Chapter 7
A Fuzzy Design of Single and Double Acceptance Sampling Plans

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Abstract In this chapter, we briefly introduce the topic of acceptance sampling. We also examine acceptance sampling plans with intelligent techniques for solving complex quality problems. Among intelligent techniques, we focus on the application of the fuzzy set theory in the acceptance sampling. Moreover, we propose multi-objective mathematical models for fuzzy single and fuzzy double acceptance sampling plans with illustrative examples. The study illustrates how an acceptance sampling plan should be designed under fuzzy environment.

Keywords Acceptance sampling · Double sampling · Single sampling · Fuzzy sets

7.1 Introduction

In manufacturing industries, sampling inspection is a common practice for quality assurance and cost reduction. Acceptance sampling is a practical and economical alternative to costly 100 % inspection. Acceptance sampling offers an efficient way to assess the quality of an entire lot of product and to decide whether to accept or reject it. The basic decisions in sampling inspection are how many manufactured items to be sampled from each lot and how many identified defective items in the sample to accept or reject each lot (Wang and Chankong 1991). The application of

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acceptance sampling minimizes product destruction during inspection and testing as well as increases the inspection quantity and effectiveness.

Practically, acceptance sampling is a form of testing that involves taking random samples of lots or batches of finished products and measuring them against predetermined standards. Acceptance sampling pertains to incoming batches of raw materials or purchased parts and to outgoing batches of finished goods.

Acceptance sampling is useful when one or more of the following conditions is available: a large number of items must be processed in a small amount of time; the cost of passing defective items is low; destructive testing is required; or the inspectors may experience boredom or fatigue in inspecting large numbers of items.

Acceptance sampling plans are useful tools for quality control practices, which involve quality contracting on product orders between the vendor and the buyer. Those sampling plans provide the vendor and the buyer rules for lot sentencing while meeting their preset requirements on product quality. Nowadays, sampling plans are the primary tools for quality and performance management in industry. Sampling plans are used to decide either to accept or reject a received batch of items. In any acceptance sampling plan, there are two possible error, which are producer's risk and consumer's risk. Producer's risk is the rejection of a good lot. Consumer's risk is the acception of a bad lot.

Acceptance sampling plans provide the vendor and buyer the decision rules for product acceptance to meet the present product quality requirement. In practice, proper design of an acceptance sampling planning is based on the true quality level required by customers. However, it is sometimes not possible to determine this quality level with certain values. Especially in production, it is not easy to determine the parameters of acceptance sampling such as proportion of defective items, sample size, acceptable defective items.

Classical acceptance sampling plans have been studied by many researchers. In different acceptance sampling plans the proportion of defective items, is considered as a crisp value. The proportions of defective items are estimated or provided by experiment. According to Fountoulaki et al. (2008), approaches employing machine learning techniques in acceptance sampling are limited and mainly focused on the design of acceptance sampling plans. Sampath (2009) emphasized that in the manufacturing processes, quantities such as the proportion of defective items in a production lot may not be precisely known and usually the practitioners have to compromise with some imprecise or approximate values. Prior knowledge of such quantities is required to evaluate the quality of a produced lot.

The vagueness present from personal judgment, experiment or estimation can be treated formally with the help of fuzzy set theory. Among other intelligent techniques, fuzzy set theory is known as a powerful mathematical tool for modeling uncertainity in classical attribute quality characteristics (Jamkhaneh et al. 2009).

There are many other investigations and many other publications related to acceptance sampling plans. In this chapter, we briefly introduce the topic of acceptance sampling. Also, we examine acceptance sampling plans with intelligent techniques for solving important as well as fairly complex problems related to acceptance sampling. A lot or batch of items can be inspected in several ways including the use of single, double, multiple, sequential sampling. Among other intelligent techniques, we focus on the application of fuzzy set theory in the acceptance sampling. We propose mathematical models for fuzzy single and fuzzy double acceptance sampling plans with illustrative examples.

The rest of this chapter is organized as follows. In Sect. 7.2, acceptance sampling basic concepts, terminology and plans are given. In Sect. 7.3, intelligent techniques in acceptance sampling are briefly reviewed. Design of fuzzy acceptance sampling plans and their illustrative examples are provided in Sect. 7.4. In Sect. 7.5 proposed fuzzy multi-objective mathematical models are explained with illustratives examples. Finally conclusion, discussions as well as recommendations for further studies are provided in the last section.

7.2 Acceptance Sampling Basic Concepts and Terminology

Acceptance sampling inspection is part of statistical practice concerned with sampled items to produce some quality information about the inspected products, especially to check whether products have met predetermined quality specifications (Schilling 1982). The complexity of the sampling inspection process gives rise to challenges for the definition of data quality elements, determination of sample item, sample size and acceptance number, and a combination of quality levels required by the producer and the consumer. Here are the top 10 reasons why acceptance sampling is still necessary:

- Tests are destructive, necessitating sampling.
- Process not in control, necessitating sampling to evaluate product.
- 100 % sampling is inefficient, 0 % is risky.
- Special causes may occur after process inspection.
- Need for assurance while instituting process control.
- Rational subgroups for process control may not reflect outgoing quality.
- Deliberate submission of defective material.
- Process control may be impractical because of cost, or lack of sophistication of personnel.
- 100 % inspection does not promote process/product improvement.
- Customer mandates sampling plan.

The principle of acceptance sampling to control quality is the fact that it is not checked all units (N), but only selected part (n). Acceptance sampling plan is a specific plan that clearly states the rules for sampling and the associated criteria for acceptance or otherwise. Acceptance sampling plans can be applied for inspection of end items, components, raw materials, operations, materials in process, supplies in storage, maintenance operations, data or records and administrative procedures. There are two essential issues in acceptance sampling inspection theory. The first is the determination of the acceptance sampling plan, which is characterized by sample size and acceptance number. The main goal of designing an optimal sampling plan is to obtain a high accuracy of product inspection and to reduce the inspection cost (Von Mises 1957). The second is to determine the method to select samples from the lot, which refers to the sampling method. Commonly used sampling methods include simple random sampling, system sampling, stratified sampling, and cluster sampling (Cochran 1977; Degroot 1986; Wang et al. 2010).

Acceptance-sampling plans classify to different ways. One major classification is by attributes and variables. Acceptance-sampling plans by attributes are single sampling plan, double sampling plan, multiple-sampling plan, and sequential sampling plan (Schilling 1982).

Single sampling is undoubtedly the most used of any sampling procedure. The simplest form of such a plan is single sampling by attributes which relates to dichotomous situations, i.e., those in which inspection results can be classified into only two classes of outcomes. This includes go, no-go gauging procedures as well as other classifications. Applicable to all sampling situations, the attributes single sampling plan has become the benchmark against which other sampling plans are judged. It is employed in inspection by counting the number of defects found in sample (Poisson distribution) or evaluating the proportion defective from processes or large lots (binomial distribution) or from individual lots (hypergeometric distribution). It involves taking a random sample size n from a lot size N. The number of defectives (or defects) d found is compared to an acceptance number c. If the number found is less than or equal to c, the lot is accepted. If the number found is greater than c, the lot is rejected.

Often a lot of items are so good or so bad that we can reach a conclusion about its quality by taking a smaller sample than would have been used in a single sampling plan. In double sampling if the results of the first sample are not definitive in leading to acceptance or rejection, a second sample is taken which then leads to a decision on the disposition of the lot. In brief, if the number of defects in this first sample (d_1) is less than or equal to some lower limit (c_1), the lot can be accepted. If the number of defects first and second sample (d_2) exceeds an upper limit (c_2), the whole lot can be rejected. But if the number of defects in the n1 sample is between c_1 and c_2, a second sample is drawn. The cumulative results determine whether to accept or reject the lot. The concept is called double sampling.

Multiple sampling involves the inspection of specific lots on the basis of k successive samples as needed to make a decision, where k varies from 1 to K (i.e. a whole number). It is an extension of double sampling, with smaller samples used sequentially until a clear decision can be made. In multiple sampling by attributes, more than two samples can be taken in order to reach a decision to accept or reject the lot. The main advantage of multiple sampling plans is a reduction in sample size for the same protection.

Single, double, and multiple plans assess one or more successive samples to determine lot acceptability. Sequential sampling involves making a decision as to disposition of the lot or resample successively as each item of the sample is taken and it may be regarded as multiple-sampling plan with sample size one and no upper limit on the number of samples to be taken. It is often applied where sample size critical so that a minimum sample must be taken. Under sequential sampling, samples are taken, one at time, until a decision is made on the lot or process sampled. After each item is taken a decision is made to (7.1) accept, (7.2) reject, or (7.3) continue sampling. Samples are taken until an acceptance or rejection decision is made. Thus, the procedure is open ended, the sample size not being determined until the lot is accepted or rejected. Selection of the best sampling approach (single, double, multiple or sequential) depends on the types of products being inspected and their expected quality level. A very low-quality batch of goods, for example, can be identified quickly and more cheaply with sequential sampling. This means that the inspection, which may be costly and/or destructive, can end sooner. On the other hand, in many cases a single sampling plan is easier and simpler for workers to conduct even though the number sampled may be greater than under other plans.

7.3 Operating Characteristic Curves

The operating characteristic (OC) curve plots the probability of acceptance against possible values of proportion defective. OC curve describes how well an acceptance plan discriminates between good and bad lots. A curve pertains to a specific plan, that is, a combination of n and c. It is intended to show the probability that the plan will accept lots of various quality levels. The curves for different sampling plans are shown in Fig. 7.1. The OC curve sketches the performance of a plan for various possible proportions defective. It is plotted using appropriate probability functions for the sampling situation involved. The curve shows the ability of a sampling plan to discriminate between high quality and low quality lots. With acceptance sampling, two parties are usually involved: the producer of the product and the

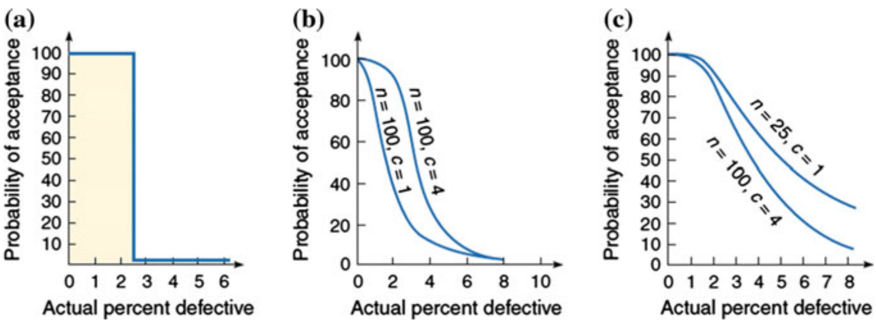


Fig. 7.1 **a** Perfect discrimination for inspection plan. **b** OC curves for two different acceptable levels of defects (c = 1, c = 4) for the same sample size (n = 100). **c** OC curves for two different sample sizes (n = 25, n = 100) but same acceptance percentages (4 %). Larger sample size shows better

consumer of the product. When specifying a sampling plan, each party wants to avoid costly mistakes in accepting or rejecting a lot.

