Adaptive Neural Network Control for a Class of Stochastic Nonlinear Strict-Feedback Systems*

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Abstract. An adaptive neural network control approach is proposed for a class of stochastic nonlinear strict-feedback systems with unknown nonlinear function in this paper. Only one NN (neural network) approximator is used to tackle unknown nonlinear functions at the last step and only one actual control law and one adaptive law are contained in the designed controller. This approach simplifies the controller design and alleviates the computational burden. The Lyapunov Stability analysis given in this paper shows that the control law can guarantee the solution of the closed-loop system uniformly ultimate boundedness (UUB) in probability. The simulation example is given to illustrate the effectiveness of the proposed approach.

Keywords: adaptive control, neural networks, stochastic nonlinear strict-feedback system.

1 Introduction

Backstepping technique has been a powerful method for synthesizing adaptive controllers for deterministic strict-feedback nonlinear systems, and some useful control schemes have been developed [1-3]. However, little attention has been paid to the stabilization problem for the stochastic nonlinear systems until recently. Efforts toward stabilization of stochastic nonlinear systems have been initiated in the work of Florchinger [4]. By employing the quadratic Lyapunov functions and the Itô differentiation rule Deng and Krstić[5] gave a backstepping design for stochastic strict-feedback system with the form of quartic Lyapunov function.

As well known, both neural network (NN) and fuzzy logic system (FLS) have been found to be particularly useful for controlling nonlinear systems with nonlinearly

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parameterized uncertainties. The main advantage is that the unknown nonlinear functions can be approximated by the neural networks [6-8]. For simplifying the complexity of control design and alleviating the computation burden, numerous control approaches have been developed. For instance, Chen [9] and Li [10] introduced the adaptive neural network control schemes to the output-feedback stochastic nonlinear strict-feedback systems, and only an NN to compensate for all upper bounding functions depending on the system output. A novel direct adaptive neural network controller was proposed to control a class of stochastic system with completely unknown nonlinear functions in [11]. For the purpose of solving the problem of the explosion of neural network learning parameters, Yang *et al.* first solved the problem in their pioneering work [12], where the so-called "minimal learning parameter (MLP)" algorithm containing much less online adaptive parameters were constructed by fusion of traditional backstepping technique and radial-basis-function (RBF) NNs. By combining dynamic surface control (DSC) and MLP techniques, Li et al in [13] first proposed an algorithm which can simultaneously solve both problems of the explosion of learning parameters and the explosion of computation complexity. However, many approximators are still used to construct virtual control laws and actual control law and all the virtual control law also must be actually implemented in the process of controller design. In order to eliminate the complexity growing problem and deduce the computation burden mentioned above completely, Sun *et al.* proposed a new adaptive control design approach to handle the problems mentioned above [14], only one NN is used to approximate the lumped unknown function of the system.

Motivated by the aforementioned discussion, in this paper, a single neural network approximation based adaptive control approach is presented for the strict-feedback stochastic nonlinear systems. The main contributions lie in the following: (i) only one NN is used to deal with those unknown system functions, those virtual control law are not necessary to be actually implemented in the process of control design; (ii) there is only one adaptive law proposed in this paper, which make the computational burden significantly alleviated and the control scheme more easily implemented in practical applications.

2 Preliminaries and Problem Formulation

Consider an n-dimensional stochastic nonlinear system

$$dx = f(x)dt + \psi(x)dw \tag{1}$$

where $x \in R^n$ is the system state, w is an r-dimensional standard Brownian motion defined on the complete probability space (Ω, F, P) with Ω be a sample space, F being a σ-field. $f(x): R^n \to R^n$, $\psi(x): R^n \to R^{n\times r}$ are locally Lipschitz.

