Chapter 3
Supersymmetry Breaking

Abstract With the previously obtained classification of potentially supersymmetric models in noncommutative geometry we now address the question on how to naturally *break* supersymmetry. In this chapter we will shortly review *soft supersymmetry breaking* and analyze the question which soft supersymmetry breaking terms are present in the spectral action. We find that all possible soft supersymmetry breaking terms can be generated by simply taking into account additional contributions to the action that arise from introducing gaugino masses. In addition there can be contributions from the second Seeley-DeWitt coefficient that is already part of the spectral action.

3.1 Soft Supersymmetry Breaking

Already shortly after the advent of supersymmetry (e.g. [20]) it was realized [19] that if it is a real symmetry of nature, then the superpartners should be of equal mass. This, however, is very much not the case. If it were, we should have seen all the sfermions and gauginos that feature in the Minimal Supersymmetric Standard Model (MSSM, e.g. [7]) in particle accelerators by now. In the context of the MSSM we need [14] a supersymmetry breaking Higgs potential to get electroweak symmetry breaking and give mass to the SM particles. Somehow there should be a mechanism at play that *breaks* supersymmetry. Over the years many mechanisms have been suggested that break supersymmetry and explain why the masses of superpartners should be different at low scales. Ideally this should be mediated by a *spontaneous* symmetry breaking mechanism, such as D-term [17] or F-term [9] supersymmetry breaking. But phenomenologically such schemes are disfavoured, for they require that 'in each family at least one slepton/squark is lighter than the corresponding fermion' [7, Sect. 9.1]. Alternatively, supersymmetry can be broken *explicitly* by means of a supersymmetry breaking Lagrangian. In order for the solution to the hierarchy problem that supersymmetry provides to remain useful, the terms in this supersymmetry breaking Lagrangian should be *soft* [10]. This means that such terms have couplings of positive mass dimension, not yield quadratically divergent loop corrections that would spoil the solution to the hierarchy problem (the enormous

W. Beenakker et al., *Supersymmetry and Noncommutative Geometry*,
SpringerBriefs in Mathematical Physics, DOI 10.1007/978-3-319-24798-4_3

sensitivity of the Higgs boson mass to perturbative corrections) that supersymmetry provides.

More precisely, consider a simple gauge group G, a set of scalar fields $\{\widetilde{\psi}_\alpha, \alpha = 1, \ldots, N\}$, all in a representation of G, and gauginos $\lambda = \lambda_a T^a$, with T^a the generators of G. Then the most general renormalizable Lagrangian that breaks supersymmetry softly is given [12] by

$$\mathscr{L}_{\text{soft}} = -\widetilde{\psi}^*_\alpha (m^2)_{\alpha\beta} \widetilde{\psi}_\beta + \left(\frac{1}{3!} A_{\alpha\beta\gamma} \widetilde{\psi}_\alpha \widetilde{\psi}_\beta \widetilde{\psi}_\gamma - \frac{1}{2} B_{\alpha\beta} \widetilde{\psi}_\alpha \widetilde{\psi}_\beta + C_\alpha \widetilde{\psi}_\alpha + h.c. \right) - \frac{1}{2}(M \lambda_a \lambda_a + h.c.), \tag{3.1}$$

where the combinations of fields should be such that each term is gauge invariant. This expression contains the following terms:

- mass terms for the scalar bosons $\widetilde{\psi}_\alpha$. For the action to be real, the matrix m^2 should be self-adjoint;
- trilinear couplings, proportional to a symmetric tensor $A_{\alpha\beta\gamma}$ of mass dimension one;
- bilinear scalar interactions via a matrix $B_{\alpha\beta}$ of mass dimension two;
- for gauge singlets there can be linear couplings, with $C_\alpha \in \mathbb{C}$ having mass dimension three;
- gaugino mass terms, with $M \in \mathbb{C}$.

