A New Proposal of Defuzzification of Intuitionistic Fuzzy Quantities

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Abstract In this paper we propose a method to defuzzify an intuitionistic fuzzy quantity that, depending on two parameters, recover previous methods and leaves freedom to the user.

Keywords Fuzzy sets · Fuzzy quantities · Intuitionistic fuzzy sets · Evaluation

1 Introduction

In many practical applications the available information corresponding to a fuzzy concept may be incomplete, that is the sum of the membership degree and the non-membership degree may be less than one. A possible solution is to use "Intuitionistic fuzzy sets" (IFSs) introduced by Atanassov [3–5].

Working in a fuzzy context, for different reasons, an optimization problem, a decision making problem and a control system need to transform the fuzzy result into a crisp value. In a fuzzy intuitionistic context the same problem occurs. This step is classically called "defuzzification". Many results are present in several papers for fuzzy numbers. There are in literature different approaches. One of the most recurring consists of the choice of a function that maps fuzzy numbers into the reals. This idea offers two opportunities. The first is to associate a real number to a fuzzy set, the second is to transfer the total order present into the reals to fuzzy numbers permitting to choose the better solution. One of the more used function is called the "centroid" that is the abscissa of the centre of gravity of output membership function hypograph. This method has the advantage to be useful even for a general fuzzy set. In the intuitionistic context the literature is not so wide. The problems in this context

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are more than one. First of all an IFS is defined by two membership functions, the second one is that the fuzzy sets they identify an IFS may be non-normal and/or non-convex and so any result of defuzzification method introduced for fuzzy numbers is unusable. In [6, 7] the Authors present two methods, they call “de-i-fuzzification”, to transform the IFS to evaluate in a fuzzy set. The solution to the problem they propose, justified by an optimization problem, is an average of the membership function (μ) and one minus the non membership function (ν). Starting from this idea, Yager in [9] propose a defuzzification way, in the discrete case, that matches the centroid.

In this paper we propose a different idea and approach for the second step. Even if the two membership functions that individuate an IFS are non-normal and non-convex we use an α-cut method and, solving an optimization problem, we introduce a defuzzification method that has its generality in the presence of two sets of weights. For a particular case of the weights we recover the centroid. But changing the weights we found other methods present in literature for non-normal and non-convex fuzzy sets. Our result is a sort of “generator” of defuzzification methods that, thanks to the presence of the two families of weights, leaves to the user a wide choice depending on his preferences and perceptions.

In Sect. 2 we give basic definitions and notations. In Sect. 3 we deal with defuzzification of intuitionistic fuzzy sets. In Sect. 4 we introduce intuitionistic fuzzy quantities. In Sect. 5 we propose an evaluation of intuitionistic fuzzy quantities.

2 Preliminaries and Notation

2.1 Fuzzy Sets

Let X denote a universe of discourse. A fuzzy set A in X is defined by a membership function $\mu_A : X \to [0, 1]$ which assigns to each element of X a grade of membership to the set A. The height of A is $h_A = height\, A = \sup_{x \in X} \mu_A(x)$. The support and the core of A are defined, respectively, as the crisp sets $supp(A) = \{x \in X; \mu_A(x) > 0\}$ and $core(A) = \{x \in X; \mu_A(x) = 1\}$. A fuzzy set A is normal if its core is nonempty. The union of two fuzzy sets A and B is the fuzzy set $A \cup B$ defined by the membership function $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$, $x \in X$. The intersection is the fuzzy set $A \cap B$ defined by $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$.

The α-cut of a fuzzy set A, with $0 \le \alpha \le 1$, is defined as the crisp set $A_\alpha = \left\{x \in X; \mu_A(x) \ge \alpha\right\}$ if $0 < \alpha \le 1$ and as the closure of the support if $\alpha = 0$.

We say that $A \subseteq B$ if $\mu_A(x) \le \mu_B(x)$ for each $x \in X$. Note that $A \subseteq B \iff A_\alpha \subseteq B_\alpha\ \forall \alpha$.

A fuzzy set is called convex if each α-cut is a closed interval $A_\alpha = [a_L(\alpha), a_R(\alpha)]$, where $a_L(\alpha) = \inf A_\alpha$ and $a_R(\alpha) = \sup A_\alpha$.

