A Kinetic Study of Opinion Dynamics in Multi-agent Systems

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Abstract. In this paper we rephrase the problem of opinion formation from a physical viewpoint. We consider a multi-agent system where each agent is associated with an opinion and interacts with any other agent. Interpreting the agents as the molecules of a gas, we model the opinion evolution according to a kinetic model based on the analysis of interactions among agents. From a microscopic description of each interaction between pairs of agents, we derive the stationary profiles under given assumption. Results show that, depending on the average opinion and on the model parameters, different profiles can be found, with different properties. Each stationary profile is characterized by the presence of one or two maxima.

1 Introduction

This paper describes a model for opinion formation in multi-agent systems. Many kinds of approaches have been investigated in the literature to study opinion evolution among agents based, e.g., on graph theory [1], cellular automata [2], and thermodynamics [3]. Recently, social interactions in multi-agent systems have been described according to microscopic models based on kinetic theory. Typically, kinetic theory is used to derive macroscopic properties of gases by analyzing the details of the collisions of the molecules [4]. Similarly, from the description of the details of each interaction between pairs of agents, the global opinion can be described from a macroscopic point of view [5].

The research interest related to the application of kinetic approaches to the description of multi-agent systems gave birth to new disciplines, namely *econophysics* and *sociophysics* [6]. Econophysics deals with the description of the evolution of market economy and wealth distribution in a society [7]. Sociophysics, instead, aims at characterizing the evolution of social features, such as opinion, in a society [8]. These disciplines are based on the fact that the formalism that describes the interactions between molecules in a gas can be adapted to describe the effects of interactions between agents. In particular, the kinetic framework can be used to outline agent-based cooperation models, such as that in [9], to study large scale systems, such as those in [10], and to model wireless sensor networks (see, e.g., [11]).

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In this paper we focus on studying the opinion evolution in a multi-agent system on the basis of a kinetic formulation. Under the assumption that each agent is associated with an opinion $v \in I \subseteq \mathbb{R}$, we investigate how the opinion of the considered system changes on the basis of given rules that describe the effects of single interactions. The model that we consider is introduced in [12]. In particular, we assume that each agent can interact with any other agent in the system and that the opinion of each changes due to two different reasons, namely compromise and diffusion [13]. Compromise is related to the fact that, as a consequence of an interaction, an agent can be persuaded to change its opinion in favour of that of the interacting agent. This phenomenon is modeled as a deterministic process. Diffusion is instead modeled as a random process and it is related to the fact that agents can change their opinions autonomously.

The paper is organized as follows. Section 2 describes the considered kinetic model from an analytical viewpoint. Section 3 derives explicit formulas for the stationary profiles in a specific case. Section 4 shows simulation results for different values of the parameters of the model. Section 5 concludes the paper.

2 Kinetic Model of Opinion Formation

Sociophysics is based on the idea that the same laws that describe the interactions among molecules can be generalized to describe the effects of social interactions among agents. As a matter of fact, while molecules are typically associated with their velocities, agents can be associated with an attribute which represents one of its characteristics that can be, for instance, its opinion. In the following, we associate to each agent a parameter v defined in the interval $I = [-1, 1]$. According to such an assumption, ± 1 represent extremal opinions, while 0 correspond to the middle point of the interval of interest I.

Kinetic theory relies on the definition of a function $f(v,t)$ which represents the density of opinion v at time t and which is defined for each opinion $v \in I$ and for each time $t \geq 0$. Since $f(v,t)$ is a density function, the following equality needs to hold

$$\int_I f(v,t)\mathrm{d}v = 1. \tag{1}$$

In order to describe the opinion evolution using a kinetic approach, we assume that the function $f(v,t)$ evolves on the basis of the Boltzmann equation. In particular, we consider the following formulation of the Boltzmann equation

