Methods and Practices of Three-Way Decisions for Complex Problem Solving

Hong Yu[1(✉)], Guoyin Wang[1], Baoqing Hu[2], Xiuyi Jia[3], Huaxiong Li[4], Tianrui Li[5], Decui Liang[6], Jiye Liang[7], Baoxiang Liu[8], Dun Liu[5], Jianmin Ma[9], Duoqian Miao[10], Fan Min[11], Jianjun Qi[12], Lin Shang[4], Jiucheng Xu[13], Hailong Yang[14], Jingtao Yao[15], Yiyu Yao[15], Hongying Zhang[16], Yanping Zhang[17], and Yanhui Zhu[18]

[1] Chongqing University of Posts and Telecommunications, Chongqing 400065, China
yuhong@cqupt.edu.cn
[2] Wuhan University, Wuhan 430072, China
[3] Nanjing University of Science and Technology, Nanjing 210094, China
[4] Nanjing University, Nanjing 210093, China
[5] Southwest Jiaotong University, Chengdu 610031, China
[6] University of Electronic Science and Technology of China, Chengdu 610054, China
[7] Taiyuan Normal University, Taiyuan 030006, China
[8] North China University of Science and Technology, Tangshan 063009, China
[9] Chang'an University, Xi'an 710064, China
[10] Tongji University, Shanghai 201804, China
[11] Southwest Petroleum University, Chengdu 610500, China
[12] Xidian University, Xi'an 710071, China
[13] Henan Normal University, Xinxiang 453007, China
[14] Shaanxi Normal University, Xi'an 710119, China
[15] University of Regina, Regina, SK S4S 0A2, Canada
[16] Xi'an Jiaotong University, Xi'an 710049, China
[17] Anhui University, Hefei 230601, China
[18] Hunan University of Technology, Zhuzhou 412007, China

Abstract. A theory of three-way decisions is formulated based on the notions of three regions and associated actions for processing the three regions. Three-way decisions play a key role in everyday decision-making and have been widely used in many fields and disciplines. A group of Chinese researchers further investigated the theory of three-way decision and applied it in different domains. Their research results are highlighted in an edited Chinese book entitled "Three-way Decisions: Methods and Practices for Complex Problem Solving." Based on the contributed chapters of the edited book, this paper introduces and reviews most recent studies on three-way decisions.

1 Introduction

The essential ideas of three-way decisions are commonly used in everyday life and are widely applied in many fields and disciplines, including medical decision-making, social judgement theory, hypothesis testing in statistics, management

D. Ciucci et al. (Eds.): RSKT 2015, LNAI 9436, pp. 255–265, 2015.
DOI: 10.1007/978-3-319-25754-9_23

sciences, classifications, machine learning and data mining, and peer review processes. By extracting the common elements of decision-making methods in these fields, Yao [37] introduced the notion of three-way decisions, consisting of the positive, boundary and negative rules, as a new paradigm of decision-making. A key feature of three-way decisions is to take different actions and to make different decisions for the three regions. Three-way decision approaches have been applied to areas such as decision support systems, email spam filtering, clustering analysis and so on [6,17,18,41].

The main idea of three-way decisions is to divide a universe into three disjoint regions and to process the different regions by using different strategy. By using notations and terminology of rough set theory [26,27,38], we give a brief description of three-way decisions as follows [41].

Definition 1. *Suppose U is a finite nonempty set of objects or decision alternatives and C is a finite set of conditions. Each condition in C may be a criterion, an objective, or a constraint. The problem of three-way decisions is to divide, based on the set of conditions C, U into three pair-wise disjoint regions by a mapping f:*

$$f : U \longrightarrow \{\mathrm{POS}, \mathrm{BND}, \mathrm{NEG}\}. \tag{1}$$

The three regions are called the positive, boundary, and negative regions, respectively.