2.2 Intuitionistic Fuzzy Sets

An intuitionistic fuzzy set (IFS) A in X is defined as

$$A = \left\{\langle x, \mu_A(x), \nu_A(x)\rangle\,;\; x \in X\right\}$$

where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ satisfy the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1\,.$$

The numbers $\mu_A(x), \nu_A(x) \in [0, 1]$ denote the degree of membership and a degree of non-membership of x to A, respectively. For each IFS A in X, we denote

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$$

the degree of the indeterminacy membership of the element x in A, that is the hesitation margin (or intuitionistic index) of $x \in A$ which expresses a lack of information of whether x belongs to A or not. We have $0 \le \pi_A(x) \le 1$ for all $x \in X$.

3 Defuzzification of IFSs

We now deal with the problem to defuzzificate an IFS A. A way to associate to an IFS A a real number may be described by the following procedure:

(i) transform the IFS A into a (standard) fuzzy set;
(ii) evaluate the standard fuzzy set by using a defuzzification method.

For step (i), in [7] the Authors called "de-i-fuzzification" a procedure to obtain a suitable fuzzy set starting from an IFS. Furthermore, they proposed to use the operator introduced in [4]

$$D_\lambda(A) = \left\{\langle x, \mu_A(x) + \lambda\pi_A(x), \nu_A(x) + (1 - \lambda)\pi_A(x)\rangle;\; x \in X\right\}$$

with $\lambda \in [0, 1]$. Note that $D_\lambda(A)$ is a standard fuzzy subset with membership function

$$\mu_\lambda(x) = \mu_A(x) + \lambda\pi_A(x)$$

In particular, they proposed $\lambda = 0.5$, as solution of the minimum problem

$$\min_{\lambda\in[0,1]} d(D_\lambda(A), A)$$

where d is the Euclidean distance. In this case the fuzzy set $D_{0.5}(A)$ is characterized by the membership function

$$\mu(x) = \frac{1}{2}(1 + \mu_A(x) - \nu_A(x)). \tag{1}$$

For step (ii), in agreement with the approach suggested in [9, Sect. 10], we may evaluate the IFS A by computing the center of gravity (COG) of the obtained fuzzy set, that is

$$Val_\lambda(A) = \frac{\int_{-\infty}^{+\infty} x\,\mu_\lambda(x)\,dx}{\int_{-\infty}^{+\infty} \mu_\lambda(x)\,dx} \tag{2}$$

with $\lambda = 0.5$.

Our aim is to propose a defuzzification method for an IFS A using α-cuts. In the following we will present a defuzzification procedure for intuitionistic fuzzy quantities.

4 Intuitionistic Fuzzy Quantities

4.1 Fuzzy Quantities

We now introduce the concept of fuzzy quantity as defined in [1, 2].

Definition 1 Let N be a positive integer and let $a_1, a_2, \dots, a_{4N}$ be real numbers with $a_1 < a_2 \le a_3 < a_4 \le a_5 < a_6 \le a_7 < a_8 \le a_9 < \cdots < a_{4N-2} \le a_{4N-1} < a_{4N}$. We call fuzzy quantity

$$A = (a_1, a_2, \dots, a_{4N};\ h_1, h_2, \dots, h_N,\ h_{1,2}, h_{2,3}, \dots, h_{N-1,N}) \tag{3}$$

where $0 < h_j \le 1$ for $j = 1, \dots, N$ and $0 \le h_{j,j+1} < \min\{h_j, h_{j+1}\}$ for $j = 1, \dots, N-1$, the fuzzy set defined by a continuous membership function $\mu : \mathbb{R} \to [0, 1]$, with $\mu(x) = 0$ for $x \le a_1$ or $x \ge a_{4N}$, such that for $j = 1, 2, \dots, N$

(i) μ is strictly increasing in $[a_{4j-3}, a_{4j-2}]$, with $\mu(a_{4j-3}) = h_{j-1,j}$ and $\mu(a_{4j-2}) = h_j$,
(ii) μ is constant in $[a_{4j-2}, a_{4j-1}]$, with $\mu \equiv h_j$,
(iii) μ is strictly decreasing in $[a_{4j-1}, a_{4j}]$, with $\mu(a_{4j-1}) = h_j$ and $\mu(a_{4j}) = h_{j,j+1}$,

and for $j = 1, 2, \dots, N-1$

(iv) μ is constant in $[a_{4j}, a_{4j+1}]$, with $\mu \equiv h_{j,j+1}$,

where $h_{0,1} = h_{N,N+1} = 0$. Thus the height of A is $h_A = \max_{j=1,\dots,N} h_j$.

