a gain. In my opinion it might well be a gain:

- by recognizing and integrating diverse FKs chances grow that more students will be able to progress in their mathematical education, since their diverging insights will be used and taken in during the learning processes. Chances are that dropout will decrease as a consequence of this line of working.
- solving problems by means of a variety of approaches will most likely yield more engagement and more productive thinking than formatting the diversity into uniformity on the basis of the tenets of AM. I write this with the successes of IT and engineer researchers in the back of my mind: they do not do ‘proper’ or ‘pure mathematics’ as it should be done, but rather use what they need and think useful for the problems they are working on. And there is no way we should call their research unproductive, useless or ‘wrong’, except when measured by means of an orthodox ‘ruler’ they think a waste of time and unproductive themselves.
- finally, my approach is pragmatic in the sense that I hypothetically project a ‘Frame of Reference’ of mathematical activities (FORMA), along the lines of the UFOR I devised for the ethnographic and the comparative anthropological study of space in another instance (see Pinxten et al. 1983, 1987). As will be clear from the presentation of this approach, relativism is not rampant, but a rather surprising balance of potentially universal and local or culture-specific activities emerges.

I will offer some detail on this FORMA concept first.

2 FORMA: A Frame of Reference of Mathematical Activities

This idea draws on two sources: the first one is my own former work on anthropology of knowledge, especially focused on spatial thinking in non-western traditions. The second one is the ominous work by mathematicians, namely Bishop (1988) and Barton (2006).

(a) the Frame of Reference (FOR) approach:

All human beings orient themselves in space; they all attribute features of volume, bulkiness or slenderness, width and depth to places and phenomena. All of us develop concepts about the cardinal directions, and about the sun and the stars. Reasoning along these lines, I looked at a series of disciplines, which offer data and models on spatial thinking, talking and imagining. This resulted in a set of over one hundred generic terms, ordered in three subsets: physical space, social space and cosmological space. The subsets were construed on the basis of the different types of human interaction with each of these spaces: physical space has to do with all these aspects of and phenomena in reality we can manipulate, grasp, and eventually modify physically as humans. E.g., the form of a stone, but also bodily spatial features, and so on belong here. Social space refers to the type of space we are in, or we share with other phenomena. We are hardly ever confronting this space while trying to have impact on it, because we are always 'part' of it. Here one finds architectural and environmental spaces, but also other strictly social spatial relationships (e.g., distance, perspective). Finally, there is a third subset of space, the elements of which we can only interact with in a virtual way: we can only look at, pray to or otherwise indirectly interact with the sun or the stars. We can never touch or manipulate them, except in the fantasy of a myth, for instance. So, the three subsets are distinguished from one another on the basis of the different types of action and interaction human beings have vis-à-vis each of them.

The list of this FOR in the realm of spatial notions (Pinxten et al. 1983) thus comprises potentially all distinctions human beings will or can deal with in their ways of positioning or orienting themselves, and of defining spatial features of persons, things and situations. However, the items of the frame are formulated in such a way that they are links, hints, or reference points to start filling in or defining the corresponding items in language X or culture Y. The FOR items are generic concepts. Any particular and empirically found spatial notion will be culturally and locally specific.

For example, the body structure of human beings (together with their stereoscopic vision) yields that we all share some basic intuition of three-dimensionality: our body and our body motions divide space in three dimensions, with our spine in the upright position as the vertical axis. However, in Athapascan languages, to stick to that example, the three dimensions are thought of in terms of movements in three intersecting voluminous spaces, and this differs markedly from the Cartesian view we learn in schools in the West (with three straight lines intersecting in one point). Staying with this particular instance, one can see that the generic notion of three-dimensionality works like a teaser or a minimal input to start research into the particular phrasings and differentiations of the particular cultural tradition and the language one is researching on. Put differently, I have a handle with the FOR entry by means of which I can begin to explore, discuss and understand the corresponding Navajo notion of three-dimensionality. Finally, the entry of the FOR will be checked after each ethnographic study and will eventually be adapted to become even more generic and culture-indifferent along the way.

In the present study I want to make a proposal about mathematically relevant activities and basic intuitions in order to start building a similar Frame for mathematics education, now dubbed FORMA (Frame of Reference for Mathematical Activities).

(b) Inspiration in Bishop and Barton:

Apart from the many detailed ethnographic studies on the innumerable languages and cultures we know, I detect two substantial synthetic books, which are useful for the present endeavour. I treat the works, that inspired me, in some detail.

Bishop (1988a, b) poses the problem of mathematics education in terms of a possible conflict between Mathematics or (big) M) on the one hand (what I call consistently AM), and mathematical activities or (small m), on the other hand. He chooses for an emphasis on activities, for which he finds support in the ethnographic studies: across the world adults and children engage in all sorts of activities that have relevance for or even use mathematics. Bishop distinguishes between six activities: I consider them to be core-activities for mathematical education.

- counting,
- locating,
- measuring,
- designing (shape, size, scale, proportion and other geometric concepts),
- playing as a tool for exploration,
- and explaining (through underlying structures and rules).

I want to add a few more, which would rather be facilitating with respect to the development of mathematical notions and practices:

- moving, especially dancing and rhythmic moves; ceremonial actions,
- generalizing by comparing,
- logically operating,
- exchanging and market activities,
- making music,
- and story telling.

Bishop did not intend to ascribe any universal quality to the mathematical activities he isolated, but rather held that one way of reasoning would be strong and phrased in a particular way in one cultural tradition, and another one in a different group. I am convinced that his point is well taken: indeed, a child (or even an adult) in a hunter-gatherer culture living in a tropical rainforest will have quite different concepts on location than their counterparts in the desert, let alone in a city context. The emphasis on moving rather than seeing of the former two will be strong, whereas the reliance on perceived lines and structural elements in a city context will be obvious in that particular case. So, the relevance and the differential conceptualization will vary across cultures and languages, and I claim that it is sensible to respect those differences in the mathematics classes, or during the initial steps of learning in formal thinking.

I borrow the list of activities from Bishop, with the addendum I mentioned, and will use it as a point of departure in the construction of FORMA.

Barton's book (Barton 2006) offers interesting elaborations on particular notions with mathematical relevance: e.g., path, navigation, positioning (location in his words), quantity. The richness of the details is amazing. On the other hand, Barton wants to demonstrate that mathematics as such is a form of language or communication. A lot of the book's argument focuses on that topic. For reasons outlined above, I am convinced that the emphasis on language is not easy and in fact may be rather cumbersome to use in the variety of cultural settings I want to invite to my perspective. My point is that action and interaction is the more generic category, and talking (and writing) are particular forms of action. In education, it then might prove beneficial to start from so-called mathematical activities rather than mathematical labelling and communication. An example will illustrate my point: a Navajo child is never instructed verbally about weaving patterns (or about pretty much anything at all). Most interaction is nonverbal: children are put upright in their wooden cradle, from birth on, facing the mother in all the activities the latter is doing. The child watches the mother weaving, while staying strapped to the cradle. When at age five or six the child feels like it, (s)he starts a small loom that serves to weave belts, with the typical double-sided geometric patterns on them. No verbal instruction is ever given, and the child learns by imitation, and basically by seeing and doing. In fact, no adult weaver was able to tell me with any precision what the different patterns in their tapestry were called, nor explain in words how the weaving is performed in order to reach the high quality it is renown for. Moreover, the symmetry and the precise geometric figures in the rugs is not reached or checked by means of verbal procedures, but rather by acts: the weaver would regularly 'measure' width or distance by using her fingers, and the perfect symmetry is reached by visual checks and by holding up and folding the rug in such a way that one half would overlap the other half in a perfect way.

Entries of FORMA (Frame of Reference for Mathematical Activities):

(a) COUNTING

Several studies have been published on the many different counting systems in the world. The more known research would be that on Africa (Zaslavsky 1990), on American Indians (Urton 1997) and a wonderful PhD on more than 400 counting systems in the Pacific area (Lean 1995; but also Lean 1986). But there is much more. In fact, we can safely say that all cultures in the world have some kind of counting tradition: some count on their fingers (and toes), some would develop a row starting from one fingertip and ending in the opposite hand while visiting shoulders, neck and elbows along that path. Many (like the Navajo) would focus on major numbers like three and four, reaching ten and then transferring to 'many' for larger amounts. Aboriginal Australian counting systems usually contain only two or three cardinal numbers (Harris 1991), and body-counting (spots all over the human body are linked with particular amounts) is widespread.

David Lancy, who worked in the line of cultural psychologist Michael Cole, differentiates between 225 different counting systems in Papua New Guinea. He distinguishes between four types of counting systems in that area of the world (Lancy 1983):

- body parts are drawn into a counting system, with 12–68 different parts being counted,
- the use of counters, like sticks. The base number will vary from 2 to 5.
- mixed bases of 5 and 20: compound numbers will be labelled as for example 'two hands, one foot', meaning 15.
- base 10 system of numbers.

Throughout Papua territory particular instances of one of these types will be found.

The mathematical activity of counting involves that separate or discrete phenomena are put together in an order, such that each consecutive item is recognized to be distinct from each other one, but at the same time is seen to belong with the former items in a larger group of entities.

(b) LOCATING

This activity has to do with 'finding one's way around, knowing one's home area' (Bishop 1988b: 147) and so on. In Bishop the notion refers to anything spatial, including navigation and even astronomy. Personally, as indicated above, I think this category of Bishop is too large, since it encompasses activities of very diverse nature: navigation implies notions of comparison, of star lore and so on, where locating oneself in physical space (vis-à-vis concrete physical objects and persons) is much less complex. The cultural variation in the former case is relatively straightforward, whereas odour, colour, and even mythology may intervene as constitutive elements in the latter. Therefore I would prefer to restrict the term to locating an ego in relationship to another person or object, or locating an object in relationship to another one or to a person. In more complex spatial activities location will be a part, but will not stand for the whole.

The intriguing discussion of the positioning of ego in the Maori-Tahiti (in Barton 2006) comes to mind here: whereas Cartesian thinkers of the West would define any position in terms of the ego overlooking or confronting reality (the 'external world' as a given outside of the subject), the Maori would take into consideration the positions of all interlocutors involved vis-à-vis the phenomenon. In that case, locating will amount to finding a position where the perspectives of all partners are combined. The very idea of an external, outsider's frame with ego as the sole origin of positioning within this 'objective' frame is foreign to the Maori view: their approach constructs a frame involving the relative positions of all participating in the act of positioning. It is the sum of all these positions that is finally considered to be the position 'in reality'. For example, when thinking about a canoe on the open sea in the Pacific, the island in the distance is not a fixed set of values in an external frame (like the Cartesian grid), but rather a series of positions for each partner who is playing a role in the moving along: different canoes sailing the ocean at the same time, the moving sun, a regular current in the ocean, and so on. The combination of all these make up the position of an island for the sailors in the canoe.

Locating can hence be defined as describing one's spatial relationship vis-à-vis other phenomena such as other human beings, animals, objects or places. This activity can be part of a movement through space, but see below.

(c) MEASURING

In the West measuring has come to imply high precision, to such a degree that positivists in social sciences, economy and even in politics proposed the slogan: 'to measure is to know'. In other words, there is a tendency to rely on precise measurements in order to develop irrefutable arguments, i.e. with the aura of proof. In practice, however, most of the time probability reasoning is what is used when dealing with complex phenomena. Survey research, large questionnaires and marketing research most often work with samples of subjects, and produce statistical results. Notwithstanding such imprecise measures, instead of 100 % certainty, ample use is made of such studies to make decisions in private life (what car will I buy? What phone is good for me?) and in public policy ('a majority of the Flemish people want X or Y'). Programming of media is dependent on what large groups of the clients are believed to want. The latter are followed in their choices on the basis of regular sampling research: soaps with a large audience will be continued regardless of their surreal picturing of the world, and even news programs counting will be rated in terms of consumer tastes. I am critical about this overrating of measuring in the West, but at the same time I want to stress that measures and measuring are important societal activities, with a political impact.

Bishop (1988b) rightly objects to a narrow definition of measuring, when considering the worldwide use of this activity. He states that almost always measuring is qualitative and imprecise, rather than precise and expressed in strict numbers. People would make estimates, compare on the basis of looking at things, or depend on customs: for example, a goat of one year old is worth a bucket of wheat seeds. Or illiterate people will be able to buy a fitting dress for a relative, just by looking at it and eventually comparing it with the size of one's own body. Bishop: 'So accuracy is not necessarily to be valued highly…' (Bishop 1988b: 148).

Several informants in Navajo country told me that they measure distances in terms of time, meaning in fact bodily activities: for example, anything that can be touched is near, whereas something I have to walk up to is distant. When the distance is considerable, markers along the way will be referred to (rocks, washes, bushes, houses) and the path of the sun may be invoked: 'I have to walk from sunrise until midday to reach that place'. Anything that cannot be readily seen is referred as '*unleidi*', meaning 'far away'. This always implies a long trip by foot or by horse, further than you can see with the naked eye.

Even when I grant that accuracy is not so important in measuring activities across the board (and hence that precision is a locally prominent feature, in the western tradition), it is probably so that other aspects of measuring are intrinsic and hence universal. One feature that stands out is that of comparing: the famous essay by M. Mauss on exchange systems, 'The Gift' (Mauss 1924) gives an insight in the ways people make estimates on value, honour and shame and translate these to material goods. When during the potlatch cycle a gift is prepared by village B, the actors hold in mind the value of the opponent's gift of village A. The return gift should be so impressive, so valuable that it matches or outclasses the one received and because of that, it will have to be accepted by village A. Basically, provided that A has to recognize the importance and the value of the gift, A will have to accept it and thus recognize the symbolic power and status of B. This recognition puts shame on A, and the obligation to return the gift with one that cannot be refused by B. During this reciprocal exchange comparison is of the essence. But of course the measuring of value and status is qualitative and imprecise, by standards of AM: one should outsmart the other, one should aim at excellence in beauty, in wit and so on. And the gift is successful when the other party has to come to the conclusion that the gift is excellent, and it has to be accepted because of that, which will hence cause shame for the recipient. Of course, the cognitive processes and principles involved here are not readily recognized by the mathematician of AM.

But, basically, measuring activities based on comparison are the core of the activities mentioned, and hence a BK of mathematical reasoning is at work here.

What does measuring come down to? It is the mental activities, often cast in social, religious or other cultural actions, where features of one phenomenon are compared with or matched against similar features of a second phenomenon. The unit of value or of a measuring rod which is 'defined' by the features selected can vary from any immaterial item (like status or beauty) to the materialised metric unit the West has invested in (like a meter, an ounce, etc.).

(d) DESIGNING

All cultural traditions in the world make things, from shelters to artefacts and art pieces. Some anthropologists have formulated the proposition that being human might well be equal to making tools and artefacts. The cultural psychology of Michael Cole (1996) builds on a similar premise: in his view humans are first and foremost artefact producers and users. He distinguishes between primary artefacts (being material things), secondary artefacts (beings designs and models for the production of things, like language) and tertiary artefacts (encompassing all pure things of fantasy or imagination).

A particular intuitive or 'natural' way of generalizing is funnelled along with designing activities: a designed object often proves to become a model for other objects. The tremendously prolific production of types of pottery and of textiles can be understood this way: some of the Pueblo potters of the Southwest in the USA have a longstanding high reputation. When looking into their style tradition it proves the case that a typical Zuni or Hopi piece of pottery can easily be recognized as belonging to these traditions, as distinguished from Acoma or any other, on the basis of the sophistication of the design model within a small range of variation. The technique and the particular figures are restricted to that area of the Southwest, showing that a small set of designs come to act as constraining models for a whole tradition. When doing a bit of fieldwork on Navajo tapestry (which is their domain of artistic excellence, rather than pottery) it became clear that throughout this large reservation particular figures would be specific for a region and even for a group of families living next to each other, while another design would be characteristic for another part of the reservation. This is a translation of the principle mentioned: a particular design becomes a model, which then inspires a long tradition of making.

Another case can be found in the tremendous work of Rubinstein (2004), who distinguishes between a myriad of textile designs, sometimes particular for one village each, spread out across the many islands of the Pacific region. Thanks to a substantial grant from NSF he has been enabled to collect, analyse and integrate into educational programs of EM the clearly distinct designs. Finally, the many analyses of architectural designs in almost every culture of the world show how design of a particular group, clan or people is turned into a model. We can distinguish between architectural styles, particular for any cultural tradition, precisely because the design of a group X was turned into the model of how to build, with slight variations, clearly distinguishable from neighbours or rival groups (Oliver

1987). This brings me to the conclusion Bishop (1988a, b) formulates on the relevance of design for mathematics education:

> What is important mathematically is the plan, the structure, the imagined shape, the perceived spatial relationship between object and purpose, the abstracted form and the abstracting process. (Bishop 1988b: 149).

Summarizing, I define design as that mathematically relevant activity that is embedded in the development, planning, trying out in a direct or indirect way in the making of artefacts. Designing implies abstracting forms in matter and objects in order to manipulate mentally and physically the actual form in view of transforming it in another one.

(e) PLAYING

Anthropologists have been studying childhood for some time, including playing of children in different parts of the world. In earlier research the focus was on education, in the sense of growing up in culture x or y. Thus, Margaret Mead was one of the first to try and show what it meant to grow up in Samoa (in 1928 she published her famous monograph 'Coming of Age in Samoa'), but the work of the Whitings was the first and path-breaking study of a comparative nature (Whiting and Whiting 1975). In the first systematically comparative study on the various ways children and childhood are conceived and transferred over generations, Lancy (2009) added a substantial dossier for discussion. Not only did he study on mathematical thinking in Papua New Guinea earlier, but he also published articles and a book on the role of playing in different cultures. This kind of multifaceted and deep informed work is recent, and classifies former work (like Mead's) at best as initial or beginner's literature on the subject. In that respect Bishop's focus on play as mathematically relevant activity can be substantiated more thoroughly today.

Play is most certainly important for a variety of reasons: it gives motor training for the child (legs and hands, speed, precision). Several studies in Flanders point out that children with little or no opportunity for play at home enter school with clear handicaps on this dimension. For example, learning to write implies skilful use of the hand(s), which is trained during play and by the use of open-ended tools for play. But, play also gives a social training: the child learns how to negotiate, to anticipate, to perform in team, and so on. A negative effect of computer games, which are solitary and may build mistrust against others in the child's mind (e.g., all sorts of war games), is documented recently in work of moral philosophers and psychologists (Commers 2011; Verhaeghe 2012). I call this trend in the virtual world 'negative', because social education and the skills of empathic collaboration, negotiation and the like seem to be dwindling when children from early age on spend hours a day with the latest generations of aggressive computer games. In technical terms there the socialization through play is conceived in a world of one human being in interaction with a group of inimical powerful creatures, within a worldview that promotes the attitude of one-against-all and kill-or-get-killed. This is a form of socialization, of course, but one that anthropologists can hardly find anywhere else in the world. The formats that come closest to this notion of human beings and society have been documented in so-called disaster studies within anthropology: when a cultural group is suddenly and deeply uprooted (by a natural disaster, a war, etc.), disaster studies showed that a very systematic, almost methodical destruction of solidarity, of values of empathy and collaboration occurs, starting with the starvation and/or killing of the elderly and the infants, and eventually yielding fights of all survivors against each other (Avruch 1998).

Play and mathematics: negotiating in itself implies notions such as an estimate, the weighing of interests, and the balancing of need and power. All of these are not directly mathematical, but they all involve comparison, and a qualitative notion of measuring. Apart from that, play entails imagination: the player should at the very least be able to imagine a world that is different from the readily accessible one. It involves the willingness, but also the capacity to think and act in a virtual world. Notwithstanding that, concrete plays and tools for playing often prepare the child for later, so-called adult life: dolls are representing relatives for whom one learns to care through playing, huts, tents or buildings are constructed first in playtime, and the exploration of nature by a child carries information on the plant and animal world that is shared by the adult community. Apart from all that, play often involves singing and dancing, with all the counting, rhythmic movements and so on that are themselves structured and often numerical.

Play need not be defined here. It suffices to say that acts of imagining, generalizing, counting and comparing are very much present in children's playing, and that socialization in numerous categories of adult life is mediated by play.

(f) EXPLAINING

The last activity Bishop (1988a, b) singled out was explaining. What is meant is the epistemological activity of explanation: we understand a particular phenomenon when we can describe it in such a way that it becomes a particular case or instance of an encompassing process or complex entity. Put very simply: water boils at 100 ° C can be described on the basis of a particular instance of a concrete kettle of water. But the explanatory sentence 'water boils at 100 °C' has the character of a law, a general statement that captures all particular instances, and their descriptions and even has the quality of predicting all future instances of this process. That is why Bishop places explaining among his six basic mathematical activities: 'it is this activity which gives mathematics its meta-conceptual characteristic.' (Bishop 1988b: 150).

Defining explaining I focus on generalisation first. The distinction between particular instances and a formulation of a rule or a generic feature, which can safely be ascribed to any and all similar phenomena, is a prerequisite here. It is safe to say that all mathematical reasoning is presupposing the awareness and the ability to use that distinction, and that therefore explaining is intrinsic in mathematical activities.

Expanded list of FORMA

I suggest to add six supplementary activities. They are universal in the sense that all peoples in the world know and perform them. On the other hand, just as is the case with Bishop's list, each particular cultural tradition will show a differential content and use of them: more or less, elaborate of succinct, in isolation or in combinations. It is good to mention that, in a casual way, some of the list below can be detected in other authors: Bishop (1988) speaks of western values of progress, objectivity, logic, and context independence. Older sources hint at such deeply cultural values, without thorough analysis, let alone without their grounding in one or more cultures (e.g., Kline 1972; Wilder 1978). Indeed, almost all treatment of such questions focuses on the western tradition only. Of course, the latter has great records to show: the systematic and deeply trained way of solving problems without reference to any context whatsoever has shown to be very powerful, but there is no convincing evidence that good formal thinking is restricted to this tradition.

The list the reader will find here comprises those activities which have (high) relevance for mathematical reasoning, but not exclusively so. That is to say, they might play an important role in other domains of life and of reasoning as well, and in that sense they are not 'typical' of or exclusive for mathematical thinking. Nevertheless, they often have relevance and they serve as an entry to formal thinking for this or that tradition. In view of these arguments, and especially with an eye for educational relevance, I include them here.

(g) MOVING/DANCING

Anyone who has seen an Afro-American person in the midst of Native Americans and some Anglo's (white people) will understand what I mean by this entry. The Afro-American seems to dance when he is only walking with the Native Americans, while the white person is "loud" in his movements, but in an oddly different way from the others. Anyone who is used to circulate amongst Japanese people as a westerner will have felt awkward (and may even have been warned by the Japanese) because he takes such a lot of space while just standing, communicating or walking. Japanese people seem to restrict their personal space to just a few inches beyond their skin, where westerners appear to need yards in all directions. The body movements are markedly contrasting and the westerner will bump into people and things constantly in a Japanese context.

These casual remarks point to a broad experience that people from different traditions have different body spaces and diverging interpersonal spaces. Hall (1979) was one among several scholars who got intrigued by this and tried to develop an intercultural tool to study these differences. It became known as the approach of 'proxemics'. Hall observed that people have a different body position, but also other interpersonal distances in communication and interaction, depending on the cultural tradition they came from: Moroccans touch each other on the arms or the chest and are only inches apart when they have a casual conversation. Northern Europeans and white Americans would restrict from touching each other and keep an arm's length between each other when having the same social talk. Navajo never look into each other's eyes when communicating, which is considered ill-mannered by the common westerner. And so on. These registers of hidden or implicit interpersonal space (the 'hidden dimension' in Hall's words) are specific for a language/culture. Since it is learned and used at a subconscious level, it constitutes part of the shared, but unspoken knowledge of a cultural or linguistic community.

In that sense movement and bodily behaviour express spatial knowledge, and hence holds mathematical knowledge.

In dancing the intercultural differences are even more outspoken. The notational system which was developed in the West (including Russia in this particular case) is inadequate to capture other traditions, like Hopi dances, for instance (Kealiinohomoku 2008). In a sense, the choreography of Acogny or Béjart in Senegal recognizes this point: African dance movements are deconstructed in their own terms and then reconstructed on the scene as “ballet”. But neither plot, nor the decidedly upward movement of western ballet with dancers seemingly trying to escape from the earth and propelling themselves in the air, are present in the West African dance style. Rather, the feet are firmly planted on the earth and the horizontal plane seems to be more important than the vertical dimension in movements.

Together with all this, the rhythm of a Native American, a Hindu, a Black African or a western musical piece and dance performance are quite different. The rather robust drumming rhythm of the Black African music is not found in the Native American monotonous rhythm (to a western trained ear) or the strict metric of the European classical music. Since music is a widespread medium in social events, such as rituals and festivities, it can be estimated that it has an important place in the educational or socialization processes of each new generation. Whatever else, rhythm certainly structures time and movements through time. It thus belongs to the FK of children who enter an educational process aiming at their sophistication of mathematical or formal thinking. Therefore, when different cultures use diverging rhythms it is important to take account of these differences and use them explicitly in the educational process, is my suggestion.

(h) STORY TELLING

Socialization is without doubt the most important process for the continuation of a cultural tradition (apart from material survival). We are biological specimens, but we are more than that: we develop knowledge, religious actions and beliefs, art, social networks, values and so on; all of this has to be learned through transfer from previous generations. Story telling is beyond doubt one of the main ‘instruments’ the human species developed to accomplish all that. Of course, language as such is part of that complex transfer system, which grants the point of the superior position in knowledge building (including mathematical knowledge), which Barton emphasizes (Barton 2006). But it is not merely language that is at play: Bruner, the world renown Harvard educational psychologist of the past century devoted some of his later publications to the study of the particular form of language use, which is called story telling. Without stories we have no human beings. Put differently, education is not only and maybe even not primarily, cognitive expansion or deepening. The development of the whole person is the central concern. And that person is a social and cultural being, in whom the cognitive processes are embedded. Story telling in the many cultural traditions we know about serves that purpose: children are introduced in almost every aspect of the culture, including many ‘cognitive’ features, of the world through stories. Good education, also through the school format, should take this into account (Bruner 2004). Put differently, education that focuses exclusively on the cognitive functions (building up the brain, and nothing but the brain) fails miserably, because it disregards the social and cultural embeddedness of cognition. Which refers back to the socio-cultural school of thought I have been mentioning (Lave, Rogoff, Cole, etc.): situated knowledge, cultural learning and the like.

In mathematics education some scholars have recognized this point. For example, in his thorough study of Quechua number systems Urton (1997) explains at length how the cardinal numbers in Quechua are associated in the learning procedures and in the language with social relations: 1 is associated with mother, 2 with the first born infant, 3 with the second born infant, and so on. The title of the book refers to this particularity: 'the social life of numbers'. Now, social relationships are typically not transferred through 'instruction' in oral traditions: they are learned through stories, which tell about social relations, often by means of stories about violations of social relationships. Stories can be allegoric (e.g., animals substitute for human beings or for characteristics of them), or cosmological or hero stories. Thus, the spatial relations in the universe will be learned through stories about the beginning of time, when the celestial bodies were 'placed' (as in the Navajo origin myths, Reichard 1950) or when a set of natural phenomena came down to earth in a basket which somehow fell from the sky (as in the Dogon myth: Griaule and Dieterlen 1965), when they washed ashore after a flood or any other way. Quite often myths are embedded in rituals, and the telling of the story comes closer to a performance than to a mere linguistic act. The point is that a great amount of cognitive 'data' are transferred in and through stories, which can be enacted or integrated in ceremonial and other cultural actions, rather than being the subject matter of context-free cognitive instructions. This is important: stories will make use of a different type of speech acts, like metaphors, and seldom go for purely referential meaning in language, where the latter is so typical for knowledge transfer in the school context (Lakoff and Johnson 2003).

My point is, of course, that the story telling style of reasoning (metaphorical, in a dialogical setting) is part of the FK of many children who get in touch with mathematics education for the first time. My plea then, is that we should not shy away from that FK, but rather try to integrate it in the educational process. In the mixed classes of our city schools this will imply that exploration of diverging stories, coming from a variety of cultural backgrounds, would be a more 'natural' (i.e., culture-sensitive) way of mathematics education, rather than the type of the strictly cognitive problem solving format I found e.g., in the OECD assessment questionnaires.

The point being made here is that story telling is a rich source for education. But, since we deal with mixed classes in the urban parts of the world, different stories, with diverging metaphors and images will have to be looked into in order to build a varied and powerful tool for curriculum building.

(i) MARKET BEHAVIOR

Mathematical skills are a bonus in market or exchange situations. In fact, they will probably get a boost when they are needed in such contexts. Since capitalism and traditional exchanges (the gift and other formats) are widespread in the interconnected world of today, it is safe to suppose that mathematical skills will be a benefit to ever more people. And lack of these skills and that knowledge will yield a handicap in the world market life we are facing. Of course, this does not imply that the OECD norm (in the PISA assessments) with its free market ideological program is the best humanity has come up with. Nor does it entail that the whole world should adapt to the uniform capitalist skills OECD promotes. The only thing I am saying in this paragraph is that selling and buying, exchanging and sharing do imply mathematical activities, and that hence the educational value of such contexts and activities should be recognized in mathematics education as well. But the world of market and exchange is much vaster, and ethically more diverse than the rather recent western capitalist version of it.