$$\frac{\partial f}{\partial t} = \mathcal{Q}(f,f)(v,t) \tag{2}$$

where $\frac{\partial f}{\partial t}$ represents the temporal evolution of the distribution function and $\mathcal{Q}$ is the *collisional operator* which takes into account the effects of interactions. In order to derive an explicit formula for the collisional operator $\mathcal{Q}$, the details of the binary interactions need to be described. In the considered model, the post-interaction opinions of two interacting agents are obtained by adding to their pre-interaction opinions a contribution related to compromise and a contribution related to diffusion, according to the following formula

$$\begin{cases} v' = v + \gamma C(|v|)(w - v) + \eta_* D(|v|) \\ w' = w + \gamma C(|w|)(v - w) + \eta D(|w|). \end{cases} \tag{3}$$

where the pair (v', w') denotes the post-interaction opinions of the two agents, whose pre-interaction opinions were (v, w). In (3) the second terms on the right hand side of the two equations are related to compromise, according to the parameter γ, which is defined in $(0, \frac{1}{2})$, and the function $C(\cdot)$; the third terms are related to diffusion, through the random variables η and η_* and the function $D(\cdot)$. The functions $C(\cdot)$ and $D(\cdot)$, which describe the impact of compromise and diffusion, respectively, depend on the absolute value of the opinion, namely they are symmetrical with respect to the middle point of I. Moreover, we assume that both functions are not increasing with respect to the absolute value of the opinion, coherently with the fact that, typically, extremal opinions are more difficult to change. Finally, we assume that

$$0 \leq C(|v|), D(|v|) \leq 1 \qquad \forall v \in I.$$

From (3), since both γ and $C(\cdot)$ are positive, the contribution of compromise is positive each time an agent interacts with another agent whose opinion value is greater while it is negative otherwise. Hence, the idea of compromise is respected, since the difference between the opinions of the two agents is reduced after the considered interaction if the diffusion term is neglected.

The contribution of diffusion, instead, can be either positive or negative depending on the value of the random variables η and η_*. In the following, we assume that such random variables have the same statistics. In particular, we assume that their average value is 0 and their variance is σ^2, namely

$$\begin{aligned} \int \eta \vartheta(\eta) \mathrm{d}\eta &= \int \eta_* \vartheta(\eta_*) \mathrm{d}\eta_* = 0 \\ \int \eta^2 \vartheta(\eta) \mathrm{d}\eta &= \int \eta_*^2 \vartheta(\eta_*) \mathrm{d}\eta_* = \sigma^2. \end{aligned} \tag{4}$$

where $\vartheta(\cdot)$ is the probability density function. In order to take into account the effects of diffusion we need to define the *transition rate*

$$W(v, w, v', w') = \vartheta(\eta)\vartheta(\eta_*)\chi_I(v')\chi_I(w') \tag{5}$$

where χ_I is the indicator function relative to the set I (equal to 1 if its argument belong to I, and to 0 otherwise) and it is meant to make sure that the post-interaction opinions are in I.

Under these assumptions, the explicit expression of the collisional operator $\mathcal{Q}$ defined in (2) can be finally written as

$$Q(f, f) = \int_{\mathbb{B}^2} \int_I \left[{}'W \frac{1}{J} f('v) f('w) - W f(v) f(w) \right] \mathrm{d}w \mathrm{d}\eta \mathrm{d}\eta_*$$

where $\mathbb{B}$ is the support of ϑ, $'v$ and $'w$ are the pre-interaction variables which lead to v and w, respectively, $'W$ is the transition rate relative to the 4−uple $('v,'w,v,w)$ and J is the Jacobian of the transformation of $('v,'w)$ in (v,w) [12].