The producer wants to avoid the mistake of having a good lot rejected (producer's risk) because he or she usually must replace the rejected lot. Conversely, the customer or consumer wants to avoid the mistake of accepting a bad lot because defects found in a lot that has already been accepted are usually the responsibility of the customer (consumer's risk). The producer's risk α is the probability of not accepting a lot of acceptable quality level (AQL) quality and the consumer's risk β is the probability of accepting a lot of limiting quality level (LQL) quality. The term acceptable quality level (AQL) is commonly used as the 95 % point of probability of acceptance, although most definitions do not tie the term to a specific point on the OC curve and simply associate it with a "high" probability of acceptance. The term is used here as it was used by the Columbia Statistical Research Group in preparing the (Von Mises 1957) input to the JAN-STD-105 standard. LTPD refers to the 10 % probability point of the OC curve and is generally associated with percent defective. The advent of plans controlling other parameters of the distribution led to the term limiting quality level (LQL), usually preceded by the percentage point controlled. Thus, "10 % limiting quality" is the LTPD (Schilling 1982).

In most sampling plans, when a lot is rejected, the entire lot is inspected and all of the defective items are replaced. Use of this replacement technique improves the average outgoing quality in terms of percent defective.

The average outgoing quality (AOQ) can be explained as the expected quality of outgoing product following the use of an acceptance sampling plan for a given value of the incoming quality. For the lots accepted by the sampling plan, no screening will be done and the outgoing quality will be the same as that of the incoming quality p. For those lots screened, the outgoing quality will be zero, meaning that they contain no nonconforming items. Since the probability of accepting a lot is P_a, the outgoing lots will contain a proportion of pP_a defectives. If the nonconforming units found in the sample of size n are replaced by good ones, the average outgoing quality (AOQ) will be (Kahraman and Kaya 2010):

$$AOQ = \frac{N-n}{N} pP_a \tag{7.1}$$

For large N,

$$AOQ \cong pP_a \tag{7.2}$$

The maximum value of AOQ over all possible values of fraction defective, which might be submitted, is called the AOQ limit (AOQL). It represents the maximum long-term average fraction defective that the consumer can see under operation of the rectification plan. It is sometimes necessary to determine the average amount of inspection per lot in the application of such rectification schemes, including 100 % inspection of rejected lots. This average, called the average total inspection (ATI), is made up of the sample size n on every lot plus the remaining (N-n) units on the rejected lots, so that the ATI for single sampling is calculated as following Eqs. (7.3 and 7.4).

$$ATI = n + (1 - P_a)(N - n) \tag{7.3}$$

$$ATI = P_a n + (1 - P_a)N \tag{7.4}$$

The ATI for the double sampling plan can be calculated from the following Eqs. (7.5–7.7). In Eq. (7.5), the average sample number (ASN) is the mean number of items inspected per lot. The concept of ASN is very useful in determining the average number of samples that will be inspected in using more advanced sampling plans. For a single sampling plan, one takes only a single sample of size n and hence the ASN is simply the sample size n. In single sampling, the size of the sample inspected from the lot is always constant, whereas in double sampling, in double sampling plans, for example, the second sample is taken only if results from the first sample are not sufficiently definitive to lead to acceptance or rejection outright. In such a situation the inspection may be concluded after either one or two samples are taken and so the concept of ASN is necessary to evaluate the average magnitude of inspection in the long run.

$$ATI = ASN + (N - n_1)P(d_1 > c_2) + (N - n_1 - n_2)P(d_1 + d_2 > c_2) \tag{7.5}$$

where

$$P(d_1 > c_2) = 1 - P(d_1 \leq c_2) \tag{7.6}$$

$$P(d_1 + d_2 > c_2) = 1 - P_a - P(d_1 > c_2) \tag{7.7}$$

A general formula for the average sample number in double sampling is

$$ASN = n_1 P_1 + (n_1 + n_2)(1 - P_1) = n_1 + n_2(1 - P_1) \tag{7.8}$$

where P_1 is the probability of making a lot dispositioning decision on the first sample. This is calculated as following equation:

$$P_1 = P\{\text{lot is accepted on the first sample}\} + P\{\text{lot is rejected on the first sample}\} \tag{7.9}$$

Acceptance sampling is useful for screening incoming lots. When the defective parts are replaced with good parts, acceptance sampling helps to increase the quality of the lots by reducing the outgoing percent defective. Sampling plans and OC curves facilitate acceptance sampling and provide the manager with tools to evaluate the quality of a production run or shipment.

7.4 Literature Review on Acceptance Sampling

In recent years, there are some studies concentrated on acceptance sampling in the literature. Figure 7.2 shows the publication frequencies of acceptance sampling according to years between 2004 (including 2004 and earlier) and 2013.

Some of these publications are journal articles, books/e-books, and so on. Figure 7.3 shows the distribution of these publications according to publication categories. According to this figure, most of the studies on acceptance sampling are published in journals with a rate of 69 %. For example, Baklizi (2003) developed acceptance sampling plans assuming that the life test is truncated at a pre-assigned time. The minimum sample size necessary to ensure the specified average life was obtained and the operating characteristic values of the sampling plans and producer's risk were presented. Kuo (2006) developed an optimal adaptive control policy for joint machine maintenance and product quality control. It included the interactions between the machine maintenance and the product sampling in the search for the best machine maintenance and quality control strategy to find the optimal value function and identify the optimal policy more efficiently in the value iteration algorithm of the dynamic programming.

Borget et al. (2006) applied a single sampling plan by attributes with an acceptance quality level of 2.2 % was evaluated. A prognostic study using a logistic regression model was performed for some drugs to identify risk factors associated with the non-conformity rate of preparations to determine if it was necessary to assay all therapeutic batches produced, or to calculate an individual control rate for each cytotoxic drug, according to various parameters (like number of batches or drug stability). The sampling plan allowed a reduction of almost 8000 analyses with respect to the number of batches analysed for 6 drugs. Pearn and Wub (2007) proposed an effective sampling plan based on process capability index, C_{pk}, to deal with product acceptance determination for low fraction non-conforming products

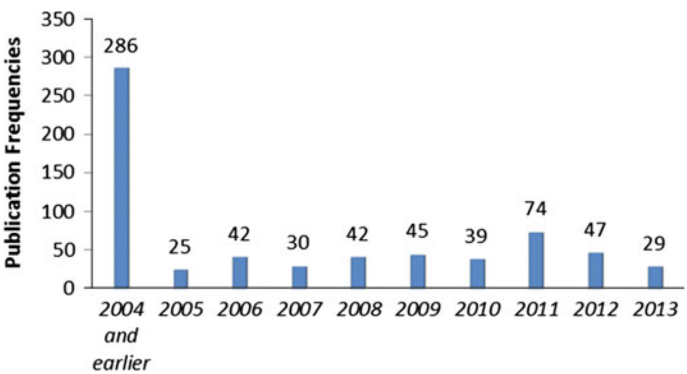


Fig. 7.2 Publication frequencies of acceptance sampling

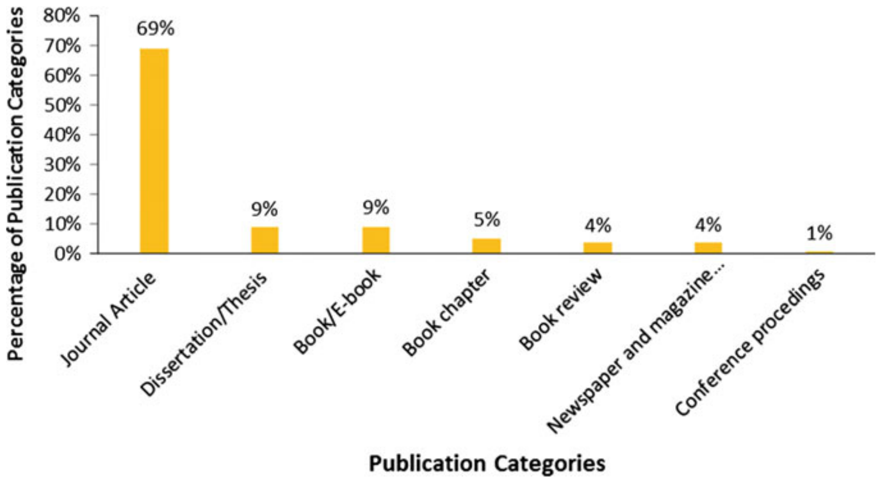


Fig. 7.3 Percentages of publication categories of acceptance sampling

based on the exact sampling distribution rather than approximation. Practitioners could use this proposed method to determine the number of required inspection units and the critical acceptance value, and make reliable decisions in product acceptance. Aslam (2008) evolved a reliability acceptance plan assuming that the lifetime of a product follows the generalized Rayleigh distribution with known value of the shape parameter. Tsai et al. (2009) explained ordinary and approximate acceptance sampling procedures under progressive censoring with intermittent inspections for exponential lifetimes. Jozani and Mirkamali (2010) demonstrated the use of maxima nomination sampling (MNS) technique in design and evaluation of single AQL, LTPD, and EQL acceptance sampling plans for attributes. They exploited the effect of sample size and acceptance number on the performance of their proposed MNS plans using operating characteristic (OC) curve. Aslam et al. (2010) developed the double sampling and group sampling plans and determined the design parameters satisfying both the producer's and consumer's risks simultaneously for the specified reliability levels in terms of the mean ratio to the specified life. Nezhad et al. (2011) introduced a novel acceptance-sampling plan is proposed to decide whether to accept or reject a receiving batch of items. In this plan, the items in the receiving batch are inspected until a nonconforming item is found. When the sum of two consecutive values of the number of conforming items between two successive nonconforming items falls underneath of a lower control threshold, the batch is rejected. If this number falls above an upper control threshold, the batch is accepted, and if it falls within the upper and the lower thresholds then the process of inspecting items continues. Fernández and Pérez-González (2012) presented for determining optimal failure-censored reliability sampling plans for log-location–scale lifetime models. The optimization procedure to decide the acceptability of a product is usually sufficiently accurate for the most widely used parametric lifetime models, such as the Weibull and log-normal distributions, and fairly robust to small deviations in the prior knowledge. Hsieh and Lu (2013) developed a risk-embedded model via conditional value-at-risk that allows a decision maker to choose an acceptance sampling plan with minimal expected excess cost in accordance with his or her attitude towards risk to gain insights into the role of a decision maker's risk aversion in the determination of Bayesian acceptance sampling plans.

