Chapter 13
From Elementary Pragmatic Model (EPM) to Evolutive Elementary Pragmatic Model (E^2PM)

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1 Introduction

The Elementary Pragmatic Model (EPM) was developed in the 1970s [9, 31, 37], following Gregory Bateson's constructivist participant observer concept in the "second order cybernetics", to arrive to what was called "new cybernetics", according to cybernetics classical historical categorization. EPM is a high operative and didactic, versatile tool and new application areas are envisaged continuously [14]. Quite recently, EPM intrinsic Self-Reflexive Functional Logical Closure contributed to find an original solution to the dreadful double-bind problem in classic information and algorithmic theory (i.e. our contemporary systemic tools and classic information computational and communication algorithms are totally unable to discriminate the difference between an optimal encoded information-rich message and a random jumble of signs that we call "noise" usually) [13, 15]. In turn, this new awareness has allowed to enlarge our panorama for neurocognitive system behavior understanding, and to develop information conservation and regeneration systems in a numeric self-reflexive/reflective evolutive reference framework [13]. Accordingly, new methods and models to build effective applications and strategies, from new forms of inter- and trans-disciplinarity, can be conceived conveniently. Rational human thinking is like a solid archipelago emerging out of an ocean of unaware intuitions. Human brain is an harmonization machine fed by unaware intuitions to produce learning and rational awareness about our environment. Our "Eulogic Thought" emerges out of

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G. Minati et al. (eds.), *Towards a Post-Bertalanffy Systemics*, Contemporary Systems Thinking, DOI 10.1007/978-3-319-24391-7_13

a continuous harmonization interaction between Paleologic and Neologic Thought components. “Emotional Intelligence” (EI) and “Emotional Creativity” (EC) [19] coexist at the same time with “Rational Thinking”, (RT) sharing the same input environment information. Different forms of knowledge representations inducing a non-deductive procedure during inferences exist. For instance, the metaphor is a representation modality that bypasses deductive procedure. The source of the metaphor brings a structure exhibiting explicit and implicit attributes of the target. The metaphor allows direct inferences and learning about new things (target) by extending what it is already known about the source [1, 2, 24]. Similarly, the process of hierarchical inheritance in semantics networks allows direct inferences on properties shared by classes, subclasses and instances of objects without effort of explicit deduction [6]. Another form of direct inference is heuristics that take advantage of specialized knowledge to permit inferential short-cuts [4, 18]. Mental imagery also provides with detailed representation of objects that conveys implicit information without deductive effort [11]. In logic, diagrammatic notation is the main form of knowledge representation that excludes a deductive procedure through the process of spatial inference. EPM provides us with a reliable closed logic starting scheme to face “unknown known” situations [16, 17]. If we like to use it on more and more complex applications with ability to capture natural emergent phenomena dynamics, we need to extend it to be able to face even unpredictable perturbation (“unknown unknown”) at design level [16, 17]. In this case, an evolutive structure to manage unexpected dynamics is needed to EPM. To cope with ontological uncertainty effectively at system level, it is possible to use two coupled irreducible information management subsystems, based on the following ideal coupled irreducible asymptotic dichotomy: Information Reliable Predictability and Information Reliable Unpredictability. In this way, to behave realistically, overall system must guarantee both Logical Closure (Reactive Information Management, “to learn and prosper”) and Logical Aperture (Proactive Information Management, “to survive and grow”), both fed by environmental “noise” (better … from what human beings call “noise”) [13]. So, a natural operating point can emerge as a new Trans-disciplinary Reality Level, out of the Interaction of Two Complementary Irreducible Information Management Subsystems. Building on this idea, it is possible to envisage an Evolutive Elementary Pragmatic Model (E^2PM) able to profit by both classic EPM intrinsic Self-Reflexive Functional Logical Closure and new evolutive Self-Reflective Functional Logical Aperture.

