Chapter 6
An Integrated Framework to Analyze the Performance of Process Industrial Systems Using a Fuzzy and Evolutionary Algorithm

Harish Garg

Abstract In the design of critical combinations and complex integrations of large engineering systems, their reliability, availability and maintainability (RAM) analysis of the inherent processes in the system and their related equipments are needed to be determined. Although there have been tremendous advances in the art and science of system evaluation, yet it is very difficult to assess these parameters with a very high accuracy or precision. Basically, this inaccuracy in assessment stems mainly from the inaccuracy of data, lack of exactness of the models and even from the limitations of the current methods themselves and hence management decisions are based on experience. Thus the objective of this chapter is to present a methodology for increasing the performance as well as productivity of the system by utilizing these uncertain data. For this an optimization problem is formulated by considering RAM parameters as an objective function. The conflicting nature between the objectives is resolved by defining their nonlinear fuzzy goals and then aggregate by using a product aggregator operator. The failure rates and repair times of all constituent components are obtained by solving the reformulated fuzzy optimization problem with evolutionary algorithms. In order to increase the performance of the system, the obtained data are used for analyzing their behavior pattern in terms of membership and non-membership functions using intuitionistic fuzzy set theory and weakest t-norm based arithmetic operations. A composite measure of RAM parameters named as the RAM-Index has been formulated for measuring the performance of the system and hence finding the critical component of the system based on its performance. Finally the computed results of the proposed approach have been compared with the existing approaches for supremacy the approach. The suggested framework has been illustrated with the help of a case.

H. Garg (✉)
School of Mathematics, Thapar University Patiala, Patiala 147004, Punjab, India
e-mail: harishg58iitr@gmail.com
URL: http://sites.google.com/site/harishg58iitr/

C. Kahraman and S. Yanık (eds.), *Intelligent Decision Making in Quality Management*, Intelligent Systems Reference Library 97,
DOI 10.1007/978-3-319-24499-0_6

6.1 Introduction

Today's competitive business environment requires manufacturers to design, develop, test, manufacture, and deploy high-reliability products in less time at lower cost. For achieving this, billions of dollars are being spent annually worldwide to develop reliable and efficient products. With the advance in technology, a designer always wants to manufacture the equipment and systems of greater capital cost, complexity and capacity which results in increasing the reliability of the system. Also at the same time the unfortunate penalty of low availability and high maintenance cost need to be improved for their survival. Thus, for this reason, there is a growing interest in the investigations of the reliability, availability and maintainability (RAM) principles in various industrial systems during last decades which affects on the system performance directly. A brief literature review regarding the reliability/availability evaluation using evolutionary as well as fuzzy methodology is given below.

6.1.1 Reliability/Availability Analysis Using Evolutionary Algorithm

With the advances in technology and need of the modern society, the job of the system analyst and plant personnel becomes so challenging in order to maintain the profile of the system so that it becomes operating continuously for a longer time. This is happening so because failure is an inevitable phenomenon for all industrial systems. Therefore, it is difficult, if not impossible, to construct their mathematical and statistical model so as to reduce the number of likelihood failures. Thus there is a need of developing a suitable methodology for analyzing the performance of the complex systems so that necessary action should be initiated for enhancing the performance as well as achieving the goal of higher targets. For this, generally, system performance can be improved either by incremental improvements of component reliability/availability or by provision of redundant components in parallel; both methods result in an increase in system cost. Traditionally analytical and Monte-Carlo simulation techniques have been used for analyzing the system reliability. While analytical techniques are potentially faster, they tend to get difficult as system size and complexity increases. Monte Carlo methods, on the other hand, afford tremendous modeling flexibility, and can be used for systems with large size and complexity. However, Monte Carlo methods tend to be extremely time consuming, particularly for reliable systems. Therefore, optimization methods are necessary to obtain allowable costs at the same time as high availability levels. Extensive reliability design techniques have been introduced by the researchers during the past two decades for solving the optimization problem on the specific applications. Comprehensive overviews of these models have been addressed in Gen and Yun (2006), Kuo et al. (2001).

As demonstrated in the literature, the aforementioned optimization techniques are successfully applied to various reliability optimization problems and show a significant difference in getting an optimal or near optimal solution. However, the previously-developed algorithms, as stochastic optimization techniques, heuristic algorithm have some weakness such as the lower robustness, premature convergence of the solution, not using a prior knowledge, not exploiting local search information, difficultly in dealing with large scale reliability problems. Also, the heuristic techniques require derivatives for all non-linear constraint functions that are not derived easily because of the high computational complexity. To overcome this difficulty metaheuristics/evolutionary algorithms have been selected such as Genetic Algorithm (GA) (Goldberg 1989; Holland 1975), Particle Swarm optimization (PSO) (Eberhart and Kennedy 1995; Kennedy and Eberhart 1995), Artificial bee colony (ABC) (Karaboga 2005; Karaboga and Akay 2009; Karaboga and Basturk 2007) etc., and have proved itself to be able to approach the optimal solution against these problems.

In that direction, Bris et al. (2003) attempted to optimize the maintenance policy, for each component of the system, minimizing the cost function, with respect to the availability constraints using genetic constraints. Barabady and Kumar (2005a, b) had presented a methodology for improving the availability of a repairable system by using the concept of important measures. The empirical data of two crushing plants at the Jajarm bauxite mine of Iran are used as a case study for reliability and availability analysis. Zavala et al. (2005) proposed a PSO-based algorithm, to solve a bi-objective redundant reliability problem with the aim of maximizing the system reliability, and minimizing redundant components' cost for three types of systems as series, parallel, and k-out-of-N systems. Juang et al. (2008) proposed a genetic algorithm based optimization model to optimize the availability of a series parallel system where the objective is to determine the most economical policy of component's MTBF and MTTR. Liberopoulos and Tsarouhas (2002) presented a case study of chipitas food processing system, based on the simplified assumption that the failure and repair times of the workstations of the lines have exponential distributions. Kumar et al. (2007) developed an optimization model for optimizing the reliability, maintainability and supportability under performance based logistics using goal programming. Their model simultaneously considered multiple system engineering metrics during the design stage of the product development. Khan et al. (2008) presented a two step risk-based methodology to estimate optimal inspection and maintenance intervals which maximize the system's availability. Sharma and Kumar (2008) presented the application of RAM analysis in a process industry by using a Markovian approach as a tool to model the system behavior. Rajpal et al. (2006) explored the application of artificial neural networks to model the behavior of a complex, repairable system. A composite measure of RAM parameters called as the RAM—Index has been proposed for measuring the system performance by simultaneously considers all the three key indices which influence the system performance directly. Their index was static in nature while Garg et al. (2012, 2013) introduced RAM-Index which was time dependent and used historical uncertain data for its evolution. Yeh et al. (2011) presented an approximate model for predicting the network reliability by combining the ABC algorithm and Monte Carlo simulation. Yeh and Hsieh (2011) and Hsieh and Yeh (2012) presented a penalty guided artificial bee colony algorithm to solve system reliability redundancy allocation problems with a mix of components. Garg and Sharma (2012) had discussed the two-phase approach for analyzing the reliability and maintainability analysis of the industrial system. The crankcase unit of the two wheeler manufacturing industry has been taken as an illustrative example and gave a recommendation to the system analyst. Garg and Sharma (2013) have investigated the multi-objective reliability-redundancy allocation problem by using PSO and GA while Garg et al. (2012, 2014) have solved the reliability optimization problem with ABC algorithm and compared their performance with other evolutionary algorithm.

6.1.2 Reliability Analysis Using a Fuzzy Algorithm

Engineering systems are usually complex, involve a lot of detail, and operate in unpredictable environments which leads to the job of system analysts has become more challenging, as they have to study, characterize, measure and analyze the uncertain systems' behavior, using various techniques, which require the component failure and repair pattern. Further, age, adverse operating conditions and the vagaries of the system, affect each unit of the system differently. Thus, one comes across the problem of uncertainty in reliability assessment. To this effect, fuzzy-theoretic approach (Zadeh 1965) has been used to handle the subjective information or uncertainties during the evaluation of the reliability of a system than the probabilistic approach. After their successful applications, a lot of progress has been made in both theory and application and hence several researches were conducted on the extensions of the notion of fuzzy sets. Among these extensions the one that have drawn the attention of many researches during the last decades is the theory of intuitionistic fuzzy sets (IFS) introduced by Attanassov (1986, 1989). The concepts of IFS can be viewed as an appropriate/alternative approach to define a fuzzy set in the case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. IFS add an extra degree to the usual fuzzy sets in order to model hesitation and uncertainty about the membership degree of belonging. In fuzzy sets, the degree of acceptance is considered only but IFS is characterized by a membership function and a non-member function so that the sum of both values is less than or equal to one. Gau and Buehrer (1993) extended the idea of fuzzy sets by vague sets. Bustince and Burillo (1996) showed that the notion of vague sets coincides with that of IFSs. Therefore, it is expected that IFSs could be used to simulate any activities and processes requiring human expertise and knowledge, which are inevitably imprecise or not totally reliable. As far as reliability field is concerned, IFSs has been proven to be highly useful to deal with uncertainty and vagueness, and a lot of work has been done to develop and enrich the IFS theory given in Chang et al. (2006), Chen (2003), Garg and Rani

(2013), Garg et al. (2014), Kumar et al. (2006) Kumar and Yadav (2012) Taheri and Zarei (2011) and their corresponding references.

All the above researchers have used only reliability index during their analysis. But it is quite common that other reliability parameters such as failure rate, repair time, mean time between failures (MBTF) etc. are simultaneously affect the system behavior and hence on its performance. This idea is highlighted by Knezevic and Odoom (2001) and Garg (2013) in the fuzzy and intuitionistic fuzzy set theory respectively. In their approaches, system are modeled with the help of Petri nets and uncertainties which are present in the data are handled with the help of triangular fuzzy numbers and hence various reliability parameters of interest are computed in the form of membership and nonmembership functions. But it has been analyzed from their study that their approach is limited to a small size structural system. Thus when their approaches are applied to a complex structural system then the computed reliability indices contains a wide range of uncertainties in the form of support (spread) (Garg et al. 2013; Garg and Sharma 2012). This is due to the use of various fuzzy arithmetic operations involved in the analysis. Thus these approaches are no longer suitable for constructing the membership functions of IFS and hence do not give the accurate trend of the system as the uncertainty level increases. Therefore, there is a need of suitable methodology that can be used for computing the membership function of the reliability index up to a desired degree of accuracy. For this, by taking the advantages of evolutionary algorithms, the formulated reliability optimization problem has been solved with the Cuckoo search algorithm and compares their results with other algorithms. Since most of the collected data are imprecise and vague, so increase the relevance of the study, the obtained desired parameters are represented in the form of fuzzy numbers by taking different level of uncertainties. Based on these numbers, an analysis has been conducted for finding the most critical component of the system so that proper maintenance actions should be implemented for increasing the performance of the system.

Thus in the nutshell, the motive of this chapter is to devise a method to chalk out the performance measures of any repairable system by utilizing limited, vague and imprecise data. For this, the methodology has been proposed which is an amalgam of two techniques, EAs and intuitionistic fuzzy set theory, which can be described in stepwise as, (i) develop an optimization model by considering reliability, availability and maintainability of the system. The conflict naturalists between the objectives are resolved with the help of defining their fuzzy goals by using a nonlinear (sigmoidal) functions) (ii) obtain optimal MTBF and MTTR for the main component of the system using EAs and optimize the reliability parameters, and (iii) use their optimal parameters for computing various performance measures such as failure rate, repair time, ENOF etc. by using intuitionistic fuzzy set theory and weakest t-norm based arithmetic operations. Sensitivity analysis has been conducted on system MTBF for various combinations of reliability parameters. Finally, a composite measure of RAM parameters called RAM-Index has been used for finding the critical components of the system based on their variations of failure rate and repair time individually as well as simultaneously on its index. Results obtained from proposed technique are compared with the existing fuzzy and intuitionistic fuzzy set theory result. Plant personnel may use the results and can give guidelines to improve the system's performance by adopting suitable maintenance strategies. An example of the washing unit in a paper mill is taken into account to demonstrate the proposed approach.

