An Improved Grey Forecasting Models: Case in China's Coal Consumption Demand

Chia-Nan Wang and Van-Thanh Phan(✉)

Department of Industrial Engineering and Management,
National Kaohsiung University of Applied Sciences, Kaohsiung, Taiwan
cn.wang@cc.kuas.edu.tw, thanhkem2710@gmail.com

Abstract. In order to improve the application area and the prediction accuracy of classical GM (1, 1) and Non Linear Grey Bernoulli Model (NGBM (1, 1)), a Fourier Grey model FRMGM (1, 1), and Fourier Non Linear Grey Bernoulli Model (abbreviated FRMNGBM (1, 1)) are proposed in this paper. These proposed models were built by using Fourier series to modify their residual values. To verify the effectiveness of these proposed models, the total coal consumption demand in China during period time from 1980 to 2012 was used to exam the forecast performance. The empirical results demonstrated that the accuracy of both GM (1, 1) and NGBM (1, 1) forecasting models after using Fourier series revised their residual error provided more accuracy than original ones. Furthermore, this paper also indicated that the FRMNGBM (1, 1) is the better model with MAPE=0.003.

Keywords: GM (1, 1) · Nonlinear grey bernoulli model · Fourier series · Coal consumption demand · China

1 Introduction

Grey system theory, founded by Prof. Deng in the 1980s [1], is a quantitative method for dealing with grey systems that are characterized by both partially known and partially unknown information [2, 3, 4]. As a vital part of Grey system theory, grey forecasting models with their advantages in dealing with uncertain information and using as few as four data points [5, 6]. Therefore, it has been successful in applied to various fields such as tourism [7, 8], sea transportation [9, 10], financial and economic [11, 12], integrated circuit industry [13, 14]. Especially, in the energy industry GM (1, 1) has also been used [15, 16].

In the recent years, many scholars have been proposed new procedures or new models to improve the precision of GM (1, 1) model and NGBM (1, 1), such as Wang and Phan [17], proposed FRMGM (1, 1) to forecast the GDP growth rate in Vietnam by used Fourier function. Wang et al [18] proposed optimized NGBM (1, 1) model for forecasting the qualified discharge rate of industrial wastewater in China by improved background interpolation value p and exponential value n is put forward in an NGBM (1, 1). Shang Ling [19] provided GAIGM (1, 1) by used genetic algorithm combined with improved GM (1, 1) to forecast the agriculture output from 1998 to 2010. Truong and Ahn [20] investigated a novel Grey model named "Smart Adaptive Grey Model, SAGM (1, 1)", etc.

M. Núñez et al. (Eds.): ICCCI 2015, Part II, LNCS 9330, pp. 544–553, 2015.
DOI: 10.1007/978-3-319-24306-1_53

In order to improve the quality of the forecasting models, in this study, a new approach to minimize the errors obtained from the conventional models is suggested by modifying the residual series with Fourier series. Practical example in coal consumption forecasting shows that the proposed FRMNGBM (1, 1) model has higher performance on model prediction. The remaining of this paper was organized as follows. In section 2, the modeling and methodology of GM (1, 1), NGBM (1, 1), and Fourier Residual Modification of their models was presented. Based on the fundamental function of GM (1, 1), NGBM (1, 1) and new two models FRMGM (1, 1) and FRMNGBM (1, 1) are created by using Fourier function to modified their residual , the case study and empirical results was shown in section 3. Finally, section 4 concluded this paper.

2 Modeling and Methodology

2.1 Classical GM (1, 1) Model

GM (1, 1) is the basic model of Grey forecasting modeling, a first order differential model with one input variable which has been successfully applied in many different researches. It is obtained as the following procedure.

Step 1: Let raw matrix $X^{(0)}$ stands for the *non-negative* original historical time series data

$$X^{(0)} = \{x^{(0)}(t_i)\}, \quad i = 1,2,..., \quad n \tag{1.1}$$

Where $x^{(0)}(t_i)$ is the value at time t_i, and n is the total number of modeling data

Step 2: Construct $X^{(1)}$ by one time accumulated generating operation (1-AGO), which is

$$X^{(1)} = \{x^{(1)}(t_i)\}, \quad i = 1,2,..., n \tag{1.2}$$

Where $$x^1(t_k) = \sum_{i=1}^{k} x^{(0)}(t_i), k = 1,2,..., \quad n \tag{1.3}$$

Step 3: $X^{(1)}$ is a monotonic increasing sequence which is modeled by the first order linear differential equation:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \tag{1.4}$$

Where the parameter *a* is called the developing coefficient and *b* is the named the grey input.

Step 4: In order to estimate the parameter *a* and *b*, Eq. (1.4) is approximated as:

$$\frac{\Delta X^{(1)}(t_k)}{dt_k} + aX^{(1)}(t_k) = b \tag{1.5}$$

Where $$\Delta X^{(1)}(t_k) = x^{(1)}(t_k) - x^{(1)}(t_{k-1}) = x^{(0)}(t_k) \tag{1.6}$$

$$\Delta t_k = t_k - t_{k-1} \tag{1.7}$$

If the sampling time interval is units, then let $\Delta t_k = 1$, Using

$$z^{(1)}(t_k) = px^{(1)}(t_k) + (1-p)x^{(1)}(t_{k-1}), \quad k = 2, 3, \ldots, n \tag{1.8}$$

To replace $X^{(1)}(t_k)$ in Eq. (1.5), we obtain

$$x^{(0)}(t_k) + az^{(1)}(t_k) = b, \quad k = 2, 3, \ldots, n \tag{1.9}$$

Where $z^{(1)}(t_k)$ in Eq. (1.8) is termed background value, and p is production coefficient of the background value in the range of (0, 1), which is traditionally set to 0.5.

