Chapter 1
Introduction: Perspectives on Didactics of Mathematics through Michèle Artigue's Contributions

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In 2012, a memorable conference on the didactics of mathematics was organised and held in Paris, a result of the determination of an entire community to honour one of its most distinguished members, Michèle Artigue. Given the exceptional character of their highly esteemed colleague, the organisers of the meeting expected the attendance of numerous researchers in the field, but the success largely exceeded their expectations: more than three hundred people from many parts of the world kindly participated so as to express both their admiration and their affection for Michèle. An especially emotional event in the symposium was undoubtedly its conclusion when, moving smoothly and graciously between French, Spanish and English, Michèle Artigue evoked her family origins in her native Pyrenees and her commitment to the development of education in all its forms. The importance of this commitment was recognised in 2013 by the International Commission on Mathematical Instruction (ICMI)—of which Michèle was herself Vice-President (1999–2006) and President (2007–2009)—when she was awarded the Felix Klein Medal, "*in recognition of her more than thirty years of sustained, consistent, and outstanding lifetime achievements in mathematics education research and development*". More recently, she was awarded the 2015 Luis Santaló Medal by the Latino-American educational community (*Comité Interamericano de Educación Matemática*—CIAEM), showing again her influence beyond borders.

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The book *The Didactics of Mathematics: Approaches and Issues. A Homage to Michèle Artigue* is the outcome of this conference, and it utilises the same general structure. However, it offers more than merely a reflection of the event, as various well-known researchers from the field have been invited to summarise the main topics where the importance of Artigue's contribution is widely recognised. Her multiple interest areas, as a researcher involved in a wider community, give to this volume its unique flavour of diversity. In the preparation of each chapter, authors were given the opportunity to use one important paper by Artigue to initiate their reflections about a given topic. It was not always easy to identify a clear or pre-established order among this abundant and diverse material, as the different themes discussed during the conference drew on the extremely rich scientific journey of our colleague. As a conclusion to the book, Michèle Artigue—who needless to say was given *carte blanche*—offers a few personal keys to understanding the main elements that guided her throughout her scientific choices.

Since the early 1970s and up to the present day, Michèle Artigue has been closely linked to the emergence and the development of the didactics of mathematics. By observing her exemplary professional history, one can witness a new and specific research domain taking form, as well as see the difficulties that accompanied its recognition by both the academic community and, more generally, the whole education community. Academic recognition relies on the elaboration of a research programme with a specific basis, both methodological and theoretical. Such recognition also passes through the definition of themes specific to the domain and the identification of its links and differences with related disciplines likewise interested both in an epistemological and cognitive perspective on mathematics. Finally, recognition from the education community implies that researchers abandon the comfort of their research labs to become involved in social and cultural debates.

Following this conception of recognition, we have organised this opening chapter around some of the major issues related to the past, the present, and the future of the didactics of mathematics, and more generally of mathematics education: didactics as a specific research domain, the role of theoretical frameworks, the relationship to connected fields of research, and finally, the way didactics considers its relationship with the outside world of mathematics teaching and learning.

1.1 Didactics of Mathematics as a Specific Research Domain

1.1.1 At the Very Beginning: The Year 1968

In order to honour Michèle Artigue, to delineate her scientific and academic path, and because of her close association with the genesis and development of didactics of mathematics, we have to review the evolution of this research domain, both in France and more generally around the world.

In 1993, a conference entitled *Twenty years of didactics of mathematics in France* was held in Paris. As pointed out in its title, the conference dated the birth of this research domain in France to 1973. In his address to that conference, Kilpatrick (1994) stressed the arbitrariness of such a date. Mentioning the outstanding figure of Sylvestre-François Lacroix in the XVIIIth century, he wondered whether we should not trace this birth to 200 years earlier.

Sylvestre-François Lacroix is the author of a remarkable publication about mathematics teaching, as well as of a mathematics textbook intended for high school teachers of that time. Inspired by Kilpatrick, we now consider this period of the history of teaching, making connections with today's concerns.

At the end of the eighteenth century, after the French Revolution, a first "massification" of education—actually rather relative—required enlarging the number of trained teachers. The solution to this problem was the establishment of the *École Normale de Paris* in the year III (Dhombres 1992). The implemented model was that of a pyramidal system disseminating knowledge from top to bottom, with students from districts all over France following the courses in the École Normale, and then redistributing knowledge to prospective teachers in their own districts.