7.5 Intelligent Techniques in Acceptance Sampling

In sampling inspection, the fundamental decisions are how many manufactured items to be sampled from each lot and how many identified defective items in the sample to accept or reject each lot. The problem of determining an optimal sampling plan is NP-complete (2008). The reason is the combinatorial nature of alternative solutions on the sample sizes and acceptance criteria possessing the combinatorial nature. From this standpoint, in recent years, there are a number of researches that merge acceptance sampling with intelligent techniques. In this section, we briefly examine the researches regarding acceptance sampling plans with intelligent techniques for solving important as well as fairly complex problems related to acceptance sampling.

Wang and Chankong (1991) proposed a neurally-inspired approach to generating acceptance sampling inspection plans. They formulated a Bayesian cost model of multi-stage-multi-attribute sampling inspections for quality assurance in serial production systems. The proposed model can accommodate various dispositions of rejected lot such as scraping and screening. Besides, the model can reflect the relationships between stages and among attributes. To determine the sampling plans based on the formulated model, they developed a neurally-inspired stochastic algorithm, which simulates the state transition of a primal-dual stochastic neural network to generate the sampling plans. The simulated primal network is responsible for generation of new states whereas the dual network is for recording the generated solutions. Starting with an arbitrary feasible solution, this algorithm is able to converge to a near optimal or an optimal sampling plan with a sequence of monotonically improved solutions.

Tabled sampling schemes such as MIL-STD-105D offer limited flexibility to quality control engineers in designing sampling plans to meet specific needs. Vasudevan et al. (2012) attempted to find a closed form solution for the design of a single sampling plan for attributes to determine the accepted quality level (AQL) indexed single sampling plan using an artificial neural network (ANN). They used the data from tabled sampling schemes and determined the sample size and the acceptance number by training ANNs, namely with feed forward neural networks with sigmoid neural function by a back propagation algorithm for normal, tightened, and reduced inspections. From these trained ANNs, they obtained the relevant weight and bias values and the closed form solutions to determine the sampling plans using these values. They provided the examples for using these closed form solutions to determine sampling plans for normal, tightened, and reduced inspections. The proposed method does not involve table look-ups or complex calculations. Sampling plan can be determined by using this method, for any required acceptable quality level and lot size. They provided suggestions to duplicate this idea for applying to other standard sampling table schemes process.

Cheng and Chen (2007) suggested a Genetic Algorithm (GA) mechanism to reach a closed form solution for the design of a double sampling plan. In order to design the double sampling plan, the operating characteristic curve has to satisfy some specific criteria. As the parameters of the sampling plan have to be integers, the solution has to be optimal in each case. The GA mechanism is responsible for providing the optimal solution in contrast to the trial-and-error method that has been used so far. This approach seeks for the minimum sample number, even when the initial criteria are not satisfied. Its disadvantage is the relatively large number of the proposed solutions, from which the quality engineer has to decide the optimal one by changing the criteria that were predetermined at the beginning of the process.

Designing double sampling plan requires identification of sample sizes and acceptance numbers. Sampath and Deepa (2012) designed a genetic algorithm for the selection of optimal acceptance numbers and sample sizes for the specified producer's risk and consumer's risk. Implementation of the algorithm has been illustrated numerically for different choices of quantities involved in a double sampling plan.

Fountoulaki et al. (2008) proposed methodology for Acceptance Sampling by Variables, dealing with the assurance of products quality, using machine learning techniques to address the complexity and remedy the drawbacks of existing approaches. Their methodology exploited ANNs to aid decision making about the acceptance or rejection of an inspected sample. For any type of inspection, ANNs are trained by data from corresponding tables of a standard's sampling plan schemes. Once trained, ANNs can give closed-form solutions for any acceptance quality level and sample size, thus leading to an automation of the reading of the sampling plan tables, without any need of compromise with the values of the specific standard chosen each time. Their methodology provides enough flexibility to quality control engineers during the inspection of their samples, allowing the consideration of specific needs, while it also reduces the time and the cost required for these inspections.

In acceptance sampling plans, the decisions on either accepting or rejecting a specific batch is still a challenging problem. In order to provide a desired level of protection for customers as well as manufacturers, Fallahnezhad and Niaki (2012) proposed a new acceptance sampling design to accept or reject a batch based on Bayesian modeling to update the distribution function of the percentage of non-conforming items. They utilized the backwards induction methodology of the decision tree approach to determine the required sample size. They carried out a sensitivity analysis on the parameters of the proposed methodology showing the optimal solution is affected by initial values of the parameters. Furthermore, they determined an optimal (n, c) design when there is a limited time and budget available and they specified the maximum sample size in advance.

In many practical cases it is difficult to classify inspected items as conforming or nonconforming. This problem rather frequently can be faced when quality data come from users who express their assessments in an informal way, using such expressions like “almost good”, “quite good”, “not so bad”, and etc.

Ohta and Ichihashi (1988), Kanagawa and Ohta (1990), Tamaki et al. (1991), and Grzegorzewski (1998), Grzegorzewski et al. (2001) discussed single sampling by attributes with relaxed requirements.

Ohta and Ichihashi (1988) presented a procedure for designing a single sampling plan using fuzzy membership functions for both the producer’s risk and consumer’s risk, with the aim of finding a reasonable solution for the trade-off between the sampling size and the producer’s and consumer’s risks. This design methodology is deficient in the sense that it does not explicitly takes into account of minimizing the sample size n. The desire for smaller sample size is imposed by choosing triangular membership functions for the risks. However, this choice does not make sense for the part of the membership functions where the risks are higher than their nominal values.

Kanagawa and Ohta (1990) selected trapezoidal membership functions for risks, and taking into account a membership function for the grade of satisfaction for the sample size. They stated that the membership function must be a monotonically decreasing function of the sample size n, however, no method for constructing this function is proposed.

Sampling plan by attributes for vague data were considered by Hryniewicz. Hryniewicz (1992) attempted to cope with the statistical analysis of such quality data.

If the quality characteristic monitored is a variable, acceptable quality level and rejectable quality level (AQL and RQL) are identified for evaluating the acceptance or rejection of an inspection lot. Otherwise, when the quality characteristic is an attribute, the number of defectives is compared to a specific limit of number of allowed defectives for the decisions of accept/reject. In the former case, acceptable quality level and rejectable quality level may not be specified as a crisp value because rigid values of AQL and RQL may not necessarily give a sampl plan. Besides, these values are commonly not very precise but rather descriptive. Thus, the crisp values of AQL and RQL may be relaxed as fuzzy values.Much more practical procedure, namely the fuzzy version of an acceptance sampling plan by variables, has been proposed by Grzegorzewski (2002). Grzegorzewski et al. (2001), Grzegorzewski (2002) considered sampling plan by variables with fuzzy requirements. General results from the theory of fuzzy statistical tests have been used for the construction of fuzzy sampling plans when the quality characteristic of interest is described by a fuzzy normal distribution.

Hryniewicz (2003) has shown why in the case of imprecise input information optimal inspection intervals are usually determined using additional preference measures than strict optimization techniques.

Krätschmer (2005) proposed a mathematically sound basis for the sampling inspection by attributes in fuzzy environment. According to Hryniewicz (2008), no new practical SQC procedures have been proposed using that general model.

Jamkhaneh et al. (2009) proposed a method for designing acceptance single sampling plans with fuzzy quality characteristic with using fuzzy Poisson distribution. They presented the acceptance single sampling plan when the fraction of nonconforming items is a fuzzy number and modeled by fuzzy Poisson distribution. Their plans are well defined since if the fraction of defective items is crisp they reduce to classical plans. They showed that the operating characteristic curve of the plan is like a band having a high and low bounds whose width depends on the ambiguity proportion parameter in the lot when that sample size and acceptance numbers is fixed. They showed that the plan operating characteristic bands are convex with zero acceptance number.Then, they compared the operating characteristic bands of using of binomial with the operating characteristic bands of using of Poisson distribution.

Sampath (2009) considered the properties of single sampling plan under situations involving both impreciseness and randomness using the Theory of Chance. For fuzzy random environment, the process of drawing an operating characteristic curve and the issue of identifying optimal sampling plans are also addressed in the study called hybrid single sampling plan.

In a single stage sampling plan, the decision to accept or reject a lot is made based on inspecting a random sample of certain size from the lot. Conventional designs may result in needlessly large sample size. The sample size n can be reduced by relaxing the conditions on the producer's and consumer's risks. Ajorlou and Ajorlou (2009) proposed a method for constructing the membership function of the grade of satisfaction for the sample size n based on the shape of the sampling cost function. They found a reasonable solution to the trade-off between relaxing the conditions on the actual risks and the sample size n. For three general sampling cost functions, they derived the membership function of the grade of satisfaction for the sample size.

Kahraman and Kaya (2010) handled two main distributions of acceptance sampling plans which are binomial and Poisson distributions with fuzzy parameters and they derived their acceptance probability functions. Then fuzzy acceptance sampling plans were developed based on these distributions.

