A New Model of a Fuzzy Associative Memory

Irina Perfilieva(✉)

Centre of Excellence IT4Innovations, Institute for Research and Applications of Fuzzy Modeling, University of Ostrava, 30. dubna 22, 701 03 Ostrava 1, Czech Republic
Irina.Perfilieva@osu.cz

Abstract. We propose a new theory of implicative fuzzy associative memory. This memory is modeled by a fuzzy preorder relation. We give a necessary and sufficient condition on input data that guarantees an effective composition of a fuzzy associative memory, which is moreover, insensitivity to a certain type of noise.

Keywords: Fuzzy associative memory · Fuzzy preorder · Upper set · Noise

1 Introduction

In this contribution, we are focused on knowledge integration in uncertain environments and especially on data storage in the form of fuzzy associative memory (FAM) and retrieval. The latter is considered even in the case of damaged, incomplete or noisy requests.

The first attempt to construct a *fuzzy associative memory* (FAM) has been made by Kosko - [4]. This approach presented FAM as a single-layer feedforward neural net containing nonlinear matrix-vector product. This approach was later extended with the purpose to increase the storage capacity (e.g. [2]). Significant progress was achieved by the introduction of the so called learning implication rules [1,3], that afterwards led to *implicative fuzzy associative memory* (IFAM) with *implicative fuzzy learning*. Theoretical background of IFAM were discussed in [12].

In our contribution, we give a new theoretical justification of IFAM that is based on the notion of a fuzzy preorder relation. This enables us to discover conditions on input data that guarantee that IFAM works properly. We constructively characterize all types of noise that do not influence the successful retrieval.

2 Preliminaries

2.1 Implicative Fuzzy Associative Memory

In this Section, we explain background of the theory of fuzzy associative memories and their implicative forms. We choose database $\{(\mathbf{x}^1, \mathbf{y}^1), \dots, (\mathbf{x}^p, \mathbf{y}^p)\}$ of

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input-output objects (images, patterns, signals, texts, etc.) and assume that they can be represented by couples of normal fuzzy sets so that a particular fuzzy set $\mathbf{x}^k$, $k = 1, \dots, p$, is a mapping $\mathbf{x}^k : X \to [0,1]$, and similarly, $\mathbf{y}^k : Y \to [0,1]$ where $X = \{u_1, \dots, u_n\}$, $Y = \{v_1, \dots, v_m\}$.

A model of FAM is associated with a couple (W, θ), consisted of a fuzzy relation $W : X \times Y \to [0,1]$ and a bias vector $\theta \in [0,1]^m$. A model of FAM connects every input $\mathbf{x}^k$ of a corresponding database with the related to it output $\mathbf{y}^k$, $k = 1, \dots, p$. The connection can be realized by a sup $-t$ composition[1] $\circ$, so that

$$\mathbf{y}^k = W \circ \mathbf{x}^k \vee \theta,\ k = 1, \dots, p, \tag{1}$$

or by a one level fuzzy neural network endowed with Pedrycz's neurons. The first one is represented by the following expression

$$\mathbf{y}_i^k = \bigvee_{j=1}^{n} (w_{ij} \text{ t } \mathbf{x}_j^k) \vee \theta_i,\ i = 1, \dots, m, \tag{2}$$

where $\mathbf{x}_j^k = \mathbf{x}^k(u_j)$, $\mathbf{y}_i^k = \mathbf{y}(v_i)$ and $w_{ij} = W(u_i, v_j)$. The second one is a computation model which realizes (2). In the language of fuzzy neural networks, we say that $W = (w_{ij})$ is a synaptic weight matrix and p is a number of constituent input-output patterns.

In practice, the crisp equality in (1) changes to

$$\mathbf{y}^k \approx W \circ \mathbf{x}^k, k = 1, \dots, p, \tag{3}$$

where the right-hand side is supposed to be close to $\mathbf{y}^k$. Moreover, FAM is supposed to be tolerant to a particular input noise.

In [12], a model (W, θ) of implicative fuzzy associative memory (IFAM) has been proposed where

$$w_{ij} = \bigwedge_{k=1}^{p} (\mathbf{x}_j^k \to \mathbf{y}_i^k), \tag{4}$$

$$\theta_i = \bigwedge_{k=1}^{p} \mathbf{x}_i^k,$$

and $\to$ is an adjoint implication with respect to the chosen continuous t-norm.