It is important to note that the Lagrangian (3.1) corresponds to a theory that is defined on a Minkowskian background. Performing a Wick transformation $t \to i\tau$ for the time variable to translate it to a theory on a Euclidean background, changes all the signs in (3.1):

$$\mathscr{L}^{\text{E}}_{\text{soft}} = \widetilde{\psi}^*_\alpha (m^2)_{\alpha\beta} \widetilde{\psi}_\beta - \left(\frac{1}{3!} A_{\alpha\beta\gamma} \widetilde{\psi}_\alpha \widetilde{\psi}_\beta \widetilde{\psi}_\gamma - \frac{1}{2} B_{\alpha\beta} \widetilde{\psi}_\alpha \widetilde{\psi}_\beta + C_\alpha \widetilde{\psi}_\alpha + h.c. \right) + \frac{1}{2}(M \lambda_a \lambda_a + h.c.). \tag{3.2}$$

This expression can easily be extended to the case of a direct product of simple groups, but its main purpose is to give an idea of what soft supersymmetry breaking terms typically look like.

3.2 Soft Supersymmetry Breaking Terms from the Spectral Action

As was mentioned at the end of Sect. 1.2.2, we have to settle with the terms in the action that the spectral action principle provides us. The question at hand is thus whether noncommutative geometry can give us terms needed to break the

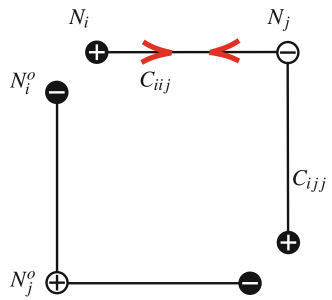

Fig. 3.1 A building block of the second type that defines a fermion—sfermion pair $(\psi_{ij}, \widetilde{\psi}_{ij})$. Contributions to the mass term of the sfermion correspond to paths going back and forth on an edge, as is depicted on the *top edge*

supersymmetry. In Chap. 2 we have disregarded the second to last term ($\propto \Lambda^2$) in the expansion (1.24) of the spectral action. Here we *will* take this term into account.

In the following sections we will check for each of the terms in (3.2) if it can also occur in the spectral action (1.21) (with (1.24) for the expansion of its second term) in the context of the building blocks. We will denote scalar fields generically by $\widetilde{\psi}_{ij} \in C^\infty(M, \mathbf{N}_i \otimes \mathbf{N}_j^o)$, fermions by $\psi_{ij} \in L^2(M, S \otimes \mathbf{N}_i \otimes \mathbf{N}_j^o)$ and gauginos by $\lambda_i \in L^2(M, S \otimes M_{N_i}(\mathbb{C}))$, with $M_{N_i}(\mathbb{C}) \to su(N_i)$ after reducing the gaugino degrees of freedom, Sect. 2.2.1.1.

3.2.1 Scalar Masses (E.g. Higgs Masses)

Terms that describe the masses of the scalar particles such as the first term of (3.2) are known [15, Sect. 5.4] to originate from the square of the finite Dirac operator (c.f. (1.24)). In terms of Krajewski diagrams these contributions are given by paths such as depicted in Fig. 3.1.

Then the contribution to the action from a building block of the second type is:

$$-\frac{1}{2\pi^2}\Lambda^2 f_2 \operatorname{tr}_F \Phi^2 = -\frac{1}{2\pi^2}\Lambda^2 f_2\big(4N_i|C_{iij}\widetilde{\psi}_{ij}|^2 + 4N_j|C_{ijj}\widetilde{\psi}_{ij}|^2\big) \qquad (3.3)$$

where $N_{i,j}$ are the dimensions of the representations $\mathbf{N_{i,j}}$ and $\widetilde{\psi}_{ij}$ is the field that is generated by the components of D_F parametrized by C_{iij} and C_{ijj}. Their expression depends on which building blocks are present in the spectral triple.

In the case that there is a building block $\mathscr{B}_{ijk}$ of the third type present (parametrized by—say—$\Upsilon_i{}^j$, $\Upsilon_i{}^k$ and $\Upsilon_j{}^k$ acting on family-space), we can both get the correct fermion–sfermion–gaugino interaction and a normalized kinetic term for the sfermion $\widetilde{\psi}_{ij}$ by on the one hand setting

$$C_{iij} = \varepsilon_{i,j}\sqrt{\frac{r_i}{\omega_{ij}}}(N_k \Upsilon_i{}^{j*}\Upsilon_i{}^j)^{1/2}, \quad C_{ijj} = s_{ij}\sqrt{\frac{r_j}{r_i}}C_{iij}, \quad s_{ij} = \varepsilon_{i,j}\varepsilon_{j,i} \qquad (3.4)$$

