Queueing System $GI|GI|\infty$ with n Types of Customers

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Abstract. The research of the queuing system with renewal arrival process, infinite number of n different types servers and arbitrary service time distribution is proposed. Expressions for the characteristic function of the number of busy servers for different types of customers in the system under the asymptotic condition that service time infinitely grows equivalently to each type of customers are derived.

Keywords: Queuin system · Renewal arrival process · Different types servers · Arbitrary service time · Characteristic function · Asymptotic analysis

1 Introduction

The results of research of the queuing system with infinite number of servers can be found in articles of A.V. Pechinkin [1–3], A.A. Nazarov, P. Abaev, R. Razumchik [4], B. D'Auria [5], D. Baum and L. Breuer [6,7], J. Bojarovich and L. Marchenko [8], E.A. van Doorn and A.A. Jagers [9], N.G. Duffield [10], C. Fricker and M. R. Jaïbi [11], E. Girlich [12], A. K. Jayawardene and O. Kella [13], M. Parulekar and A. M. Makowski [14] and others.

Numerous studies of real flows in various subject areas, in particular, telecommunication flows and flows in economic systems led to the conclusion about the inadequacy of the classic models of flows of random events to real data. There is an interest in investigation of flows, in which the customers are not identical and therefore require fundamentally different services [23,24]. The queuing systems with heterogeneous devices include systems of parallel service, which can be found in articles of G.P. Basharin, K.E. Samuylov [15], A. Movaghar [16], M. Kargahi [17], J.A. Morrisson, C. Knessl [18], D.G. Down [19], N. Bambos, G. Michalidis [20] and others. In these works, all systems have a Poisson input and exponential service time. In the papers [21,22], systems with parallel service of MMPP and renewal arrivals with paired customers are investigated.

In this paper, we study a queueing system with renewal arrival process and heterogeneous service. The main difference between the system in the paper from the previously considered ones is that when the customer comes in the system it is marked by i-th$(i = 1,\ldots,n)$ type in order to given probabilities. Service times for customers of different types has different arbitrary distribution function.

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2 Statement of the Problem

Consider the queuing system with infinite number of servers of n different types and arbitrary service time. Incoming flow is a renewal arrival process with n types of customers. Recurrent incoming flow is determined by the distribution function $A(x)$ of the lengths of the intervals between the time of occurrence of renewal arrival process. At the time of occurrence of the event in this stream only one customer comes in the system. The type of incoming customer is defined as i-type with probability p_i $(i = 1, \dots, n)$. It is servicing during a random time having an arbitrary distribution function B_i corresponding to the type of the customer.

Set the problem of analysis of n-dimensional stochastic process $\{l_1(t), l_2(t), \dots, l_n(t)\}$ of the number of busy servers of each type at the moment t. Incoming stream is not Poisson, therefore the n-dimensional process $\{l_1(t), l_2(t), \dots, l_n(t)\}$ is non-Markov. Consider a $(n+1)$-dimensional Markov process $\{z(t), l_1(t), l_2(t), \dots, l_n(t)\}$,where $z(t)$ —the remaining time from t until the occurrence of the following event of renewal arrival process.

Denote: $\{r_1(T), \dots, r_n(T)\}$ —the number of customers who have not completed service at time T and enrolled in at the time t, $t < T$;

$S_i(t) = P\{\tau_k^{(i)} > T - t\} = 1 - B_i(T - t)$ —the probability of non-completion of the service application type i, $(i = 1, \dots, n)$;

$1 - S_i(t)$ —the probability of completion of the service application type i, $(i = 1, \dots, n)$.

Let at the initial moment of time $t_0 < T$ the system is empty, i.e. $l_1(t_0) = \dots = l_n(t_0) = 0$. Then $l_1(T) = r_1(T), \dots, l_n(T) = r_n(T)$. Thus to study the process $\{l_1(t), \dots, l_n(t)\}$ it is necessary to investigate the n-dimensional process $\{r_1(t), \dots, r_n(t)\}$ at any point of time $t_0 \le t \le T$ and put $t = T$.