One important case of IFAM is specified by identical input-output patterns. This memory is called an *autoassociative fuzzy implicative memory* (AFIM), and it is aimed at memorizing patterns as well as error correction or removing of noise.

For a given input $\mathbf{x}$, AFIM returns output $\mathbf{y}$ in accordance with (2). If $\mathbf{x}$ is close to some pattern $\mathbf{x}^k$, then $\mathbf{y}$ is close to the same pattern $\mathbf{x}^k$. In the ideal case, patterns from $\{\mathbf{x}^1, \dots, \mathbf{x}^p\}$ are eigen vectors of an AFIM model.

[1] t is a t-norm, i.e. a binary operation on $[0,1]$, which is commutative, associative, monotone and has 1 as a neutral element.

One of the main benefits of AFIM is its error correction ability. By this we mean that if an input $\mathbf{x}$ is close to some pattern $\mathbf{x}^k$, then the output $\mathbf{y}$ is equal to the same pattern $\mathbf{x}^k$.

In the proposed contribution, we analyze the AFIM retrieval mechanism with respect to two goals: (a) to have patterns from $\{\mathbf{x}^1, \dots, \mathbf{x}^p\}$ as eigen vectors of that fuzzy relation W, which constitutes a model of AFIM; (b) to correct a certain type of noise. We find a necessary and sufficient condition on input patterns that guarantees that the goal (a) is fulfilled and moreover, we characterize a noise that can be successfully removed by the retrieval procedure.

Our technical platform is more general than that in [12]: we replace $[0, 1]$ by an arbitrary complete residuated lattice $\mathcal{L}$ and consider initial objects as fuzzy sets with values in $\mathcal{L}$. This allows us to utilize many known facts about fuzzy sets and fuzzy relations of particular types.

2.2 Algebraic Background

In this Section, we will step aside from the terminology of associative memories and introduce an algebraic background of the technique proposed below.

Let $\mathcal{L} = \langle L, \vee, \wedge, *, \rightarrow, 0, 1\rangle$ be a fixed, complete, integral, residuated, commutative l-monoid (*a complete residuated lattice*). We remind the main characteristics of this structure: $\langle L, \vee, \wedge, 0, 1\rangle$ is a complete bounded lattice, $\langle L, *, \rightarrow, 1\rangle$ is a residuated, commutative monoid.

Let X be a non-empty set, L^X a class of *fuzzy sets* on X and $L^{X\times X}$ a class of *fuzzy relations* on X. Fuzzy sets and fuzzy relations are identified with their membership functions, i.e. elements from L^X and $L^{X\times X}$, respectively. A fuzzy set A is *normal* if there exists $x_A \in X$ such that $A(x_A) = 1$. The (ordinary) set $Core(A) = \{x \in X \mid A(x) = 1\}$ is the *core* of the normal fuzzy set A. Fuzzy sets $A \in L^X$ and $B \in L^X$ are *equal* ($A = B$), if for all $x \in X$, $A(x) = B(x)$. A fuzzy set $A \in L^X$ is *less than or equal* to a fuzzy set $B \in L^X$ ($A \le B$), if for all $x \in X$, $A(x) \le B(x)$.

The lattice operations $\vee$ and $\wedge$ induce the union and intersection of fuzzy sets, respectively. The binary operation $*$ of $\mathcal{L}$ is used below for set-relation composition of the type sup-*, which is usually denoted by $\circ$ so that

$$(A \circ R)(y) = \bigvee_{x\in X} (A(x) * R(x, y)).$$

Let us remind that the $\circ$ composition was introduced by L. Zadeh [15] in the form max – min.

3 Fuzzy Preorders and Their Eigen Sets

In this Section, we introduce theoretical results which will be used below in the discussed application. The results are formulated in the language of residuated lattices. We will first recall basic facts about fuzzy preorder relations as they were presented in [5]. Then we will characterize eigen sets of fuzzy preorder relations and how they can be reconstructed.