The anthropological studies mentioned in previous sections of this book make clear that mathematical skills and the urge to learn them increase when market activities are engaged in. Gay and Cole (1967) relate that the Kpelle they worked with did not have a formal mathematics education. Still, they were very able to apply mathematical skills in market situations: for example, when estimating body proportions the street tailors showed a high degree of precision and were thus able to make a perfectly fitting suit. On top of that they could competently set a price for it, and enter negotiation when need be. Urton (1997) mentions that counting is very much around with illiterate Quechua, when they get involved in dealing processes in the market place.

It does not take a lot of imagination to understand that people use mathematics in a market context: a price is set, money is counted, averages are calculated, and amounts are measured, eventually with the multiplication of the price per piece. Negotiation on the price or on the amount of goods, and a long term book keeping system may be honoured. All of these aspects of market behaviour imply mathematical activities. But there is more. A market is institutionalised over time, and this means that conventions emerge: the group of dealers define what is the unit of measure, what equations will be acceptable, and so on. So, over time and with the growth of partnerships, more mathematical concepts and procedures will be found. On the other hand, different sorts of markets may grow. For example, when I think of the street children in Brazil's cities, it is clear that they are very capable of using their market competences in their particular environment: they know the escape routes and measure the distances they are separated from them (in case the police shows up), they can calculate the street values of the things they trade and sell, and they can negotiate about prices like nobody else (Mesquita et al. 2012). Still, when put in a classroom their competences are not recognized and they fail miserably. Their notions of relevance, their detachment vis-à-vis concrete contexts and so on will be different or even in contradiction with the official market logic of the dominant society.

This implies that market competences with mathematical features can be recognized, but that at the same time different notions of market (and exchange systems) may apply. Returning to education I claim that it is important to recognize the range of market notions and mechanisms and integrate them as might seem suitable in the curriculum and the learning activities. Depending on the FK and BK of the particular children in a group, different notions will be used explicitly as the point of departure for mathematics education.

(j) GENERALIZING BY COMPARING

Several authors mentioned that generalizing is a prerequisite for mathematical reasoning. Both Bishop (1988) and Barton (2006) emphasized this point. Freudenthal (1970, 1979), who is rightly known for his 'reality based' mathematics education stresses that this branch of education cannot stay stuck in playing, imitating and such, but should lead to abstractions. Jurow (2004) describes a thoroughly empirical study on population modelling, in this case with guppy fish in a bowl. However concrete and particular the case may be, Jurow emphasizes that comparison of population sizes is always present, and that hence the making of generalizations becomes the core of the empirical experiments. More than this: the students (of middle school level) decided in the course of the project that they needed to generalize in order to cope with the problem of population size (and eventual overpopulation in a particular bowl) in an adequate way. Thus, while starting with concrete situations the mathematics students soon made room for comparison and, by doing so, almost automatically learned about notions like average, comparison, weighing, even scales. In a rather inductive way generalizations are developed on the basis of empirical and descriptive work. The fact that students worked on a problem in groups may be a real advantage, since the process of negotiation and decision making is then enhanced by the shared insights of each of the group members.

The important point with this entry is that 'situated learning' on the one hand and 'generalization' on the other hand do not contradict each other. It is clear that it helps to start from situated learning and use the insights the learner has, his or her FK, in the process of mathematics education in a more integrated way (than would happen in spontaneous learning). On the other hand it is important to understand that sophistication in thinking benefits the learner. Thus, generalization is a necessary step towards more formal reasoning. The gradual steps which lead the learner from her being embedded in her particular socio-cultural context towards a more detached position, where a line of reasoning or a problem solving procedure will be used beyond the context it was born in, is what concerns me here. I suggest generalization is necessary, but the steps involved should start explicitly and deliberately from the particular context of experience and meaning generation, which is the learner's. That context is particular and is part of the FK of the learner. In a classroom or, more generally, in an urban environment a variety of 'particular contexts' will coexist. My point is that the steps towards generalization should recognize this plurality: in terms of content, of specific problem identifications and so on, this plurality should be central in the educational program, curriculum and problem solving procedures that are allowed. Better still in all of these that are promoted through mathematics education. Only along that way the principles of 'multimathemacy' will be realized in a manner that allows for emancipation rather than alienation through mathematics education.

(k) MUSIC

Although I have a separate category on dance/moving, several readers and discussants urged me to include music as an independent entry as well.

Without doubt music teaches the performer and the listener about rhythm, but also about tones and the space between them in different tonal systems. In a more general approach of lately, Leman tries to understand what is music by looking at the embodiment of rhythms: we all know that people tend to nod their head, or tap with their fingers or move their whole body when music is playing. In a rather systematic research on this issue Leman found that physical or rather bodily movements, from nearly invisible or slight to exuberant actions, are in fact setting a base score for the listener and for the performer alike. The often more complicated, sometimes counterintuitive actual melody or theme is then read and performed ‘over it’ so to speak. Certainly with the often unusual avant-garde music in the West, the latter is an activity of the mind, which has to be somehow interpreted and performed as a second, separate cognitive line of music which additional to the basic rhythmic, bodily line. Also professional musicians use this embodied baseline to have the regular rhythm established in a controllable way in order to devote their full cognitive attention to the melody or to the actual printed score (‘text’): the typical rhythmic movement of a foot, or the nod of the head or a slight and continuous movement of the torso sets the basic rhythm (Leman 2013).

Obviously, in other cultures the situation can be different. But basically, the same bodily framing appears. In an overview Sels (2014) lists the following differences between classical western and Middle Eastern folk music:

– in the western tone system, established roughly between 1700 and 1900, an octave is the standard range of tones, subdivided in 12 equal parts. This contrasts with many other systems, like the Middle Eastern one. There we see an unequal division of the octave with intervals between the tones of 4/5 or other fractions.
– the time economy in musical systems differs also: in the western tradition we find regular and symmetric metres, with double, triple or quadruple divisions. With a fixed metre (e.g., 2/4, 3/4 or 4/4) the musical content is fitted in, with each bar having the same length.

In the Middle Eastern case one finds both regular and irregular, symmetric and asymmetric metres. Thus, the asymmetric metre can allow for units of unequal length, like one bar is (3 + 2 + 2 + 3) or it can also be (2 + 2/3 + 2 + 2). The way this works out is that time can show an additive structure with a flexible metre which follows the rhythm of the poem, instead of ‘fitting the content into a fixed bar’.

– finally, the texture differs. In the western tradition harmony is crucial, and the skills consists in producing a melody within the dominant rules of harmony. In the Middle Eastern case the melody is dominant, linked with a rather horizontal view on music. Hence the music produced will easily become monophonic, with adaptation of the rhythms and the tones to the needs dictated by the melody (and the poem).

A net result of all these differences is, according to Sels (2014) that the textual score is guiding in the western tradition: the performer has to learn to execute the score first and foremost. In the Middle Eastern oral-aural practice written scores are at best helping one's memory.

When this type of comparison would be expanded to cover many different musical traditions in the world (following older work like Merriam and others in musicology), it is clear that different ways of learning and performing musical rhythms, tone systems, and melodies can be explicitly and purposefully used in the mathematics teaching process. Once children will be made conscious of the structural and generally relational aspects of tones and rhythms in their particular tradition, they can be invited to explore and invent beyond their own tradition by using the mathematics in the musical structures deliberately.

(l) LOGICAL OPERATIONS

A final mathematically relevant entry is that of logical operators and their use in different cultures. Classification is most probably a basic use of logical operators, which is universal. Ever since the path-breaking work of Durkheim and Mauss (1901-3) we know that all cultures of the world classify phenomena by making use of elementary class logic: phenomena are put together in one indiscriminate set on the basis of one or more parameters, contrasting with adjacent but distinct sets which differ from the first one (and from each other) with regard to the values on these parameter(s). For example, all edible phenomena in nature form one class, which I contrast on this one feature (edibility) with all inedible or poisonous phenomena.

However, at surface level this class logic does not necessarily manifest itself in a direct or universally unique way. Instead, Durkheim and Mauss showed that classes can be embedded in a kind of associative networks: for example, edible plants may be associated with clans, with colours, with stars in the sky, with rituals and so on in one cultural tradition, and it may be embedded in a different complex with social and religious actions and obligations in a second one. The net result of these diverse ways of classifying in concrete traditions had many scholars believe that only westerners (and first of all the Ancient Greek tradition) really knew about classification logic. The analysis by Durkheim and Mauss at the turn of the 20th century showed that one should deconstruct the cultural embeddedness in order to find the same logic of classification worldwide. That is to say, the same major logical operators seem to obtain, but they may not be readily recognizable across cultures. However, the fact that cognition is culturally situated does not deny at all that classifications obtain worldwide. Hence class logic in one form or another is likely to be universal (Lehman 1985).

Another aspect is that not all possible operators can be found always and everywhere. Obviously, a group of operators that have been developed in recent decades in so-called logistics or symbolic logic (Carnap 1953) will not be found in a myriad of oral traditions. Neither do the different branches of formal logic have a parallel in them. On the other hand, surprises prove to pop up from time to time. For example, MIT linguist and polyglot Ken Hale describes how the Warlpiri Aboriginals in Australia show a deliberate and sophisticated use of paradoxes and similar avoidance procedures in their conversations. In an intriguing contribution Hale (1971) illustrates how the Warlpiri test the less or indeed unknown interlocutor by presenting the latter with paradoxical or deliberately ambivalent utterances. If the interlocutor is able to recognize this conversational move and hence to 'turn around' the statement and indicate that he refuses the paradox, the conversation on the theme is carried on. If not, then the interlocutor is rated as not knowledgeable and is not allowed in the further exchanges of knowledge. This sort of testing through purposeful ambivalence and paradox in the communication process shows the mastering of the logical operator of contradiction, and of the more complex one of paradox. Such data from linguistics and anthropology again point to the universal knowledge and use of at least some logical operators, albeit they are not (or not necessarily) used in the disciplinary way of the logician in western tradition.

The question to be answered then becomes not whether universal logical operators exist, but rather which ones we can discern with that status? And how will we decide o what classification and what operator is sensible, workable or otherwise 'natural' or 'obvious'?

The second question will allow me to give an answer to the first one. The best answers I have come across for this second question refer to the 'utilitarian factor'. D.T. Campbell coined the term 'entitativity' to explain why all over the world plant and animal classifications (among others) can be found, and why they are so comparable to each other as well as to the scientific classifications of biologists (Atran 1990). Entitativity points to the fact that human beings make distinctions (classify) according to the needs they feel: distinguishing edible from inedible plants is not a peripheral issue, but a basically relevant point for survival. Some classifications sort of force themselves upon people, because they frame a fundamental distinction for the survival of human beings. In Campbell's terms: some categories have a high degree of 'naturalness' or 'obviousness' in terms of survival of human beings. Put differently they have a high degree of entitativity, I;e. of presenting themselves as an entity (Campbell 1989). Philosophers such as Quine have used the term 'natural kind' to install the same notion, but I will stick with Campbell for the purpose of this book. Rephrasing the issue, I claim that the degree of entitativity points to the degree to which a classification or categorisation forces itself upon the knower as the most obvious or most 'natural' way to classify. Evolutionists and functionalists will find reasons why the entitativity is so obvious in such cases, but is of secondary relevance for the present argument (see Hunn 1985).

Having clarified my position on the second question, I can return to the first one. There are several ways to formulate an answer on this question, depending on the logic one wants to use. I will voice the issue in the most common symbolic logic today, that is in propositional logic. The probably most general connective signs, corresponding to the most generic logical operations, are:

- disjunction: the class defined by disjunction is the encompassing set of phenomena including elements from set A and set B. The formal expression is: A v B.
- conjunction: here the relationship identifies common elements in two sets or configurations A and B. the subset is identified as the sum of all shared elements of A and B: formally, A. B.
- negation: a classification or gathering of phenomena is based on the opposition or contrast: instead of A, a particular series or configuration is identified as anything not-A, formally −A.
- implication: this is best known in the form of cause-effect relationships, but other types of implication were discussed over the years. In the most common form it captures relationships between phenomena of the type 'when A occurs, then B will follow', meaning that A is the cause of B. Formally: A > B.

In the history of logic not less than a hundred different types of implication have been discussed in the literature, with the strong material implication as the most obvious one: when I exert a certain power on a body or mass, it will move in the opposition direction of the power (kicking, pushing, etc.). From there on, a myriad of different implications can be identified, a lot of them probabilistic rather than strong causal relationships (Quine 1982),

- equivalence: the content, meaning or domain of one proposition is equivalent of that of the second one, or the domains of both overlap completely or are identical. Formally, A = B.

The material equivalence would point to the identity of both sets of phenomena, whereas the propositional equivalence refers to equivalence of meaning or content.

These basic logical operators most likely are universal. However, their use and appearance will differ to the extent that their universal presence will be overlooked or denied by shallow observations:

(a) the premises on the basis of which the operators will be used will differ. If a certain tradition starts from the premise that 'mind' and 'matter' are two different and quite separate manifestations of reality, then a whole series of statements can be produced (in fact a learned tradition of scholarship was started in Christianity) defining cause-effect relations between both 'classes of phenomena'. If, on the contrary, the 'mindbody' unity of human reality is the basic premise (as in Japanese thinking about humans, Shaner et al. 1991), then the consecutive steps in reasoning and the relations between phenomena will be quite different.

It is clear in mathematical education the FK and BK of learners will exhibit different insights and intuitions, yielding quite diverging premises for further formal thought.

(b) secondly, I make the claim that different cultural traditions will have a preference for diverging logical operators (at least to some extent). Concretely, the emphasis on implication as the preferred and hence most esteemed logical

> operator in terms of knowledge is remarkable. We would state in the West that knowledge is only serious or founded or worthwhile provided the relationships between phenomena can be expressed in terms of implications, preferably even of causal relationships. The ‘if-then’ relation is central in western understanding of genuine knowledge. The tradition of ‘proof’ (in mathematics, and in other sciences to a lesser extent) can only be understood against the background of this preference for the operator of implication.

In other traditions this insistence on implications, let alone on cause-effect reasoning may be less prominent or even absent. For example, when working with the Navajo in the USA I was often struck by the fact that a preference for conjunction and disjunction operators was dominant. Informants would observe for a long time and in minute detail a series of events and then conclude that ‘when X occurs, then Y is often happening as well’.

Concretely, Curly Mustache, an old speaker-philosopher who was held in very high esteem throughout the reservation developed an interpretation of the expansion of English among his people at the expense of Navajo language. He described how seven different languages came to be among the people over time, with English as the last addition. According to him the next step would be the gradual decrease of languages used, ending with English monolingual Navajos. In the reasoning, and more particularly in the terms used (Pinxten et al. 1983), there is no causal relationship to be found. It is the juxtaposition of languages in the total process over a long time span that is described.

Again, the traditional preference for one or the other logical operator is not neutral, nor universal. In AM the preference for implications is implicit (shown clearly in the high evaluation of proof), and it is important for mathematics education that the teacher is aware of this and reflects differences in the curriculum and in the learning procedures allowed.

Chapter 8
Learning Formal Thinking in a Culture-Specific Context

1 Real Children and Genuine Learning

The What ifs of the first sections of this book phrased a particular and indeed a straightforward position. What perspectives do I advocate?

Recapitulating, I claim that children are learners in real contexts. The latter are culturally, religiously, socially and historically tainted. The discussion on whether such extra-cognitive dimensions are determining or constraining, merely influencing or having no impact at all (as the deterministic socio-biologist might have it) will not be decided in the near future. What we have to conclude in the present phase of the history of schooling and development planning is that the culture-independent rationalistic approach to mathematics education is not an overall success. Notably in the West the growing shortage of sufficiently trained 'brains' for engineering and natural sciences (for industrial jobs, let alone for academic jobs) is reaching an alarming level, which caused such official initiatives as the National Mathematics Advisory Panel in the USA, for example, to become a lobby group for more and supposedly better mathematics education. Whatever the quality and effectiveness of such initiatives may be (see Greer 2012), they certainly indicate the awareness of a deep problem: mathematics education has too much dropout, and this may be threatening the future of our civilization. The brain drain from Asia, which allowed to alter the shortage for a while, has basically stopped, since those brains are now put to use in Asia, and not in the West.

The position I defend on this issue is that education in formal thinking (mathematics or broader) should move away from the rationalistic perspective, which is still very much present and start from the worldview and the FK of the children. Only by choosing that way will insightful steps in the learning processes become more common, and hence may dropout rates be expected to dwindle. I side with Freudenthal's approach on this point (Freudenthal 1985), but will expand the 'reality' he was talking about to one, which encompasses the great variety of

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cultural knowledge traditions in the world. With globalisation such an expansion looks like a natural choice to make for me.

Two major points need to be emphasized, once this road is taken:

(1) education in formal thinking and mathematics which starts from the FK and BK of the children (or is 'realistic' in Freudenthal's words) should still be education. That is to say, the notions and procedures found at the start of the process are not to be mistaken for or substituted by the goals of it: the former are initial insights and notions, which should be developed and sophisticated through education. But taking the psycho-genetically original and local seriously in the educational process will allow for insightful learning. That is my tenet.

(2) an organisational problem arises, which has been mentioned as a side-issue already when I discussed the OECD views on education: when advocating the cultural groundedness of mathematics education curricula and learning processes, I necessarily have to opt for plurality and against uniformity. That is precisely what multimathemacy aims for, and why it is a political choice as well. Rephrasing the notion by means of a visual metaphor, I offer the following image: mathematics is to be found in the world as a rather large city with a variety of buildings. Maybe the unique beautiful skyscraper would be the building of AM, with a great variety of rooms organized in a fairly logical order. But next to that tower, we see great buildings of Chinese, Islamic and old Aztec mathematics, and a huge amount of little houses and huts of local practices and knowledge shops with the most exotic ware, offering material on trade, costume making, designs in tapestry or navigation techniques. The whole city stands as a metaphor for what is around as mathematical operations and skills (with the 'small m' next to the capital M, in Bishop's famous distinction, Bishop 1988). In view of this visual metaphor of the 'city of m buildings', multimathemacy professes that education should be organized in such a way that children or learners all live in the small dwellings of this city, and education will start from the local knowledge and search for the best path from there to a particular room or flight of the AM building. Each path will be slightly different from the next one. Each search will be decided upon in terms of functionality and values, both of and by the 'clients'. All paths should allow for trajectories, which lead from one insight to the next, as much as possible. Rote learning may be a necessity at some point in the process, but sparingly. The conviction behind multimathemacy is that this way dropout from math classes will be reduced substantially and hence chances for emancipation through the learning of formal thinking in general and mathematics in particular will be enhanced considerably. Obviously, allowing for a diversity of trajectories is again a reaction against the colonial Eurocentric perspective in math education, and a choice for the capabilities approach of Sen and Nussbaum (see Chap. 12).

A consequence of the combination of both remarks for the remainder of this book is that uniformity in teaching procedures and in curriculum material is not a good idea. It is not excluded that general and universally usable textbooks will continue to be found in mathematics courses. But they are more likely to be useful and beneficial at a later stage, with higher forms of mathematics, than at the level of initial steps in formal thinking and mathematics. When the ‘culture’ of mathematics of the AM skyscraper is sufficiently understood by the student, then particularity or differentiation of insights will probably be unnecessary, but this only applies for the advanced learner (or at least for most of them). At the primary levels the growth of knowledge should be accomplished by means of steps of insightful learning, and those will draw to a very large extent on the BK and the FK of the learners, that is on knowledge and understanding of the world which is rather more than less contextual, cultural and particular. Hence, the ‘multi-’in multimathemacy should be taken seriously as much as possible in the first stages of the processes of learning to generalize, to use formal logic and to explore the world of what is colloquially understood as mathematics.

In this book then, it is impossible to develop a general, uniform or universally applicable curriculum, nor to describe the unique and necessary path of acquisition of procedures and rules of learning. What I can do is present the FORMA as a tool of analysis and ethnographic description (see the above chapter), and then offer particular, local, culture-specific and intriguing examples on several elementary mathematical notions, each of them tied to the particular cultural context where they should be situated. So, the remainder of the book will present some examples of steps toward formal reasoning, and mathematics in particular, at the elementary level. It is there that the dropout finds its onset and the first stumbling stones are to be situated. Not understanding yields backlash and finally dropout, when education does not remedy by building bridges of insight. What I try to do in the remainder of the body of this book is to elaborate on this point in a variety of particular, non-universal and hence necessarily partial examples, which stem from different parts of the world. It is up to the curriculum designer and the teacher and learner to eventually pick up one or the other example, turn it around and adapt it to the context they are living and working in and thus multiply material and procedures along the way. That is what I would term genuine learning with real children, while being conscious that we give up of the economic comfort of uniformity along the way.

2 What Sort of Material and What Kind of Learning Processes Can I Think of?

In the literature of ethnomathematics it has become common knowledge now that ICT people, physicians, and even engineers use mathematics in unorthodox ways. They pick and choose what they find necessary for the task at hand, and they hardly ever bother to go through all the tedious steps to produce proofs in the correct way, as prescribed by AM. Without any doubt their knowledge is very important and has tremendous impact on the world as we know it. Even more, survival, the quality of life and a lot of wealth depends on their thinking in the globalized world that is ours.

When looking at survival strategies, at ways of understanding and describing reality, and at formal reasoning in other cultures I see a lot of similarities. Navigation strategies and maps of the great seafarers in the Pacific area are not precise, let alone they would testify of a strong notion of proof. But they ‘worked’ to have people survive for ages, and the mathematics and logic in them is intriguing in the way they instantiate examples of formal thinking. Measuring of land patches in agricultural groups, as well as estimating distances by the hunter on foot who leaves parts of his catch behind with ‘accessible’ spaces in between during weeks of hunting, are other cases. The many geometric designs in tapestry and on pottery should be looked into as well. But dances of a ceremonial nature with the Hopi in North America, with Africans anywhere and indeed with ballet dancers in the western tradition (De Keersmaecker 2013) equally qualify. I will use examples from such particular contexts and from what people have developed as more formal notions and thought processes on the basis of them.

3 Counting: Some Examples

People count and use numbers all over the world. There are a series of studies in anthropology about counting systems. This is not the place to give an overview, however. Let me suffice by pointing to just one ethnographic doctorate to show the detail of what we have available in the literature: Lean (1995) presented a delightful study of over two thousand pages, giving an overview of several hundred counting systems in Papua New Guinea. The mere span of this kind of study makes you realize how vast and varied the human ways of thinking are.

In mathematics education my plea is to take this diversity into account. At the very least this results in two different advices for the organization of education:

- the BK of the pupils will express this variety of counting systems, and
- in a lot of cases learning will happen in a sphere (to quote Ingold, Chap. 1 of the present book).

(a) the BK: Urton (2013) has it that a rationalistic view on mathematics and mathematics education is actually often used as a political weapon of subordination. In his words: ‘the ‘state’ accounting, as realized in the practices of alphanumeric, double-entry bookkeeping in Europe and in khipu (knotted-string) record keeping in the Inca empire constituted highly effective strategies for the exercise of social control in the two settings’ (Urton 2013: 17–18). The accounting system of number use and counting as well as the khipu system (Ascher and Ascher 1997) standardized the use of numbers to express values, and introduced the general practicing of addition, subtraction, multiplication and division.

The khipu is a set of strings with knots in them. All strings are tied to each other in a bundle. The bundle can be carried around; it is a bookkeeping system which 'memorizes' and processes data about sales, debts and such by means of strings (registering and summarizing the balance for a client or supplier, or for a certain type of deals), knots (specifying individual transactions) and an overarching mother string to which all particular strings are attached and which allows for an overview of transactions at any moment. Adding or deleting a knot or a string is always possible. Thus, procedures of adding, subtracting, but also multiplying and dividing can be carried out by means of the khipu as a whole, for individuals and for groups.

This example is well known by now. The Papua study (of Lean, o.c.) is less known, but should be understood in the same way: people use the different points on fingers, elbows, shoulders, back of the head, ears, top of the head and anything imaginable on this trajectory from left to right in order to produce a concatenation of numbers. On this 'line' between the left and the right little finger sums can be made.

Or there is the Navajo case I studied to some extent, where the sexual features of the male produce the number three, and those of the female genitals yield four as sacred numbers. Not surprisingly, this 'four' plays a major role in other aspects of nature, such as the cardinal directions, the four main winds, the four colours and so on (Pinxten et al. 1983). Diverging aspects of the world are counted in threes and fours.

Many similar examples on counting from around the world could be cited. Even the European tradition did not start with the decimal system either: the toes and fingers were used as a first major counting device, adding up to twenty, as is still witnessed in some expressions (like the French number for 80, which is 'quatre vingt' or 4 time 20). But there is more: 10, 6, 12, 20 all have been the standard unit for counting systems throughout history (Brown 2012). Counting then yielded such educational devices as the board of five, which I was taught in my earliest school years and is still used sometimes today. The child learns to count by filling in small knobs, which fit into tiny holes in a board. The holes are grouped in fives. Two sets of fives form a neat larger board. For example:

Figure 1 represents 5 knobs in their holes on the board, clearly referring to the digits of one hand. Adding 3 then amounts to start a new similar box with three digits out of five filled in, adding it on to the first board and thus making 8 in all (one hand and three more fingers). The open holes are represented by a dot (.), and the filled in ones by ° (Fig. 2).

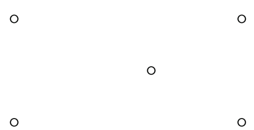

Fig. 1 Five

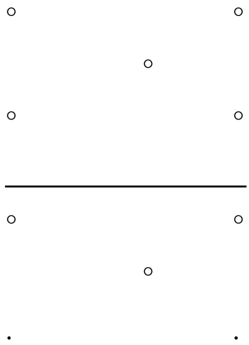

Fig. 2 Eight

Obviously, the use of number blocks is another, but similar semi-abstract way of counting. The blocks, which are widely used, have the following properties:

- a unit (or 1) is a little 1 × 1 wooden or plastic chip, of a particular colour, here depicted in Fig. 3:
- ten units form the number 10 and are materially available as a block of 10 cm, in a different colour from the 1 block (Fig. 4):
- 100 will be a square block measuring ten by ten.

Counting then is learned by actively adding or subtracting units of ones, tens or eventually hundreds, which can be visualized clearly in the manipulation of the blocks.

My point is that European children started from visual, contextual material which referred clearly to the instruments they used themselves, namely the fingers on their hands (and eventually toes on their feet). In a second stage the concrete and local 'manipulatives' are left behind and abstract integral numbers 1, 2, etc. take their place.

When I first met A. Bishop in 1987 at Cambridge University, he was doing research in British classrooms, showing how children solved mathematical problems of counting in a variety of ways. That is to say, there was and is an orthodox, or standard, or proper way to solve a problem in mathematics, according to the national policy services, referring to the AM tradition. In the classroom, teachers will strive to implement that way, which offers them an easy overview of the progress of the whole group at any time. It makes the learning process uniform. It

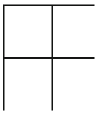

Fig. 3 Four digits of 1

Fig. 4 One digit of 10

allows for easier and faster correction of the exercises and assignments, and is considered to be the preferred way of mathematicians (because it is standard, or elegant, or rewarding in the long run, for example). But children tend to do things differently from mathematicians: they feel uneasy with the prescribed way, or do not grasp the meaning of the procedure, or they might find the orthodox way too cumbersome (which is what I heard many times, including from my own son, at elementary level).

The point of these remarks is that there might well be arguments from the mathematician's and from the teacher's points of view, but if the prescribed procedures are not understood or may be even rejected by the pupils, they are likely to cause or induce dropout. Then it is more than worthwhile to allow for different avenues in the learning process. Indeed, children in Bishop's classroom and in my own informal investigations at elementary school level were able to find the good solution for a problem, following problem solving paths that differed from the standard one. In terms of 'multimathemacy' this diversity should be allowed for and even invited by the teacher. That does not amount to saying that the AM way is wrong. Indeed, after years of winnowing and applying the chisel mathematicians will probably have found the best way to solve a particular problem (in terms of rationality, elegance, and so forth). But that is not the issue. My point is that it is of great importance in the learning process (and maybe only there) that children will understand, and will take insightful steps toward the solution of a problem. Whatever their diverging way to solve the problem can be esteemed to be the preferential one to start with, since that way is most likely based on insights for the child at work. Solving the problem will hence give self-confidence to the child, which is a major issue in the end (see Hersh and John-Steiner 2011).