Instead of solving (2) we consider its weak form. In functional analysis, the weak form of a differential equation is obtained by multiplying both sides of the considered equation by a test function $\phi(v)$, namely a smooth function with compact support, and then integrating the obtained equation with respect to v. The weak form of the Boltzmann equation can be derived from (2) and, using a proper change of variable in the integral, it can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_I f(w,t)\phi(v)\mathrm{d}v = \int_{\mathbb{B}^2}\int_{I^2} W f(v)f(w)(\phi(v') - \phi(v))\mathrm{d}v\mathrm{d}w\mathrm{d}\eta\mathrm{d}\eta_* \tag{6}$$

If we consider $\phi(v) = 1$ in (6) then the following equation is obtained

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_I f(v,t)\mathrm{d}v = 0 \tag{7}$$

which says that the number of agents is constant. This property is analogous to mass conservation of the molecules in a gas.

Considering $\phi(v) = v$ as a test function in (6) and recalling (3) we obtain

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_I f(w,t)v\mathrm{d}v = \gamma\int_{\mathbb{B}^2}\int_{I^2} W f(v)f(w)C(|v|)(w-v)\mathrm{d}v\mathrm{d}w\mathrm{d}\eta\mathrm{d}\eta_* \\ + \int_{\mathbb{B}^2}\int_{I^2} W f(v)f(w)\eta D(|v|)\mathrm{d}v\mathrm{d}w\mathrm{d}\eta\mathrm{d}\eta_*\end{aligned} \tag{8}$$

Defining the average value of the opinion at time t as

$$u(t) = \int_I f(w,t)v\,\mathrm{d}v \tag{9}$$

the left hand side of (8) corresponds to the derivative $\dot{u}(t)$ of the average opinion. The first integral in the right hand side of (8) can be written as

$$\gamma\int_I f(v)C(|v|)\mathrm{d}v\int_I vf(v)\mathrm{d}v - \gamma\int_I f(v)C(|v|)v\mathrm{d}v. \tag{10}$$

The second integral in (8) is 0 because the average value of ϑ is 0, according to (4). Therefore, from (8) and (10) it can be obtained that the variation of the average opinion u can be written as

$$\dot{u}(t) = \gamma\int_I f(v)C(|v|)\mathrm{d}v\int_I vf(v)\mathrm{d}v - \gamma\int_I f(v)C(|v|)v\mathrm{d}v. \tag{11}$$

Observe that if C is constant then (10) is 0 for symmetry and (11) becomes

$$\dot{u}(t) = 0 \tag{12}$$

i.e., the average opinion is conserved, namely $u(t) = u(0)$. This property corresponds to the conservation of momentum.

We are interested in studying the behaviour of the distribution function $f(v,t)$ for large values of the time t and to derive, eventually, stationary profiles. In order to simplify notation we first define a new temporal variable τ

$$\tau = \gamma t \tag{13}$$

where γ is the coefficient related to compromise which appear in (3). Assuming that $\gamma \simeq 0$, namely that each interaction causes small opinion exchange,

$$g(v,\tau) = f(v,t) \tag{14}$$

describes the asymptotic behaviour of $f(v,t)$. The weak form of a Fokker-Planck equation can be derived by substituting $f(v,t)$ with $g(v,\tau)$ in (6) and using a Taylor series expansion of $\phi(v)$ around v in (6) [12]:

$$\frac{\mathrm{d}g}{\mathrm{d}\tau} = \frac{\lambda}{2}\frac{\partial^2}{\partial v^2}(D(|v|)^2 g) + \frac{\partial}{\partial v}((v-u)g) \tag{15}$$

where

$$\lambda = \sigma^2/\gamma. \tag{16}$$

We are now interested in studying stationary solutions g_∞ of (15), which satisfy

$$\frac{\mathrm{d}g_\infty}{\mathrm{d}\tau} = 0. \tag{17}$$

In next section we analyze these solutions for different values of λ.

3 Stationary Behaviour of Opinion Distribution

In this section we derive some stationary profiles for the opinion density g. Such profiles are defined as solutions of (17) and, therefore, they depend on parameters u and λ and on the choice of the diffusion function.