By definition, the three regions POS, BND and NEG are subsets of U with $U = \mathrm{POS} \cup \mathrm{BND} \cup \mathrm{NEG}$, $\mathrm{POS} \cap \mathrm{BND} = \emptyset$, $\mathrm{BND} \cap \mathrm{NEG} = \emptyset$, and $\mathrm{NEG} \cap \mathrm{POS} = \emptyset$. Because one or two of the three regions may be the empty set $\emptyset$, $\{\mathrm{POS}, \mathrm{BND}, \mathrm{NEG}\}$ might not be a partition of U. In order to facilitate the discussion, we usually call $\{\mathrm{POS}, \mathrm{BND}, \mathrm{NEG}\}$ a tripartition of U by slightly loosening the definition of a partition. Corresponding to these three regions, the complements are as follows: $\mathrm{POS}^c = \mathrm{BND} \cup \mathrm{NEG}$, $\mathrm{BND}^c = \mathrm{POS} \cup \mathrm{NEG}$, and $\mathrm{NEG}^c = \mathrm{POS} \cup \mathrm{BND}$.

Depending on the construction and interpretation of the mapping f, there are qualitative three-way decisions and quantitative three-way decisions. In qualitative three-way decision models, the universe is divided into three regions based on a function f that is of a qualitative nature. Quantitative three-way decision models are induced by that is of a quantitative nature. An evaluation-based three-way decision model uses an evaluation function that measures the desirability of objects with reference the set of criteria.

It should be pointed out that we can have a more general description of three-way decisions by using more generic labels and names. For example, in an evaluation-based model of three-way decisions [37], we can use a pair of thresholds to divide a universe into three regions. If we arrange objects in an increasing order with lower values at left, then we can conveniently label the three regions as the left, middle, or right regions, respectively, or simply L, M, and R regions [39]. In a similar way, strategies for processing three regions can be described in more generic terms [40].

Originally, the concept of three-way decisions was proposed and used to interpret probabilistic rough set three regions. Further studies show that a theory of three-way decision can be developed by moving beyond rough set theory. In fact, many recent studies went far beyond rough sets. An edited Chinese book entitled "Three-way Decisions: Methods and Practices for Complex Problem Solving" provides a snapshot of such research efforts from a group of Chinese researchers. In order to go further insights into three-way decisions and promote further research, this paper gives a brief of results on three-way decisions reported in the edited book.

2 Studies on the Basic Issues of Three-Way Decisions

Cost-Sensitive Models Based on DTRS. Three-way decisions originate from the studies on the decision-theoretic rough set (DTRS) model. It presents a semantics explanation on how to decide a concept into positive, negative and boundary regions based on the minimization of the decision cost, rather than decision error. Therefore, three-way decisions can be viewed as a cost-sensitive decision. Li et al. incorporated the three-way decisions into cost-sensitive learning and proposed a three-region cost-sensitive classification [14]. The key content of three-region cost-sensitive classification is to classify an instance into boundary region when precise classification (decide positive instance and negative instance decisions) cannot be immediately decided [15]. It is evident that the boundary decision may achieve lower cost/risk than positive and negative decisions do, if available information for immediate decision is insufficient, which is consistent with human decision process [14,16].

Cost-Sensitive Sequential Three-Way Decisions. In real-world applications, the available information is always insufficient, or it may associate with extra costs to get available information, which leads to frequent boundary decision. However, if the available information continuously increases, the previous boundary decisions may be converted to positive or negative decisions, which forms a sequential decision process [35,36]. Li et al. proposed a cost-sensitive sequential three-way decision strategy [12,13], which simulates the human decisions on such dynamic sequential decision: from rough granule to precise granule process. The applications of cost-sensitive sequential three-way decisions in face recognition were investigated in [57], which indicated that cost sequential three-way decisions achieve lower misrecognition costs than traditional two-way decisions in the dynamic sequential decision process of face recognition.

Compared to two-way decisions approaches, three-way decisions approaches introduces deferment decision through a pair of thresholds (α, β). Therefore, for three-way decision models, a great challenge is acquirement of a set of pairs of thresholds (α, β).

Determination of the Thresholds Through Optimization. Shang and Jia [7,8] studied this problem from an optimization viewpoint. An optimization problem is constructed to minimize the decision cost. Through solving the

optimization problem, the thresholds and corresponding cost functions for making three-way decisions can be learned from given data without any preliminary knowledge. An adaptive algorithm with computational complexity $O(n^2)$ and other evolutionary algorithms were also proposed. They also studied the problem of how to obtain a two-way decisions result through a three-way decisions procedure. A two-phase classification approach is proposed by importing ensemble learning method [19].