Jamkhaneh et al. (2011) discussed the single acceptance sampling plan, when the proportion of nonconforming products is a fuzzy number and also they showed that the operating characteristic (OC) curve of the plan is a band having high and low bounds and that for fixed sample size and acceptance number, the width of the band depends on the ambiguity proportion parameter in the lot. Consequently they explained when the acceptance number equals zero, this band is convex and the convexity increases with n.

Turanoglu et al. (2012) analyzed when main parameters of acceptance sampling plan were assumed triangular and trapezoidal fuzzy numbers and also operating characteristic curve (OC), AOQ, average sample number (ASN), and ATI were obtained for single and double sampling plans under fuzzy environment.

In the latter case, when the fraction of the defective items is needed to be used due to the nature of the quakity characteristic, the non-conforming items may not be specified exactly. Thu, the fraction of the non-conforming items, the fraction of the non-conforming items is generally not known exactly in practical cases. The general approach is to replace the value with a crisp estimate value. Due to the uncertainty of the estimation or the experimentation procedure for the estimation, there exists a vagueness of the value of the fraction of defective items. In order to model the vagueness, fuzzy set theory has been used in the literature. The number of defective items in a sample has a binomial distribution. When we use a fuzzy approach in order to model the uncertainty, the binomial distribution is defined with a fuzzy parameter $\widetilde{p}$. If the number of defective items in a sample is small, the common approach is to use fuzzy Poisson distribution to approximate the fuzzy binomial (Turanoglu 2012).

Acceptance sampling applications are classified into two based on the nature of the quality characteristics inspected. If the items can only be identified as disjoint categories such as good and bad, acceptance sampling by attributes are applied. In cases where quality characteristics can be continuously measured such as weight, strength, we apply acceptance sampling by variables. The fuzzy approaches for both of these types of acceptance sampling have been studied in the literature. Sampling by attributes with relaxed requirements were discussed by Ohta and Ichihashi (1988), Kanagawa and Ohta (1990), Tamaki et al. (1991), and Grzegorzewski (1998), Grzegorzewski et al. (2001), Hryniewicz (1992). Grzegorzewski et al. (2001), Grzegorzewski (2002), Jamkhaneh and Gildeh (2010) considered sampling plan by variables with fuzzy requirements.

Another classification of acceptance sampling applications is based on the number of samples taken until a decision is made related to the lot. A sequential sampling consists of a sequence of samples from the lot and the number of samples to be taken is identified based on the results of the sampling process. A random sample if drawn from the lot and the actual quality level of the sample is compared with the limit levels. Based on the results of this comparison, three decisions can be made: (i) the lot can be accepted, (ii) the lot can be rejected; (iii) a new sample is taken and inspected to make a decision. When only one sample is inspected at each sampling stage, the procedure is named as item sequential sampling. When only two decisions, accept and reject is defined after the inspection of the first sample, the sampling is named as single sampling plan. Single, double and sequential sampling plans with fuzzy parameters have also been studied in the literature. Single sampling plans with fuzzy parameters are investigated by Ohta and Ichihashi (1988), Kanagawa and Ohta (1990), Tamaki, Kanagawa and Ohta (1991), and Grzegorzewski (1998), Grzegorzewski et al. (2001), Jamkhaneh et al. (2010), Jamkhaneh et al. (2011). Sequential sampling plans with fuzzy parameters are discussed by Jamkhaneh and Gildeh (2010).

7.6 Design of Acceptance Sampling Plans Under Fuzzy Environment

Acceptance sampling procedures can be applied to lots of items when testing reveals non-conformance or non-conformities regarding product functional attributes. It can also be applied to variables characterizing lots, thus revealing how far product quality levels are from specifications. These applications have the main purpose of sort outing a lot as accepted or rejected, given the quality levels required for it. Generally, there are two major assumptions made when creating sampling plans. The first is that the sampling parameters are crisp, such as the fraction of nonconformities which is the rate of the observed nonconformities in the inspected samples, and sample rate which is a compromise between the accuracy and cost of the inspection. The second is that these parameters are vague values, particularly in the case where they can only be expressed by linguistic variables. According to Literature Review (Kahraman and Kaya 2010; Ohta and Ichihashi 1988; Kanagawa and Ohta 1990) some of the acceptance sampling studies have concentrated on fuzzy parameters. Some of these are given with the illustrative examples in the following subsections.

7.6.1 Design of Single Sampling Plans Under Fuzzy Environment

The single attribute sampling plan provides a decision rule to accept or reject a lot based on the inspection results obtained from a single random sample. The procedure corresponds to taking a random sample from the lot with size n_1 and inspects each item. If the number of non-conformities or nonconforming items does not exceed the specified acceptance number c_1, the entire lot is accepted. Many different acceptance plans meet the requirements of both the producer and the consumer. However, the producer is also interested in keeping the average number of items inspected to a minimum, aiming to reduce the costs of sampling and inspection, and economic aspects of the sampling plans must also be considered in practical implementations (Duarte and Saraiva 2008).

Kahraman and Kaya (2010) analyzed single and double sampling plans by taking into account two fuzzy discrete distributions such as binomial and Poisson distribution. They developed a single sampling plan assuming that a sample whose size is a fuzzy number $\tilde{n}$ is taken and 100 % inspected. The fraction nonconforming of the sample is also a fuzzy number $\tilde{p}$. The acceptance number is determined as a fuzzy number $\tilde{c}$. The acceptance probability for this single sampling plan can be calculated as follows:

$$\tilde{P}_a = P(d \le \tilde{c}|\tilde{n}, \tilde{c}, \tilde{p}) = \sum_{d=0}^{\tilde{c}} \frac{\tilde{\lambda}^d e^{-\tilde{\lambda}}}{d!} \tag{7.10}$$

where $\tilde{\lambda} = \tilde{n}\tilde{p}$.

$$P_a(\alpha) = \left[P_{al,d,\tilde{\lambda}}(\alpha), P_{ar,d,\tilde{\lambda}}(\alpha)\right] \tag{7.11}$$

$$\begin{aligned} P_{al,d,\tilde{\lambda}}(\alpha) &= \min\left\{ \sum_{d=0}^{c} \frac{\lambda^d e^{-\lambda}}{d!} \middle| \lambda \in \lambda(\alpha), n \in n(\alpha), c \in c(\alpha) \right\} \\ P_{ar,d,\tilde{\lambda}}(\alpha) &= \max\left\{ \sum_{d=0}^{c} \frac{\lambda^d e^{-\lambda}}{d!} \middle| \lambda \in \lambda(\alpha), n \in n(\alpha), c \in c(\alpha) \right\} \end{aligned} \tag{7.12}$$

If the binomial distribution is used, acceptance probability can be calculated as follows:

$$\tilde{P}_a = \sum_{d=0}^{\tilde{c}} \binom{\tilde{n}}{d} \tilde{p}^d \tilde{q}^{\tilde{n}-d} \tag{7.13}$$

$$\begin{aligned} \tilde{P}_a &= \sum_{d=0}^{\tilde{c}} \binom{\tilde{n}}{d} \tilde{p}^d \tilde{q}^{\tilde{n}-d} \\ &= \left\{ \sum_{d=0}^{\tilde{c}} \binom{\tilde{n}}{d} \tilde{p}^d \tilde{q}^{\tilde{n}-d} \middle| p \in p(\alpha), q \in q(\alpha), n \in n(\alpha), c \in c(\alpha) \right\} \end{aligned} \tag{7.14}$$

$$P_a(\alpha) = [P_{al}(\alpha), P_{ar}(\alpha)] \tag{7.15}$$

$$\begin{aligned} P_{al}(\alpha) &= \min\left\{ \sum_{d=0}^{c} \binom{n}{d} p^d q^{n-d} \middle| p \in p(\alpha), q \in q(\alpha), n \in n(\alpha), c \in c(\alpha) \right\}, a \\ P_{ar}(\alpha) &= \max\left\{ \sum_{d=0}^{c} \binom{n}{d} p^d q^{n-d} \middle| p \in p(\alpha), q \in q(\alpha), n \in n(\alpha), c \in c(\alpha) \right\} \end{aligned} \tag{7.16}$$

AOQ values for fuzzy single sampling can be calculated as follows:

$$A\tilde{O}Q \cong \tilde{P}_a \tilde{p} \tag{7.17}$$

$$AOQ(\alpha) = [AOQ_l(\alpha), AOQ_r(\alpha)] \tag{7.18}$$

$$\begin{aligned} AOQ_l(\alpha) &= \min\{P_a p | p \in p(\alpha), P_a \in P_a(\alpha)\}, \\ AOQ_r(\alpha) &= \max\{P_a p | p \in p(\alpha), P_a \in P_a(\alpha)\} \end{aligned} \tag{7.19}$$

ATI curve can also be calculated as follows:

$$A\tilde{T}I = \tilde{n} + \left(1 - \tilde{P}_a\right)\left(\tilde{N} - \tilde{n}\right) \tag{7.20}$$

$$ATI(\alpha) = [ATI_l(\alpha), ATI_r(\alpha)] \tag{7.21}$$

$$\begin{aligned} ATI_l(\alpha) &= \min\{n + (1 - P_a)(N - n) | p \in p(\alpha), P_a \in P_a(\alpha), p \in N(\alpha), N \in N(\alpha)\}, \\ ATI_r(\alpha) &= \max\{n + (1 - P_a)(N - n) | p \in p(\alpha), P_a \in P_a(\alpha), p \in N(\alpha), N \in N(\alpha)\} \end{aligned} \tag{7.22}$$

Numerical Example-1

Suppose that a product is shipped in lots of size "Approximately 5000". The receiving inspection procedure used is a single sampling plan with a sample size of "Approximately 50" and an acceptance number of "Approximately 2". If fraction of nonconforming for the incoming lots is "Approximately 0.05", calculate the acceptance probability of the lot. Based on Eq. (7.16), the acceptance probability of the sampling plan is calculated as $\tilde{P}_a = P\left(d \leq \tilde{2}\right) = \text{TFN}(0.190,\ 0.544, 0.864)$ and its membership function is shown in Fig. 7.4.