Our interest to fuzzy preorder relations came from the analysis of the expression (4) – a representation of a fuzzy relation in a model of AFM. In the particular case of autoassociative fuzzy implicative memory, expression (4) changes to

$$w_{ij} = \bigwedge_{k=1}^{p} (\mathbf{x}_j^k \rightarrow \mathbf{x}_i^k).$$

This is a representation of the Valverde (fuzzy) preorder (see Remark 1 below).

3.1 Fuzzy Preorders and their Upper and Lower Sets

The text in this Section is an adapted version of [8].

A binary fuzzy relation on X is a $*$-*fuzzy preorder* of X, if it is reflexive and $*$-transitive. The fuzzy preorder $Q^* \in L^{X\times X}$, where

$$Q^*(x,y) = \bigwedge_{i\in I} (A_i(x) \rightarrow A_i(y)), \tag{5}$$

is *generated* by an arbitrary family of fuzzy sets $(A_i)_{i\in I}$ of X.

Remark 1. The fuzzy preorder Q^* (5) is often called the Valverde order on X determined by a family of fuzzy sets $(A_i)_{i\in I}$ of X (see [14] for details).

If Q is a fuzzy preorder on X, then the fuzzy set $A \in L^X$ such that

$$A(x) * Q(x,y) \leq A(y) \quad (A(y) * Q(x,y) \leq A(x)), \quad x,y \in X,$$

is called an *upper set* (a *lower set*) of Q (see [5]). Denote $Q^t(x) = Q(t,x)$ $(Q_t(x) = Q(x,t))$, $x \in X$, and see that Q^t (Q_t) is an upper set (lower set) of Q. The fuzzy set Q^t (Q_t) is called a *principal* upper set (lower set).

If Q is a fuzzy preorder on X, then $Q^{op} \in L^{X\times X}$ such that $Q^{op}(x,y) = Q(y,x)$ is a fuzzy preorder on X as well. It follows that an upper set of Q is a lower set of Q^{op} and vice versa. For this reason, our results will be formulated for upper sets of respective fuzzy preorders.

The necessary and sufficient condition that a family of fuzzy sets of X constitutes a family of upper sets of some fuzzy preorder on X has been proven in [5]. In Theorem 1 [8], given below, we characterize principal upper sets of a fuzzy preorder on X. Let us remark that assumptions of Theorem 1 are different from those in [5].

Theorem 1. *Let I be an index set, $(A_i)_{i\in I} \subseteq L^X$ a family of normal fuzzy sets of X and $(x_i)_{i\in I} \subseteq X$ a family of pairwise different core elements such that for all $i \in I$, $A_i(x_i) = 1$. Then the following statements are equivalent:*

- *(i) There exists a fuzzy preorder Q on X such that for all $i \in I$, $x \in X$, $A_i(x) = Q(x_i, x)$ (A_i is a principal upper set of Q).*

(ii) *For all $i \in I$, $x \in X$, $A_i(x) = Q^*(x_i, x)$ (A_i is a principal upper set of Q^*) where Q^* is given by (5).*

(iii) *For all $i, j \in I$,*

$$A_i(x_j) \leq \bigwedge_{x \in X} (A_j(x) \to A_i(x)). \tag{6}$$

Corollary 1. *Let $(A_i)_{i \in I} \subseteq L^X$ be a family of normal fuzzy sets of X and $(x_i)_{i \in I} \subseteq X$ a family of pairwise different core elements such that for all $i, j \in I$, (6) holds true. Then Q^* is the coarsest fuzzy preorder on X such that every fuzzy set A_i, $i \in I$, is a principal upper set of Q^*.*

Remark 2. On the basis of Theorem 1 and its Corollary 1, we conclude that a family of normal fuzzy sets $(A_i)_{i \in I} \subseteq L^X$ with pairwise different core elements $(x_i)_{i \in I} \subseteq X$, such that (6) is fulfilled, generates the coarsest fuzzy preorder Q^* on X such that every family element A_i is a principal upper set of Q^* that corresponds to its core element x_i.