6.2 Overview of IFS and EAs

A brief overview about the intuitionistic fuzzy set theory (IFS) and evolutionary algorithm (EA) have been given here.

6.2.1 Intuitionistic Fuzzy Set Theory

Intuitionistic fuzzy set (IFS) is one of the widely used and successful extension of the concept of fuzzy set. In order to model the hesitation and uncertainty about the degree of membership, Atanassov in (1986) add an extra degree, called as degree of non-membership, to the notion of the fuzzy set. Mathematically, if we define X be a universe of discourse then

$$\tilde{A} = \{ <x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x) > | x \in X\} \tag{6.1}$$

where $\mu_{\tilde{A}}, v_{\tilde{A}} : X \to [0, 1]$ be the degree of membership and nonmembership of the element x in the fuzzy set $\tilde{A}$, respectively such that $\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1$. The function $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x)$ is called the degree of hesitation or uncertainty level of the element x in the set $\tilde{A}$. Especially, if $\pi_{\tilde{A}}(x) = 0$ for all $x \in X$, then the IFS is reduced to a fuzzy set.

(α, β)–cut of the IFS set is defined as

$$A_{(\alpha,\beta)} = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha \quad and \quad v_{\tilde{A}}(x) \leq \beta\} \tag{6.2}$$

In other words, $A_{(\alpha,\beta)} = A_\alpha \cap A^\beta$ where $A_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$ and $A^\beta = \{x \in X | v_{\tilde{A}}(x) \leq \beta\}$

Definition: Convex Intuitionistic fuzzy set An IFS $\tilde{A}$ in universe X is convex if and only if membership functions of $\mu_{\tilde{A}}(x)$ and $v_{\tilde{A}}(x)$ of $\tilde{A}$ are fuzzy—convex and fuzzy—concave respectively i.e.,

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in U, 0 \leq \lambda \leq 1 \tag{6.3}$$

and

$$v_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq \max(v_{\tilde{A}}(x_1), v_{\tilde{A}}(x_2)) \forall x_1, x_2 \in U, 0 \leq \lambda \leq 1 \tag{6.4}$$

Definition: Normal Intuitionistic fuzzy set Let $\tilde{A}$ be an IFS with universe $\mathbb{R}$, then $\tilde{A}$ is said to be normalized if there exist at least two points $x_1, x_2 \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_1) = 1$ and $v_{\tilde{A}}(x_2) = 1$ otherwise it is said to subnormal IFS.

Definition: Intuitionistic fuzzy number (IFN) An IFN $\tilde{A}$ is a normal, convex membership function on the real line $\mathbb{R}$ with bounded support i.e. $\{x \in X | v_{\tilde{A}}(x) < 1\}$ is bounded and $\mu_{\tilde{A}}$ is upper semi-continuous and $v_{\tilde{A}}$ is lower semi-continuous. Let $\tilde{A}$ be IFS denoted by $\tilde{A} = <[(a, b, c); \mu, v]>$, where $a, b, c \in \mathbb{R}$ then the set $\tilde{A}$ is said to be triangular intuitionistic fuzzy number (TIFN) if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu\left(\frac{x-a}{b-a}\right); & a \leq x \leq b \\ \mu; & x = b \\ \mu\left(\frac{c-x}{c-b}\right); & b \leq x \leq c; \\ 0 & \text{otherwise} \end{cases};$$

$$1 - v_{\tilde{A}}(x) = \begin{cases} (1-v)\left(\frac{x-a}{b-a}\right); & a \leq x \leq b \\ 1-v; & x = b \\ (1-v)\left(\frac{c-x}{c-b}\right); & b \leq x \leq c \\ 0; & \text{otherwise} \end{cases}$$

where the parameter b gives the modal values of A and a, c are the lower and upper bounds of available area for the evaluation data. A triangular vague set defined by the triplet (a, b, c) with α-cuts, given in Fig. 6.1 is defined below for membership and non-membership functions respectively.

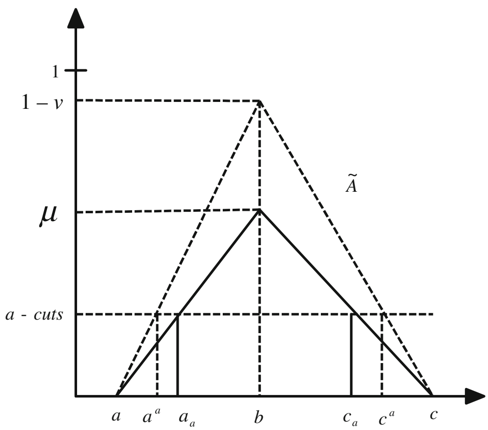

Fig. 6.1 Representation of α-cut of the IFS set

$$A_\alpha = [a_\alpha, c_\alpha] \quad \text{and} \quad A^\alpha = [a^\alpha, c^\alpha] \tag{6.5}$$

Here a_α, a^α are the increasing functions, c_α, c^α are decreasing functions of cut set given as follows.

$$a_\alpha = a + \frac{\alpha}{\mu_i}(b-a); \quad a^\alpha = a + \frac{\alpha}{1-v_i}(b-a)$$
$$c_\alpha = c - \frac{\alpha}{\mu_i}(c-b); \quad c^\alpha = c - \frac{\alpha}{1-v_i}(c-b)$$

Definition: T-norm and weakest t-norm A triangular norm (t-norm) T is a binary operation on $[0,1]$, i.e. a function $T:[0,1]^2 \to [0,1]$ such that (i) T is associative, (ii) T is commutative, (iii) T is nondecreasing, and (iv) T has 1 as a neutral element such that $T(x,1) = x$ for each $x \in [0,1]$.

A t-norm is called the weakest t—norm iff

$$T(x,y) = \begin{cases} 0; & \max(x,y) < 1 \\ \min(x,y); & \text{otherwise} \end{cases} \tag{6.6}$$

The basic arithmetic operations i.e. addition, subtraction, multiplication and division of IFNs depends upon the arithmetic of the interval of confidence. The four main basic arithmetic operations for the n triangular IFSs using T_ω—based approximate intuitionistic fuzzy arithmetic operations and with α—cuts arithmetic operations on triangular fuzzy numbers (TFNs), with $\mu = \min(\mu_i)$ and $v = \max(v_i)$, are defined as follow.

1. Addition of T_w ($\oplus$)

$$\tilde{A}_1 \oplus^\alpha_{T_w} \cdots \oplus^\alpha_{T_w} \tilde{A}_n = \begin{cases} [\sum\limits_{i=1}^n a_{i1}^{(\alpha)}, \quad \sum\limits_{i=1}^n a_{i3}^{(\alpha)}] & \text{if} \quad \tilde{A}_i \in \text{TFNs} \\ \left[\sum\limits_{i=1}^n a_{i2} - \max\limits_{1 \le i \le n}\Big((a_{i2} - a_{i1}^{(\alpha)})\Big), \right. & \\ \left. \sum\limits_{i=1}^n a_{i2} + \max\limits_{1 \le i \le n}\Big((a_{i3}^{(\alpha)} - a_{i2})\Big)\right] & \text{otherwise} \end{cases} \tag{6.7}$$

2. Subtraction of T_w ($\ominus$):

$$\tilde{A}_1 \ominus^\alpha_{T_w} \cdots \ominus^\alpha_{T_w} \tilde{A}_n = \begin{cases} [a_{11}^{(\alpha)} - \sum\limits_{i=2}^n a_{i3}^{(\alpha)}, a_{13}^{(\alpha)} - \sum\limits_{i=2}^n a_{i1}^{(\alpha)}] & \text{if} \quad \tilde{A}_i \in \text{TFNs} \\ \left[a_{12} - \sum\limits_{i=2}^n a_{i2} - \max\limits_{1 \le i \le n}\Big((a_{i2} - a_{i1}^{(\alpha)})\Big), \right. & \\ \left. a_{12} - \sum\limits_{i=2}^n a_{i2} + \max\limits_{1 \le i \le n}\Big((a_{i3}^{(\alpha)} - a_{i2})\Big)\right] & \text{otherwise} \end{cases} \tag{6.8}$$

3. Multiplication of $T_w\,(\otimes)$: Here, multiplication of the approximate fuzzy operations are shown for $\tilde{A}_i \in \mathbb{R}^+$

$$\tilde{A}_1 \otimes_{T_w}^{\alpha} \cdots \otimes_{T_w}^{\alpha} \tilde{A}_n = \begin{cases} \left[\prod_{i=1}^{n} a_{i1}^{(\alpha)}, \prod_{i=1}^{n} a_{i3}^{(\alpha)}\right] & \text{if} \quad \tilde{A}_i \in \text{TFNs} \\ \left[\prod_{i=1}^{n} a_{i2} - \max_{1 \le i \le n}\left((a_{i2} - a_{i1}^{(\alpha)}) \prod_{\substack{j=1 \\ j \neq i}}^{n} a_{j2}\right), \right. & \\ \left. \prod_{i=1}^{n} a_{i2} + \max_{1 \le i \le n}\left((a_{i3}^{(\alpha)} - a_{i2}) \prod_{\substack{j=1 \\ j \neq i}}^{n} a_{j2}\right)\right] & \text{otherwise} \end{cases} \tag{6.9}$$

4. Division of $T_w\,(\varnothing)$: Here, division of the approximate fuzzy operations are shown for $\tilde{A}_i \in \mathbb{R}^+$

$$\tilde{A}_1 \varnothing_{T_w}^{\alpha} \cdots \varnothing_{T_w}^{\alpha} \tilde{A}_n = \tilde{A}_1 \otimes_{T_w}^{\alpha} \frac{1}{\tilde{A}_2} \cdots \otimes_{T_w}^{\alpha} \frac{1}{\tilde{A}_n}; \quad \text{if } 0 \notin \tilde{A}_i, i \ge 2 \tag{6.10}$$

6.2.2 *Evolutionary Algorithms: GA, PSO, ABC, CS*

6.2.2.1 Genetic Algorithm

Genetic Algorithms (GAs) (Goldberg 1989; Holland 1975) are adaptive heuristic search algorithms introduced in the evolutionary themes of natural selection. The fundamental concept of the GA design is to model processes in a natural system that is required for evolution, specifically those that follow the principles posed by Charles Darwin to find the survival of the fittest. GAs constitutes an intelligent development of a random search within a defined search space to solve a problem. GAs was first pioneered by John Holland in the 1960s, and has been widely studied, experimented, and applied in numerous engineering disciplines. GAs was introduced as a computational analogy of adaptive systems. They are modeled loosely on the principles of the evolution through natural selection, employing a population of individuals that undergo selection in the presence of variability-inducing operators such as mutation and recombination (crossover). A fitness function is used to evaluate individuals, and reproductive success varies with fitness. The pseudo code of the GA algorithm is described in Algorithm 1:

Algorithm 1 Pseudo code of Genetic algorithm (GA)

```
Objective function: f(x)
Define Fitness F (eg. F ∝ f(x) for maximization)
Initialize population
Initial probabilities of crossover (p_c) and mutation (p_m)
repeat
    Generate new solution by crossover and mutation
    if p_c >rand, Crossover; end if
    if p_m >rand, Mutate; end if
    Accept the new solution if its fitness increases.
    Select the current best for the next generation.
until requirements are met
```