Once this insightful mastering of mathematical problem solving is sufficiently developed in the child the time may be ripe and the need may present itself to substitute one's way of problem solving for the orthodox one of AM. But that is, in my opinion, at best a secondary problem, when looked at from the point of view of diminishing the tremendous dropout rates. In my terms: the plurality of approaches and the recognition of the BK of the child come first, and the tradition or preferences of the mathematician of AM should leave room for them (provided the right solutions to a problem can be found by the child in a so-called 'unorthodox' way).

So, counting with reference to one's fingers and toes (as in the 4×5 digits counting system) is legitimate to start with, although it is by far not abstract enough to be the end result of learning.

(b) the sphere: In Tibetan country (Gold 1994), in the Navajo context in North America (Witherspoon 1977), in Australian Aboriginal desert lands (Isaacs 1980) and in possibly any of the thousands of oral traditions numerous children around the world grow up in, the intuitions of the local cultural environment are used to build concepts about natural phenomena in their out-of-school world view or BK (Ingold 2004). In all these cultural groups it is likely that the world of experience is primarily lived and seen as a sphere in which one is embedded and which moulds the mind of the human subjects.

> That is to say, the external world or nature is the vast and encompassing network of which one is an integral part in such traditions. In the western intuition of the 'God's Eye View' the world is taught to be the set of things which is unified by the fact that all of it is outside of me and 'hence' can be looked upon as if from the outside. Most probably, this intuitive stand is deeply entrenched in generations of western culture bearers by continuous education in a religious cosmology, which thinks of the Creator God as the unique outsider (indeed a transcendent entity), with humans as the sole creatures who are capable of at the least mentally adopting this outsider's objectifying intuition. It is precisely this particular mental act of intuition, which is called 'the God's Eye View' (Pinxten 2010).

If the hypothesis is true that deeply diverging intuitions about 'being in reality' obtain, then the cosmology of oral traditions such as hunter-gatherers (Saami in Finland, Navajo or Kaiapo in the Americas, and so on) teaches about the world as an encompassing sphere to children of those traditions. In the dual cosmos of the western religions, children are reared in the mentality of Adam from the Old Testament, who sides with the Creator God (rather than being immersed with every other being in the sphere of reality) and is explicitly granted the right to use the rest of natural beings according to his insight. That is, the child adopts the mentality that humans are somehow 'above' all other creatures, or outside of the sphere of all natural beings, and manipulate the latter as objects rather than as subjects. When I agree on this distinction between intuitive worldviews, then it follows that mathematical learning strategies for the children from many oral cultures may benefit from adopting an accompanying mindset of sphere, rather than the extremely context-free and objectifying mode of thinking that is typical of AM.

In educational settings, storytelling can help here since it involves sharing of a world of experience for listener and storyteller to a large extent (see below). But a more general use of the action modes and the communication networks of the children will probably yield better results for a large group of children, than the exclusive and rather algorithm-driven reasoning of the rationalistic type of mathematician (raised in a long history of God's Eye View intuitions).

Again, many examples can be cited here. A more exotic one in this realm is the example of street mathematics (Mesquita 2013). Beyond any doubt street children in Brazil are counting and measuring in their street environment: they negotiate deals with their 'customers', they have their own rather accurate bookkeeping on suppliers and buyers, and they move about in a way that safeguards their possible exit to a subway or a nearby street when police is spotted or a rival gang appears on the premises. At the same time, when forced to perform similar cognitive problem solving tasks in a classroom setting, seemingly unrelated to their out-of-school contextual experiences, they fail miserably. The lack of familiarity with middle class reasoning, and the context-independent use of mathematical skills in a regular school makes them feel so alienated that any recognition of the sphere they do feel at home with (i.e., the street) proves impossible. My interpretation of this failure is that one of the reasons for the inability to 'use the knowledge which they obviously have in the street' is that the encompassing frame is alienating: there is no sphere in the school context in which they can interact with things and people the way they are able to in their street context. Rather, the mental gap between the learner and knowledge, but also other people, is a prerequisite for performance, which they lack. Other parameters play a role (like differences in language, other social rules, etc., Mesquita 2013), but the cosmologically different mindset will be a remarkably difficult one to integrate in the schooled learning tradition.

A conclusion I draw from such cases is that differentiation and diversity are important in the learning of numbers and of the procedures of counting. One line of approach will be closer to or more overlapping with the child's BK and will hence make for security and recognition of what Vygotsky called the 'zone of proximal learning' of a particular pupil. Whereas a standard school approach will be alienating and hence dissuading from further learning. To be sure, the literature shows that the variety in cultural notions and procedures is vast, but that in itself is only a hindrance for the rigid thinker and teacher, and should not be a major threshold for the one who places the enhancement of understanding and skills first.

So what does 'counting' really mean? Or, put differently, what can we safely use in as transcultural notion of counting, regardless of cultural contexts? To the best of my knowledge, this is a wrong question. We simply do not have such a neutral, transcultural, a priori universal notion of counting. The example from the Quechua tradition shatters the belief in such a universal notion. I use the translations to English by Urton, although it would be more accurate (but less understandable) to use the native terms. In Quechua a distinction is made between four word groups, according to Urton's elaborate analysis (Urton 1983):

- 'augment'
- 'unite'
- 'expand' and
- 'part-to-whole' (Urton 1983: 152–153).

Obviously, any of these recalls in the westerner a particular aspect of the notion of counting: the 'augmenting' of a set of things will yield a larger set, and hence something like an operation of addition seems at play. But then 'to expand' is at the very least similar for that westerner: it may sound more like a spatial sort of addition, going from a particular length or surface or volume to a larger instance of any of these. However, one can readily understand that expanding and augmenting might belong in different categories: I cannot 'augment' a line, nor can I 'expand' a bundle of dollar bills. The two other word groups add more differentiation still: 'uniting' something will imply that two or more separate units can fit into one greater encompassing complex. On the other hand, the 'part-to-whole' movement is clearly pointing to a recognizable (and maybe somehow preconceived) whole, which is filled in with the parts. One may argue that these word groups somehow distinguish at a more 'concrete' level what is caught in a more abstract way by the generalization 'addition in counting'. But that does not really yield a representative or fully respectful interpretation. What seems to be at stake is that in the Quechua cosmology counting is a rectification of what is out of balance: 'the main forms of rectification include addition, subtraction, multiplication, and division' (Urton 1983, 146). So, the word groups point to the major forms of 'rectification of the imbalance through addition', which the Quechua differentiate. In our western cosmology a range of phenomena is seen as existing on or in themselves: the 'res' (in Latin,), the 'things' or 'objects' which are believed to have a certain degree of entitativity (as Campbell put 1989) or are conceived as 'natural kinds' to a large extent (in Quine's view 1987). Of course, both cosmologies have different implications for reasoning and for counting, but neither is true or false in itself. They are different, but not necessarily true or false. For example the Quechua way proves very handy in the market, and hence is true, valuable, relevant and so on for the purposes it is used for.

Of course, the same reasoning applies to the other 'forms of rectification' for the Quechua: subtraction is to be found in a set of four word groups, going from 'reducing/removing' (in size or quantity), over to 'reduce/lighten' (of weight), to 'disunite or disaggregate' and finally to yield a 'remainder, a part of a whole'. It is easy to see that the last two groups seem to act as the opposites of 'unite' and 'part-to-whole movement'.

Multiplication as a form of rectification is expressed in two different word groups: one is called 'turn' and points to a repetitive action, like in 'x times y'. On the other hand, what Urton translates as 'multiplication' points towards the enlargement or growth of what is multiplied.

Division is more complex again: there is an action of 'separation' like in the unravelling of threads on a string. Then there is the word group of 'divide/fragment' which looks like a fork-movement: aspects/things/properties are torn apart from each other. Finally, there is 'repartition/distribution' as a word group, which indicates a division of parts from a whole and their eventual spreading or redistribution.

I have elaborated a bit on this example, because it shows what deep and possibly pervasive aspects of BK (worldview, out-of-school concepts, etc.) should be taken into account when planning mathematics education. On top of that, it is obvious that the FK (foreground knowledge) of the pupil will be significantly different in a Quechua, a western, a Chinese or any other context of BK.

Urton (1983) makes the plea that these different cosmologies, and hence the different notions of counting, have their value. None should be disqualified because it is different from the next one. In the contribution cited elsewhere (Urton 2013) he draws the line more sharply: counting systems have a political importance, since they are used by the ruling minority in order to control the subordinated groups. So, along this line of reasoning, there is no neutrality to begin with, but there are contexts of power play in which mathematical skills and practices play a role (see also François 2011). Hence, mathematics education implies a choice about society, about democracy or power imbalance. This meets my point on multimathemacy: if one allows for different notions, procedures and problem formulations to begin with, the chances increase for all those children who come with a different cosmology or mindset than the one that is shared by AM. This does not downplay AM in any way, but it speaks about allowing maximum opportunities to those who come to the table from a different cultural world.

Chapter 9
Complex Mathematical Activities

1 What Are 'Complex Activities/Operations'?

In the analytical frame of reference I distinguish between twelve distinct and analytically separable activities, which yield mathematical operations. It is obvious that real human beings do not necessarily and maybe not commonly reason within the confines of analytical differentiations. It may even be part of a main goal of education (especially in the so-called modern perspective of western history) to learn to reason in distinctive analytical categories, which are not found in so-called natural or vernacular thinking.

My suggestion is that it is good to recognize this and to work with an open mind when planning education. Thus, combinations of operations may be beneficial to education, whereas they may appear to be clumsy or lack precision and rigour from the perspective of the scientist. Concretely, when Micronesians navigate the oceans in their canoes (Gladwin 1973) they use spatial orientation, logical operations, counting and maybe more operations in combination. When Navajo medicine persons perform ceremonies they use geometric notions of symmetry, dance and rhythm, but also counting (especially the number four: repeat verses four times, refer regularly to the four cardinal directions, etc. Reichard 1939). And so on. So, in view of 'realistic' mathematics education in Freudenthal's terms, I want to take this seriously and introduce 'combined' or 'complex operations'.

2 Locating and Representing

(a) *Ethno-geography*

The Ifugao on one of the islands of the Philippines present a very intriguing case of combined or complex mathematically relevant activities. They live in a mountainous area. Over the centuries these rice growing people reshaped the

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> valley they settled in, in such a way that the anthropologist Hal Conklin invented the notion of "ethnographic atlas" (Conklin 1980) to describe the elaborate multidisciplinary folk knowledge he found in their tradition. The magnificent book with that title looks like a 'normal' geographic atlas with maps and some data on production and traffic in the valley of the Ifugao. This is how it appears from the outside; until one starts to look closer. As an anthropologist Conklin describes in great detail the year cycle of Ifugao as rice growers: the preparation of the rice fields, the work on the irrigation canals throughout the valley, the construction of dykes, the carving of wood beams for the houses and storage buildings, the planting, replanting and harvesting of the rice, the collection of wood for cooking and heating on the ridges on top of the valley. In the same breath he gives a detailed overview of the ritual system of the Ifugao: throughout the year the valley is the scenery of a great diversity of rituals for 191 out of the regular 365 days in a year. Some of these ceremonies go on for several days and involve nearly everybody in the valley; some are local and kinship-linked (like puberty rites, funerals, etc.). Almost 500 chicken, 80 pigs and some other animals are sacrificed each year. The important point for my purpose in the present book is that this ceremonial cycle structures time and of course community life, but also that it defines and redefines the spatial organization of the valley. Fields are allocated and redistributed on these occasions, because each death and each birth reshuffle the needs of the persons involved and hence necessitate reconsidering locations, fields and duties in the community. The deep relations between social and economic life on the one hand (agriculture) and religion on the other hand is illustrated by the fact that the 22 stages of the agricultural year are paralleled by 23 rituals or ritual complexes: 'the designated ritual events constitute the most significant points marking progression of the agricultural year.' (Conklin 1980: 13). Some rituals in this elaborate set are exclusively accompanying 'field marking', every year.

What Conklin (1980) does in his remarkable ethnographic atlas is to translate loads of information from all of these areas of Ifugao life into tens of maps, drawn in the way of a geographer, and printed in what looks like a common atlas. The maps relate data from the Ifugao lore about land use, ritual cycles, communication and interaction lines, terraces linked to kinship ties, irrigation ditches and canals and interdependencies within the irrigation system, plots, parcels, common land (the forest on the ridges), families. Hence we get something like human geographic maps, but from the point of view of the Ifugao. Put differently, what Conklin adds to the knowledge of the local people is the graphic representation in geographic maps. What the maps represent is the folk knowledge. To make this stand out in its full potential I go into some detail.

In order to survive the Ifugao have reasoned that their cosmos is interdependent: they are immersed in an environmental sphere wherein everybody and everything is interlinked tightly. Thus, the encompassing system of irrigation is designed and continuously modified in view of the needs of all families who live in and off the valley. Ownership, size of the fields, scale of the crops and so on are not private matters, but are seen as part and parcel of an overarching and interactive whole, which is covered or accompanied by a ritual cycle of continuous redefinition of kin and village duties, rights and needs. All of this together forms a dynamic equilibrium that is meant to guarantee the survival of the community of the valley as a whole. There is no formal bookkeeping (as was mentioned by Urton with regard to western and Aztec political control through bookkeeping), but allocations and reshufflings are not done randomly either. For one thing, it is impossible to imagine the sustainability of such a system without the Ifugao using comparison: e.g., the need of a family with one child and two grandparents will change drastically when the latter die, or when another child is born. Upon such a change in household, the whole valley will reallocate fields and rights to wood from the common grounds accordingly. Moreover, to make things manageable adjacent fields with adjacent irrigation canals should preferably be destined for the needy family: obviously, the acquisition of a field implies also the acceptance of the duty to work on dykes and canals for that field, in collaboration with the neighbours and their fields and irrigation canals and such. In a valley with hundreds of little fields it is quite a task to keep track of everything. Without doubt counting, comparing, orientation and other mathematical activities and competences are involved.

When I want to engage the Ifugao children in a mathematics class, it seems obvious to me that it would be a shame not to use all this out-of-school knowledge to begin with. This is part of the BK of the children there. Their FK can be mapped on the basis of the latter: the teacher should make the BK relating to the ethno-geography explicit, use examples from the children's family life as problems in the math class, go into the procedures of map drawing of the valley (the way Conklin did) and determine its usefulness and potential with the pupils, and so on.

For the sake of argument, let me show how a curriculum can be developed in this particular case: not the market values of CD's are known and cherished by the Ifugao children (as is supposed to be universally so according to the OECD 'assessment' of PISA), but the needs and rights of particular families within the constraints of the valley. The following tasks can be defined in this particular case:

– list the parcels and fields which are in use now, and draw a list of the tasks with reference to irrigation of the fields (water supply, dykes, etc.),
– negotiate in group about sensible measures for allocating fields: we need an overview of the families and their needs, and an overview of available fields and their potential crops, and an idea of the capacities to raise animals in the hammocks and their surroundings,
– how will we compare? This question will inevitably lead to a discussion on a unit of comparison (of needs and crops),
– from there, a clear problem can be formulated and the search can begin to find useful and understandable mathematical procedures which will be helpful for as many inhabitants of the valley as possible,
– draw maps and paths to make a more systematic overview, available to all, of the communication, interaction and exchange patterns within the valley. This will be a major project in itself, which involves a lot of geometric notions in a

kind of 'natural way'. It will be intriguing to see what are the most relevant or important notions to begin with, according to the children: the ritual paths and spots, the trajectories from the house to the fields, the borders/markers of the dikes, the 'territories' of each family, etc.? Next comes the question how the map is built: from a panoramic view as a starting point and filling in the details vis-à-vis the panorama? Or should it start from the particular family house and its spatial and ceremonial links with particular spots and paths in the valley? Or is there still another way? The work on 'mental maps' comes to mind here: western subjects in cities were shown to build up their own mental map of the city by starting from markers along the trajectory they were used to follow. They selectively reconstruct the environment in view of their needs and habits (Gould and White 1974). The techniques of 'mental mapping', which are quite straightforward, might be helpful for the students in other cultural contexts.

– generalizing from these particular experiences in order to reach negotiation and decision procedures which are accessible and shared by the community. At this point experiences from other cultural settings can be introduced.

(b) *architectural knowledge*

Oliver (1987) is a reference work on this topic, because it focused on 'dwelling' as a central notion in human housing/building/shaping operations. That is to say, human beings seek for shelter in a variety of ways. They build shelters, but at a primary level they seek out and adopt certain spaces as 'dwelling': that is, they redefine natural settings in a social space, as a place to live in. Only at a secondary level they adapt the space they found and eventually build a totally new spatial construct, known as a building. In that last case, human beings have been extremely innovative, much more than the remarkably good builders like birds, monkeys and some other species. Of course, this is (again) a vast and fascinating subject in itself. It ranges from cages and simple dugouts over the pyramids, palaces and temples of great civilizations to the high-tech buildings and space-labs of today. Mathematics, and especially geometry, can be detected in the Ancient Greek buildings, but also for over three millennia earlier in China, India, Mesopotamia and Egypt. Apart from those we are at a loss to set any definite temporal borders in other continents (the Zimbabwe monument, the Meso-American civilizations, and others come to mind). My point is, once more, that a vast amount of knowledge about space and use of spatial and arithmetical concepts is to be expected with children from around the world. When we plan mathematics education in a realistic perspective, the least we can do is integrate some, a lot, maybe most of this out-of-school knowledge in education and curriculum material.

To make things digestible I will restrict myself to a few examples. The French philosopher-anthropologist Pierre Bourdieu wrote a famous paper in a double volume published in honour of Claude Lévi-Strauss. It is known as the paper on the Kabyle house, and has been republished a number of times. Kabyle is an area in central Algeria, with little houses, inhabited by small farmers and herders. Bourdieu shows how the house and the social rules and values are neatly intertwined: the exterior-interior distinction is matched by the male-female gender separation, with the interior of the house as female area. The closed backyard of the house, isolated from the outside world by walls and thus acting as an extension of the interior of the house, is female territory. The threshold of the house is the border between male and female worlds, as well as between public and private space. The animals are integrated in the house and hence in the female territory. Social life and politics are 'male' and belonging to the outside world. Bourdieu (1970), von Bruck (1997) gives a lot of spatial and geometric details that concur with the gender distinctions cited:

- the house is rectangular, with a front and a back door; the front door opens up to the exterior world, while the back door allows to go into a secluded little garden;
- the southern side is destined for the stable and the storage of food, whereas the north side has the loom (female), the sleeping place and the kitchen (female);
- the east side is bright and turned towards the outside world (hence it is male); whereas the west side is darker and turned towards the inner world of the house (and hence female),
- between north and south is a central pillar;
- a newly wed bride is carried on the back of a non-kin elder over the threshold, symbolizing the way of the scapegoat which comes from outside and will be the locus for things bad.

The spatial organization is defining a fixed structure, which thus guarantees continuity and prosperity through procreation. The tight control of the female factor is a structural feature of the whole. Rituals underline the gender distinction and thus add to the continuity. In his later work Bourdieu (1998) stressed that, in contradistinction to Lévi-Staruss's structuralism 'in the mind', he aimed at contextualizing and historicizing the meanings of the Berber house: the strict gender divisions, strengthened through the absolute spatial differentiations express power relations, where the male aims to control the female in a nearly absolute way.

I treated Bourdieu's description of one particular cultural elaboration on architecture and dwelling in great detail, because it offers us a refined and nuanced view on how architectural space is invested with meaning in concrete cultural traditions. Oliver (1987) is full of other examples, and the encyclopaedia that author has been working on for decades offers a tremendous overview of the richness of building and housing practices and principles around the world. (But see also Egenter (1990), amongst many other).

In terms of mathematics, it is obvious that children living in agricultural traditions will share similar worlds of experience as the one described by Bourdieu. Apart from the political aspects in the example cited (the gender separation and the control of one over the other), the way cosmological, geometric and other features are intertwined with economic and social characteristics makes up the BK of the child, and probably influences its FK. Indeed, on the one hand, these notions and conventions at home are more or less consciously 'known' by the child, and hence they can be expected to direct and limit its interest and perspectives on further knowledge. Simply put, it is to be expected that girls who are raised in a situation as described by Bourdieu will be less motivated to cognitively explore concepts and dimensions that belong in the vast external world: they have nothing more or less sophisticated to say about them and it is likely they would consider them irrelevant for the female world of experience. Any education, which disregards this sort of constraints, will most probably be alienating and only yield more dropout. But on the positive side, it is obvious to me that the BK that children do bring along to the mathematics education can be used as the base on which to work in order to reach more sophistication, more conscious and elaborate development of one's grasp of mathematical notions.

The example from yet another part of the world can be referred to here: in recent work Hardaker (1998) made a very detailed analysis of the way kivas have to be constructed. In the Southwest of the USA archaeologists found a lot of kivas (or temples) dating back to the Anasazi period. Anasazi is the name of the people who inhabited the area in present-day New Mexico and Arizona till the 13th century A.D., when they rather suddenly disappeared from the territory. The archaeological sites are fabulous, with large cities of almost intact housing and temple complexes (e.g., Chaco Canyon in New Mexico and Mesa Verde in Colorado). The round kiva or temple is what intrigued Hardaker in terms of 'native geometry'. He makes a geometric analysis of several of the kivas, and rebuilt some manually: drawing, laying out the foundations and physically making the ground circle of the buildings. His point is that geometric notions grow in the process of the making of the building: 'Simply stated, when the art or design is competently carried out within the limitations of the rule, the math happens whether you are aware of it or not' (Hardaker 1990: 18, see also below: 6). Obviously, this point rather automatically leads to education: in the classroom or elsewhere the (re)making of buildings has educational potential (see also Pinxten et al. 1987). So, the taking into account of the worldview of the children will allow for an insightful trajectory of learning. I will not dwell on this example any more, since a lot of curriculum material is available, using building and similar activities. Even a tremendous amount of toys can be used in this regard, some of it with high educational standards (e.g., packages of UNESCO material on dwellings around the world).

Similar activities where geometric and geographical knowledge is involved will be looked into now.

(c) *geographical notion*

The amazing knowledge and skills on boat building and on navigation of peoples of the Pacific has been the subject of several studies. In anthropology the early work on starlore and knowledge about waves and currents comes to mind: the inhabitants of the Carolines produced a sidereal compass on the one hand, giving them sailing directions. On the other hand, they 'developed an almanac and calendar…to predict seasonal winds, currents, rains, and overcasts' (Goodenough 1953: 3). The horizon and the sky full of stars (and the Milky Way, of course) are studied and represented on a circle. Notwithstanding the fact that such compasses are not very precise, they do carry a lot of knowledge, which is at least accessible for some members of the group. The carving of a canoe and its stabilizing (yielding the precursor of what is now known as a 'catamaran' boat) allowed the Micronesians to travel the ocean over vast distances, literally hundreds of miles (Gladwin 1973). With the Second World War these islands were integrated in the American sphere of interest: some island became the locus for an American army base, and on all of the Pacific islands the motor boat was introduced as a token of modernity, relegating the traditional canoe to the museum or the waste dump. In recent years, however, it became clear to the local leaders of the islands that oil was becoming too expensive to be imported without future risk: the islanders are literally threatened with starvation if they continue to depend on oil for their boats, which are a major instrument for fishing and hence for their survival. Indeed, oil has to be imported and is more and more costly after the so-called oil-peak. As a consequence some of the islands decided to reintroduce the canoes, this time equipped with GPS (on solar batteries). The production of the boats, the more general and conscious teaching of the navigation techniques and the exploration of the ocean currents and the cloudy skies all become subject to education in mathematics today. The simple fact that the island context presents a different world of experience yields the consciousness that relevant problems and the skills to formulate and to solve them has an enormous potential, will enable the children from these contexts to become motivated for the myriad of mathematical questions relating to this intriguing world.

Let me give just a few examples of what this knowledge amounts to and how it could be put to work in education.

Following Freudenthal's general principles of reality-based mathematics education (Freudenthal 1990) I suggest the following sort of curriculum should be developed:

(c.a) mapping B(ackground) K(nowledge)

Children all over the world live in a series of environments, which are more or less interlocked with each other. One lives in a plain, or on a mountain, or in a forest, or in a city neighbourhood. In any one of these types of environments it is a first step in education, I claim, to explore and consciously map the relevant features of the habitat. For the Ifugao children this would mean to draw or otherwise represent the patchwork of little rice fields, linked to the kinship and ritual groups attached to them. The series of problems, which will emerge in the process of making explicit the knowledge, is rich stuff for mathematics education:

- rice fields are adjacent to one another or not, linked by means of a direct canal of the irrigation system or not;
- the fields form a set belonging to one kinship group, and could be spatially grouped because of that;
- each field has particular measures, referring to possible crops it may yield: how are we to measure and estimate all this? In order to distribute and reallocate fields to families one needs to know what needs the family has

(or stopped having) because of the family's size and work capacities. One needs counting both working and non-working members of a household to start with: two columns obtain (mouths, working hands), and we need to define a relationship between both. In order for the whole valley to secure the survival of all the families living off the valley's crops, we need to have a means of comparison: given the workload, the mouths to feed, the hands that can work, the elderly and the youth, etc. we can map the needs and the input of labour. However, the fields have differential value in themselves: some are high up the ridge and thus demand more effort to be kept up and be profitable than others, some are larger (and thus yield proportionately more) than others, and so on. What sort of counting, measuring and partitioning can be devised to take most of this into account and offer a means for comparative estimates on which to base durable and correct management of the totality of fields?

- it is obvious that measuring the fields will be a first prerequisite action: not simply the geometric surface alone, but adding (if this is what people want, as the Ifugao do) such dimensions as distance from the homestead, accessibility, quality of crops, and so on. So, the spatial and geometric notions will be embedded in richer cultural and social contexts of meaning (Conklin 1981). On top of that the ritual layers of meaning within the valley will be added: a family or homestead with three puberty rites and a funeral in the course of one year will see these factors to be taken into account when determining the value of their rice fields. One with a marriage ritual only will have a different value measured.
- referring to a different context (Navajo Indian, USA—the geography and geometry of the land will be determined by the accessibility for large or small flocks of goat and sheep, depending on the availability of water, the shrubs and grasses around and the physical accessibility of the latter (Pinxten and François 2011). Children clearly demonstrate this type of knowledge about the environment on the survival values of the place they live in. In mathematics education such knowledge should first of all be made explicit and further used in a comparative way.
- finally, the child living in an urban environment carries with her a BK about distances (of walking or riding), places of interest (for food, rest, and so on), danger or whatever. The child recognizes some places and paths (from home to school, from home to grandparents, or examples), avoids particular areas (e.g., migrant children will typically circumvent the city centre in Ghent, Belgium rather than going right through it, because that area belongs to a local upper-class they are not familiar with) and will make a 'mental map' filled with relevant cultural, economic and social-political data attached to spatial and geometric forms. Again, this BK can be made explicit, put into drawings, filmed or marked in different ways. We used mental mapping techniques a lot with primary school children in our own research, and this technique yielded very interesting material: for one thing, the maps by children differ widely and this allows

for comparative study and for discussions on how and where to opt for uniformity in the representation. Moreover, attached to points or marks on the map stories about people, about the places and about other issues deemed relevant can be added to he material.

(c.b) developing abstract notions
In mathematics and mathematics education making tacit or hidden knowledge explicit is a first step. Notions, principles, problem solving procedures and so on may be hidden below the surface of conscious knowledge, because there is no need for explicit knowledge. One learns skills through imitation, by watching the elders do what they do and then try it out oneself. This is certainly a major principle of education in many oral cultures: one does not explain or instruct, but the learner is supposed to 'pay attention' (Farrer 1981) and then imitate the performance. It is this way that children learn to weave in Navajo culture, and it is also the way Ifugao children are picking up knowledge about the irrigation of patches of land, the planting and harvesting of the crops, and so on. My suggestion is that mathematics teachers will do the same: let us move away from the typical instruction format, with the clearly defined problem and the accompanying procedures to solve the problem. Let us first do things, show skills and have the children from oral cultures watch and imitate.

I take the case of designing the irrigation canals and the patchwork of fields of the Ifugao settlement. After a while, giving the richness of the case, discussions will emerge. It proves possible to design one rice field or one local network of irrigation canals. But it is impossible, or at the very least hard, to organize durable and possibly efficient collaboration at the level of several families, let alone the whole valley. In the classroom the children can be organized in temporary 'kin groups', each having to take care of 'their' rice fields and irrigation canals. Nevertheless, since they all belong to the same valley, collaboration is in order, and that involves explicit negotiations, discussions about needs, measures, and so on. At this point the teacher or guide can come in (if need be: maybe children will decide about this step on their own) and suggest we negotiate notions and terms first, so that all use more or less the same content when using particular words or even action procedures. Making this step is, of course, going beyond the particular into the abstract. The step is not made because of a school norm or a literate tradition, but because it serves a purpose: it helps to use sufficiently general meanings when making projects and deciding on action at a larger scale, involving more people with their particularities and stretching over a longer period than the local and temporary event one is living. I suggest this shift towards abstraction needs to be discussed and negotiated repeatedly and very explicitly; it should be obvious to teacher and pupil that the shift serves a purpose, that it is functional. Hence it is clear that function and abstraction are tightly linked in action.