In mathematics, the courses were entrusted to the best mathematicians, who in practice were not really concerned about the "art of teaching", but rather eager to communicate their own creations, often the most advanced of their time. For instance, Monge explained the theory and methods of descriptive geometry, of which he was the inventor. "Pedagogical" concerns, and more precisely concerns for a "method", are nevertheless present in the work of Lacroix, who was Lagrange's assistant during that period. Lacroix published two books on algebra, in which he adopted two radically different positions relative to student work. The first book, published in 1797, is an adaptation of a book by Clairaut chosen because its structure follows "the process of invention" (Ehrhardt 2009). Writing a second book two years later, Lacroix then adopted a synthetic structure, saying that he was now convinced "that it is necessary to reduce the part of invention, and that after the student overcomes the first difficulties […], he does not need a presentation of contents following the way they were invented" (Ehrhardt 2009, p. 15).

Two centuries later, with the New Math reform, the same tendency was observed to assign the development of curricula and the organisation of education to distinguished mathematicians. Again, the main orientation was provided by books for teachers produced by these mathematicians, such as Dieudonné rejecting the geometry of the triangle in favour of a geometry based on linear algebra—as is the case in advanced mathematics. The same problems and tensions were thus encountered when developing education to adapt to the rapidly changing world of the 1960s. During that period, France aimed at a deep transformation of school education, and substantial financial and institutional support was provided. Prominent mathematicians were given the task of educational design, thus triggering conflicts between school mathematics and research mathematics. The dilemma again arose of choosing between learning approaches based on invention, and more systematic methods that logically sequenced mathematical propositions.

Kilpatrick concludes by favouring the year 1968 as the birth of didactics of mathematics, when all these conditions helped the emergence of a research domain about mathematics teaching/learning: 1968 was the year of "tranquil revolutions" in a booming Western world, of the emergence of New Math in the curricula, as well as an important increase in the number of secondary school students. In France it is also the year that the *Instituts de Recherche sur l'Enseignement des Mathématiques* (IREMs) network was established and, most importantly with regard to this book, 1968 marks Michèle Artigue's first year of teaching.

1.1.2 Reproducibility of Didactical Situations: Towards a Normal Science

At its inception, didactics benefited from solid institutional supports, among them the network of the above-mentioned IREMs, thus involving university teaching staff and facilitating the preparation and development of teachers in charge of implementing the new mathematics reform.

Nowadays (2015), didactics of mathematics—even though this precise term may still be a subject of debate within the international community—seems to be widely recognised as a research field. We can even speak of a "normal science", or more precisely of a normal domain of research.

Indeed, a common assertion by the instigators of didactics of mathematics, in France if not elsewhere, pertains to the scientific nature of the research project. For instance, Brousseau declared that:

> The didactics of mathematics presents itself, a priori, quite naturally, as the science of specific conditions for the provoked acquisition of mathematical knowledge. (Brousseau 1994, p. 51).

Can we however speak of a "normal science" the way Kuhn defines it? Such a question leads us to consider the issue of "scientificity" and experimenting in didactics of mathematics. The notion of falsifiability introduced by the philosopher Popper (1935) allows to draw a line between science and non-science. Certain explicative or predictive statements of a scientific theory should be tested through experiments. Therefore, contrarily to the *Magna Didactica* written by Comenius in 1638, the new didactics is experimental and seeks to take into account contingency. This in turn raises the issue of what is an experiment in didactics.

Michèle Artigue's contribution to this matter is crucial, questioning the reproducibility of didactical situations and developing the notion of *ingénierie didactique* in her "thèse d'état" and her frequently quoted 1990 paper (Artigue 1990a) as a method for validating hypotheses in didactics. Hence, a method of research and validation (and so of refutation) of didactical approaches was initiated.

> An experimental design based on the didactical achievements in class, i.e., the design, implementation and analysis of teaching sequences. (Artigue 1990a, p. 285).