where $\varepsilon_{i,j}, \varepsilon_{j,i}, s_{ij} \in \{\pm 1\}$, $r_i := q_i n_i$ with $q_i := f(0) g_i^2/\pi^2$, n_i the normalization constant for the generators T_i^a of $su(N_i)$ in the fundamental representation and $\omega_{ij} := 1 - r_i N_i - r_j N_j$. On the other hand we scale the sfermion according to

$$\widetilde{\psi}_{ij} \to \mathcal{N}_{ij}^{-1} \widetilde{\psi}_{ij}, \quad \text{with} \quad \mathcal{N}_{ij}^{-1} = \sqrt{\frac{2\pi^2 \omega_{ij}}{f(0)}} (N_k \Upsilon_i{}^{j*} \Upsilon_i{}^{j})^{-1/2}. \tag{3.5}$$

There is an extra contribution from $\text{tr}_F\, \Phi^2$ to $|\widetilde{\psi}_{ij}|^2$ compared to that of the building block of the second type. This contribution corresponds to paths going back and forth over the rightmost and bottommost edges in Fig. 2.6. In the parametrizations (3.4) and upon scaling according to (3.5) these together yield

$$-\frac{1}{2\pi^2} \Lambda^2 f_2 \Big(4N_i |C_{iij} \widetilde{\psi}_{ij}|^2 + 4N_j |C_{ijj} \widetilde{\psi}_{ij}|^2 + 4N_k |\Upsilon_i{}^j \widetilde{\psi}_{ij}|^2 \Big) \to -4\Lambda^2 \frac{f_2}{f(0)} |\widetilde{\psi}_{ij}|^2, \tag{3.6}$$

and similar expressions for $|\widetilde{\psi}_{ik}|^2$ and $|\widetilde{\psi}_{jk}|^2$. Interestingly, the pre-factor for this contribution is universal, i.e. it is completely independent from the representation $\mathbf{N}_i \otimes \mathbf{N}_j^o$ the scalar resides in.

Note that, for $\Lambda \in \mathbb{R}$ and $f(x)$ a positive function (as is required for the spectral action) in both cases the scalar mass contributions are of the wrong sign, i.e. they have the same sign as a Higgs-type scalar potential would have. The result would be a theory whose gauge group is broken maximally. We will see that, perhaps counterintuitively, we can escape this by adding gaugino-masses.

3.2.2 Gaugino Masses

Having a building block of the first type, that consists of two copies of $M_N(\mathbb{C})$ for a particular value of N, allows us to define a finite Dirac operator whose two components map between these copies, since both are of opposite grading. On the basis $\mathcal{H}_F = M_N(\mathbb{C})_L \oplus M_N(\mathbb{C})_R$ this is written as

$$D_F = \begin{pmatrix} 0 & G \\ G^* & 0 \end{pmatrix}, \qquad G : M_N(\mathbb{C})_R \to M_N(\mathbb{C})_L,$$

since it needs to be self-adjoint. This form for D_F automatically satisfies the order one condition (1.12) and the demand $JD = DJ$ (see (1.10)) translates into $G = JG^*J^*$. If we want this to be a genuine mass term it should not generate any scalar field via its inner fluctuations. For this G must be a multiple of the identity and consequently we write $G = M\, \text{id}_N$, $M \in \mathbb{C}$. This particular pre-factor is dictated by how the term appears in (3.2).

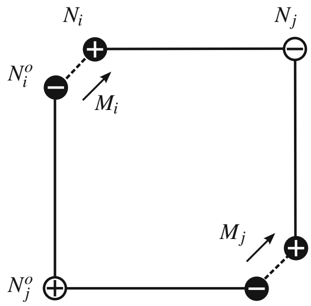

Fig. 3.2 A building block of the second type that defines a fermion—sfermion pair $(\psi_{ij}, \widetilde{\psi}_{ij})$, dressed with mass terms for the corresponding gauginos (*dashed edges*, labeled by $M_{i,j}$)

For the fermionic action we then have

$$\frac{1}{2}\langle J(\lambda_L, \lambda_R), \gamma^5 D_F(\lambda_L, \lambda_R)\rangle = \frac{1}{2}M\langle J_M\lambda_R, \gamma^5\lambda_R\rangle + \frac{1}{2}\overline{M}\langle J_M\lambda_L, \gamma^5\lambda_L\rangle, \tag{3.7}$$

where $(\lambda_L, \lambda_R) \in \mathscr{H}^+ = L^2(S_+ \otimes M_N(\mathbb{C})_L) \oplus L^2(S_- \otimes M_N(\mathbb{C})_R)$, with $S_\pm$ the space of left- resp. right-handed spinors. This indeed describes a gaugino mass term for a theory on a Euclidean background (cf. [2], Eq. 4.52).