Without doubt in the course of this process it will prove necessary to make, construct or negotiate terms that will be used for the abstractions one starts using. I remember that teachers on the board of the Bicultural School of Rough Rock, Arizona on the Navajo Reservation, who had engaged themselves for the elementary geometry project I had been introducing there, took some time out at this point. They withdrew in a group amongst themselves, not allowing me to join in the process, because they decided a set of geometric terms needed to be decided upon, and defined. This set would function only to refer to the abstractions we had come to at that point. After a few days, I was presented with a short list of terms (equivalents of point, line, and so on) and had to promise I would be careful they would not be used loosely or by just anybody (Pinxten et al. 1987). The point is, of course, that using language is always directly impacting on reality for the Navajo tradition (Witherspoon 1977). Hence, coining the abstract terms and giving them free for use in the learning context was still considered to carry some danger.

Just like descriptions in Navajo (and other oral traditions) will show minute detail and take care of all possible ramifications of what is said in order to control the possible impact on reality, so the abstract terms were presented and given up for use with serious hesitation. This is, obviously, different from the use of abstract terms in a western school tradition. There, abstract terms are considered to be completely detached, context-free. Hence, their use is not believed to be potentially harmful or violating in themselves. Putting knowledge (and abstract terms along with products of thinking altogether) to use so that people or nature may be harmed by it is a moral issue for the westerner, which is not intrinsic in knowledge or in abstraction as such. For the Navajo it is: thinking, speaking, profane and ritual acting are sides of the same coin, and all are seen to have direct impact on reality and can hence be harming human beings as well (Witherspoon, o.c.). One conclusion I draw from such examples (Ifugao and Navajo, to restrict myself to just these two cases) is that abstraction is closely linked with functionality, even with beneficial and harmful effects in these oral traditions. Hence, mathematics education should take this point fully into account and grant that abstract thinking in and of itself is not necessarily a value for these traditions because of that. What I think one can learn from such cases is that the linking of abstraction and functionality in the learning process should be a central concern. This is probably true for most of the western children as well, but the learned tradition of AM tends to forget that and considers abstraction to be a value *sui generis.* Oral traditions teach us differently, and I suggest we pick up this point, especially since the traditional genuinely literate format is waning in western culture of the present generations.

(c.c) combining mathematical activities into a complex curricular entry

In the examples hinted at in the previous paragraphs it is clear that in actual practice hardly a single mathematical operation will be found on its own, isolated from other ones. Nevertheless, most of the textbook curriculum material has children learn mathematics along different lines: one lesson is devoted to one particular operation, separated from the next one. And spatial thinking, geometry and arithmetic are rarely joined for one exercise. Of course, educationalists will argue that focus is important for learning. My argument against this, at the very least at the elementary level, is that isolating materials and problems from each other because they would belong to

> different branches or sub-disciplines of mathematics, will most probably alienate the child. This might be even more the case if the general worldview (spherical versus global view, for example: Ingold 2004) of the child will be further away from the intuitions and the worldview, which underlie western mathematics. Hence, I call for project-based education: it is sensible to work with problems and cases that are meaningful, recognizable and rich in content from the point of view of the particular children (their BK, so to speak). Most of the time, and quite naturally, this entails that questions and procedures for problem solving from different mathematical sub-disciplines will be on the table during the course of one and the same project. The examples above have shown this in an indirect way: measuring the size of a rice field involves counting, measuring distances, but also comparing needs. Graphic representations and discussions about the weight of different parameters cut across the disciplines again. The main gain in such an approach is that the relevance or the functionality of particular mathematical operations stands out for the children: they will pick and use from anywhere whatever will be most convenient and useful. They will not select problem-solving procedures according to the discipline.

Obviously, in the course of higher studies the usefulness or the advantage of using procedures from other disciplines (e.g., algorithms from algebra) may be sensible for the child: when her or his ‘zone of proximal learning’ allows for more abstract and hence more convenient procedures, it will be time to stress that path of abstraction then.

An intriguing critique on the presuppositions and ‘biases’ of pedagogical perspectives of AM can be found in Harel (2014). The author questions some basic a prioris or ‘biases’ of mathematics education. He discusses the ways geometry is taught. His argument runs as follows: some of the most difficult, and often also abstract concepts of mathematics are quite often considered as just another step in the curriculum by mathematicians. Instead, Harel states, ‘we believe that it is absolutely essential that teachers and curriculum developers attend to the question of how to motivate abstract geometric concepts and reasoning to their students.’ (Harel 2014: 24). Consequently, Harel makes a plea to take into account the students’ intellectual needs: i.e., what has meaning in their worldview at any moment in their development. He states that all too often teachers work in the line of ‘*definitional reasoning*’, that is to say ‘the ability to characterize objects and prove assertions in terms of mathematical definitions’ (idem: 25). But we learned, Harel says, that this way of thinking is difficult to acquire. Quoting Poincaré he recites: ‘What is a good definition? For the philosopher or the scientist, it is a definition which applies to all objects to be defined, and applies only to them; it is that which satisfies the rules of logic. But in education it is not that; it is one that can be understood by the pupils…’ (Poincaré 1952 in Harel 2014: 26).

Harel then goes on to develop a sort of story, where fundamental geometric abstractions (like line, translation, etc.) are gradually conceptualized in a dialogue between a mathematician and a blind man who lacks all knowledge of geometry, called Euclid. Obviously, Euclid is the smarter one, who always points to presuppositions, references to "known" data or concepts, drawings and other visual aids, all of which he lacks in his blindness. The dialogue reveals how much mathematicians in the AM school take for granted, presuppose as common knowledge and hence disregard as possible thresholds for pupils. In a masterly way Harel shows how the concepts can be acquired with an absolute minimum of presuppositions. This amounts to a different pedagogy: instead of continuously building on the basis of logical rules and axioms the learner searches systematically for meaning, for insightful steps. Thus, not one single concept is taken for granted 'because it was defined in abstract terms': the learner and the teacher continuously look for empirical grounding in action (such as drawing, but also bodily actions) and for explicit discussion on the notion. The protagonist Euclid in the dialogue puts the mathematics teacher firmly with his feet on the ground, repeatedly: 'I cannot see, so what you say is senseless to me', or 'ah yes, you are drawing again'. For one thing, the teacher and learner thus become very conscious of their own presuppositions and are made aware that other interlocutors might not share these. In other words, they are not given or a priori, but prove to be conventions which are known and understood (or not) by all involved in the dialogue (see also Hersh (2002) on the origin of geometry). Of course, in a more general way Davies and Hersh (1981) already made a forceful plea for the combination of algorithmic and 'dialectic' mathematics, where the latter focuses on the search for alternatives in any given context.

In actual mathematical education I invite teachers and learners to invest a lot of time in this approach of moving on through the field of concepts and procedures by understanding. This will imply going back and forth to experiential data and the exploration of content and meaning, and shying away from purely abstract, indeed often formalistic algorithmic progressing through the curriculum. At least, during the primary years of education this would sound like a wise advice in view of avoiding dropout.

3 Measuring

Paulus Gerdes[1] has published literally thousands of pages on mathematical activities in Black Africa. His overview of anthropological work in Black Africa with relevance for mathematical thinking is an important bit of scholarly work in itself (Gerdes 2014). Some of the books he brought out focus exclusively or in large part on the graphic representations of measuring and counting. For example, some will deal with 'Pythagoras in Africa' (e.g., Gerdes 2011). In the latter Gerdes tries to make the point that the designs in drawings in the sand, or in the baskets woven in every village in practice show the mastery of the principles of Pythagoras: the weaver 'knows' how to fold and ply reed or cloth strips in order to produce the

[1] See /www.lulu.com/spotlight/pgerdes.

decorative motif in a basket we all know. The author makes a distinction, which resounds the famous tacit versus explicit knowledge of anthropology: artisans, he claims, exemplify the 'frozen mathematics' in their work (Gerdes 2014, Chap. 4). That is to say, they master strict action procedures which produce geometrically correct patterns in the artefacts without being able to formulate the geometry exhibited in any explicit way: it is a form of 'hidden geometrical thinking'. School mathematics, on the other hand, often tends to exclude and frustrate students because it disregards these 'frozen' forms of knowledge and focuses exclusively on the abstract, context-free way of problem solving. Very soon the pupil looses track, because what he might know from out-of-school contexts is not valued in the school context, and the child cannot possibly make the link between his or her practical daily knowledge and the logic driven abstract world of the mathematician.

In terms of education Gerdes suggests that we should take time as teachers, to work with concrete material and focus on making or fabricating forms and things for a substantial time. Also, coming back to this concrete level is a good thing to do anytime the pupil gets stuck and shows puzzlement or lack of understanding. The author starts working in the classroom with sticks and ropes, which quite easily allow for making things, laying out forms (geometric as well as other ones) and so on. When actions like these are repeated, discussions on patterns and regularities, that is on abstraction, are coming up almost spontaneously. In my own work I presented similar 'reality-based' procedures: children (especially boys) on the Navajo reservation dream of becoming a rodeo cowboy (or cowgirl). In that sense, the preferred entertainment during summer is an emblematic 'all Indian cowboy rodeo', meaning all participant cowboys are Native Americans. Hence, the arena of the rodeo is part of the world of experience of the child: the form of the arena (oval shape, no corners) and the scaffold form of the places for the audience are 'known' to the children. It is important to use this rather implicit knowledge as BK for the classes in geometry (Pinxten et al. 1987). I organized working classes with children from primary school where small-scale rodeo grounds were built from sticks and raw material. In the action the children had to make explicit the notions they carried around, negotiate on actions and measures in order to reach a reasonable result. One class even produced an arena with the size of the classroom, using waste material. Thus, they were able to try out in actions what was needed with themselves as protagonists of the classroom rodeo.

A similar area of action is that of weaving. Countless cultures have weaving practices, with wool (Witherspoon 1977) or with reed (Gerdes 2014). Whereas for the outsider it is striking that in oral cultures the learning of weaving does not involve instruction, anthropologists have shown how observation and imitation of adult activities by the child is essential here: children watch adults weave, and start imitating at a certain moment. Sometimes this involves 'child's looms', like the ones for weaving belts in Navajo tradition. Obviously, for a cloth or a basket to be woven in the right way, some measuring needs to be done: here again, using the caterpillar measure (walking the index and the thumb over a distance) is often used as an aid. Although this measure unit is not conventionalized as such (all fingers are slightly different from one another), the ideas of regularity and of unit of measurement are present.

Actions that are notable in the course of such exercises are:

- determining a centre for the construction,
- negotiating a circumference, and a radius,
- agreeing on a unit of measurement, from the 'caterpillar of one's hand', which progresses over a certain distance to using a particular stick as conventional measuring rod for all,
- orienting the construction in terms of the cardinal directions (a primary act for Navajo),
- counting: so much units in a row, and so on.

4 Designing

This is another one of Bishop's basic mathematical activities. It is quite obvious that geometry makes use of graphic design. But the same can be said about architecture, irrigational engineering, and even counting systems. Finally, ICT and computer simulations use a lot of design techniques and graphic forms.

Segaud (2010) lined up what is known from anthropology of architecture and anthropology of spatial knowledge. It is striking how all relevant operations of generalization, abstraction, classification and the like are all present in this summary, and—on purpose—how they are linked to concrete and action-linked forms of imagination.

Segaud states at the beginning of the book that space is dealt with in a thousand different ways across the world, but four operations seem to appear on a universal basis: dwelling ('*habiter*': living in a spatial context), founding, distributing and transforming. They are the basic operations in what is grouped under the heading of 'designing'. Obviously, peoples around the world manipulate space in order to make parts of it habitable. Architects are gradually recognizing the relevance of anthropological knowledge on this issue, as is illustrated by Segaud for France. Thus, the mere geographical and physical space is explored in view of making it habitable. This is far from easy in many cases: a desert context forces dwellers to either judge the environment in terms of safety (keep clear of winds, make escape roads easy, etc.) or focus on the availability of water. Peasant groups will be knowledgeable about the quality of the soil for crops they want to raise, and water supply. Traders will be mapping the environment in terms of travel and communication opportunities: river banks, road crossings and the protection from pillaging groups will be important parameters in their imagining of a good spot for a settlement. Some cultural groups have excelled in making mountainous and cliff areas habitable.

The Dogon of Mali are masterful cliff dwellers: they constructed ingenious villages on dangerous slopes and tiny levelled patches in a mountain range.

The designing is apparent from the way the buildings are fitted into a symbolically formatted total frame: the basic design of the whole village should be that of a human figure: the blacksmiths live in the northern 'head' of the figure, and the agricultural families occupy parts of the village which are situated where the legs and the arms of the encompassing figure would be. The main sacrificing altar of the village, in an open space in the centre, is situated where the 'penis' of the design is to be found (Griaule and Dieterlen 1965).

The next operation in designing would be '*fonder*' (founding) in Segaud's view. This activity has been described already to some extent: it consists in preparing the surface space in order to erect a tent or a building. In many cases this involves determining how the construction will be oriented in view of the cardinal directions.

Segaud refers to the universal use of classifications (as shown convincingly in Durkheim and Mauss 1901–1903, and see Conklin 1971): natural surroundings, cosmological phenomena and living things are classified in a variety of ways across the world, but forms of classification obtain universally. In a second cognitive moment some of the classificatory content is used as the foundation for the dwelling: the construction is oriented according to the cardinal directions, the openings in the building make use of natural sources for light and/or for moisture, and so on. In Navajo culture, where people live as semi-nomads herding sheep and goats, any building is oriented toward the East, the place of the rising sun.

Distribution ('*distribuer et classer*') is the next higher operation: dwellings are oriented with respect to each other, and in connection with the natural resources of the community. This is most important with villages and cities: it is then that distribution plays a greater role. Space, resources, measuring distances, measuring and comparison of parcels and volumes all become very important when designing a durable settlement for a larger amount of people, not necessarily related through kinship.

A very informative example was the distribution of land, irrigation means and housing space in Ifugao. Another one, in the heart of western culture, is that of the land and settlement policies of villages in the French Ardennes (Karnoouh 1980). Karnoouh shows in a longitudinal study how the distribution of land and of building lots in a village in the north of France was kept absolutely unchanged for over a millennium. The baptism records of the village from around 800 C.E. until the last quarter of the 20th century were studied. This poor agricultural area had the main houses of the village, with the best pieces of farm land adjacent to them, distributed to a fixed set of families over the whole period. Through a keen marriage policy of these families nobody within the village was able to 'move up' to the centre of power or acquire land from the privileged families. Neither the French revolution, nor the industrialization of the area (which was rich of coal and saw steel companies moving in) altered this situation. Hence, throughout the ages, the local power remained in the hands of the same small set of families. It was only with the '60s of the past century that city people managed to buy a 'house in the country' from villagers. From that moment on the power relations within the village changed markedly. What this beautiful example shows is how distribution of land, with clear estimates of the value of parcels, of the importance of location and so on, is at work in a community, which is turned onto itself. It is obvious that children in the village bring this kind of knowledge to the mathematics classroom, more or less consciously and explicitly processed in their minds. Approximate measures, distances and comparative procedures about wealth and power attached to specific parcels and locations, will be known by every child in the village: it will be part of their out-of-school knowledge. But moreover, the relevance of this type of mathematically rich contexts need not be explained to the pupils, since the social system of power and esteem, and the constraints on each individual's own marriage and power ambitions guarantee that this sort of BK will be highly stimulating for otherwise possibly alienating mathematics teaching.

The former example also illustrates what Segaud means by '*transformer, reformuler, représenter*' (transforming, reformulating, representing): as was shown in the example of Bourdieu's description of the Kabylian house in central Algeria, space and measures, orientation and borders are not neutral or abstract in real life. They are invested with meanings of all sorts: gender distinctions, power relations, inheritance and religious meanings are attached to the mere spatial notions. That is what is meant by this last type of operation. In modern cities the actual or particular meanings may differ, but the same sort of transformations and reformulations obtain: the rich families live in the centre of town and mark their territory by fences or heavy doors and eventually barred windows. The better situated of today live in gated communities, with guards or cameras marking forbidding thresholds. The poor will travel for hours from their modest houses and sheds in order to reach the rich houses, where they earn a living as house personnel. In some cities of Latin America the rich have private railways or even private highways to connect their offices in the city high rise buildings with their gated community in the wealthy suburb where they live (Davis 2007). Again this sort of BK is well installed in the mind of the child coming from either of the family backgrounds mentioned. It is rich material that can readily be used in order to develop insightful and not alienating mathematics learning. Moreover, some authors have been travelling along this pedagogical road already and have shown how present this kind of foreknowledge is for the children, and how it can be used for insightful development of mathematical concepts and reasoning procedures (Frankenstein 1989; Mesquita et al. 2011). Of course, children from different social backgrounds will have different particular notions and competencies, but that in itself constitutes a good argument to use these in the process of mathematical development. It will clearly show all children how grounded or rooted notions are, and how abstractions can be powerful means to escape from initial constraints by mere power of mind. It may thus dawn on children that one's particular initial condition of life need not necessarily or fully be determining one's chances. However, when neglecting to take into account the diversity of worlds of experience of the children, chances are that the dominant values and notions will work in alienating ways on the children coming from a lower and less valued family. Indeed, the bourgeois or upper class view on the world will tend to dominate, thus elevating the threshold for the children of a lesser background.

Ethnographic literature offers a lot of useful material on designing. Turning to religious activities of the Navajo Indians in the USA, one is immediately struck by the amazing refinement of the hundreds of sandpaintings they have developed over the years. These larger ceremonial drawings (with a diameter of 3–6 m) represent cosmological figures, plants, animals, mountains and mythical agents. They are ordered in colourful constructions with a high degree of symmetry. Typically, the cardinal directions orient the sandpainting and allows to align figures and other phenomena according to the celestial course of the sun. The patient, who will be subjected to the healing ceremony, is put right in the centre of the sandpainting, where (s)he is induced to play the role of a mythical hero in his symbolic interaction with a variety of natural forces and figures. The signs and figures in the set of sandpaintings are numerous, amounting to several hundred distinct drawings (Reichard 1950; Witherspoon 1977). Ceremonies are meant to heal Navajo subjects. At the same time, they are unique occasions for hundreds of people to gather during the five to nine nights a ceremony will take: they are the major means of learning about the cultural tradition for the Navajo, who tend to live rather isolated from each other in family dwellings, which lie miles apart in a wide desert context (Pinxten 2010). Similar things can be found in the description and interpretation of mandala pictures in Tibet. It was the primary aim of Gold (1994) to focus on the similarity between Navajo and Tibetan ceremonial paintings and designs. With respect for the particularity of both traditions, it is striking how the ceremonial drawings in both have this same emphasis on symmetry, on cosmological figures and on cardinal directions, given the fact that both groups live in high mountain areas with vast horizons and a dominant role of sun and star sky. Moreover, both traditions use their ceremonial artefacts as instruments to learn about their worldview.

It is obvious for me that such material artefacts, as well as the learning procedures associated with them, would be recognized as valuable and can be actively used in mathematics education of the children from these traditions (Fig. 1).

The example of a Navajo sandpainting can illustrate some of the points I make. As can be seen here the whole field is divided in a symmetric way: the three big figures are surrounded on three sides by a border figure (a rainbow, protecting the ritual space). On the fourth side, the East, the rainbow is open. On that side two bats (animals with horns and hooks) guard the entrance to the ritual inner space. The central figure is accompanied in the north (black snake) and the south (blue snake) and in the northeast (black bird) and the southeast (blue bird) by ceremonial animals.

In sandpainting shown in Fig. 2, the design on the basis of cardinal directions and symmetry principles is even more striking.

In this case, the cardinal directions structure the ritual universe, which is depicted: the East is the channel of communication between the ritual space and the outside world (nature). Natural forces/beings are said to come in and go out again exclusively through the East. The three other directions are contained by the overarching rainbow figure.

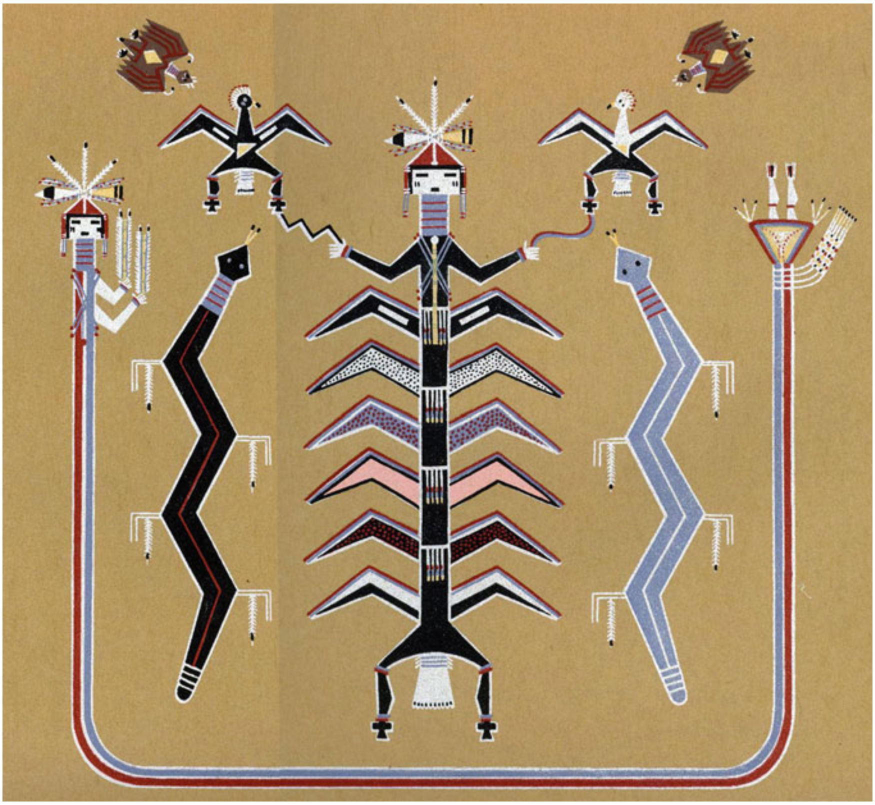

Fig. 1 Reichard (1939) (Color figure online)

- the centre of the ritual space is represented as the centre of the earth, indicated by the water hole through which 'all things came upon the earth' at the time of origin. A little ladder shows symbolically how the phenomena climbed out, so to speak.
- in each cardinal direction a sort of hunting figure stands out. Between each of them a particular plant (with the colour of the cardinal direction) divides the empty space.
- left and right (or north and south) of the East two dragon flies guard the communication channel in and out of the ritual space.
- the whole sandpainting is structured by a series of diameters, each structuring the ritual space in symmetric parts.

It is quite obvious that the mathematics in such sandpaintings, which is known by the child through watching the paintings being constructed under their very eyes (and eventually participating in the construction themselves), is a rich source for mathematics education. The knowledge in the structures of sandpaintings is part of the BK of the pupils.

Fig. 2 Reichard (1939)

5 Traditional Building

Jewish and Christian traditions have a variety of ceremonial buildings. The Temple and other mythic buildings have been studied to great lengths. In a recent analysis, based on some of the Dead Sea scrolls, Antonissen (in press) shows how five different mathematical concepts are combined in the descriptions of the structure of the New Jerusalem. Obviously, this is just one case of ancient architectural knowledge, and many others could be used in mathematics education (pyramids, Asian temples, Ancient Greek buildings and so on). I pick out the case of the city of Jerusalem because we have remarkable detail here in this newly discovered material of the Mediterranean tradition.

In the Dead Sea scrolls the construction of the old Jerusalem, and of the Great temple within it, is described in detail. Antonissen analysed the texts and found five mathematical concepts in the development of the grid plan of the city. They are combined in order to allow for the believer-builder to construct the city plan and the temple as the central nucleus within it. The concepts are:

- planimetry (as a part of geometry)
- the *pars pro toto* operation
- the creation of a grid by plane division

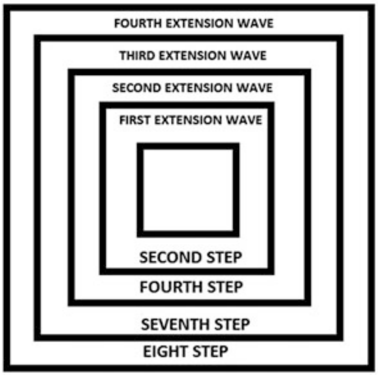

Fig. 3 After Antonissen, (in press, Fig. 16)

– the use of graphs
– the notion of similarity

The schematic representation of the overall grid plan that is reached looks like Fig. 3.

The first step is the construction of the inner square, representing the inner court of the main temple. The construction of the grid plan is an instance of planimetry, making use of the other four mathematical concepts. One conceptual move, which is repeated at least four times in the conceptualisation of the temple complex is that of 'radial expansion' thus instantiating the notion of similarity. That is to say, from this inner square a similar, but bigger square is designed which embeds the inner square in a regular way. One can 'do this expansion' by expanding each of the sides of the inner square along the two radial axes (Fig. 4).

This is repeated four times, reaching the outer and largest square in the end. However, this movement of expansion is not done in one step. Rather it is reached through a systematic construction of paths linking a particular corner with the radial axis and thus drawing a graph between the corner and the doorway in each of the sides of the square (Fig. 5).

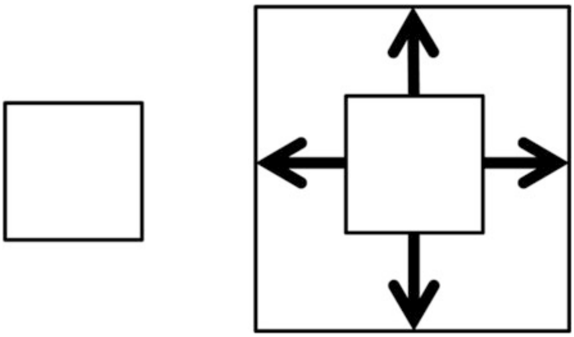

Fig. 4 After Antonissen, (idem, Fig. 1)

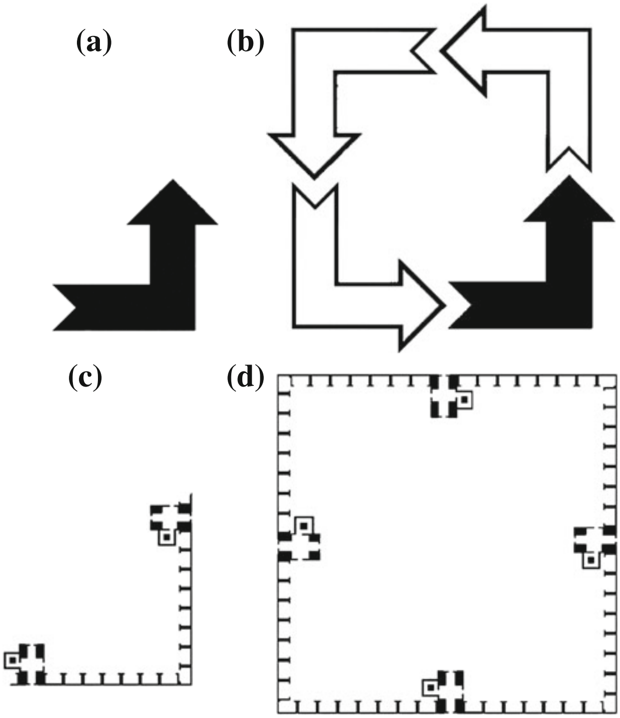

Fig. 5 **a** Partial presentation: partial information. **b** Partial presentation: figure to be constructed. **c** Partial ground plan (quadrant) of an *insula.* **d** Complete ground plan of an *insula.* After Antonissen, (in press)

Thus, the consecutive *pars pro toto* steps of reasoning (each corner and two straight lines attached to it) define the square, first for the inner court and then in the expanded squares. In the process the grid is constructed by subdividing the plane in a very systematic way.

At the level of the city, a similar way of division of the plane is seen, designing a grid for the whole city, which reminds one of the geometric pattern of modern cities (like Manhattan, for example). These are in contrast with the organically structured old cities of Europe, where straight lines and exact geometric figures are rare. On the contrary, bends and irregular adaptations of roads and buildings along the way are the rule.

