Inverse-Free Scheme of G1 Type to Velocity-Level Inverse Kinematics of Redundant Robot Manipulators

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Abstract. With the superiority of owning more degrees of freedom than ordinary robot manipulators, redundant robot manipulators have gotten much attention in recent years. In order to control the trajectory of the robot end-effector with a desired velocity, it is very popular to apply the inverse kinematics approaches, such as pseudo-inverse scheme. However, calculating the inverse of Jacobian matrix requires a lot of time. Thus base on gradient neural dynamics (GND), an inverse-free scheme is proposed at the joint-velocity level. The scheme is named G1 type as it uses GND once. In addition, two path tracking simulations based on five-link and six-link redundant robot manipulators illustrate the efficiency and the accuracy of the proposed scheme. What is more, the physical realizability of G1 type scheme is also verified by a physical experiment based on the six-link planar redundant robot manipulator hardware system.

Keywords: redundant robot manipulators, control, inverse-free scheme, gradient neural dynamics, path tracking.

1 Introduction

In recent years, there have been numerous investigations of robot manipulators [1,2,3,4], especially the redundant robot manipulators. Compared with the ordinary robot manipulators, the redundant one has more degrees of freedom than necessary for position and orientation. It is worth noting that this characteristic improves the kinematic and the dynamic performance of the robot manipulator such as increasing dexterity, avoiding obstacles and singularities, and optimizing joint velocity, which makes redundant manipulators widely applied in the field of robotic manipulator control [5]. As one of the central issues in robot control, path tracking refers to making the end-effector move as expected by controlling the joints of robot manipulators. When the end-effector moves in a desired speed, it is often called path tracking control in the velocity level, which can fit in with

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the needs of the operators well with two advantages [6,7]. One is that the high velocity can economize the execution time while the other one is that the low velocity can enhance the objective precision [8]. So the control in the velocity level is particularly suited to the tasks well for machining operations, such as cutting and milling [9].

Robot kinematics which studies the relationships between joint space and cartesian space, is an very effective way to control the robot manipulator and attracts numberous researchers to study [10,11,12]. The velocities of all joints play a decisive role in realizing the desired speed of the end-effector. So the most fundamental problem is how to use the kinematic equations to calculate the homologous joint velocities, which is also called inverse kinematics. However, because most kinematic equations involve complex (inverse) trigonometric functions, the inverse kinematics mapping has no closed-form solutions for most manipulators and animation figures [13]. Conventionally, most of researchers exploit pseudo-inverse approaches to obtain a simple general-form solution [14,15]. However, these approaches not only need expensive time in calculating the inverse of Jacobian matrix, but also require the Jacobian matrix to be of full rank which may be away from the reality. Thus various approaches have been proposed, investigated and developed to avoid the calculation of the inverse of Jacobian matrix, such as the quadratic programming method [16].

Gradient neural dynamics (GND) [17,18] is a significant neural dynamic method which attracts many researchers to investigate and develop it. Now GND method is proved to be useful and effective, and it is widely acknowledged in scientific and engineering field, thus generalizing such a GND method has become the primary work [16,19,20,21,22]. In this paper, based on the advantage of GND method that it can help find a minimum of a nonnegative objective function effectively, we propose and investigate a scheme named G1 type for path tracking in the redundant robot manipulator at the joint-velocity level. Besides, in the framework of Zhang-gradient (ZG) method, G1 type scheme is a special Z0G1 situation which only uses the GND method once. The ZG method is an effective method built by combining Zhang neural dynamics and GND to solve the tracking-control and singularity problems [23,24].

The rest of this paper is organized into the following sections. The inverse scheme formulation is presented in Section 2. In Section 3, the G1 type scheme is proposed and analyzed for the redundant robot manipulators at the joint-velocity level. Section 4 illustrates the effectiveness and the accuracy through two simulations, and Section 5 further illustrates the effectiveness and physical realizability of G1 type scheme based on a six-link planar redundant robot manipulator hardware system. Finally, Section 6 concludes the whole work with final remarks. Before ending this introductory section, the main contributions of this paper are listed as follows.