In the remaining of the paper, we assume that the compromise function $C(|v|)$ is constant and equal to 1. As observed in Section 2, this choice leads to a constant value of the average opinion, which is denoted as u in the following. We consider the following distribution function

$$D(|v|) = 1 - v^2 \tag{18}$$

which is a non increasing function of $|v|$, as discussed at the beginning of the previous section. According to this assumption, the effects of the interactions between pairs of agents described in (3) are

$$\begin{cases} v' = v + \gamma(w-v) + \eta(1-v^2) \\ w' = w + \gamma(v-w) + \eta_*(1-w^2) \end{cases} \tag{19}$$

In order to guarantee that the post-interaction opinions still belong to the interval of interest I we need to define the support $\mathbb{B}$ of the distribution function $\vartheta(\cdot)$ of η and η_*. Considering the first equation in (19), we can conclude that

$$|v'| \leq (1-\gamma)|v| + \gamma + |\eta|(1-v^2)$$

from which it can be derived that if $|\eta| \leq M = \frac{1-\gamma}{1+|v|}$ then $|v'| \leq 1$. Analogous considerations hold for $|w'|$ when taking into account the second equation of (19). Since the minimum value of M is obtained in correspondence of the maximum values of $|v'|$ and γ, namely when $|v| = 1$ and $\gamma \simeq 1/2$, then it can be concluded that if $|\eta| \leq \frac{1}{4}$, then $|v'| \leq 1$ independently of the pre-interaction opinion v. The same holds for $|w'|$, therefore from now on we assume that $\mathbb{B} = (-1/4, 1/4)$. We are now interested in finding the stationary solutions, namely the functions which satisfy (17). From (15) the stationary solutions satisfy

$$\frac{\lambda}{2}\frac{\partial}{\partial v}\big((1-v^2)^2 g\big) + (v-u)g = C \tag{20}$$

where u is the average opinion (which is constant) and C is a constant. Observe that the constant C must be 0. As a matter of fact, by integrating (20) one obtains

$$\frac{\lambda}{2}\int_{-v_1}^{v_2}\frac{\partial}{\partial v}\big((1-v^2)^2 g\big) + \int_{-v_1}^{v_2}(v-u)g = C(v_2+v_1). \tag{21}$$

From the previous equation, if $v_1 \to 1$ and $v_2 \to 1$ then the first integral is 0 for symmetry and the second integral can be written as

$$\int_I vg\mathrm{d}v - u\int_I g\mathrm{d}v = u - u = 0.$$

It can then be concluded from (21) that $C = 0$.

Using classical analysis in (20), one obtains

$$\frac{g'}{g} = \frac{4v}{1-v^2} + \frac{2(u-v)}{\lambda(1-v^2)^2}. \tag{22}$$

The left hand side of the previous equation is the derivative of $\log g$. Integrating the right hand side of (22) leads to an explicit expression of $\log g$, and, therefore, of g. The first added on the right hand side of (22) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}v}\left(-2\log(1-v^2)\right) \tag{23}$$

Concerning the remaining terms in (22), first observe that

$$\frac{2u}{\lambda}\frac{1}{(1-v^2)^2} = \frac{\mathrm{d}}{\mathrm{d}v}\left(\frac{u}{2\lambda}\log\left(\frac{1+v}{1-v}\right) + \frac{uv}{\lambda(1-v^2)}\right). \tag{24}$$

Moreover one can calculate

$$\begin{aligned} -\frac{2v}{\lambda(1-v^2)^2} &= \frac{1}{2\lambda}\left(\frac{1}{(1+v)^2} - \frac{1}{(1-v)^2}\right) = -\frac{1}{2\lambda}\frac{\mathrm{d}}{\mathrm{d}v}\left(\frac{1}{1+v} + \frac{1}{1-v}\right) \\ &= -\frac{1}{\lambda}\frac{\mathrm{d}}{\mathrm{d}v}\frac{1}{1-v^2}. \end{aligned} \tag{25}$$