A gaugino mass term in combination with building blocks of the second type (for which two gaugino pairs are required), gives extra contributions to the spectral action. From the set up as is depicted in Fig. 3.2, one can see that tr D_F^4 receives extra contributions coming from paths that traverse two edges representing a gaugino mass and two representing the scalar $\widetilde{\psi}_{ij}$. In detail, the extra contributions are given by:

$$\begin{aligned}\frac{f(0)}{8\pi^2}\operatorname{tr}_F \Phi^4 &= \frac{f(0)}{\pi^2}\left(N_i|M_i|^2|C_{iij}\widetilde{\psi}_{ij}|^2 + N_j|M_j|^2|C_{ijj}\widetilde{\psi}_{ij}|^2\right)\\ &\to 2\Big(r_iN_i|M_i|^2 + r_jN_j|M_j|^2\Big)|\widetilde{\psi}_{ij}|^2,\end{aligned} \tag{3.8}$$

upon scaling the fields.

This means that there is an extra contribution to the scalar mass terms, that is of opposite sign (i.e. positive) as compared to the one from the previous section. When

$$2r_iN_i|M_i|^2 + 2r_jN_j|M_j|^2 > 4\frac{f_2}{f(0)}\Lambda^2,$$

then the mass terms of the sfermions have the correct sign, averting the problem of a maximally broken gauge group that was mentioned in the previous section. Comparing this with the expression for the Higgs mass(es) raises interesting questions about the physical interpretation of this result. In particular, if we would require the mass terms of the sfermions and Higgs boson(s) to have the correct sign already at the scale Λ on which we perform the expansion of the spectral action, this seems to suggest that at least some gaugino masses must be very large.

Note that a gauge singlet $\psi_{\rm sin} \in L^2(M, S \otimes \mathbf{1} \otimes \mathbf{1}'^o)$ (such as the right-handed neutrino) can be dressed with a Majorana mass matrix $\Upsilon_{\rm m}$ in family space (see [2, Sect. 2.6] and Fig. 3.3). This yields extra supersymmetry breaking contributions:

$$\frac{f(0)}{8\pi^2}\,\mathrm{tr}\left[4(C_{111'}\widetilde{\psi}_{\rm sin})\overline{\Upsilon_{\rm m}}(C_{111'}\widetilde{\psi}_{\rm sin})\overline{M} + 4(C_{11'1'}\widetilde{\psi}_{\rm sin})\overline{\Upsilon_{\rm m}}(C_{11'1'}\widetilde{\psi}_{\rm sin})\overline{M'}\right] + h.c.$$
$$\to r_1(\overline{M} + \overline{M'})\,\mathrm{tr}\,\overline{\Upsilon_{\rm m}}\widetilde{\psi}^2_{\rm sin} + h.c. \tag{3.9}$$

where M and M' denote the gaugino masses of the two one-dimensional building blocks $\mathscr{B}_1$, $\mathscr{B}_{1'}$ of the first type respectively and the trace is over family space. This expression is independent of whether there are building bocks of the third type present.

Note furthermore that the gaugino masses do not give rise to mass terms for the gauge bosons. In the spectral action such terms could come from an expression featuring both $D_A = i\gamma^\mu D_\mu$ and D_F twice. We do have such a term in (1.24) but since it appears with a commutator between the two and since we demanded the gaugino masses to be a multiple of the identity in $M_N(\mathbb{C})$, such terms vanish automatically. (In contrast, the Higgs boson does generate mass terms for the $W^\pm$- and Z-bosons, partly since the Higgs is not in the adjoint representation.)