1) By exploiting the GND method, a G1 type scheme in an inverse-free manner is proposed and investigated at the joint-velocity level.
2) The proposed G1 type scheme can solve the inverse kinematics problem effectively but avoid calculating the inverse of Jacobian matrix.

3) The path-tracking simulations and the physical experiment are conducted to further illustrate the effectiveness, the high accuracy and the physical realizability of the G1 type scheme.

2 Inverse Scheme Formulation

For a redundant robot manipulator with n joints, the end-effector pose (or position in this paper) vector $\boldsymbol{r} \in \mathbb{R}^m$ can be described by the following equation:

$$\boldsymbol{r} = f(\boldsymbol{\theta}), \tag{1}$$

where $\boldsymbol{\theta} \in \mathbb{R}^n$ refers to the variables (or angles in this paper) of the n joints, and $f(\cdot)$ is a differentiable nonlinear function with a known structure and parameters for a given manipulator. Then by differentiating (1), the end-effector velocity is

$$\dot{\boldsymbol{r}} = J\dot{\boldsymbol{\theta}}, \tag{2}$$

where $\dot{\boldsymbol{r}} \in \mathbb{R}^m$ refers to the end-effector velocity, and $\dot{\boldsymbol{\theta}} \in \mathbb{R}^n$ refers to the velocities of all joints. Note that $J \in \mathbb{R}^{m\times n}$ is the Jacobian matrix defined as $J = \partial f(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$. According to (2), for tracking the desired path $\boldsymbol{r}_\mathrm{d} \in \mathbb{R}^m$ via the desired speed $\dot{\boldsymbol{r}}_\mathrm{d} \in \mathbb{R}^m$, the velocities of all joints can be obtained by the following equation if J is a square matrix and of full rank:

$$\dot{\boldsymbol{\theta}} = J^{-1}\dot{\boldsymbol{r}}_\mathrm{d}, \tag{3}$$

If J is rectangular, the velocities of joints may be computed by the following equation of generalized inverse [6]:

$$\dot{\boldsymbol{\theta}} = J^{+}\dot{\boldsymbol{r}}_\mathrm{d}, \tag{4}$$

where J^+ denotes the pseudoinverse of Jacobian matrix J. Note that, the solution obtained by using (4) is a least square solution. However, the inverse scheme theoretically requires the Jacobian matrix to be of full rank, which, in a real world application, may be unavailable sometimes in practice. What is more, the expensive calculating time of Jacobian matrix is also not suited in industry.

3 G1 Type Scheme Formulation

In this section, we generalize the GND method to obtain an inverse-free scheme at the joint-velocity level. By following the GND method, the design procedure of such an inverse-free scheme can be presented detailedly in the following steps.

Firstly, to monitor and control the process of solving the time-varying inverse kinematics problem of redundant manipulators, we define a scalar-valued norm-based energy function according to (1):

$$E = \|\boldsymbol{r}_\mathrm{d} - f(\boldsymbol{\theta})\|_2^2/2, \tag{5}$$

where $\|\cdot\|_2$ denotes the two-norm of a vector.

Secondly, a computational rule is designed to evolve along a descent direction of this energy function until the minimum point is reached. The typical descent direction is the negative gradient of E, i.e.,

$$-\partial E/\partial \boldsymbol{\theta} = J^{\mathrm{T}}(\boldsymbol{r}_{\mathrm{d}} - f(\boldsymbol{\theta})). \tag{6}$$

Then we combine the aforementioned negative gradient (6) and the following GND design formula [17,18]:

$$\dot{\boldsymbol{\theta}} = -\alpha \partial E/\partial \boldsymbol{\theta}, \tag{7}$$

where the design parameter $\alpha > 0$ is used to scale the convergence rate of the GND method.