2 EPM Logical Structure

EPM was conceived as a Boolean model of two binary inputs x_0, P, and x_1, Q, for three distinct elements. In two-valued logic there are 2 nullary operators (constants), 4 unary operators, 16 binary operators, 256 ternary operators, and, in general, 2^{2^n} n-ary operators. Classically the domain and range of a truth function are {truth, falsehood}, but they may have any number of truth values, including an infinity of these. A concrete function may be also referred to as an operator. EPM "elementary interaction coordinates" (unary operators) are $2^2 = 4$, and they were named: (001) Antifunction, (011) Acceptance, (101) Maintenance and (111) Sharing. So, there are sixteen possible truth functions (binary operators), also called Boolean functions for two inputs. Any of these functions corresponds to a truth table of a certain logical connective in classical logic, including several degenerate cases such as a function not depending on one or both of its arguments. The arity of a function or operation is the number of arguments or operands the function or operation accepts. The arity of a relation (or predicate) is the dimension of the domain in the corresponding Cartesian product. A function of arity "a" thus has arity $n = a + 1$, if considered as a relation. Usually, truth and falsehood is denoted as 1 and 0. An example of Boolean operations is illustrated in full, for $a = 2$ in Fig. 13.1. Finally, EPM can accommodate $2^8 = 256$ ternary operators. Please, note, that in this case, those 256 values can be thought as EPM power set $P(W)$, where $W = 2^3 = 8$. $P(W)$ can be even interpreted as EPM intrinsic Self-Reflexive Functional Logical Closure.

Therefore, EPM associated Boolean algebra is B_3. The Boolean algebra B_3 can be represented as a cube C_3 in three-dimensional Euclidean space R^3. This is done by the "conventional" Left-To-Right (LTR) coordinate mapping $c : \{0;1\}^3 \rightarrow R^3$, (Fig. 13.2).

It is well-known that a Boolean algebra can always be visualized by means of a Hasse diagram that is centrally symmetric (with all complementary pairs of elements ordered around the center of symmetry) [8]. Furthermore, a finite Boolean algebra can be partitioned into "levels" L_0, L_1, L_2, which are recursively defined as follows: $L_0 = [\bot]$, and

$$L_{k+1} = \{x \,|\, \exists y \in L_k : y \triangleleft x\}. \tag{13.1}$$

x_0	x_1	${}^2f_0$	${}^2f_1$	${}^2f_2$	${}^2f_3$	${}^2f_4$	${}^2f_5$	${}^2f_6$	${}^2f_7$	${}^2f_8$	${}^2f_9$	${}^2f_{10}$	${}^2f_{11}$	${}^2f_{12}$	${}^2f_{13}$	${}^2f_{14}$	${}^2f_{15}$
0	**0**	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
1	**0**	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	**1**	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	**1**	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

Fig. 13.1: Example of Boolean operations of arity 2

Hasse diagrams for the Boolean algebra $\phi(\{1,2,3\})$, and for a Boolean algebra of formulas from the modal logic $S5$ are shown in Fig. 13.3. It is immediate to map cube C_3 to Hasse diagram on left side of Fig. 13.3. Boolean algebras are locally finite and their word problem is always decidable (closed logic).

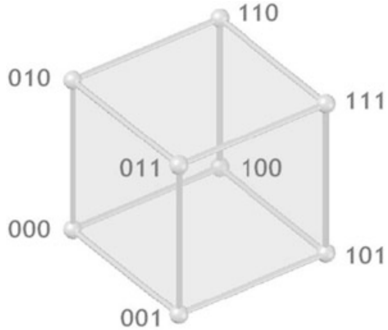

Fig. 13.2: EPM associated Boolean algebra B_3 is represented (LTR) by cube C_3 with its bitstring decoration in R^3

