Chapter 4
Reversing Time

4.1 The History of *i*

When you square any number by multiplying it by itself, whether it is positive or negative, the result is always a positive number. Therefore, the square root of minus one, or i in the usual mathematical notation, in some sense is meaningless or imaginary, because no real number has that mathematical property. But i turns out to be useful in mathematics, physics, and engineering as a mathematical object. Nevertheless, it required some time for i to assume its proper place in mathematics.

In 1545, Gerolamo Cardano published his ground-breaking book on algebra, *Ars Magna*, in which he investigated the solutions of some cubic equations. He introduced a new idea, that the square root of a negative number should be taken seriously and used in algebraic procedures rather than discarded as nonsense. In Rafael Bombelli's 1569 book *Algebra*, he developed the rules of complex algebra and pointed out that terms involving the square root of a negative number had to be segregated and treated separately from real numbers when performing addition or multiplication. Mathematicians subsequently realized the importance of i as the key to a whole new branch of mathematics, the theory of functions of a complex variable, and this became an important part of the discipline of mathematics.

About 1750, Leonhard Euler used his skill with power-series expansions to introduce his famous formula: $\exp(i\theta) = \cos(\theta) + i\sin(\theta)$. The geometrical interpretation of this formula has found broad uses in physics and engineering. It makes the one-dimensional line of real numbers into a two-dimensional "complex plane" with the real numbers on the horizontal axis, the imaginary numbers on the vertical axis, with the variable θ representing the angle that a line from the origin to given point in the complex plane makes with the real axis. Entities like waves that have sinusoidal behavior can be very compactly described mathematically as rotations in the complex plane.

Electrical engineers often describe the time-dependent sinusoidal voltages on AC power lines as $V(t) = V_0 \sin(\omega t) = V_0\, \Im\{\exp[i(\omega t)]\}$ where "$\Im\{\}$" means the imaginary part, i.e., using only the sin-function part of Euler's formula, V_0 is the

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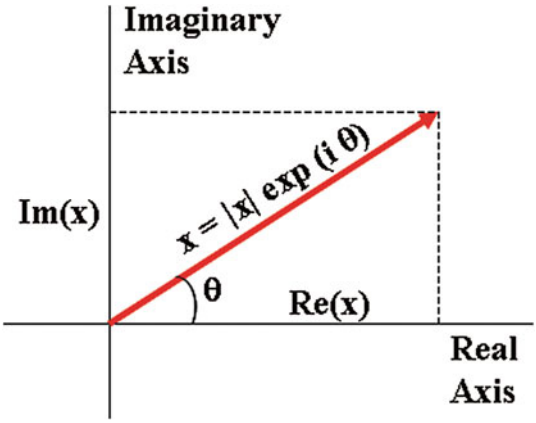

Fig. 4.1 Representation of a complex variable $x = |x|\exp(i\theta)$ on the complex plane. The real part of x projects on the horizontal axis, and the imaginary part of x projects on the imaginary axis. The variable makes an angle θ with the real axis

peak voltage of the wave form, and ω is the angular frequency in radians per second of the voltage sine wave, i.e., the rate at which θ in Fig. 4.1 is changing with time. It is convenient, when engineers are dealing with combinations of such waves, to keep the whole exponential (not just the imaginary part) throughout a calculation, and to apply the extraction of the imaginary part only to the final result. We also note that electrical engineers typically use the symbol "j" rather than "i" to denote $\sqrt{-1}$ because "i" is already used by them as the symbol for electrical current.

In classical physics, a function describing a pressure wave or a displacement wave moving along the x-axis of a system, perhaps a sound wave or a wave on a string, is often written as $y(x,t) = y_0 \cos(kx - \omega t) = y_0\, \Re\{\exp[i(kx - \omega t)]\}$. Here "$\Re\{\}$" means the real part of the function, y_0 is the amplitude of the displacement or pressure variation, and k the wave number ($k = 2\pi/\lambda$), ω is the angular frequency ($\omega = 2\pi f$), f is the frequency in Hz, and λ is the wavelength of the wave.

In the wave functions of quantum mechanics, the extraction of the real part of the function is dropped and the whole complex function is used. Thus, the probability-amplitude wave function of a wave moving on the x-axis has the schematic form: $\psi(x,t) = \psi_0 \exp(ikx - i\omega t)$. Quantum mechanics is unique among physics theories in "eating whole" the mathematical formalism of functions of a complex variable.