The notion of *ingénierie didactique*, and more generally issues related to empirical studies in didactics, remain vibrant in the field and recently (in 2009) the French Didactics Summer School was entirely devoted to this topic. Specifically, questions of validity remain important regarding the number of individuals involved in an empirical study, the duration, the influence of contextual factors, the variability of curricula, and so on. In addition, the influence of settings in an empirical study, distinguishing between "laboratory experiment" and "ordinary classroom", should not be left aside.

1.1.3 The French School of Didactics of Mathematics Within an International Community

As already mentioned above, the mere use of the expression 'didactics of mathematics' may still be seen as a source of debate within the international community. In spite of such a lack of consensus and without entering into the linguistic or practical issues that may be attached to it, this term is most helpful in the present book in order to stress the specificities and the impact of the approach through which issues concerning the teaching and learning of mathematics are generally addressed in France. As is well known, one can even speak of a 'French School' of didactics of mathematics (or of mathematics education, in the usual English parlance). Most of Michèle Artigue's work can be seen as taking its source from this French tradition of *didactique des mathématiques*, and it was considered important in this book to propose some reflections on the connections with other approaches and contexts encountered in 'mathematics education'—connections in which Michèle has herself played a crucial role.

In a paper orchestrated by Tommy Dreyfus and Kenneth Ruthven, and inspired by the metaphor of travel—*Didactique goes travelling: its actual and potential articulations with other traditions of research on the learning and teaching of mathematics* (Chap. 2)—four colleagues, with interest and experience in research in the didactics of mathematics from both a French and an international perspective, were invited to reflect on the main issues underlying the actual and potential connections between the French approach and others. Arcavi first stresses the importance, in spite of the inherent difficulties, of establishing an extensive and intensive 'dialogue' between different research traditions. Kilpatrick then uses the notion of translation as a paradigm to address these difficulties, but emphasises that what is at stake is more than a mere translation of language, from French to English (or eventually other languages), but also, and more importantly, a translation between cultures. Boero takes these comments further, pointing to the fact that even a certain proximity of languages, such as may be the case with French and Italian, does not necessarily eliminate all potential obstacles for communication. Finally, Radford reminds us of key epistemological approaches on which didactique drew to develop its analysis of the genesis of new knowledge and draws our attention to two

very different strands, one emphasising the internal logic of a discipline as the motor of its development, another urging attention to the broader sociocultural context.

1.2 The Multiplicity of Theoretical Frameworks: Networking Theories

In an article dealing with the notion of example spaces, Goldenberg and Mason (2008) ended their presentation with a fierce attack against the vagueness of definitions in mathematics education, due to the absence of an axiomatic framework. They reiterate that we frequently come across multiple usages of similar terms, and usages of multiples terms having similar meanings. This attack may come as a surprise to French researchers, as for a long time the French didactics has already been poking fun at its own propensity to generate theoretical frameworks—actually motivated by its scientific orientation which implies strong theoretical work.

The problems began with the proliferation of frameworks and paradigms. However this can be seen as the normal state for an emerging science, or as an intrinsic characteristic of a field which is so extended and complex that it needs different theoretical approaches to report properly. Conducting crucial experiments would then determine which, among all the theories, would lead to a "normal science" based on a paradigm (or paradigms) accepted by the majority.

Evidence of the field's scientificity would arise from its ability to avoid fragmenting into several, and sometimes ideological, chapels, and to construct the field upon a coherent networking of various approaches and methods. In order to make progress on the question of theorisation and the multiplicity of frameworks, it is useful to go back to simple questions often repeated by Michèle Artigue in order to communicate beyond unconnected frameworks:

What is the issue that the researcher wants to address?

What is the nature of the institutional context in which this issue arises?

Normally, the theoretical framework should adjust to the questions and explain certain essential differences. This constant return to the initial issue motivating research should help to stay clear of two frequent obstacles:

- On the one hand, accumulating research within theoretical frameworks, without referring to questions.
- On the other hand, omitting the context by applying exotic or exogenous theoretical frameworks.

In this book, Kidron and Bikner-Ahsbahs present, in their chapter entitled *Networking different theoretical perspectives* (Chap. 3), the efforts of mathematics education researchers in understanding how theories can be successfully connected while respecting their underlying conceptual and methodological assumptions, a process called "networking theories". Both authors had the privilege of collaborating

with Michèle Artigue and other colleagues in exploring ways of handling the diversity of theories. They describe and explain the reasons for networking, as well as the expected difficulties of the networking process. They characterise different cases of networking and provide methodological reflections on the difficulties and benefits that accompany the networking.