3 Extending EPM as a Boolean-Valued Model

To provide EPM with an evolutive structure (open logic), we can follow many different approaches. In previous section, we saw that EPM associated Boolean algebra B_3 can be represented LTR by cube C_3 in R^3. By remembering the notions of "logical space" proposed by Wittgenstein [43] and of "hypercube" proposed by Pólya [34], it is straightforward to consider a Boolean-valued model, as the simplest extension of EPM. A Boolean-valued model is a generalization of the ordinary Tarskian notion of structure from model theory [41]. In a Boolean-valued model, the truth values of propositions are not limited to "true" and "false", but instead take values in some fixed complete Boolean algebra. Boolean-valued models were introduced by Dana Scott, Robert M. Solovay, and Petr Vopěnka in the 1960s in order to help understand Paul Cohen's method of forcing, presented in 1963 [5]. They are also related to Heyting algebra semantics in intuitionistic logic [42]. The problem of whether a given equation holds in every Heyting algebra was shown to

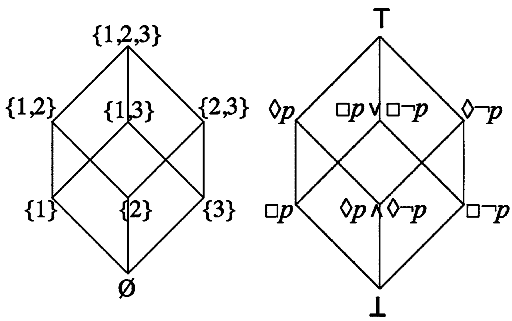

Fig. 13.3: Hasse diagrams for the Boolean algebra $\phi(\{1,2,3\})$ on the left side and for a Boolean algebra of formulas from the modal logic $S5$ with modality $(\Box, \Diamond)$, on the right side [10]

be decidable by S. Kripke in 1965 [28]. The precise computational complexity of the problem was established by R. Statman in 1979, who showed it was PSPACE-complete [38] and hence at least as hard as deciding equations of Boolean algebra (shown NP-complete in 1971 by S. Cook [7]) and conjectured to be considerably harder. The elementary or first-order theory of Heyting algebras is undecidable [20]. It remains open whether the universal Horn theory of Heyting algebras, or word problem, is decidable [26]. For the word problem it is known that Heyting algebras are not locally finite (no Heyting algebra generated by a finite nonempty set is finite), in contrast to Boolean algebras which are locally finite and whose word problem is decidable. It is unknown whether free complete Heyting algebras exist except in the case of a single generator where the free Heyting algebra on one generator is trivially completable by adjoining a new top. A Boolean algebra of order 2^n, called B_n, is graded of rank n, [29] and can be represented as a hypercube or n-cube C_n, in n-dimensional Euclidean space R^n, for $n = 0, 1, 2, 3, 4, \cdots, \infty, n \in N$. Our main idea is to achieve EPM open logic model behavior (logic dynamics) by providing EPM with the asymptotic process of the structured sequence of locally finite Boolean algebras for $n \to \infty$ theoretically. In Fig. 13.4, the process to obtain successive n-dimensional hypercubes up to $n = 4$ is depicted. For $n = 4$, the Boolean algebra is represented as a four-dimensional hypercube C_4. We again can employ the "conventional" LTR coordinate mapping $c : \{0; 1\}^4 \to R^4$. This can be generalized to any number of dimensions. In fact, the process of sweeping out volumes (Fig. 13.4) can be formalized mathematically as a Minkowski sum: the n-dimensional hypercube is the Minkowski sum of n mutually perpendicular unit-length line segments, and is therefore an example of zonotope. Then, n-dimensional hypercubes geometrical information can be projected to convenient projection planes to study the local behavior of their connection components as graphs. For instance, in Fig. 13.5, the related Petrie polygon Orthographic projections up to $n = 8$ are shown. Then, the graph of the n-hypercube's edges is isomorphic to the Hasse diagram of the $(n-1)$-simplex's face lattice. This can be seen by orienting the n-hypercube so that two opposite vertices lie vertically, corresponding to the $(n-1)$-simplex itself and the null polytope, respectively.

Each vertex connected to the top vertex then uniquely maps to one of the $(n-1)$-simplex's facets ($n-2$ faces), and each vertex connected to those vertices maps to one of the simplex's $n-3$ faces, and so forth, and the vertices connected to the bottom vertex map to the simplex's vertices. This relation may be used to generate the face lattice of an $(n-1)$-simplex efficiently, since face lattice enumeration algorithms applicable to general polytopes are more computationally expensive.