6.2.2.2 Particle Swarm Optimization Algorithm

In 1995, Eberhart and Kennedy (1995), Kennedy and Eberhart (1955) developed PSO, a population based on stochastic optimization strategy, inspired by social behavior of a flock of birds, schools of fish, a swarm of bees and even sometimes social behavior of human. Though PSO is similar to Genetic Algorithms (GA) in terms of population initialization with random solutions and searching for global optima in successive generations, PSO does not undergo crossover and mutation, whereas the particles move through the problem space following the current optimum particles. The underlying concept is that, for every time instant, the velocity of each particle also known as the potential solution, changes between its personnel best (pbest) and global best (gbest) locations. Mathematically, swarm of particles is initialized randomly over the search space and move through D—dimensional space to search new solutions. Let x_k^i and v_k^i respectively be the position and velocity of ith particle in the search space at kth iteration then the position of this particle at $(k+1)$th iteration is updated through the equation,

$$x_{k+1}^i = x_k^i + v_{k+1}^i \tag{6.11}$$

where v_{k+1}^i is the updated velocity vector of ith particle at $(k+1)$th iteration and are adjusted according to their swarm own experience and that of its neighbors and are given as follow.

$$v_{k+1}^i = \underbrace{w \cdot v_k^i}_{\text{inertia}} + \underbrace{c_1 \cdot r_1 \cdot (p_k^i - x_k^i)}_{\text{personal influence}} + \underbrace{c_2 \cdot r_2 \cdot (p_k^g - x_k^i)}_{\text{social influence}} \tag{6.12}$$

where v_k^i is the velocity vector at kth iteration, r_1 and r_2 represent random numbers between 0 and 1; p_k^i represents the best ever position of ith particle, and p_k^g corresponds to the global best position in the swarm up to kth iteration.

The essential steps of the particle swarm optimization can be summarized as the pseudo code given in Algorithm 2.

Algorithm 2 Pseudo code of Particle swarm optimization (PSO)

1: Objective function: $f(\mathbf{x}), \quad \mathbf{x} = (x_1, x_2, \ldots, x_D)$;
2: Initialize particle position and velocity for each particle and set $k = 1$.
3: Initialize the particle's best known position to its initial position i.e. $p_k^i = x_k^i$.
4: **repeat**
5: Update the best known position (p_k^i) of each particle and swarm's best known position (p_k^g).
6: Calculate particle velocity according to the velocity equation (12).
7: Update particle position according to the position equation (11).
8: **until** requirements are met.

6.2.2.3 Artificial Bee Colony Algorithm

The artificial bee colony (ABC) optimization algorithm was first developed by Karaboga in 2005. Since then Karaboga and Basturk and their coauthors (2005), Karaboga and Akay (2009) have systematically studied the performance of the ABC algorithm and its extension on unconstrained optimization problems. In ABC algorithm, the bees in a colony are divided into three groups: employed bees (forager bees), onlooker bees (observer bees) and scouts. For each food source, there is only one employed bee. That is to say, the number of employed bees is equal to a number of food sources. The employed bee of a discarded food site is forced to become a scout for searching new food source randomly. The whole process of the algorithm may also be explained through the Algorithm 2.2.3. In this, the first stage is the initialization stage in which food source positions are randomly selected by the bees and their nectar amounts (i.e. fitness function, f) is determined. Then, these bees come into the hive and share the nectar information of the sources with the bees waiting for the dance area with a probability $p_h = f_h / \sum_{h=1}^{N} f_h$ where N is the number of food sources and $f_h = f(x_h)$ is the amount of nectar evaluated by its employed bee. After a solution is generated, that solution is improved by using a local search process called greedy selection process carried out by an onlooker and employed bees and is given by

$$Z_{hj} = x_{hj} + \phi(x_{hj} - x_{kj}) \tag{6.13}$$

where $k \in \{1, 2, \ldots, N\}$ and $j \in \{1, 2, \ldots, D\}$ are randomly chosen index such that k is different from h and ϕ is a random number between $[-1, 1]$ and Z_h is the solution in the neighborhood of x_h. Here, except for the selected parameter j, all other parametric value of Z_h are same as that of x_h. If a particular food source solution does not improve for a predetermined iteration number then it becomes a scout and hence discovers a new food source with the randomly generated food source within its domain. So this randomly generated food source is equally assigned to this scout and changing its status from scout to employ and hence other iteration/cycle of the algorithm begins until the termination condition, maximum cycle number (MCN) or relative error, is not satisfied.

Algorithm 3 Pseudo code of Artificial Bee Colony (ABC) optimization

```
1: Objective function: f(x),   x = (x_1, x_2, ..., x_D);
2: Initialization Phase
3: repeat
4:     Employed Bee Phase
5:     Onlooker Bee Phase
6:     Scout Bee Phase
7:     Memorize the best position achieved so far.
8: until requirements are met.
```

6.2.2.4 Cuckoo Search Algorithm

CS is a meta-heuristic search algorithm which has been proposed recently by Yang and Deb (2009) getting inspired from the reproduction strategy of cuckoos. At the most basic level, cuckoos lay their eggs in the nests of other host birds, which may be of different species. The host bird may discover that the eggs are not its own so it either destroys the eggs or abandons the nest all together. This has resulted in the evolution of cuckoo eggs which mimic the eggs of local host birds. CS is based on three idealized rules:

(i) Each cuckoo lays one egg at a time, and dumps it in a randomly chosen nest.
(ii) The best nests with high quality of eggs (solutions) will carry over to the next generations.
(iii) The number of available host nests is fixed, and a host can discover an alien egg with a probability $p_a \in [0, 1]$. In this case, the host bird can either throw the egg away or abandon the nest so as to build a completely new nest in a new location.

To make the things even more simple, the last assumption can be approximated by the fraction of p_a of n nests that are replaced by new nests with new random solutions. The fitness function of the solution is defined in a similar way as in other evolutionary techniques. In this technique, egg presented in the nest will represent the solution while the cuckoo egg represent the new solution. The aim is to use the new and potentially better solutions (cuckoos) to replace worse solutions that are in the nests. Based on these three rules, the basic steps of the cuckoo search is described in Algorithm 4.

Algorithm 4 Pseudo code of Cuckoo Search (CS)

Objective function: $f(\mathbf{x}), \quad \mathbf{x} = (x_1, x_2, \ldots, x_D)$;
Generate an initial population of n host nests x_i ; $i = 1, 2, \ldots, N$;
While (t < MaxGeneration) or (stop criterion)
  Get a cuckoo randomly (say, i)
  Generate a new solution by performing Lévy flights;
  Evaluate its fitness f_i
  Choose a nest among n (say, j) randomly;
  if ($f_i > f_j$)
    Replace j by new solution
  end if
  A fraction(p_a) of the worse nests are abandoned and new ones are built;
  Keep the best solutions/nests;
  Rank the solutions/nests and find the current best;
  Pass the current best solutions to the next generation;
end while

The new solution $x_i^{(t+1)}$ of the cuckoo search is generated, from its current location x_i^t and probability of transition, with the following equation

$$x_i^{(t+1)} = x_i^{(t)} + \alpha \oplus \text{Lévy}(\lambda) \tag{6.14}$$

where α, $(\alpha > 0)$ represents a step size and we can use $\alpha = O(L/10)$ where L is the characteristic scale of the problem of interest. This step size should be related to the problem specification and t is the current iteration number. The product $\oplus$ represents entry-wise multiplications as similar to other evolutionary algorithms like PSO but random walk via Lévy flight is much more efficient in exploring the search space as its step length is much longer in the long run.

The Lévy flight essentially provides a random walk whose random step length drawn from a Lévy distribution

$$Lévy \sim u = t^{-\lambda}, (1 < \lambda \leq 3) \tag{6.15}$$

which has an infinite variance with an infinite mean. Here the steps essentially form a random walk process with a power-law step length distribution with a heavy tail.

6.3 Methodology

The present methodology is divided into two folds for analyzing the behavior of an industrial system. In the first fold, optimal design parameters for system performance has been computed by formulating and solving reliability optimization model with EAs. On the other hand, second fold deals with the determination of the various reliability parameters by using the obtained optimal desired parameters—MTBF and MTTR in terms of membership and non-membership functions of IFS.

The following tools are adopted for this purpose, which may give better results (closer to real conditions)

- The reliability optimization model has been constructed for optimal design of systems parameters i.e. MTBF and MTTR by considering reliability, availability and maintainability functions as an objective.
- Sigmoidal membership functions has been used for handling the conflictness between the objectives.
- CS is used for finding the optimal (or near to) values as it always give a global solution as compared to other EAs.
- For increasing the efficiency of the methodology, the weakest t-norm based arithmetic operations has been used for computing the various reliability parameters in terms of membership functions.
- Sensitivity and performance analysis of the components of the system has been addressed for ranking the components as per preferential order for increasing the productivity of the system

The strategy followed through this approach is shown by the flow chart in Fig. 6.2 and both the phases are described as below under the following assumptions.

(i) component failure and repair rates are statistically independent, constant and obey exponential distribution.
(ii) after repair, the repaired component is considered as good as new.
(iii) separate maintenance facility is available for each component
(iv) standby units are of the same nature and capacity as the active unit.
(v) system structure is precisely known.

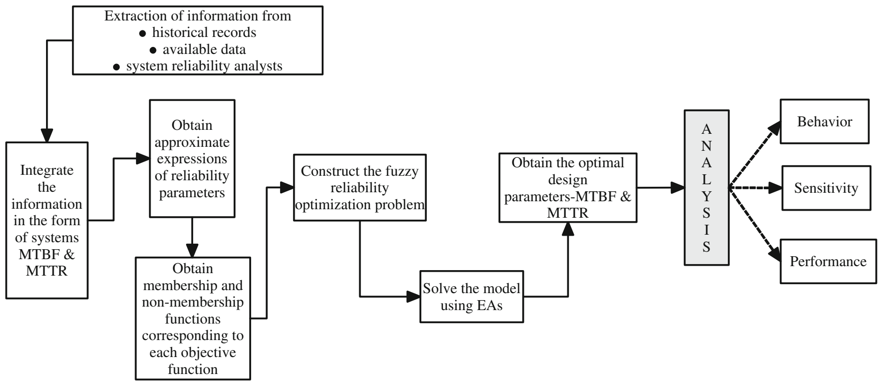

Fig. 6.2 Flow chart of the methodology

6.3.1 Obtaining the Optimal Values of Design Parameters

The main motive of this fold is to compute the design parameters—MTBF and MTTR—of each component of the system so that the design efficiency will be maximized. System reliability, maintainability and availability have assumed great significance in recent years due to a competitive environment and overall operating and production costs. Performance of equipment depends on the reliability and availability of the equipment used, operating environment, maintenance efficiency, operation process and technical expertise of operators, etc. When the reliability and availability of systems are low, efforts are needed to improve them by reducing the failure rate or increasing the repair rate for each component or subsystem. Thus, reliability, availability and maintainability are the important key features for keeping the production and productivity of the system high. The given industrial system is divided into its constituent components and based on the reliability block diagram (RBD), the expressions for the availability, failure rate and repair rates are obtained from Birolini (2007). The basic parameters for series and parallel system are shown in Table 6.1. In this table, λ_i and μ_i represent respectively the failure and repair rates for the ith component of system while λ_s and μ_s represent the same for system's. Av_s and Av_i represent the system and ith component availability. Based on the expressions in Table 6.1, the approximate reliability (R_s), availability (Av_s) and maintainability (M_s) expression for the system can be written as:

$$\mathrm{R}_s = \exp(-\lambda_s t) \tag{6.16}$$

$$\mathrm{Av}_s = f(\mathrm{MTBF}_1, \ldots \mathrm{MTBF}_n, \mathrm{MTTR}_1, \ldots \mathrm{MTTR}_n) \tag{6.17}$$

$$\mathrm{M}_s = 1 - \exp(-\mu_s t) \tag{6.18}$$

The conflict between the objectives (f_t's) are resolved by defining their fuzzy goals corresponding to $f_t(x) \le m_t$ and $f_t(x) \ge M_t$ where m_t and M_t are the lower and upper bound of the objective functions respectively. For defining of this, we make use of the standard logarithm sigmoid function $\psi(a) = \frac{1}{1+e^{-a}}$ and arbitrarily take the