A random $(n+1)$-dimensional process $\{z(t), r_1(t), \dots, r_n(t)\}$ is a $(n+1)$-dimensional non-stationary Markov chain. Write the system of Kolmogorov differential equations for the joint probability distribution $P\{z, r_1, \dots, r_n, t\}$

$$\frac{\partial P(z, r_1, \dots, r_n, t)}{\partial t} = \frac{\partial P(z, r_1, \dots, r_n, t)}{\partial z} + \frac{\partial P(0, r_1, \dots, r_n, t)}{\partial z}(A(z) - 1)$$
$$+ \frac{\partial P(0, r_1 - 1, \dots, r_n, t)}{\partial z} p_1 S_1(t) A(z) + \dots + \frac{\partial P(0, r_1, \dots, r_n - 1, t)}{\partial z} p_n S_n(t) A(z)$$
$$- \frac{\partial P(0, r_1, \dots, r_n, t)}{\partial z} A(z) \sum_{i=1}^{n} p_i S_i(t). \quad (1)$$

Introduce the characteristic function of the form:

$$H(z, u_1, \dots, u_n, t) = \sum_{r_1=0}^{\infty} \cdots \sum_{r_n=0}^{\infty} e^{ju_1 r_1} \times \cdots \times e^{ju_n r_n} P(z, r_1, \dots, r_n, t),$$

where $j = \sqrt{-1}$ – imaginary unit.

Using (1) write the system of differential equations for the characteristic function $H(z, u_1, \ldots, u_n, t)$

$$\frac{\partial H(z,u_1,\ldots,u_n,t)}{\partial t} = \frac{\partial H(z,u_1,\ldots,u_n,t)}{\partial z} + \frac{\partial H(0,u_1,\ldots,u_n,t)}{\partial z}(A(z)-1)$$

$$+ \frac{\partial H(0,u_1,\ldots,u_n,t)}{\partial z} A(z) \sum_{i=1}^{n} p_i S_i(t)(e^{ju_i} - 1), \qquad (2)$$

$$H(z, u_1, \ldots, u_n, t_0) = R(z),$$

where $R(z)$ - stationary probability distribution of the stochastic process $z(t)$.

3 Method of the Asymptotic Analysis

3.1 Asymptotics of the First Order

We will solve the basis equation for the characteristic function (2) in the asymptotic condition that service time on appliances growths equivalently to each other, viz. $b_i \to \infty$, where $b_i = \int_0^\infty (1 - B_i(x))dx,\ i = 1, \ldots, n$ —the average value of the service time customer such as the i-th.

Denote

$$t\varepsilon = \tau,\ t_0\varepsilon = \tau_0,\ b_i = \frac{1}{q_i\varepsilon},\ u_i = \varepsilon x_i, \qquad (3)$$

$$S_i(t) = \tilde{S}_i(\tau),\ i = 1, \ldots, n,\ H(z, u_1, \ldots, u_n, t) = F_1(z, x_1, \ldots, x_n, \tau, \varepsilon).$$

Taking into account (3) we can write (2) as

$$\varepsilon \frac{\partial F_1(z,x_1,\ldots,x_n,\tau,\varepsilon)}{\partial \tau} = \frac{\partial F_1(z,x_1,\ldots,x_n,\tau,\varepsilon)}{\partial z} \qquad (4)$$

$$+ \frac{\partial F_1(0,x_1,\ldots,x_n,\tau,\varepsilon)}{\partial z}(A(z)-1) + \frac{\partial F_1(0,x_1,\ldots,x_n,\tau,\varepsilon)}{\partial z} A(z) \sum_{i=1}^{n} p_i \tilde{S}_i(\tau)(e^{j\varepsilon x_i} - 1).$$

Lemma 1. *Limit value function $F_1(z, x_1, \ldots, x_n, \tau, \varepsilon)$ at $\varepsilon \to 0$ has the form*

$$\lim_{\varepsilon \to 0} F_1(z, x_1, \ldots, x_n, \tau, \varepsilon) = F_1(z, x_1, \ldots, x_n, \tau)$$

$$= R(z) \exp\left\{ j\lambda \sum_{i=1}^{n} p_i x_i \int_{\tau_o}^{\tau} \tilde{S}_i(w) dw \right\}, \qquad (5)$$

where $\lambda = \frac{\partial R(0)}{\partial z}$.

