Chapter 5
Trends on Process Capability Indices in Fuzzy Environment

Abbas Parchami and B. Sadeghpour-Gildeh

Abstract After the fuzzy set theory was introduced and developed, many studies have been realized to combine quality control methods and fuzzy set theory. This chapter is including the categorization of most essential works on fuzzy process capability indices in the following four main categories:

(1) Lee et al.'s method and its extensions: This class deals with the method of modeling and estimating the membership function of process capability indices where all data and specifications are fuzzy numbers;
(2) Parchami et al.'s method and its extensions: This class deals with the problem of obtaining fuzzy process capability indices based on fuzzy specification limits and crisp data by extension principle approach;
(3) Kaya and Kahraman's method and its extensions: This class deals with the problem of estimating the classical process capability indices by a triangular shaped fuzzy number when both specifications and data are crisp;
(4) Yongting's method and its extensions: This class deals with introducing process capability indices based on fuzzy quality where the data and parameters are crisp.

After presenting the basic idea of the main works, all related studies briefly reviewed in each class. Some numerical examples are presented to show the applicability of the proposed methods.

Keywords Confidence interval · Statistical quality control · Fuzzy data · Fuzzy specification limits · Fuzzy event · Fuzzy quality

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5.1 Preliminaries: Process Capability Indices

A common way to measure performance of a manufacturing process is using process capability indices based on a random sample which taken from the production line. In fact, process capability ratio or process capability index (PCI) is a shorthand numerical comparison, which measured the capability and effectiveness of the quality characteristic with respect to the specification limits. In other words, PCI is a statistical measure to calculate the ability of a process to produce output within specification limits. Several PCIs introduced in the literature such as C_p, C_{pk}, C_{pm}, C_{pmk} and so on (Kotz 1993). When univariate measurements concerned, we will denote the corresponding random variable (quality characteristic) by X. Expected value and standard deviation of X will be denoted by μ and σ, respectively. The commonly recognized PCIs are:

$$C_p = \frac{USL - LSL}{6\sigma}, \tag{5.1}$$

where USL and LSL are respectively the upper and lower specification limits. This C_p is used when $\mu = M$ with $M = (U + L)/2$.

$$C_{pk} = \frac{USL - LSL - 2|\mu - M|}{6\sigma} = \frac{\min\{USL - \mu, \mu - LSL\}}{3\sigma}, \tag{5.2}$$

and

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{USL - LSL}{6\sqrt{E[(X - T)^2]}}, \tag{5.3}$$

where T is the target value and $E[.]$ denotes the expected value. There is also the hybrid index

$$C_{pmk} = \frac{USL - LSL - 2|\mu - M|}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{USL - LSL - 2|\mu - M|}{6\sqrt{E[(X - T)^2]}}. \tag{5.4}$$

Usually, $T = M$. If $T \neq M$ the situation sometimes described as "asymmetric tolerances", see (Boyles 1994). Introduction of C_p ascribed to Juran (1974); that of C_{pk} to Kane (1986); that of C_{pm} for the most part to Hsiang and Taguchi (1985), and C_{pmk} to Pearn et al. (1992). Substituting the sample mean and standard deviation provides a point estimate for any of PCIs. For more details on conventional and classical PCIs see (Kotz and Johnson 2002), (Montgomery 2005) and (Kotz and Lovelace 1998).

Although PCIs are effective tools for quality assurance and they have been proposed to provide numerical measures on process capability in a precise environment, but we may confront imprecise concepts in a manufacturing process. If we introduce vagueness into some crisp assumptions (such as data, quality set, specification limits and target value), then we face quite new and interesting processes, where the ordinary capability indices are not appropriate for measuring the capability of these processes. However, classical PCIs extension to fuzzy environment is concomitant with some computational difficulties. Classical statistical quality control is based on crisp data, random variables, control charts, decision rules, capability indices, and so on. As there are many different situations in which the above assumptions are rather unrealistic, there have been some attempts to analyze these situations with the fuzzy set theory. In the present chapter, we try to briefly overview some works on the applications of fuzzy set theory and fuzzy logic for extending process capability indices in quality control researches. For simplify, we classify this overview in four categories that are presented in four Sects. 5.2, 5.3, 5.4 and 5.5. It must be mentioned that the assumptions of theses four categories are different and so they are not comparable.