Finally, we thus have the following generalized G1 type scheme for solving the time-varying inverse kinematics problem of redundant robot manipulators:

$$\dot{\boldsymbol{\theta}} = \alpha J^{\mathrm{T}}(\boldsymbol{r}_{\mathrm{d}} - f(\boldsymbol{\theta})). \tag{8}$$

Evidently, the above scheme (8) does not require the Jacobian inversion appearing in (3) and (4). Besides, since scheme (8) is obtained by applying the GND method only once, it is named G1 type (or Z0G1 type in the ZG framework).

4 Simulations

In this section, the corresponding path-tracking simulations (square path tracking and "Z" path tracking) are performed on five-link and six-link redundant robot manipulators respectively to illustrate the effectiveness and the accuracy of the proposed G1 type scheme. Note that, when we apply such a scheme to solving the time-varying inverse kinematics problem in an inverse-free manner, the design parameter $\alpha = 10^5$ is used throughout this section. For visualized reading, two error functions are defined as follows:

$$\begin{aligned} \varepsilon_{\mathrm{X}} &= r_{\mathrm{dX}} - p_{\mathrm{X}}, \\ \varepsilon_{\mathrm{Y}} &= r_{\mathrm{dY}} - p_{\mathrm{Y}}, \end{aligned} \tag{9}$$

where r_{dX} and r_{dY} refer to the desired positions in the X direction and the Y direction respectively, and p_{X} and p_{Y} refer to the actual positions in the X direction and the Y direction respectively. Accordingly, $\dot{\varepsilon}_{\mathrm{dX}}$ represents the velocity error in the X direction, while $\dot{\varepsilon}_{\mathrm{dY}}$ represents the velocity error in the Y direction.

4.1 Square Path Tracking

In this first example, G1 type scheme (8) is applied to tracking a square path via a desired velocity. The side length of the square path is 0.8 m, and the path tracking is simulated on a five-link redundant robot manipulator, with an initial state $\boldsymbol{\theta}(0)$ being $[\pi/3, \pi/3, \pi/2, -\pi/4, \pi/4]^{\mathrm{T}}$ rad. The corresponding simulation