Grugeon-Allys, Godino and Castela accept these differences between researchers, each engaged in a different theoretical reflection not so easy to bring closer one to the other. While acknowledging the fact that the creativity of researchers, which gave birth to a number of theories, may have created problems in the community, they offer a triple viewpoint on these matters in their chapter, *Three perspectives on the issue of theoretical diversity* (Chap. 4). The first perspective examines the richness of a multidimensional approach based on the mobilisation and networking of various well-identified theories, enabling a segmentation of reality that is well suited to the study of didactic phenomena. The second considers a possible methodology for reducing theoretical diversity based on an upward integration within an onto-didactical framework. Finally, the third perspective examines from a social viewpoint the multiplicity of theories in the didactics of mathematics and the search for connections.

1.3 Didactics of Mathematics and Related Research Fields

Didactics of mathematics covers a wide area related both to mathematics and to the conditions of its transmission and appropriation by various institutions. This naturally involves borrowing from other already highly structured fields—of course from mathematics itself, as well as from its epistemology and history—and from more distant fields such as psychology, semiotics and sociology.

1.3.1 Mathematics

The early development of didactics of mathematics was supported by leading mathematicians, such as Freudenthal in the Netherlands, Rouche in Belgium, and Revuz and Glaeser in France. These initial and very close links between mathematics didactics and mathematics itself were maintained throughout the years, evidenced by didactics research teams in France which are often still today part of university scientific departments. Such is the case, for instance, with the *Laboratoire de Didactique André Revuz*, of which Michèle Artigue is a member, which belongs both to the mathematics and the physics departments of *Université Paris Diderot*. This relationship with researchers in mathematics ensures the epistemological vigilance necessary to any didactic development.

These comments about the importance of mathematics per se clearly apply to Michèle Artigue herself. She started her own academic career as a mathematician

with research projects in mathematical logic. But she very early became interested in the teaching of analysis, with a particular attention to the research conducted by Reeb and Lutz (see Lutz et al. 1996) concerning the use of non-standard analysis in teaching. Her interest in the teaching of mathematical analysis remained unfailing throughout the years and justifies the choice of this theme in the symposium.

Analysis is considered in this book from a didactical viewpoint in Oktaç and Vivier's *Conversion, change, transition… in research about analysis* (Chap. 5). In this chapter, the authors offer a personal and original synthesis of research in the didactics of mathematical analysis. The presentation stresses once more the necessity of cross-approaches to this specific mathematics domain, where students are led to embrace a number of complex and diverse notions—e.g., real numbers, functions, limits—that are both central to and emblematic of analysis. The authors show how theories such as socio-epistemology, APOS, or the use Duval's semiotic registers or Chevallard's praxeologies can enrich didactical as well as epistemological questioning.

Two further chapters of this book pertaining not only to analysis, but also to the field of mathematics as a whole, deal with Digital technology and mathematics education. The general background to this theme was clearly captured by the title of the very first so-called 'ICMI Study', *The influence of computers and informatics on mathematics and its teaching* (Howson and Kahane 1986). Artigue's contribution to this theme was of primary importance.

Issues concerning the role and impact of digital technologies in the teaching and learning of mathematics are addressed here under two headings. In their chapter, *Core ideas and key dimensions of Michèle Artigue's theoretical work on digital tools and its impact on mathematics education research* (Chap. 6), Kieran and Drijvers revisit Artigue's classical paper (2002) by drawing out what they consider to be the core theoretical ideas and key dimensions of the body of work on tools and tool use that Michèle not only elaborated, but also inspired others to develop further. They trace the evolutionary path of these core ideas, noting the ways in which they theorise the four general key dimensions: learner, teacher, tool, and mathematics. They focus on core theoretical ideas that have been central to Michèle's work and that have impacted in various ways on the research of others: the instrumental approach to tool use, instrumental genesis, the pragmatic-epistemic duality, the technical-conceptual connection, the paper-and-pencil versus digitally-instrumented-technique relationship, the institutional aspect, and the networking of theories.