Then, the Hasse diagram for any B_n can be seen as $(n-1)$-dimensional vertex-first projections of these hypercubes. Although Hasse diagrams are simple as well as intuitive tools for dealing with finite posets, it turns out to be rather difficult to draw "good" diagrams. The reason is that there will in general be many possible ways to draw a Hasse diagram for a given poset.

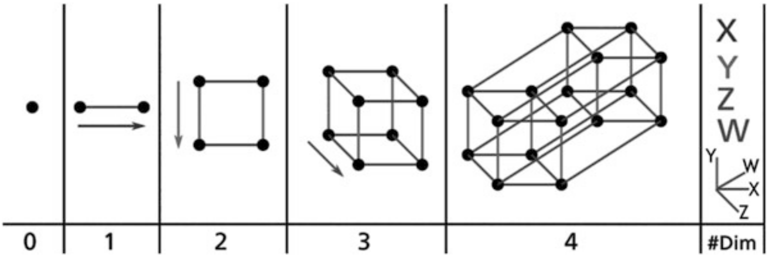

Fig. 13.4: (*0*)—A point is a hypercube of dimension zero. (*1*)—If one moves this point one unit length, it will sweep out a line segment, which is a unit hypercube of dimension one. (*2*)—If one moves this line segment its length in a perpendicular direction from itself; it sweeps out a two-dimensional square. (*3*)—If one moves the square one unit length in the direction perpendicular to the plane it lies on, it will generate a three-dimensional cube. (*4*)—If one moves the cube one unit length into the fourth dimension, it generates a four-dimensional unit hypercube (a unit tesseract)

The simple technique of just starting with the minimal elements of an order and then drawing greater elements incrementally often produces quite poor results: symmetries and internal structure of the order are easily lost. In Fig. 13.6 this issue is demonstrated by considering the power set of a 4-element set ordered by inclusion ($\subseteq$) of a 4-cube or tesseract. There are four different Hasse diagrams for this partial order. Each subset has a node labelled with a binary encoding that shows whether

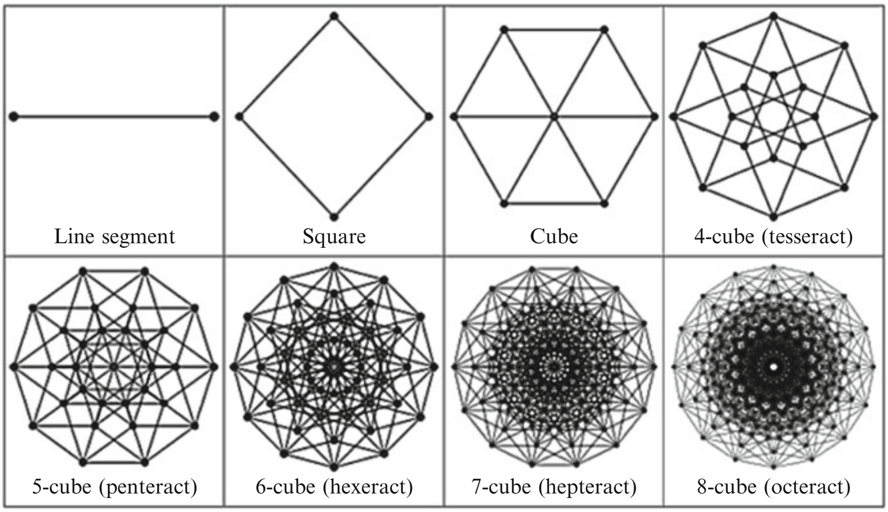

Fig. 13.5: *N*-dimensional Hypercube Petrie polygon Orthographic projections from $n = 1$ up to $n = 8$, as graphs