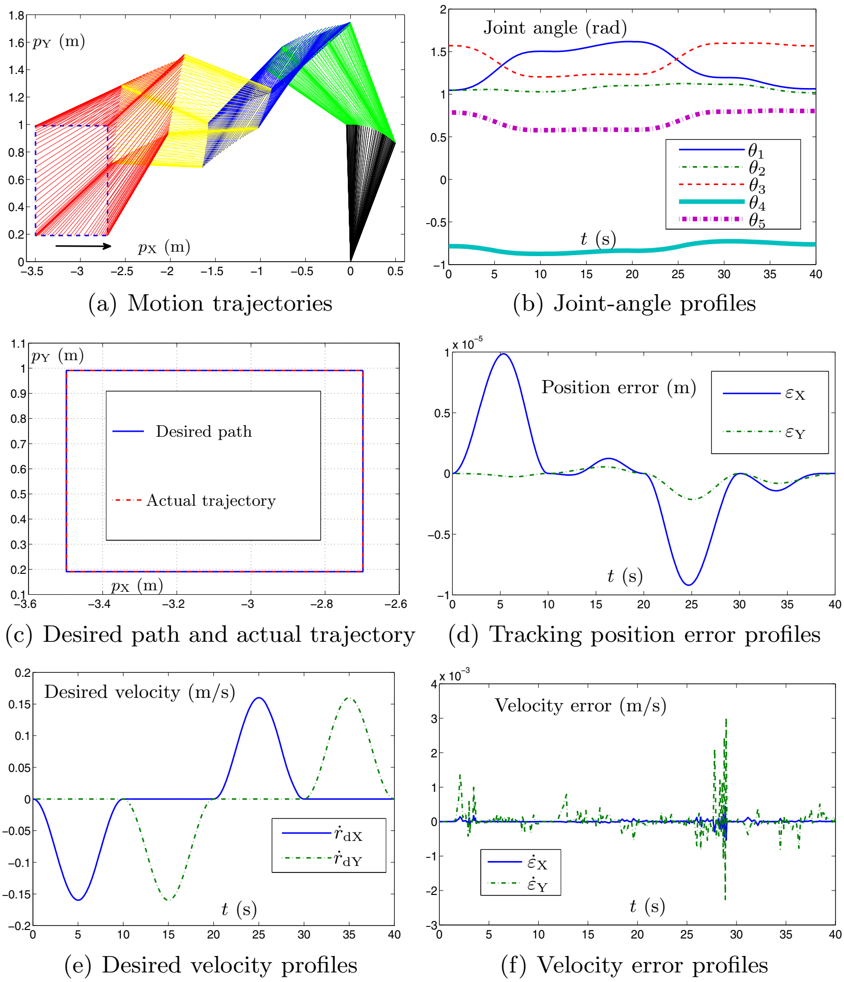


Fig. 1. Simulation results of the five-link redundant robot manipulator tracking the given square path synthesized by G1 type scheme (8).

results are shown in Fig. 1, which illustrate the effectiveness and high accuracy of the proposed G1 type scheme (8) for solving the time-varying inverse kinematics of robot manipulators.

Specifically, the results can be seen from Fig. 1(a) and Fig. 1(b), from which we can see how the manipulator tracks the desired path with five joints. It is easily found that the actual end-effector trajectory coincides with the desired square path. Besides, Fig. 1(c) and Fig. 1(d) show us more details about the trajectory. That is, Fig. 1(c) presents the desired path and the actual trajectory, which illustrates the effectiveness and the accuracy intuitively. The corresponding errors shown in Fig. 1(d) are all less than 1×10^{-5} m. This implies that

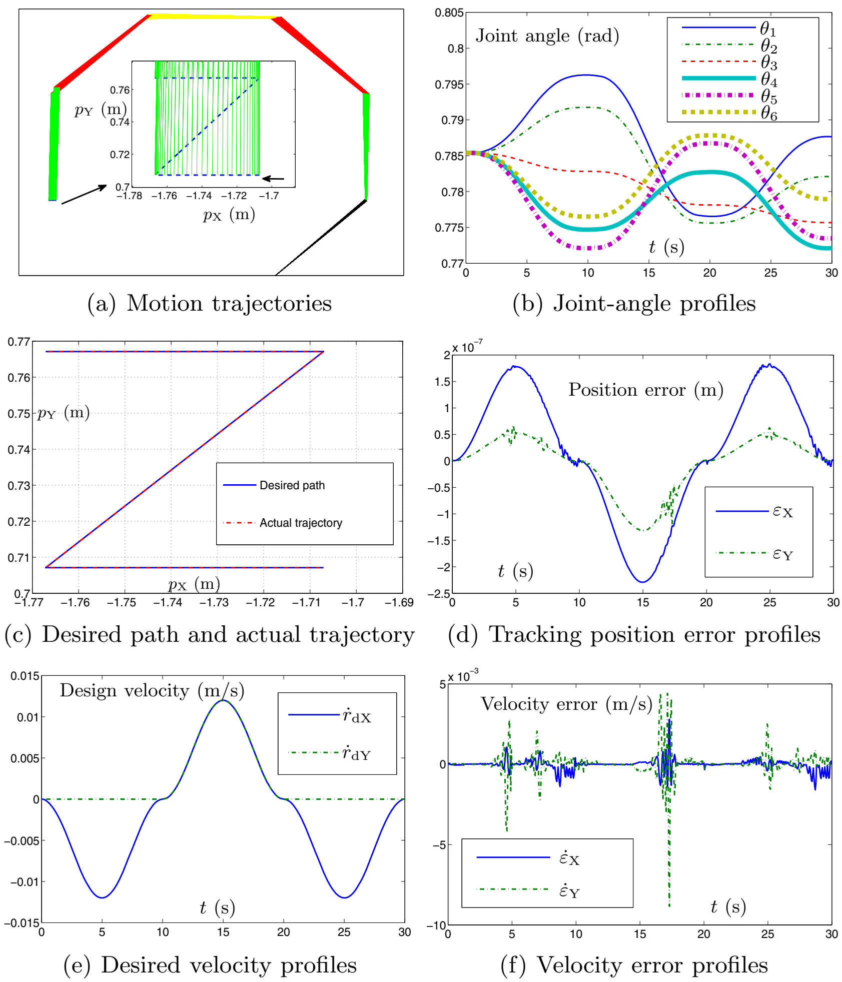
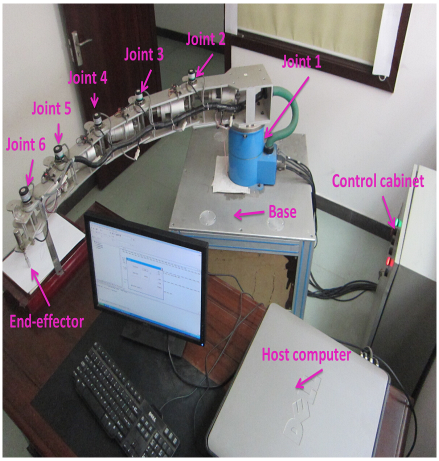