Proof. If $\varepsilon \to 0$ in (4), then obtain:

$$\frac{\partial F_1(z,x_1,\ldots,x_n,\tau)}{\partial z} + \frac{\partial F_1(0,x_1,\ldots,x_n,\tau)}{\partial z}(A(z)-1) = 0. \qquad (6)$$

Then we look for $F_1(z, x_1, \ldots, x_n, \tau)$ as

$$F_1(z, x_1, \ldots, x_n, \tau) = R(z)\Phi_1(x_1, \ldots, x_n, \tau), \tag{7}$$

where $\Phi_1(x_1, \ldots, x_n, \tau)$ - the desired function.

If $z \to \infty$ in (4), then obtain:

$$\varepsilon \frac{\partial F_1(\infty, x_1, \ldots, x_n, \tau, \varepsilon)}{\partial \tau} = \frac{\partial F_1(0, x_1, \ldots, x_n, \tau, \varepsilon)}{\partial z} \sum_{i=1}^{n} p_i \tilde{S}_i(\tau)(e^{j\varepsilon x_i} - 1). \tag{8}$$

Expand exponents in the Eq. (8) into a Taylor series, divide the left and right side of it by ε, substitute into the received expression the function $F_1(z, x_1, \ldots, x_n, \tau)$ in the form (7) and let $\varepsilon \to 0$:

$$\frac{\partial \Phi_1(x_1, \ldots, x_n, \tau)}{\partial \tau} = j \frac{\partial R(0)}{\partial z} \Phi(x_1, \ldots, x_n, \tau) \sum_{i=1}^{n} p_i \tilde{S}_i(\tau) x_i. \tag{9}$$

Taking into account the initial condition $\Phi_1(x_1, \ldots, x_n, \tau_0) = 1$ we obtain the following expression

$$\Phi_1(x_1, \ldots, x_n, \tau) = \exp\left\{ j\lambda \sum_{i=1}^{n} p_i x_i \int_{\tau_0}^{\tau} \tilde{S}_i(w) dw \right\}. \tag{10}$$

Thus,

$$F_1(z, x_1, \ldots, x_n, \tau) = R(z) \exp\left\{ j\lambda \sum_{i=1}^{n} p_i x_i \int_{\tau_0}^{\tau} \tilde{S}_i(w) dw \right\}.$$

□

Taking into account Lemma 1 and substitutions (3) we can write the asymptotic approximate equality ($\varepsilon \to 0$):

$$H(z, u_1, \ldots, u_n, t) = F_1(z, x_1, \ldots, x_n, \tau, \varepsilon) \approx F_1(z, x_1, \ldots, x_n, \tau)$$
$$= R(z) \exp\left\{ j\lambda \sum_{i=1}^{n} p_i u_i \int_{t_0}^{t} S_i(w) dw \right\}. \tag{11}$$

For the characteristic function of process $\{l_1(t), \ldots, l_n(t)\}$ at $t = T = 0$ denote

$$h_1(u_1, \ldots, u_n) = \exp\left\{ j\lambda \sum_{i=1}^{n} p_i u_i \int_{-\infty}^{0} (1 - B_i(-w)) dw \right\}$$
$$= \exp\left\{ j\lambda \sum_{i=1}^{n} p_i u_i b_i \right\}. \tag{12}$$

The function $h_1(u_1, \ldots, u_n)$ will be called the asymptotics of the first order for the system $GI|GI|\infty$ with heterogeneous service.