Determination of the Thresholds Through Gini Coefficients. Zhang and Yao [42] investigated the relationship between changes in rough set regions and their impacts on the Gini coefficients of decision regions. Effective decision regions can be obtained by satisfying objective functions of Gini coefficients of decision regions. Three different objective functions are discussed in the book. The example shows that effective decision regions can be obtained by tuning Gini coefficients of decision regions to satisfy a certain objective function. It is suggested that with the new approach more applicable decision regions and decision rules may be obtained.

Determination the Thresholds by a Constructive Covering Algorithm. Zhang and Zou et al. [44] proposed a cost-sensitive three-way decisions model based on constructive covering algorithm (CCA). Zhang and Xing et al. [43] introduced CCA to three-way decisions procedure and proposed a new three-way decisions model based on CCA. According to the samples, POS, NEG and BND are gotten automatically. The new model does not need any given parameters to form the regions. This model has three advantages: (1) it is easier to process multi-categories classification; (2) it can process discrete type data and continuous type data directly; (3) the most important one is that it provides a new method to form three regions automatically for three-way decisions.

Three-Way Decision Spaces. Hu [5] established three-way decision space based on the proposed decision measurement, decision condition and evaluation function. That is, we use fuzzy lattice (complete distributive lattice with an inverse order and involutive operator) as a decision measurement tool, decision condition is unified by a mapping from universe to fuzzy lattice and evaluation function is unified through three axioms, i.e. (E1) Minimum element axiom, (E2) Monotonicity axiom and (E3) Complement axiom. Thus, three-way decisions based on fuzzy sets, interval-valued fuzzy sets, random sets and rough sets are the special examples of three-way decision spaces. At the same time, multi-granulation three-way decisions space and its corresponding multi-granulation three-way decisions are also established. We also introduce novel dynamic two-way decisions and dynamic three-way decisions based on three-way decisions spaces and three-way decisions based on bi-evaluation functions.

Three-Way Attribute Reduction. Miao and his group constructed three-way weighted entropies from the concept level to classification level, and further explore three-way attribute reduction by a novel approach of Bayesian inference [45]; established region-based hierarchical attribute reduction [46] and reduction target structure-based hierarchical attribute reduction [48] for the two-category decision

theoretic rough set model; and built an expanded double-quantitative model by logically integrating probabilities and grades and the relevant double-quantitative reduction by hierarchically preserving specific regions [47].

3 Three-Way Decisions with Rough Sets

Incremental Approaches with DTRS for Incomplete Data. Considering that incomplete data with missing values are very common in many data-intensive applications. Luo et al. [20] proposed an incremental approach for updating probabilistic rough approximations, i.e., positive, boundary and negative regions, with the variation of objects in an incomplete information system. Four different maintenance strategies of the three-way decision rules with an incremental object based on the rough approximations were implemented, respectively [21].

Dynamic Three-Way Decisions with DTRS. Considering the addition of new objects in an information system, some new attributes may appear simultaneously. Chen et al. [1] investigated the dynamic DTRS approach for updating rough approximations with respect to the variation of objects and attributes simultaneously. On consideration of the semantic explanations of three-way decisions, Liu et al. [22] proposed a "four-level" approach and integrated the existing probabilistic rough set models to a generalized research framework. In general, three-way decisions build a bridge to connect the rough sets and decision theory, and promote the development of the both two research fields. Liu et al. [23] considered the dynamic change of loss functions in DTRS with the time, and further proposed the dynamic three-way decision model. Liu et al. [24] investigated multiple-category classification problems with three-way decisions, and further proposed a dynamic two-stage method to choose the best candidate classification.

4 Three-Way Decisions with Other Theories

Three-Way Decisions with Dempster-Shafer Theory. Faced with a multi-criteria group decision making with more attributes and a great deal of alternatives, the traditional decision making methods usually have the intensive computations or too many parameters to determine. To overcome these limitations, Wang et al. [29] proposed a Dempster-Shafer theory based intelligent three-way group sorting method. The method constructs the decision evidences from computing the fuzzy memberships of an alternative belonging to the decision classes and aggregates these evidences by using the famous Dempster combination approach. The proposed method has two merits: only one parameter to determine and the decision evidences obtained from decision makers' assessments.