Fig. 2. Simulation results of the six-link redundant robot manipulator tracking the given "Z" path synthesized by G1 type scheme (8).

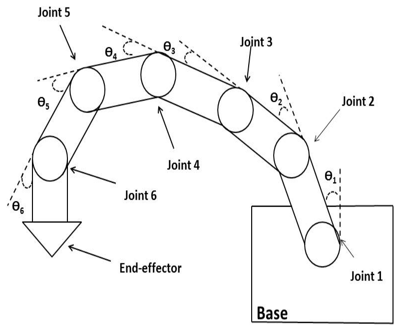
the five-link robot manipulator completes the given square-path tracking task well. In addition, the desired velocities and the corresponding velocity errors are shown in Fig. 1(e) and Fig. 1(f). We see that the velocity errors are less than 4×10^{-3} m/s, validating the high accuracy of such an inverse-free type scheme.

4.2 "Z" Path Tracking

In this (second) example, G1 type scheme (8) is applied to tracking another path task, i.e., a "Z" path task with the side length being 0.8 m. The corresponding simulation results based on a six-link redundant robot manipulator are shown



(a) Hardware system



(b) Manipulator-joint structure

Fig. 3. The hardware system of the six-link planar redundant robot manipulator with its structure planform.

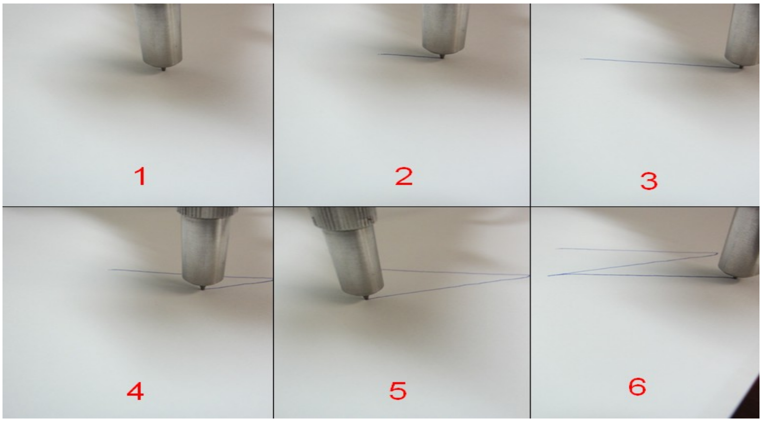
in Fig. 2, where an initial state $\boldsymbol{\theta}(0)$ is selected as $[\pi/4, \pi/4, \pi/4, \pi/4, \pi/4, \pi/4]^{\mathrm{T}}$ rad. These results illustrate once more the effectiveness and the high accuracy of the proposed G1 type scheme (8) for solving the time-varying inverse kinematics of robot manipulators.