At the end of this section, we clarify our notation for triangular fuzzy numbers which used through paper. As an especial case of fuzzy numbers, triangular fuzzy number defined by membership function

$$T(x) = \begin{cases} \frac{x-a}{b-a} & if \quad a \leq x < b \\ \frac{c-x}{c-b} & if \quad b \leq x < c \\ 0 & \quad elsewhere \end{cases} \tag{5.5}$$

and it symbolically noted by $T(a, b, c)$. The real number b called the core value and the positive real numbers $b - a$ and $c - b$ called left and right spreads of triangular fuzzy number, respectively. $F_T(R)$ denotes the set of all triangular fuzzy numbers where R is the set of all real numbers.

Four basic methods on generalization of process capability indices for fuzzy environment and their extensions are reviewed and discussed in this paper.

The rest of the paper is organized as follows. An introduction and a brief review of Lee et al.'s (1999) method, Parchami et al.'s (2005) method, Kaya and Kahraman's (2009) method and Yongting's (1996) method are presented in Sects. 5.2, 5.3, 5.4 and 5.5, respectively. The last section concludes the paper and gives suggestions for further research.

5.2 Lee et al.'s Method and Its Extensions

Lee et al. (1999) generalized the capability index C_p by extension principles based on fuzzy specifications and fuzzy data. Under a similar conditions, Lee (2001) follows his approach to generalize capability index C_{pk}. Based on triangular fuzzy observations $\tilde{x}_j = T(o_j, p_j, q_j) \in F_T(R), j = 1, \ldots, n,$ and considering triangular fuzzy target value $\tilde{t} = T(w, y, z) \in F_T(R)$ and also triangular fuzzy specification limits $\widetilde{LSL} = T(l, m, n) \in F_T(R)$ and $\widetilde{USL} = T(o, p, q) \in F_T(R)$, Lee proposed the following approximation for the membership function of C_{pk} index

$$U_{C_{pk}}(I) \simeq \begin{cases} \frac{-B_1}{2A_1} + \left[\left(\frac{B_1}{2A_1}\right)^2 - \frac{C_1 - I}{A_1}\right]^{1/2} & if \quad C_1 \leq I \leq C_3 \\ \frac{B_2}{2A_2} - \left[\left(\frac{B_2}{2A_2}\right)^2 - \frac{C_2 - I}{A_2}\right]^{1/2} & if \quad C_3 \leq I \leq C_2 \\ 0 & elsewhere, \end{cases} \tag{5.6}$$

in which

$$A_1 = (b-a)(e-d)\,, \quad A_2 = (c-b)(f-e),$$
$$B_1 = a(e-d) + d(b-a)\,, \quad B_2 = c(f-e) + f(c-b),$$
$$C_1 = ad\,, \quad C_2 = cf\,, \quad C_3 = be\,,$$

$$a = 1 - \left(\frac{\sum_{j=1}^{n} o_j}{n} - z\right)\left(\frac{2}{q-l}\right),$$
$$b = 1 - \left(\frac{\sum_{j=1}^{n} p_j}{n} - y\right)\left(\frac{2}{p-m}\right),$$
$$c = 1 - \left(\frac{\sum_{j=1}^{n} q_j}{n} - w\right)\left(\frac{2}{o-n}\right),$$
$$d = (o-n)\left(\frac{1}{6C_2}\right),$$
$$e = (p-m)\left(\frac{1}{6C_3}\right),$$
$$f = (q-l)\left(\frac{1}{6C_1}\right).$$

After computing the membership function of fuzzy PCI, he fuzzified the proposed fuzzy PCI for making final decision in the examined manufacturing process. The major advantage of the proposed method is using extension principle approach. Complex calculations, low speed of process and presenting non-exact approximates for capability indices are weakness points of Lee's method which cause increasing the progress of the proposed method.

A similar approach to solve this problem based on extension principle presented by Shu and Wu (2009) by fuzzy data. In their approach, which is easier and fasten than Lee's method, the α-cuts of fuzzy index C_{pk} was calculated based on the α-cuts of fuzzy data for $0 \leq \alpha \leq 1$. Meanwhile, they investigated on the capability of the LCD monitors assembly line using their generalized indices. In this regard, the capability test on the generalized capability index C_p with fuzzy data have been investigated by Tsai and Chen (2006).