3.2 Eigen Sets of Fuzzy Preorders and their "Skeletons"

In this Section, we show that if the assumptions of Theorem 1 are fulfilled, and if the fuzzy preorder Q^* on X is generated (in the sense of (5)) by normal fuzzy sets $(A_i)_{i \in I} \subseteq L^X$ with pairwise different core elements $(x_i)_{i \in I} \subseteq X$, then these fuzzy sets are the eigen (fuzzy) sets of Q^* (see [10]), i.e. they fulfill

$$A_i \circ Q^* = A_i,\ i \in I. \tag{7}$$

Proposition 1. *Let family $(A_i)_{i \in I} \subseteq L^X$, $i \in I$, of normal fuzzy sets of X with pairwise different core elements $(x_i)_{i \in I} \subseteq X$ fulfill (6) and generate fuzzy preorder Q^* in the sense of (5). Then every A_i, $i \in I$, is an eigen set of Q^*.*

Proof. Let us choose and fix A_i, $i \in I$. By Theorem 1, $A_i(x) = Q^*(x_i, x)$. Then

$$A_i(x) \circ Q^*(x, y) = \bigvee_{x \in X} (Q^*(x_i, x) * Q^*(x, y)) \leq$$
$$Q^*(x_i, y) = A_i(y).$$

On the other hand,

$$A_i(x) \circ Q^*(x, y) = \bigvee_{x \in X} (Q^*(x_i, x) * Q^*(x, y)) \geq$$
$$Q^*(x_i, y) * Q^*(y, y) = Q^*(x_i, y) = A_i(y).$$

Corollary 2. *Let the assumptions of Propositions 1 be fulfilled and fuzzy set $\bar{A}_i \in L^X$, $i \in I$, be a "skeleton" of A_i, where*

$$\bar{A}_i(x) = \begin{cases} 1, & \text{if } x \in Core(A_i), \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Then A_i can be reconstructed from $\bar{A}_i$, i.e.

$$\bar{A}_i \circ Q^* = A_i. \tag{9}$$

Proof. By Proposition 1, and the inequality $\bar{A}_i \leq A_i$, we have $\bar{A}_i \circ Q^* \leq A_i \circ Q^* = A_i$. On the other hand,

$$(\bar{A}_i \circ Q^*)(y) = \bigvee_{x \in X} (\bar{A}_i(x) * Q^*(x, y)) \geq Q^*(x_i, y) = A_i(y).$$

Corollary 3. *Let the assumptions of Propositions 1 be fulfilled and fuzzy set $\widetilde{A}_i \in L^X$ be "in between" $\bar{A}_i$ and A_i, i.e.*

$$\bar{A}_i \leq \widetilde{A}_i \leq A_i,$$

where $i \in I$. Then A_i can be reconstructed from $\widetilde{A}_i$, i.e.

$$\widetilde{A}_i \circ Q^* = A_i. \tag{10}$$

Proof. The proof follows from the following chain of inequalities:

$$A_i = A_i \circ Q^* \geq \widetilde{A}_i \circ Q^* \geq \bar{A}_i \circ Q^* = A_i.$$

The following proposition is important for the below considered applications. It shows that under the assumptions of Propositions 1, every A_i, $i \in I$, is an eigen set of another fuzzy preorder Q^r, which is composed from all these constituent fuzzy sets. By saying "composed", we mean that opposite to Q^*, Q^r does not require any computation.

Proposition 2. *Let family $(A_i)_{i \in I} \subseteq L^X$, $i \in I$, of normal fuzzy sets of X with pairwise different core elements $(x_i)_{i \in I} \subseteq X$ fulfill (6). Then every A_i, $i \in I$, is an eigen set of the following fuzzy preorder*

$$Q^r(x, y) = \begin{cases} A_i(y), & \textit{if } x = x_i, \\ 1, & \textit{if } x = y, \\ 0, & \textit{otherwise} . \end{cases} \tag{11}$$

4 Fuzzy Preorders and AFIM

In this Section, we will put a bridge between the theory, presented in Section 3, and the theory of autoassociative fuzzy implicative memories (AFIM), presented in Section 2. We will see that in the proposed below model of AFIM, a connecting fuzzy relation (denoted above by W) is a fuzzy preorder relation.