Table 6.1 Basic parameters of availability for series-parallel systems

Type of system	Expression
Series configuration	$\mathrm{Av}_s = \mathrm{Av}_1 \cdot \mathrm{Av}_2 \cdots \mathrm{Av}_n \approx 1 - (\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \cdots + \frac{\lambda_n}{\mu_n})$
	$\lambda_s \approx \lambda_1 + \lambda_2 + \cdots + \lambda_n;\ \mu_s \approx \frac{\lambda_1 + \lambda_2 + \cdots + \lambda_n}{\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} + \cdots + \frac{\lambda_n}{\mu_n}}$
Parallel configuration	$\mathrm{Av}_s \approx 1 - \frac{\lambda_1 \cdot \lambda_2 \cdots \lambda_n}{\mu_1 \cdot \mu_2 \cdots \mu_n}$ $\lambda_s \approx \frac{\lambda_1 \cdot \lambda_2 \cdots \lambda_n (\mu_1 + \mu_2 \cdots \mu_n)}{\mu_1 \cdot \mu_2 \cdots \mu_n};\ \mu_s \approx \mu_1 + \mu_2 + \cdots + \mu_n$

domain of this function as [−5, 5]. The corresponding membership functions are given as (Garg and Sharma 2013)

$$\mu_{f_t}(x) = \begin{cases} 1, & f_t(x) \le m_t \\ \frac{\psi(5)-\psi(\{f_t(x)-\frac{M_t+m_t}{2}\}\delta_t)}{\psi(5)-\psi(-5)}, & m_t \le f_t(x) \le M_t \\ 0, & f_t(x) \ge M_t \end{cases} \tag{6.19}$$

and

$$\mu_{f_t}(x) = \begin{cases} 1, & f_t(x) \ge M_t \\ \frac{\psi(\{f_t(x)-\frac{M_t+m_t}{2}\}\delta_t)-\psi(-5)}{\psi(5)-\psi(-5)}, & m_t \le f_t(x) \le M_t \\ 0, & f_t(x) \le m_t \end{cases} \tag{6.20}$$

where $\delta_t = \frac{10}{M_t - m_t}$. The membership function μ_{f_t} are on the same scale and are discontinuous at the points m_t, f_t, M_t. Here $(M_t + m_t)/2$ is the crossover point of the sigmoidal membership functions.

Using the achieved objective functions of the system, the optimization model is formulated as

$$\begin{aligned} & \textit{Maximize}\ \mu_D = \mu_{R_s} \times \mu_{A_s} \times \mu_{M_s} \\ & \textit{subject to}\ \mathrm{LbMTBF}_i \le \mathrm{MTBF}_i \le \mathrm{UbMTBF}_i \\ & \qquad \mathrm{LbMTTR}_i \le \mathrm{MTTR}_i \le \mathrm{UbMTTR}_i \\ & \qquad i = 1, 2 \ldots n \quad \text{All variables} \ge 0 \end{aligned} \tag{6.21}$$

where LbMTBF_i, UbMTBF_i, LbMTTR_i, UbMTTR_i are respectively the lower and upper bound of MTBF and MTTR for ith component of the system. The optimization model (6.21) thus obtained is solved by the evolution strategies techniques, namely as GA, PSO, ABC and CS.

6.3.2 *Analyzing the Behavior of the System*

In this fold, the optimal values of design parameters, obtained in previous folds/phase are used to calculate the various reliability parameters using weakest t—norm based arithmetic operations on vague lambda-tau methodology, so as to increase the efficiency of the methodology. The procedural steps of the methodology can be described as follows:

Step 1 The technique start with the information extraction phase in which data related to the failure rate and repair time of the main component of the system are collected or extracted from various resources. In the present study, the data related to failure rate and repair time, are obtained using phase Sect. 6.3.1 of the proposed technique

Step 2 To handle the uncertainties or vagueness in the data, the obtained data are converted into intuitionistic triangular vague numbers with some spread as suggested by the DMs on both sides of the data. For instance, the failure rate and repair time for the ith component of the system are converted into ITFNs with $\pm 15\ \%$ spreads are depicted in Fig. 6.3 where $\widetilde{\lambda_{ij}}$ and $\tilde{\tau}_{ij}$ are the vague failure rate and repair time, of component i, with $j = 1, 2, 3$, being lower, middle (crisp) and upper limit of a triangular membership function, respectively. As soon as the input data are represented in the form intuitionistic fuzzy numbers then their corresponding values for their top event of the system are calculated using the extension principle coupled with $\alpha-$ cuts and interval weakest t- norm based arithmetic operations on conventional AND/OR expression, as listed in Table 6.2. The weakest t-norm based interval expression for the triangular vague number, for the failure rate $\tilde{\lambda}$ and repair time $\tilde{\tau}$, for AND/OR-transitions are as follow

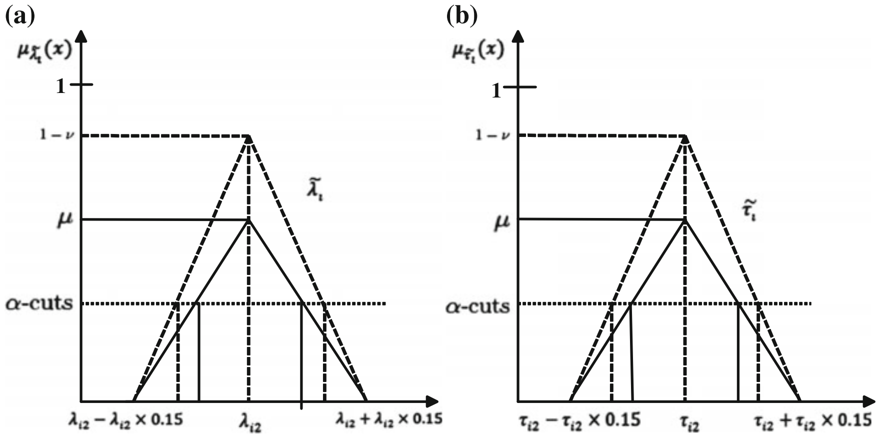

Fig. 6.3 Input intuitionistic triangular fuzzy number. **a** Membership Functions of $\tilde{\lambda}_i$ **b** Membership Functions of $\tilde{\tau}_i$

Table 6.2 Basic expressions of lambda tau methodology

Gate	λ_{AND}	τ_{AND}	λ_{OR}	τ_{OR}
Expression	$\prod_{j=1}^{n} \lambda_j [\sum_{i=1}^{n} \prod_{j=1, i \neq j}^{n} \tau_j]$	$\frac{\prod_{i=1}^{n} \tau_i}{\sum_{j=1}^{n} [\prod_{i=1, i \neq j}^{n} \tau_i]}$	$\sum_{i=1}^{n} \lambda_i$	$\frac{\sum_{i=1}^{n} \lambda_i \tau_i}{\sum_{i=1}^{n} \lambda_i}$

For truth membership functions:

Expressions for AND-Transitions

$$\lambda^{(\alpha)} = \left[\prod_{i=1}^{n} \{ (\lambda_{i2} - \max_{1 \le i \le n} \left((\lambda_{i2} - \lambda_{i1}^{(\alpha)}) \right) \} \cdot \sum_{j=1}^{n} \left[\prod_{\substack{i=1 \\ i \neq j}}^{n} \{ (\tau_{i2} - \max_{1 \le i \le n} \left((\tau_{i2} - \tau_{i1}^{(\alpha)}) \right) \right], \right.$$

$$\left. \prod_{i=1}^{n} \{ \lambda_{i2} + \max_{1 \le i \le n} \left((\lambda_{i3}^{(\alpha)} - \lambda_{i2}) \right) \} \cdot \sum_{j=1}^{n} \left[\prod_{\substack{i=1 \\ i \neq j}}^{n} \{ (\tau_{i2} + \max_{1 \le i \le n} ((\tau_{i3}^{(\alpha)} - \tau_{i2})) \right] \right]$$

$$\tau^{(\alpha)} = \left[\frac{\prod_{i=1}^{n} \{ \tau_{i2} - \max_{1 \le i \le n} \left((\tau_{i2} - \tau_{i1}^{(\alpha)}) \right)}{\sum_{j=1}^{n} [\prod_{\substack{i = 1 \\ i \neq j}}^{n} \{ \tau_{i2} + \max_{1 \le i \le n} \left((\tau_{i3}^{(\alpha)} - \tau_{i2}) \right) \}]}, \frac{\prod_{i=1}^{n} \{ \tau_{i2} + \max_{1 \le i \le n} \left((\tau_{i3}^{(\alpha)} - \tau_{i2}) \right) \}}{\sum_{j=1}^{n} [\prod_{\substack{i = 1 \\ i \neq j}}^{n} \{ (\tau_{i2} - \max_{1 \le i \le n} \left((\tau_{i2} - \tau_{i1}^{(\alpha)}) \right) \}]} \right]$$

Expressions for OR-Transitions

$$\lambda^{(\alpha)} = \left[\sum_{i=1}^{n} \{ \lambda_{i2} - \max_{1 \le i \le n} \left((\lambda_{i2} - \lambda_{i1}^{(\alpha)}) \right) \}, \sum_{i=1}^{n} \{ \lambda_{i2} + \max_{1 \le i \le n} \left((\lambda_{i3}^{(\alpha)} - \lambda_{i2}) \right) \} \right]$$

$$\tau^{(\alpha)} = \left[\frac{\sum_{i=1}^{n} [\{ \lambda_{i2} - \max_{1 \le i \le n} \left((\lambda_{i2} - \lambda_{i1}^{(\alpha)}) \right) \} \cdot \{ \tau_{i2} - \max_{1 \le i \le n} \left((\tau_{i2} - \tau_{i1}^{(\alpha)}) \right) \}]}{\sum_{i=1}^{n} \{ \lambda_{i2} + \max_{1 \le i \le n} \left((\lambda_{i3}^{(\alpha)} - \lambda_{i2}) \right) \}}, \right.$$

$$\left. \frac{\sum_{i=1}^{n} [\{ \lambda_{i2} + \max_{1 \le i \le n} \left((\lambda_{i3}^{(\alpha)} - \lambda_{i2}) \right) \} \cdot \{ \tau_{i2} + \max_{1 \le i \le n} \left((\tau_{i3}^{(\alpha)} - \tau_{i2}) \right) \}]}{\sum_{i=1}^{n} \{ \lambda_{i2} - \max_{1 \le i \le n} \left((\lambda_{i2} - \lambda_{i1}^{(\alpha)}) \right) \}} \right]$$