In this paper, the following RBF NN will be used to approximate any unknown continuous function $h(Z)$, namely $h_{nn}(Z) = W^T S(Z)$, where $Z \in \Omega_Z \subset R^q$ is the input vector with q being the input dimension of neural networks,

$W=[w_1,\ w_2,\cdots,\ w_l]^T\in R^l$ is the weight vector, $l>1$ is the neural networks node number, and $S(Z)=\left[s_1(Z),\cdots,\ s_l(Z)\right]^T$ means the basis function vector, $s_i(Z)$ is the Gaussian function of the form $s_i(Z)=\exp\left[-(z-\mu_i)^T(z-\mu_i)/\varsigma^2\right]$, $i=1,2,\cdots,l$, where $\mu_i=[\mu_{i1;},\mu_{i2},\cdots,\mu_{iq}]^T$ is the center of the receptive field and $\varsigma>0$ are the width of the basis function.

It has been proven that neural network can approximate any continuous function over a compact set $\Omega_Z\subset R^q$ to arbitrary any accuracy such as $h(Z)=W^{*T}S(Z)+\delta(Z)$, where W^* is the ideal constant weight vector and $\delta(Z)$ denotes the approximation error and satisfies $\left|\delta(Z)\right|\le\varepsilon$.

Assumption 1 [15]. There exist constants b_m and b_M such that for $1\le i\le n$, $\forall\bar{x}_i\in R^i$, $0<b_m\le g_i\left(\bar{x}_i\right)\le b_M<\infty$.

Assumption 2. The desired trajectory signal $y_d(t)$ is continuous and bounded, and its time derivatives up to the nth order are also continuous and bounded.

Lemma 1 [16]. Consider the stochastic system (1). If there exists a positive definite, radially unbounded, twice continuously differentiable Lyapunov function $V:R^n\to R$, and constants $a_0>0$, $\gamma_0\ge 0$, such that

$$LV(x)\le -a_0V(x)+\gamma_0 \tag{2}$$

Then, the system has a unique solution almost surely, and the system is bounded in probability.

Consider the following stochastic nonlinear strict-feedback system

$$\begin{cases} dx_i=(g(\bar{x}_i)x_{i+1}+f_i(\bar{x}_i))dt+\psi_i(\bar{x}_i)dw \\ dx_n=(g(\bar{x}_n)u+f_n(\bar{x}_n))dt+\psi_n(\bar{x}_n)dw \\ y=x_1 \end{cases} \tag{3}$$

where $x=\left[x_1,\cdots,\ x_n\right]^T\in R^n$, $u\in R$ and $y\in R$ are the state variable, the control input, and the system output respectively, $\bar{x}_i=\left[x_1,\cdots,\ x_i\right]^T\in R^i$, $f_i(\cdot)$, $g_i(\cdot):R^i\to R$ and $\psi_i(\cdot):R^i\to R^r,(i=1,\cdots,n)$ are unknown smooth nonlinear functions with $f_i(0)=0$, $\psi_i\left(0\right)=0\left(1\le i\le n\right)$ $\left(i=1,2,\cdots,n\right)$.

3 Controller Design

Step 1: Define the first error surface as $z_1=x_1-y_d$, where y_d is the desired trajectory. Its differential is

$$dz_1=\left[g_1\left(x_1\right)x_2+f_1\left(x_1\right)-\dot{y}_d\right]dt+\psi_1\left(x_1\right)dw \tag{4}$$

Define the virtual controller α_2 as follows

$$\alpha_2 = -k_1 z_1 - F_1(x_1, y_d, \dot{y}_d) \tag{5}$$

where $k_1 > 0$ is a positive real design constant, $F_1(x_1, y_d, \dot{y}_d)$ is an unknown smooth function in the following form

$$F_1(x_1, y_d, \dot{y}_d) = \frac{1}{g_1(x_1)}\left[f_1(x_1) - \dot{y}_d + \frac{3}{4} l_1^{-2} z_1 \left\| \varphi(x_1) \right\|^4 \right] \tag{6}$$

where $\varphi(x_1) = \psi(x_1)$. Define the second error surface as $z_2 = x_2 - \alpha_2$. Then, we have

$$z_2 = x_2 - \dot{y}_d + k_1(x_1 - y_d) + F_1^*(x_1, y_d, \dot{y}_d) \tag{7}$$

where $F_1^*(x_1, y_d, \dot{y}_d) = F_1(x_1, y_d, \dot{y}_d) + \dot{y}_d$.

