What Is Fuzzy Natural Logic
Abstract

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Natural Logic. In 1970, G. Lakoff published a paper [8] in which he introduced the concept of *natural logic* with the following goals:

– to express all concepts capable of being expressed in natural language,
– to characterize all the valid inferences that can be made in natural language,
– to mesh with adequate linguistic descriptions of all natural languages.

Natural logic is thus a collection of terms and rules that come with natural language and that allows us to reason and argue in it. According to G. Lakoff's hypothesis, natural language employs a relatively small finite number of atomic predicates that take sentential complements (sentential operators) and are related to each other by meaning-postulates that do not vary from language to language. The concept of natural logic has been further developed by several authors (see, e.g., [2,9] and elsewhere).

In this paper, we will briefly overview a special extension of the mathematical fuzzy logic in narrow sense (FLn) that is called *Fuzzy Natural Logic* (FNL). Its goal stems from the above Lakoff's characterization and can be specified as follows: to develop a *formal theory of human reasoning that includes mathematical models of the semantics of certain classes of special expressions of natural language and generalized quantifiers with regard to presence of the vagueness phenomenon.* The main difference from the Lakoff's characterization is that FNL is a *mathematical* theory which, in addition, includes also the model of the vagueness phenomenon using tools of FLn. At the same time it must follow results of the logical analysis of natural language (see, e.g., [3]).

Fuzzy Logic in Narrow Sense. Recall that FLn is a generalization of classical mathematical logic (see [6,14]) in the sense that it has formally established syntax and semantics. The syntax consists of precise definitions of a formula, proof, formal theory, provability, model, etc. It is extended by more connectives and more logical axioms. Semantics of this logic is many-valued. The completeness theorem says that a formula A is provable iff it is true in the degree 1 in all models.

There are many formal calculi in FLn, for example MTL (monidal t-norm-based logic), BL (basic logic), Łukasiewicz, product, ŁΠ and other logics. They differ from each other by the considered structure of truth values. For FNL,

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the most important is higher-order fuzzy logic called Fuzzy Type Theory (FTT) because the experience indicates that first-order logical systems are not powerful enough for the proper formalization of linguistic semantics which is a necessary constituent of FNL.

Paradigm of FNL. The fuzzy natural logic consists of several formal theories developed on a unique formal basis:

(a) Formal theory of evaluative linguistic expressions [16]; see also [15].
(b) Formal theory of fuzzy/linguistic IF-THEN rules and linguistic descriptions; approximate reasoning based on them [5,13,20,21].
(c) Formal theory of intermediate and generalized quantifiers [4,7,10,11,17].

Fuzzy Type Theory. This is a higher-order fuzzy logic being generalization of classical type theory introduced by B. Russel, A. Church and L. Henkin (for extensive presentation of it see [1]). The generalization consists especially in replacement of the axiom stating "there are two truth values" by a sequence of axioms characterizing structure of the algebra of truth values.

The fuzzy type theory (FTT) is the basic formal tool for FNL. We work with a special case of the Łukasiewicz FTT, which is based on the algebra of truth values forming the standard Łukasiewicz MV$_\Delta$-algebra $\mathcal{L} = \langle [0,1], \vee, \wedge, \otimes, \oplus, \Delta, \rightarrow, 0, 1\rangle$. Important concept in FTT is that of a *fuzzy equality*, which is a reflexive, symmetric and $\otimes$-transitive binary fuzzy relation on some set M, i.e. it is a function $\doteq\colon M \times M \rightarrow L$.

Syntax of FTT is a generalization of the lambda-calculus constructed in a classical way, but differing from classical type theory by definition of additional special connectives and larger list of axioms. It has been proved that the fuzzy type theory is complete. The details can be found in [12,18].

Evaluative Linguistic Expressions. These are expressions of natural language, for example, *small, medium, big, roughly one hundred, very short, more or less deep, not tall, roughly warm or medium hot, quite roughly strong, roughly medium size*, etc. They form a small, syntactically simple, but very important part of natural language which is present in its everyday use any time. The reason is that people regularly need to evaluate phenomena around them and to make important decisions, learn how to behave, and realize various kinds of activities based on evaluation of the given situation. In FNL, a special formal theory of FTT has been constructed using which semantics of the evaluative expressions including their vagueness is modeled. The details can be found in [16].

Fuzzy/Linguistic IF-THEN Rules. In FNL, these are taken as genuine conditional clauses of natural language with the general form

$$\mathsf{IF}\ X \text{ is } \mathcal{A}\ \mathsf{THEN}\ Y \text{ is } \mathcal{B}, \tag{1}$$

where "X is $\mathcal{A}$", "Y is $\mathcal{B}$" are the, so called, evaluative linguistic predications. A typical example is

IF *temperature* is *extremely small* THEN *the amount of gas* is *very big*.

A finite set of rules (1) is called *linguistic description* and it is construed as a special text. The method of *perception-based logical deduction* enables to derive conclusion from linguistic descriptions, thus simulating reasoning of people.

Intermediate and Fuzzy Quantifiers. These are natural language expressions such as *most, a lot of, many, a few, a great deal of, large part of, small part of*. In correspondence with the analysis given by P. Peterson in [22] they are in FNL modeled as special formulas of fuzzy type theory in a certain extension of the formal theory of evaluative linguistic expressions. Typical elaborated quantifiers are

"Most (Almost all, Few, Many) B are A".

The developed model stems from the assumption that intermediate quantifiers are classical general or existential quantifiers for which the universe of quantification is modified and the modification can be imprecise.

Intermediate quantifiers occur also in generalized Aristotle syllogisms, for example:

$$\textbf{PPI}\text{-III:}\ \begin{array}{l} \textit{Major premise } P_1\textit{:}\ \text{Almost all employed people have a car} \\ \textit{Minor premise } P_2\textit{:}\ \text{Almost all employed people are well situated} \\ \hline \textit{Conclusion } C\textit{:}\ \text{Some well situated people have a car} \end{array}$$

Formal validity of more than 120 such syllogisms was already proved. This means that the implication $P_1 \Rightarrow (P_2 \Rightarrow C)$ is true in the degree 1 in all models.

Modeling Human Reasoning. Human reasoning is typical by employing natural language. We argue that formalism of FNL is rich enough to be able to develop a sufficiently well working model of human reasoning. One such possibility was described in [19] where a model of non-monotonic reasoning based on a series of linguistic descriptions characterizing a criminal case faced by a detective Columbo was developed.

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