3.2.3 Linear Couplings

The fourth term of (3.2) can only occur for a gauge singlet, i.e. the representation $\mathbf{1}\otimes\mathbf{1}^o$ (or, quite similarly, the representation $\overline{\mathbf{1}}\otimes\overline{\mathbf{1}}^o$). The only situation in which such a term can arise is with a building block of the second type—defining a fermion–sfermion pair $(\psi_{\rm sin}, \widetilde{\psi}_{\rm sin})$ and their antiparticles (see Fig. 3.3). Moreover in this case a Majorana mass $\Upsilon_{\rm m}$ is possible, that does not generate a new field.

Any such term in the spectral action must originate from a path in this Krajewski diagram consisting of either two or four steps (corresponding to the second and fourth

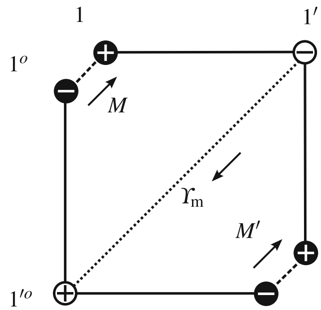

Fig. 3.3 A building block of the second type that defines a gauge singlet fermion–sfermion pair $(\psi_{\rm sin}, \widetilde{\psi}_{\rm sin})$. Moreover, a Majorana mass term $\Upsilon_{\rm m}$ is possible

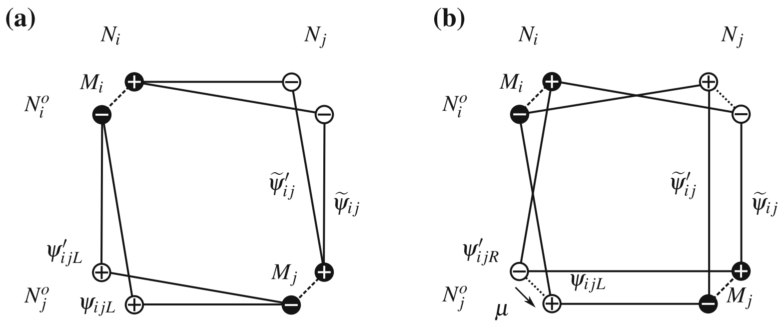

Fig. 3.4 Two building blocks of the second type defining two fermion–sfermion pairs $(\psi_{ij}, \widetilde{\psi}_{ij})$ and $(\psi'_{ij}, \widetilde{\psi}'_{ij})$ in the same representation. **a** When the gradings of the representations are equal. **b** When the gradings of the representations differ

power of the Dirac operator), ending at the same vertex at which it started (if it is to contribute to the trace) and traversing an edge labeled by $\widetilde{\psi}_{\text{sin}}$ only once. From the diagram one readily checks that such a contribution cannot exist.

3.2.4 Bilinear Couplings

If a bilinear coupling (such as the third term in (3.2)) is to be a gauge singlet, the two fields $\widetilde{\psi}_{ij}$ and $\widetilde{\psi}'_{ij}$ appearing in the expression should have opposite finite representations, e.g. $\widetilde{\psi}_{ij} \in C^\infty(M, \mathbf{N}_i \otimes \mathbf{N}_j^o)$, $\widetilde{\psi}'_{ij} \in C^\infty(M, \mathbf{N}_j \otimes \mathbf{N}_i^o)$. We will rename $\widetilde{\psi}'_{ij} \to \overline{\widetilde{\psi}}'_{ij}$ for consistency with Sect. 2.2.5.2. The building blocks of the second type by which they are defined are depicted in Fig. 3.4.

The gradings of both representations are either the same (left image of Fig. 3.4), or they are of opposite eigenvalue (the right image). A contribution to the action that resembles the third term in (3.2) needs to come from paths in the Krajewski diagram of Fig. 3.4 consisting of either two or four steps, ending in the same point as where they started and traversing an edge labeled by $\widetilde{\psi}_{ij}$ and $\widetilde{\psi}'_{ij}$ only once.