Finally, using (23), (24), and (25), equation (22) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}v}\log g(v) = \frac{\mathrm{d}}{\mathrm{d}v}\left[-2\log(1-v^2) + \frac{u}{2\lambda}\log\left(\frac{1+v}{1-v}\right) + \frac{uv-1}{\lambda(1-v^2)}\right]$$

and, therefore,

$$\log g(v) = \log(1-v^2)^{-2} + \log\left(\frac{1+v}{1-v}\right)^{\frac{u}{2\lambda}} + \frac{uv-1}{\lambda(1-v^2)} + \alpha_{u,\lambda}. \tag{26}$$

where $\alpha_{u,\lambda}$ is a constant depending on the average opinion u and on the value of λ. Taking the exponential of (26) the following expression for the stationary solution is derived

$$g_\infty(v) = c_{u,\lambda}(1+v)^{-2+\frac{u}{2\lambda}}(1-v)^{-2-\frac{u}{2\lambda}}\exp\left(\frac{uv-1}{\lambda(1-v^2)}\right) \tag{27}$$

where $c_{u,\lambda}$ must be determined in order to satisfy

$$\int_I g_\infty(w) = 1. \tag{28}$$

Observe that if $u = 0$, then $g_\infty(v)$ is an even function.

In order to see if the stationary profile is characterized by maxima and/or minima, we now aim at studying the derivative of g_∞. From (22) the derivative of g_∞ can be written as

$$g'_\infty(v) = g_\infty(v)\left(\frac{4\lambda v(1-v^2) + 2(u-v)}{\lambda(1-v^2)^2}\right) \tag{29}$$

and, therefore,

$$g'_\infty(v) = 0 \quad\iff\quad g_\infty(v) = 0 \quad\vee\quad 4\lambda v^3 + (2-4\lambda)\,v - 2u = 0. \tag{30}$$

From (27), $g_\infty(v) = 0$ if and only if $v = \pm 1$, namely in the extremes of the considered interval I. Hence, we are interested in finding the solutions of the second condition in (30), namely the solutions of

$$v^3 + \left(\frac{1}{2\lambda} - 1\right)v - \frac{u}{2\lambda} = 0. \tag{31}$$

Observe that equation (31) is a polynomial equation of degree 3 and therefore it always admits at least one real solution.

If $u = 0$, namely if the average opinion is the middle point of I, (31) becomes

$$v^3 + \left(\frac{1}{2\lambda} - 1\right)v = 0 \tag{32}$$

and in this case the solutions are

$$v_1 = 0 \qquad v_{2,3} = \pm\sqrt{1 - \frac{1}{2\lambda}} \tag{33}$$

Observe that if $\lambda \leq \frac{1}{2}$ the only real root of (32) is $v_1 = 0$ and its multiplicity is 1 if $\lambda < \frac{1}{2}$ while it is 3 if $\lambda = \frac{1}{2}$. In these cases, v_1 is a maximum point. If $\lambda > \frac{1}{2}$, instead, equation (32) admits three real roots. In this last case, v_1 is a point of minimum while v_2 and v_3 are points of maximum.

If $u \neq 0$ the solution of (31) requires the use of Cardano's formula for the solution of polynomial equations of degree 3, according to which

$$v_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}} \tag{34}$$

is a real root of (31), where

$$\Delta = \frac{q^2}{4} + \frac{p^3}{27} \qquad p = \left(\frac{1}{2\lambda} - 1\right) \qquad q = -\frac{u}{2\lambda} \tag{35}$$

If $\Delta \leq 0$ then equation (31) has three real roots, which, besides v_1, are

$$v_{2,3} = -\frac{v_1}{2} \pm \frac{1}{2}\sqrt{-4p - 3v_1^2}. \tag{36}$$

Hence, if $\Delta < 0$, the stationary profile g_∞ has three singular points. If $\Delta = 0$, then from (34) it can be concluded that v_1 has the following simplified expression

$$v_1 = 2\sqrt[3]{-\frac{q}{2}}.$$

Substituting this result in (36) one obtains that $-4p - 3v_1^2 = 0$ and, therefore, $v_2 = v_3 = -v_1/2$, namely equation (31) has three real roots, two of which are coincident. In the case with $\Delta = 0$ the singular points of g_∞ are two and one of them is also an inflection point. Finally, if $\Delta > 0$ then equation (31) has v_1 as the only real root, hence g_∞ has only one singular point.