For false membership functions (i.e. non-membership functions):
Expressions for AND-Transitions

$$\lambda^{(\beta)} = \left[\prod_{i=1}^{n} \{ (\lambda_{i2} - \max_{1 \le i \le n} \left((\lambda_{i2} - \lambda_{i1}^{(\beta)}) \right) \} \cdot \sum_{j=1}^{n} [\prod_{\substack{i=1 \\ i \ne j}}^{n} \{ (\tau_{i2} - \max_{1 \le i \le n} \left((\tau_{i2} - \tau_{i1}^{(\beta)}) \right) \right],$$

$$\left. \prod_{i=1}^{n} \{ \lambda_{i2} + \max_{1 \le i \le n} \left((\lambda_{i3}^{(\beta)} - \lambda_{i2}) \right) \} \cdot \sum_{j=1}^{n} [\prod_{\substack{i=1 \\ i \ne j}}^{n} \{ (\tau_{i2} + \max_{1 \le i \le n} \left((\tau_{i3}^{(\beta)} - \tau_{i2}) \right)] \right]$$

$$\tau^{(\beta)} = \left[\frac{\prod_{i=1}^{n} \{ \tau_{i2} - \max_{1 \le i \le n} \left((\tau_{i2} - \tau_{i1}^{(\beta)}) \right)}{\sum_{j=1}^{n} [\prod_{\substack{i=1 \\ i \ne j}}^{n} \{ \tau_{i2} + \max_{1 \le i \le n} \left((\tau_{i3}^{(\beta)} - \tau_{i2}) \right) \}]}, \frac{\prod_{i=1}^{n} \{ \tau_{i2} + \max_{1 \le i \le n} \left((\tau_{i3}^{(\beta)} - \tau_{i2}) \right) \}}{\sum_{j=1}^{n} [\prod_{\substack{i=1 \\ i \ne j}}^{n} \{ (\tau_{i2} - \max_{1 \le i \le n} \left((\tau_{i2} - \tau_{i1}^{(\beta)}) \right) \}]} \right]$$

Expressions for OR-Transitions

$$\lambda^{(\beta)} = \left[\sum_{i=1}^{n} \{ \lambda_{i2} - \max_{1 \le i \le n} \left((\lambda_{i2} - \lambda_{i1}^{(\beta)}) \right) \}, \sum_{i=1}^{n} \{ \lambda_{i2} + \max_{1 \le i \le n} \left((\lambda_{i3}^{(\beta)} - \lambda_{i2}) \right) \} \right]$$

$$\tau^{(\beta)} = \left[\frac{\sum_{i=1}^{n} [\{ \lambda_{i2} - \max_{1 \le i \le n} \left((\lambda_{i2} - \lambda_{i1}^{(\beta)}) \right) \} \cdot \{ \tau_{i2} - \max_{1 \le i \le n} \left((\tau_{i2} - \tau_{i1}^{(\beta)}) \right) \}]}{\sum_{i=1}^{n} \{ \lambda_{i2} + \max_{1 \le i \le n} \left((\lambda_{i3}^{(\beta)} - \lambda_{i2}) \right) \}}, \right.$$

$$\left. \frac{\sum_{i=1}^{n} [\{ \lambda_{i2} + \max_{1 \le i \le n} \left((\lambda_{i3}^{(\beta)} - \lambda_{i2}) \right) \} \cdot \{ \tau_{i2} + \max_{1 \le i \le n} \left((\tau_{i3}^{(\beta)} - \tau_{i2}) \right) \}]}{\sum_{i=1}^{n} \{ \lambda_{i2} - \max_{1 \le i \le n} \left((\lambda_{i2} - \lambda_{i1}^{(\beta)}) \right) \}} \right]$$

Step 3 In order to analyze the system behavior quantitatively, various reliability parameters such as system failure rate, repair time, MTBF, reliability etc. are analyzed in terms of membership and non-membership functions at various membership grades with an increment of 0.1 confidence level

Step 4 In order to obtain a crisp result from fuzzy output, defuzzification is carried out. In the literature various techniques for defuzzification such as centroid, bisector, middle of the max, weighted average exists. The criterion's for their selection are disambiguated (result in unique value), plausibility (lie approximately in the middle of the area) and computational simplicity (Ross 2004). In the present study, the centroid method is used for defuzzification as it gives mean value of the parameters

6.4 An Illustrative Example

To demonstrate the application of the proposed methodology, a case from a paper mill, situated in the northern part of India is taken which produces approximately 200 tons of paper per day. The paper mills are large capital oriented engineering systems, comprising of various subsystems namely, feeding, pulping, washing, screening, bleaching and paper formulation system, arranged in a predefined configuration (Garg 2013; Garg and Sharma 2012). The present analysis is based on the study of one of the important unit i.e. washing unit whose brief description is as follows.

6.4.1 System Description

The Washing of prepared pulp is done in three to four stages, shown in systematic diagram in Fig. 6.4, to get it free from blackness and to prepare the fine fibers of the pulp. The system consists of four main subsystems, defined as:

- **Filter (A)**: It consists of single unit which is used to drain black liquor from the cooked pulp.
- **Cleaners (B)**: In this subsystem three units of cleaners are arranged in parallel configuration. Each unit may be used to clean the pulp by centrifugal action. Failure of anyone will reduce the efficiency of the system as well as quality of paper.
- **Screeners (C)**: Herein two units of screeners are arranged in series. These are used to remove oversized, uncooked and odd shaped fibers from pulp through straining action. Failure of any one will cause the complete failure of the system.
- **Deckers (D)**: Two units of deckers are arranged in parallel configuration. The function of deckers is to reduce the blackness of pulp. Complete failure of decker occurs when both the components will fail.

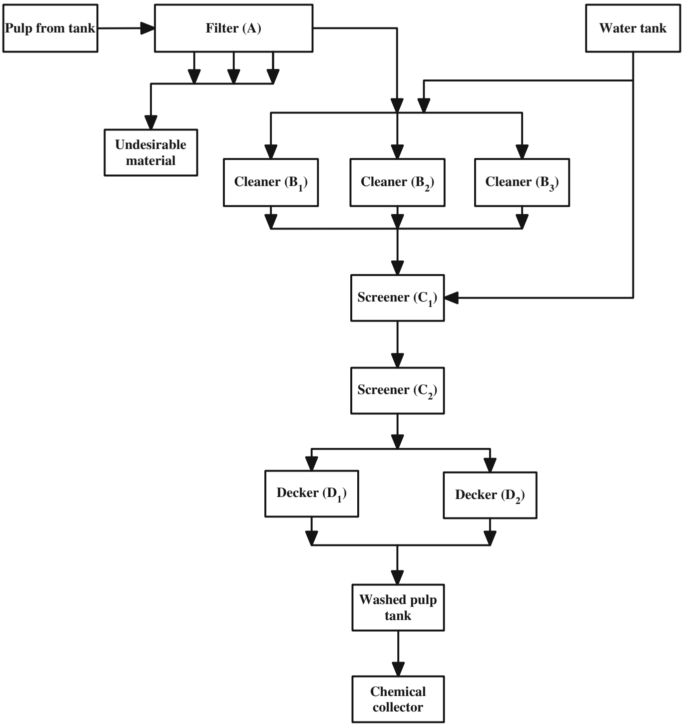

Fig. 6.4 Systematic diagram of the washing system

6.4.2 Formulation of Optimization Model

Let MTBF_i and MTTR_i be the mean time between failures and mean time to repair of the *i*th component of the system then the approximate expressions of system parameters in the form of reliability, availability and maintainability are expressed as below

$$R_s = \exp(-\lambda_s t)$$
$$A_s = 1 - \left[\frac{MTTR_1}{MTBF_1} + \left(\frac{MTTR_2}{MTBF_2}\right)^3 + 2 \cdot \frac{MTTR_3}{MTBF_3} + \left(\frac{MTTR_4}{MTBF_4}\right)^2\right]$$
$$M_s = 1 - \exp(-t/\tau_s)$$

where λ_s and τ_s are given as

$$\lambda_s = \lambda_1 + \lambda_2\lambda_3\lambda_4(\tau_2\tau_3 + \tau_3\tau_4 + \tau_4\tau_2) + \lambda_5 + \lambda_6 + \lambda_7\lambda_8(\tau_7 + \tau_8)$$
$$\tau_s = \frac{\lambda_1\tau_1 + \lambda_2\lambda_3\lambda_4\tau_2\tau_3\tau_4 + \lambda_5\tau_5 + \lambda_6\tau_6 + \lambda_7\lambda_8\tau_7\tau_8}{\lambda_s}$$

Table 6.3 Variance range of MTBF and MTTR of components

Component	MTBF in hrs		MTTR in hrs	
	Lb	Ub	Lb	Ub
Filter	2995	3150	2	4
Cleaners	1850	1950	2	5
Screeners	1880	1920	2	4
Deckers	1860	1910	2	5

As the information collected related to systems' parameter—MTBF and MTTR, are mostly imprecise in nature because these data are collected from various historical records, logbooks etc. which represents the past behavior of the system but unable to represent the future behavior. Thus for handling this issue and to resolve the conflictness between the objective, the membership functions corresponding to objectives are defined by using log-sigmoidal membership functions as given in Eq. (6.20) and hence an optimization model (6.21) is formulated for the considered system. Variance range of the main components' of the system in the form of MTBF and MTTR are summarized in Table 6.3.

6.4.2.1 Parametric Setting

In all algorithms, the values of the common parameters used in each algorithm such as population size and total evaluation number are chosen to be the same. Population size and the maximum evaluation number are taken as $20 \times D$ and 500 respectively for the function, where D is the dimension of the problem. The method has been implemented in Matlab (MathWorks) and the program has been run on a T6400 @ 2 GHz Intel Core (TM) 2 Duo processor with 2 GB of Random Access Memory (RAM). In order to eliminate stochastic discrepancy, 30 independent runs has been made that involves 30 different initial trial solutions. The termination criterion has been set either limited to a maximum number of generations or to the order of relative error equal to 10^{-6}, whichever is achieved first. The other specific parameters of algorithms are given below:

GA Settings: In our experiments, we employed a real coded standard GA having an evaluation, fitness scaling, crossover, mutation units. Single point crossover operation with the rate of 0.85 was employed. Mutation operation restores genetic diversity lost during the application of reproduction and crossover. Mutation rate in our experiments was 0.02.

PSO Settings: Cognitive and social components (c_1 and c_2 in (6.12)) are constants that can be used to change the weighting between personal and population experience, respectively. In our experiments cognitive and the social components were both set to 1.5. Inertia weight (w), which determines how the previous velocity of the particle influences the velocity in the next iteration, was defined as the linear decreases from initial weight $w_{\max} = 0.9$ to final weight $w_{\min} = 0.4$ with the relation $w = w_{\max} - (w_{\max} - w_{\min})(iter/iter_{\max})$. Here iter$_{\max}$ represents the maximum generation number and 'iter' is used a generation number as recommended in Clerc and Kennedy (2002), Shi and Eberhart (1998).

ABC Settings: Except common parameters (population number and maximum evaluation number), the basic ABC used in this study employs only one control parameter, which is called *limit*. A food source will not be exploited anymore and is assumed to be abandoned when *limit* is exceeded for the source. This means that the solution of which "trial number" exceeds the limit value cannot be improved anymore. The *limit* value is defined by using the dimension of the problem and the colony size as (Karaboga and Akay 2009) $limit = SN \times D$, where SN is the number of food sources or employed bees.

CS Settings: Except common parameters, CS employ only one control parameter called probability (p_a) of a host for discovering an alien egg. Here p_a is set to be randomly 0.25 (Yang and Deb 2009).

6.4.2.2 Computational Results

By using these settings, the optimal design parameters for the system performance optimization are obtained and their corresponding results are tabulated in Table 6.4. The estimation of optimal design parameters will generally help the maintenance engineers to understand the behavioral dynamics of the system. However, by using these optimal designs—MTBF and MTTR—results, the plant personnel may change their initial goals so as to reduce the operational and maintenance cost by adopting suitable maintenance strategies from their design results. This methodology will assist the plant managers to carry out design modification, if any, required to achieve minimum failures, and to help in maintenance (repair and replacement actions) decision making.