If we want a wave that moves in the negative space ($-x$) direction, we reverse the sign of the k-term to give $\psi_{x-rev}(x,t) = \psi_0 \exp[-ikx - i\omega t]$. If we want a wave that moves in the negative time direction, we reverse the sign of the ω-term to give $\psi_{t-rev}(x,t) = \psi_0 \exp[ikx + i\omega t]$. The mathematical operation of complex conjugation, which replaces i with $-i$ everywhere in a mathematical function, performs both of these reversals, so that $\psi^*(x,t) = \psi_0 \exp[-ikx + i\omega t]$. In 1932, Eugene Wigner showed that in quantum mechanics, the operation of complex conjugation is the time-reversal operator, an operation that reverses the direction of time in the description of a quantum system.

Thus, the role of i in quantum mechanics is a subtle one. Quantum wave functions are complex because their time structure is encoded in their complexity, and the operation of complex conjugation reverses that time structure, making time run backwards in the quantum mechanical description.

4.2 Dirac and Time Symmetry

Paul Dirac (Fig. 4.2) was among the originators of the standard formalism of quantum mechanics, developing a general formalism and showing that both Heisenberg's matrix mechanics and Schrödinger's wave mechanics were included in it [1]. In 1928, he introduced the Dirac equation, a wave equation compatible with relativity that is specific to "fermion" particles [2]. Fermions (examples: electrons, μ-leptons, protons, quarks, ...) have an intrinsic angular momentum or "spin" that is a half-integer fraction of $\hbar$ (typically, $\hbar/2$) and obey the rules of Fermi-Dirac statistics, a generalization of the Pauli Exclusion Principle. In particular, fermions obey statistical rules that are qualitatively different from those observed by "boson" particles (examples: photons, α-particles, π-mesons, gluons, ...), which have a spin of 0 or some integer

Fig. 4.2 In 1933, Paul A.M. Dirac (1902–1984) received the Nobel Prize in Physics for his work on quantum mechanics

multiple of $\hbar$ and obey the statistical rules of Bose–Einstein statistics. It is interesting and puzzling to note that if bosons are rotated by 360° they return to the same state, while fermions must be rotated by 720° to return to the same state.

One of the unexpected predictions of the Dirac equation was the existence of anti-matter. Each fermion particle described by the Dirac equation should have a mirror-twin "antiparticle" that has the same mass and spin and an opposite electrical charge and parity. Dirac predicted that the negative-charge electron should have a positive-charge antimatter twin, the positron. In 1932, four years after Dirac's prediction, the positron was observed by Carl Anderson in cosmic-ray events in a cloud chamber.

But the Dirac equation raised other problems. In 1938, Dirac attempted to address one-such problem that had been troubling physics for many years, the "self-energy problem". Particles like electrons and positrons are described by the Dirac equation as having no structure, so in essence they are point-like spin-$^1/_2$ objects with an electric charge, a spin, and a dipole magnetic field.

However, electric and magnetic fields represent energy stored in space, with the energy in a given volume of space proportional to the square of the field strength in that region. If we imagine approaching an electron and measuring its field as we approach it, we would find that the field grows as the inverse square of the diminishing distance. If we reduce our distance from the electron by a factor of $^1/_2$, the electric field should grow by a factor of 4 and the energy per unit volume contained in the field should grow by a factor of 16. If we take the limit of this process as our distance from the electron goes to zero (because it is a point with no outer surface), the field and energy density should become infinite. Therefore, using Einstein's relation between mass and energy ($m = E/c^2$), each electron should, by this argument, have an infinite rest mass. As we know, electrons do not have infinite mass, so something must be wrong with this argument.

This is called the self-energy problem, and it represented a puzzle for both classical physics and quantum mechanics. It was more serious in quantum mechanics, however. In classical physics one could treat the electron as a sphere of charge with a radius and an outer surface at which the electric field stopped growing and adjust that radius to give the known mass of the electron. In quantum mechanics one is not allowed to give objects arbitrary unobserved properties, so electrons must be treated as point-like objects with no structure.

In his 1938 paper, Dirac suggested that perhaps the classical problem of self-energy was being handled incorrectly [3]. He pointed out that Maxwell's wave equation for electromagnetism had not one but two independent solutions (see Sect. 1.1). One of these solutions is the commonly-used "retarded" solution describing radiation that requires a time-delay to travel a spatial distance. The other solution is commonly ignored. It is the "advanced" solution describing radiation that requires a negative time-delay to travel a spatial distance. It can be thought of as an incoming rather than an outgoing wave, or as a wave that is traveling *backwards in time*. He proposed an

alternate time-symmetric formalism for classical electrodynamics that used solutions of Maxwell's wave equation that are half of the retarded solution added to half of the advanced solution. In other words, instead of choosing the retarded solution by invoking causality, he assumed an even-handed treatment of time with no preferred time direction built into the formalism. He applied this to a description of how the electron would move in response to a force from a pulse of an external electric field. Dirac found that this permitted the elimination of the self-energy problem and led to a point-like electron with a finite mass.