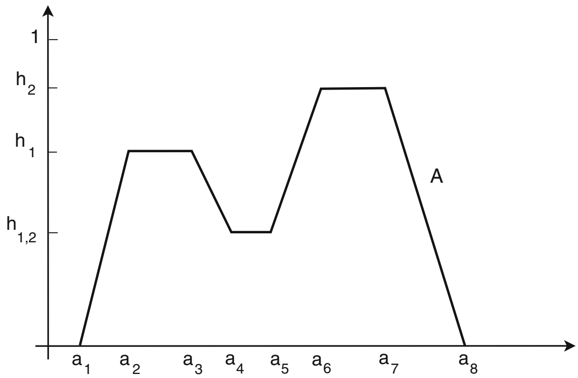


Fig. 1 Piecewise linear T1 FQ ($N = 2$)

We observe that in the case $N = 1$ the fuzzy quantity defined in (3) is fuzzy convex, that is every α-cut A_α is a closed interval. If $N \geq 2$ the fuzzy quantity defined in (3) is a non-convex fuzzy set with N humps and height $h_A = \max_{j=1,\dots,N} h_j$.

Figure 1 shows an example of piecewise linear fuzzy quantity with $N = 2$.

Proposition 1 *Let A be the T1 FQ defined in (3) with height h_A. Then for each $\alpha \in [0, h_A]$ there exist an integer n_α^A, with $1 \leq n_\alpha^A \leq N$, and $A_1^\alpha, \dots, A_{n_\alpha^A}^\alpha$ disjoint closed intervals such that*

$$A_\alpha = \bigcup_{i=1}^{n_\alpha^A} A_i^\alpha = \bigcup_{i=1}^{n_\alpha} [a_i^L(\alpha), a_i^R(\alpha)], \tag{4}$$

where we have denoted $A_i^\alpha = [a_i^L(\alpha), a_i^R(\alpha)]$, with $A_i^\alpha < A_{i+1}^\alpha$ (that is $a_i^R(\alpha) < a_{i+1}^L(\alpha)$). Thus n_α^A is the number of intervals producing the α-cut A_α.

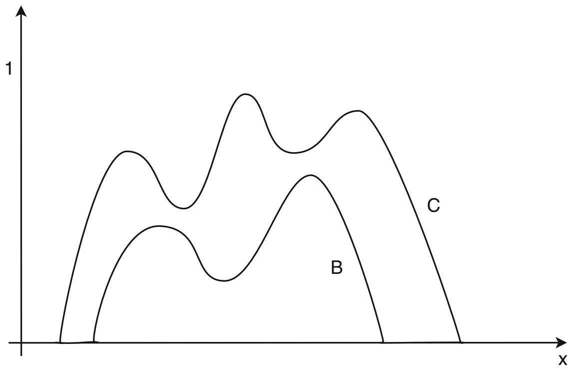
From decomposition theorem for fuzzy sets and using previous result, we get the representation

$$A = \bigcup_{\alpha \in [0, h_A]} \alpha A_\alpha = \bigcup_{\alpha \in [0, h_A]} \alpha \bigcup_{i=1}^{n_\alpha^A} A_i^\alpha = \bigcup_{\alpha \in [0, h_A]} \bigcup_{i=1}^{n_\alpha^A} \alpha A_i^\alpha. \tag{5}$$

4.2 Intuitionistic Fuzzy Quantities

Definition 2 We call *intuitionistic fuzzy quantity* (IFQ) an IFS $A = \langle \mu_A, \nu_A \rangle$ of the real line such that μ_A and $1 - \nu_A$ are membership functions of fuzzy quantities.