One can easily check that in the left image of Fig. 3.4 no such paths exist. In the second case (right image of Fig. 3.4), however, there arises the possibility of a component μ of the finite Dirac operator that maps between the vertices labeled by ψ_{ij} and ψ'_{ij} (and consequently also between $\overline{\psi}_{ij}$ and $\overline{\psi}'_{ij}$). This corresponds to a building block of the fifth type (Sect. 2.2.5.2). There is a contribution to the action (via tr D_F^4) that comes from loops traversing both an edge representing a gaugino mass and one representing μ. If the component μ is parameterized by a complex number, then the contribution is

$$\frac{f(0)}{8\pi^2}\Big(8N_i \operatorname{tr} M_i \overline{\widetilde{\psi}}_{ij} C^*_{iij}\mu C'_{iij}\widetilde{\psi}'_{ij} + 8N_j \operatorname{tr} M_j \overline{\widetilde{\psi}}_{ij} C^*_{ijj}\mu C'_{ijj}\widetilde{\psi}'_{ij}\Big) + h.c.$$
$$\to 2\big(r_i N_i M_i + r_j N_j M_j\big)\mu \operatorname{tr} \overline{\widetilde{\psi}}_{ij}\widetilde{\psi}'_{ij} + h.c., \tag{3.10}$$

where the traces are over $\mathbf{N}_j^{\oplus M}$, with M the number of copies of $\mathbf{N}_i \otimes \mathbf{N}_j^o$. This indeed yields a bilinear term such as the third one of (3.2).

3.2.5 Trilinear Couplings

Trilinear terms such as the second term of (3.2) might appear in the spectral action. For that we need three fields $\widetilde{\psi}_{ij} \in C^\infty(M, \mathbf{N}_i \otimes \mathbf{N}_j^o)$, $\widetilde{\psi}_{jk} \in C^\infty(M, \mathbf{N}_j \otimes \mathbf{N}_k^o)$ and $\widetilde{\psi}_{ik} \in C^\infty(M, \mathbf{N}_i \otimes \mathbf{N}_k^o)$, generated by the finite Dirac operator. Such a term can only arise from the fourth power of the finite Dirac operator[1] which is visualized by paths in the Krajewski diagram consisting of four steps, three of which correspond to a component that generates a scalar field, the other one must be a term that does not generate inner fluctuations, e.g. a mass term. Non-gaugino fermion mass terms were already covered in Chap. 2 and were seen to generate potentially supersymmetric trilinear interactions, so the mass term must be a gaugino mass.

If the component of the finite Dirac operator that does not generate a field is a gaugino mass term (mapping between—say—$M_{N_i}(\mathbb{C})_R$ and $M_{N_i}(\mathbb{C})_L$), then two of the three components that do generate a field must come from building blocks of the second type, since they are the only ones connecting to the adjoint representations. If we denote the non-adjoint representations from these building blocks by $\mathbf{N}_i \otimes \mathbf{N}_j^o$ and $\mathbf{N}_i \otimes \mathbf{N}_k^o$ then we can only get a contribution to $\operatorname{tr} D_F^4$ if there is a component of D_F connecting these two representations. If $\mathbf{N_j} = \mathbf{N_k}$, such a component could yield a mass term for the fermion in the representation $\mathbf{N}_i \otimes \mathbf{N}_j^o$, and we revert to the previous section. If $\mathbf{N_j} \neq \mathbf{N_k}$ then the remaining component of D_F must be part of a building block of the third type, namely $\mathscr{B}_{ijk}$. This situation is depicted in Fig. 3.5. It gives rise to three different trilinear interactions corresponding to the paths labeled by arrows in the figure. Each of these three paths actually represents four contributions: one can traverse each path in the opposite direction, and for each path one can reflect it around the diagonal, giving another path with the same contribution to the action.

Calculating the spectral action we get for each building block $\mathscr{B}_{ijk}$ of the third type the contributions

$$\frac{f(0)}{\pi^2}\Big(N_i\overline{M_i} \operatorname{tr} \Upsilon_j{}^k \widetilde{\psi}_{jk}\overline{\widetilde{\psi}}_{ik} C^*_{iik} C_{iij}\widetilde{\psi}_{ij} + N_j\overline{M_j} \operatorname{tr} C_{jjk}\widetilde{\psi}_{jk}\overline{\widetilde{\psi}}_{ik}\Upsilon_i{}^k C_{ijj}\widetilde{\psi}_{ij}$$
$$+ N_k\overline{M_k} \operatorname{tr} C_{jkk}\widetilde{\psi}_{jk}\overline{\widetilde{\psi}}_{ik} C^*_{ikk}\Upsilon_i{}^j\widetilde{\psi}_{ij}\Big) + h.c. \tag{3.11}$$

[1]Here we assume that each component of the finite Dirac operator generates only a single field, instead of—say—two composite ones.