Eventually, the solution to the self-energy problem was supposedly solved in quantum mechanics in another way that involved using vacuum polarization, a concept also originated by Dirac. In this approach, the bare charge of the electron is shielded at small distances by virtual particles of the opposite charge emerging from the vacuum, thereby reducing the mass-energy of the electric field to the observed value.[1] Nevertheless, Dirac's foray into time-symmetric electrodynamics was valid physics, it provided the basis for later work, and it should perhaps be reexamined in view of contemporary cosmological issues.

4.3 Wheeler–Feynman Absorber Theory

Before the beginning of World War II, Richard Feynman had been John Wheeler's graduate student at Princeton, and they had followed up Dirac's work with their own brand of time-symmetric electrodynamics. During WWII and the Manhattan Project, Feynman (Fig. 4.3) had led the calculation group at Los Alamos and Wheeler (Fig. 4.4) had been deeply involved in the production of plutonium at the first major nuclear reactor in Richland, Washington. At the end of WWII in 1945, they jointly published their work on time-symmetric electrodynamics, which became known as Wheeler–Feynman Absorber Theory [4, 5].

The classical electrodynamics described by Wheeler and Feynman (WF) was intended to deal with the problem of the self-energy of the electron in an innovative way. Assuming the time symmetric formalism of Dirac [3] combined with the *ad hoc* assumption that an electron does not interact with its own field, WF was able to formally eliminate the self-energy term from their electrodynamics. But along with self energy, these assumptions also removed the well observed energy loss and recoil processes (i.e., radiative damping) arising from the interaction of the radiating electron with its own radiation field.

However, WF accounted for these well-known damping effects by allowing the emitting electron to interact with the advanced waves sent by other electrons that would ultimately, at some time future, absorb the retarded radiation. Thus the energy

[1] We note that solving the self-energy problem by invoking vacuum polarization brings with it some unwanted baggage: the theory leads to an overestimate of the energy content of the vacuum, otherwise known as the cosmological constant, by a factor of 10^{120}.

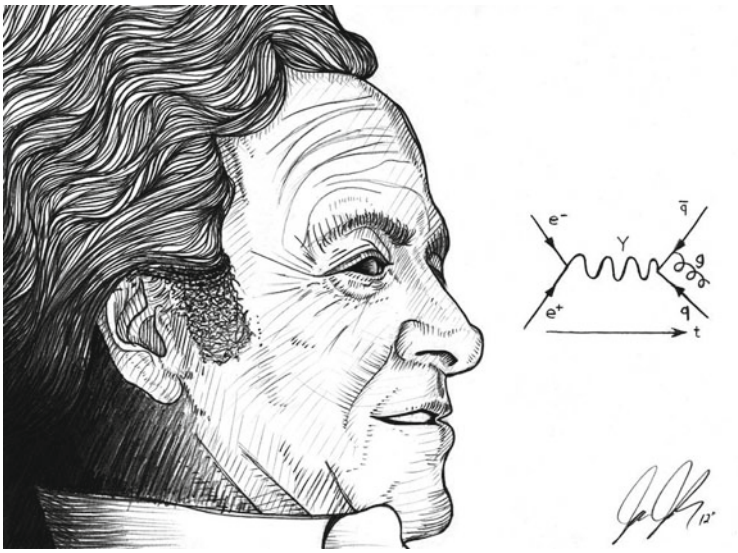

Fig. 4.3 In 1965, Richard P. Feynman (1918–1988) received the Nobel Prize in Physics for his work on quantum electrodynamics, in which he introduced Feynman diagrams like the one shown here

loss and recoil of the emitter were accounted for without having it interact with its own field. Moreover, the calculation succeeded in describing electrodynamic interactions in a completely time-symmetric way. Effectively, the retarded and advanced waves together did a "handshake" that arranged for the transfer of energy and momentum (Fig. 4.5). To account for the observed asymmetric dominance of retarded radiation, WF invoked the action of external boundary conditions arising from thermodynamics. They thus avoided resort to the usual *ad hoc* "causality" condition usually needed to eliminate the advanced radiation solutions.

Regrettably, the work of Wheeler and Feynman, while mathematically correct, proved to be an invalid way of dealing with self-energy. As Feynman [6] later pointed out, the self-interaction they eliminated is a necessary part of electrodynamics, needed, for example, to account for the Lamb shift. And it is relevant that the WF *ad hoc* assumption of non-interaction is not needed in their recoil calculations because, as later authors have pointed out [7, 8], the electron cannot undergo energy loss or recoil, which are intrinsically asymmetric-in-time processes, as a result of interacting with its own (or any other) locally time-symmetric field.