AOQ is calculated as $A\tilde{O}Q = \text{TFN}(0.008,\ 0.027, 0.052)$ by using Eq. (7.19). ATI is also calculated as $ATI = \text{TFN}(707.163,\ 2308.125, 4140.47)$ by using Eq. (7.22) and its membership function is illustrated in Fig. 7.5.

Kanawaga and Ohta (1990) presented a design procedure for the single sampling attribute plan based on the fuzzy sets theory. They improved the fuzzy design

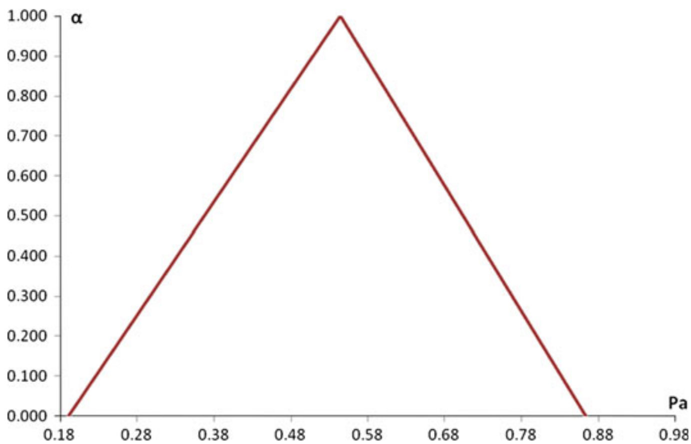


Fig. 7.4 Membership function of acceptance probability

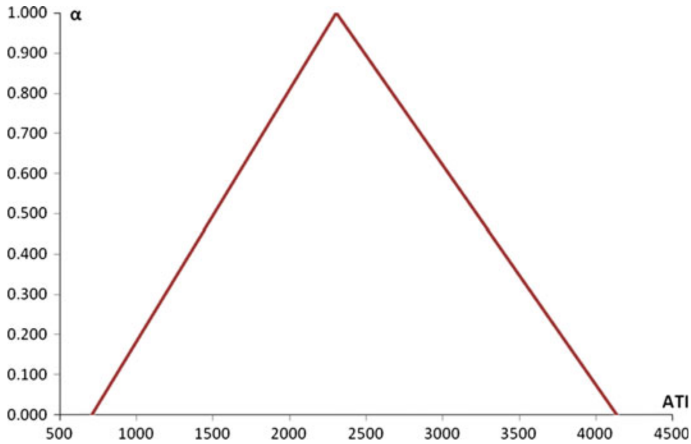


Fig. 7.5 Membership function of ATI for single sampling

procedure proposed by Ohta and Ichihashi (1988) with getting rid of the imbalance between the producer's and consumer's risks and the case in which a large sample size is (needlessly) required, both of which often arise in traditional crisp formulation, by means of the orthodox formulation as fuzzy mathematical programming with several objective function. They proposed the following formulation for the fuzzy design of the single sampling attribute plan.

$$P(p_1) \gtrsim 1 - \alpha, \quad P(p_2) \lesssim \beta, \tag{7.23}$$

$$n \rightarrow 0 \tag{7.24}$$

where the symbols $\gtrsim$ and $\lesssim$ stand for fuzzy inequlities. The membership functions $\mu_A(\alpha^*)$ and $\mu_B(\beta^*)$ in this case are shown in Fig. 7.6. The membership function $\mu_n(n)$ which represents the grade of satisfaction for the sample size must monotonically decrease as n increases as shown in Fig. 7.7. The fuzzy formulation can be written as the following fuzzy mathematical programming problem:

Problem 1 Find (n, c) so that

$$\min\{\mu_A(\alpha^*), \mu_B(\beta^*), \mu_n(n)\}$$

is maximized.

These membership functions in Fig. 7.6 are as follows:

$$\mu_A(\alpha^*) = \begin{cases} 1 & (\alpha^* \leq \alpha) \\ \frac{\alpha_u - 1 + P(p_1)}{\alpha_u - \alpha} & (\alpha \leq \alpha^* \leq \alpha_u) \\ 0 & (\alpha_u \leq \alpha^*) \end{cases} \tag{7.25}$$

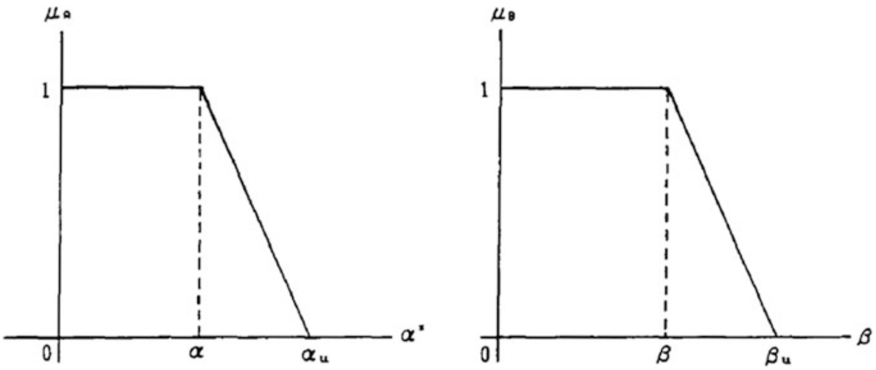
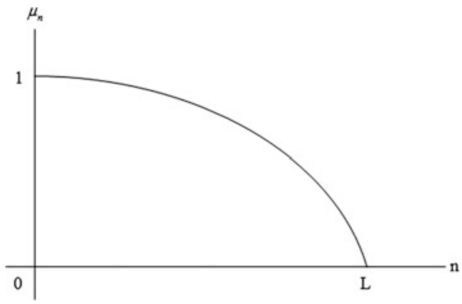


Fig. 7.6 Membership functions μ_A and μ_B

Fig. 7.7 Membership function μ_n which represents the grade of satisfaction for the sample size n



$$\mu_B(\beta^*) = \begin{cases} 1 & (\beta^* \leq \beta) \\ \frac{\beta_u - 1 + P(p_2)}{\beta_u - \beta} & (\beta \leq \beta^* \leq \beta_u) \\ 0 & (\beta_u \leq \beta^*) \end{cases} \tag{7.26}$$

where

$$P(p) = \sum_{k=0}^{c} \binom{n}{k} p^k (1-p)^{n-k} \tag{7.27}$$

Figure 7.8 shows the graphs of the membership functions $\mu_A(\alpha^*)$ and $\mu_B(\beta^*)$ with respect to the sample size n. In this figure, $n_{\alpha u}$ and $n_{\beta u}$ are real numbers which satisfy respectively.

$$2\lambda(n_{\alpha_u}, c, p_1) = 2g(n_{\alpha_u}, c)h(p_1) = \mathrm{X}^2_{1-\alpha_u}(2c+2), \tag{7.28}$$

$$2\lambda(n_{\beta_u}, c, p_2) = 2g(n_{\beta_u}, c)h(p_2) = \mathrm{X}^2_{1-\beta_u}(2c+2), \tag{7.29}$$

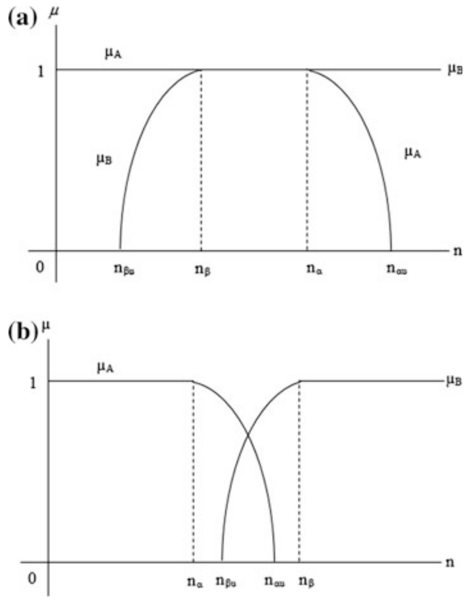


Fig. 7.8 Membership functions $\mu_A(\alpha^*)$ and $\mu_B(\beta^*)$ with respect to sample size n

Let c_u be the minimum integer which satisfies $\geq R(c_u; \alpha, \beta)$. If the membership functions μ_A and μ_B were decided as in Fig. 7.6, that is, in Fig. 7.8, it is found that acceptance number c in Problem 1 is less than or equal to c_u. Because when c is greater than or equal to c_u, it is $r \geq R(c_u; \alpha, \beta)$, then the membership functions μ_A *and* μ_B are shown in Fig. 7.8a. Accordingly, which maximizes min $\{\mu_A, \mu_B, \mu_n\}$ depends on only the intersection of μ_A *and* μ_B. The grade of max min $\{\mu_A, \mu_B, \mu_n\}$ decreases with c, because μ_n is monotonically decreasing. So c is found to be less than or equal to c_u. Let the sample size n expand to a real number. Setting $n^* \in R$, which satisfies

$$\mu_B(\beta^*(n^*, c)) = \mu_n(n) \tag{7.30}$$

It is obvious that when c is equal to c_u, n^* belongs to interval $[n_{\beta_u}, n_\beta]$. If n^* is found, the integer solution n is either $[n^*]$ or $[n^*] + 1$. Note that when c is less than c_u, the relation is changed to $n_\beta \geq n_\alpha$. In the both cases, the integer solution n will be found by means of searching in the integer interval $[[n_{\beta_u}] - 1, [n_{\beta_u}] + 1]$. Finally sample size n is selected as

$$n_i = \max_i \{\mu_0(\mathrm{n}_0), \mu_1(\mathrm{n}_1), \mu_2(\mathrm{n}_2), \ldots, \mu_i(\mathrm{n}_i), \ldots\} \tag{7.31}$$

and acceptance number is selected as $c_u - i$.