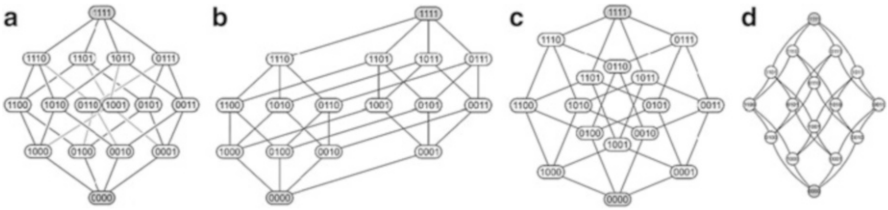

Fig. 13.6: Hasse diagrams (see text). The (**a**) first diagram on the left makes clear that the power set is a graded poset. The (**b**) second diagram has the same graded structure, but by making some edges longer than others, it emphasizes that the four-dimensional cube is a union of two three-dimensional cubes. The (**c**) third diagram shows some of the internal symmetry of the structure. In the (**d**) fourth diagram, on the right, the vertices are arranged like the fields of a 4×4 matrix

a certain element is in the subset (1) or not (0). Therefore, whether the diagram is Hasse or Aristotelian depends on our choice of the projection axis [10, 32]. For a full definition of these structures, by a theoretical perspective, one should look to Judson, at least [27]. Finally, please, note, that the power set $P(B_n)$ of any locally finite Boolean algebra B_n can be thought even as a Self-Reflective Functional Logical Closure for the power set $P(B_{n-1})$ of preceding locally finite Boolean algebra $P(B_{n-1})$. According to brand new Computational Information Conservation Theory (CICT), this property is fundamental to achieve overall model systemic resilience and antifragility behaviour) [13, 16, 17].

3.1 Ontological Uncertainty Modeling

To behave realistically (i.e. to capture natural event dynamics), system must guarantee both Logical Closure (to get RT, to learn and prosper) and Logical Aperture (to get EI and EC, to survive and grow), both fed by environmental noise (better … from what human beings call “noise”) [13]. All the while almost everything, classically approached and traditionally studied, about social life focuses on the “normal”, particularly with “bell curve” methods of inference that tell you close to nothing about natural events. Why? Epistemic uncertainty sources are still treated with the traditional approach of risk analysis only, which provides an acceptable cost/benefit ratio to producer/manufacturer, but in some cases it may not represent an optimal solution to end user [16, 17]. In fact, deep epistemic limitations reside in some parts of the areas covered in decision making. In fact, the bell curve ignores large deviations, cannot handle them, yet makes us confident that we have tamed uncertainty [40]. On the other end, almost everything in social life is produced by rare but consequential shocks and jumps. As the experiences of the 1970s, 1980s, 1990s and 2000s have shown, unpredictable changes can be very disorienting at enterprise level. These major changes, usually discontinuities referred to as fractures in the environment rather than trends, will largely determine the long-term future of organization. They need to be handled, as opportunities, as positively as possible. Model developers must concentrate on not ignoring or double counting uncertainties and clearly documenting the process in which they represent and quantify uncertainties. Traditionally, uncertainties are characterized as epistemic, if the modeler sees a possibility to reduce them by gathering more data or by refining models. Uncertainties are categorized as aleatory if the modeler does not foresee the possibility of reducing them. From a pragmatic standpoint, it is useful to categorize the uncertainties within a model, since it then becomes clear as to which uncertainties have the potential of being reduced. Influences of the two types of uncertainties in reliability assessment, codified design, performance-based engineering and risk-based decision-making are always present. More importantly, epistemic uncertainties may introduce dependence between events, which may not be properly noted if their character is not correctly modeled. Unfortunately, decision theory, based on a "fixed universe" or a model of possible outcomes (closed logic), ignores and minimizes the effect of events that are "outside model". In fact, human made systems can be quite fragile to unexpected perturbation because Statistics can fool you [39]. While the advantage of differentiating between natural (aleatoric) and epistemic uncertainty in analysis is clear, the necessity of distinguishing between them is not, at operational or operative level. As a matter of fact, epistemic and aleatory uncertainties are fixed neither in space nor in time. What is aleatory uncertainty in one model can be epistemic uncertainty in another model, at least in part. And what appears to be aleatory uncertainty at the present time may be cast, at least in part, into epistemic uncertainty at a later date [21]. It is much better to consider ontological uncertainty [30] as an emergent phenomenon out of a complex system [16, 17]. Then, our ontological perspective can be thought only as an emergent, natural operating point out of, at least, the interaction dichotomy of two coupled irreducible complementary ideal asymptotic concepts: (a) reliable predictability (closed logic subsystem) and (b) reliable unpredictability (open logic subsystem). From top-level management perspective, the reliable predictability concept can be referred to traditional system reactive approach (fixed logic subsystem) and operative management techniques, while the reliable unpredictability concept can be associated to system proactive approach (open logic subsystem) and strategic management techniques (Fig. 13.7).