Defenition 1. *The functions*

$$h_1{}^{(i)}(u_i) = Me^{ju_il_i(t)} = h_1(0,\dots,u_i,\dots,0) = \exp\{j\lambda p_i u_i b_i\},\ i=1,\dots,n,$$

will be called the asymptotics of the first order for the characteristic function of the busy servers of any type in system $GI|GI|\infty$ with heterogeneous service.

Consider the asymtotics of the second order for more accurate approximation.

3.2 Asymptotics of the Second Order

Consider the function $H(z,u_1,\dots,u_n,t)$ in the form of

$$H(z,u_1,\dots,u_n,t) = H_2(z,u_1,\dots,u_n,t)\exp\left\{j\lambda\sum_{i=1}^{n}p_iu_i\int_{t_0}^{t}S_i(w)dw\right\}. \quad (13)$$

Using (13) in (2) obtain the expression for $H_2(z,u_1,\dots,u_n,t)$:

$$\frac{\partial H_2(z,u_1,\dots,u_n,t)}{\partial t} + H_2(z,u_1,\dots,u_n,t)j\lambda\sum_{i=1}^{n}p_iS_i(t)u_i$$
$$= \frac{\partial H_2(z,u_1,\dots,u_n,t)}{\partial z} + \frac{\partial H_2(0,u_1,\dots,u_n,t)}{\partial z}(A(z)-1) \quad (14)$$
$$+\frac{\partial H_2(0,u_1,\dots,u_n,t)}{\partial z}A(z)\sum_{i=1}^{n}p_iS_i(t)(e^{ju_i}-1),$$

where $\lambda = \frac{\partial R(0)}{\partial z}$.

Substitute the following in (14):

$$t\varepsilon^2=\tau,\ t_0\varepsilon^2=\tau_0,\ b_i=\frac{1}{q_i\varepsilon^2},\ u_i=\varepsilon x_i, \quad (15)$$

$$S_i(t)=\tilde{S}_i(\tau),\ i=1,\dots,n,\ H_2(z,u_1,\dots,u_n,t)=F_2(z,x_1,\dots,x_n,\tau,\varepsilon)$$

and obtain:

$$\varepsilon^2\frac{\partial F_2(z,x_1,\dots,x_n,\tau,\varepsilon)}{\partial \tau} + F_2(z,x_1,\dots,x_n,\tau,\varepsilon)j\lambda\varepsilon\sum_{i=1}^{n}p_ix_i\tilde{S}_i(\tau)$$
$$= \frac{\partial F_2(z,x_1,\dots,x_n,\tau,\varepsilon)}{\partial z} + \frac{\partial F_2(0,x_1,\dots,x_n,\tau,\varepsilon)}{\partial z}(A(z)-1) \quad (16)$$
$$+\frac{\partial F_2(0,x_1,\dots,x_n,\tau,\varepsilon)}{\partial z}A(z)\sum_{i=1}^{n}p_i\tilde{S}_i(\tau)(e^{j\varepsilon x_i}-1).$$

Theorem 1. *Limit value function $F_2(z, x_1, \ldots, x_n, \tau, \varepsilon)$ at $\varepsilon \to 0$ has the form*

$$\lim_{\varepsilon \to 0} F_2(z, x_1, \ldots, x_n, \tau, \varepsilon) = F_2(z, x_1, \ldots, x_n, \tau)$$
$$= R(z) \exp\left\{ j^2 \left[\lambda \sum_{i=1}^{n} p_i \frac{x_i^2}{2} \int_{\tau_0}^{\tau} \tilde{S}_i(w) dw \right.\right. \tag{17}$$
$$\left.\left. + \sum_{i=1}^{n} p_i^2 x_i^2 \frac{\partial f_i(0)}{\partial z} \int_{\tau_0}^{\tau} \tilde{S}_i^2(w) dw + \sum_{i=1}^{n} \sum_{g=1, g \neq i}^{n} p_i p_g x_i x_g \int_{\tau_0}^{\tau} \tilde{S}_i(w) \tilde{S}_g(w) dw \right] \right\},$$

where $\lambda = \frac{\partial R(0)}{\partial z}$ and functions $f_i(z)$ are defined by the following system of equations

$$\frac{\partial f_i(z)}{\partial z} + \frac{\partial f_i(0)}{\partial z}(A(z) - 1) + \lambda A(z) = \lambda R(z), \; i = 1, \ldots, n. \tag{18}$$