Although this particular finding in the Dead Sea scrolls may be the exception, since most cities in ancient times have indeed been growing rather organically on rough or uneven surfaces, the mental processes shown here may not have been as rare as might be expected. In a very recent study on the Mystic Lamb by Van Eyck one knowledgeable scholar (Schmidt 2006) pointed out that the central panel shows the same radial expansion of the central geometric figure in the painting. Indeed, the central panel of the Mystic Lamb shows the structure of the Christian cosmos, as it was believed to be laid out in the Gospels: a vertical axis runs from the central octagonal fountain at the bottom, the self-sacrificing lamb in the centre and the Holy Spirit near the upper edge of the painting. They are situated exactly on one straight vertical line dividing the cosmos in two halves in the panel. The left halve shows Old Testament figures, and the Apostles, with John the Baptist closest to the fountain. The right halve of the painting shows the Church Fathers and major clergy, as well as important churches of the time. The lore seems to be that through baptism (in the water of the fountain, which has a little outlet pouring towards the believers in the church, watching the painting) one has to move from the old and doomed world towards the new era, going from left to right. The main structure is this double radial structure of vertical and horizontal orders. However, Schmidt discovered that the octagonal structure of the fountain structures the division of the space of the whole panel through radial expansion of the octagon: the many figures on the left and the right side, the angels defining a spatial boundary between human beings and the rest of nature, and so on are precisely situated on the outer expanded octagons. So, a similar sort of playful use of mathematical concepts (as in the case of the city and temple grid from the Dead Sea scrolls) may have been on Van Eyck's mind when he was representing the spatial logic in the Christian cosmos. I leave the question open whether and to what extent such mathematical concepts were consciously and purposefully used in ancient times. It may be that the contemporary interpretation 'reads mathematics into the ancient texts'; nevertheless, the systematic use of mathematical concepts and terms in Ezekiel, an Old Testament author of the Babylonian time, seems to allow present-day scholars to safely assume that more mathematical knowledge was around than most historians have taken for granted.[2]

It is clear to me that this type of making plans of building and cities will be enthusiastically used by children of a rather young age (maybe 11–15). It offers great opportunities to see mathematical concepts at work, to understand the function of graphic representation, of similarity and of radial structures. On the other hand, using such concepts in a variety of cases, designed as projects by different groups of children on different concrete cases, will certainly induce a better understanding on what generalisation means.

6 Archaeological Digging

In many cultures old sites or remnants of ancestors are around. In many cases, as we came to know in archaeology and anthropology, such places are treated with great respect by the inhabitants. The scholars working on them have often been accused of stealing or disrespectfully disturbing the sites. A former student and now friend of mine, a young woman from Cherokee descent expressed this double attitude

[2]I thank my friend and Christian theologian with Old Testament roots, Peter Schmidt, for this remark.

towards members of our disciplines by opening our first meeting with the words: 'My kind of people do not like your kind of people.' Subsequently, she explained that the distrust vis-à-vis archaeologists and anthropologists primarily circled around experiences of disrespect, which had led to 'stealing' or removing artefacts and knowledge from the cultures under study. Meanwhile, a substantial ethical code has been passed in the professional associations, but the feeling expressed by her has historical roots. The reaction shows something else, as well, and that is of interest to those concerned with mathematical education.

Often some secret or restricted knowledge about the sites will be found. That is to say, most people (including children) might know about the objects or remains on the site, through occasional festivities they have experienced. However, the more advanced and the symbolic meaning may not be shared by them. However, the 'lay' experiences matter here, since children will be interested in the objects and in the sites through their experience. I will describe very shortly a few anthropological cases.

In the Dogon culture of Mali, West Africa, long carved masks are kept in special caves. These Great Masks (Dieterlen 1971) are carved from full-grown trees and kept for centuries in a common cave, even past their physical deterioration. Every 60 years a particular ceremony of total rebirth is performed, involving all members of the Dogon people. For that occasion a new Great Mask is carved and is carried around through all the villages of the Dogon region in a cycle that takes eight years. Each of the years another village is visited by the Great Mask. The event is accompanied by weeks of dancing and feasting. The rebirth ritual is timed on the basis of a celestial cycle (referring to the circling of a small star around Sirius, every 60-odd years). Literally everybody—including foetuses of pregnant women—is 'reborn', which is marked by the symbolic confrontation with the Great Mask. The initiated old men will know every detail about the Great mask and how to produce and move it. But the rest of the people possesses mathematically relevant lay knowledge about it, which is useful for education:

- the Great mask is the length of a grown tree, which in itself is remarkable in this desert area. In order to move it around on the cliffs of the Dogon and have it 'dance', particular spatial calculations need to be made. One can use this case and measure the Great mask, make a trajectory and maybe a map for its journey, devise supports on which to lay it out in the village, and so on.
- the Great Mask is carved and starts its journey when the rebirth of reality is about to happen: this is a cycle of 60-odd years, and its calculation is based on star lore about Sirius, one of the brightest stars in the sky. Obviously, knowledge about the stars, about their position, about the smallest star circling Sirius, etc. is a rich reservoir for mathematical exploration. The regularity of the event,—every generation of the Dogon people—, is a step up for generalisation.
- the ceremonial trajectory through Dogon country, with visits of all the villages over a couple of years, is again relevant for mathematics.

To cite just one more example, out of a list which looks endless, I turn towards a classical site: the Anasazi Kiva ruins in the Southwest of the United States. There has been a number of archaeologists working in that area. An in-depth study by Hardaker (1998) focuses on the geometric knowledge in kiva construction of past generations. In general, the Anasazi culture is believed to be extinct since the 12th or 13th century C.A. Anyone who has been in or around a kiva has been struck by the geometry of it: the basic structure is a circular form, with an entrance (most of the time) from the centre of the roof. Of course, the circle can be drawn or laid out or designed manually in another way (reminding me of Dingler's interpretation of Euclid—Dingler 1933). Nevertheless, because of the almost perfect circles we find with kivas in the Southwest, people started asking questions about hidden or lost or secretly transferred mathematical knowledge. Hardaker (1998) sums up what can simply never be known about such issues:

- whether or not the Anasazi knew about rational or irrational numbers,
- whether they used a decimal number system,
- whether they had a concept of degrees, or a notion of zero,
- whether they were able to translate proportions in exact numbers,
- whether they had an analogue of Pythagoras (as is claimed for African traditions, see Gerdes 2011, 2014),

These are all questions we will never be able to answer, Hardaker claims. However, this is of minor relevance, when looked at from the point of the mathematics educator. The division of a circle in quarters and in six equal segments provides the agent with a 'replicable framework…grounded by the physical operation of its own exercise' (Hardaker 1998: 20).

Of course, exactly this is what any anthropologist can witness till this day, when a contemporary Navajo living in the same region, builds his hooghan (or local traditional dwelling). And it is what a comparative study on 300 Native American sites concluded years back when examining Hopewell 'numerous large enclosures of earth and stone, some in the form of circles, squares, and octagons' (Marshall 1987: 36). The conclusion was that some sort of measuring must have been grasped, using a sort of 'unit of measure and considerable knowledge of land and geometry' (idem: 36). But because of the lack of written sources on these questions, the only way the investigator could work is by physically doing the work himself, thus using the knowledge and making the 'errors made by white American land surveyors and …carpenters, masons and bricklayers…' (idem: 37).

My suggestion in mentioning these studies is that this is exactly what the mathematics educator should start with: appeal to and use the FK in children, which is comparable to that of artisans and lay people. Make the mathematics in the actions and in the products explicit and invite the children to move from their particular and local knowledge to generalizations: they know and are conscious of the implicit knowledge, now made explicit, and can hence compare their notions with those of other laymen, of ancestors and of people from other traditions. When that consciousness is explicitly elaborated on, then the way to generalisations is safely paved in the minds of the children.

The curriculum item that can be elaborated here will most probably fascinate children of primary schools: how can we build a settlement, starting from the remains one finds in a particular ruin?

The mathematically relevant actions are numerous: how can we choose a good spot? It has to be more or less flat and levelled. How much surface do we need? Depending on how many people we want to give a dwelling place.

How are we going to provide water for the settlement? Eventually, an irrigation canal must be built. And how about storage room, depending on the amount of mouths to be fed? Suppose we want to bury the dead: do we provide room in the neighbourhood? Or do we bury them in the houses of the settlement (as is suggested by some studies of the Mesa Verde and the Chaco Canyon ruins)?

What would be a good place for ceremonial buildings: Hopi use the kiva, as well as the market place in the centre of the village, for ceremonial purpose, as I witnessed myself. But Mesa Verde ruins in Colorado, USA, as well as many of the Ancient Greek excavations suggest that temples have their own place, somewhat removed from and quite distinct from the houses. Whatever will be the choice: will sacred places or buildings be different from dwellings for families, and how can one define these differences? E.g., the orientation of sacred building according the cardinal dimensions is a very common feature (Pinxten et al. 1983). Saying that implies, obviously, that more or less systematic knowledge of the cardinal directions should be built up by children who want to provide for a sacred place in their design of a site. It is obvious that projects on 'archaeological sites' offer tremendous opportunities for mathematics teaching and learning. Moreover, all sorts of skills will be put to work in such a project, allowing for many children to bring in their particular competences in the joint project.

7 Music

Dance is agreed upon by many to be an interesting domain of human activities with mathematical potential (see Sect. 8, below). But what can be done with music in general? Can music be seen as a realm of action, and hence of mathematically relevant or potent activity? Intuitively, one may agree easily: a lot of mathematicians and pure theoretical scientists proved to be fairly good musicians as well. Or, Bach's work sometimes sounds like an algorithm in tones. But who is researching this topic explicitly?

To begin with, it is important to try and look at music in terms of activity, rather than as a domain of the mind. The research group around M. Leman at the IPEM (Institute for Psychoacoustics and Electronic Music) at Ghent University in Belgium has a worldwide reputation of doing just that. The researchers of this institute claim that music works, and has impact on people, precisely because it is first and foremost embodied (Lesaffre and Leman 2013). To put it bluntly: they focus on the body rather than the mind, and on the performance aspects rather than on listening (after Leman 2013: 18). They develop this approach in a series of steps:

- complex musical interactions (e.g., a piece played by a performer for an audience) are decomposed ‘into the mechanisms and information streams that form the basis of these interactions’ (idem: 19).
- the body is the primary mediator: both listeners and performers follow or set the rhythm by body movements, registered in a systematic way by the researchers;
- the action repertoire can be identified for a particular piece of music. That is: ‘the set of action commands and their perceived outcomes that is somehow kept in memory and used for the next action and the interpretation of the next perceptions’ (idem: 22). To give another example: when a Navajo medicine man performs a certain song during a ceremony, (s)he will automatically follow sequences of four musical units, four lines of a song also. After that a new set is started. At a higher level groups of four times four lines/sequences can be identified in the progression of the ceremony, and so on. This regularity is embodied in such a way that is becomes automatic, not consciously chosen.
- gestures are the next constitutive element: they are ‘spatial-temporal patterns that move and that may carry expressive information’ (idem: 22). They can be seen both as body movements (as in anthropology) and as musical gestures in the sound.
- entrainment refers to the way ‘resonant systems adapt their resonance rate to each other so that they sweep each other along in their flow’ (idem: 23). In performances this can be seen in the way the sounds of instruments (including the voice) are made to resonate in unison, so that the desired right timing and harmony obtains. However, the researchers looked at the way the body rhythm and the timing of the instruments are brought into a unified mode of beats and times. Again, the activity aspect is central,
- finally, the perception and the action modes are interacting in music: hearing, moving along, incorporating the sound and the rhythm are important aspects of what is called ‘listening to music’. But the same is happening in the musician. And the incorporated and acted-out rhythms, tones and modes of producing and perceiving the sounds mirror each other through the body actions. Moreover, musicians (especially in jazz and pop music) are known to talk about the way they keep to a rhythm by ‘delegating’ the task to their foot or to little nods of the head. That is to say to purely bodily motions.[3]

What is the relevance of all this?

This type of research indicates that counting, organizing regular intervals in sound series in action and in perception, are first and foremost a type of subliminal action, which is known to subjects in a particular tradition or culture. The reference to the mirroring is significant here: listener and performer mirror each other’s almost subconscious spatio-temporal sequencing. As pointed out by Leman (2013) this speaks more about ‘doing’ mathematical spacing, repetition, and so on, than about thinking, let alone textual learning. You have to ‘feel the mathematics’ in your body, rather than reason about it in an abstract way, laid down in a written format.

[3] I am grateful to Marc Leman for this observation.

If this approach holds, then music becomes even more promising for mathematics education than one should have expected: indeed, it might be a more embodied and hence trans-cultural vehicle for learning mathematics. The implicit mathematics in the bodily expressed rhythms, the moving along physically with high and low tones (even in a slight way), and so on should be made explicit and used as a means for exploration of formal concepts and relations. It might prove that music is thus a nonverbal realm, close to body experiences, which is much less hampered by cultural and institutional particularism than explicit notions and forms in any particular language. Obviously, music playing, even genuine performances, with children from a variety of cultural backgrounds may be a promising avenue for entering an open curriculum on mathematics.

8 Dancing

Dance is, obviously a corporeal activity, embodied in the real sense of the word. But it is much more than that. It is a way of expressing the conscious and the unconscious experience of reality. Along that dimension, dance could be seen as a form of knowledge as well. In numerous rituals around the world this may be happening indeed: in the ceremonial movements, in the dances, knowledge about the world is expressed, and transferred to the participants of the event. Most certainly, rituals are ways of trans-generational transmission of knowledge, with direct impact on nature and on the participants alike. In this I follow Bourdieu's view on ritual behaviour (Bourdieu 1981; Pinxten 1991): social praxis is the core of human interaction, and it is nearly always a way of learning about and impacting on reality. Ritual dances will be known to a larger or smaller extent to children, who participate in them or watch them in their own tradition. But, obviously, not all that is to be seen in occasional performances will be understood or participated in by children. That makes the use of such ritual dance material in classrooms far from easy.

Still, dance in itself is a great source for learning mathematics through action, I claim. Any example from ballet may illustrate this point. The example I picked is from contemporary dance, which is likely to be more accessible to young people than classical ballet. The choreographer Anne Theresa De Keersmaeker is world famous with her group Rosas. This is the kind of ballet that looks more like a sports event, with group action by dancers who run, jump and slide a lot, rather than show the sweet, harmonious movements of a star and her surrounding choir-dancers in classical ballet. Lately, this dancer and choreographer published books and DVDs on her own work (De Keersmaeker and Cvejic 2013, 2014). I will draw on this work to illustrate how and why dance has mathematical potential.

With each and every production De Keersmaeker starts drawing or painting a series of geometric figures on the floor of the *bühne*. These drawings define as it were a little cosmos for the dancers: within this cosmos the movements will happen. In a typical combination of traditional ballet and contemporary dance, this cosmos allows for the creation of a spatial universe in the eyes of the audience, who does not see the figures on the floor. In traditional ballet dancers seem to try and detach themselves from the earth and fly as high as possible: the male dancer then often seems to act as a support for the ballerina, who is reaching repeatedly for the sky, for something higher than the earth. In striking contrast African dancers, also the women, are firmly moving on the earth: they seem to explore or to occupy the earth most of the time, and hardly ever fly into the air like a bird or a butterfly. In De Keersmaeker's ballet an intriguing combination of both is happening.

But, obviously, this 'cosmological' signifying is not the primary concern of the choreographer. In one of the books referred to (De Keersmaeker and Cvejic 2013) it is stated that a basic feature of her work is a certain type of formalism: 'The composition of space, time, music, bodily movement, and arrangement of bodies in movement reveals the aesthetic principle of form in a double sense.' (idem: 5). One sense is what I called the 'cosmological' element: 'It is the *geometrical* form that designs the space: rectangles, circles, and spirals; parallel, perpendicular, or diagonal lines; pentagons or five-pointed stars in various combinations.' (idem: 9). It is clear that for the dancers all of these geometric notions are sketched on the floor. On the other hand, in a three-dimensional performance these geometric notions structure the danced movements, which the audience can see. This 'mapping' of surface geometry into the three-dimensional universe is in itself an interesting case for mathematics leaning.

The second sense is expressed as follows: 'The *arithmetic* form configures the order of appearance and arrangement of the dancers in space following the Fibonacci sequence (solos, duets,…). The body yields its own *architecture in forms and proportions…*' (idem: 9). That is to say, De Keersmaeker conceives of the dancing platform as a cube where the dancers move in accordance with the proportions of Leonardo's *Vitruvian man*: the centre of the human body is the main focus, and the movements should always respect that central position in the movements of the dancers. This entails that dancers can only detach themselves so much from the floor, or move so much in any direction on the scene as is allowed according to Leonardo's principles: the head is at the most 1/7 of the total length of a person, the centre of the body structures the standing figure in such a way that a circle might be constructed around the extended hands and legs of the human body, stretched in all cardinal directions. When this proportional system is mapped onto the 'cube' of the dancing space, Leonardo's body structure becomes the basis of a three-dimensional universe of movement, which limits and structures the possible dance movements of the performers.

Both principles or senses can be used as basic insights by learners of mathematics. The enormous advantage of the dance medium is that children can make the movements, draw the designs on the floor and explore through actual bodily actions and reactions the mathematical notions involved. In a second time the understanding and the insights can be made explicit in group discussion, aided by actual drawing. The fact that De Keersmaeker (in the tradition of many other choreographers, like M. Béjart, Nijinski and others) explicitly refers to sacred geometry and sacred space of other traditions (idem: 10) can only help to use dance in a variety of cultural groups.

In the latest book of De Keersmaeker's trilogy (De Keersmaeker and Cvejiic 2014) she discusses the choreography she developed for Steve Reich's "*Drumming*". She started out with the basic phrase of the music, and translated this in the rigorous geometric organization of the space of movement for each of the dancers: 'The basic phrase traces a trajectory of a spiral unfolding in eight squares whose sizes result from dividing one rectangle into golden section proportions. This means that each square has different dimensions according to the Fibonacci sequence of progression. The center of the spiral is what I refer to as the "house" of the dancer, and each dancer has his or her house.' (De Keersmaeker and Cvejic 2014: 20). The floor where the dancing will be happening is filled with the geometric figures for each dancer, fitting within the general structure of the unfolding spiral. Dancing is then performed within these painted boundaries by each dancer, and the speed or rhythm of one dancer will be adapted in such a way that (s)he meet a second (slower or faster) dancer at the end of the musical phrase in the common geometric point of the great spiral.

What is mathematical about dance? I can point to registers of FK children will carry with them from their lay acquaintance with dance:

– rhythm: the bodily movements in themselves guide, hold and express knowledge about rhythm (Leman 2013),
– space: dance is necessarily a sort of activity that explores and structures space. Since dance is learned by seeing and doing (imitating), it is a means to learn about space and about movement through space. In a highly sophisticated way De Keersmaeker's geometric expression of space through dancing/bodily movements is just that. But of course, one need not use 'academic' geometry in other types of dance. What is happening all the time, though, is that space and spatial relations are explored and learned through dance,
– distances, notions of proportion, of nearness and of overlapping are most certainly used and learned in dance,
– direction, and situating oneself in an encompassing environment (eventually cosmos) is an obvious skill for dancing,
– counting is most of the time part of dance: so many steps, so many beats of the drum, etc. structure time and space through dance,
– finally, designing and planning are intrinsic dimensions in any dance: in order to qualify as dance instead of simply moving the body, movements must be planned in advance, often in some sort of symmetry or convergence with other moving bodies.

Many films about gangs and street children focus on these elements (starting with *West Side Story* as an easily accessible example), implying that the recognition of the knowledge embedded in the children's dances is the stepping stone to the further educational approach to them. In a way, the explicit use of what became known as 'street mathematics' in street youth of Brazilian cities shows a parallel reasoning: poor performers in the school context show remarkable mathematical skills in their street context. The recognition of the latter will be experienced by the children as a positive appreciation of one's worth and will hence open the door for them towards other educational topics and programs (Mesquita et al. 2011).

9 Computer Design

Ron Eglash has done substantial research on technology and its impact on cognitive development (Eglash et al. 2004). His focus is not only computer technology, but I single this out in the present book because computer skills are quite obviously becoming ever more important in the ICT world we are living.

It is good to refer to Raju's critical analysis of 'Math Wars' again. He wants to decolonize the status of mathematics. In his book on this issue (Raju 2007) he claims that mathematics was 'sanitized' in the Christian West in two 'Math Wars' (respectively around 1000 and around 1600 CE) which is shown in the 'complete elimination of the empirical from mathematics, as in the current notion of mathematical proof…' (Raju: 413–14) . In the present era the third 'Math War' is raging, triggered by the rise of ICT. It is obvious that computers are turned on empirical, practical knowledge first and foremost. Like engineering the ICT branch is not particularly interested in purely theoretical proofs, but focuses on usefulness and relevance primarily. It is obvious that the impact on life and survival of the engineering frame is ominous. This in itself may persuade scientists and educators to move away from the so-called anti-empirical and proof-dominated approach of the past centuries.

The computer revolution of the past decades has made the PC available to many households and to schools. Along with that the computer-assisted education mode has been introduced, and a host of programs became available for design, for calculations and for map drawing. Without doubt the implications for mathematics education are vast, and have not yet been fully explored. Finally, Internet and the web offer a world of information on ideas, products and projects, all of which potentially revolutionize mathematics education. It is on these issues, combined with the perspective on the world as a complex of many cultural traditions, that Eglash has been working. For years now he runs the website 'Culturally situated design tools: teaching math and computing through culture' (http://csdt.rpi.edu/). The website publishes research result, but also curriculum material from a variety of cultures: from African fractals (in architectural designs, in basketry, and in dreadlocks) over graffiti graphs in the West, to Native American arts. In the latter category the pyramids of pre-Columbian South America figure prominently, but also star lore, basket weaving and rug weaving in North America offer a lot of algebraic and geometric examples of mathematical reasoning in non-western cultures.

What Eglash demonstrates abundantly is that each and every culture offers designs and craft products within which algebraic, geometric and/or counting traditions are apparent. In a first move, the mathematician recognizes the implicit mathematics in the cultural products. He then makes the latter explicit and formulates in technical terms what is done by the local cultural group. This conscious and explicit knowledge can then be used in the classroom, showing how what is known in the making and (re)producing can become a stepping stone to mathematics proper (or rather AM) in the school setting. Computer techniques of design can help along the road. For example, Coyote is a mythological hero in many Native American cultures. He is known as the one animal/natural force who seems to create ‘an “irregular” complexity’ (Eglash 2002: 3). All other animals in the myth are carefully picking out elements, stars, plants and placing them in an orderly way in nature: e.g., the bear has a fixed constellation of stars put in the sky, and arranges for bees and flowers on the earth. The coyote is nervous and grabs the sack of stars from the paws of the bear, throwing the contents in one big swing towards the sky. What comes out is a randomly ordered mass of stars known as the Milky Way. Eglash then goes on to say that irregularity and randomness are important notions in modern western mathematics as well: ‘a foundational concept for certain measures of complexity used in modern mathematics.’ (Eglash: idem).

Although I see the relevance of this kind of approach, I feel one should go beyond the a prioris of this outlook. That is to say, the perspective is primarily one of translation of non-western material into the western frame of mind, and then concluding: ‘see, you have the same as we have’. The basic political reasoning is one of justification or recognition of sameness on the basis of translation. However, such a translation is, even in the best of cases, a reduction and hence a betrayal. That is to say: while translating one necessarily selects and orders the contents by means of the criteria of one’s own cultural constraints and perspectives. In other words: the richness in the difference is reduced or erased in order to safeguard a universal truth, which happens to be the western insight (referring to Raju’s criticism once more, Raju 2007). Neither the relevance or usefulness of the non-western notion, nor its embedded nature for the tradition it stems from, nor its possible networks of associations are respected. All these aspects are stripped away by the act of translation, making the native knowledge most of all an instance of implicit and more or less concrete or contextual use of notions only westerners recognize in their full potential of abstractions. I grant that this position can be defended, and that it might be a powerful point of view in the globalized world we are living in. But my point is that it carries with it a lot of tacit humiliation when used in an educational context. Thus, in order to proceed along this road, I suggest it is of great importance to be very conscious about the implications of any particular translation and to explain the process and its consequences throughout the pedagogical process.

10 Storytelling

It is very well known that most of the learning formats in the school perspective on education take for granted that learning happens in the head (mainly) and can be understood most profitably in terms of abilities, skills and the like to solve problems. Implied in such a perspective is that texts become central in the transfer of knowledge. Experience with other cultures teaches us that storytelling has an extremely central place in the socialization of children, and even of adults. Storytelling is intrinsically different from textual instruction. In fact, instruction rarely happens in oral cultures, let alone that texts or other 'fixed' messages would be all important in the educational processes.

In the western tradition of learning or preparing for adult life through schooling it took a very long time, and many psychology 'wars' (e.g., between behaviourists and cognitivists) until a major figure in the field, Jerome Bruner, turned around and started telling psychologists and pedagogues as the 'professionals' on learning, that storytelling is a firmly and tragically underrated way of learning in the school format. Indeed, socialization happens first and foremost through storytelling, with all the contextual, random and audience-dependent aspects attached to it. There is no textual way one can choose to make a human being part of a group, or of society at large. The only way is to learn by doing things together, and construct the procedures and the contents of what is learned along the way and in collaboration with others. And that is precisely where storytelling differs from textual instruction (Bruner 2004). I learned from anthropological fieldwork how storytelling is quite different from textual instruction in a number of ways:

- the storyteller adapts continuously to the audience (s)he is addressing, and invites the hearers to repeat, offer alternative versions or otherwise build on the story (s)he is telling. It is definitely an interactive event.
- any particular story will vary according to the qualities of the teller (who dresses, intones, selects themes etc., for the particular audience) and of the listeners. For example, coyote stories will be good for children and foreigners with insufficient knowledge about the culture or the religion in the perception of the Navajo. I personally had to prove how knowledgeable I was with particular informants to get access to more delicate, more sophisticated and more important stories;
- a story is not textual in yet another way: there is no strict canon, let alone an unchangeable text as in the written tradition of the Mediterranean area. Instead, depending on the mood and the dramatic qualities of the storyteller, on the context, on the qualities of the audience, and on the occasion a story will treat different themes, and eventually have other outcomes. It is this aspect that became clear to Hymes (1981), when his informant grew tired of his 'textual prejudice' in checking on her 'version' of a story, asking for the relationship with presumed 'mother' versions, and so on. She exclaimed: 'In vain I tried to tell you', stressing that orthodoxy, textuality, even the binding status of presumably invariable tradition were all western projections, which came from the (holy) tradition of the book religions, but were absolutely absent in the storytelling practice and concept of knowledge transfer.
- the school curricula owe a lot to the religious tradition of the West: textuality is omnipresent, and there is believed to be a mother version or orthodox truth which could and should be transferred from generation to generation by means of the 'right texts'. The debates between schools in the psychology of learning

> then focused on the procedures to transfer the orthodoxy by means of texts, but were blind to the local and highly particular nature of the presumed indubitable principles of the learning procedures. It is against this background that Bruner's courageous 'conversion' to storytelling should be appreciated.

The mathematics teacher M.S. Schiro is the first one, to my knowledge, who took up this challenge and actively explored how oral storytelling can be used for teaching mathematics. In his book Schiro (2004) explains how he came to meet Doris Lawson, a fourth grade teacher who loves to tell stories in her classroom. Together they developed a series of stories for mathematics classes. The general trajectory is as follows: the teacher starts telling a story, which can take any theme as its subject matter. All of a sudden a problem arises in the course of the story: for example, the hero has to estimate a distance to reach the goal of his journey. Or a wall has to be constructed in order to keep potential enemies out: how many bricks do we need? What amount of glue will be used? How can we measure height and width of the wall, and then come to know the surface? And where do we go from this measure to determine the amount of bricks that will go into the wall? Or which is the shortest and hence safest way for an animal to circumvent a farm where the peasant is also a hunter, in order to reach the forest behind the farm?

The amount of stories is endless. In Schiro the teacher chooses regularly to start a story and then leave the plots largely to the students. The complications added trigger the students even more to look into mathematical problems, and to look up and learn the mathematical notions and procedures needed to solve the problems they encounter in the story, they write themselves. Without question, the motivation in these mathematics classes is very high, and the 'naturalness' of the use and exploration of mathematical knowledge is a great benefit here. Quite obviously, children do not go for the easiest and least interesting plots, but enjoy complicating their world of fantasy. This makes the story telling line such a rich and fulfilling avenue for mathematics learning.

I will restrict from giving or developing any other examples of stories. It suffices to say that the enormous resources of myths and legends are a real treasure to start with.

11 Exchange and Market Activities

Finally, I want to draw attention to the importance of incentives for using mathematics and mathematical notions and procedures in different cultures. When the *quipu* became known (e.g., Ascher and Ascher 1997), they told me the story of how people in that Latin American area wanted to keep track of buying and selling, of dues and revenues. When Michael Cole and his collaborators wanted to introduce Modern Mathematics with the Kpelle of Liberia, who had no schooling experience until that time, they first concluded that the Kpelle were without any mathematical knowledge. Until Cole started doubting this 'finding': indeed, illiterate tailors were able to make a costume that fitted perfectly, thus showing that tremendous knowledge of proportions, but also of accounting were around for whom was able to spot them (Gay and Cole 1967; Cole et al. 1971). Any visitor to the rural areas of China, Africa or Latin America will tell you that markets flourish, and that people know how to make deals, keep track of their earnings and savings and puzzle the visitor with their mathematical skills, although they hardly know how to read and write. Lately, the very idea of 'street mathematics' of the Favella kids in Brazil points to a similar mathematical knowledge: unschooled children are very able to strike deals, make the calculations they need for their business and at the same time keep track of the distance to the nearest escape street -should the police show up. When put in school, they fail miserably, since they seem not to possess any mathematical skills, the way they are required there (Mesquita et al. 2011). Nevertheless, in the business context of their natural survival context they do show the latter skills.