4 Concluding Remarks

Now, traditional EPM can be thought as a reliable starting subsystem (closed logic, operative management, Fig. 13.7) to initialize a process of continuous self-organizing and self-logic learning refinement (open logic, strategic management subsystem, Fig. 13.7). As already described in previous sections, this method can capture natural logic dynamics behavior, as function of specific unpredictable perturbation, unknown at system design level. Though the hypercube logical geometry

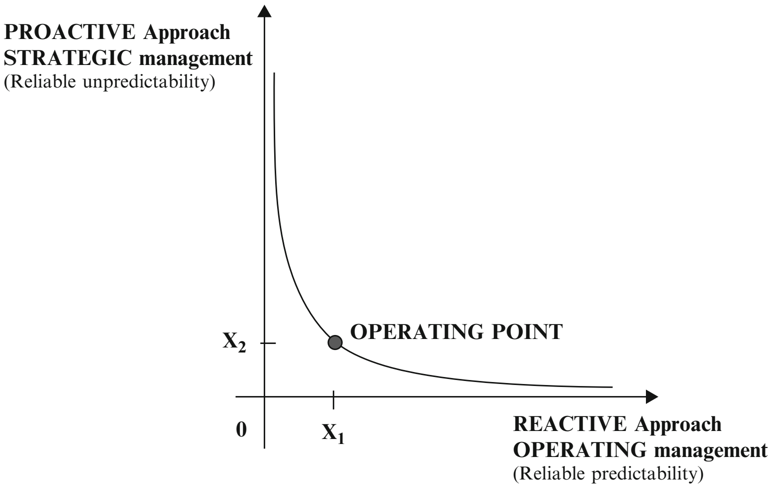

Fig. 13.7: Operating Point can emerge as a new Trans-disciplinary Reality Level, based on the Interaction of Two Complementary Irreducible Management Subsystems

seems to be a straight-forward method to depict the logical relations in propositional logic, further research must be planned to go beyond this first approach of the notation. Future studies ought to validate empirically the contribution of this logical geometry approach to the understanding of logical relationships, notably in educational settings. The intuitive character of the related algebra to apprehend logical relations must be tested in comparison with classical methods of learning. Through the hypercube geometric algebra, we propose a notation that goes beyond a format distinction and constructed with the purpose to facilitate inferences either on a diagrammatic representation, or a lexical one. The latter particularly allows operations on complex propositions within hypercube with more than three dimensions, mentally difficult to imagine. This algebra, by posting directly configurations in which a complex proposition is true, can explicitly represent all mental models, in the sense of Johnson-Laird [25], necessary for the apprehension of a proposition in all its complexity. In agreement to Morineau [33], we think that this algebra could represent a tool for assisting work activities that involve inductive reasoning, like problem- and case-based reasoning in medical diagnosis [12], and subject profiling in psychiatry and psychotherapy [3, 22, 23]. More specifically, from a biomedical engineering perspective, fault diagnosis task [35] and troubleshooting on logical networks [36] could be areas of application for reliable testing and validation of the presented EPM extension as "Evolutive Elementary Pragmatic Model" (E^2PM). E^2PM presents a relevant contribute to models and simulations offering an example of new forms of evolutive behavior inter- and trans-disciplinarity modeling.

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