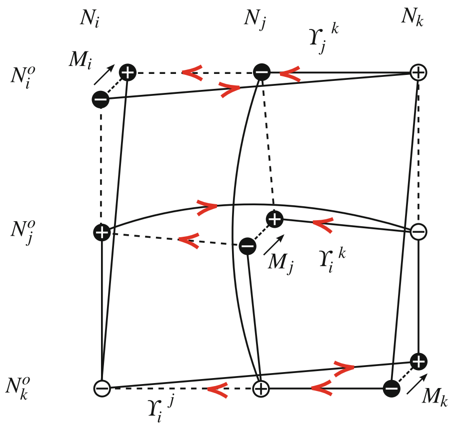

Fig. 3.5 A situation in which there are three building blocks $\mathscr{B}_{i,j,k}$ of the first type (black vertices), three building blocks $\mathscr{B}_{ij,jk,ik}$ of the second type and a building block $\mathscr{B}_{ijk}$ of the third type. Adding gaugino masses (*dashed edges*) gives rise to trilinear interactions, corresponding to the paths in the diagram marked by *arrows*

where all traces are over $\mathbf{N}_j^{\oplus M}$. A careful analysis of the demand for supersymmetry in this context (see Sect. 2.2.3) requires the parameters $\Upsilon_i{}^j$, $\Upsilon_i{}^k$ and $\Upsilon_j{}^k$ to be related via

$$C^*_{ikk}\Upsilon_j{}^k = -\Upsilon_i{}^k C_{jkk}, \quad \Upsilon_i{}^k C_{iij} = -C^*_{iik}\Upsilon_i{}^j, \quad \Upsilon_i{}^j C_{jjk} = -\Upsilon_j{}^k C_{ijj} \qquad (3.12)$$

where C_{iij} and C_{ijj} act trivially on family space if $\widetilde{\psi}_{ij}$ is assumed to have $R = 1$. From this relation we can deduce that $s_{ij}s_{ik}s_{jk} = -1$ for the product of the three signs defined in (3.4). If we replace $C_{iik} \to C_{ikk}$, $C_{iij} \to C_{ijj}$, $C_{jjk} \to C_{jkk}$ and $C_{ijj} \to C_{iij}$ in the first two terms of (3.11) using (3.4), employ (2.55), then (3.11) can be written as

$$\frac{f(0)}{\pi^2}\left(N_i\overline{M_i}\frac{r_i}{r_k} + N_j\overline{M_j}\frac{r_j}{r_k} + N_k\overline{M_k}\right) \operatorname{tr} C_{jkk}\widetilde{\psi}_{jk}\overline{\widetilde{\psi}}_{ik}C^*_{ikk}\Upsilon_i{}^j\widetilde{\psi}_{ij} + h.c.$$

We then scale the sfermions according to (3.5), again using (3.4) for C_{jkk} and C^*_{ikk} to obtain

$$2\kappa_k g_l\sqrt{2\frac{\omega_{ij}}{q_l}}\left(r_i N_i\overline{M_i} + r_j N_j\overline{M_j} + r_k N_k\overline{M_k}\right) \operatorname{tr} \widetilde{\Upsilon}\,\widetilde{\psi}_{ij}\widetilde{\psi}_{jk}\overline{\widetilde{\psi}}_{ik} + h.c., \qquad (3.13)$$

where we have written

$$\widetilde{\Upsilon} := \Upsilon_i{}^j(N_k \operatorname{tr} \Upsilon_i{}^{j*}\Upsilon_i{}^j)^{-1/2}$$

for the scaled version of the parameter $\Upsilon_i{}^j$, $\kappa_k := \varepsilon_{k,j}\varepsilon_{k,i}$ and the index l can take any of the values that appear in the theory.