In details, we choose a residuated lattice with the support $L = [0, 1]$ and a database $\{\mathbf{x}^1, \ldots, \mathbf{x}^p\}$ of initial objects that are represented by fuzzy sets or fuzzy relations. In the first case, we have a database of signals, while in the second one, we have 2D gray-scaled images. The second case can be easily reduced to the first one - it is enough to represent an image as a sequence of rows. Below, we assume that our objects are normal fuzzy sets identified with their membership functions, i.e. they are elements of $[0, 1]^X$, where X is a finite universe. The assumption of normality does not put any restriction, because any given finite collection of fuzzy sets on a finite universe can be normalized. Because we illustrate the proposed technique by images, we refer to the initial objects as to images.

In accordance with (5), we construct the fuzzy preorder relation Q^*, such that

$$Q^*(i,j) = \bigwedge_{k=1}^{p} (\mathbf{x}^k(i) \to \mathbf{x}^k(j)).$$

We remark that this is the reverse fuzzy preorder with respect to that given by (4). In the terminology of the theory of autoassociative memories, the results from Section 3 show that under condition (6),

- each constituent input image $\mathbf{x}^k$, $k = 1, \dots, p$, can be retrieved, if the weight matrix W is equal to Q^* and the computation of the output is based on the simpler version of (2), i.e.

$$\mathbf{y}_i = \bigvee_{j=1}^{n} (\mathbf{x}_j^k \ \mathrm{t} \ w_{ij}),\ i = 1, \dots, m, \tag{12}$$

  which does not involve bias θ;
- each constituent input image $\mathbf{x}^k$, $k = 1, \dots, p$, can be retrieved, if the weight matrix W is equal to Q^r (see (11)) with the subsequent computation of the output by (12);
- each constituent input image $\mathbf{x}^k$, $k = 1, \dots, p$, can be fully reconstructed from its binary "skeleton" (see (8) in Corollary 2).

Let us remark that from the second result, listed above, it follows that there exists a weight matrix W which can be assembled from constituent input images in accordance with (11) and by this, no computation is needed. This fact leads to a tremendous saving of computational complexity.

Moreover, from Corollary 3 we deduce a complete characterization of a noise N_k that can be "added to" (actually, subtracted from) a constituent input image $\mathbf{x}^k$ without any corruption of the output. In details,

$$N_k(t) = \begin{cases} n_k(t), & \text{if } t \notin Core\mathbf{x}^k, \\ 0, & \text{otherwise}, \end{cases} \tag{13}$$

where for $t \notin Core\mathbf{x}^k$, the value $n_k(t)$ fulfills the requirement $0 \le n_k(t) \le \mathbf{x}^k(t)$, $k = 1, \dots, p$.

Below, we demonstrate how the presented above theory works in the case of some benchmark input images.

5 Illustration

The aim of this Section is to give illustrations to the theoretical results of this paper. We used gray scaled images with the range $[0, 1]$, where 0 (1) represents the black (white) color. We chose two different sets of images, both were artificially created from available databases. The sets contain 2D images of 20×20 and 32×32 pixels, respectively. All images are represented by vectors that are comprised by successive rows. Each image corresponds to a fuzzy set on $\{1, \ldots, 20\} \times \{1, \ldots, 20\}$ or $\{1, \ldots, 32\} \times \{1, \ldots, 32\}$ with values in $[0, 1]$.

5.1 Experiments with Abstract Images

We have created three databases A, B, and C of 2D images of 20×20 pixels, where condition (6) is/is not fulfilled, details are below.

- **Database A** - contains three images (see figure 1) such that (6) is fulfilled.
- **Database B** - contains four images (see figure 2) such that (6) is not fulfilled with the non-separability degree as follows:

$$D_B = \bigwedge_{x \in X} (A_j(x) \to A_i(x)) \to A_i(x_j). \tag{14}$$

.

- **Database C** - contains eight images (see figure 3) such that (6) is not fulfilled, and the corresponding non-separability degree D_C is less than D_B.

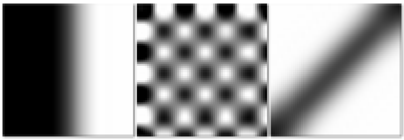

Fig. 1. Database A contains three images such that (6) is fulfilled.