Table 7.1 The grade of fuzzy union set with respect to n on each c

c:	Grade of fuzzy product set with respect to n on each c
4:	Max min $\{\mu_B, \mu_n\}$ in $[73, 89] = \mu_n(79) = 0.4846$
4-1:	Max min $\{\mu_A, \mu_B, \mu_n\}$ in $[60, 73] = \mu_n(66) = 0.5310$
4-2:	Max min $\{\mu_A, \mu_B, \mu_n\}$ in $[46, 58] = \mu_n(48) = 0.1853$
4-3:	Fuzzy product set does not exist

Numerical Example-2

It is set up the membership functions μ_A *and* μ_B as follows Kahraman and Kaya (2010):

$$\alpha = 5\,\%, \quad \alpha_u = 8\,\%, \quad \beta = 10\,\%, \quad \beta_u = 20\,\%.$$

For the membership function functions $\mu_n(n)$, it will be accepted the following function:

$$\mu_n(n) = 1 - \left(\frac{n}{L}\right)^m$$

where L is the tolerance limit of the sample size so that n should be smaller than L. it is better to select L to be less than N/10 for use of the binomial distribution. m is the shape parameter of the membership function, and m is selected so that $0 \le m \le 1$. In the case where L = 300, m = 0.5, $p_1 = 0.02$, $p_2 = 0.09$, the solving procedure of Problem 1 is as follows:

$$r = (h(p_2)/h(\mathrm{p}_1)) = 4.665$$

So that $c_u = 4$. Then the grade of fuzzy union set with respect to n on each c is shown in Table 7.1. After all it is c = 3, n = 66, $\alpha^* = 0.04338$, $\beta^* = 0.1441$.

7.6.2 *Design of Double Sampling Plans Under Fuzzy Environment*

In double sampling by attributes, an initial sample is taken, and a decision to accept or reject the lot is reached on the basis of this first sample if the number of nonconforming units is either quite small or quite large. A second sample is taken if the results of the first sample are not decisive. Since it is necessary to draw and inspect the second sample only in borderline cases, the average number of pieces inspected per lot is generally smaller with double sampling. It has been demonstrated to be simple to use in a wide variety of conditions, economical in total cost, and acceptable psychologically to both producer and consumer (Juran 1998).

Kahraman and Kaya (2010) used a double sampling plan with fuzzy parameters $(\tilde{n}_1, \tilde{c}_1, \tilde{n}_2, \tilde{c}_2)a$. $\tilde{N}$ and $\tilde{p}$ are also fuzzy. If the Poisson distribution is used, the acceptance probability of double sampling can be calculated as follows:

$$P_a = P(d_1 \leq \tilde{c}_1) + P(\tilde{c}_1 < d_1 \leq \tilde{c}_2)P(d_1 + d_2 \leq \tilde{c}_2) \tag{7.32}$$

$$\tilde{P}_a = \sum_{d_1=0}^{\tilde{c}_1} \frac{\lambda^{d_1} e^{-\tilde{n}_1\tilde{p}}}{d_1!} + \sum_{d_1 > \tilde{c}_1}^{\tilde{c}_2} \left(\frac{\lambda^{d_1} e^{-\tilde{n}_1\tilde{p}}}{d_1!} \times \sum_{d_2=0}^{\tilde{c}_2-d_1} \frac{\lambda^{d_2} e^{-\tilde{n}_2\tilde{p}}}{d_2!} \right) \tag{7.33}$$

$$P_a(\alpha) = \left[P_{al,d;\tilde{\lambda}}(\alpha), P_{ar,d;\tilde{\lambda}}(\alpha) \right] \tag{7.34}$$

$$\begin{aligned} P_{al,d;\tilde{\lambda}}(\alpha) &= \min\left\{ \sum_{d_1=0}^{c_1} \frac{\lambda^{d_1} e^{-n_1 p}}{d_1!} + \sum_{d_1 > c_1}^{c_2} \left(\frac{\lambda^{d_1} e^{-n_1 p}}{d_1!} \times \sum_{d_2=0}^{c_2-d_1} \frac{\lambda^{d_2} e^{-n_2 p}}{d_2!} \right) \right\} \\ P_{ar,d;\tilde{\lambda}}(\alpha) &= \max\left\{ \sum_{d_1=0}^{c_1} \frac{\lambda^{d_1} e^{-n_1 p}}{d_1!} + \sum_{d_1 > c_1}^{c_2} \left(\frac{\lambda^{d_1} e^{-n_1 p}}{d_1!} \times \sum_{d_2=0}^{c_2-d_1} \frac{\lambda^{d_2} e^{-n_2 p}}{d_2!} \right) \right\} \end{aligned} \tag{7.35}$$

where $p \in p(\alpha), n \in n(\alpha)$, and $c \in c(\alpha)$.

If the binomial distribution is used, acceptance probability can be calculated as follows:

$$P_a = \sum_{d_1=0}^{\tilde{c}_1} \binom{\tilde{n}_1}{d_1} \tilde{p}^{d_1}(1-\tilde{p})^{\tilde{n}_1-d_1} + \sum_{d_1 > \tilde{c}_1}^{\tilde{c}_2} \left(\binom{\tilde{n}_1}{d_1} \tilde{p}^{d_1}(1-\tilde{p})^{\tilde{n}_1-d_1} \times \sum_{d_2=0}^{\tilde{c}_2-d_1} \binom{\tilde{n}_2}{d_2} \tilde{p}^{d_2}(1-\tilde{p})^{\tilde{n}_2-d_2} \right) \tag{7.36}$$

$$\begin{aligned} P_{al}(\alpha) &= \min\left\{ \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} + \sum_{d_1 > c_1}^{c_2} \left(\binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \times \sum_{d_2=0}^{c_2-d_1} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \right) \right\}, \\ P_{ar}(\alpha) &= \max\left\{ \sum_{d_1=0}^{c_1} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} + \sum_{d_1 > c_1}^{c_2} \left(\binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} \times \sum_{d_2=0}^{c_2-d_1} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \right) \right\} \end{aligned} \tag{7.37}$$

where $p \in p(\alpha), q \in q(\alpha), n_1 \in n_1(\alpha), c_1 \in c_1(\alpha), n_2 \in n_2(\alpha)$, and $c_2 \in c_2(\alpha)$.

AOQ values for fuzzy double sampling can be calculated as in Eqs. (7.17–7.19).

ASN for double sampling can be calculated as follows:

$$\begin{aligned} A\tilde{S}N &= \tilde{n}_1\tilde{P}_I + (\tilde{n}_1 + \tilde{n}_2)\left(1 - \tilde{P}_I\right) \\ &= \tilde{n}_1 + \tilde{n}_2\left(1 - \tilde{P}_I\right) \end{aligned} \tag{7.38}$$

$$ASN(\alpha) = [ASN_l(\alpha), ASN_r(\alpha)] \tag{7.39}$$

$$\begin{aligned} ASN_l(\alpha) &= \min\{n_1 + n_2(1 - P_I) | p \in p(\alpha), n_1 \in n_1(\alpha), n_2 \in n_2(\alpha), P_I \in P_I(\alpha)\}, \\ ASN_r(\alpha) &= \max\{n_1 + n_2(1 - P_I) | p \in p(\alpha), n_1 \in n_1(\alpha), n_2 \in n_2(\alpha), P_I \in P_I(\alpha)\} \end{aligned} \tag{7.40}$$

ATI curve for fuzzy double sampling can also be calculated as follows:

$$A\tilde{T}I = A\tilde{S}N + \left(\tilde{N} - \tilde{n}_1\right)P(d_1 > \tilde{c}_2) + \left(\tilde{N} - \tilde{n}_1 - \tilde{n}_2\right)P(d_1 + d_2 > \tilde{c}_2) \quad (7.41)$$

$$ATI(\alpha) = [ATI_l(\alpha), ATI_r(\alpha)] \quad (7.42)$$

$$\begin{aligned} ATI_l(\alpha) &= \min\{ASN + (N - n_1)P(d_1 > c_2) + (N - n_1 - n_2)P(d_1 + d_2 > c_2)\}, \\ ATI_r(\alpha) &= \max\{ASN + (N - n_1)P(d_1 > c_2) + (N - n_1 - n_2)P(d_1 + d_2 > c_2)\} \end{aligned} \quad (7.43)$$

where $p \in p(\alpha), ASN \in ASN(\alpha), n_1 \in n_1(\alpha), N \in N(\alpha), n_2 \in n_2(\alpha)$, and $c_2 \in c_2(\alpha)$.

Numerical Example-3

Let us reconsider Numerical Example-1 for the case of fuzzy double sampling. The sample sizes are determined as “Approximately 75” and “Approximately 300” for the first and second samples, respectively. Also the acceptance numbers are determined as “Approximately 0” and “Approximately 3” for the first and second samples, respectively. Based on Eq. (7.35), acceptance probability of the double sampling plan is calculated as follows:

$$\begin{aligned} \tilde{P}_a &= P\left(d_1 \le \tilde{0}\right) + \left[P\left(d_1 = \tilde{1}\right) \times P\left(d_2 \le \tilde{2}\right)\right] + \left[P\left(d_1 = \tilde{2}\right) \times P\left(d_2 \le \tilde{1}\right)\right] + \left[P\left(d_1 = \tilde{3}\right) \times P\left(d_2 \le \tilde{0}\right)\right] \\ &= (0.0105,\ 0.0235,\ 0.2052) + [(0.0105,\ 0.0882,\ 0.227) \times (0,\ 0,\ 0.0024)] \\ &\quad + [(0.0477,\ 0.1654,\ 0.224) \times (0,\ 0,\ 0.0005)] + [(0.1088,\ 0.2067,\ 0.227) \times (0, 0, 0.0001)] \\ &= (0.0105,\ 0.0235,\ 0.2052) + (0,\ 0,\ 0.0005) + (0,\ 0,\ 0.0001) + (0,\ 0,\ 0) \\ \tilde{P}_a &= (0.0105,\ 0.0235,\ 0.2058). \end{aligned}$$

Its membership function is shown in Fig. 7.9.