Proof. Desirable solution of the Eq. (16) should be like the following:

$$F_2(z, x_1, \ldots, x_n, \tau, \varepsilon) = \Phi_2(x_1, \ldots, x_n, \tau)$$
$$\times \left\{ R(z) + j\varepsilon \sum_{i=1}^{n} p_i x_i f_i(z) \tilde{S}_i(\tau) \right\} + O(\varepsilon^2). \tag{19}$$

Using (19) in (16), obtain:

$$R(z) j\varepsilon\lambda \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) = \frac{\partial R(z)}{\partial z} + \frac{\partial R(0)}{\partial z}(A(z) - 1) \tag{20}$$

$$+ j\varepsilon \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) \left\{ \frac{\partial f_i(z)}{\partial z} + (A(z) - 1)\frac{\partial f_i(0)}{\partial z} + \lambda A(z) \right\} + O(\varepsilon^2).$$

Hence taking into account $\frac{\partial R(z)}{\partial z} + \frac{\partial R(0)}{\partial z}(A(z) - 1) = 0$ may earn the following system of equations for the functions $f_i(z)$, $i = 1, \ldots, n$ when $\varepsilon \to 0$:

$$\frac{\partial f_i(z)}{\partial z} + \frac{\partial f_i(0)}{\partial z}(A(z) - 1) + \lambda A(z) = \lambda R(z),$$

which coincides with (18).

Expand exponents in the Eq. (16) into a Taylor series:

$$\varepsilon^2 \frac{\partial F_2(z, x_1, \ldots, x_n, \tau, \varepsilon)}{\partial \tau} = (j\varepsilon)^2 A(z) \sum_{i=1}^{n} p_i \frac{x_i^2}{2} \tilde{S}_i(\tau) \frac{\partial F_2(0, x_1, \ldots, x_n, \tau, \varepsilon)}{\partial z}$$
$$+ (j\varepsilon) \left[A(z) \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) \frac{\partial F_2(0, x_1, \ldots, x_n, \tau, \varepsilon)}{\partial z} - \lambda \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) F_2(z, x_1, \ldots, x_n, \tau, \varepsilon) \right]$$
$$+ \frac{\partial F_2(z, x_1, \ldots, x_n, \tau, \varepsilon)}{\partial z} + (A(z) - 1) \frac{\partial F_2(0, x_1, \ldots, x_n, \tau, \varepsilon)}{\partial z} + O(\varepsilon^3).$$

Substitute into received expression (19). Since $\frac{\partial R(z)}{\partial z} + \frac{\partial R(0)}{\partial z}(A(z)-1) = 0$ we can write

$$\varepsilon^2 \frac{\partial \Phi_2(x_1,\ldots,x_n,\tau)}{\partial \tau} R(z) = (j\varepsilon)^2 \Phi_2(x_1,\ldots,x_n,\tau)$$
$$\times \left[A(z)\lambda \sum_{i=1}^{n} p_i \frac{x_i^2}{2} \tilde{S}_i(\tau) + A(z) \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) \sum_{g=1}^{n} p_g x_g \tilde{S}_g(\tau) \frac{\partial f_g(0)}{\partial z} \right.$$
$$\left. -\lambda \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) \sum_{g=1}^{n} p_g x_g \tilde{S}_g(\tau) f_g(z) \right] + j\varepsilon \Phi_(x_1,\ldots,x_n,\tau) \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau)$$
$$\times \left[\lambda A(z) - \lambda R(z) + \frac{\partial f_i(z)}{\partial z} + (A(z)-1)\frac{\partial f_i(0)}{\partial z} \right] + O(\varepsilon^3).$$