The point I want to make here is that market activities, selling and buying, comparing prices and profits and so on, seem to be beneficial in promoting and developing mathematical skills. Hence, making use of such activities in mathematics education is obvious to me: they are part and parcel of the FK and BK of the children who come from different socio-economic contexts, and promoting such activities may benefit all children in the development of mathematical knowledge. Of course, the practical knowledge instances I will come up with in the following paragraphs do not in themselves qualify as mathematical activities. However, they offer mathematically rich material, which can and will have to be made explicit, drawn into comparisons and brought to more generalized statements on quantification, on measurement, on scale, and so on in order to serve in the mathematics educational settings. But, as ever, it is important to use these instances because they offer the concrete and rich contexts and activities, which can serve as the base for the insightful learning of generalized, more abstract notions in the minds of the children.

I will explore some examples a bit more here.

Along the coast of the North Sea, from France over Belgium to the Netherlands, it is common practice for kids on a beach holiday to start making paper flowers, build a small wall of sand to serve as a counter, and open shop on the beaches. Children (and adults) would stroll along the beach and visit the shops occasionally in order to buy flowers. Quality, size, colours are compared and a price is negotiated for each particular flower that one wants to purchase. Whatever 'money' to be used is carried along by the buyer in a small bucket: the collection of the best shells one could find on the beach. Different shells have different value: rare ones are worth more than common ones, pure white ones may have the highest value, and damaged ones would only qualify as copper money. The negotiation is double then: both the value of the currency is discussed, and the exchange value of the flower being purchased. Everybody trades with everybody else; after a while the shop is closed or abandoned, and children will engage in another play. However, the discussions on value, on fair exchanges, on the equation between high quality and high price are most certainly full of mathematics, and the FK of the children is what matters most in these transactions.

A quite different example stems from Native Americans. It is common in the Southwest of the USA to see old men or women walk on the road and eventually hitchhike to reach a distant place. Quite often these old people are carrying their most precious belongings with them: they are wearing expensive necklaces, rings or earrings for example. When the silver they are carrying around is of high quality, the value of the jewels can easily equal that of a car, as I was told repeatedly. They get picked up and driven near the place they want to go without ever being touched, let alone robbed of their belongings. Yet, everybody knows they walk around like a bank with the doors of the vault wide open, to venture a daring comparison. It often happened that native collaborators, and even children, would indicate to me that a particular necklace was meant to pay for a ceremony the old person had ordered. For example, (s)he would pay the flock of 50 or more sheep with the necklace. Alternatively, a woman came up to me and offered to trade a rug she just finished. After negotiation we agreed on a price, partly in food. Afterwards I heard that this excellent weaver traded a few years earlier a rug twice this size, of higher quality, in Santa Fe for a new pickup truck. In the deal with me, she was on the reservation and knew that her rug was not as nice as the ones she could sell at Santa Fe, so she settled for less. At the same time, she could not afford to go to the city outside of the reservation because she needed money urgently. If she would have been able to go to the city, she told me, the price would easily be ten times more, because she was the one who made the rug and therefore it was authentic (and not a Mexican rug). This information was confirmed to me later by the owner of a local trading post, who is a well-known collector of rugs. Again, the market relationships taught people to make estimates, determine the value of things and negotiate about the price. Obviously, a lot of mathematical activity is involved here: the size of the rug must be known in some detail, the difficulty of the design matters, and the location or context co-defines the market conditions.

Finally, an example of a new local practice, almost a hype in my part of the world. Since the advent of consumer society people in the West have been gathering goods, cloths, household appliances and other material cultural things in enormous amounts. With the gradual decline of wealth since the latest economic crisis (the bank crisis of 2008) a phenomenon of second hand markets started booming. From April till the end of October every quarter or neighbourhood in France, Belgium or Germany has garage sales and second hand markets. Mothers and fathers, but also young children from the age of six or seven on, take to the street or to a local park and set up shop. They offer the cloths they have grown out of, the toys they do not use anymore, the electric household equipment they decide to dump, and even computer and game boards. The turnout is massive, and flocks of people come round to look at the material that is on offer by their neighbour or fellow citizen. People will bring along some snacks and drinks, and organize a little social gathering on the side. Over the year millions of citizens are participating in this exchange system: goods are traded for very small amounts of money, which is then often spent to buy things a few stands down the road. Enormous amounts of children cloths are traded this way: stacks of baby material change hands, and even adults will purchase dresses, shirts or shoes at ridiculously low prices. Nevertheless, calculations are important. I was told by several friends that some of the markets offered better quality than others. Also, it is common to negotiate about the price one is willing to give, in contrast to the condition in a regular shop. Discussing about the price indicated is never done in a warehouse or shop in Belgium. In the second hand market these negotiations are common, and children learn the tricks of the market swiftly and are allowed to make some extra pocket money by trading there. For certain types of goods the impact of the tens of second hand markets in any city is substantial: shoes and cloths of children are expensive, and children hardly ever wear them until they are genuinely worn out. They simply grow out of them. The markets offer a tremendous amount of this material at very low cost, in fact at a symbolic price (because of the amount of the supply). I have very regularly seen mothers inspect and negotiate about children's goods, comparing and calculating with great competence. Chances are that this type of 'outlet economy' makes a serious difference in the household of an average citizen by now. In fact, the government tolerates this substantial exchange system, although it totally escapes the taxman and can be seen as a type of 'black market economy'.

A research group of political economists has been studying the impact of this and similar sorts of informal exchanges of goods and services for years now. The international, European-funded research group of 'social innovation' (e.g., Klein and Harrison 2007) does longitudinal studies on informal economy in different parts of the world. Among other things they started counting what parts of the budget of well to do families in the East Coast USA suburbs is in fact outside of any official or regular markets: neighbours helping each other with food, exchanges services, doing repairs in the house for one another, and so on. According to the researchers up to 42 % of the total budget of these white middle class groups are informal, unofficial deals and trades. Obviously, here again, people estimate the value of a good or a service, negotiate about compensations, compare exchange values and keep some informal balance. In a sense, the *quipu* sort of reasoning and accounting can be thought of in such cases.

From the perspective of mathematics education it is obvious that the curriculum can easily make room for projects with potential here. Any primary level textbook today will offer some instance of exercises on shopping, or on household tasks.

A possible item could be to design a project 'start a shop': what goods do we need? What prices should we set? What is reasonable profit? How do we keep track of selling and buying activities? How do we measure amounts of goods: 1 kg costs X, but I only need 200 g. What if I trade one good against another one, or against a service? How to handle this?

Another example is to start from the perspective of a family household: what do we need in terms of food and other goods, for a family of four? How much does this cost? Each week, each month? How much does the parent have to earn in order to match that cost? And so on. When this problem is expanded to a whole neighbourhood, we come close to the mathematical problems found in the Ifugao valley, which Conklin (1980) described in detail. Following Conklin's work children can make maps of the neighbourhood, in order to have a continuous overview of goods, of their trajectory, of the needs and supplies available, allowing for a just and adequate satisfaction of the needs of all. One step beyond this stage, one can decide that equal distribution should be strived for: what would be the outcome of such a plan, and how can one keep track of the deployment of such a plan? Some of the work of ethnomathematics and of radical mathematics (e.g., Frankenstein 1989) offers examples of this approach, as well as Freudenthal's reality-based curriculum proposals (Freudenthal 1970). But as far as I know no systematic exploration has been carried through in a full-fledged curriculum, let alone that a pluri-cultural perspective has been taken seriously.

12 Conclusion

It will be clear for the reader that I embrace Bishop's original six activities of mathematics. However, my contention is that more mathematically significant activities can be identified and used in order to develop and expand mathematical knowledge by means of a concatenation of insights. The latter obviously (to my mind) start with those activities that belong to the FK and the BK of the child, over the many and diversified cultural traditions it comes from. Hence, the elaboration in the preceding paragraphs and sections. On top of that, no exhaustive list is offered here, but I focus on what looks like the most salient or the richest types of activities in the child's world of experience. Put differently, the world view of each individual child, in every particular culture, will show a specific selection of the whole range, phrased in ever so many different voices. This emphasis on individual or personal optimal development is in line with the new humanism, which is thought through by Sen and Nussbaum (see Chap. 12).

Chapter 10
Education in a School Context

1 Education Happens (also) in Schools

In the history of Europe, and since the period of colonization in most parts of the world, education has been systematized to a large extent in an institutional setting. Since the 9th century the particular institution of the school has been successfully implanted in ever more countries of the Christian world. In Christian evangelization outside of Europe (roughly from Columbus on) the spreading of western Christian education has been incorporated by clerical orders, who specialized in schooling. So, education got intimately linked to and almost identified with schooling.

Of course, in the present era severe criticism on this identification has been voiced. Paolo Freire's appeal to 'de-school society' may be one of the better-known examples, but the critical studies are not limited to his work. In the present book I mentioned several critical voices in mathematics education. Moreover, the progressive reduction of education to schooling in the modern era has produced the shift to an even narrower instrumentalist view on educational schooling recently: the norm issued by OECD and its PISA assessment studies aim to make education through schooling subordinated to a capitalist market view on person and on knowing. With globalisation reaching out to the world at large, this development has the side-effect of producing massive dropout and hence loss of chances in real life, where diplomas tend to have a growing importance for job opportunities. Within the market economy salaried work is the main avenue to a so-called decent life, and the school has become the gatekeeper.

Notwithstanding the criticism on schools and on the inequality despite of or maybe even through schooling, it remains a fact that schooling still is and will probably continue to be an important format for learning in the life of generations to come around the world. Therefore attention should be paid to attempts to reshape learning and education outside, but also within the school contexts, even though I certainly do not advocate looking at the latter contexts exclusively in view of education. Having said this, I will not go for a systematic overview of school-bound learning

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principles, but rather build on the expertise gained from over twenty years of involvement with the Centre of Excellence 'Intercultural Education' at Ghent University (acting as the chair of that centre, www.diversiteitenleren.be). In the wake of the tremendous amount of empirical studies and of nationwide training sessions emanating from this centre, it became clear that an interactive or participatory perspective on education in schools is to be preferred to the unidirectional, teacher-dominated and canon-focused practice of traditional (Christian and secular) school educational format. I subscribe to this view (see e.g., Verlot and Pinxten 2000), but add that the increasing cultural and linguistic mix in the schools of cities and larger urban areas urges educationalists to adopt an intercultural perspective on school education as well. I discuss both aspects consecutively: the dialogical or interactive view, and the intercultural perspective.

2 Visible Learning

Within the broader field of alternative theories of school learning, I choose the synthetic, widely tested and democratic proposal of 'Visible learning' (Hattie 2012). The New Zealand research group of Hattie proposes a straightforward and more efficient approach to learning in a school context by making 'learning more visible'. This implies that the constraints, the attitudes and capacities of all involved, the constitutive features of the physical and social setting where the learning takes place, as well as the curricular aspects of schoolish education are all made visible, assessed and discussed by all participants. That is the aim of 'making visible': understand, make explicit, negotiate in a continuous dialogue with all involved all the ins and outs of what is happening in the classroom. That way, everybody will be more engaged in the process of learning and actively collaborate to enhance its quality. Hattie and his collaborators did thousands of interviews, analyzed up to 800 reports, organized sessions of training and developed a system of continuous dialogue between teachers, between teachers and school officials and between teachers and pupils in order to follow the quality of the learning process and the degree of wellbeing of all partners on a very regular basis. That way, they could show that it worked.

What are the main principles of this approach?

- the central focus in schools is and should be on learning, and not on instruction,
- learning should be made visible, meaning that the teachers should look at their own teaching and discuss it with each other regularly; that the focus, the progress or the lack of it through teaching should be watched and spoken about; that teachers and pupils recognize, discuss and remediate problems and choices in the learning processes in mutual dialogue,
- the communication and interaction between all involved (pupils, teachers, school direction, even policy people) are of the essence in order to allow for high quality in learning in a school context. Obviously, this emphasis breaks

away from the traditional practice, where school personnel has the role of the provider and the authority and the pupils a more or less passive receiver of knowledge, and finally:

- the learning process of each individual pupil is the focus of attention: the particular capacities and skills of each one should be known and taken into account, the emotional and attitudinal setup of each pupil is important and should be respected in the process. This involves dialogue again and tailor-made learning processes, rather than the universalistic and canon-based approach of the traditional top-down organization of the school curriculum (which is still dominant in the assessment studies of PISA, by the OECD, for instance). On the one hand this implies that generational differences in interest and worldview are guiding in this perspective: e.g., school knowledge in the era of Internet will show children to develop other skills of information management, with other skills and expectations at the cognitive and at the social level. It might even need more or a different emphasis on interpersonal skills, but certainly yields a greater need to develop selectors in this vast and a-historical supply of information in a predominantly virtual world.

In actual detail, teachers are invited to look at their own teaching, reconsider the goals of each lesson, discuss about it with pupils, and have the latter 'educate themselves' by allowing feedback on the lesson taught. That way, the child discovers the goal, the trajectory of the learning process, the likely drawbacks and benefits of a particular step in that process. In the dialogue with the teacher, the pupils make explicit what they think they observe and recognize. Doing all this is how pupils educate themselves. Obviously, pupils bring to the school different kinds and different levels of out-of-school knowledge, constituting their differential FK (as explained several times in the course of the present book). Thus, the approach of Hattie allows for differentiation of the learning process in line with the FK, the inspiration and the capacities of the pupils involved. Before each lesson the teacher is invited to use a practical guide for the preparation of the lesson (with definition of the goal, weighing of the material used, and so on); also, the insights of the pupils are briefly tested. After each lesson, a short assessment will be gone into, allowing to understand what is the result of the lesson in terms of insights, understanding the main points in the learning process, and so on. Hattie stresses that understanding is much more important than just learning facts. The latter he calls 'superficial learning', in opposition to 'deep learning' of relationships between facts and contexts, and 'conceptual learning' as the accomplished understanding of the content in abstract terms, beyond the context of experience.

It goes without saying that reaching the conceptual level with the variety of pupil's abilities and interests mentioned will take time, and will involve very regular discussion and dialogue. Moreover, one will easily grasp that starting the learning process from the BK and FK (or worldview categories) of each pupil in this sort of approach will individualize the learning process, and will demand time and effort in the school context. Again and rather obviously, the curriculum, let alone a pre-established and dominant corpus or canon of knowledge-to-be-acquired, is not the best way to efficient and insightful learning, according to Hattie; in lengthy and very systematic assessments on the quality and the depth of learning along the lines advocated in the 'visible learning' project, they offer very convincing arguments on this point.

3 Visible Learning and Multimathemacy

What Hattie and his research team do not take into consideration is the impact of a culturally mixed background of pupils on the quality of learning. I advocate that we should add culturally salient parameters to the theoretical frame offered by his group. If the capacities and the BK and the FK of the pupils matter and have to be integrated in the dialogue and the continuous readjustment within the school according to Hattie (2012), then I emphasize that part of these can be identified as the linguistic and cultural features which distinguish many students throughout the world from the mainstream middle class western children most of the school logic has used as reference and basis for assessment. My choice throughout this book is to allow for the optimal development of each and every pupil, along the lines of the capability theory (Nussbaum 2012: see Chap. 12). This could be the broader humanistic frame for Hattie's more focused pedagogical perspective.

In a general sense, beyond the culture specific conditions of any particular school population, Hattie successfully argues that those projects which take into account the worldview and cognitive capacities (the 'level') of individual pupils work and yield better learning results, whereas those which are thought out and implemented top-down produce more failure in the learning process. The former ones are based on BK, motivations, ambitions and insights of the pupils to start with, allowing for the continuous dialogue between all in the school population, which Hattie calls for. The latter have a shallow base of shared motivations and insights—or sometimes none at all—and are likely to produce more alienation, frustration and resentment against the material presented by the school authority. Translated towards the culturally mixed school population I am concerned about, the problem is not qualitatively different, I suspect. These are not 'other kinds' of students in the schools, but because of the neglect of added diversity, the cultural mix will yield more frustration and alienation with more members of the school population because a greater variety of backgrounds, motivations, learning habits and so on are disregarded. In a uniform, top-down approach to school learning the translation and informal remediation (among students, most of the time) will be less likely the more mixed the class population will become. When schools in urban areas now have children from up to 75 different cultures and many different home languages (e.g., the high school 'Atheneum' in Antwerp), the dropout will predictably be massive if interculturality is not a focus. But first grade schools in many cities in Western Europe, which are probably the most decisive steppingstones for a school career, have children from more and more diverse linguistic and cultural backgrounds. In my own observations with such primary schools in Ghent, I witnessed how the increase in diversity should be met from day one and be followed up continuously, lest more and more pupils from non-middle class and from foreign origin will progressively abdicate.

From those same observations I can testify that the work of the teachers is not easy: schools will offer language courses for the mothers, have individualized teaching along the lines of Freinet or Dalton pedagogies and will then be able to narrow the gap between middle class white kids and the rest of the class. But PISA researches show, over the years, an ever-widening gap between good and bad performers in our countries (especially Flanders, Belgium, but also elsewhere in Europe: PISA, 2010) when these efforts are not fully integrated in regular school education. With this point I am entering the political arena once more: school reform programs run into serious opposition from right wing, neoliberal political formations, who feed the anxiety of middle class groups that mixing a school population will 'water down the quality of schooling' (Jacobs 2014), while progressive voices from the field plea for a better education for all by differentiation and diversification of the learning processes in schools (Schmidt 2014).

Let me illustrate the relevance of such an approach for mathematics learning in a school context. I choose two examples:

(a) Radical mathematics and poor neighbourhoods:

In her work on radical mathematics Frankenstein (1989) situates mathematics teaching in the world of experience of the children. And more often than not, this is not the world of the well-to-do, the rich people. Even when the proportion between rich and poor would not be as steep as the Occupy movement has it (speaking of the 99 versus the 1 %), it is clear that the gap between haves and have-nots is growing again, all over the world, also in the West. Hence, radical mathematics teaching has a point in focusing on the worldview and the world of experience of the latter group in the population. A very competent proof reader of this book (B. Greer) suggested in a comment on the manuscript that the present book should be written from the perspective of the majority, much as Zinn's 'People's History of the United States' presents a history from below. I will try to show what this could mean for mathematics teaching by developing an example.

Example: budgeting for the neighbourhood.

It has become rather common now in European cities that some purchases of basic goods are done in a temporary collaborative structure, a sort of informal cooperation. Especially social democratic and green parties have thus been able to negotiate a cheaper price for gas, electricity and water from the large corporations providing these by acting as a temporary group of consumers. The customers who join have a benefit, and the provider is certain of an interesting deal by dropping a small amount of the usual profit with individual customers. Of course, the cost is less for the provider when a large deal for a group of several hundred clients can be reached, instead of doing the bookkeeping for each and every individual client.

One could use this successful example of a social practice and expand it to other areas of consumption. In the cities everybody knows that the proportion of poor people is growing (see Agorakring 2012: Armoede door kinderogen/Poverty through children's eyes: www.agorakring.be, n.d.).

The classroom gathers information in four groups:

- group A interviews each other on what is needed in a household (food and non-food) on a weekly basis, and what this means in terms of budget,
- group B visits a local centre for the poor and interviews personnel and clients on what the clients need and what the budget amounts to,

Groups A and B compare notes and make up a list of needs, and a global budget for the purchases.

- group C visits the local supermarket, the detail shop and retail shop: they check on the prices of the goods, focusing on basic products first,
- group D develops a plan to distribute the goods over all customers in an efficient and cost-friendly way.

All groups have one or two reporters, an accountant, and a chair to supervise the discussions within the group. All members of the groups keep notes on the data of interviews, on prices, and so on.

All groups gather in a weekly general session, reporting on the progress of each group. Children discuss the pros and cons of particular lists of purchases, and decide on what would be the best bargains for people with low budgets. Here a reporter makes the minutes of the meeting, and two accountant-pupils develop a schedule in view of a realistic budget.

The teacher acts as a provider of mathematical skills and procedures, on request of the pupils. In a less radical way, this pedagogy is used already in the Freinet schools, which are very popular in Belgian cities.

The end result of this exercise will be that the class will be able to match a low budget of the clients with the 'ideal' purchase policy for each of them. Mathematically the following operations will be learned and used in the course of this exercise (which may take weeks to be sure): addition and subtraction, multiplication and division, measuring proportion, calculating the mean, making lists for the budget, bookkeeping. Other operations may be involved as well. A major benefit of this sort of larger exercise is that other cognitive and social skills are integrated and sophisticated in a rather natural way here: children learn to pose questions in real life, with real interlocutors. They learn to collaborate with each other and with 'informants'. They learn to weigh and interpret the meanings of terms, as well as the impact of speech acts during the fieldwork parts of the exercise. In the processing of the data the children experience what it means to work with social scientific data, and how negotiation is an integral part of any human decision making in a group.

(b) designing and building a catamaran:

The Pacific Islanders are famous for having developed a sailing boat, which is the reference for the present-day catamaran. For ages the peoples from Truk, Guam, and many other Micronesian and adjacent islands have been sailing the ocean with their simple canoes, which they basically cut from a tree. To sail the high sea they understood they should add a stabilizing ski, which would glide on the water (Gladwell 1967). The technology they thus developed was the basis for the now well-known catamaran. With the Second World War some of these islands became the locus for American military bases, and in the wake of that they discontinued the 'primitive' fishing techniques they had and exchanged their canoes for American motorboats. However, with the recent rise of the oil prices worldwide they threaten to starve on the islands, because the fishing is becoming too costly now. Recently, some of the islands decided to start looking at the old boat technology again, as well as at the navigation techniques that went with them. Obviously, the old canoes can be made in other materials today, and can be equipped with modern information devices on solar energy. Thus their old ecological knowledge might help them to survive on the islands, and the addition of modern ecological tools may enhance the success of the seafaring trips. A large research project is now being run in order to see what knowledge from the island is still known and can be taught to the next generation (Rubinstein 2014).

By way of example for other and similar projects in mathematics teaching, my suggestion is to have one or more courses of 12–13 year olds work on the project 'Build and sail your own catamaran'.

My first inspiration is, of course, that young people would dream of having and even more of building their own boat. Attached to this, the ethnographic material abounds in information about the strategies and techniques for navigation, about the parameters in the sky and in the sea that should be read in order to go on a safe journey, and so on.

Steps I see in this project:

– read ethnographic material (e.g., Gladwin 1967) and technological booklets on making a canoe; making a script on the basis of this: what are basic features of the material (it should be resistant to salt/sweet water, it should be durable, it should be light enough to float and to be carried by only a few people, etc.)? What sort of materials are available and can be managed by teenagers (wood, polyester, new composite materials)? How is stability of a boat to be reached: the balance between the floor, the fin and the width of the boat. Try this out in a scale model; then calculate it for the real boat. How to conceive of, produce and attach the stabilizer? For all of these issues the ethnographic and the modern technological data are available.
– how to devise the interior of the boat? How to install benches, a rear steering system, a compass, a GPS or other additions?
– what can be learnt about navigation from the seafarers?

Spatial orientation: defining reference points in the sky, on a compass, or taking yourself as moving object as reference (such as Micronesians do).

Spatial distance: defining distance in terms of the sun cycle/the stars/the islands on the horizon. Measuring with sea currents, the colour of the water (as Micronesians do).

- how does a sextant operate?
- developing a sea roadmap: defining objectives, departure and destiny points.
- defining a trajectory.

It is in this way that curricular material, but also learning procedures can be conceived. From the sketches of the two examples it will be clear that the way of working is deeply dialogical and intercultural. Children and teachers are in constant dialogue, organize work and task conventions together, and define goals and trajectories in mutual negotiations. That is the dialogical dimension of such an approach. At the same time, the traditional and indeed colonial way of working (Johnston and Yasukawa 2010) is left behind and a genuine intercultural perspective is adopted: all are involved in the learning process, actively search in the literature and even in real life or out-of-school contexts for relevant knowledge, which is useful, or interesting for the goals and tasks agreed upon. This search is not bound, nor directed by the status or the power of one particular cultural tradition. That is to say, the western canon of ideas and aspects of knowledge is not the dominant one, as is the case in almost all mathematics curricula today. But on the other hand, not any other particular tradition is dominant either, as is the case in some of the older cultural curricula (e.g., detailing the native categories exclusively: Hardaker 1992) or in some of the older multicultural curriculum material in the past (where aspects of x cultural traditions would be presented next to each other). What I aim for here is the insight that all types of cultural material, whatever its origin, is potentially useful for the mathematics classroom. The choice will be made by the group of children and teachers, in light of the needs for a particular project, of relevance, beauty or even mere availability of the particular knowledge to be used. That is an open, gradual, and inclusive intercultural perspective.

4 Assessment Procedures

Johnston and Yasukawa (2010) present a synthetic overview of the elaborate studies they did in unison, focusing on the social and cultural differences of learning processes in mathematics classes, but even on the cultural differences in the appraisal of rationality (let alone ‘proof’) of knowledge and mathematics in particular, in countries like Australia and Japan. The overall point they make is that numeracy, as the competence in mathematical knowledge, has become a major instrument of control and imperialism. Cultural differences will yield less competence, or a lower output in the typical western tests and assessments to which children from a different cultural background will be submitted. Instead of uncritically accepting these ‘measures’ as scientific so-called objective truth, the authors come to the conclusion that this type of assessment and testing is simply instrumental in the control and subordination of people from a different cultural background than the contemporary western school tradition.

Of course, this sounds a lot like the Vygotsky perspective mentioned before, but it is important to emphasise the implicit imperialistic attitude in this presumably neutral or objective assessment practice. The sociocultural perspective elaborated by Johnston and Yasukawa is taken up by another leading figure from Asia. Ma (2010) presents an overview of the attempts to remediate the growing dropout rates of the past decades. In rather rare longitudinal research on the matter Ma comes to the conclusion that mathematics education in the mind of the student largely builds on former successes and failures: the most important factors for participation to mathematics classes in high school are '…prior mathematics achievement and prior attitude toward mathematics…' (Ma 2010: 224). Hence, investment in insightful learning procedures and attention for the student's attitude and self image vis-à-vis mathematics classes is of the essence. Pure mathematics is an unlikely candidate for success in this realm, whereas situated learning and exploration in so-called 'trivial mathematics' will most probably allow for smooth and efficient progress in the student career.

What is needed instead, then, is an intercultural version of the approach of visual learning, as described by the Hattie group, where the constant assessment and dialogical (self)steering would invite children and teachers to constantly compare data and perspective and include intercultural perspectives in all choices negotiated and adopted. To my knowledge such assessment and test batteries are not available yet. On the other hand, the high degree of urbanization in the world and the inalterable cultural mixing that follows from such a global trend, will make such an intercultural assessment battery a matter of urgency. We cannot just keep ignoring the imperialist perspective in the dominant-cultural assessments we have, when we find that over 60 % of the world population is living in urbanized contexts (Castells 2002) and cities in the US and in Europe count people within their 'walls' of more than one hundred different non-local cultural backgrounds (Corijn 2013).

5 Theoretical Conclusions

This short chapter pointed to a few possible lines of (applied) research in the school context. Although it is not at all my view that everybody in the world has to be taught and reared in a school system, I am realistic enough to recognize that schools are and will continue to be a major medium in the educational trajectory of the vast majority of children. Hence, it is important to look for the problems and the remedies within that educational context.

In that line of reasoning I suggested that modern pedagogical approaches such as 'visible learning' should be screened and possibly adapted to the needs of an urbanised and mixed (culturally, religiously, linguistically and socially mixed) school population. I suggest it is urgent and necessary to scrutinize the bottom-up procedures of visible learning. The fact that the pupil's perspective, as well as the teacher's and the administrator's knowledge and views are integrated in this practical approach is a plus. It should be taken seriously in order to have the school context adapted in a more systematic way to the real world of all these agents in the process of education. However, this Australian perspective is too much culture-insensitive to my mind. In the deeply urbanised world this point of view is far from neutral, since it proves to exclude and produce dropout on a worldwide scale. Therefore, I suggest to integrate the bulk of notions and procedures of the present book into the visible learning paradigm, so that a deeply mixed world of experience constitutes the frame of reference, and that the FK of children from different origins will define a platform of both learning traditions (with an added comparative attitude) and curriculum materials.

Chapter 11
General Conclusions

This is a book about mathematics education by a non-mathematician. In itself this should not be odd, let alone be a cause for alarm. But in the present era, and in the West, it might raise some eyebrows. Hence, I want to state this up front.