Step i $(2 \le i \le n-1)$ **:** A similar procedure is recursively employed for each step i , from the former step, it can be obtained that

$$z_i = x_i - y_d^{(i-1)} + \sum_{j=1}^{i-1} k_j k_{j+1} \cdots k_{i-1}\left(x_j - y_d^{(j-1)}\right) + F_{i-1}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i-1)}\right) \tag{8}$$

where

$$F_{i-1}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i-1)}\right) = k_{i-1} F_{i-2}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i-2)}\right) + F_{i-1}\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i-1)}\right) + y_d^{(i-1)} \tag{9}$$

The differential of z_2 is

$$\begin{aligned} dz_i = \Bigg[& g_i(\overline{x}_i) x_{i+1} + \sum_{j=1}^{i-1} \frac{\partial F_{i-1}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial x_j}\left(g_j(\overline{x}_j) x_{j+1} + f_j(\overline{x}_j)\right) \\ & + f_i(\overline{x}_i) - y_d^{(i)} + \sum_{j=1}^{i-1} k_j k_{j+1} \cdots k_{i-1}\left(g_j(\overline{x}_j) x_{j+1} + f_j(\overline{x}_j) - y_d^{(j)}\right) \\ & + \sum_{j=1}^{i} \frac{\partial F_{i-1}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial y_d^{(j-1)}} y_d^{(j)} \Bigg] + \varphi_i(\overline{x}_i) dw \end{aligned} \tag{10}$$

where

$$\varphi_i(\overline{x}_i) = \psi_i(\overline{x}_i) + \sum_{j=1}^{i-1} k_j k_{j+1} \cdots k_{i-1} \psi_j(\overline{x}_j) + \sum_{j=1}^{i} \frac{\partial F_{i-1}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial y_d^{(j-1)}} \psi_j(\overline{x}_j)$$

The virtual control law α_{i+1} is chosen as follows:

$$\alpha_{i+1} = -k_i z_i - F_i\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i)}\right) \tag{11}$$

where k_i is a positive real design constant, $F_i\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i)}\right)$ is an unknown smooth function in the following form

$$F_i\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i)}\right) = \frac{1}{g_i\left(\overline{x}_i\right)}\left[\sum_{j=1}^{i-1}\frac{\partial F_{i-1}^*\left(\overline{x}_{i-1}, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial x_j}\left(g_j\left(\overline{x}_j\right)x_{j+1} + f_j\left(\overline{x}_j\right)\right)\right.$$
$$+ f_i\left(\overline{x}_i\right) - y_d^{(i)} + \sum_{j=1}^{i-1}k_j k_{j+1}\cdots k_{i-1}\left(g_j\left(\overline{x}_j\right)x_{j+1} + f_j\left(\overline{x}_j\right) - y_d^{(j)}\right)$$
$$\left. + \sum_{j=1}^{i}\frac{\partial F_{i-1}^*\left(\overline{x}_{i-1}, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial y_d^{(j-1)}}y_d^{(j)} + \frac{3}{4}l_i^{-2}z_i\left\|\varphi_i\left(\overline{x}_i\right)\right\|^4\right] \quad (12)$$

Define the $(i+1)$th error surface as $z_{i+1} = x_{i+1} - \alpha_{i+1}$, Substituting α_{i+1} into z_{i+1}, it can be obtained that

$$z_{i+1} = x_{i+1} - y_d^{(i)} + \sum_{j=1}^{i}k_j k_{j+1}\cdots k_i\left(x_j - y_d^{(j-1)}\right) + F_i^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i)}\right) \quad (13)$$

where $F_i^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i)}\right)$ is also an unknown function in the following form

$$F_i^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i)}\right) = k_i F_{i-1}^*\left(\overline{x}_{i-1}, y_d, \dot{y}_d, \cdots, y_d^{(i-1)}\right) + F_i\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(i)}\right) + y_d^{(i)} \quad (14)$$