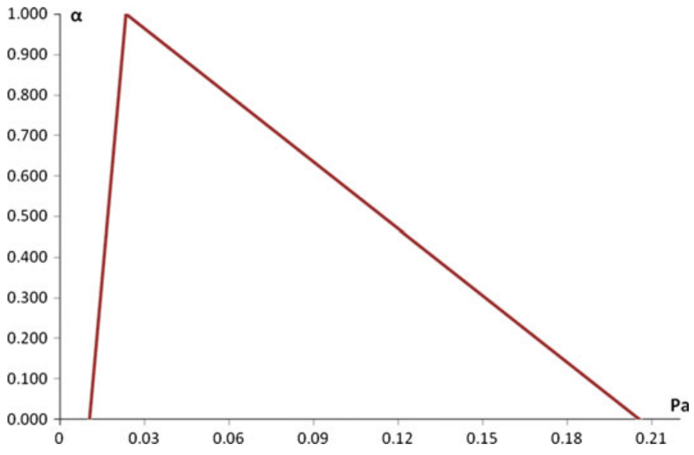


Fig. 7.9 Membership function of acceptance probability for double sampling

ASN is calculated as ASN = TFN(74.00, 213.08, 320.24) by using Eqs. (7.38–7.40). Also AOQ is calculated as AOQ = TFN(0.00042, 0.001175, 0.01235).

Wang and Chen (1997) formulated the problem of determining the Dodge-Romig LTPD double sampling plan under the fuzzy environment satisfies the consumer's risk closely around β using fuzzy mathematical programming. They proposed a model to minimize the ATI at the process average p_m subject to satisfying the consumer's risk closely around b, the Dodge-Romig LTPD double sampling plan finds a non-fuzzy non-negative integer pair (n_1, n_2, c_1, c_2) that minimizes:

$$I(p_m, n_1, n_2, c_1, c_2) = n_1 + n_2 * (1 - G(c_1, n_1 p_m)) + (N - n_1 - n_2)* \left[1 - G(c_1, n_1 p_m) - \sum_{j=1}^{c_2 - c_1} g(c_1 + j, n_1 p_m) * G(c_2 - c_1 - j, n_2 p_m)\right] \tag{7.44}$$

subject to

$$K(p_n, n_1, n_2, c_1, c_2) \lesssim \beta \tag{7.45}$$

$$\mathrm{n}_1, \mathrm{n}_2, \mathrm{c}_1, \mathrm{c}_2 \geq 0, \text{ integer} \tag{7.46}$$

where

$$K(p_n, n_1, n_2, c_1, c_2) = G(c_1, n_1 p_n) + \sum_{j=1}^{c_2 - c_1} g(c_1 + j, n_1 p_m) * G(c_2 - c_1 - j, n_2 p_n) \tag{7.47}$$

where $g(x, np)$ represents the probability of the Poisson distribution for the randım variable x with parameter np, i.e. $g(x, np) = \mathrm{e}^{-np}(np)^x/x!$ and $G(x, np)$ is its cumulative distribution. $I(p_m, n_1, n_2, c_1, c_2)$ is the ATI. The symbol $\lesssim$ stands for fuzzy inequality.

The symmetry of the decision model in a fuzzy environment rests essentially on the assumption that the objective function as well as the constraint can be fuzzy sets, and that the degree of membership of solutions to the objective function and to the constraint could be considered comparable. The above model shown using Eqs. (7.44–7.46) is a non-linear integer fuzzy mathematical programming problem. Because the objective function in Eq. (7.44) is a crisp set and the constraint in Eq. (7.45) is a fuzzy set, it is the optimization problem is a non-symmetrical fuzzy model. So Wang and Chen (1997) used the studies of Zimmermann (1985) and Chakraborty (1988, 1992) with the minimum operator to aggregate the membership functions of fuzzy sets, they obtained the following model for Dodge-Romig LTPD double sampling plan problem:

$$Maximize\left\{\begin{array}{l}\min n_1, n_2, c_1, c_2 \geq 0, \text{ integer} \\ \left[\dfrac{\inf\limits_{R_1} I - I(p_m, n_1, n_2, c_1, c_2)}{\inf\limits_{R_1} I - \inf\limits_{S(R)} I} = \lambda_1, \dfrac{\beta_u - K(p_n, n_1, n_2, c_1, c_2)}{\beta_u - \beta} = \lambda_2\right] = \lambda\end{array}\right\} \tag{7.48}$$

Subject to

$$\inf_{S(R)} I \leq I(p_m, n_1, n_2, c_1, c_2) \leq \inf_{R_1} I \tag{7.49}$$

$$\beta \leq K(p_n, n_1, n_2, c_1, c_2) \leq \beta_u \tag{7.50}$$

$$0 \leq \lambda \leq 1 \tag{7.51}$$

$$\mathrm{n}_1, \mathrm{n}_2, \mathrm{c}_1, \mathrm{c}_2 \geq 0, \text{ integer} \tag{7.52}$$

where R is a fuzzy feasible region, $S(R)$ support of R, and $R_{1\_}-$ level cut of R for $\alpha = 1$.

This problem can be rewritten in the equivalent optimization problem to find $n_1, n_2, c_1, c_2, \lambda$ that maximize λ and subject to

$$\lambda \leq \frac{\inf\limits_{R_1} I - I(p_m, n_1, n_2, c_1, c_2)}{\inf\limits_{R_1} I - \inf\limits_{S(R)} I} \tag{7.53}$$

$$\lambda \leq \frac{\beta_u - K(p_n, n_1, n_2, c_1, c_2)}{\beta_u - \beta} \tag{7.54}$$

and inequalities (7.49) through (7.52).Let $I_1 = \inf\limits_{R_1} I$ and $I_0 = \inf\limits_{S(R)} I$. Then the above optimization problem can be expressed as: find $n_1, n_2, c_1, c_2, \lambda$ that maximize λ and subject to

$$R^{(I_1 - I_0)} + I(p_m, n_1, \mathrm{n}_2, c_1, c_2) \leq I_1 \tag{7.55}$$

$$R^{(\beta_u - \beta_0)} + K(p_n, n_1, \mathrm{n}_2, c_1, c_2) \leq -_U \tag{7.56}$$

$$I_0 \leq I(p_m, n_1, n_2, c_1, c_2) \leq I_1 \tag{7.57}$$

and inequalities (7.50), (7.51) and (7.52).

Numerical Example-4

It is considered the example given in Hald (1981): $N = 2000$, $p_m = 0.02$, $p_n = 0.10$, and $\beta = 0.10$. The optimum LTPD double sampling plan$(n_1^*, n_2^*, c_1^*, c_2^*)$ is to be found for $\beta_u = 0.15$. For a LTPD double sampling plan, it is obtained $(n_1^a, n_2^a, c_1^a, c_2^a) = (55, 132, 2, 10)$ with $I_1 = 70.30$ and $(n_1^b, n_2^b, c_1^b, c_2^b) = (36, 111, 1, 8)$ with $I_0 = 58.45$. The DSP giving the largest value of λ for each c_1 near $\{c_1^a 1,\ \ c_2^a = 2\}$ is found. It is concluded that the optimum LTPD double sampling plan is; $(n_1^*, n_2^*, c_1^*, c_2^*) = (40, 104, 1, 8)$ with $\lambda^* = 0.52I(p_m, n_1^*, n_2^*, c_1^*, c_2^*) = 64.12$ and $K(p_n, n_1^*, n_2^*, c_1^*, c_2^*) = 0.1235$.

In this particular example, the difference between the solution of the traditional Dodge-Romig LTPD double sampling plan and their is that the decision maker takes an additional consumer's risk of 2.35 % for a lot being rejected against a saving of inspection effort per lot.

7.7 Proposed Fuzzy Multi-objective Mathematical Models for Design of Acceptance Sampling Plans

In this section, multi-objective mathematical models for designing of single and double sampling by attributes are developed and the optimal results are obtained by considering the various constraints under fuzziness. As a result it is obtained that the lower sample sizes in developed single and double sampling plans under fuzzy environment.

7.7.1 Proposed Fuzzy Multiobjective Models for Design of Single Acceptance Sampling Plan

The case in which a large sample size is needlessly required often arises in conventional design. The sample size n can be reduced as desired by relaxing the conditions on the consumer's risks. Hence, the tradeoff between the reduction of sample size and the relaxation of the conditions becomes a serious problem. So we developed a design procedure based on fuzzy multi-objective mathematical model for single sampling plans. In practical applications, LTPD cannot be known precisely. Hence the following model is developed to find the most appropriate sample size n with minimizing of ATI and AOQ. Also in this model, LTPD and consumer's risks β are defined as fuzzy numbers. The closed form of the model is given following equations:

Objective function

Min ATI

Min AOQ

Subject to

$$P_a(n, c; \widetilde{LTPD}) \leq \widetilde{\beta} \tag{7.58}$$

$$n \geq c \tag{7.59}$$

$$\text{n} > 0, \text{ integer; c} \geq 0, \text{ integer} \tag{7.60}$$

The open form of the model is given by using Eqs. (7.61–7.64).
Objective Function
Min ATI

$$n + (N - n) \times \left[1 - \sum_{x=0}^{c} \frac{e^{-n \times \widetilde{LTPD}} \times (\text{n} \times \widetilde{LTPD})^x}{x!}\right] \tag{7.61}$$

Min AOQ

$$\frac{\widetilde{LTPD}}{N} \times (N - n) \times \left[\sum_{x=0}^{c} \frac{e^{-n \times \widetilde{LTPD}} \times (\text{n} \times \widetilde{LTPD})^x}{x!}\right] \tag{7.62}$$

Subject to

$$\sum_{x=0}^{c} \frac{e^{-n \times \widetilde{LTPD}} \times (\text{n} \times \widetilde{LTPD})^x}{x!} \leq \widetilde{\beta} \tag{7.63}$$

$$n \geq c$$

$$\text{n} > 0, \text{ integer; c} \geq 0, \text{ integer} \tag{7.64}$$

Numerical Example-5
Developed fuzzy multi-objective mathematical model for single sampling plan shown in Eqs. (7.61–7.64) is solved for N = 500, $\widetilde{\text{LTPD}} = \text{TFN}(0.02, 0.03, 0.04)$ and. The obtained results for *n*, *ATI* and *AOQ* are given in Table 7.2.