I have been doing anthropology throughout my career, as a result of the philosophical questions I started formulating as a student in ethics and philosophy. Since my main focus was on theory of knowledge and on the way(s) worldviews and conceptions of space, time and relationships between humans and nature took shape in the history that we westerners were a little bit acquainted with (that is the Judaic-Greek-Christian heritage basically), I grew ever more worried and even annoyed by the unique 'voice' we intellectuals were raised in. That is the 'master's voice', which held that "real" thinking, like "real" mathematics was only developed in this local tradition, which therefore saw itself as superior, more genuine and "evidence based" than any other human tradition. Throughout colonial times, this attitude was firmly established through schooling and the civilization projects vis-à-vis all other parts of the world. With the oncoming of decolonization, and certainly with the shift of power after the 80s of the past century, at least some space for alternative views on knowledge, on superiority and inferiority of cultures, on difference and on the intrinsically mixed nature of humanity now emerges, or rather is secured in a severe intellectual battle.

In the "mathematized" world we live in today, according to some (Atweh et al. 2010), globalisation also entails the rather sudden halt of the long brain drain of mathematically sophisticated personnel from India, Korea and other so-called Third World countries. This brain drain had compensated a well-known failure of mathematics education in the West: a large percentage of dropout was documented by alarming reports (e.g. 'Nation at Risk' in 1983, followed by many other studies: Xin 2010). With the decreasing brain drain from elsewhere in the world, the dropout in the West rapidly grew into a huge problem. Where would the engineers come from? Who would continue the production of wealth-through-knowledge in what is now called the 'knowledge society'?

Within that climate of 'crisis studies' on STEM, and especially on mathematics education, I take the stand that it is wise to sit back and rethink the problem area.

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I started to look at some of the 'weighty' voices in the arena, like OECD and its policy on mathematics education. Rather than take its criteria for assessment and even its notion of mathematical competence for granted, I try to look at the value of these in the perspective on human beings and on just society as they emerged from a broad humanistic point of view (Sen and Nussbaum, in Chap. 12). That is to say, I ask myself the question what would follow from a perspective on humans as varied and complex beings, living within their many different cultural and social contexts, with their different psychological setups. In other words, suppose human beings are not by nature or in any context-independent and intrinsic way '(economic) market players', or at the very least not only that. And suppose the plurality and diversity we witness across the many hundreds of cultural traditions anthropology speaks about are only retarded or 'un-developed' in the eyes of a few amongst them. The latter might base this view not on solid ground, but only on religious convictions and on a few generations of local and temporary economic, military and political power. But empires rise and fall, as we all know, and each of them held that they were the absolute and unique high point of humanity. In the apparent shifts and seizures of our time, with tremendous urbanisation and hence substantial increase of cultural diversity, blindness for one's own local and temporary status may cost dearly. And of course, blindness for the unfairness of privileges may end up as the choice for stupidity and for horror wars.

When mathematical literacy (Gellert et al. 2010) now seems to gain a crucial position in the world, it is important to sit back and look at the nature of it, and see what choices are made or could be called for in a realistic, a fair and a sustainable interpretation of reality. After all, when the era of colonial dominance may have come to end, and the mixing of people in the heavily urbanised world is on the agenda, it is likely that reconsideration of educational positions and procedures will be high on the agenda too.

In this broad 'feel of the world' I want to listen to many messages and opinions on mathematics education, considering that activity as a deeply political one. Of course, I have some research competence to allow me to at least enter the arena and try to speak up there. As an anthropologist I witnessed, did research and developed some remedial material in this area. My research was both 'pure' research as an anthropologist working on spatial notions with particular cultural groups. But that is, obviously limited to only a few cultural groups, in my case basically three (Navajo Indians in the USA, Turkish migrants in Belgium, and mixed schools in Flanders, Belgium). My scientific work also consisted in action research with the same groups: it is one thing to do in-depth linguistic analysis, for example, but it is better to also intervene through the research process, I hold. This is certainly also the mentality which is subjacent in this book, where I mention a variety of researches by psychologists, mathematicians, anthropologists and so on, but also try to formulate means, processes and curriculum perspectives from a variety of origins, which may remediate this unfair situation of the large percentage of dropouts in the mathematics classes, thus barring them from better positions and from a more fulfilling way of life. Starting with the 'Whatifs', I invite researchers and pedagogues in this field to freely explore with me the unwarranted constraints and presuppositions of the dominant view on mathematics education and look at the whole project of education from another perspective.

In a way what I propose is rather straightforward: I hold that the mathematics of the professional mathematicians is a huge and quite remarkable achievement. I call it the skyscraper of AM. But in the city of mathematical knowledge, to stick to that visual metaphor of buildings, I find a lot of buildings: there is the AM tower, but also some other substantial buildings like Hindu, Islamic, or Chinese mathematical traditions. But the city also has many smaller buildings, from middle sized houses to mere huts, with local and often very specific efficient mathematical knowledge on building, on art forms, on navigation, on calculating systems and so on. My basic call now is to take all this seriously when engaging in mathematical education. So, where (to put it in an oversimplified way) AM claims to know what mathematics is all about—and that is 'pure' mathematics, to quote Hardy—and hence how children from anywhere should be programmed in a systematic way from nothing right up to the structural and procedural universe of the AM skyscraper, I want to turn this reasoning around over 180°. It is the children, with their diverse out-of-school knowledge and their differential trajectories of learning and developing who should be the anchor point for the educationalist, not the AM pure mathematician. That is to say, my educated guess is that dropout rates in the mathematics classes will be reduced if we start mathematics learning at the elementary level while thinking along with the children and their out-of-school knowledge. Disregarding this knowledge may be the main cause of the large dropout we witness, even more so when the context of learning gets deeply culturally mixed, as is the case in the urbanised world we enter.

In a more general way, I claim we should reinvestigate whether 'trivial' mathematics (to quote Hardy 1967, again, although with the opposite appreciation he attached to this qualification) might not better be the first and maybe the main goal of mathematics education for all. The implicit mathematics in dance, play, building and what have you, should be the focus at the elementary level: these notions should be explored, made explicit, discussed and negotiated to begin with, not the concepts of the AM tower. The gradual steps toward presumably 'pure' mathematics at a later stage in the learning trajectory may then become optional for a minority, but a basic layer of insightful and 'useful' mathematical knowledge will be a right for all. Moreover, my stand is that children 'know' a lot already when entering the school, and that this knowledge should better be taken seriously. It should be the basis for explicit and further sophisticated levels in mathematical education, rather than being looked upon as mistaken ideas or misguided intuitions. In this book I develop this argument, draw in studies from the socially and culturally sensitive learning theory of cultural psychology, and try to open the educational horizon to include such not-strictly cognitive activities as dance, music, and so on.

What these initial intuitions entail for the remedial curricula and for the learning strategies in mathematics education is explained, sometimes hinted at, often locally gone into in the bulk of this book. Obviously I cannot and do not give a systematic alternative, nor even a source book for mathematics education throughout a world of diversity. I can and do point to the major obstacles of the AM view, and I indicate how a tremendously colourful and varied set of cases and insights exists and waits to be used in a broad and again diversified approach on an emancipating road to formal thinking. But the nature of my endeavour is to not present a uniform or universal alternative, but rather invite mathematics teachers to allow for many trajectories and to use all sorts of sensitive and rich experiential material they prove to have available.

The book invites the field of professionals to reconsider, and discuss what may be concrete, viable and promising avenues, with respect for the diversity in the classrooms and in the streets. Because the children are the subjects of the future, not the objects of today.

Part V
Learning and Capabilities

Chapter 12
Appendix: Human Beings as Learners-in-Context: An "Engine" for the Capability Approach

Goedele A.M. De Clerck

This chapter sketches the broader panorama of a new humanistic philosophy (my terms), which is developed to a large extent by Nobel laureate Amartya Sen and political philosopher Martha Nussbaum. As I mentioned repeatedly in this book, I situate mathematics and mathematics education in a socio-cultural and historical context. The changes in this context over recent decades give us the opportunity to rethink our foundations and practices. I mean, we have to reconsider critically the views on human beings and on society we adopted in the past. This is an exercise that we will have to do continuously, and which goes beyond the disciplinary frame we are used to think and work with. Therefore, I present my choices and search results on these issues as an appendix, subjacent to the 'object level', which is predominant in this book. This appendix is the product of the collaboration between my assistant researcher (G.DC) and myself (R.P.).

Sen (2000, 2008) and Nussbaum (2000, 2006b, 2011), developers of the capability approach (also referred to as the "capabilities approach"), instantiate a humanistic view on human beings in their striving for a decent life. Looking into the anthropological and philosophical perspectives embedded in this proposal, we make the view on humankind and personhood that is articulated in the capability approach more explicit. Further, we explore in what sense this primarily "structural" proposal can be equipped with a processual component (an "engine" or "gear mechanism") so as to guarantee the achievement of capabilities. We argue that only if this occurs will Sen and Nussbaum's humanistic intentions be realised. The "engine" we see as best suited to this purpose is found within the so-called socio-historical or socio-cultural approach to learning theory, as expressed by Lev Vygotsky (1978). For the capability approach to be realised in a globalised and diversified context, a cosmopolitical reading of the dialogical self may be necessary. Our argument is illustrated with empirical data from case studies of emancipation processes in deaf community members in diverse contexts (Western and non-Western). We opt for this particular example, because a diversity driven perspective should include these minorities as well.

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In the context of this book capability theory offers a theoretical frame, geared on sustainability, inclusiveness and participation of all concerned. For these reasons we want to elaborate on it in this book as the best frame to work with. At the same time, it is not (yet) or not enough taking into account in the deeply urbanised and the culturally mixed nature of the world today. These aspects, which are central to the approach in this book, will be added here.

1 Introduction

The capability approach (also referred to as the "capabilities approach") is an ethically normative theory of humankind, which formulates 10 capabilities that need to be realized for human lives and societies to be 'decent'. The approach conceptualises human beings as individuals, who are able to develop their lives in alignment with what they value as decent ways of being and doing (Sen 2000, 2008). The capability approach is oriented towards freedom expansion "both as the primary and as the principal means of development" (Sen 2000, p. xii). As Nussbaum (2000) points out, the capabilities approach presupposes an ethical conception of humankind:

> The basic intuition from which the capabilities approach begins, in the political arena, is that certain human abilities exert a moral claim that they should be developed. Human beings are creatures such that provided with the right educational and material support, they can become fully capable of all these human functions. (p. 83)

In the present chapter, we argue that, in its focus on the objective of capability expansion, the point of departure of the capability approach has been under-theorized. In contrast to social contract theories, the capability approach "starts from the Aristotelian/Marxian conception of the human being as a social and political being, who finds fulfilment in relations with others" (Nussbaum 2006b: 85). If one takes these ethical presuppositions of the capability approach and its perspective on humankind for granted, one risks falling into the trap of romanticism. The first section of this chapter will introduce the capability approach and highlight the views of humankind and personhood that are at the basis of the theory.

Looking at the relevance of education in the capability approach, Nussbaum (2011) deepens this basis further, emphasizing the role of conflicts within individuals of moral emotions such as empathy and compassion and their anti-moral counterparts such as shame and greed in shaping the world, in democratic citizenship, and in choosing for human development and the capability approach. We argue that, for the capability approach to become "not just a theoretical construct" but "a way of life" (Nussbaum 1995, p. 15), the ethical conception of human beings needs to be supplemented by an anthropological understanding of human beings as learners in a social, cultural, and political context. Drawing on the learning theory of Vygotsky (1978) and Cole (1996), we provide the capability approach with a scientifically based "motor" (engine) that can foster capability expansion and the achievement of functionings (realized activities of being and doing). This theory of learning, the relation of human development and learning, and the development of the theory are worked out in the second and third sections of the chapter.

Learning processes are often mentioned in discussions of the capability approach and its applications in the educational context; however, a broader conception of human beings as learners is not only a prerequisite to further insight into these processes, it also supports comprehension of how people can identity and act as world citizens. Another interpretation of personhood, which contributes to further understanding of how the capability approach can be actualised in an increasingly diversified global setting, is found in a cosmopolitical reading of the dialogical self. This is worked out in the fourth section.

In this chapter, we aim to contribute to the further exploration of the capability approach by exploring interdisciplinary and cross-cultural perspectives and empirical research. This argument is illustrated by the application of our proposal to case studies of emancipation processes in deaf community members in Cameroon and in international deaf people at Gallaudet University, the world's only university for deaf people, located in Washington, DC. Obviously, deaf communities are minorities within a dominant hearing society. Not surprisingly very similar mechanisms of dropout and/or exclusion are documented with these communities, as with other social or cultural minorities. That is the reason we dare to present this case (which is the main focus of research of De Clerck) in the present analysis.

2 Philosophical Points

Before we embark on an explication of the concrete issues we want to address, a few philosophical remarks may be helpful. The capability approach is an anthropological theory (of humankind) with a normative edge, projecting a new humanistic perspective on "good" human life and social life. Though this is perhaps an interesting and even enticing notion, it might be considered as an ultimately moral statement, beyond the strictly empirical realm. Therefore, it may prove edifying to scrutinise in a scientific way what the approach involves, before an ethical stand is taken. First, the perspective on humankind within the capability approach holds that humans are growing, developing creatures, living in social networks. They have capabilities at the psychological and social psychological levels of existence, which have been discerned by Sen and Nussbaum. The viewpoints of these two scholars differ somewhat, but these distinctions are not relevant to our present argument (see Robeyns 2005; Unterhalter et al. 2007). We draw more on Nussbaum, perhaps in line with the more narrative approach in her work (see Robeyns 2005).

Nussbaum (2006b) states that a certain form of human decency should underlie all societal rules and agreements. Beneath a certain level of decency, human functioning becomes insufficient or even impossible. In her work, the minimal conditions for a decent life are described by means of a list of 10 capabilities:

1. life, that is, a normal life expectancy
2. bodily health
3. bodily integrity
4. senses, imagination, and thought
5. emotions
6. practical reason, including freedom of conscience and religion
7. affiliation, that is, (a) living with and toward others, (b) having self-respect, and avoiding humiliation
8. other species
9. play
10. control over one's environment: (a) political; (b) material

It follows that any society can hence be assessed on its human decency level, or its "fair" character (social justice), on the basis of how well it enables the realisation of these 10 capabilities for each of its members. Thus, the capability theory of humankind is normative in the sense that the decent, just, or ethical society is that which functions to fulfil the maximum capabilities of all human beings. Nussbaum (2011) distinguishes between capability and *functioning*: "A functioning is an active realization of one or more capabilities. Functionings need not be especially active (…). Functions are beings and doings that are the outgrowths or realizations of capabilities" (pp. 24–25).

The critical point we want to introduce here is that if one takes for granted the basic presuppositions in the ethical realm of the theory, one may risk falling prey to naive, romanticist, Rousseauan views of humankind and education. During the Enlightenment, Jean Jacques Rousseau split from the atheistic and agnostic humanist group of Denis Diderot and Baron d'Holbach (Paul Heinrich Dietrich), who were trying to develop a scientific (or rational) theory-cum-ethics of humankind. Adhering to his religious belief, Rousseau advanced the first romantic view of humanity: that, in essence, humans are morally pure when they are born, and are corrupted only through education and politics. He developed this idea in his famous work *Emile: Or on Education* (1762/1979), but also used it to uphold his proposal that inherently free and good humans agree to organise society on the basis of a deliberate contract, that is, a "social contract."

While Sen and Nussbaum are critical of Rousseau's social contract as it pertains to modern political theory (see Nussbaum 2006b; Sen 2008), they nonetheless seem to fall into the romantic trap when it comes to education. Conscious of the need for education if the capability approach is to work, the two scholars explicitly draw on Rousseau's philosophy: "Rousseau argues that a good education, which acquaints one with all the usual vicissitudes of fortune, will make it difficult to refuse acknowledgement to the poor and sick, or slaves or members of lower classes" (Nussbaum 2003, p. 92). In this citation it appears that the mere adoption of the Rousseauan approach will automatically yield capability expansion and the achievement of functioning, and the amelioration of life for all. This view can only be embraced if one takes a moralistic view of humankind to begin with.

Paraphrasing the tenets of capability theory as it is now known, and as it is adopted in pedagogical discourse (see, e.g., Unterhalter et al. 2007; Walker and Unterhalter 2007), we find that 10 minimal conditions should be guaranteed for human beings to grow and act in; when these are realised, a fair and decent society will ensue. This perspective smacks of the same sort of romanticism we find in Rousseau's view (even more clearly in his idea of the "noble savage"), which embodies a retreat from the Enlightenment and a return to the "natural philosophy" of the Christian era (Blom 2010). We find arguments for our critique in other texts—for example, in a chapter in Nussbaum (2006b) on "moral sentiments":

> Our basic equipment would appear to be more Rousseauan than Hobbesian: If we are made aware of another person's suffering in the right way, we will go to his or her aid. The problem is that most of the time we are distracted, not well educated to understand the plights of other people, and (what both Rousseau and [C. Daniel] Bateson emphasize in different ways) not led, through an education of the imagination, to picture these sufferings vividly to ourselves. (412)

Nussbaum (2006b) highlights her sense that if the capability approach is to be made a realistic alternative, social change is required. Regarding the way which education of some sort will yield the moral person she so highly praises, Nussbaum admits that her theorizing has limits:

> A liberal society may foster, and make central, conceptions of the person and of human relations that support its basic political principles. A society aspiring to justice in the three areas I have discussed must devote sustained attention to the moral sentiments and their cultivation—in child development, in public education, in public rhetoric, in the arts. I have not shown that the extension of sentiment required by the normative project of this book is possible. And I have certainly not shown *how* it is possible. …. Even though I have not yet shown that the realization of justice as I construe it is possible, I do believe that my argument here removes one obstacle to seeing it as possible. For it establishes that a particular picture of who we are and what political society is has for some time imprisoned us, preventing us from imagining other ways in which people might get together and decide to live together. (414)

In our rephrasing, the moral program remains only a vow or a wish and it cannot explain how human beings develop agency, identity, and citizenship in real life settings, processes which account of a capability approach in practice. This is somewhat ironic in the case of Sen (2008) and Nussbaum (2006b), since they so meticulously criticised the social contract view of the same Rousseau (in the guise of the adapted theory of justice advanced by John Rawls and others).

The "model of education for democratic citizenship" presented by Nussbaum (2002, 2006a) expands on her conception of personhood. Three capacities are particularly relevant for education towards global citizenship: critical thinking (reflecting on one's own position, traditions, etc., and reasoning with respect for oneself and others); thinking as a "global citizen" (instead of a citizen of a group or region); and "narrative imagination" (being able to imagine taking a different position than one's own, i.e., standing in someone else's shoes). Also in this context, Nussbaum does not show the conception of personhood and humankind presumed in her theory; neither does she provide insight into the process of interaction of individuals in social, cultural, and political contexts, conceptualised in the notion of "combined capabilities" (Nussbaum 2000; see also the notion of "conversion," Sen 1995), which is a necessary condition for turning capabilities into achieved functioning. The capability approach is agency oriented, which also brings us to the relationship between education and freedom, and between agency, identity formation, empowerment, and social change (see also Walker and Unterhalter 2007). In working towards evaluating capabilities (rather than functionings), it is crucial to understand how these processes take place in human beings in a real-life context. Hence, we want to recognize these critical points and try to remediate them in the light of the culturally diverse population we focus on in the present book.

We notice that the following questions have not been sufficiently answered: How do human beings actually become moral persons? How are capabilities turned into achieved functionings by learners in social and cultural contexts? What psychological and anthropological processes are involved? How can education be conceptualised as "a basic capability that affects the development and expansion of other capabilities?" (Walker and Unterhalter 2007, p. 8). How can human beings become democratic world citizens who practice the capability approach? In our contribution, we aim to hint at fundamental responses to these questions.

Our perspective on the matter is that this (implicit) romanticism in the humanistic view captured in the capability approach (see also Flores-Crespo 2007) can easily be overcome by developing one more element of the anthropology we find here: human beings are learners. They do not mature and become full humans automatically in a process that can be guided by constraints of fairness and decency in the social surroundings, but they learn to behave and think in a just and fair way, or in a crooked and derailed way. By learning strategies and controlling mechanisms, they find their growth into adults and citizens. The implicit and hence assumed expansion of capability and promotion of "functioning" in a just and decent society in the case of the present state of capability theory (which, to us, is what smacks to some extent of natural law and romanticism) can be replaced with a robust theory about the mechanisms operating in society. In other words, the "motor" in the fulfilment process is not automatic or "natural," but can be chosen, implemented, and actively assessed and adapted. Scientific knowledge about the workings of this "motor" is available and critically scrutinised in at least one learning theory that has been developed over the years: the socio-historical (or, recently, socio-cultural) theory of learning. We develop this view here and show how it works in the "motor" function.

In relation to a socio-historical theory of learning, we also provide insight into the process of agency and social change, which is necessary for the actualisation of the capability approach. In a cosmopolitical conceptualisation of the "dialogical self," we provide a view of personhood that is able to support this actualisation.

3 Socio-cultural Learning Theory

The scientific study of learning has produced a few learning theories so far. One of them, the so-called socio-cultural theory, is the most relevant to the questions we deal with here (see Chap. 2). It exists under three different labels: in the first decades of the twentieth century, Lev Vygotsky advanced the original Russian *socio-historical theory*. It has been reintroduced in the West since the late 1960s and slightly broadened into what is now known as the *socio-cultural theory*. With his 1996 book *Cultural Psychology: A Once and Future Discipline*, Michael Cole offered his attempt at a synthesis of the whole field, going back to the founding father of scientific psychology, Wilhelm Wundt. Cole relabelled the field *cultural psychology* in his book. The basic features under the three labels remain the same, and they concern us here. This type of learning theory sees learning as mediated: the learner does not exist in a void, but is situated in a social and cultural setting. Furthermore, learning occurs in the interaction of biological, psychological, and socio-cultural (or socio-historical, in a narrower sense) factors. This conception breaks away from the organicism of Rousseau, in which education seems to serve primarily as a safeguard of the "nature" of the child throughout what can best be described as a development cycle. In the socio-cultural perspective the interactions between organism and contexts, and between the person and the learning procedures, can be studied with scientific rigour. Later on, it can be directed or managed to a large extent through control and manipulation of the context of learning. We start with some of Vygotsky's ideas to point to scientific views on learning, the social context of learning, and the like.

(a) Development and Learning

Vygotsky aimed at several fundamental issues in his work. One major point, which is hinted at in the discussion of the Rousseauan character of "education" in the capability approach as we understand it, is that of the distinction between development and learning. In texts on this issue, which were published in translation from the Russian originals in *Mind in Society: Development of Higher Psychological Processes* (1978), Vygotsky distinguishes between different approaches that were applied in his time: some theories claimed that, basically, children mature and thus become adults. This is the exclusive development view, as can be found in the work of Jean Piaget and others. The cycle of development has its own logic, since it is basically an organismic process of maturation. Learning is external, and premature learning (offering materials for which the child is not yet ready) is considered useless or even harmful. Rousseau's corruption of the nature of a child through contact with society can be situated here. A second approach (with behaviourism) identifies development as learning: you add stimuli, and hence the cycle of development starts up. A third view, which incorporates Gestalt theory, combines behaviourism and the cycle of development, focussing on the development of the brain as an organism and situating learning as a development process in its own right. It can be classified with either of them or it can stand as a third approach, which is akin to both former ones. In the bulk of this book we placed this perspective under the two former ones (disregarding its relative identity); here we shed a bit more light on it by making it a category on its own. However, where development and maturation are primarily organismic processes, learning in this approach is always in and through contexts. In the terminology of the socio-cultural approach, however, learning happens through semiotic mediation. Contexts are carriers of meaning, and learning occurs through mediation of those meaningful aspects of the contexts one lives in. Vygotsky thus makes a clear distinction between development and learning. The important element for the sake of our present contribution is that he therefore opened a line of research that continues to the present day, with a cascade of scientific studies, both sound empirical work and deep conceptual analysis, linking nature, mind, and society in a unique way (e.g., Cole 1996; Holland and Lachicotte 2007).

Beyond the three proposals he scrutinises, Vygotsky then offers a fourth avenue, which has become a core element of the cultural psychology school as we now know it. A central concept in this Vygotskyan approach is that of the "zone of proximal development": Vygotsky states that each individual has a so-called actual level of development, or "mental age." That is, the level of development of a person at a given moment is a result of the series of stages or processes the person has transited so far. On top of that, or rather together with that, every person has a zone of proximal development: "It is the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky 1978, p. 86). Thus, an understanding of reasoning that acknowledges the zone of proximal development defines a place for learning processes (with a learner in social and cultural-context) as a dynamic complement to the processes of development and maturation. The processes of learning in and through this zone provide exactly the "motor" we have been looking for. It is through this situated motor that development continues, not as a sui generis process but as a socially and culturally guided and manipulated one: More capable peers help the process along in the learner, for example. In the past half century, a tremendous amount of experimental and observational data have been gathered within this perspective, thus producing a solid scientific theory of the interplay between the "natural" (in the sense of organismic) development or maturing processes and the learning and instruction procedures and contexts. Finally, in humans, learning can be identified first and foremost in the more complex processes of higher psychological functioning. It is at that level of complexity that humans typically show attributes such as agency and rational reasoning (Vygotsky 1978). With the synthesis of Cole (1996) and several other partial theories that came afterwards, a solid scientific framework is now in place. It is our suggestion to integrate this scientific theory within the capability approach.

(b) Development of the Scientific Theory

The combination of development and learning, in the form of Vygotsky's model, paved the way to develop the scientific theory, with the usual scientific assessment procedures. Cole's *Cultural Psychology* (1996) is a major work in this respect: he picks up the "two roots" of psychology in Wundt. One is the experimental tradition, which has been abundantly elaborated in observational and laboratory studies for a century and a half; the other root is the "*Völkerpsychologie*" of Wundt (1900–1920), which was left underdeveloped for a long time. Only the secondary and almost peripheral studies in cross-cultural psychology focussed on this second root, at least until the socio-cultural synthesis came along (with the revival of Vygotsky and the Russian school, up to Mikhail Bakhtin, starting in the 1960s). This line of study forms a solid body of scientific research with both a tremendous amount of empirical work (e.g., in excellent journals such as *Mind, Culture, and Activity* and *Culture and Psychology*) and ambitious theoretical work.

In his synthesising work Cole (1996) develops a theory of artefacts: human beings continuously make use of different types of artefacts in the development and learning processes. Cole distinguishes between primary artefacts (material things that can be found in nature, e.g., a cave), secondary artefacts (those that refer to the primary ones, e.g., a home in the shape of a cave), and tertiary artefacts. The latter are products of imagination and are as such largely detached from reality as observed or experienced: For example, an architect designs a home in the shape and style of his imagination. Cultural artefacts are overwhelmingly made or produced by humans: language, art, social categories, etc. The notion of culture then "represents the species-specific environment of human life that is constituted of the accumulated *artefacts* of prior generations [original emphasis], extending back to the beginning of the species" (Cole and Gajdamashko 2008, p. 131).

In the present chapter, we want to link this line of scientific research to the ethically inspired capability theory: fitting in the socio-cultural approach on development and learning provides the capability theory with a "motor" mechanism that makes it possible to understand the systematic workings of learning processes. Moreover, it then becomes possible to control and assess the manipulation (in a technical sense) of learning processes in a scientific manner. The advantage of this line of approach is that the argument on behalf of the capability theory is strengthened because it is less dependent on voluntarism, let alone on a Rousseauan romantic idea of education of and by merely well-meaning people.

Learning theories within the cultural-historical tradition also take into account the fact that individuals participate in groups and actively contribute to the creation of groups; such theories also acknowledge culture and social change: "Development is a process of *people's changing participation in socio-cultural* *activities of their communities*. People contribute to the process involved in socio-cultural activities at the same time that they inherit practices invented by others" (Rogoff 2003; cited in Cole and Gajdamashko 2008, p. 138, italics in the original). As such, the capability approach can become a tertiary artefact that is produced and transmitted inter-generationally.

Theories of learning contribute to insight into agency and identity formation, thus additionally illuminating how the capability approach can be put into practice, and how the process of imagining this alternative way of life can inspire people's agency and identity formation. Holland, Lachicotte, Skinner, and Cain (1998) draw on Bakhtin's theorizing on power, status, conflicts, and struggles and situations of heteroglossia to complement Vygotsky's socio-genetic concept of the self. Agency and identity are, for example, found in the label of "self-authoring":

> A Bakhtinian "space of authoring" is then very much a particular "zone of proximal development," and one that is extremely important in an explication of the development of identities as aspects of history-in-person. Bakhtin does not take development as the centre of his concerns, as does Vygotsky. Yet he does write about differences between the neophyte, given over to a voice of authority, and the person of greater experience, who begins to rearrange, reword, rephrase, reorchestrate different voices, and, by this process, develops her own "authorial stance (183).