5.3 Parchami et al.'s Method and Its Extensions

A process with fuzzy specification limits, which Parchami et al. (2005) called a fuzzy process for short, is one which approximately satisfies the normal distribution condition and its specification limits are fuzzy. They extend the classical PCIs (1-4) by extension principle for fuzzy processes as follows

$$\tilde{C}_p = T\left(\frac{a_u - c_l}{6\sigma}, \frac{b_u - b_l}{6\sigma}, \frac{c_u - a_l}{6\sigma}\right), \tag{5.7}$$

$$\tilde{C}_{pk} = T\left(\frac{a_u - c_l - 2|\mu - m|}{6\sigma}, \frac{b_u - b_l - 2|\mu - m|}{6\sigma}, \frac{c_u - a_l - 2|\mu - m|}{6\sigma}\right) \tag{5.8}$$

$$\tilde{C}_{pm} = T\left(\frac{a_u - c_l}{6\sqrt{\sigma^2 + (\mu - t)^2}}, \frac{b_u - b_l}{6\sqrt{\sigma^2 + (\mu - t)^2}}, \frac{c_u - a_l}{6\sqrt{\sigma^2 + (\mu - t)^2}}\right) \tag{5.9}$$

and

$$\tilde{C}_{pmk} = T\left(\frac{a_u - c_l - 2|\mu - m|}{6\sqrt{\sigma^2 + (\mu - t)^2}}, \frac{b_u - b_l - 2|\mu - m|}{6\sqrt{\sigma^2 + (\mu - t)^2}}, \frac{c_u - a_l - 2|\mu - m|}{6\sqrt{\sigma^2 + (\mu - t)^2}}\right), \tag{5.10}$$

where t is target value, $m = (b_u + b_l)/2$, $a_u \geq c_l$ and the fuzzy numbers $U(a_u, b_u, c_u) = T(a_u, b_u, c_u) \in F_T(R)$ and $L(a_l, b_l, c_l) = T(a_l, b_l, c_l) \in F_T(R)$ are the upper and lower engineering specification limits, respectively. It is obvious that the proposed fuzzy indices $\tilde{C}_p$, $\tilde{C}_{pk}$, $\tilde{C}_{pm}$, $\tilde{C}_{pmk}$ are exactly triangular fuzzy numbers and they are applied when the data are crisp and specification limits are two triangular fuzzy numbers.

Example 1 For a special product suppose that the lower and upper specification limits are considered to be "approximately 4" and "approximately 8", which are characterized by $L(2,4,6) = T(2,4,6) \in F_T(R)$ and $U(7,8,9) = T(7,8,9) \in F_T(R)$; respectively (see the left graph of Fig. 5.1). Assume that the process mean and the process standard deviation are 6 and $\frac{2}{3}$, respectively. Also, let the target value be equal to 7. By Eq. (5.7) one can easily calculate the fuzzy index $\tilde{C}_p = T\left(\frac{1}{4}, 1, \frac{7}{4}\right)$. Therefore, $\tilde{C}_p$ is "approximately one", as shown in the right graph of Fig. 5.1. Also by Eq. (5.9), one can similarly calculate the fuzzy index $\tilde{C}_{pm} = T\left(\frac{1}{2\sqrt{13}}, \frac{2}{\sqrt{13}}, \frac{7}{2\sqrt{13}}\right)$. Moreover, considering Eqs. (5.7–5.10), we can expect that $\tilde{C}_{pk} = \tilde{C}_p = T(0.25, 1, 1.75)$ and $\tilde{C}_{pmk} = \tilde{C}_{pm} = T(0.139, 0.555, 0.971)$ which are drawn in the right graph of Fig. 5.1, since $m = \mu$.