According to Table 7.2, when $\widetilde{LTPD}$ and $\widetilde{\beta}$ re defined as fuzzy numbers, the smaller values of *n*, *ATI* and *AOQ* are obtained. Table 7.3 gives the comparison of

Table 7.2 The results of n, ATI, and AOQ for single sampling plan given values for $\widetilde{\text{LTPD}}$ and $\tilde{\beta}$ and N = 500

N	$\widetilde{\text{LTPD}}$	Single sampling plan	Acceptance number (c)				
			1	2	3	4	5
500	TFN(0.02, 0.03, 0.04)	n	34	66	100	136	174
		ATI	104.158	131.915	160.017	190.611	223.688
		AOQ	0.0158	0.0147	0.0136	0.0124	0.011

Table 7.3 The comparison of the values of n, *ATI* and *AOQ* for the crisp values of LTPD and $\beta = 0.10$ and the fuzzy values of $\widetilde{\mathrm{LTPD}}$ and $\tilde{\beta}$

Parameters		Single sampling plan	LTPD			$\widetilde{\mathrm{LTPD}} = \mathrm{TFN}(0.02, 0.03, 0.04)$
N	C		0.02	0.03	0.04	
500	1	n	195	130	98	20
		ATI	469.748	463.301	460.757	40.422
		AOQ	0.0012	0.0022	0.0031	0.0183
	2	n	267	178	134	44
		ATI	476,983	468.191	464.342	72.644
		AOQ	0.0009	0.0019	0.0028	0.0171
	3	n	335	223	168	71
		ATI	483,697	473.459	467.603	97.211
		AOQ	0.0006	0.0016	0.0025	0.0161
	4	n	400	267	200	101
		ATI	490.036	476.919	470.110	125.551
		AOQ	0.0004	0.0014	0.0024	0.0148
	5	n	464	311	232	132
		ATI	496.410	481.654	473.276	154.429
		AOQ	0.0001	0.0011	0.0021	0.0138

the values of *n, ATI* and *AQO* for the crisp values of LTPD and $\beta = 0.10$ and fuzzy values of $\widetilde{LTPD}$ and $\tilde{\beta}$.

7.7.2 Proposed Fuzzy Multiobjective Models for Design of Double Acceptance Sampling Plan

In the presented model for double sampling plan, the decision maker specifies consumer's riskβ and *LTPD* as fuzzy numbers to find the most appropriate sample sizes n_1 and n_2 with minimizing *ATI* and *AOQ*. The closed form of the model is given following equations:

Objective function

Min ATI

Min AOQ

Subject to

$$P_a(n_1, n_2; c_1, c_2; \widetilde{LTPD}) \leq \tilde{\beta} \tag{7.65}$$

$$\mathrm{n}_1, \mathrm{n}_2 > 0, \text{ integer}; \mathrm{c}_1, \mathrm{c}_2 \geq 0, \text{ integer} \tag{7.66}$$

The open form of the model is given by using Eqs. (7.67–7.70).

Objective function

Min ATI

$$
\begin{aligned}
& n_1 + n_2 \times \left[\sum_{x=0}^{c_2} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} - \sum_{x=0}^{c_1} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} \right] \\
& \quad + (N - n_1) \times \left[1 - \sum_{x=0}^{c_2} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} \right] + (N - n_1 - n_2) \times \left[\sum_{x=0}^{c_2} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} \right] \\
& \quad - \left(\begin{array}{l}
\displaystyle \sum_{x=0}^{c_1} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{(c_1+1)}}{(c_1+1)!} \times \sum_{x=0}^{c_2-c_1-1} \frac{e^{-n_2 \times \widetilde{LTPD}} \times (n_2 \times \widetilde{LTPD})^x}{x!} \\
\displaystyle + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{(c_1+2)}}{(c_1+2)!} \times \sum_{x=0}^{c_2-c_1-2} \frac{e^{-n_2 \times \widetilde{LTPD}} \times (n_2 \times \widetilde{LTPD})^x}{x!} + \cdots + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{c_2}}{c_2!} \\
\times\ e^{-n_2 \times \widetilde{LTPD}}
\end{array} \right)
\end{aligned} \tag{7.67}
$$

Min AOQ

$$
\frac{\widetilde{LTPD}}{N} \times \left[\begin{array}{l}
\displaystyle \sum_{x=0}^{c_1} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} \times (N - n_1) + (N - n_1 - n_2) \\
\displaystyle \times \sum_{x=0}^{c_2} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} - \left(\begin{array}{l}
\displaystyle \sum_{x=0}^{c_1} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{(c_1+1)}}{(c_1+1)!} \\
\displaystyle \times \sum_{x=0}^{c_2-c_1-1} \frac{e^{-n_2 \times \widetilde{LTPD}} \times (n_2 \times \widetilde{LTPD})^x}{x!} + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{(c_1+2)}}{(c_1+2)!} \\
\displaystyle \times \sum_{x=0}^{c_2-c_1-2} \frac{e^{-n_2 \times \widetilde{LTPD}} \times (n_2 \times \widetilde{LTPD})^x}{x!} + \cdots + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{c_2}}{c_2!} \\
\times\ e^{-n_2 \times \widetilde{LTPD}}
\end{array} \right)
\end{array} \right] \tag{7.68}
$$

Subject to

$$
\left[\begin{array}{l}
\displaystyle \sum_{x=0}^{c_2} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} \\
- \left(\begin{array}{l}
\displaystyle \sum_{x=0}^{c_1} \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^x}{x!} + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{(c_1+1)}}{(c_1+1)!} \\
\displaystyle \times \sum_{x=0}^{c_2-c_1-1} \frac{e^{-n_2 \times \widetilde{LTPD}} \times (n_2 \times \widetilde{LTPD})^x}{x!} + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{(c_1+2)}}{(c_1+2)!} \\
\displaystyle \times \sum_{x=0}^{c_2-c_1-2} \frac{e^{-n_2 \times \widetilde{LTPD}} \times (n_2 \times \widetilde{LTPD})^x}{x!} + \cdots + \frac{e^{-n_1 \times \widetilde{LTPD}} \times (n_1 \times \widetilde{LTPD})^{c_2}}{c_2!} \\
\times\ e^{-n_2 \times \widetilde{LTPD}}
\end{array} \right)
\end{array} \right] \leq \widetilde{\beta} \tag{7.69}
$$

$$
\mathrm{n}_1, \mathrm{n}_2 > 0,\ \text{integer};\ \mathrm{c}_1, \mathrm{c}_2 \geq 0,\ \text{integer} \tag{7.70}
$$

Table 7.4 The results of n for n_1, n_2 *ATI* and *AOQ* for double sampling plan given values for $\widetilde{\text{LTPD}}$ and $\tilde{\beta}$ and N = 500

$\widetilde{\text{LTPD}}$	Double sampling plan	Acceptance numbers (c_1 and c_2)				
		0–1	1–2	2–3	3–4	4–5
TFN(0.02, 0.03, 0.04)	n_1	14	29	49	71	94
	n_2	14	29	49	71	94
	ATI	112.956	135.238	175.544	216.570	257.793
	AOQ	0.0218	0.0246	0.0243	0.0234	0.0221

Numerical Example-6

Developed fuzzy multi-objective mathematical model for double sampling plan shown in Eqs. (7.67–7.70) is solved for

N = 500, $\widetilde{\text{LTPD}} = \text{TFN}(0.02, 0.03, 0.04)$ and $\tilde{\beta} = \text{TFN}(0.10, 0.15, 0.20)$. The obtained results for n_1, n_2*ATI* and *AOQ* are given in Table 7.4.

According to Table 7.4, when $\widetilde{\text{LTPD}}$ and $\tilde{\beta}$ are defined as fuzzy numbers, the smaller values of n_1, n_2 and *ATI* are obtained. Table 7.5 shows the comparison of

Table 7.5 The comparison of the values of n_1, n_2, *ATI* and *AQO* for the crisp values of LTPD and $\beta = 0.10$ and fuzzy values of $\widetilde{\text{LTPD}}$ and $\tilde{\beta}$

Parameters		Double sampling plan	LTPD			$\widetilde{\text{LTPD}}$
c_1	c_2		0.02	0.03	0.04	TFN(0.02, 0.03, 0.04)
0	1	n_1	16	11	8	14
		n_2	16	11	8	14
		ATI	89.294	80.528	70.926	112.956
		AOQ	0.0151	0.0230	0.0310	0.0218
1	2	n_1	36	24	17	29
		n_2	36	24	17	29
		ATI	125.790	100.756	88.238	135.238
		AOQ	0.0166	0.0257	0.0347	0.0246
2	3	n_1	62	41	31	49
		n_2	62	41	31	49
		ATI	176.329	132.972	113.837	175.544
		AOQ	0.0162	0.0256	0.0349	0.0243
3	4	n_1	92	61	46	71
		n_2	92	61	46	71
		ATI	234.181	171.887	142.884	216.570
		AOQ	0.0152	0.0247	0.0342	0.0234
4	5	n_1	124	83	62	94
		n_2	124	83	62	94
		ATI	294.655	214.481	172.498	257.793
		AOQ	0.0140	0.0235	0.0331	0.0221

the values of n_1, n_2, ATI and AQO for the crisp values of LTPD and $\beta = 0.10$ and fuzzy values of $\widetilde{\text{LTPD}}$ and $\widetilde{\beta}$.

7.8 Conclusion

The complexity of industrial manufacturing is growing and the need for higher efficiency, greater flexibility; better product quality and lower cost have changed the face of manufacturing practice. Statistical Quality Control is a tool for developing required resolution plans against problematic areas of manufacturing practice. One of the most important subjects of the Statistical Quality Control is acceptance sampling. Proper design of an acceptance sampling planning usually depends on knowing the true level of quality required by customers. However, it is sometimes not possible to determine this quality level with certain values. Especially in production, it is not easy to determine the parameters of acceptance sampling such as proportion of defect items, sample size, acceptable defect items.

A lot or batch of items can be inspected in several ways including the use of single, double, multiple, sequential sampling. In this chapter, the parameters used in acceptance sampling are defined with the help of linguistic variables and fuzzy set theory has successfully been applied to acceptance sampling to eliminate uncertainty and lack of knowledge mentioned above. We propose fuzzy multi-objective mathematical models for single and double sampling schemes. As a result it is obtained that the lower sample sizes in developed single and double sampling plans under fuzzy environment. For further studies, multi-objective mathematical models for multiple and sequential sampling schemes can be developed under fuzzy environment. Also decision trees that identify the causes of the non conformities of a rejected sample and indicate the appropriate interventions in the manufacturing process are worthwhile to study for acceptance sampling.

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