***Step n*:** The differential of z_n is

$$dz_n = \left[g_n\left(\overline{x}_n\right)u + f_n\left(\overline{x}_n\right) + \sum_{j=1}^{n-1}\frac{\partial F_{n-1}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial x_j}\left(g_j\left(\overline{x}_j\right)x_{j+1} + f_j\left(\overline{x}_j\right)\right)\right.$$
$$- y_d^{(n)} + \sum_{j=1}^{n-1}k_j k_{j+1}\cdots k_{n-1}\left(g_j\left(\overline{x}_j\right)x_{j+1} + f_j\left(\overline{x}_j\right) - y_d^{(j)}\right)$$
$$\left. + \sum_{j=1}^{n}\frac{\partial F_{n-1}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial y_d^{(j-1)}}y_d^{(j)}\right] + \varphi_n\left(\overline{x}_n\right)dw \quad (15)$$

where

$$\varphi_n\left(\overline{x}_n\right) = \psi_n\left(\overline{x}_n\right) + \sum_{j=1}^{n-1}k_j k_{j+1}\cdots k_{n-1}\psi_j\left(\overline{x}_j\right) + \sum_{j=1}^{n}\frac{\partial F_{n-1}^*\left(\overline{x}_i, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial y_d^{(j-1)}}\psi_j\left(\overline{x}_n\right)$$

Chose the desired control law as

$$u^* = -k_n\left[x_n - y_d^{(n-1)} + \sum_{j=1}^{n-1}k_j k_{j+1}\cdots k_{n-1}\left(x_j - y_d^{(j-1)}\right)\right] - F_n^*\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right) \quad (16)$$

where $F_n^*\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right) = k_n F_{n-1}^*\left(\overline{x}_{n-1}, y_d, \dot{y}_d, \cdots, y_d^{(n-1)}\right) + F_n\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right) + y_d^{(n)}$ is an unknown smooth function. Where k_n is a positive real design constant, $F_n\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right)$ is an unknown smooth function in the following form

$$F_n\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right) = \frac{1}{g_n\left(\overline{x}_n\right)}\left[\sum_{j=1}^{n-1}\frac{\partial F_{n-1}^*\left(\overline{x}_{n-1}, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial x_j}\left(g_j\left(\overline{x}_j\right)x_{j+1} + f_j\left(\overline{x}_j\right)\right)\right.$$
$$+ f_n\left(\overline{x}_n\right) - y_d^{(n)} + \sum_{j=1}^{n-1} k_j k_{j+1}\cdots k_{n-1}\left(g_j\left(\overline{x}_j\right)x_{j+1} + f_j\left(\overline{x}_j\right) - y_d^{(j)}\right)$$
$$\left. + \sum_{j=1}^{n}\frac{\partial F_{n-1}^*\left(\overline{x}_{n-1}, y_d, \dot{y}_d, \cdots, y_d^{(j-1)}\right)}{\partial y_d^{(j-1)}} y_d^{(j)} + \frac{3}{4} l_n^{-2} z_n \left\|\varphi_n\left(\overline{x}_n\right)\right\|^4\right] \tag{17}$$

Since function $F_n^*\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right)$ is unknown, an RBF neural network can be used to approximate it. That is

$$F_n^*\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right) = W^{*T} S\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right) + \varepsilon \tag{18}$$

Then the actual control law u is chosen as follows:

$$u = -k_n\left[x_n - y_d^{(n-1)} + \sum_{j=1}^{n-1} k_j k_{j+1}\cdots k_{n-1}\left(x_j - y_d^{(j-1)}\right)\right] - \hat{W}^T S\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right) \tag{19}$$

where $\hat{W}$ is the estimation of W^* and is updated as follows:

$$\dot{\hat{W}} = \Gamma\left(z_1^3 S\left(\overline{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right) - \gamma\hat{W}\right) \tag{20}$$

with a constant matrix $\Gamma = \Gamma^T > 0$, and a real scalar $\gamma > 0$.