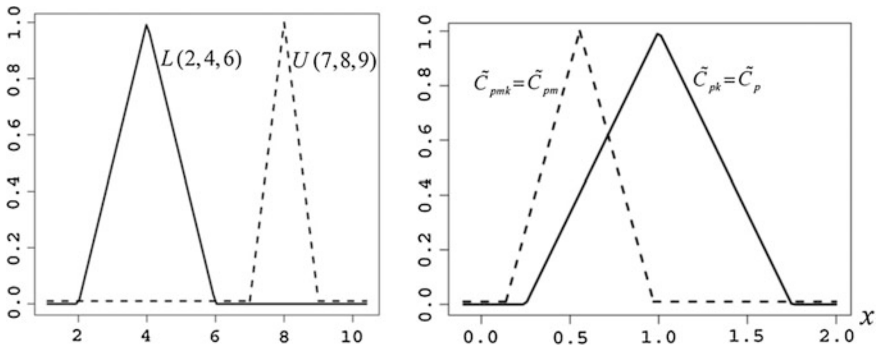

Fig. 5.1 The membership functions of fuzzy specification limits (*left graph*) and the membership functions of fuzzy process capability indices based on Parchami et al.'s (2005) method (*right graph*) in Example 1

In recent years, some papers have been concentrated on different statistical fields of fuzzy process which we briefly review them. Moeti et al. (2006) introduced the fuzzy process capability indices based on *LR* specification limits. Ramezani et al. (2011) and Parchami et al. (2006), (2011) constructed several fuzzy confidence regions for fuzzy PCIs $\tilde{C}_p$ and $\tilde{C}_{pm}$, respectively. Testing the capability of fuzzy processes are investigated by Parchami and Mashinchi (2009) where specification limits are triangular fuzzy numbers. Extending other classical and conventional PCIs are followed by Kaya and Kahraman (2010) based on this method. Also, after extending this method by Kaya and Kahraman (2008) for trapezoidal fuzzy specification limits, they applied their extended PCIs to compare several educational and teaching processes (also see Mashinchi et al. 2005).

5.4 Kaya and Kahraman's Method and Its Extensions

An another prevalent method for PCIs estimation is constructed on the basis of Buckley's estimation approach. Buckley (2004), (2006) propose a general estimation approach to estimate any unknown parameter by a triangular shaped fuzzy number whose α-cuts are equal to the $100(1-\alpha)\%$ confidence intervals of the parameter. Recently, several authors used Buckley's estimation approach to PCIs estimation by a triangular shaped fuzzy number when both specifications and data are crisp. Parchami and Mashinchi (2007) estimated classical PCIs C_p, C_{pk} and C_{pm} by Buckley's approach and they proposed a method for the comparison of the estimated PCIs. For instance in their approach, the α-cut of the fuzzy estimation for C_p is equivalent to

$$\left[\hat{C}_p\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}},\ \hat{C}_p\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right],\quad 0<\alpha<1, \tag{5.11}$$

in which $\hat{C}_p = \frac{USL-LSL}{6s}$ is the point estimation of C_p and $\chi^2_{n,\alpha}$ is the α-quantile of Chi-square distribution with n degrees of freedom. So, the proposed estimations for PCIs contain both point and interval estimates and so provide more information for the practitioner. Kahraman and Kaya (2009) introduced fuzzy PCIs for quality control of irrigation water. Wu (2009) proposed an approach for testing process performance C_{pk} based on Buckley's estimator with crisp data and crisp specification limits. Also, after introducing Buckley's fuzzy estimation for capability index, Wu and Liao (2009) investigated on testing process yield assuming fuzzy critical value and fuzzy p-value. It must be clarified that both data and specification limits have considered crisp in two recent works and the presented concepts are also illustrated in a case study on the light emitting diodes manufacturing process. In this regard, Kaya and Kahraman (2009) introduced fuzzy robust capability indices and they evaluated the air pollution's Istanbul by their fuzzy PCIs. For instance, the α-cut of the presented fuzzy estimation in Eq. (5.11) modified in their method as follows

$$\left[\hat{C}_p\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}} + \left(\hat{C}_p - \hat{C}_p\sqrt{\frac{\chi^2_{n-1,0.5}}{n-1}}\right),\right. \\ \left.\hat{C}_p\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}} + \left(\hat{C}_p - \hat{C}_p\sqrt{\frac{\chi^2_{n-1,0.5}}{n-1}}\right)\right],\qquad 0<\alpha\le 1. \tag{5.12}$$