Using (18) we obtain the following expression:

$$\varepsilon^2 \frac{\partial \Phi_2(x_1,\ldots,x_n,\tau)}{\partial \tau} R(z) = (j\varepsilon)^2 \Phi_2(x_1,\ldots,x_n,\tau)$$
$$\times \left[A(z)\lambda \sum_{i=1}^{n} p_i \frac{x_i^2}{2} \tilde{S}_i(\tau) + A(z) \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) \sum_{g=1}^{n} p_g x_g \tilde{S}_g(\tau) \frac{\partial f_g(0)}{\partial z} \right. \quad (21)$$
$$\left. -\lambda \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) \sum_{g=1}^{n} p_g x_g \tilde{S}_g(\tau) f_g(z) \right] + O(\varepsilon^3).$$

Divide both sides of the expression (21) by ε^2 and pass to the limit provided $\varepsilon \to 0$ and $z \to \infty$:

$$\frac{\partial \Phi_2(x_1,\ldots,x_n,\tau)}{\partial \tau} = j^2 \Phi_2(x_1,\ldots,x_n,\tau)$$
$$\times \left[\lambda \sum_{i=1}^{n} p_i \frac{x_i^2}{2} \tilde{S}_i(\tau) + \sum_{i=1}^{n} p_i x_i \tilde{S}_i(\tau) \sum_{g=1}^{n} p_g x_g \tilde{S}_g(\tau) \frac{\partial f_g(0)}{\partial z} \right]. \quad (22)$$

Solution of the differential Eq. (22) corresponding to the initial condition $\Phi_2(x_1,\ldots,x_n,\tau_0) = 1$ is the function $\Phi_2(x_1,\ldots,x_n,\tau)$ of the form:

$$\Phi_2(x_1,\ldots,x_n,\tau) = \exp\left\{ j^2 \left[\lambda \sum_{i=1}^{n} p_i \frac{x_i^2}{2} \int_{\tau_0}^{\tau} \tilde{S}_i(w)dw + \sum_{i=1}^{n} p_i^2 x_i^2 \frac{\partial f_i(0)}{\partial z} \int_{\tau_0}^{\tau} \tilde{S}_i^2(w)dw \right.\right.$$
$$\left.\left. + \sum_{i=1}^{n} \sum_{g=1, g\neq i}^{n} p_i p_g x_i x_g \frac{\partial f_i(0)}{\partial z} \int_{\tau_0}^{\tau} \tilde{S}_i(w)\tilde{S}_g(w)dw \right] \right\}. \quad (23)$$

□

Taking into account the approximate equations of the form

$$H_2(z,u_1,\ldots,u_n,t) = F_2(z,x_1,\ldots,x_n,\tau,\varepsilon)$$
$$\approx F_2(z,x_1,\ldots,x_n,\tau) = R(z)\Phi_2(x_1,\ldots,x_n,\tau).$$

Using (15) write expression for the function $H_2(z, u_1, \ldots, u_n, t)$:

$$H_2(z, u_1, \ldots, u_n, t) = R(z) \exp\left\{ j^2 \left[\lambda \sum_{i=1}^{n} p_i \frac{u_i^2}{2} \int_{t_0}^{t} S_i(w)dw \right.\right.$$
$$\left.\left. + \sum_{i=1}^{n} p_i^2 u_i^2 \frac{\partial f_i(0)}{\partial z} \int_{t_0}^{t} S_i^2(w)dw + \sum_{i=1}^{n} \sum_{g=1, g\neq i}^{n} p_i p_g u_i u_g \frac{\partial f_i(0)}{\partial z} \int_{t_0}^{t} S_i(w) S_g(w)dw \right] \right\}.$$