The statistical simulation results after 30 independent results in terms of values of the mean, best, worst, standard deviation (S.D) and median of the objective functions are obtained by CS algorithm and compared with respect to other algorithms are summarized in Table 6.5. It has also been observed from the table that the S.D. by proposed one are pretty low, and it further implies that the approach seems reliable to solve the reliability optimization problems.

In order to analyze whether the results as obtained in the above tables are statistically significantly with each other or not, we performed t—test on pair of algorithms. For this firstly equality of variances will be tested, since the t—test assumes equality of variances, by using an F—test on the pair of algorithms. For this, two tailed F—test has been performed with significant level of $\alpha = 0.05$ for checking equality of variances of the two results based on their variances values after 30 independent runs. The two-tailed version tests against the alternative that the variances are not equal. Under the null hypothesis, no difference in population variances, the calculated values of F-statistics are 1.227887, 1.189696 and 1.697688 respectively for GA, PSO and ABC when pair with CS. As the critical

Table 6.4 Optimal design parameters for the System

Method → Components ↓	GA		PSO		ABC		CS	
	MTBF	MTTR	MTBF	MTTR	MTBF	MTTR	MTBF	MTTR
Digester	3103.629984	2.153718	3121.631510	2.055150	3104.871834	2.010746	3116.751764	2.026474
Knotter	1910.838263	3.539449	1857.405859	2.363101	1880.633596	2.027245	1938.555352	4.245118
Decker	1899.390049	2.345644	1896.305893	2.003680	1880.054203	2.000236	1906.594880	2.003558
Opener	1895.796417	3.835511	1866.889014	2.123994	1869.688537	2.891751	1865.039620	2.407312
Obj. function	0.9965286		0.9970178		0.9970131		0.9970375	

Table 6.5 Statistics analysis for the optimization problem

Methods	Mean	Best	Worst	Median	SD $(\times 10^{-5})$
GA	0.9963972	0.9965286	0.9961616	0.9963215	3.5109
PSO	0.9969629	0.9970177	0.9969189	0.9969829	3.4017
ABC	0.9969330	0.9970130	0.9968258	0.9969427	4.8542
CS	0.9969831	0.9970375	0.9968965	0.9969861	2.8593

values for testing null hypothesis against the alternative hypothesis at level of significance $\alpha = 0.05$ are given by

$$\begin{aligned} & F > F_{29,29}(\alpha/2) = F_{29,29}(0.025) = 0.475964 \\ \textit{and} \quad & F < F_{29,29}(1-\alpha/2) = F_{29,29}(0.975) = 2.100995 \end{aligned}$$

Since, the calculated value of F-statistics (= 1.227887, 1.189696 and 1.697688) lies between 0.475964 and 2.100995, it is not significant and hence null hypothesis of equality of population variances may be accepted at level of significance $\alpha = 0.05$. Now a single-tail t-test has been performed with the null hypothesis that their mean difference is zero at 5 % significance level in the case of CS results with other results. The results computed are tabulated in Table 6.6 and it indicates that the value of their t-stat is much greater than the t-critical values. Also the p-value obtained during the test is less than the significance level. Thus it is highly significant and null hypothesis i.e. mean of the two algorithms is identical is rejected. Hence the two types of means differ significantly. Further, since mean of the performance function value of the system with CS is greater than others, we conclude that CS is definitely better than others results and this difference is statistically significant.

Table 6.6 t-test for Statistical analysis

	GA	PSO	ABC	CS
Mean	0.99639724	0.9969629	0.9969330	0.9969831
SD	3.5109×10^{-4}	3.4017×10^{-5}	4.8542×10^{-5}	2.8593×10^{-5}
Variance $(\times 10^{-8})$	0.123264	0.115715	0.235632	0.081756
Observation	30	30	30	30
Pooled variance $(\times 10^{-8})$	0.1025101	0.0987357	0.1586942	
Hypothesized mean difference	0	0	0	
Degree of freedom	58	58	58	
t—stat	70.868982	2.4897725	4.8708273	
$P(T \le t)$ one tail	0	0.007835	4.475094×10^{-6}	
T-critical one tail	1.6772241	1.6772241	1.6772241	

6.4.3 Behavior Analysis

The behavior of the system has been analyzed by using the above computed design parameters in the vague set [0.6, 0.8] i.e. degree of acceptance $\mu = 0.6$ and degree of rejection is $v = 1 - 0.8 = 0.2$ so that efficiency of the vague lambda-tau methodology may increase. In this, the computed failure rate and repair time of each of the components are represented in the form of vague triangular numbers with ± 15 % spread and hence various reliability parameters of the system are computed in the form of membership and non-membership functions with the left and right spreads. These behavior plots are shown graphically in Fig. 6.5 along with the existing methodologies results.

(i) The results computed by the traditional or crisp methodology are independent of the uncertainty level α. Hence their results will be suitable only for a system with precise data.
(ii) The results computed by FLT methodology (Knezevic and Odoom 2001) are not that much practical as it contains a wide range of uncertainties during the analysis. Also domain of confidence level is taken to be one and there is a 0° of hesitation between the membership functions.
(iii) The above shortcomings during the analysis has been taken into account by Garg (2013) in their analysis and hence proposed a new technique named as Vague Lambda-Tau methodology (VLTM). In their approach, the domain of confidence level is taken to be ≤ 0.8 instead of one and the intuitionistic fuzzy set theory has been used for representing the uncertainties in the data in the form of membership and non-membership functions. In their approach, interval level uncertainty has been considered with 0.2° of hesitation between the membership functions. However, their results gave more maintenance strategy for the decision maker for increasing the performance of the system as it gives an interval value of a reliability parameter for a particular level of significance (α) as compared to point value. Since in their analysis fuzzy arithmetic operations have been used for computing the system's reliability parameters and hence the level of uncertainties has not been reduced so much.
(iv) The proposed approach provides an improvement over the above shortcoming by considering 0.2° of hesitation between the degree of membership and non-membership functions. In the proposed approach the domain of confidence level is clearly $\alpha \leq 0.8$. The graphical results show that if the uncertainty in input data is described by means of triangular fuzzy numbers, then the possibility distribution of failure rate and repair time is a distorted triangle because after applying the fuzzy operations, the linear sides of triangle changes to parabolic one. These results obtained by weakest t-norm based arithmetic operations on vague set theory are more suitable than the other existing methods. To sustain the analysis for different spreads say ± 15, ± 25 and ± 50 % and to import the results to the system analysts it is necessary that the obtained fuzzy output is converted into crisp value so that decision maker/system analyst may implement these results into the system. For this

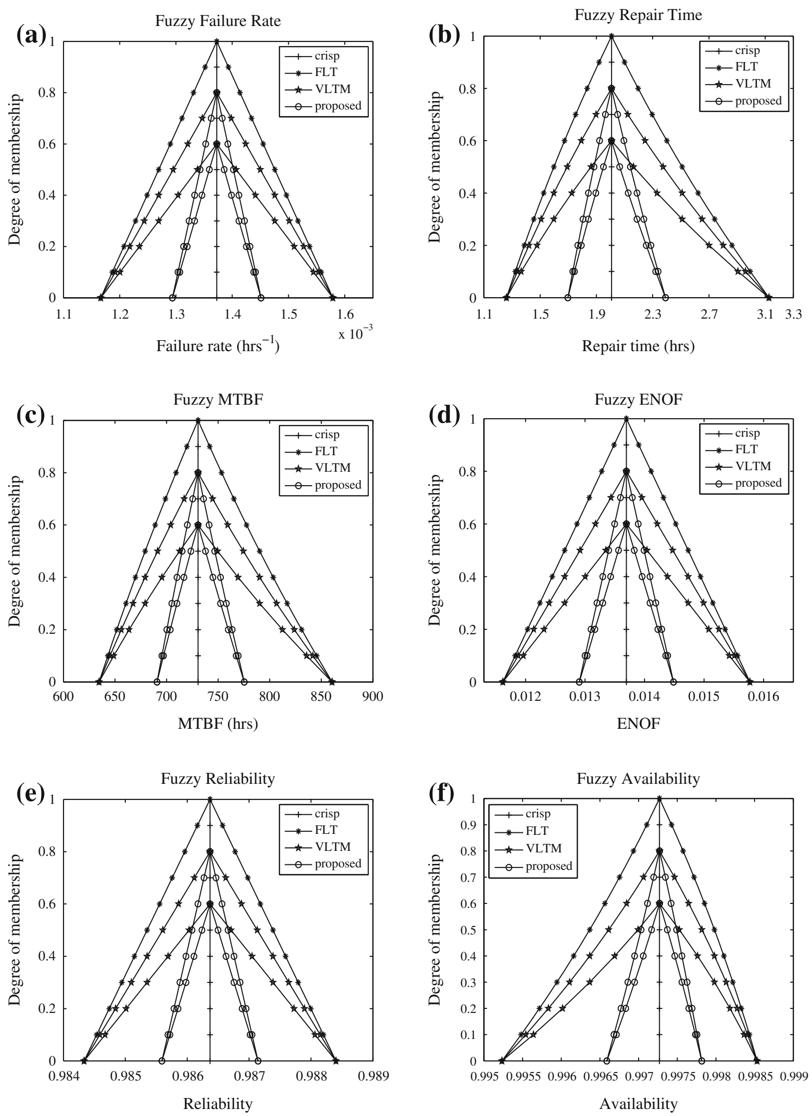

Fig. 6.5 Reliability plots for the system at ±15 % spreads

defuzzification has been done by using the center of gravity method and their corresponding values at different level of uncertainties along with their crisp values are tabulated in Table 6.7. It has been concluded from the table that crisp values do not change with the change of spread while defuzzified values change with change of spreads.

Table 6.7 Defuzzified values of the reliability parameters

Spread	Technique	Failure rate ($\times 10^{-3}$)	Repair time	MTBF	ENOF	Reliability	Availability
	Crisp		0.0081063	30.58174	0.0136952	0.9863683	0.9972699
		Defuzzified values for reliability indices					
±15 %	I	1.3725926	2.1010431	739.06560	0.0136948	0.9863688	0.9970756
	II	A: 1.3725929	2.1016184	739.11816	0.0136948	0.98636890	0.9970745
		B: 1.3725927	2.1012252	739.08223	0.0136948	0.9863688	0.9970753
	III	A: 1.3725448	2.0256469	731.81321	0.0136950	0.9863684	0.9972364
		B: 1.3725444	2.0255711	731.80823	0.0136950	0.9863684	0.9972366
±25 %	I	1.3726777	2.2761652	754.99016	0.0136941	0.9863699	0.9967136
	II	A: 1.3726785	2.2779755	755.15528	0.0136941	0.9863699	0.9967099
		B: 1.3726780	2.2767388	755.04247	0.0136941	0.9863699	0.9967124
	III	A: 1.3726328	2.0592971	734.02191	0.0136947	0.9863687	0.9971736
		B: 1.3726324	2.0590829	734.00783	0.0136947	0.9863687	0.9971740
±50 %	I	1.3730771	3.3173456	848.32954	0.0136898	0.9863747	0.9945536
	II	A: 1.3730803	3.3314687	849.59655	0.0136898	0.98637480	0.9945225
		B: 1.3730781	3.3218515	848.73335	0.0136898	0.9863747	0.9945437
	III	A: 1.3729649	2.2224341	744.68031	0.0136931	0.9863700	0.9968725
		B: 1.3729614	2.2215040	744.61915	0.0136931	0.9863700	0.9968742

I: FLT (Knezevic and Odoom 2001), II: VLTM (Garg 2013), III: proposed approach
A: membership function, B: nonmembership function

6.4.3.1 Sensitivity Analysis

To analyze the impact of the reliability parameters on system MTBF, an analysis has been done in which various combinations of reliability, availability and failure rate parameters has been taken. Throughout the combinations, ranges of repair time and ENOF are fixed and have been varied respectively in the range computed from their membership functions at cut level $\alpha = 0$. For instance, the first three combinations of the reliability parameters states that when reliability and availability of the system has been fixed to 0.9855 and 0.9964 respectively and failure rate are changed from 0.0008 to 0.0013 and further to 0.0018 then the predicted range of the system MTBF has been reduced to 56.7185, 56.7614 and 56.8040 % from Garg (2013) approach when proposed approach has been applied. A similar effect is observed for other combinations too and their ranges are tabulated in Table 6.8. The major advantage of this analysis is that based on their results the system analyst may preserve the particular index and hence seen the effect of taking wrong combinations of the reliability parameters on its MTBF. Also it shows that how the slightest change of failure rate will effect on system MTBF and hence on its performance.