Fig. 2 IFQ $A = (B, C)$



If A is an IFQ we denote by A^+ the fuzzy quantity with membership function $\mu_{A^+} = \mu_A$ and by A^- the fuzzy quantity with membership function $\mu_{A^-} = 1 - \nu_A$. An IFQ A may be indifferently denoted by $A = \langle \mu_A, \nu_A \rangle$ or $A = (A^+, A^-)$.

For the sake of notation simplicity, in the following an IFQ $A = (A^+, A^-)$ will be denoted by

$$A = (B, C).$$

Thus, B and C are fuzzy quantities with membership functions $\mu_B = \mu_{A^+} = \mu_A$ and $\mu_C = \mu_{A^-} = 1 - \nu_A$, respectively (Fig. 2).

5 Evaluation of Intuitionistic Fuzzy Quantities

A useful tool for dealing with fuzzy subsets are their α-cuts. In the case of an IFQ $A = (B, C)$ we have followed the procedure suggested in [8] and we call

$$B_\alpha = \left\{ x \in X; \mu_A(x) \geq \alpha \right\}$$

and

$$C_\alpha = \left\{ x \in X; 1 - \nu_A(x) \geq \alpha \right\} .$$

From (5) the α-cuts of fuzzy quantities B and C can be decomposed as

$$B_\alpha = \bigcup_{i=1}^{n_\alpha^B} B_i^\alpha, \qquad C_\alpha = \bigcup_{j=1}^{n_\alpha^C} C_j^\alpha.$$

Such decompositions enables us to introduce the family $\mathcal{A}$ of all the closed intervals B_i^α, C_j^α, that is

$$\mathcal{A} = \{B_1^\alpha, \dots, B_{n_\alpha^B}^\alpha, C_1^\alpha, \dots, C_{n_\alpha^C}^\alpha;\ 0 \leq \alpha \leq h\}$$

where

$$h = \max\{h_B, h_C\} = h_C.$$

For convenience, by defining

$$A_i^\alpha = \begin{cases} B_i^\alpha & i = 1, \ldots, n_\alpha^B \\ C_{i-n_\alpha^B}^\alpha & i = n_\alpha^B + 1, \ldots, n_\alpha^A \end{cases} \qquad \alpha \in [0, h] \tag{6}$$

where

$$n_\alpha^A = n_\alpha^B + n_\alpha^C, \tag{7}$$

we can represent $\mathcal{A}$ as the family of all the closed intervals $A_i^\alpha = [a_i^L(\alpha), a_i^R(\alpha)]$, that is

$$\mathcal{A} = \{A_i^\alpha;\ i = 1, \ldots, n_\alpha^A,\ 0 \leq \alpha \leq h\}. \tag{8}$$

Our idea is to associate to IFQ A the nearest point to $\mathcal{A}$ respect to the Euclidean distance that depends on two parameters p and f. These two parameters will work as weights so we can say that we are looking for the real number $k^* = k^*(A; p,f)$ which minimizes the weighted mean of the squared distances.

Definition 3 We say that the real number k^* is an evaluation of the IFQ A with respect to (p,f) if it minimizes the weighted mean of the squared distances

$$\mathcal{D}_{p,f}^{(2)}(k; \mathcal{A}) = \int_0^{h_A} \sum_{i=1}^{n_\alpha^A} \left[(k - a_i^L(\alpha))^2 + (k - a_i^R(\alpha))^2\right] p_i(\alpha) f(\alpha)\, d\alpha \tag{9}$$

among all $k \in \mathbb{R}$, where, for each level α, the weights $p(\alpha) = (p_i(\alpha))_{i=1,\ldots,\tilde{n}_\alpha}$ satisfy the properties

$$p_i(\alpha) \geq 0 \qquad \sum_{i=1}^{n_\alpha^A} p_i(\alpha) = 1, \tag{10}$$

the weight function $f : [0, 1] \to [0, +\infty[$ fulfil the condition

$$\int_0^h f(\alpha)\, d\alpha = 1. \tag{11}$$

The weights we have introduced work in a different manner: $p(\alpha)$ gives the possibility to evaluate in a different way the several intervals that produce anye α-cut, the weighting function f offers the possibility to give different importance to each α-level.