Three-Way Decisions with Fuzzy Sets. Zadeh had introduced the concept of fuzzy sets to address the uncertainty and fuzziness due to ambiguity and

incomplete information in real world. Zhang and her team mainly study uncertainty measures, ranking and their applications of fuzzy sets and its generalized model, including interval-valued fuzzy sets and hesitant fuzzy sets [50,51]. A new axiomatic definition of entropy of interval-valued fuzzy sets was proposed and its relation with similarity measure was discussed in [50]. A hybrid monotonic inclusion measure was proposed in [51] to quantitative ranking of any two fuzzy sets. A general model of ranking of any two interval sets and the uncertainty and ambiguity measures were discussed in detail [49] and their applications to three-way decisions was presented. Yang et al. [30] had devoted to construct the basic frame of fuzzy three-way decisions by extending three-way decisions to fuzzy case based on fuzzy set theory. In fuzzy three-way decisions, one universe is divided into three fuzzy regions satisfying some conditions rather than three pair-wise disjoint crisp regions. Some research results about fuzzy three-way decisions have been contained in the published book [41]. They introduced notions of several kinds of evaluation functions based fuzzy three-way decisions by using fuzzy set theory, and illustrated the relationships among these notions. In future research, they will focus on how to apply fuzzy three-way decisions to real-world applications by using fuzzy mathematic theory and methods. Besides, three-way approximations of intuitionistic fuzzy sets is also this group's research issue.

Three-Way Decisions with Formal Concept Analysis. Qi et al. [28] proposed the three-way concept analysis based on combining three-way decisions [37] and formal concept analysis [2]. In the framework of formal concept analysis, given a formal context (U, V, R), a formal concept (X, A) means that X contains just those objects sharing all the attributes in A and the attributes in A are precisely those common to all the objects in X. This expresses the semantics of "*jointly possessed*" between an object subset and an attribute subset in a formal context. But such concept cannot express the semantics of "*jointly not possessed*" which also exists in a formal context. The extension (or/and intension) of a three-way concept is equipped with two parts: positive and negative ones, where the positive part is used to express "*jointly possessed*" and the negative one is used to express "*jointly not possessed*". For example, a three-way concept $(X, (A, B))$ implies that all the objects in X jointly possessed every attribute in A and jointly not possessed every attribute in B.

5 Applications of Three-Way Decisions

Clustering Analysis. Yu and her group studied overlapping clustering [31], determining the number of clusters [32], incremental clustering [34] and so on, based on the three-way decision theory. In their work, a cluster is represented by an interval set instead of a single set. They use three regions to represent a cluster. Objects in the POS region belong to the cluster definitely, objects in the NEG region do not belong to the cluster definitely, and objects in BND region are fringe elements of the cluster. The advantages of the representation are that it is not only show which objects just belong to this cluster but also show which

objects might belong to the cluster intuitively. Through the further work on the BND region [33], we can know the degree of an object influences on the form of the cluster intuitively, which is very helpful in some practice of applications such as community evaluations.

Frequent Itemsets Mining. Min and his group applied three-way decisions to the incremental mining of frequent itemsets [25,52]. In these works, as new data added, the algorithm only need to check and update itemsets in the boundary region. All possible itemsets are divided into three regions, namely the positive, the boundary and the negative region. Itemsets in the positive region are already frequent. Itemsets in the boundary region are infrequent, however may be frequent after data increment in the near future. Itemsets in the negative region will not be frequent even after data increment. Therefore to keep the frequent itemsets up-to-date, one only need to check those in the boundary region, and the runtime is saved.

Text Classification. Shang and Jia combined the three-way decisions solution with text sentiment analysis to improve the performance of sentiment classification [53], and they also applied it to filter spam email to obtain a lower misclassification rate and a less misclassification cost [9]. Miao and his group established an instance-centric hierarchical classification framework based on decision-theoretic rough set model [10]. Furthermore, Zhu et al. [55] presented a two-stage three-way decision classifiers based on integration of three-way decision and traditional machine learning algorithms. Zhang et al. [56] applied three-way decisions to sentiment classification with sentiment uncertainty to deal with the problems of context-dependent sentiment classification and topic-dependent sentiment classification.

Image Processing, Video Analysis and Others. Shang and Jia have combined three-way decisions solution with fuzzy sets to propose a new fuzzy rough set model and applied it in image segmentation [3,4]; Miao and his group have proposed a novel algorithm for image segmentation with noise in the framework of decision-theoretic rough set model [11]. Shang and Jia have adopted three-way decisions to analyze video behaviors and detect video anomaly behaviors [54].