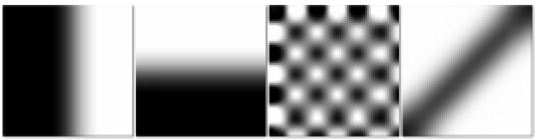

Fig. 2. Database B contains four images such that (6) is not fulfilled with the degree D_B.

For each database of images, we computed he corresponding fuzzy preorder Q^* and its reduction Q^r.

In Fig. 4, we demonstrate the influence of condition (6) on the quality of retrieval by the AFIM mechanism, where the computation of the output is based on (12) and the weight matrix W is equal to Q^* or Q^r. For this purpose we choose

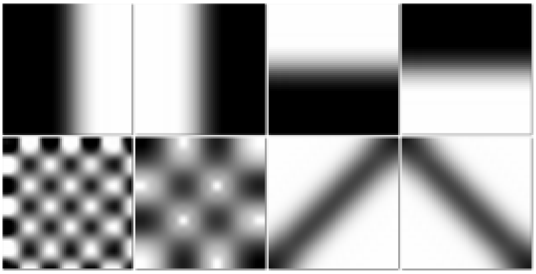

Fig. 3. Database C contains eight images such that (6) is not fulfilled with the degree D_C such that $D_C \leq D_B$.

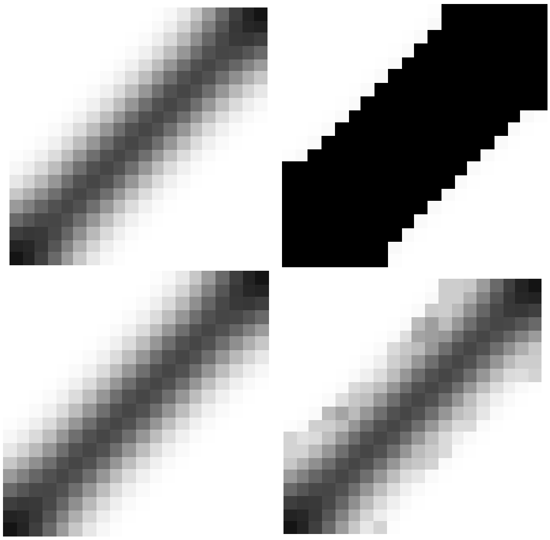

Fig. 4. *Top left* Third image from database A. *Top right* Binary "skeleton" of the third image from database A. *Bottom left* Output image retrieved from the binary "skeleton" of the third image and database A - identical with the third image. *Bottom right* Output image retrieved from the third image and database C - different from the third input image.

the third image from database A as an input and retrieve it from each of three databases A, B and C.

In Fig. 4, we see that if the weight matrix W is computed (comprised) from database A as fuzzy preorder Q^* (Q^r), then the output coincides with the identical to it input. Moreover, any image from database A can be retrieved from its binary "skeleton". If W is computed from database C as fuzzy preorder Q^*, then the output differs from the input, i.e. images from database C cannot be retrieved precisely.

Fig. 5. *Left* Third image from database A with 70% density of eroded noise. *Right* The output, retrieved by AFIM with the weight matrix Q^r. Eroded noise has been completely removed by the AFIM retrieval.

In Fig. 5, we demonstrate how eroded noise (13) can be removed by the AFIM retrieval. We added 70% dense erosion to the third image from database A and process the obtained eroded image by the AFIM with the weight matrix W that corresponds to fuzzy preorder Q^r, computed from database A. In the right-hand side of Fig. 5, we show the output, retrieved by IFAM with the weight matrix Q^r. This output coincides with the original (non-eroded) third image.

6 Conclusion

A new theory of implicative fuzzy associative memory has been proposed. We showed that

1. every database pattern can be successfully retrieved,
   - if all database patterns are well separated, the weight matrix W corresponds to a certain fuzzy preorder relation and the computation of the output is based on a composition with W, which does not involve bias θ;
   - if additionally to the above conditions, the weight matrix W corresponds to a certain reduction of the fuzzy preorder relation.
2. the weight matrix W does not require computation, if the above mentioned conditions are fulfilled.

We discovered a necessary and sufficient condition that guarantees insensitivity to a certain type of noise. The latter is precisely characterized.

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