Theorem 1 *The real number* k^* *which minimizes (9) with respect to* (p, f) *is given by*

$$k^* = \int_0^h \sum_{i=1}^{n_\alpha^A} mid(A_i^\alpha)\, p_i(\alpha) f(\alpha)\, d\alpha, \tag{12}$$

where

$$mid(A_i^\alpha) = \frac{a_i^L(\alpha) + a_i^R(\alpha)}{2}$$

denotes the middle point of the interval $A_i^\alpha = [a_i^L(\alpha), a_i^R(\alpha)]$.

Proof We have to minimize the function $g : \mathbb{R} \to \mathbb{R}_+$ defined by

$$g(k) = \int_0^{h_A} \sum_{i=1}^{n_\alpha^A} \left[(k - a_i^L(\alpha))^2 + (k - a_i^R(\alpha))^2\right] p_i(\alpha) f(\alpha)\, d\alpha.$$

We have

$$g'(k) = 2 \int_0^{h_A} \sum_{i=1}^{n_\alpha^A} \left[2k - a_i^L(\alpha) - a_i^R(\alpha)\right] p_i(\alpha) f(\alpha)\, d\alpha.$$

By solving $g'(k) = 0$, taking into account that p and f satisfy conditions (10) and (11), respectively, we easily obtain that the solution k^* is given by (12). Moreover we get $g''(k) = 4 > 0$ and thus k^* minimizes g. □

In the following we indicate the evaluation of the IFQ $A = (B, C)$ as

$$V(A) = V(A; p, f) = \int_0^h \sum_{i=1}^{n_\alpha^A} mid(A_i^\alpha)\, p_i(\alpha) f(\alpha)\, d\alpha. \tag{13}$$

5.1 The Centroid as Particular Case

Let us consider an IFQ $A = (B, C)$. We show that defuzzification (2) with $\lambda = 0.5$, that is

$$Val(A) = \frac{\int_{-\infty}^{+\infty} x\, \mu(x)\, dx}{\int_{-\infty}^{+\infty} \mu(x)\, dx}$$

where μ is defined as $\mu(x) = (\mu_B(x) + \mu_C(x))/2$, may be obtained by the evaluation (13) we propose choosing particular values of the parameters involved.

In order to achieve this, we recall a previous result [2, Proposition 9.3] for fuzzy quantities.

Lemma 1 *Let A be a fuzzy quantity as defined in (3) with membership function* μ_A*, height* h_A *and* α*-cuts given by (4). Then for* $t \geq 0$

$$\int_{-\infty}^{+\infty} x^t \, \mu_A(x)\, dx = \frac{1}{t+1} \int_0^{h_A} \sum_{i=1}^{n_\alpha^A} \left(a_i^R(\alpha)^{t+1} - a_i^L(\alpha)^{t+1} \right) \, d\alpha.$$

In particular, for $t = 0$

$$\int_{-\infty}^{+\infty} \mu_A(x)\, dx = \int_0^{h_A} \sum_{i=1}^{n_\alpha^A} |A_i^\alpha| \, d\alpha = \int_0^{h_A} |A_\alpha| \, d\alpha \tag{14}$$

and, for $t = 1$

$$\int_{-\infty}^{+\infty} x \, \mu_A(x)\, dx = \int_0^{h_A} \sum_{i=1}^{n_\alpha^A} mid(A_i^\alpha)\, |A_i^\alpha| \, d\alpha, \tag{15}$$

where $|A_i^\alpha| = a_i^R(\alpha) - a_i^L(\alpha)$ *is the length of interval* A_i^α *and* $|A_\alpha|$ *is the Lebesgue measure of* A_α*.*