6 Conclusions

The notion of three-way decisions was introduced for meeting the needs to properly explain three regions of probabilistic rough sets. A theory of three-way decisions moves far beyond this original goal. We begin to see a more general theory that embraces ideas from many fields and disciplines. An edited Chinese book entitled "Three-way Decisions: Methods and Practices for Complex Problem Solving" marks a cornerstone in the development of three-way decisions. It provides a snapshot of most recent research on three-way decisions. This paper is a summary of the main results reported in the book. A reader is encourage to read the book for more detailed descriptions.

For studies on basic issues of three-way decisions, we review cost-sensitive models based on decision-theoretic rough sets, cost-sensitive sequential three-way decisions, approaches to computing the thresholds α and β, three-way decision spaces, and algorithms of three-way attribute reductions. In the context of rough set theory, we discuss multigranulation decision-theoretic rough set models, incremental approaches to rough set approximations, and dynamic three-way decisions. With respect to other theories, we examine three-way decisions with Dempster-Shafer theory of evidential reasoning, fuzzy sets and formal concept analysis. Finally, we touch upon applications of three-way decisions in clustering analysis, frequent itemsets mining, text classification, sentiment analysis, image processing and so on.

By introducing a sample of most recent studies on three-way decisions, we want to demonstrate the value and power, as well as the great potentials, of three-way decisions. Considering the significance and generality of three-way decisions, continued research efforts are needed. We welcome you to join us and to work on an emerging and exciting theory of three-way decisions. For additional information, please consult the homepage of three-way decisions at: http://www2.cs.uregina.ca/~twd/.

Acknowledgements. The authors would like to thank the organizers of IJCRS 2015 for inviting and encouraging them to present their results.

References

1. Chen, H.M., Li, T.R., Luo, C., Horng, S.J., Wang, G.Y.: A decision-theoretic rough set approach for dynamic data mining. IEEE Trans. Fuzzy Syst. (2015). doi:10.1109/TFUZZ.2014.2387877
2. Ganter, B., Wille, R.: Formal Concept Analysis. Mathematical Foundations. Springer, Berlin, Heidelberg (1999)
3. Guo, M., Shang, L.: Selecting the appropriate fuzzy membership functions based on user-demand in fuzzy decision-theoretic rough set model. In: Proceedings of FUZZ-IEEE, pp. 1–8 (2013)
4. Guo, M., Shang, L.: Color image segmentation based on decision-theoretic rough set model and fuzzy C-means algorithm. In: Proceedings of FUZZ-IEEE, pp. 229–236 (2014)
5. Hu, B.Q.: Three-way decisions space and three-way decisions. Inf. Sci. **281**, 21–52 (2014)
6. Jia, X.Y., Shang, L., Zhou, X.Z., Liang, J.Y., Miao, D.Q., Wang, G.Y., Li, T.R., Zhang, Y.P.: Theory of Three-Way Decisions and Application. Nanjing University Press, Nanjing, China (2012). (In Chinese)
7. Jia, X.Y., Li, W.W., Shang, L., Chen, J.J.: An optimization viewpoint of decision-theoretic rough set model. In: Yao, J.T., Ramanna, S., Wang, G.Y., Suraj, Z. (eds.) RSKT2011. LNCS, vol. 6954, pp. 457–465. Springer, Heidelberg (2011)
8. Jia, X.Y., Tang, Z.M., Liao, W.H., Shang, L.: On an optimization representation of decision-theoretic rough set model. Int. J. Approximate Reasoning **55**, 156–166 (2014)