6.4.3.2 Performance Analysis Using RAM-Index

As the time passes then the reliability of the system would gradually decrease if no preventive maintenance action has been taken within a regular interval of time. Thus it is necessary for the system analyst to perform a necessary maintenance action in order to increase the performance of the system. But it is difficult, if not impossible, to find the component from the system on which more attention should be given for saving the money, time and manpower so that the efficiency of the system may increase. For such analysis, a composite measure of the system reliability, availability and maintainability parameter named as the RAM—Index has been used for finding the critical component, as per preferential order, of the system.

The mathematical expression of the RAM-Index is defined as

$$RAM(t) = w_1 \times R_s(t) + w_2 \times A_s(t) + w_3 \times M_s(t) \tag{6.22}$$

where $w_i \in (0, 1), i = 1, 2, 3$ are the weights corresponding to reliability, availability and maintainability respectively such that $\sum_{i=1}^{3} w_i = 1$. Here $w_1 = 0.36$, $w_2 = 30$ and $w_3 = 0.34$ have been used during the analysis. The major advantage of using this index is that by varying the components failure and repair rate parameters, the impact onto the system's performance by the change in its behavior can be analyzed effectively to make the future course of action. Since RAM parameters are represented in the form of membership functions and hence consequently RAM-Index will come as a fuzzy membership function. In order to analyze the system performance, firstly the effect of uncertainties on RAM-Index

Table 6.8 Change in MTBF for various combination of reliability parameters

		(Reliability, failure rate, availability)								
Methods	Range of MTBF	(0.9855, 0.0008, 0.9964)	(0.9855, 0.0013, 0.9964)	(0.9855, 0.0018, 0.9964)	(0.9863, 0.0008, 0.9972)	(0.9863, 0.0013, 0.9972)	(0.9863, 0.0018, 0.9972)	(0.9869, 0.0008, 0.9977)	(0.9869, 0.0013, 0.9977)	(0.9869, 0.0018, 0.9977)
I	Min:	886.06863	545.35771	393.93063	836.80659	515.02378	372.00920	799.89237	492.29557	355.58589
	Max:	1574.22222	969.83379	701.21671	1486.29795	915.48618	661.79206	1420.47933	874.83218	632.32234
II	Min:	1118.23302	688.28734	497.20038	1056.04747	649.98732	469.51615	1009.45175	621.29304	448.77806
	Max:	1416.07603	871.82499	629.93564	1337.23570	823.22040	594.76916	1278.17571	786.82113	568.44131

I: VLTM approach II: proposed approach

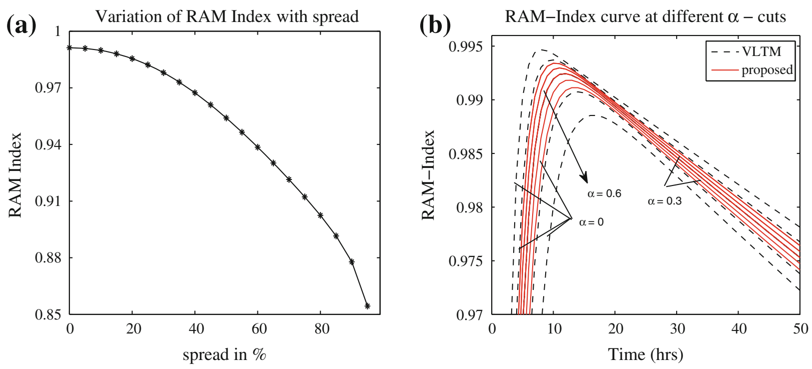

Fig. 6.6 Variation of the RAM-Index plots

has been investigated by varying their spread from 0 to 100 % and their corresponding variation of their index has been plotted in Fig. 6.6a which indicates that RAM-Index decreases with the increase in the uncertainty level. It means to achieve higher performance of the systems, involved uncertainties should be minimized. On the other hand, at different α—cut (0, 0.3, 0.6) the long-run period behavior of the RAM-Index for the system has been shown in Fig. 6.6b which shows that

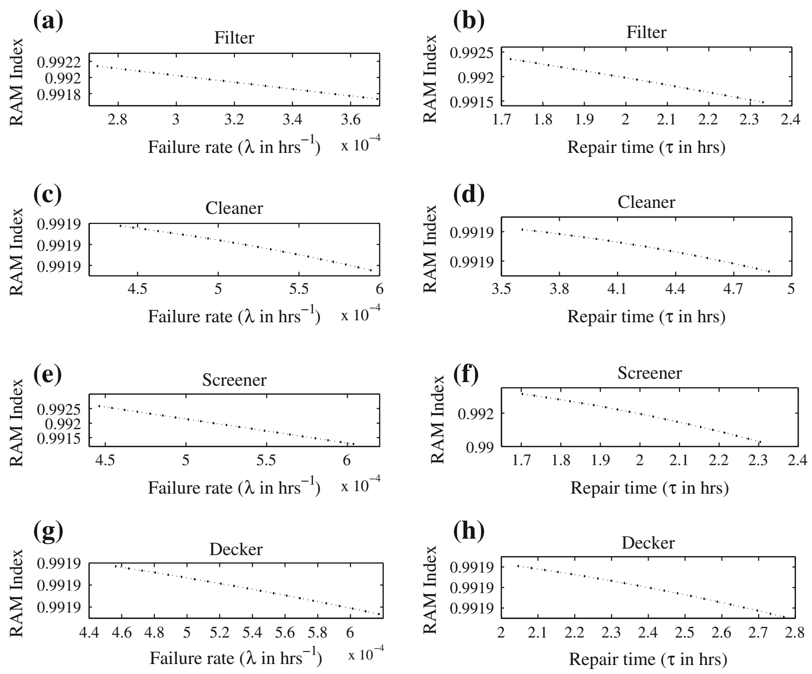

Fig. 6.7 Effect of individual varying components parameters on RAM-Index

RAM-Index of the system increases within the time interval from t = 0 to 13 h and attain its maximum value at t = 13 h in the interval 0.9918217–0.9929697 and after that system performance reduces exponentially. Thus it is found that for increasing the performance of the system, a necessary action should be taken after time t = 13 h.

As the performance of the system is directly depends upon its components and hence the effect on its index has been investigated by varying the failure rate and repair time of each component separately at t = 10 h and simultaneously fixing the other component parameter in Fig. 6.7. In this figure, each plot contains two sub-plots against variations in failure rate and repair time of the each component while their corresponding maximum and minimum values are summarized in Table 6.8. On the other hand, the effect of the simultaneous variations of failure rate and repair time of each component is shown in Fig. 6.8. It may be observed from the Fig. 6.8b that the variation in the failure rate and repair time of the cleaner components shows the significant impact on the performance of the system i.e. an increase in their failure rate from (0.4394331 to 0.5945271) $\times 10^{-3}$ h^{-1} and repair time from 3.60835 to 4.88188 h reduce the system index by 2.6484 %. On the other hand, the variation in the failure rate and repair time of the filter components shows the insignificant impact on the performance of the system. Similar effect on system RAM-Index by the variation of the other component failure rate and repair times is

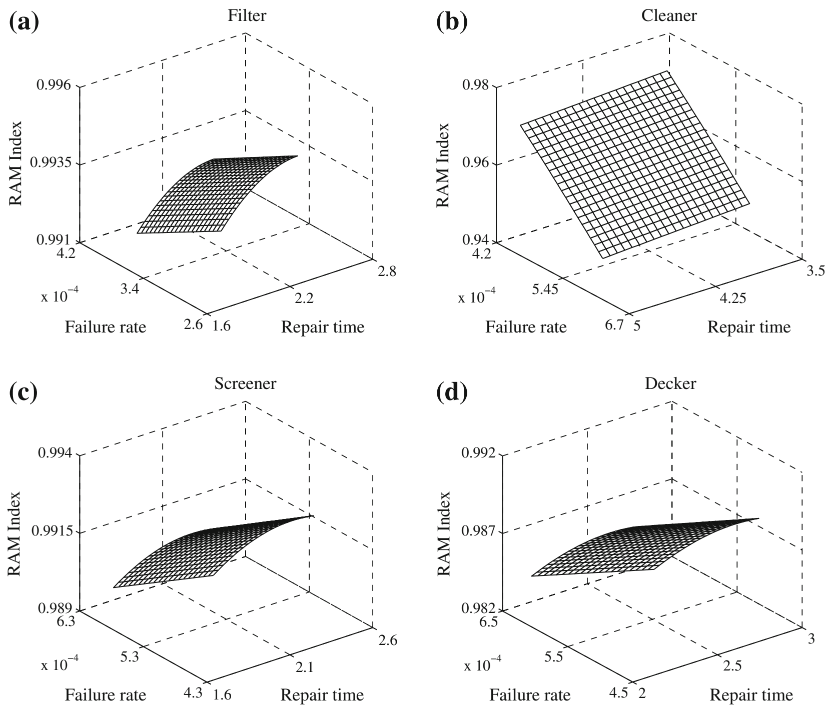

Fig. 6.8 Effect of simultaneously varying components parameters on RAM-Index

Table 6.9 Effect of variations of system's components' failure and repair times on its RAM-Index for washing system

Component	Range of failure rate $\lambda \times 10^{-3}\left(\mathrm{h}^{-1}\right)$	RAM-Index	Range of repair time $\tau(\mathrm{h})$	RAM-Index
Filter	0.2728972–0.3692139	Min: 0.99173248	1.7225029–2.33044510	Min: 0.99147413
		Max: 0.99213980		Max: 0.99235452
Cleaner	0.4394331–0.5945271	Min: 0.99193595	3.6083503–4.8818857	Min: 0.99193595
		Max: 0.99193597		Max: 0.99193598
Screener	0.4462899–0.6038041	Min: 0.99128629	1.7030243–2.3040917	Min: 0.99028515
		Max: 0.99258587		Max: 0.99313700
Decker	0.4563434–0.6174058	Min: 0.99193573	2.0462152–2.7684088	Min: 0.99193456
		Max: 0.99193616		Max: 0.99193703

Table 6.10 Effect of simultaneously variations of system's components' failure and repair times on its RAM-Index for washing system

Component	Range of failure rate $\lambda \times 10^{-3}\left(\mathrm{h}^{-1}\right)$	Range of repair time $\tau(\mathrm{h})$	RAM-Index
Filter	0.2728972–0.3692139	1.7225029–2.33044510	Min: 0.99103063
			Max: 0.99561977
Cleaner	0.4394331–0.5945271	3.6083503–4.8818857	Min: 0.94760573
			Max: 0.97270171
Screener	0.4462899–0.6038041	1.7030243–2.3040917	Min: 0.98920784
			Max: 0.99357250
Decker	0.4563434–0.6174058	2.0462152–2.7684088	Min: 0.98273166
			Max: 0.99170908

analyzed from the Fig. 6.8. The magnitude of the effect of variation in failure rate and repair times of various subsystems of the system on its performance is summarized in Table 6.10. On the basis of results tabulated, it can be analyzed that for improving the performance of the system, more attention should be given to the components as per the preferential order; cleaner, decker, screener and filter.