4 Stability Analysis

Theorem 1. Consider the system (3), and the above closed-loop systems, according to lemma 1, for any initial condition satisfying

$$\Pi = \left\{\sum_{i=1}^n z_i^4(0) + \frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W} < M_0\right\} \tag{21}$$

where M_0 is any positive constant, then there exist the control parameters k_i, Γ and γ such that all the signals in the closed-loop system are UUB in forth moment. Moreover, the ultimate boundedness of the above closed-loop signals can be tuned arbitrarily small by choosing suitable design parameters.

Proof: Consider the following Lyapunov function candidate

$$V = \frac{1}{4}\sum_{i=1}^n z_i^4 + \frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W} \tag{22}$$

According to the Itô's differential rules and Young inequality, together with equations (19) and (20), the differential of the above function V can be found as follows

$$LV \le -\left[\left(k_1-\frac{3}{4}\right)b_m-\frac{3}{4}\chi^{\frac{4}{3}}\right]z_1^4-\sum_{i=2}^{n-1}\left[\left(k_i-\frac{3}{4}\right)b_m-\frac{1}{4}b_M\right]z_i^4-\frac{1}{2}\gamma\tilde{W}^T\tilde{W}+\frac{1}{2}\left\|W^*\right\|^2$$
$$-\left[\left(k_n-\frac{3}{4}\chi^{\frac{4}{3}}-\frac{3}{4}\right)b_m-\frac{1}{4}b_M\right]z_i^4+\frac{3}{4}\sum_{i=1}^{n}l_i^2+\frac{1}{4}(1+b_M)\varpi^4+\frac{1}{4}b_M\left\|\varepsilon\right\|^4 \quad (23)$$

where $\left\|S\left(\bar{x}_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}\right)\right\| \le \chi$ [17], $\left\|\tilde{W}\right\| \le \varpi$.

Choosing the positive constants as

$$\begin{cases} c_1=\left(k_1-\frac{3}{4}\right)b_m-\frac{3}{4}\chi^{\frac{4}{3}} \\ c_i=\left(k_1-\frac{3}{4}\right)b_m-\frac{1}{4}b_M \qquad i=2,\cdots,n-1 \\ c_n=\left(k_n-\frac{3}{4}\chi^{\frac{4}{3}}-\frac{3}{4}\right)b_m-\frac{1}{4}b_M \end{cases} \quad (24)$$

Define a positive constant $a_0=\min\{4c_i, r\}$. It follows from Equation (27) that

$$LV \le -aV+D \quad (25)$$

where

$$D=\frac{1}{2}\left\|W^*\right\|^2++\frac{3}{4}\sum_{i=1}^{n}l_i^2+\frac{1}{4}(1+b_M)\varpi^4+\frac{1}{4}b_M\left\|\varepsilon\right\|^4 \quad (26)$$

From Equation (25) we can clearly observe that the first term is negative definite and the second term D is a positive constant.

Furthermore, it follows from (25) that

$$E[V(t)] \le \left(V(0)-\frac{D}{a}\right)e^{-a(t-t_0)}+\frac{D}{a} \quad (27)$$

According to lemma 2 the above analysis on the closed-loop system means that all the signals in the system (3) are UUB in the sense of probability. Furthermore, for any $\zeta_1 > \sqrt{D/a_0}$, there exists a constant $T>0$, such that $\left|z_1(t)\right| \le \zeta_1$ for all $t \ge t_0+T$. Since $\sqrt{D/a_0}$ can be made arbitrarily small if the design parameters are chosen appropriately, thus, for any given ζ_1 , one has $\lim_{t\to\infty}\left|z_1(t)\right| \le \zeta_1$. That is to say, by adjusting the design parameters, the tracking error can be made arbitrarily small.