Holland and Lachicotte (2007) invoke Vygotsky's concepts of semiotic mediation and higher psychological functions in the formulation of Meadian identities, that is, they develop a socio-genetic view of how human beings are both shaped by and give form to the social and cultural worlds in which they live. The work of Holland and Lachicotte draws on and is illustrated by an extensive literature review of identity formation and social movements in different places in the world. Their research contributes to the understanding of social change—for example, how a newly developed "authorial stance" may provide a new and adequate "answer" to a particular situation involving social relations with other persons, who, in a Meadian reading, take certain cultural and social positions (and possibly are in power). The new form of practice may become a cultural artefact that is significant to the mediation of behaviour in future activities, and as such it is a heuristic product: "A Vygotskyan approach [that] values the cultural production of new cultural resources can be seen as means, albeit a contingent one, of bringing about social and cultural change" (p. 116).

Viewing the capability approach as a cultural artefact, that is, as an alternative for a just society, which can be introduced, contextualised, transformed, put into practice in diverse contexts, and become an "authorial" stance from which people can start to act and work toward social justice, can provide a theoretical framework that leads to insight into processes of agency and can be empirically verified. This framework provides maximal room for inclusive perspectives that are sensitive to the rich diversity of (indigenous) languages, cultural practices, practices of knowledge and learning, and worldviews around the globe.

4 A Cosmopolitical Reading of the Dialogical Self

In “Not for profit” (2011), Nussbaum makes a plea for the arts and humanities and for cultural diversity and multilingualism in education for learning democratic citizenship and fostering critical thinking, empathy and imagination, and being able to enter into dialogue. Students and citizens need to be able to acquire skills to learn to deal with similarities and differences between people, groups, and communities, and to come to a shared understanding that is needed to be able to solve problems in increasingly diversified societies. Another view that may be supportive of the conception of the person in the capability approach can be found in a cosmopolitical reading of the dialogical self. We argue that this cosmopolitical reading, which provides insight into the complex, dynamic, and multilayered identities of citizens in a globalized world in which people from different life worlds increasingly come into contact and interact with each other, and in which intercultural learning is a necessary tool for democratic world citizenship (De Clerck and Pinxten 2012).

Hubert Hermans (2001b) developed the notion of the dialogical self at the intersection of William James’s psychology of the self and Bakhtin’s polyphony (multiplicity of voices and worldviews) and internal and external dialogical relationships. The notion of the dialogical self is increasingly prominent in Western culture, but may be relevant to the whole world because of urbanisation and other outcomes of globalisation.

As cultural anthropologists with a comparative perspective on learning processes in cultures, we claim that Western culture has been promoting a mono-cultural, consistency-driven view of the individual for centuries: In Western Europe, notably, a human being could only live as a person (later an individual) to the extent that he or she could become a Christian human being (Pinxten 2010). The totalistic character of Christianity induced this form of wholeness or personhood as an educational ideal for centuries (up to the 18th century and the Enlightenment, according to Blom 2010). An equally mono-cultural Enlightenment ideal in fact perpetuated a similar though, in a sense, antithetical view of the person over the next two centuries (Hermans 2001a; Pinxten 2007): the Christian tenets weakened and were gradually replaced by a secular, Eurocentric frame of reference that demythologised nature and enthroned reason as the foremost norm. It is only with a short and fashionable attack on the idea of any master discourse that postmodernist thinkers (such as Michel Foucault and Jacques Derrida) would try to shatter the ideal of the consistent person (Hermans 2005). In the present era in the West, a soft version of “a split personality” seems to have become more common: For example, young people declare that they believe in “something,” and at the same time practice Zen, but also participate in one or more lifestyle groups. They carry on a continuous dialogue within themselves and in interpersonal contacts between all these facets of identity. Empirical data are beginning to emerge on this possibly new type of personhood (see Hermans 2005).

It seems to us that the latter may be yet another and rather different path of mental growth in terms of the capability approach.

With the globalization of information and cultural goods, and with the intense urbanization occurring throughout the world, cultural and religious mixity are becoming a general trend. At the level of education, this entails a greater commonality of complex and diverse identities within a person: Through schooling, modern human beings everywhere are becoming "Newtonians," while at the same time literally hundreds of religious denominations can be found in large urban complexes or deeply urbanised regions. For example, new megalopolises in South America witness intense competition between old and new religions and lifestyles (de Theije 2009), and any historic old European city like e.g. Ghent, Belgium (population 260,000), counts residents from more than 150 non-European cultures and religions (Pinxten and Dikomitis 2009).

Within this reshuffled and reshuffling context of cultural diversity, the format of the dialogical self is on the rise. Our suggestion is that the capability approach should be able to address its ambition within this diversified context. Again, the culturally sensitive theory of learning we have been bringing into the discussion can be helpful here. In other words, the broad perspective of humanism that lies at the basis of capability theory is likely to stand as the only or the best ideal that can be devised in the world as it presents itself in its diverse and layered way. The learning processes, then, should be such that they can account for and cope with this diversity.

> If we do not insist on the crucial importance of the humanities and the arts, they will drop away, because they do not make money. They only do what is more precious than that, make a world that is worth living in, people who are able to see other human beings as full people, with thoughts and feelings of their own the deserve respect and empathy, and nations that are able to overcome fear and suspicion in favour of sympathetic and reasoned debate. (Nussbaum 2011: 143).

Although we do not know of any explicit use of the notions of dialogical self together with the socio-cultural approach to learning, mutual interest and attempts at collaboration are growing (Hermans 2005).

5 Illustrative Case Studies

In the last part of this Appendix, we illustrate the potential power of our proposed reformulation of capability theory-cum-learning theory and the dialogical self with two anthropological case studies of formal and informal learning processes in deaf communities: one set at Gallaudet University, the world's only liberal arts university for deaf people in Washington, DC, and the other in the West African nation of Cameroon.

(a) Empowerment in International Deaf People at Gallaudet University

The first case study, of international deaf people at Gallaudet University, illuminates how sociocultural theory and the dialogical self highlight the processes of capability expansion, critical thinking, and narrative imagination. It also provides insight into the process of imagining an alternative way of life that inspires (empowering) identity formation and (translocal) agency and control over one's life. This is embodied in the cultural artefact of "being a strong deaf person" that is available in the zone of proximal development that the university environment and peer context create.

In different parts of the world, deaf people experience social barriers and are limited in being social actors and managing their own lives (e.g., in marrying, graduating from school, setting up a business, communicating with hearing people). When international deaf people come to study at Gallaudet University, the world's only liberal arts university for deaf people, they enter a new cultural world. (For a full description of the Gallaudet case study, see De Clerck 2009.) Holland et al. (1998) employ the concepts of "figured worlds" and "cultural worlds" to refer to "a socially constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland 1998: 52). In the narratives of international deaf people (which are a "genre" in the cultural world of Gallaudet), the stage before arriving at Gallaudet is mostly perceived as being marked by "local," limiting or "negative" constructions of deaf identity. Gallaudet provides deaf people with cultural resources to construct positive or "empowered" ("strong" is also used) deaf identities. Gallaudet can be conceptualised as a zone of proximal development: International deaf people's participation in activities on campus and experience of peer support contribute to their personal development and identity formation.

DT, a deaf man from Colombia, narrates how he came into contact with deaf adults in the deaf club and learned about deaf lives in Colombia. He understood that deaf people's limited access to education and the cultural position of deafness in Colombia channelled deaf people into blue-collar jobs. When he learned about the life of deaf people in the United States and saw Gallaudet University on television, he realised that there was an alternative:

> My father was a doctor and very successful, while these people [who worked in factories in Colombia] weren't. That was strange, and it bothered me. Would that be my future? Because I am deaf? I didn't want that, working in a factory. Some people have got education, while other people haven't. What happened to them? They couldn't read and write. I visualised how that would impact me. I started planning, talked with friends some more, and learned that the US was good because there were interpreters, job opportunities, comfortable living, and deaf people, and so on. I had a friend who came back from the US, and he said that he had finished college. I asked how he had done that, and he told me that he had interpreters. He also told me how they used TTYs to communicate on the phone, and showed me a TTY. I thought, wow, I can do that and be successful… So that's how the US has always been in my mind. My goal was set… I knew about Gallaudet because of the

> [Deaf President Now] protest in 1988. Four months later, I flew here to the US. I remembered [seeing] the televised protest in Colombia. Deaf friends had told me about it. I saw all these deaf people so strong, and I was elated. That influenced me. (DT personal communication)

Gallaudet is related to imagining an alternative way of life and searching for a sense of belonging and an identity as a "strong deaf person." DT arrived at Gallaudet only to find out that he couldn't afford the tuition fee. He would only achieve his goal many years later.

De Clerck (2007) has employed the concept of "deaf ways of education" to refer to the transfer of "deaf cultural rhetoric", deaf cultural and visually oriented practices of knowledge and learning, language, and alternative life trajectories through informal and transnational contact with empowered deaf peers. Coming into contact with these forms of deaf knowledge and a barrier-free environment for deaf people (Jankowski 1997) raises consciousness; that is, it "wakes up" deaf people and empowers them. "Deaf cultural rhetoric" (Jankowski 1997) includes the following discourses: *Sign language* is a bona fide language. Deaf people share social and cultural patterns and traditions (*deaf culture*); they can therefore identify with a positive *deaf identity* construction as members of an ethnolinguistic minority. This rhetoric of *deaf can* focuses on the strength of deaf people to live up to their potential and counters paternalism. Recently, these discourses have become supplemented by the discourse of *deaf rights* (referring to the protection of sign languages, deaf cultures, and the linguistic and cultural identity of deaf people in the United Nations constitution and in legislation at the national level). Deaf cultural rhetoric and American Sign Language can be viewed as cultural artefacts that evoke the conceptual world of Gallaudet.

From a Vygotskyan perspective and in the theory of Holland et al. (1998), "semiotic mediation" enables individuals to liberate themselves from being determined by the environment and frees them to control their reactions and behaviour. The new cultural artefacts are viewed by international deaf people as preferred tools that provide them with an identity construction of a *strong* deaf person that (as a "higher mental function," per Vygotsky 1978) guides their interaction with other people and with the world: "The ability to organize oneself in the name of an identity… develops as one transacts cultural artefacts with others and then, at some point, applies the cultural resources to oneself" (Holland et al. 1998: 113).

Through "being involved," participating in social activities at Gallaudet, people develop empowered deaf identities and the figured world of Gallaudet continues to be figured. TS, a young deaf woman from Barbados, shares her experiences:

> It's been four years now. I feel that Gallaudet has influenced me to change, yes. My English has improved, and I've learned that diverse people have different behaviours, attitudes. International people, and Americans, too. That exposure was a shock for me. When I was home [i.e., in Barbados], I moved back and forth between work and home. I didn't socialise much, and I didn't know what people's behaviours and attitudes were. I really didn't know. My parents were quite strict and overprotective. But now that I'm at Gallaudet, I'm more independent… I also learned a lot about myself through being involved in the International Student Club. It's a good challenge to try and encourage other people. The organization

> helps me to know how to work in a businesslike environment, and learn the concept of teamwork, how to interact and see different people, and that it is important to develop relationships and interact with other people rather than just doing nothing. I learned how to work with finances, how to work in different positions, how to sell things, how to have a successful organization, and how to draw people to events. I have also learned during meetings how to disagree or agree with people, how conflicts arise and how to solve problems. (TS, personal communication)

Before her arrival at Gallaudet, TS had limited access to both the hearing and the deaf worlds. From the perspectives of the international deaf people, Gallaudet enables them to take up new social roles and discover new aspects of their personalities, a process that leads to a different understanding of the person one is, the things one likes or doesn't like, etc. "Culture is integral to self-formation: In the absence of cultural resources and cultural worlds, such identities are impossible" (Holland et al. 1998: 115).

The empowered deaf identities developed at Gallaudet also guide international deaf people's translocal agency when they return to their geographical homes. In returning to local contexts, international deaf people emphasise that they carry over the self-esteem and strength-centred view they acquired at Gallaudet. Acting as a strong deaf person includes advocating for an equal position in society for oneself and for deaf peers, and focussing on the capabilities of deaf people. The notion of "self-authoring" illuminates the negotiation and transformation of deaf identities in local practice and the production of new cultural resources that may lead to social change.

On his arrival at Gallaudet, JM, a deaf man from Botswana, was very surprised to see deaf people drive, something he had never seen in his home country. He learned to drive, and even managed to get his own car. When he visited Botswana after a couple of years during a summer break, he wanted to drive in his country, too. However, he was confronted with a sociality of spoken language and exclusion of deaf people:

> In America, people from outside [i.e., hearing people] can sign. I go to an office and people sign. When I go to my country, then I have to write—slow communication. Sometimes hearing people in my country will not help deaf people… That happened for the first time when I wanted to drive in my country. I went to the office and told that person, "I am deaf. I came here to see you because I want to drive." The woman said, "Oh." She laughed. "You are deaf?" "Yes." "Oh." And she gave me a form: "You go and fill out the form and when you are done, you come back." So I filled out the form, and when that was done, I came back. There was a line of people waiting, all hearing people, and I joined the line. I got to the desk and handed over the paper. I said, "Here you are. I am deaf." "Wait here, please." And she put me on the side: "Wait. Wait." The hearing people moved on in the line, moved on, and moved on. What is that? I became upset. I left, I gave up. Then I stayed at home and I wondered: Why are hearing people there in the line, whereas deaf can't be? Why are they different? (JM, personal communication)

Back in the United States, JM reflected on sociality conflicts (spoken language versus sign language, and deaf people who are not treated equally and not supposed to drive versus deaf people as drivers and, more generally, participants in society), and he realised that he would have to advocate for the things that are common sense at Gallaudet and in the United States. While distancing himself from his old identity construction that had developed in the time before he went to Gallaudet, he also realised that the new identity construction he developed at Gallaudet, and which inspired him to his agency, also needed transformation before it would be useful in Botswana. Dialoguing between diverse (culturally situated and conflicting) constructions of self, in relation to social positions, power, and conflict, he realised that he needed to produce new cultural resources to create an equal position as a deaf person. Exploring different strategies, he shifted to the culturality level and the discourses available at Gallaudet:

> What I see here in America is that deaf people have their own rights, the same rights as hearing people. I was thinking, and I remembered that before I arrived in America, [when I was still] in my country, I didn't know about deaf rights. All people have— must have— rights, and can do the same things as hearing people do. (JM, idem)

After his graduation, JM returned to Botswana, where he successfully employed this strategy and now drives comfortably. He feels "well equipped," explaining to officers "that deaf people are human beings and deserve equal rights and treatment as normal people." The only barrier left is the lack of interpreters at the motor vehicle department. His education (both formal and informal) and the authority status and cultural position conferred by his university degree enable him to negotiate his newly acquired identity as an equal citizen in a different environment. Just before returning to Botswana, he concluded his interview for the present case study by asserting, "I don't worry because I have my education." He expressed the wish that all deaf people in Botswana could get an education, in particular the ability to read and write, which is needed to communicate with the outside world.

As the example of JM illustrates, it may take a while before different voices of the self have been explored, rearranged, and reformulated, and an "authorial stance" (Holland et al. 1998: 183) is developed that enables genuine social agency.

(b) Emancipation Processes in the Cameroonian Deaf Community

The second case study, on emancipation processes in the Cameroonian deaf community, is illustrative of our argumentation on the acquisition of moral concepts by human beings as learners-in-context. The case study highlights Cameroonian deaf people's striving for dignified lives and touches upon the thresholds of the ten capabilities. Crucial for understanding deaf Cameroonians' pursuit of human development, is insight into indigenous concepts, knowledge, and ways of learning. Knowledge is produced by and transferred among Cameroonian deaf community members predominantly through visually oriented modes and in Cameroon Sign Language. In Cameroon, deaf community leaders are creating a zone of proximal development to enable deaf peers to become *serious* citizens and to live *good lives.* This zone of proximal development can be seen as an indigenous space of learning that has been shaped as alternative for formal educational structures that are still limitedly accessible for deaf Cameroonians. The capabilities of *practical reason* and *affiliation*, which have been given an "architectonic role" in the capabilities approach (Nussbaum 2011: 31) are under pressure, and the case study concentrates on both an indigenous perspective on a good life and the acquisition of this perspective and of control of one's life (practical reason) and on Cameroonian people's respect as human beings who are members of diverse social settings of a family, a deaf community, and broader society (For a description of the anthropological research project that took place in 2009–2012 and a broader discussion of the research findings, see De Clerck 2011, 2012, in press).

The strong call for "development"—which is literally expressed by deaf Cameroonians by this word—can be situated in the sense of moving between hope and hopelessness that is shared feeling among both hearing and deaf Cameroonians whose daily lives are entangled between the optimistic dreams of "development" that characterised times of decolonization and the structural barriers encountered in everyday reality when trying to achieve these dreams (Geschiere et al. 2008). Poverty, unemployment, the attractions of modernity have driven young people to the cities. These movements of social transition and urbanisation, have put community-based safety nets under pressure, and in combination with increased suffering and sickness, strong feelings of uncertainty are present. A long-term economical crisis has contributed to a loss of authority of failing state; and the social transitions are also marked by a moral crisis. Community elders are losing authority and respect since they are not able to provide answers to questions about the changed world in which young people today are living; indigenous knowledge, languages and cultural practices have been marginalized in the development of modern education that was introduced in colonial times (Johnson-Hanks 2007; Nsamenang 1992).

As in many other countries in Africa and elsewhere, the development of the deaf community in Cameroon is tied to the establishment of deaf schools. The history of formal deaf education in Cameroon is a recent one: The first deaf school was only established in 1972, and it was not until 1979 that a second deaf school was founded and sign language (initially, an American Sign Language-based form of Cameroonian Sign Language (CSL) became a language of instruction and communication. (For further information on CSL, see De Clerck, in press-a, b, Lutalo-Kiingi and De Clerck, in press). All deaf schools in Cameroon are private, and many parents cannot afford school fees. Consequently, there are many deaf children in Cameroon who do not have access to education. Most deaf schools only offer a primary education, after which deaf students leave school or are mainstreamed. There are no programs for interpreting training, or interpreting services. Some deaf schools offer some form of tutoring after school, but often deaf students give up.

Deaf schools have provided opportunities for deaf people to acquire a formal sign language, and to gain basic reading and writing skills. Deaf schools have brought deaf people together, and this collective life has been continued in "deaf gatherings" (Cameroonian sign) in cities. Deaf adults meet each other after church on Sundays, in deaf schools and after sports events on weekends, or any time deaf leaders organise meetings. CSL is a border marker, distinguishing members of the deaf community from "chickens," that is, deaf people who have grown up in rural areas, have not attended deaf schools, and/or have not been in contact with other deaf adults and who use gestural communication instead of Cameroonian Sign Language. Cameroonian deaf community members view the transition from gestural communication to formal signing in deaf schools or in the adult deaf community as a turning point in their lives, referring to the ideology of religion and educational and language hierarchies that were imported by deaf educational institutions, but also indicating an growth of their own pathways of human development. The first generation of deaf school graduates are in their forties and fifties now, and, together with some late-deafened people, they are leaders in the Cameroonian deaf community.

There is a sense of collective identification among deaf adults in terms of the African notion of an extended family; for example, deaf peers are called "brothers and sisters" in deaf community meetings and older deaf leaders are considered in parent-like roles. For those who have attended deaf schools, this sense of care and responsibility starts at the school, with deaf teachers or older deaf students taking care of younger students, but it is broader than this. Local deaf leaders are concerned about the well-being of deaf community members and often visit deaf people to see how they are doing or to act as negotiators in case of family problems. However, this "sense of family" is an ambiguous one, since it also characterized by a lack of confidence and collaboration among deaf adults. The broader moral crisis and collapse of the community safety net is probably also a factor, as well as the limited presence of deaf elders (and their epistemic authority) in this young deaf community who could serve as examples of role models who led good lives. Most deaf adults respond that they do not have any friends and need to solve problems on their own. Those who attended a deaf school often recall a feeling of belonging; however, this feeling seems to be overshadowed by the hard realities of adult life.

Limited educational opportunities have led to a high rate of functional illiteracy among the adult deaf community and a lack of educational degrees. Combined a lack of awareness and recognition of Cameroon Sign Language as bona fide language, deaf Cameroonians face a lot of challenges in being able to find employment, which prevents them from earning an income and from being able to meet economical conditions for being able to marry, which is an important aspect of personhood and citizenship. Although the Cameroon deaf community has benefited from transnational exposure through short term development cooperation projects and deaf schools are individually supported by international donors, a long-term and stable presence of international and national NGOs working with the community would be helpful to support the community's advocacy, to work towards adequate policy and legislation and to support structural opportunities of education and employment (also see Lutalo-Kiingi and De Clerck in press).

"Life is very hard here." "We suffer." The Cameroonian case study is, to borrow Paul Farmer's phrase, "an anthropology of human suffering" (Farmer 2010, p. 137). The social position of deaf people in society is one of dehumanization, exploitation, and exclusion: "Hearing people think that we are animals." Deaf people emphasise that they are seen as people who are not able to "reason" and learn. Consequently, they are also not involved in decision-making or in meetings with family members who are not deaf, and have only limited access to African indigenous education.

Domestic labour is part of African indigenous education and usually evolves with the growth of the child, without being exploitative (Nsamenang 1992). However, deaf adults report having had to work in the household or on the farm “as a slave.” The semantic framework of causality that explains deafness and disability in cosmological or magical terms (e.g., as a manifestation of witchcraft), and often casts the entire family in a negative light (see also Mekang 2007; Nsamenang 1992) when a child is born deaf or becomes deaf, is one of the reasons for the exclusion of deaf people. Nsamenang (1992) finds in poverty, hunger, ignorance, disease, and exploitation some of the reasons why parents may not be able to meet the needs of their children.

Despite little structural support and transnational exposure, “sites of empowerment” (Dei 2010: 75) can be identified. One of these sites of empowerment can be found in a moral safety net that is created by the Cameroonian deaf community and by deaf schools run by deaf directors in response to the challenges that are faced by many young deaf Cameroonians who are vulnerable in the lack of secondary education and room for continuous bonding, at an age when maturity and independence are being developed. This vulnerability is gendered. Deaf begging groups operate across borders in West and Central Africa take advantage of social marginalization and a general lack of education among Cameroonian deaf people. Promises of money attract naive, poor, and otherwise marginalised young deaf men and women, who are then exploited in a network of begging, stealing, slavery, and sexual violence and abuse. The begging groups also influence the formation of social networks among deaf people. When Cameroonian deaf people are asked whether they socialise with other deaf people or whether they have good friends, the response is usually negative. Socializing with deaf people is associated with the destructive behaviour of groups of deaf beggars in big cities.

Deaf leaders, both women and men, are actively employing the concept of *being serious* (expressed by the Cameroon sign SERIOUS), to keep and bring deaf adults onto the right track and to enhance their well-being. African indigenous education is oriented towards becoming *a good person* (Nsamenang 1992; Reagan 1996); this translates in practice in deaf indigenous education into an orientation towards being *a serious person*. This is particularly relevant in the light of deaf people’s limited access to African indigenous knowledge, which traditionally is transmitted in the family. The moral concept stands in opposition to the concept of *playing*, which refers to *bad* or *damaging* behaviour, at both the individual and collective levels. For example, deaf adults are encouraged to *take care of the future*. This includes saving money in the bank or through indigenous savings and credit associations known as money-go rounds (Rowlands 2009) (although these networks are mostly not accessible for deaf Cameroonians and a lack of confidence among Cameroonian deaf community members and structural support is a drawback for the community’s developing stable internal networks) instead of *eating* or *drinking* money, which is what is done in begging groups). The objective behind the concept is to create a moral network, and the directive “Take care of the future!” is applied to diverse realms of life, such as employment, education, marriage, sex, family, and leadership.

Deaf directors of deaf schools are employing the concept in education to raise deaf children to be *good people*. They keep deaf adults away from their deaf schools in order not to expose deaf children to bad behaviour (i.e., the behaviour of begging groups) before the children are mature enough to deal with this. A deaf leader explained how a deaf child can grow into a *serious* person (incidentally illustrating the need for secondary education, and micro-credits and employment opportunities):

> There are many deaf people who are wandering around. Some become beggars and thieves. They are without work; their families are poor and hearing people take advantage of them. What should they do? Because they don't have education, there is no one to follow up. They are responsible for themselves.
>
> I want deaf children to develop first and to follow them until they are grown-ups and have become responsible people. I want deaf people to have something to learn when they finish school, so that they can set up a business themselves or get another job and get paid every month. That's better than always having their family help and feed them. Then they can also feed their family. If deaf people can continue school until college, then we can follow up with them.

The concept of "serious citizenship" also touches upon the central capabilities of *bodily integrity* and *bodily health* and turning this into functioning, which requires access to information in Cameroonian Sign Language, which is informally organized through peer contact. One of the meanings included in the notion of '*Being serious* is *not wandering around'*. In a time of rampant HIV/AIDS, moral crisis, and sexual violence, older deaf women are drawing on their own life experiences and knowledge related to HIV/AIDS, relationships, marriage, and the unequal position of women in society in order to advise young deaf women on sexual behaviour and morality. This is also related to the concept of what it means to be a *good woman* in Cameroon and to the moral crisis and lack of *serious* men in Cameroon (cf. Johnson-Hanks 2007). Sexual abuse of young deaf girls is perceived as *damage,* and there is concern about increased teenage pregnancies. A young deaf woman explained how advice from older deaf women had helped her get her life back on track:

> I changed my life, and things are better now. I want to take care of the future. Before, I played, but I changed. I am afraid of AIDS as well…. Thank God that my life was saved! Others, hearing people, couldn't help me; my brothers and sisters couldn't help me. Older deaf women explained to me and helped me understand and be successful.
>
> Now I know. It is important to check a boy's character first. I don't like fighting and bad words. It is also not good to hurry up for sex.

Male leaders are also taking responsibility for advising young men—for instance, teaching them about pregnancy and how to use condoms, information that is often not available to young and "uneducated" deaf people. Unwanted pregnancies put a lot of financial pressure on a girl's parents when the boy does not have work or does not accept his responsibility. It is important that boys be well informed and be made aware of social and cultural patterns, such as marriage preparations: "They should be prepared for when they are ready. They cannot just "steal" a girl; that is not fine." While some form of safety-net is informally organized, structural barriers are encountered in accessing reproductive health care and information and services for gender-based violence; the lack of professional training of Cameroon Sign Language interpreters and of interpreting services in hospitals and health centres is a serious drawback for a population which does not have access to official spoken and written languages and is functionally illiterate. Informal organization of accessing health care with deaf friends who are not professionally trained to maintain privacy is risky. Though RIVs are available in public hospitals, HIV-testing often occurs late, and HIV- + deaf people are often not able to gain a full understanding of the disease and of treatment. Good practices of community-based programs in African countries that train and employ deaf staff members are available. Opportunities for political participation of Cameroonian deaf people and training in advocacy and human rights issues are needed to enable deaf Cameroonians to address and organize these matters adequately.

Without explicitly introducing this concept as a deaf indigenous notion, the diverse meanings of the sign "SERIOUS" were food for lively discussion during the presentation of the findings of the present case study to members of the Cameroonian deaf community. A sense of "ownership" was noticed when a leader recognised the concept as a "true" Cameroonian deaf concept, with the potential to inspire the Cameroonian deaf community to work towards the common goal of further development and action, and to inspire individual members to take responsibility for their own lives. Whether the concept is actually Cameroonian and is not used by deaf communities in neighbouring countries (there is transnational interaction) will need to be revealed by empirical research in the region.

6 Conclusion

Our intention in the present appendix has been to offer a short report of as well as a complement to the capability approach, as we know it, in order to strengthen it. We applaud its open and democratic humanism, which is possibly more adequate as a life-stance in the globalised world we enter than any alternative we knew so far. At the same time, we feel that the theory has remained stuck in moralistic intentions, which are weak in argumentative terms. Since we work with learning theories in our own research, we advocate that one particular branch of these, namely, the socio-cultural approach (linked to Vygotsky, and, more recently, Cole), is a likely candidate to offer forceful insights in the actual learning processes, which can be identified and implemented. Linking such forms of "situated learning" with the capability approach would provide a motor for the latter, and allow for assessment and rigorous scientific investigation of progress in capability expansion and the achievement of functionings. The actualisation of the capability approach in an urbanised and globalised world also benefits from a cosmopolitical interpretation of the dialogical self. Both the approaches of socio-cultural learning and dialogical self provide maximum room for cross-cultural and inclusive perspectives and for diversity of practices of language, culture, worldviews, knowledge, and learning, which is conditional to enable all human beings to live flourishing lives. In the appendix we illustrated the scope of the capabilities theory by means of the example of yet another minority group, i.e. deaf people and the opportunities and disadvantages they have to cope with. Although the example does not speak in any direct way on mathematics education, it may trigger the discussions on inclusion and emancipation, which are the same for the math dropouts as for the many deaf people in today's world. The presumed cause for discriminating treatment differs, but the educational and indeed the political problems involved are similar, if not the same.

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