In another chapter devoted to the theme of digital technologies, *The teacher perspective in mathematics education research—a long and slow journey still unfinished* (Chap. 7), Abboud-Blanchard, one of Michèle's first PhD students, draws our attention to the need for specific studies on teachers' use of digital tools for a better understanding of teaching practices in technological environments, of their determinants, and of their evolution dynamics.

1.3.2 Epistemology and History

The epistemological vigilance we just mentioned presupposes interactive exchanges with specialists in the history of mathematics and in epistemology. Michèle has clearly pointed this out in another of her fundamental papers, in which she discussed the possible connections between epistemology and didactics. Artigue (1990b) stressed the crucial need of epistemology for the researcher in didactics. She also pointed out that some knowledge of the history of mathematics is a key component of didactical research, in order either to understand the historical development of some mathematical concept, or to understand the shaping of mathematics as a cultural activity.

Historians do not provide direct answers to mathematics education research questions, for a number of structural reasons which Chorlay and de Hosson attempt to lay out in Chap. 8, *History of science, epistemology and mathematics education research*. Echoing Artigue's 1990 paper mentioned above, Chorlay and de Hosson discuss research practices at the intersection of two autonomous fields of knowledge: mathematics education research on the one hand, and history of mathematics on the other. The two main reasons they offer for the gap between history and didactics are the deep heterogeneity of the objects and contexts of study, and the epistemological differences between the two fields. Nonetheless, heterogeneity and autonomy do not imply incommensurability. The authors conclude by stressing that history does not teach, yet there is a lot to learn from it.

1.3.3 Psychology, Cognition, Semiotics and Sociology

An initial characteristic of the didactics of mathematics in France was the conspicuous presence of psychologists specialising in mathematical cognition.[1] In particular, Gérard Vergnaud, from the *Centre national de la recherche scientifique* (CNRS), participated in the first developments of didactics of mathematics. In his paper quoted above, Brousseau, in researching the link between epistemology and cognitive sciences, concluded by giving a new and enlarged definition of what didactics of mathematics was for him.

> The didactics of mathematics thus places itself within the framework of cognitive sciences as the science of the conditions specific to the diffusion of the mathematical knowledge that is useful to the functioning of human institutions. (Brousseau 1994, p. 52).

[1]This characteristic is of course not specific to France. One can also think of the *International Group for the Psychology of Mathematics Education* (PME), an ICMI-affiliated study group established in 1976, whose fifth conference was held in Grenoble in 1981. (Gérard Vergnaud was a member of the International Committee of PME at its beginning and also PME President in 1982.)

The bonds today seem to have weakened: mathematical cognition was not specifically mentioned at the conference in honour of Michèle Artigue, whereas it was very present in the 20-year of didactics conference. We assume that this gap results from the evolution of the two research fields. Today, cognitive sciences are a highly technical field of research, mainly aimed at understanding the functioning of the brain (e.g., the journal *Mind, Brain and Education*). Besides this aim, there is a societal demand for applications to education, and especially to mathematics. In recent years Michèle has stressed the need to consider this societal demand and, in concluding a booklet about gestures and embodied cognition with a special emphasis on cognitive sciences, she calls attention to "the need of working on the conceptual and methodological implications of the integration of these approaches of cognition into our research issues, reflecting on this occasion on the potentialities and limits of our theoretical frameworks to tackle these issues" (Lagrange et al. 2012, p. 36).

By contrast, the recent evolution of didactics of mathematics tends to favour research concerns centred on the forms of diffusion of specific knowledge within the institutions, with links to specific sociological approaches. Semiotics, in various forms, is a key component of most recent research studies.

1.4 Didactics of Mathematics and the Outside World of Mathematics Education

In his 1993 lecture, Kilpatrick (1994) also stressed the fact that researchers in North America tend to resist the expression *didactics of mathematics* and prefer to use instead *mathematics education* when referring either to the teaching activity or to the research field. 'Didactics of mathematics', de facto, allows a distinction between these two dimensions: research on the one hand; and its impact on teachers, on students through the education they receive, and also on curricula changes, on the other hand.

In all these dimensions, we can notice the specific input provided by Michèle Artigue and her ability to include all of them, without avoiding any. Her commitment to the implementation and assessment of digital tools in education has already been outlined above. What may be less known is that she was involved in the fashionable and institutional demand for Inquiry-Based Education (IBE), promoted today by most science and mathematics curricula on an international scale.