The proof is thus completed. □

5 Simulation Example

Consider the following third-order stochastic nonlinear system

$$\begin{cases} dx_1 = \left(1 + x_1^2\right) x_2 + x_1 \sin x_1 + x_1^3 dw \\ dx_2 = \left(1 + x_2^2\right) x_3 + x_2 e^{-0.5 x_1} + x_1 \cos x_2 dw \\ dx_3 = \left(3 + \cos\left(x_2 x_3\right)\right) u + x_1 x_2 x_3 + 3 x_1 e^{-x_3^2} dw \\ y = x_1 \end{cases} \tag{28}$$

Based on the adaptive NN controller design proposed in section 3, the true control law are designed and the adaptive law. In applied mathematics, the Wiener process can be described as the integral form of Gauss white noise, which has two main parameters, i.e., mean and variance. We choose the neuron's center and variance as $\{-5,5\} \times\{-5,5\} \times\{-5,5\} \times\{-5,5\} \times\{-5,5\}$ and 1 respectively. If we chose the desired trajectory $y_d = \sin(t)$, the suitable parameters were chosen as $k_1 = 35$, $k_2 = 2.5$, $k_3 = 100$, $\Gamma = 0.05$, $\gamma = 100$, The initial conditions are given by $[x_1(0), x_2(0), x_3(0)]^T = [0.8,0.4,0.5]^T$ and the initial weight vector $\hat{W}(0) = 0.5$ .The simulation results are shown in Figs.1~4.

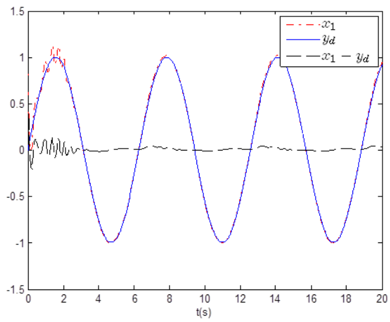

Fig. 1. The output y , the reference signal y_d

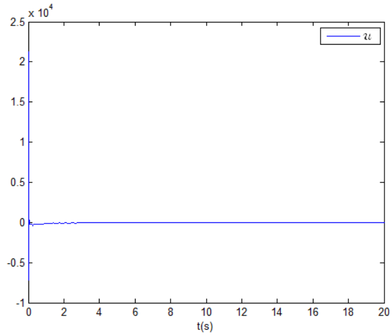

Fig.2. The control input u

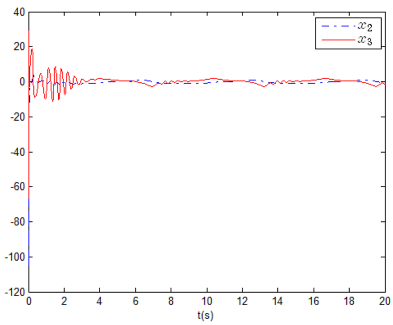

Fig.3. The state of x_2 and x_3

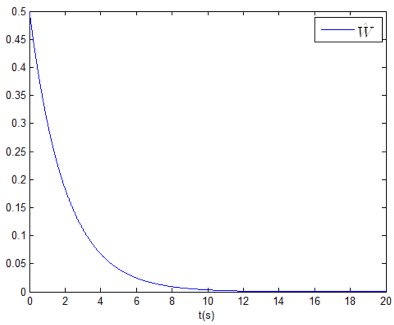

Fig.4. The adaptive law $\hat{W}$

6 Conclusion

In this paper, An adaptive NN controller has been proposed for a class of stochastic nonlinear strict-feedback systems. Using the proposed technique we can alleviate the computational burden and simplify the designed controller. Only one neural network is used to compensate the lumped unknown function at the last step. The closed-loop system has been proved UUB. The effectiveness of the proposed approach has been verified by the simulation example.

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