From (35), the value of Δ can be expressed as a function of λ and u as

$$\Delta = \frac{27u^2\lambda + 2(1 - 2\lambda)^3}{432\lambda^3} \tag{37}$$

Since, from (16), λ is defined as the ratio between two positive quantities, one can conclude that

$$\Delta < 0 \iff u^2 < M(\lambda) = \frac{2}{\lambda}\left(\frac{1 - 2\lambda}{3}\right)^3. \tag{38}$$

Since $u \in I$, then $0 \leq u^2 \leq 1$ and, therefore, if $M(\lambda) \geq 1$ the inequality on the right hand side of (38) is satisfied for all the values of u, while if $M(\lambda) < 0$ the previous inequality is never satisfied. It can be shown that

$$M(\lambda) < 0 \iff \lambda < \frac{1}{2} \qquad M(\lambda) \geq 1 \iff \lambda \geq 2. \tag{39}$$

Hence, the following considerations hold:

- if $0 < \lambda < \frac{1}{2}$ then the condition $u^2 < M(\lambda)$ is never satisfied and, therefore, $\Delta > 0$ and the stationary profile g_∞ has only one singular point
- if $\lambda \geq 2$ then the condition $u^2 < M(\lambda)$ is satisfied for all the values of the average opinion u and, therefore, $\Delta < 0$ and the stationary profile g_∞ has three singular points
- if $\frac{1}{2} \leq \lambda < 2$, the number of stationary points of g_∞ depends on the value of the average opinion u.

4 Numerical Results

In this section, various stationary profiles for different values of u and λ are shown. We start by considering $u = 0$ so that the average opinion corresponds to the middle point of I. In this case, the stationary profile g_∞ is an even function.

In Fig. 1, the stationary profiles $g_\infty(v)$ are shown for various values of λ, namely $\lambda = 1/4$ (blue line), $\lambda = 1/2$ (red line), $\lambda = 1$ (green line), and $\lambda = 3$ (black line). Fig. 1 shows that if $\lambda = 1/4$, then $g_\infty(v)$ has only one maximum (corresponding to $u = 0$), in agreement with (33). If $\lambda = 1/2$, then $v = 0$ is the only stationary point of $g_\infty(v)$, but in this case the multiplicity of $v = 0$ as a solution of (32) is 3. Observe that the value of the maximum is smaller compared to that relative to $\lambda = 1/4$. If $\lambda > 1/2$, according to (33), the function $g_\infty(v)$ admits three stationary points. In particular, Fig. 1 shows that if $\lambda = 1$ there is a minimum in correspondence of $v = 0$ and two maxima in $v = \pm 1/\sqrt{2}$. In this case, the value of the two maxima is similar to that of the maximum obtained with $\lambda = 1/2$. Fig. 1 also shows the stationary profile $g_\infty(v)$ when $\lambda = 3$. In this case, the two points of maximum are closer to the extremes of the interval I where the opinion is defined and the values of the maxima are approximately the double of those relative to $\lambda = 1$. The value of the minimum

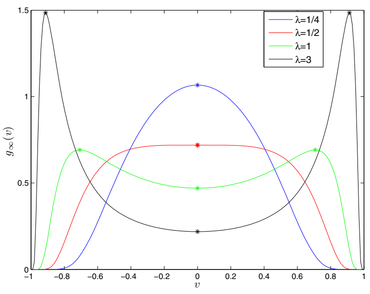

Fig. 1. The stationary profiles g_∞ relative to the average opinion $u = 0$ are shown for $\lambda = 1/4$ (blue line), $\lambda = 1/2$ (red line), $\lambda = 1$ (green line), $\lambda = 3$ (black line).