1.4.1 Inquiry-Based Education

In the chapter *Inquiry-based education (IBE): towards an analysing tool to characterise and analyse inquiry processes in mathematics and natural sciences* (Chap. 9), Ouvrier-Buffet, Bosdeveix and de Hosson (respectively from mathematics, biology and physics) offer a general overview of IBE and compare different experiences in

various scientific domains. They stress that while many guidelines have been developed for helping both teachers and teacher educators to implement teaching-learning sequences involving inquiry processes, the specificities of the scientific knowledge involved is rarely taken into account. They propose a "checklist" as a tool for analysing inquiry-based sequences that are being implemented in mathematics and science classrooms.

1.4.2 The Researcher in the Wider Community

It is impossible not to mention Michèle Artigue's exceptional contribution in being, herself, a 'Researcher in the Wider Community'. Motivated by the numerous responsibilities that Michèle took on beyond her research work, a group of authors look in practical terms, in Chap. 10, at how a researcher may come to develop this kind of involvement, and at its goals and impacts.

In the first section of that chapter, Lagrange reflects on and draws lessons from responsibilities taken by Michèle in various institutions such as the IREMs (at the time of the colloquium in her honour, she was still President of the IREMs' scientific committee). Celia Hoyles then examines her personal involvement in the National Centre for Excellence in the Teaching of Mathematics (NCETM) in the UK, discussing how the Centre started and how it has evolved since 2006. Another influential and active researcher, Jill Adler, contributes to this theme by focusing on key developments in mathematics education in Africa that have emerged through the work of the International Commission on Mathematical Instruction (ICMI), in particular under the presidency of Michèle Artigue (2007–2009). Finally, French academician Jean-Pierre Kahane discusses more generally the role and position of researchers, and especially mathematicians, in contemporary society. He concludes with remarks on the place that mathematics has in civic life and on the eminent social role played in that connection by the teaching and learning of mathematics.

One aspect concerning the issue of the researcher as a member of a wider community is related to communication, and in particular to the choice of language(s) used in various public contexts, for instance, publications or conference presentations. We have already stressed above how Michèle Artigue, in her own oral presentation at the colloquium organised in her honour, made a point of using three languages which she speaks fluently, a testimony of her sensitivity to the diversity of the attendees' cultural environments. In a preliminary note about language in his contribution to this chapter, Kahane emphasises that this colloquium included three languages, which contributed to providing a successful setting, and he warns about the use of English as the preferred only possible language, especially in educational matters.

While acknowledging the convenience of having today English as a *lingua franca*, it could be argued that the use of only English in a field so diverse as didactics of mathematics could without doubt provoke an impoverishment—a risk for which the entire educational community must be fully aware. The Artigue

symposium should be seen as a testimony to the importance of further encouraging multilingual conferences, even when editorial constraints compel using only one for the post-conference book.

1.4.3 "L'École Artigue"

In a final colloquium presentation reflected in this book, a different perspective is introduced concerning the relationship between didactics of mathematics and the global issue of the teaching and learning of mathematics: that of preparing the next generation of researchers in the field. Taking as a starting point the remarkable contribution of Michèle Artigue in this regard, Haspekian, Straesser and Arzarello propose, in their chapter, *Preparing young researchers in mathematics education: beyond simple supervising* (Chap. 11), not only a testimony to her personal accomplishments as a supervisor of numerous doctoral theses, but also a more global reflection on what it means to accompany and guide PhD candidates throughout their progression in their doctoral studies. They discuss the responsibilities of various parties involved in such an endeavour, and point to some of the main pitfalls that may occur.

1.4.4 Artigue's Didactic Aventure

In the concluding chapter written by Michèle Artigue, she underlines that the conference honouring her was a strong and emotional occasion for retrospective reflection and she uses this opportunity to convey to the new generations of didacticians some elements of a history which has shaped their field of study. She invites us to follow her on a didactic route which begins with the creation of the IREM of Paris 7 in January 1969 and ends at the conference in June 2012 in which she expresses her confidence in the future of this research domain.

Before leaving the reader to explore the chapters of this book, it is important to emphasise again the specific contribution of Michèle. She brings to us a precious existential theorem: There exists a personal and rich manner to reconcile all these points of view and develop research that interacts with social demand.

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