3.3 Summary and Conclusions

We have now considered all terms featuring in (3.2). At the same time the reader can convince himself that this exhausts all possible terms that appear via tr D_F^4 and feature a gaugino mass. As for the fermionic action, a component of D_F mapping between two adjoint representations can give gaugino mass terms (3.7). As for the bosonic action, any path of length two contributing to the trace and featuring a gaugino mass, cannot feature other fields. In contrast, a path of length four in a Krajewski diagram involving a gaugino mass can feature:

- only that mass, as a constant term (see the comment at the end of this section);
- two times the scalar from a building block of the second type, when going in one direction (3.8);
- two times the scalar from a building block of the second type, when going in two directions and when a Majorana mass is present (only possible for singlet representations, (3.9));
- two scalars from two different building blocks of the second type having opposite grading in combination with a building block of the fifth type (3.10).
- three scalars, partly originating from a building block of the second type and partly from one of the third type (3.13).

Furthermore, via tr D_F^2 there are contributions to the scalar masses from building blocks of the second and third type (3.3). We can combine the main results of the previous sections into the following theorem.

Theorem 3.1 *All possible terms that break supersymmetry softly and that can originate from the spectral action (1.24) of an almost-commutative geometry consisting of building blocks are mass terms for scalar fields and gauginos and trilinear and bilinear couplings. More precisely, the most general Lagrangian that softly breaks supersymmetry and results from almost-commutative geometries is of the form*

$$\mathscr{L}_{\text{soft}}^{\text{NCG}} = \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \mathscr{L}^{(3)} + \mathscr{L}^{(4)} + \mathscr{L}^{(5)}, \tag{3.14}$$

where

$$\mathscr{L}^{(1)} = \frac{1}{2} M_i \langle J_M \lambda_{iR}, \gamma^5 \lambda_{iR} \rangle + \frac{1}{2} \overline{M_i} \langle J_M \lambda_{iL}, \gamma^5 \lambda_{iL} \rangle \tag{3.15a}$$

for each building block $\mathscr{B}_i$ of the first type,

$$\mathscr{L}^{(2)} = 2\Big(r_i N_i |M_i|^2 + r_j N_j |M_j|^2 - 2 \frac{f_2}{f(0)} \Lambda^2 \Big) |\widetilde{\psi}_{ij}|^2, \tag{3.15b}$$

for each building block $\mathscr{B}_{ij}$ of the second type for which there is at least one building block $\mathscr{B}_{ijk}$ of the third type present (knowing that a single $\mathscr{B}_{ij}$ cannot be supersymmetric by itself, Sect. 2.2.2),

$$\mathscr{L}^{(3)} = 2\kappa_k g_l \sqrt{2\frac{\omega_{ij}}{q_l}} \left(r_i N_i \overline{M_i} + r_j N_j \overline{M_j} + r_k N_k \overline{M_k} \right) \operatorname{tr} \widetilde{\Upsilon} \widetilde{\psi}_{ij} \widetilde{\psi}_{jk} \overline{\widetilde{\psi}}_{ik} + h.c., \tag{3.15c}$$

for each building block $\mathscr{B}_{ijk}$ of the third type,

$$\mathscr{L}^{(4)} = r_1(\overline{M} + \overline{M'}) \operatorname{tr} \overline{\Upsilon_{\mathrm{m}}} \widetilde{\psi}^2_{\mathrm{sin}} + h.c. \tag{3.15d}$$

for each building block $\mathscr{B}_{\mathrm{maj}}$ of the fourth type (with the trace over a possible family index), and

$$\mathscr{L}^{(5)} = 2(r_i N_i M_i + r_j N_j M_j)\mu \operatorname{tr} \overline{\widetilde{\psi}}_{ij} \widetilde{\psi}'_{ij} + h.c. \tag{3.15e}$$

for each building block $\mathscr{B}_{\mathrm{mass}}$ of the fifth type.

It should be remarked that the building blocks of the fourth and fifth type typically already provide soft breaking terms of their own (see Sects. 2.2.5.1 and 2.2.5.2).

Interestingly, all supersymmetry breaking interactions that occur are seen to be generated by the gaugino masses (except the ones coming from the trace of the square of the finite Dirac operator) and each of them can be associated to one of the five supersymmetric building blocks. Note that the gaugino masses give rise to extra contributions that are not listed in (3.14). For each gaugino mass M_i there is an additional contribution

$$\mathscr{L}_{M_i} = \frac{f(0)}{4\pi^2}|M_i|^4 - \frac{f_2}{\pi^2}\Lambda^2|M_i|^2.$$

Since such contributions do not contain fields, they are not breaking supersymmetry, but might nonetheless be interesting from a gravitational perspective.

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