Example 2 Suppose that the lower and upper specification limits for a product are $L = 4$ and $U = 8$, respectively. By assuming $\mu = 6$, we take a random sample $X_1, X_2, \cdots, X_{41}$ from $N(6, \sigma^2)$ to estimate index C_p and assume that the estimated process standard deviation is $2/3$. According to Eq. (5.11), the α-cut of Parchami and Mashinchi's (2007) fuzzy estimation for C_p is equivalent to

$$\left[\sqrt{\frac{\chi^2_{40,\alpha/2}}{40}},\sqrt{\frac{\chi^2_{40,1-\alpha/2}}{40}}\right],\quad 0<\alpha<1, \tag{5.13}$$

in which $\hat{C}_p = \frac{8-4}{6\times\frac{2}{3}} = 1$ is computed. The graph of the membership function of Parchami and Mashinchi's (2007) fuzzy estimation is drawn in Fig. 5.2 by line. We would never expect the classical precise point estimate $\hat{C}_p = 1$ to be exactly equal to the parameter value, so we often compute a $(1-\alpha)100\%$ confidence interval for C_p. The fuzzy estimate obtained by this approach contains more information than a point

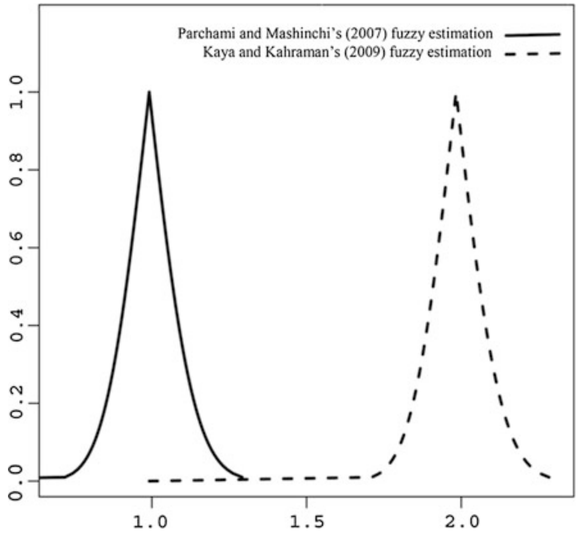

Fig. 5.2 The membership functions of fuzzy estimate for capability index by Parchami and Mashinchi (2007) and Kaya and Kahraman's (2009) methods in Example 2

or interval estimate, in the sense that the fuzzy estimate contains the point estimate and all $(1-\alpha)100\%$ confidence intervals all at once for $0<\alpha<1$, which can be useful from practical point of view. From Parchami and Mashinchi's (2007) fuzzy estimate, one can conclude that the classical crisp estimate $\hat{C}_p = 1$ belongs to their fuzzy estimate with grade of membership one. It is obvious that their fuzzy set contains more elements other than "1" with corresponding grades of membership. For example, one can says that $\hat{C}_p = 0.946$ belongs to their fuzzy estimate with grade of membership 0.68. Meanwhile according to Eq. (5.12), the α-cut of the modified Kaya and Kahraman's (2009) fuzzy estimation can be calculated as follows

$$\left[\sqrt{\frac{\chi^2_{40,\alpha/2}}{40}}+\left(1-\sqrt{\frac{\chi^2_{40,0.5}}{40}}\right),\sqrt{\frac{\chi^2_{40,1-\alpha/2}}{40}}+\left(1-\sqrt{\frac{\chi^2_{40,0.5}}{40}}\right)\right], \quad 0<\alpha\le 1, \tag{5.14}$$

where $\chi^2_{40,0.5} = 39.34$ is the median of Chi-square distribution with 40 degrees of freedom. The graph of the membership function of Kaya and Kahraman's (2009) fuzzy estimation is drawn in Fig. 5.2 by dash line.

As another work on this topic, Hsu and Shu (2008) studied on fuzzy estimation of capability index C_{pm} to assess manufacturing process capability with imprecise data. Kaya and Kahraman (2011b) estimated classical capability indices via triangular shaped fuzzy numbers by replacing Buckley's fuzzy estimations of process mean and process standard deviation. Analyzing fuzzy PCIs followed by Kaya and Kahraman (2011a) based on fuzzy measurements and also they drawn fuzzy control charts for fuzzy measurements. Moradi and Sadeghpour-Gildeh (2013) worked on fuzzy one-sided process capability plots for the family of one-sided specification limits.