Then using (13) we obtain:

$$H(z, u_1, \ldots, u_n, t) = R(z) \exp\left\{ j\lambda \sum_{i=1}^{n} p_i u_i \int_{t_0}^{t} S_i(w)dw \right.$$
$$+ j^2 \left[\lambda \sum_{i=1}^{n} p_i \frac{u_i^2}{2} \int_{t_0}^{t} S_i(w)dw + \sum_{i=1}^{n} p_i^2 u_i^2 \frac{\partial f_i(0)}{\partial z} \int_{t_0}^{t} S_i^2(w)dw \right.$$
$$\left.\left. + \sum_{i=1}^{n} \sum_{g=1, g\neq i}^{n} p_i p_g u_i u_g \frac{\partial f_i(0)}{\partial z} \int_{t_0}^{t} S_i(w) S_g(w)dw \right] \right\}.$$

Denote

$$\int_{-\infty}^{0} S_i^2(w)dw = \int_{-\infty}^{0} (1 - B_i(-w))^2 dw = \int_{0}^{\infty} (1 - B_i(w))^2 dw = \beta_i,$$
$$\int_{-\infty}^{0} S_i(w) S_g(w)dw = \int_{-\infty}^{0} (1 - B_i(-w))(1 - B_g(-w))dw$$
$$= \int_{0}^{\infty} (1 - B_i(w))(1 - B_g(w))dw = \beta_{ig},$$
$$i = 1, \ldots, n, \; g = 1, \ldots, n.$$

Then for the characteristic function of the random process $\{l_1(t), l_2(t), \ldots, l_n(t)\}$ $h_2(u_1, \ldots, u_n) = Me^{j\sum_{i=1}^{n} u_l l_i(T)} = H(\infty, u_1, \ldots, u_n, T)$ at $t = T = 0$ and $t_0 \to -\infty$ we obtain

$$h_2(u_1, \ldots, u_n) = \exp\left\{ j\lambda \sum_{i=1}^{n} p_i u_i b_i + j^2 \left[\lambda \sum_{i=1}^{n} p_i \frac{u_i^2}{2} b_i \right.\right.$$
$$\left.\left. + \sum_{i=1}^{n} p_i^2 u_i^2 \frac{\partial f_i(0)}{\partial z} \beta_i + \sum_{i=1}^{n} \sum_{g=1, g\neq i}^{n} p_i p_g u_i u_g \frac{\partial f_i(0)}{\partial z} \beta_{ig} \right] \right\}. \tag{24}$$

The expression (24) will be called the asymptotics of the second order for the system $GI|GI|\infty$ with heterogeneous service.

4 Conclusion

In this paper, we construct and investigate the mathematical model of the queuing system with the renewal arrival process and heterogeneous service. The system under consideration is studied using asymptotic analysis. Namely, the expression for the asymptotic of the first and the second order are obtained for the characteristic function of the busy servers of each type.