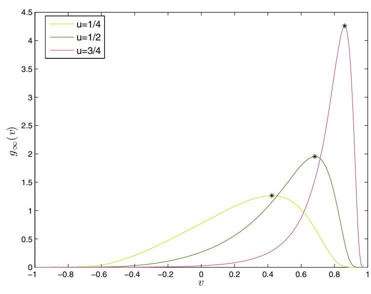

Fig. 2. The stationary profiles g_∞ relative to the value $\lambda = 1/4$ are shown for $u = 1/4$ (yellow line), $u = 1/2$ (green line), $u = 3/4$ (violet line).

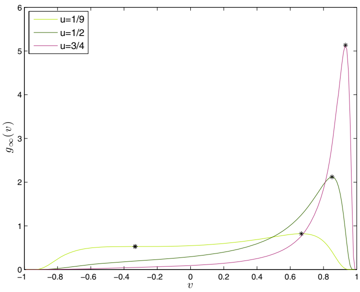

Fig. 3. The stationary profiles g_∞ relative to the value $\lambda = 3/4$ are shown for $u = 1/9$ (yellow line), $u = 1/2$ (green line), $u = 3/4$ (violet line).

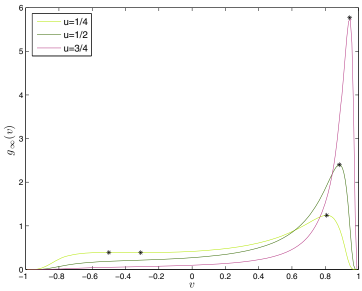

Fig. 4. The stationary profiles g_∞ relative to the value $\lambda = 1$ are shown for $u = 1/4$ (yellow line), $u = 1/2$ (green line), $u = 3/4$ (violet line).

corresponding to 0, instead, is nearly halved with respect to the previous case. Fig. 1 shows that, if $u = 0$, small values of λ, corresponding to $\sigma^2 \leq 1/2\gamma$, namely to small contributes of diffusion in (3), lead to stationary profiles where opinions are near the middle of I. At the opposite, an increase of the value of λ corresponds to stationary profiles where the agents are divided into two groups. As λ increases, the two points of maximum get closer to the extremes of I and the corresponding value of the maxima increases, showing that if the contribute of diffusion is greater than that of compromise extremal opinions tend to prevail.

From now on, we consider values of u different from 0. For symmetry reasons, we only focus on positive values of u. First, we set $\lambda = 1/4$. According to (39) and (38), in this case $\Delta < 0$ regardless of the value of the average opinion and, therefore, the stationary profile $g_\infty(v)$ always has one stationary point, namely a maximum point. Fig. 2 shows the stationary profiles for $u = 1/4$ (yellow line), $u = 1/2$ (green line), and $u = 3/4$ (violet line). The maxima are marked with a black asterisk. From Fig. 2 it can be observed that as the average opinion increases the value of the corresponding maximum also increases, in agreement with the idea that if the average opinion gets closer to 1 (namely, to one of the extremes of the interval I) the opinions of all agents tend to be more concentrated near the value of u.

We now set $\lambda = 3/4$. According to (39) and (38), in this case: $\Delta < 0$ if $|u| < 1/9$; $\Delta = 0$ if $|u| = 1/9$; $\Delta > 0$ if $|u| > 1/9$. Fig. 3 shows the stationary profiles for $u = 1/9$ (yellow line), $u = 1/2$ (green line), $u = 3/4$ (violet line), and the stationary points are marked with a black asterisk. As expected, if $u = 1/9$ the function $g_\infty(v)$ has two stationary point, namely a point of maximum in v_1 and an inflection point in $v_2 = v_3 = -v_1/2$. Greater values of u, instead, lead to a unique stationary point, namely a point of maximum.