5.5 Yongting's Method and Its Extensions

Yongting (1996), for the first time, defines fuzzy quality by substituting the indicator function $I_{\{x|\,x\in[LSL,USL]\}}$ with the membership function of the fuzzy set $\tilde{Q}$, where the membership function $\tilde{Q}(x)$ represents the degree of conformity of the measured quality characteristic with standard quality (or briefly, the degree of quality). Note that by using fuzzy quality idea, the range of quality characteristic function will be changed from $\{0, 1\}$ into $[0, 1]$, see Fig. 5.3.

Also, Yongting (1996) introduced the capability index

$$C_{\tilde{p}(Y)} = \begin{cases} \int_{-\infty}^{+\infty} \widetilde{Q}(x) f(x) dx & \text{continuous random variable} \\ \sum_{i=1}^{N} \widetilde{Q}(x_i) P(x_i) & \text{discrete random variable} \end{cases} \tag{5.15}$$

based on fuzzy quality for precise data in which f and P are p.d.f. and p.m.f. of the quality characteristic, respectively. Sadeghpour-Gildeh (2003) compared capability indices C_p, C_{pk} and $C_{\tilde{p}(Y)}$ with respect to the measurement error occurrence.

Example 3 Suppose that a random sample is taken from an assembly line of a special product under the normality assumption. The mean and standard deviation of the observed data are $\bar{x} = 0.7$ and $s = 0.15$, respectively. First, we consider a non-symmetric triangular fuzzy quality with the following membership function for product (see Fig. 5.4)

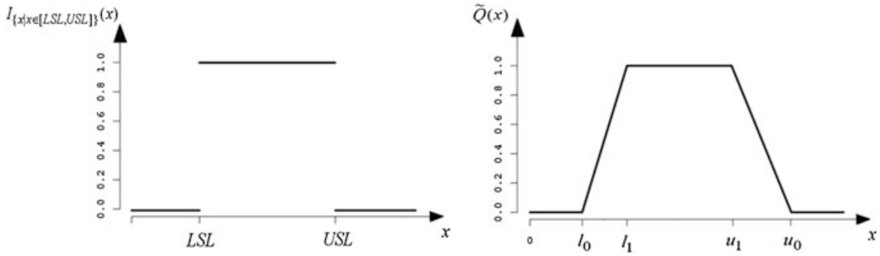

Fig. 5.3 Characterized classical quality with the indicator function of non-defective products (*left figure*), and characterized fuzzy quality with the fuzzy set of non-defective products (*right figure*)

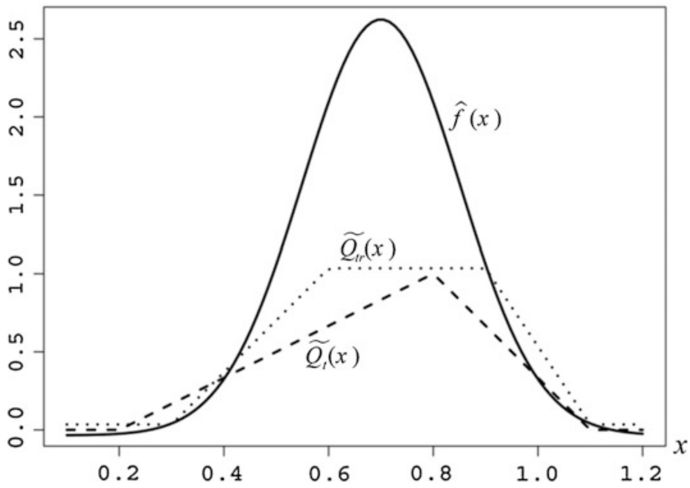

Fig. 5.4 The membership functions of triangular and trapezoidal fuzzy qualities and the estimated probability density function of the quality characteristic in Example 3

$$\widetilde{Q}_t(x) = \begin{cases} \frac{x-0.2}{0.6} & if \quad 0.2 \le x < 0.8 \\ \frac{1.1-x}{0.3} & if \quad 0.8 \le x < 1.1 \\ 0 & \quad elsewhere. \end{cases}$$