When the offending *ad hoc* assumption of non-interaction is removed from the WF formalism, what remains is a classical self-consistent and time-symmetric electrodynamics that cannot be used to deal with the problem of self energy. Further, this WF formalism is not particularly useful as an alternative method of calculating the electrodynamics of radiative processes because the mathematical description of radiation explicitly involves the interaction of the emitter with the entire future universe.

Fig. 4.4 John A. Wheeler (1911–2008) made significant theoretical contributions in nuclear physics and gravitational theory and originated the terms "black hole" and "wormhole"

Thus, a simple integration over local coordinates in the conventional formalism is replaced by an integral over all future space-time in the light cone of the emitter in the WF formalism.

However, this "difficulty" can be viewed an asset. The WF mathematics has been used to investigate the properties of cosmological models describing the future state of the universe by relating such models to radiative processes. In essence this approach provides a way of linking the cosmological arrow of time (the time direction in which the universe expands) to the electromagnetic arrow of time (the complete dominance of retarded over advanced radiation in all radiative processes). There is a considerable literature in this field, which the author has reviewed in a previous publication [9].

Although the original WF work dealt exclusively with classical electrodynamics, later authors [10–14] have developed equivalent time-symmetric quantum-electrodynamics (QED) versions of the same approach. The predictions of time-symmetric QED theories have been shown to be completely consistent with those predictions of conventional QED that can be compared with experimental observation. It has also been shown [14] that despite this similarity of prediction, the time-symmetric QED provides a qualitatively different description of electromagnetic processes.

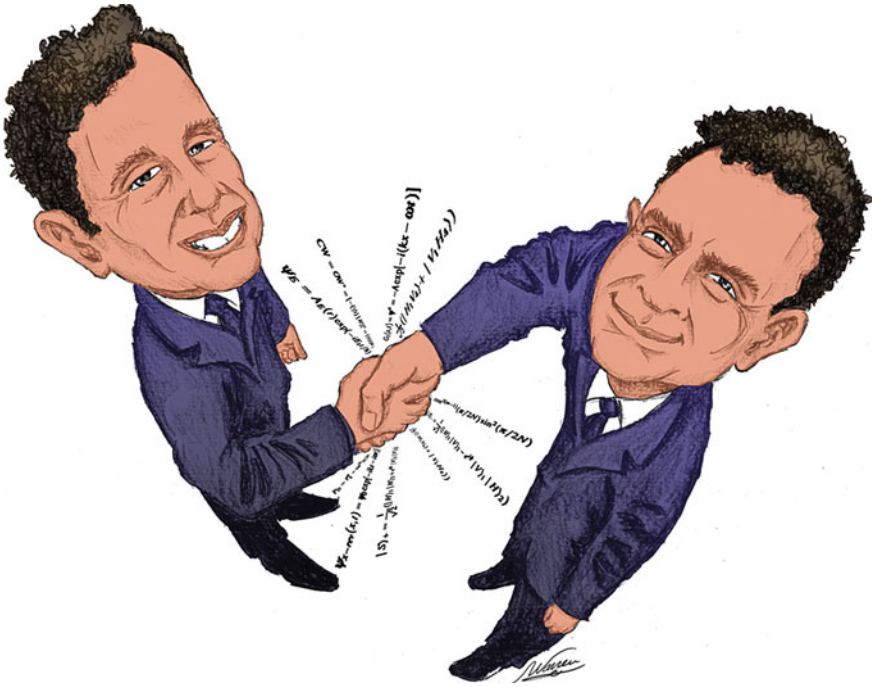

Fig. 4.5 The Wheeler–Feynman handshake, the basis of WF absorber theory and the Transactional Interpretation

It is essentially an action-at-a-distance theory with no extra degrees of freedom for the radiation fields and no second quantization. The field in effect becomes a mathematical convenience for describing action-at-a-distance processes.

There may also be another advantage to the WF approach to electrodynamics. Dirac's work [3] on time-symmetric electrodynamics, on which the WF theory is based, was introduced as a way of dealing with singularities in the radiation field in the conventional theory near a radiating electron. Emil Konopinski [15] in his Lorentz covariant treatment of the radiating electron has pointed out that this time-symmetric "Lorentz-Dirac" approach eliminates such singularities and therefore amounts to a self-renormalizing theory. This formulation may have applications in eliminating related singularities in QCD and in quantum field theory in curved space-time.

Richard Feynman's work on this problem led him to the formulation of his own brand of quantum mechanics, a framework using Lagrangians and action as starting points, instead of the more conventional approach using Hamiltonians and energy. This resulted in the application of Feynman path integrals to quantum mechanics and QED.

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