9. Jia, X., Shang, L.: Three-way decisions versus two-way decisions on filtering spam email. In: Peters, J.F., Skowron, A., Li, T., Yang, Y., Yao, J.T., Nguyen, H.S. (eds.) Transactions on Rough Sets XVIII. LNCS, vol. 8449, pp. 69–91. Springer, Heidelberg (2014)
10. Li, W., Miao, D.Q., Wang, W.L.: Hierarchical rough decision theoretic framework for text classification. In: 9th IEEE International Conference on Cognitive Informatics, pp. 484–489. IEEE Press, New York (2010)
11. Li, F., Miao, D.Q., Liu, C.H.: An image segmentation algorithm based on decision-theoretic rough set model. CAAI Trans. Intell. Syst. **9**(2), 143–147 (2014)
12. Li, H.X., Zhou, X.Z., Huang, B.: Cost-sensitive sequential three-way decisions. In: Liu, D., Li, T.R., Miao, D.Q., Wang, G.Y., Liang, J.Y. (eds.) Three-Way Decisions Granul. Comput., pp. 42–59. Science Press, Beijing (2013). (in chinese)
13. Li, H., Zhou, X., Huang, B., Liu, D.: Cost-sensitive three-way decision: a sequential strategy. In: Lingras, P., Wolski, M., Cornelis, C., Mitra, S., Wasilewski, P. (eds.) RSKT 2013. LNCS, vol. 8171, pp. 325–337. Springer, Heidelberg (2013)
14. Li, H., Zhou, X., Zhao, J., Huang, B.: Cost-sensitive classification based on decision-theoretic rough set model. In: Li, T., Nguyen, H.S., Wang, G., Grzymala-Busse, J., Janicki, R., Hassanien, A.E., Yu, H. (eds.) RSKT 2012. LNCS, vol. 7414, pp. 379–388. Springer, Heidelberg (2012)
15. Li, H.X., Zhou, X.Z., Zhao, J.B.: Cost-sensitive classification based on three-way decision-theoretic rough sets. Theory of Three-way Decisions and Application, pp. 34–45. Nanjing University Press (2012) (in Chinese)
16. Li, H.X., Zhou, X.Z.: Risk decision making based on decision-theoretic rough set: a three-way view decision model. Int. J. Comput. Intell. Syst. **4**(1), 1–11 (2011)
17. Liu, D., Li, T.R., Miao, D.Q., Wang, G.Y., Liang, J.Y.: Three-Way Decisions and Granular Computing. Science Press, Beijing, China (2013). (In Chinese)
18. Li, H.X., Zhou, X.Z., Li, T.R., Wang, G.Y., Miao, D.Q., Yao, Y.Y.: Decision-Theoretic Rough Set Theory and Recent Progress. Science Press, Beijing, China (2011). (In Chinese)
19. Li, W., Huang, Z., Jia, X.: Two-phase classification based on three-way decisions. In: Lingras, P., Wolski, M., Cornelis, C., Mitra, S., Wasilewski, P. (eds.) RSKT 2013. LNCS, vol. 8171, pp. 338–345. Springer, Heidelberg (2013)
20. Luo, C., Li, T., Chen, H.: Dynamic maintenance of three-way decision rules. In: Miao, D., Pedrycz, W., Slezak, D., Peters, G., Hu, Q., Wang, R. (eds.) RSKT 2014. LNCS, vol. 8818, pp. 801–811. Springer, Heidelberg (2014)
21. Luo, C., Li, T.: Incremental three-way decisions with incomplete information. In: Cornelis, C., Kryszkiewicz, M., Ślęzak, D., Ruiz, E.M., Bello, R., Shang, L. (eds.) RSCTC 2014. LNCS, vol. 8536, pp. 128–135. Springer, Heidelberg (2014)
22. Liu, D., Li, T.R., Ruan, D.: Probabilistic model criteria with decision-theoretic rough sets. Inf. Sci. **181**, 3709–3722 (2011)
23. Liu, D., Li, T., Liang, D.: Three-way decisions in dynamic decision-theoretic rough sets. In: Lingras, P., Wolski, M., Cornelis, C., Mitra, S., Wasilewski, P. (eds.) RSKT 2013. LNCS, vol. 8171, pp. 291–301. Springer, Heidelberg (2013)
24. Liu, D., Li, T.R., Li, H.X.: A multiple-category classification approach with decision-theoretic rough sets. Fundamenta Informaticae **115**, 173–188 (2012)
25. Li, Y., Zhang, Z.H., Chen, W.B., Min, F.: Tdup: an approach to incremental mining of frequent itemsets with three-way-decision pattern updating. Int. J. Mach. Learn. Cybern. 1–13 (2015)
26. Pawlak, Z.: Rough sets. Int. J. Comput. Inf. Sci. **11**, 341–356 (1982)
27. Pawlak, Z.: Rough Sets. Theoretical Aspects of Reasoning about Data. Kluwer Academic Publishers, Dordrecht (1991)