In this situation, one can estimate Yongting's capability index by Eq. (5.15) as follow

$$\begin{aligned} \widehat{C_{\tilde{p}\ (Y)}} &= \int_{-\infty}^{+\infty} \widetilde{Q}_t(x)\widehat{f}(x)dx \\ &= \frac{1}{\sqrt{2\pi}s} \int_{-\infty}^{+\infty} \widetilde{Q}_t(x) \exp\left(-\frac{(x-\overline{x})^2}{2s^2}\right)dx \\ &= \frac{1}{0.15\sqrt{2\pi}}\left[\int_{0.2}^{0.8} \frac{x-0.2}{0.6} \exp\left(-\frac{(x-0.7)^2}{2\times 0.15^2}\right)dx \right. \\ &\quad \left. + \int_{0.8}^{1.1} \frac{1.1-x}{0.3} \exp\left(-\frac{(x-0.7)^2}{2\times 0.15^2}\right)dx\right] \\ &= 0.543 + 0.178 = 0.721. \end{aligned}$$

Now, let us to consider a trapezoidal fuzzy quality with the following membership function for this product

$$\widetilde{Q}_{tr}(x) = \begin{cases} \frac{x-0.3}{0.3} & if \quad 0.3 \le x < 0.6 \\ 1 & if \quad 0.6 \le x < 0.9 \\ \frac{1.1-x}{0.2} & if \quad 0.9 \le x < 1.1 \\ 0 & elsewhere. \end{cases}$$

Similarly, one can estimate Yongting's capability index as follow

$$\begin{aligned} \widehat{C_{\tilde{p}\ (Y)}} &= \frac{1}{\sqrt{2\pi}s} \int_{-\infty}^{+\infty} \widetilde{Q_{tr}}(x) \exp\left(-\frac{(x-\overline{x})^2}{2s^2}\right) dx \\ &= \frac{1}{0.15\sqrt{2\pi}} \left[\int_{0.3}^{0.6} \frac{x-0.3}{0.3} \exp\left(-\frac{(x-0.7)^2}{2\times 0.15^2}\right) dx \right. \\ &\quad \left. + \int_{0.6}^{0.9} \exp\left(-\frac{(x-0.7)^2}{2\times 0.15^2}\right) dx + \int_{0.9}^{1.1} \frac{1.1-x}{0.2} \exp\left(-\frac{(x-0.7)^2}{2\times 0.15^2}\right) dx \right] \\ &= 0.178 + 0.656 + 0.060 = 0.894. \end{aligned}$$

Therefore, the process is more capable under considering the trapezoidal fuzzy quality $\widetilde{Q}_{tr}$ with respect to considering the triangular fuzzy quality $\widetilde{Q}_t$.

Amirzadeh et al. (2009) constructed a new control chart based on Yongting's fuzzy quality, and meanwhile they shown that the developed control chart has a better response to variations in both the mean and the variance of the process. Parchami and Mashinchi (2010) proved that Yongting's introduced PCI is an extension for the probability of the product is qualified. Therefore, his capability index is not a suitable extension for C_p index, since C_p is not a probability and is not always in [0,1]. Then, Parchami and Mashinchi (2010) presented the revised version of Yongting's fuzzy quality on the basis of two fuzzy specification limits $\widetilde{LSL}$ and $\widetilde{USL}$ which are able to characterize two non-precise concepts of "approximately bigger than" and "approximately smaller than" in a fuzzy process, respectively. An instance for fuzzy quality is depicted in Fig. 5.6 in which the fuzzy quality is characterized by two membership functions of fuzzy specification limits $\widetilde{LSL}$ and $\widetilde{USL}$. Figure 5.5 is shown as an instance of the classical quality by characterizing two indicator functions $I_{\{x|x \ge LSL\}}$ and $I_{\{x|x \le USL\}}$. Note that equation $\widetilde{Q} = \widetilde{USL} \cap \widetilde{LSL}$, or equivalently $\widetilde{Q}(x) = \min\{\widetilde{USL}(x), \widetilde{LSL}(x)\}$, presents the governed relation between membership functions of fuzzy specification limits in Fig. 5.6 and the membership function of Yongting's fuzzy quality in the right graph