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References

1. Pechinkin, A.V.: Boundary of change of stationary queue in queuing systems with various service disciplines. In: Proceedings of the Seminar "Problems of Stability of Stochastic Models", 109. All-Union Scientific Research Institute for System Studies, Moscow, pp. 118–121 (1985) (in Russian)
2. Pechinkin, A.V.: The inversion procedure with probabilistic priority in queuing system with extraordinary incoming flow. Stochastic processes and their applications. Mathematical research, Shtiintsa, Kishinev (1989) (in Russian)
3. Pechinkin, A.V., Sokolov, I.A., Chaplygin, V.V.: Stationary characteristics ofmultiline queuing system with simultaneous failures of devices. Comput. Sci. Appl. **1**(2), 28–38 (2007). (in Russian)
4. Abaev, P.: On mean return time in queueing system with constant service time and bi-level hysteric policy. In: Modern Probabilistic Methods for Analysis and Optimization of Information and Telecommunication Networks. Proceedings of the International Conference, Minsk, pp. 11–19 (2013)
5. Auria, B.D.: $M|M|\infty$ queues in semi-Markovian random environment. Queueing Syst. **58**(3), 221–237 (2008)
6. Baum, D.: The infinite server queue with Markov additive arrivals in space. In: Probabilistic Analysis of Rare Events. Proceedings of the International Conference, Riga, pp. 136–142 (1999)
7. Baum, D., Breuer, L.: The Inhomogeneous $BMAP|G|\infty$ queue. In: Proceedings of the 11th GI/ITG Conference on Measuring, Modelling and Evaluation of Computer and Communication Systems (MMB 2001), Aachen, pp. 209–223 (2001)
8. Bojarovich, J., Marchenko, L.: An open queueing network with temporarily non-active customers and rounds. In: Proceedings of the International Conference "Modern Probabilistic Methods for Analysis and Optimization of Information and Telecommunication Networks", Minsk, pp. 33–36 (2013)
9. Doorn, E.A., Jagers, A.A.: Note on the $GI|GI|\infty$ system with identical service and interarrival-time distributions. J. Queueing Syst. **47**, 45–52 (2004)
10. Duffield, N.G.: Queueing at large resources driven by long-tailed $M|G|\infty$-modulated processes. Queueing Syst. **28**(1–3), 245–266 (1998)
11. Fricker, C., Jaïbi, M.R.: On the fluid limit of the $M|G|\infty$ queue. Queueing Syst. **56**(3–4), 255–265 (2007)
12. Girlich, E., Kovalev, M., Listopad, N.: Optimal choice of the capacities of telecommunication networks to provide QoS-Routing. In: Proceedings of the International Conference "Modern Probabilistic Methods for Analysis and Optimization of Information and Telecommunication Networks", Minsk, pp. 93–104 (2013)

13. Jayawardene, A.K., Kella, O.: $M|G|\infty$ with alternating renewal breakdowns. Queueing Syst. **22**(1–2), 79–95 (1996)
14. Parulekar, M., Makowski, A.M.: Tail probabilities for $M|G|\infty$ input processes: I. Preliminary asymptotics. Queueing Syst. **27**(3–4), 271–296 (1997)
15. Basharin, G.P., Samouylov, K.E., Yarkina, N.V., Gudkova, I.A.: A new stage in mathematical teletraffic theory. Autom. Remote Contr. **70**(12), 1954–1964 (2009)
16. Movaghar, A.: Analysis of a dynamic assignment of impatient customers to parallel queues. Queueing Syst. **67**(3), 251–273 (2011)
17. Kargahi, M., Movaghar, A.: Utility accrual dynamic routing in real-time parallel systems. In: Transactions on Parallel and Distributed Systems (TDPS), vol. 21(12), pp. 1822–1835. IEEE (2010)
18. Knessl, C.A., Morrison, J.: Heavy traffic analysis of two coupled processors. Queueing Syst. **43**(3), 173–220 (2003)
19. Down, D.G., Wu, R.: Multi-layered round robin routing for parallel servers. Queueing Syst. **53**(4), 177–188 (2006)
20. Bambos, N., Michailidis, G.: Queueing networks of random link topology: stationary dynamics of maximal throughput schedules. Queueing Syst. **50**(1), 5–52 (2005)
21. Ivanovskaya (Sinyakova), I., Moiseeva, S.: Investigation of the queuing system $MMP^{(2)}|M_2|\infty$ by method of the moments. In: Proceedings of the Third International Conference "Problems of Cybernetics and Informatics", Baku, vol. 2, pp. 196–199 (2010)
22. Sinyakova, I., Moiseeva, S.: Investigation of queuing system $GI^{(2)}|M_2|\infty$. In: Proceedings of the International Conference "Modern Probabilistic Methods for Analysis and Optimization of Information and Telecommunication Networks", Minsk, pp. 219–225 (2011)
23. Pankratova, E., Moiseeva, S.: Queueing system $MAP|M|\infty$ with n types of customers. In: Proceedings of the 13th International Science Conference, ITMM 2014 named after A.F.Terpugov, Anzhero-Sudzhensk, pp. 356–366 (2014)
24. Pankratova, E., Moiseeva, S.: Queueing system with renewal arrival process and two types of customers. In: Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), pp. 514–517. IEEE (2014)