28. Qi, J., Wei, L., Yao, Y.: Three-way formal concept analysis. In: Miao, D., Pedrycz, W., Slezak, D., Peters, G., Hu, Q., Wang, R. (eds.) RSKT 2014. LNCS, vol. 8818, pp. 732–741. Springer, Heidelberg (2014)
29. Wang, B., Liang, J.: A novel intelligent multi-attribute three-way group sorting method based on dempster-shafer theory. In: Miao, D., Pedrycz, W., Slezak, D., Peters, G., Hu, Q., Wang, R. (eds.) RSKT 2014. LNCS, vol. 8818, pp. 789–800. Springer, Heidelberg (2014)
30. Yang, H.L., Li, S.G., Wang, S.Y., Wang, J.: Bipolar fuzzy rough set model on two different universes and its application. Knowl. Based Syst. **35**, 94–101 (2012)
31. Yu, H., Wang, Y.: Three-way decisions method for overlapping clustering. In: Yao, J.T., Yang, Y., Słowiński, R., Greco, S., Li, H., Mitra, S., Polkowski, L. (eds.) RSCTC 2012. LNCS, vol. 7413, pp. 277–286. Springer, Heidelberg (2012)
32. Yu, H., Liu, Z.G., Wang, G.Y.: An automatic method to determine the number of clusters using decision-theoretic rough set. Int. J. Approximate Reasoning **55**(1), 101–115 (2014)
33. Yu, H., Jiao, P., Wang, G.Y., Yao, Y.Y.: Categorizing overlapping regions in clustering analysis using three-way decisions. In: Proceedings of IEEE/WIC/ACM International Joint Conferences on Web Intelligence (WI) and Intelligent Agent Technologies (IAT), vol. 2, pp. 350–357 (2014)
34. Yu, H., Zhang, C., Hu, F.: An incremental clustering approach based on three-way decisions. In: Cornelis, C., Kryszkiewicz, M., Ślęzak, D., Ruiz, E.M., Bello, R., Shang, L. (eds.) RSCTC 2014. LNCS, vol. 8536, pp. 152–159. Springer, Heidelberg (2014)
35. Yao, Y.: Granular computing and sequential three-way decisions. In: Lingras, P., Wolski, M., Cornelis, C., Mitra, S., Wasilewski, P. (eds.) RSKT 2013. LNCS, vol. 8171, pp. 16–27. Springer, Heidelberg (2013)
36. Yao, Y.Y., Deng, X.F.: Sequential three-way decisions with probabilistic rough sets. In: Proceedings of IEEE ICCI*CC 2011, pp. 120–125. IEEE (2011)
37. Yao, Y.: An outline of a theory of three-way decisions. In: Yao, J.T., Yang, Y., Słowiński, R., Greco, S., Li, H., Mitra, S., Polkowski, L. (eds.) RSCTC 2012. LNCS, vol. 7413, pp. 1–17. Springer, Heidelberg (2012)
38. Yao, Y.Y.: The two sides of the theory of rough sets. Knowl. Based Syst. **80**, 67–77 (2015)
39. Yao, Y.Y., Yu, H.: An introduction to three-way decisions. In: Yu, H., Wang, G.Y., Li, T.R., Liang, J.Y., Miao, D.Q., Yao, Y.Y. (eds.) Three-Way Decisions: Methods and Practices for Complex Problem Solving. Science Press, Beijing (2015). (In Chinese)
40. Yao, Y.Y.: Rough sets and three-way decisions. In: Ciucci, D., Wang, G.Y., Mitra, S., Wu, W.Z. (eds.) RSKT 2015. LNCS (LNA), vol. 9436, pp. 62–73. Springer, Heidelberg (2015)
41. Yu, H., Wang, G.Y., Li, T.R., Liang, J.Y., Miao, D.Q., Yao, Y.Y.: Three-Way Decisions: Methods and Practices for Complex Problem Solving. Science Press, Beijing, China (2015). (In Chinese)
42. Zhang, Y., Yao, J.T.: Determining three-way decision regions with gini coefficients. In: Cornelis, C., Kryszkiewicz, M., Ślęzak, D., Ruiz, E.M., Bello, R., Shang, L. (eds.) RSCTC 2014. LNCS, vol. 8536, pp. 160–171. Springer, Heidelberg (2014)
43. Zhang, Y., Xing, H., Zou, H., Zhao, S., Wang, X.: A three-way decisions model based on constructive covering algorithm. In: Lingras, P., Wolski, M., Cornelis, C., Mitra, S., Wasilewski, P. (eds.) RSKT 2013. LNCS, vol. 8171, pp. 346–353. Springer, Heidelberg (2013)