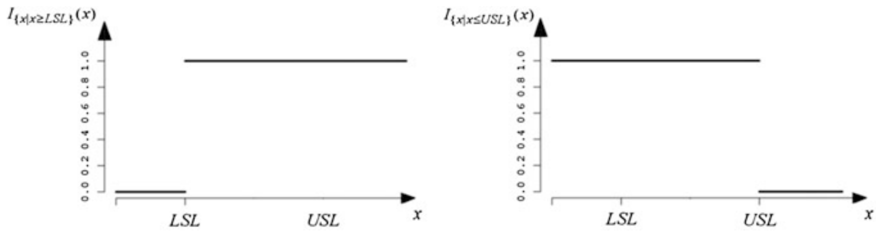

Fig. 5.5 Characterized classical quality with two indicator functions of "bigger than *USL*" (*left figure*) and "smaller than *USL*" (*right figure*)

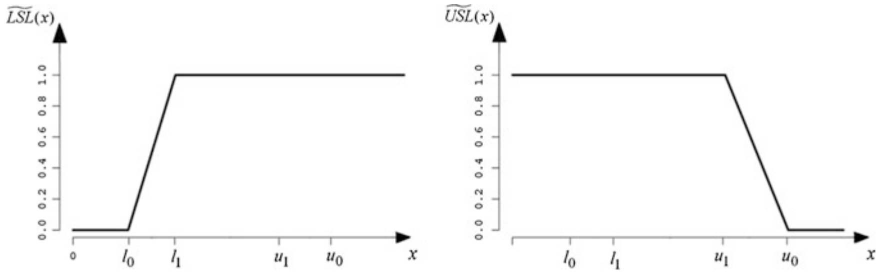

Fig. 5.6 Characterized fuzzy quality with two membership functions of "approximately bigger than" (*left figure*) and "approximately smaller than" (*right figure*)

of Fig. 5.3. Similarly, $I_{\{x|\,x\in[LSL,USL]\}}(x) = \min\{I_{\{x|\,x\leq USL\}}(x)\,, I_{\{x|\,x\geq LSL\}}(x)\}$ presents the relation between depicted indicator functions in Fig. 5.5 and depicted indicator function of classical quality $[LSL\,,\,USL]$ in the left graph of Fig. 5.3.

Motivations and merits of using fuzzy quality by considering fuzzy specification limits $\widetilde{LSL}$ and $\widetilde{USL}$, instead of applying classical quality are discussed in Parchami et al. (2014a). Parchami and Mashinchi (2010) introduced an extended version for traditional PCIs to present an alternative approach to measure the capability based on two new revised fuzzy specification limits. Their extended PCIs are used to give a numerical measure about whether a production method is capable of producing items within the fuzzy specification limits $\widetilde{LSL}$ and $\widetilde{USL}$. This new idea, provides a new methodology for measuring the fuzzy quality and also constructing confidence intervals for various PCIs, for example see Parchami and Mashinchi (2011). As an extended version of Yongting's index, Sadeghpour-Gildeh and Moradi (2012) proposed a general multivariate PCI based on fuzzy tolerance region which has not some of restriction of conventional PCIs. Another generalized version of the classical PCIs (1-4) is introduced by Parchami et al. (2014b) to measure the capability of a fuzzy-valued process in producing products based on fuzzy quality.

5.6 Conclusions and Future Research Directions

Traditional process capability indices are based on crispness of data, parameters, lower and upper specification limits, target value, and so on. As there are many different situations in which the above assumptions are rather unjustified and unrealistic. This chapter is including the classification of most essential researches on process capability indices extension for applying in fuzzy environment. After presenting the basic idea of the main works, all related studies briefly overviewed in each category. Also, some numerical examples are investigated to show how the proposed methods can be implemented in real-world cases. Some potential subjects for further research are presented in follow: (1) extending other non-extended capability indices in each category; (2) study on alternative approaches for point and interval estimation of the extended capability indices for each category; (3) construct statistical testing fuzzy quality based on Bayes, Minimax, Neyman–Pearson, Likelihood ratio, sequential and *p*-value approaches; (4) applying the extended capability indices and the presented methods in real-world cases.

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