Proposition 2 *Let* $A = (B, C)$ *be an IFQ. Let* A_i^α *be the closed intervals defined in (6). If we choose*

$$p_i(\alpha) = \frac{|A_i^\alpha|}{\sum_{j=1}^{n_\alpha^A} |A_j^\alpha|}, \qquad f(\alpha) = \frac{\sum_{j=1}^{n_\alpha^A} |A_j^\alpha|}{\int_0^h \sum_{j=1}^{n_\alpha^A} |A_j^\alpha| \, d\alpha} \tag{16}$$

then we obtain

$$V(A) = Val(A).$$

Proof Substituting the weights (p, f) given in (16) in the expression of $V(A)$ (13) we obtain

$$V(A) = \int_0^h \sum_{i=1}^{n_\alpha^A} mid(A_i^\alpha)\, p_i(\alpha) f(\alpha)\, d\alpha \; = \frac{\int_0^h \sum_{i=1}^{n_\alpha^A} mid(A_i^\alpha)\, |A_i^\alpha| \, d\alpha}{\int_0^h \sum_{i=1}^{n_\alpha^A} |A_i^\alpha| \, d\alpha}.$$

Thus from (6) and (7) we get

$$V(A) = \frac{\int_0^{h_B} \sum_{i=1}^{n_\alpha^B} mid(B_i^\alpha)|B_i^\alpha|\, d\alpha + \int_0^{h_C} \sum_{j=1}^{n_\alpha^C} mid(C_j^\alpha)|C_j^\alpha|\, d\alpha}{\int_0^{h_B} \sum_{i=1}^{n_\alpha^B} |B_i^\alpha|\, d\alpha + \int_0^{h_C} \sum_{j=1}^{n_\alpha^C} |C_j^\alpha|\, d\alpha}$$

$$= \frac{\int_{-\infty}^{+\infty} x\, \mu_B(x)\, dx + \int_{-\infty}^{+\infty} x\, \mu_C(x)\, dx}{\int_{-\infty}^{+\infty} \mu_B(x)\, dx + \int_{-\infty}^{+\infty} \mu_C(x)\, dx} = Val(A)$$

where in the second equality we have applied (14) and (15) to the fuzzy quantities B and C. □

In a similar way we show the following result.

Proposition 3 *Let $A = (B, C)$ be an IFQ. Let A_i^α be the closed intervals defined in (6). If we choose*

$$p_i(\alpha) = \begin{cases} \frac{(1-\lambda)|B_i^\alpha|}{(1-\lambda)\sum_{i=1}^{n_\alpha^B} |B_i^\alpha| + \lambda \sum_{j=1}^{n_\alpha^C} |C_j^\alpha|} & i = 1, \ldots, n_\alpha^B \\ \frac{\lambda |C_i^\alpha|}{(1-\lambda)\sum_{i=1}^{n_\alpha^B} |B_i^\alpha| + \lambda \sum_{j=1}^{n_\alpha^C} |C_j^\alpha|} & i = n_\alpha^B + 1, \ldots, n_\alpha^A \end{cases}$$

and

$$f(\alpha) = \frac{(1-\lambda)\sum_{i=1}^{n_\alpha^B} |B_i^\alpha| + \lambda \sum_{j=1}^{n_\alpha^C} |C_j^\alpha|}{(1-\lambda)\int_0^h \sum_{i=1}^{n_\alpha^B} |B_i^\alpha|\, d\alpha + \lambda \int_0^h \sum_{j=1}^{n_\alpha^C} |C_j^\alpha|\, d\alpha}$$

then we obtain

$$V(A) = Val_\lambda(A),$$

where $Val_\lambda(A)$ is defined in (2).

6 Conclusion

Our proposal of defuzzification for IFSs is given depending of two groups of parameters that are real weights. As we have said the $p_i(\alpha)$ weights act on the several intervals of every α-cut, while the second weight f works along the vertical axis changing its importance for different level of α. These two actions give a wide opportunity of freedom to the operator taking into account his behaviour more pessimistic or optimistic. Our proposal has in itself even the centroid that works on x axis. In this case, as we have seen in (16), the weight $p_i(\alpha)$, with α fixed, is the width the ith interval that forms an α-cut, suitably normalized. The weight $f(\alpha)$ is, for every α fixed, the total length of that α cut suitably normalized. As in [7] the Authors conclude their work looking for an analogous study for other operators like $F_{\alpha,\beta}$, we are also working in this direction to see if our new general defuzzification method provides some new interesting results.

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