44. Zhang, Y., Zou, H., Chen, X., Wang, X., Tang, X., Zhao, S.: Cost-sensitive three-way decisions model based on CCA. In: Cornelis, C., Kryszkiewicz, M., Ślęzak, D., Ruiz, E.M., Bello, R., Shang, L. (eds.) RSCTC 2014. LNCS, vol. 8536, pp. 172–180. Springer, Heidelberg (2014)
45. Zhang, X., Miao, D.: Three-way weighted entropies and three-way attribute reduction. In: Miao, D., Pedrycz, W., Slezak, D., Peters, G., Hu, Q., Wang, R. (eds.) RSKT 2014. LNCS, vol. 8818, pp. 707–719. Springer, Heidelberg (2014)
46. Zhang, X.Y., Miao, D.Q.: Region-based quantitative and hierarchical attribute reduction in the two-category decision theoretic rough set model. Knowl. Based Syst. **71**, 146–161 (2014)
47. Zhang, X.Y., Miao, D.Q.: An expanded double-quantitative model regarding probabilities and grades and its hierarchical double-quantitative attribute reduction. Inf. Sci. **299**, 312–336 (2015)
48. Zhang, X.Y., Miao, D.Q.: Reduction target structure-based hierarchical attribute reduction for two-category decision-theoretic rough sets. Inf. Sci. **277**, 755–776 (2014)
49. Zhang, H.Y., Yang, S.Y.: Ranking Interval Sets Based on Inclusion Measures and Applications to Three-Way Decisions, Manuscript
50. Zhang, H.Y., Zhang, W.X.: Entropy of interval-valued fuzzy sets based on distance and its relationship with similarity measure. Knowl. Based Syst. **22**, 449–454 (2009)
51. Zhang, H.Y., Zhang, W.X.: Hybrid monotonic fuzzy inclusion measure and its use in measuring similarity and distance between fuzzy sets. Fuzzy Sets Syst. **51**, 56–70 (2009)
52. Zhang, Z.H., Li, Y., Chen, W.B., Min, F.: A three-way decision approach to incremental frequent itemsets mining. J. Inf. Comput. Sci. **11**, 3399–3410 (2014)
53. Zhou, Z., Zhao, W., Shang, L.: Sentiment analysis with automatically constructed lexicon and three-way decision. In: Miao, D., Pedrycz, W., Slezak, D., Peters, G., Hu, Q., Wang, R. (eds.) RSKT 2014. LNCS, vol. 8818, pp. 777–788. Springer, Heidelberg (2014)
54. Zhao, L., Shang, L., Gao, Y., Yang, Y., Jia, X.Y.: Video behavior analysis using topic models and rough sets applications. IEEE Comput. Intell. Mag. **8**(1), 56–67 (2013)
55. Zhu, Y., Tian, H., Ma, J., Liu, J., Liang, T.: An integrated method for micro-blog subjective sentence identification based on three-way decisions and naive bayes. In: Miao, D., Pedrycz, W., Slezak, D., Peters, G., Hu, Q., Wang, R. (eds.) RSKT 2014. LNCS, vol. 8818, pp. 844–855. Springer, Heidelberg (2014)
56. Zhang, Z., Wang, R.: Applying three-way decisions to sentiment classification with sentiment uncertainty. In: Miao, D., Pedrycz, W., Slezak, D., Peters, G., Hu, Q., Wang, R. (eds.) RSKT 2014. LNCS, vol. 8818, pp. 720–731. Springer, Heidelberg (2014)
57. Zhang, L.B., Li, H.X., Zhou, X.Z., Huang, B., Shang, L.: Cost-sensitive sequential three-way decision for face recognition. In: Miao, D., Pedrycz, W., Ślęzak, D., Peters, G., Hu, Q., Wang, R. (eds.) RSKT2014. LNCS, vol. 8818, pp. 375–383. Springer, Switzerland (2014)