6.5 Conclusion

This chapter deals with the evaluation of the various reliability parameters of the industrial systems by using uncertain, vague and imprecise data. For this, a structural framework has been developed by the author, based on CS and vague set theory, to model, analyze and predict the system behavior by utilizing quantified, limited and uncertain data. The washing system of the paper industry has been taken as an illustrative example to demonstrate the approach. For this, optimal design parameters—MTBF and MTTR—of the system have been obtained firstly using reliability, availability and maintainability as an objective. The conflicting nature between the objectives is resolved by defining their nonlinear fuzzy goals and then aggregate by using a product aggregator operator. The stability of these optimal parameters is justified by means of pooled t-test statistics. These optimal design parameters will generally help the maintenance engineers to understand the behavioral dynamics of the system and to reallocate the resources. The information system stored the system designs and parameters in the knowledge base and can be retrieved by significant features, which facilitates the designer and increases design efficiency.

Due to complexity in the system configuration, the data obtained from historical records, is imprecise and inaccurate. Keeping this point in view, efficiency for analyzing the behavior of the system is increased by using computed design parameters in terms of membership and non-membership functions using weakest t-norm based arithmetic operations on vague set theory. The development of intuitionistic fuzzy numbers from the available data and using vague possibility theory can greatly increase the relevance of reliability study. The computed results are compared with the existing methodology results and have been observed that the proposed technique has compressed range of uncertainties during the analysis as compared to others and consequently the proposed approach is more flexible for the decision maker to make a more sound and effective decision in a lesser time. The crisp and defuzzified values of various reliability parameters are summarized in a tabular form. Sensitivity as well as performance analysis of the system performance index has been investigated which help the plant personnel to rank the system components. Based on their analysis, the components of the system which has excessive failure rates, long repair times or high degree of uncertainty associated with these values are identified and reported in preferential order as cleaner, decker, screener and a filter.

6.6 Future Research Direction

The present work can be done equally well to evaluate the system behavior in other process industries such as thermal power plant, sugar plant etc. as the considered methodology can overcome various kinds of problem in the area of quality, reliability and maintainability, which strongly needs the management attention. Also we can extend the present work for time varying component failure rate instead of constant rate i.e. from exponential distribution to Weibull or Normal distribution functions. The work can also be extended to devise suitable methodology for [(i)]

(i) Conducting cost analysis.
(ii) Developing inventory and spare parts maintenance management system.
(iii) Redundancy allocation problem.
(iv) suitable maintenance strategies after understanding the behavior dynamics associated with functioning of the system.

Also, the general idea presented here could also be applicable to many other systems like complex, circular, series-parallel, k-out-of-n systems and so on. The investigations on these different systems will be carried out in our future work.

References

Attanassov, K.T.: Intuitionistic fuzzy sets. Fuzzy Sets Syst. **20**, 87–96 (1986)

Attanassov, K.T.: More on intuitionistic fuzzy sets. Fuzzy Sets Syst. **33**(1), 37–46 (1989)

Barabady, J., Kumar, U.: Maintenance schedule by using reliability analysis: a case study at jajarm bauxite mine of iran. In: 20th World Mining Congress and EXPO2005, pp. 79–86. Tehran, Iran (2005a)

Barabady, J., Kumar, U.: Reliability and maintainability analysis of crushing plants in jajarm bauxite mine of iran. In: Proceedings of the Annual Reliability and Maintainability Symposium, pp. 109–115 (2005b)

Birolini, A.: Reliability Engineering: Theory and Practice, 5th edn. Springer, New York (2007)

Bris, R., Chatelet, E., Yalaoui, F.: New method to minimize the preventive maintenance cost of series-parallel systems. Reliab. Eng. Syst. Saf. **82**, 247–255 (2003)

Bustince, H., Burillo, P.: Vague sets are intuitionistic fuzzy sets. Fuzzy Sets Syst. **79**(3), 403–405 (1996)

Chang, J.R., Chang, K.H., Liao, S.H., Cheng, C.H.: The reliability of general vague fault tree analysis on weapon systems fault diagnosis. Soft. Comput. **10**, 531–542 (2006)

Chen, S.M.: Analyzing fuzzy system reliability using vague set theory. Int. J. Appl. Sci. Eng. **1**(1), 82–88 (2003)

Clerc, M., Kennedy, J.F.: The particle swarm: explosion, stability, and convergence in a multi-dimensional complex space. IEEE Trans. Evol. Comput. **6**(1), 58–73 (2002)

Dinesh-Kumar, U., Marquez, J.E.R., Nowicki, D., Verma, D.: A goal programming model for optimizing reliability, maintainability and supportability under performance based logistics. Int. J. Reliab., Qual. Saf. **14**(3), 251–261 (2007)

Eberhart, R., Kennedy, J.: A new optimizer using particle swarm theory. In: Proceedings of the Sixth International Symposium on Micro Machine and Human Science, pp. 39–43 (1995)

Garg, H.: Reliability analysis of repairable systems using Petri nets and Vague Lambda-Tau methodology. ISA Trans. **52**(1), 6–18 (2013)

Garg, H., Rani, M.: An approach for reliability analysis of industrial systems using PSO and IFS technique. ISA Trans. **52**(6), 701–710 (2013)

Garg, H., Rani, M., Sharma, S.P.: Fuzzy RAM analysis of the screening unit in a paper industry by utilizing uncertain data. Int. J. Qual., Stat. Reliab., Article ID: 203,842, 14 p (2012)

Garg, H., Rani, M., Sharma, S.P.: Predicting uncertain behavior of press unit in a paper industry using artificial bee colony and fuzzy Lambda-Tau methodology. Appl. Soft Comput. **13**(4), 1869–1881 (2013)

Garg, H., Rani, M., Sharma, S.P.: An approach for analyzing the reliability of industrial systems using soft computing based technique. Expert Syst. Appl. **41**(2), 489–501 (2014a)

Garg, H., Rani, M., Sharma, S.P., Vishwakarma, Y.: Intuitionistic fuzzy optimization technique for solving multi-objective reliability optimization problems in interval environment. Expert Syst. Appl. **41**, 3157–3167 (2014b)

Garg, H., Sharma, S.P.: Stochastic behavior analysis of industrial systems utilizing uncertain data. ISA Trans. **51**(6), 752–762 (2012)

Garg, H., Sharma, S.P.: A two-phase approach for reliability and maintainability analysis of an industrial system. Int. J. Reliab., Qual. Saf. Eng. (IJRQSE) **19**(3) (2012). doi:10.1142/S0218539312500131

Garg, H., Sharma, S.P.: Multi-objective reliability-redundancy allocation problem using particle swarm optimization. Comput. Ind. Eng. **64**(1), 247–255 (2013)

Garg, H., Sharma, S.P., Rani, M.: Cost minimization of washing unit in a paper mill using artificial bee colony technique. Int. J. Syst. Assur. Eng. Manag. **3**(4), 371–381 (2012)

Gau, W.L., Buehrer, D.J.: Vague sets. IEEE Trans. Syst., Man, Cybern. **23**, 610–613 (1993)

Gen, M., Yun, Y.S.: Soft computing approach for reliability optimization: state-of- the-art survey. Reliab. Eng. Syst. Saf. **91**(9), 1008–1026 (2006)

Goldberg, D.E.: Genetic Algorithm in Search, Optimization and Machine Learning. Addison-Wesley, MA (1989)

Holland, J.H.: Adaptation in Natural and Artificial Systems. The University of Michigan Press, Ann Arbor (1975)

Hsieh, T.J., Yeh, W.C.: Penalty guided bees search for redundancy allocation problems with a mix of components in series–parallel systems. Comput. Oper. Res. **39**(11), 2688–2704 (2012)

Juang, Y.S., Lin, S.S., Kao, H.P.: A knowledge management system for series-parallel availability optimization and design. Expert Syst. Appl. **34**, 181–193 (2008)

Karaboga, D.: An idea based on honey bee swarm for numerical optimization. Tech. rep., TR06, Erciyes University, Engineering Faculty, Computer Engineering Department (2005)

Karaboga, D., Akay, B.: A comparative study of artificial bee colony algorithm. Appl. Math. Comput. **214**(1), 108–132 (2009)

Karaboga, D., Basturk, B.: A powerful and efficient algorithm for numerical function optimization: artificial bee colony (ABC) algorithm. J. Global Optim. **39**, 459–471 (2007)

Kennedy, J., Eberhart, R.C.: Particle swarm optimization. In: IEEE International Conference on Neural Networks, vol. 4, pp. 1942–1948. Piscataway, Seoul (1995)

Khan, F.I., Haddara, M., Krishnasamy, L.: A new methodology for risk-based availability analysis. IEEE Trans. Reliab. **57**(1), 103–112 (2008)

Knezevic, J., Odoom, E.R.: Reliability modeling of repairable systems using Petri nets and Fuzzy Lambda-Tau Methodology. Reliab. Eng. Syst. Saf. **73**(1), 1–17 (2001)

Kumar, A., Yadav, S.P., Kumar, S.: Fuzzy reliability of a marine power plant using interval valued vague sets. Int. J. Appl. Sci. Eng. **4**(1), 71–82 (2006)

Kumar, M., Yadav, S.P.: A novel approach for analyzing fuzzy system reliability using different types of intuitionistic fuzzy failure rates of components. ISA Trans. **51**(2), 288–297 (2012)

Kuo, W., Prasad, V.R., Tillman, F.A., Hwang, C.: Optimal Reliability Design—Fundamentals and Applications. Cambridge University Press, Cambridge (2001)

Liberopoulos, G., Tsarouhas, P.: Systems analysis speeds up chipita's food processing line. Interfaces **32**(3), 62–76 (2002)

Rajpal, P.S., Shishodia, K.S., Sekhon, G.S.: An artificial neural network for modeling reliability, availability and maintainability of a repairable system. Reliab. Eng. Syst. Saf. **91**(7), 809–819 (2006)

Ross, T.J.: Fuzzy Logic with Engineering Applications, 2nd edn. Wiley, New York (2004)

Sharma, R.K., Kumar, S.: Performance modeling in critical engineering systems using RAM analysis. Reliab. Eng. Syst. Saf. **93**(6), 913–919 (2008)

Shi, Y., Eberhart, R.C.: Parameter selection in particle swarm optimization. evolutionary programming VII. In: EP 98, pp. 591–600. Springer, New York (1998)
Taheri, S., Zarei, R.: Bayesian system reliability assessment under the vague environment. Appl. Soft Comput. **11**(2), 1614–1622 (2011)
Yang, X.S., Deb, S.: Cuckoo search via lévy flights. In: Proceedings of World Congress on Nature & Biologically Inspired Computing (NaBIC 2009), pp. 210–214. IEEE Publications, USA (2009)
Yeh, W.C., Hsieh, T.J.: Solving reliability redundancy allocation problems using an artificial bee colony algorithm. Comput. Oper. Res. **38**(11), 1465–1473 (2011)
Yeh, W.C., Su, J.C.P., Hsieh, T.J., Chih, M., Liu, S.L.: Approximate reliability function based on wavelet latin hypercube sampling and bee recurrent neural network. IEEE Trans. Reliab. **60**(2), 404–414 (2011)
Zadeh, L.A.: Fuzzy sets. Inf. Control **8**, 338–353 (1965)
Zavala, A., Diharce, E., Aguirre, A.: Particle evolutionary swarm for design reliability optimization. In: Lecture Notes in Computer Science 3410, Presented at the Third International Conference on Evolutionary Multi-Criterion Optimization, pp. 856–869, Guanajuato (2005)