Specifically, the results can be seen from Fig. 2(a) and Fig. 2(b), from which we can see how the manipulator tracks the desired path with six joints. It is easily found that the actual end-effector trajectory coincides well with the desired "Z" path. Especially, Fig. 2(c) and Fig. 2(d) show us more details about the actual trajectory and corresponding errors. These imply that the six-link robot manipulator completes the given "Z" path tracking task well. Besides, the desired velocities and the corresponding velocity errors are shown in Fig. 2(e) and Fig. 2(f). We can find that any directional velocity error is less than 5×10^{-3} m/s, which validates the high accuracy of such an inverse-free type scheme.

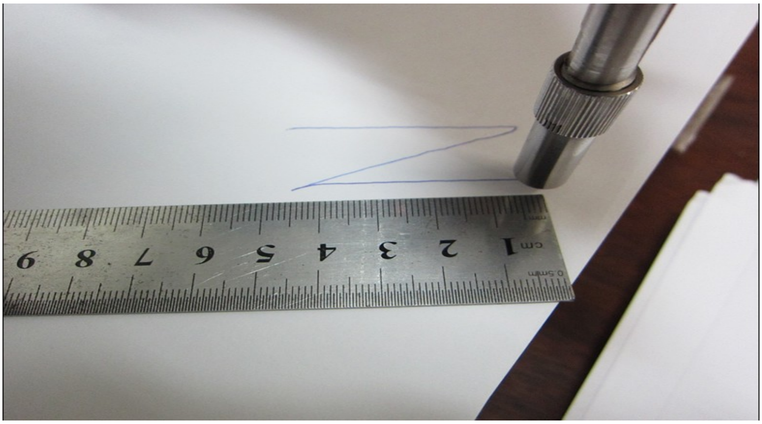
5 Physical Experiment

To verify the physical realizability of the proposed G1 type scheme (8), a six-link planar redundant robot manipulator hardware system is developed, investigated and shown. The whole manipulator system is mainly composed of a robot manipulator, a control cabinet and a host computer. Specifically, Fig. 3(a) shows this planar robot hardware system, and Fig. 3(b) depicts its manipulator-joint structure including a base and an end-effector.

For this experiment, in order to verify the proposed G1 type scheme (8) at the joint-velocity level, the end-effector is expected to move along a "Z" path with the length of 4.5 cm and an initial state $\boldsymbol{\theta}(0) = [\pi/12, \pi/12, \pi/12, \pi/12, \pi/12, \pi/12]^{\mathrm{T}}$



(a) Snapshots of task execution



(b) Measurement of experimental result

Fig. 4. The “Z” path tracking experiment of the six-link redundant redundant robot manipulator synthesized by G1 type scheme (8) at the joint-velocity level.

rad, and the design parameter α is also set as 10^5. The task execution can be seen from Fig. 4, i.e., the end-effector of the manipulator moves smoothly and draws a “Z” path precisely. Besides, the video of the process takes 24 seconds. Thus, this experiment illustrates well that the proposed G1 type scheme (8) is effective on the redundant robot manipulator’s inverse-free redundancy resolution (or say, motion planning and control).

6 Conclusion

In this paper, to solve the time-varying inverse kinematics problem for redundant robot manipulators with high efficiency and high accuracy, a special type of inverse-free scheme named G1 type scheme has been proposed and investigated at the joint-velocity level. This scheme can avoid the Jacobian inversion in traditional pseudo-inverse methods which not only costs expensive calculating time but also encounters many difficulties in practice. Besides, the corresponding path-tracking simulations have been performed on five-link and six-link redundant robot manipulators using such an inverse-free scheme. The simulation results have illustrated the effectiveness and the accuracy of the aforementioned scheme for solving the time-varying inverse kinematics problem of redundant robot manipulators in an inverse-free manner. In addition, the physical realizability of G1 type scheme has been verified further based on a six-link planar redundant robot manipulator hardware system. In the future, the scheme may be applied in the 3 dimensional case or even with kinematics uncertainties.

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