In Fig. 4 the stationary profiles for $\lambda = 1$ and for the average opinions $u = 1/4$ (yellow line), $u = 1/2$ (green line), $u = 3/4$ (violet line) are shown. If $\lambda = 1$ then: $\Delta < 0$ if $|u| < \sqrt{2/27}$; $\Delta = 0$ if $|u| = \sqrt{2/27}$; $\Delta > 0$ if $|u| > \sqrt{2/27}$. Therefore,

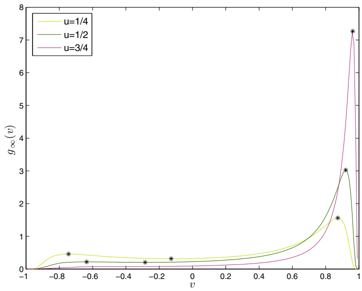

Fig. 5. The stationary profiles g_∞ relative to the value $\lambda = 3/2$ are shown for $u = 1/4$ (yellow line), $u = 1/2$ (green line), $u = 3/4$ (violet line).

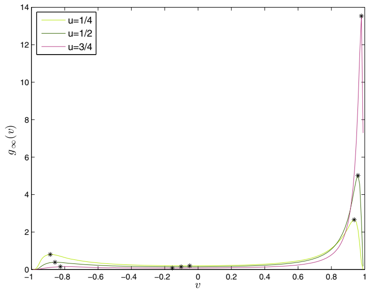

Fig. 6. The stationary profiles g_∞ relative to the value $\lambda = 3$ are shown for $u = 1/4$ (yellow line), $u = 1/2$ (green line), $u = 3/4$ (violet line).

the function $g_\infty(v)$ has three stationary points if $u = 1/4$, while it has only a stationary point if $u = 1/2$ and $u = 3/4$.

Fig. 5 shows the stationary profiles $g_\infty(v)$ for $\lambda = 3/2$. In this case: $\Delta < 0$ if $|u| < \sqrt{2}4/9$; $\Delta = 0$ if $|u| = \sqrt{2}4/9$; $\Delta > 0$ if $|u| > \sqrt{2}4/9$. We consider the same values of u as in the previous case and, since $\sqrt{2}4/9 \simeq 0.61$, it is expected that if $u = 1/4$ and $u = 1/2$ the function $g_\infty(v)$ has three stationary points while if $u = 3/4$ the stationary profile only admits a point of maximum. These results are confirmed in Fig. 5 where $g_\infty(v)$ is shown for $u = 1/4$ (yellow line), $u = 1/2$ (green line), and $u = 3/4$ (violet line).

Finally, Fig. 6 shows the stationary profiles $g_\infty(v)$ for $\lambda = 3$ and $u = 1/4$ (yellow line), $u = 1/2$ (green line), $u = 3/4$ (violet line). According to (39) and (38), in this case $\Delta < 0$ for all the possible values of the average opinion u, and, therefore, $g_\infty(v)$ always has three stationary points.

5 Conclusions

In this paper the temporal evolution of opinion in a multi-agent system is investigated through a kinetic approach. More precisely, we studied the asymptotic behaviour of the opinion distribution on the basis of a model inspired from the molecules interactions in a gas. Assuming that the opinion of each agent can change because of two reasons, namely compromise and diffusion, stationary profiles with different characteristics can be derived as the parameters of the model change. For a particular choice of the compromise function and of the diffusion function, we showed that the asymptotic distribution is characterized by one, two, or three stationary points, depending on the average opinion and on the parameters of the model.

Further analysis on this subject, which also involves simulation results, is currently under investigation. In particular, we are interested in adopting the kinetic framework in scenarios that could use general-purpose industrial strength technology (see, e.g., [14,15]) and in modeling wireless sensor networks for localization purposes (see, e.g., [16,17]).

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