Step 5: From the Eq. (1.9), the value of parameter a and b can be estimated using least-square method. That is

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_n \tag{1.10}$$

Where

$$B = \begin{bmatrix} -z^{(1)}(t_2) & 1 \\ -z^{(1)}(t_3) & 1 \\ \ldots\ldots & \ldots\ldots \\ -z^{(1)}(t_n) & 1 \end{bmatrix} \tag{1.11}$$

And $$Y_n = \left[x^{(0)}(t_2), x^{(0)}(t_3), \ldots, x^{(0)}(t_n))\right]^T \tag{1.12}$$

Step 6: The solution of Eq. (1.4) can be obtained after the parameter "a" and "b" have been estimated. That is

$$\hat{x}^{(1)}(t_k) = \left[\left(x^{(0)}(t_1) - \frac{b}{a}\right)e^{-a(t_k - t_1)} + \frac{b}{a}\right], k = 1, 2, 3, \ldots \tag{1.13}$$

Step 7: Applying inverse accumulated generating operation (IAGO) to $\hat{x}^{(1)}(t_k)$, the predicted datum of $x^{(0)}(t_k)$ can be estimated as:

$$\begin{cases} \hat{x}^{(0)}(t_1) = x^{(0)}(t_1) & (1.14) \\ \hat{x}^{(0)}(t_k) = \hat{x}^{(1)}(t_k) - \hat{x}^{(1)}(t_{k-1}) & (1.15) \end{cases}, \quad k = 2, 3, \ldots$$

2.2 Nonlinear-Grey Bernoulli Model "NGBM (1, 1)"

The procedures of deriving NGBM are as follows:

Step 1: Let raw matrix $X^{(0)}$ stands for the non-negative original historical time series data

$$X^{(0)} = \left\{x^{(0)}(t_i)\right\}, \quad i = 1, 2, \ldots, n \tag{2.1}$$

Where $x^{(0)}(t_i)$ corresponds to the system output at time t_i, and n is the total number of modeling data.

Step 2: Construct $X^{(1)}$ by one time accumulated generating operation (1-AGO), which is

$$X^{(1)} = \{x^{(1)}(t_i)\},\ i = 1,2,...,\ n \tag{2.2}$$

Where $x^1(t_k) = \sum_{i=1}^{k} x^{(0)}(t_i), k = 1,2,...,\ n$ (2.3)

Step 3: $X^{(1)}$ is a monotonic increasing sequence which is modeled by the Bernoulli differential equation:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b\left[X^{(1)}\right]^r \tag{2.4}$$

Where the parameter a is called the developing coefficient and b is the named the grey input and r is any real number excluding r=1.

Step 4: In order to estimate the parameter a and b, Eq. (2.4) is approximated as:

$$\frac{\Delta X^{(1)}(t_k)}{dt_k} + aX^{(1)}(t_k) = b\left[X^{(1)}(t_k)\right]^r \tag{2.5}$$

Where $\Delta X^{(1)}(t_k) = x^{(1)}(t_k) - x^{(1)}(t_{k-1}) = x^{(0)}(t_k)$
(2.6)

$$\Delta t_k = t_k - t_{k-1} \tag{2.7}$$

If the sampling time interval is units, then let $\Delta t_k = 1$, Using

$$z^{(1)}(t_k) = px^{(1)}(t_k) + (1 - p)x^{(1)}(t_{k-1}),\ k = 2,3,..,\ n \tag{2.8}$$

To replace $X^{(1)}(t_k)$ in Eq. (2.5), we obtain

$$x^{(0)}(t_k) + az^{(1)}(t_k) = b\left[z^{(1)}(t_k)\right]^r,\ k = 2,3,...,\ n \tag{2.9}$$

Where $z^{(1)}(t_k)$ in Eq. (2.8) is termed background value, and p is production coefficient of the background value in the range of (0, 1), which is traditionally set to 0.5.

Step 5: From the Eq. (2.9), the value of parameter a and b can be estimated using least- square method. That is

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_n \tag{2.10}$$

$$\text{Where } B = \begin{bmatrix} -z^{(1)}(t_2) & (z^{(1)}(t_2))^r \\ -z^{(1)}(t_3) & (z^{(1)}(t_3))^r \\ \ldots\ldots & \ldots\ldots \\ -z^{(1)}(t_n) & (z^{(1)}(t_n))^r \end{bmatrix} \tag{2.11}$$

$$\text{And } Y_n = \left[x^{(0)}(t_2),\ x^{(0)}(t_3),\ldots,\ x^{(0)}(t_n))\right]^T \tag{2.12}$$

Step 6: The solution of Eq. (2.4) can be obtained after the parameter a and b have been estimated. That is

$$\hat{x}^{(1)}(t_k) = \left[\left(x^{(0)}(t_1)^{(1-r)} - \frac{b}{a}\right)e^{-a(1-r)(t_k - t_1)} + \frac{b}{a}\right]^{\frac{1}{1-r}},\ r \neq 1,\quad k = 1,2,3,\ldots \tag{2.13}$$

Step 7: Applying inverse accumulated generating operation (IAGO) to $\hat{x}^{(1)}(t_k)$, the predicted datum of $x^{(0)}(t_k)$ can be estimated as:

$$\begin{cases} \hat{x}^{(0)}(t_1) = x^{(0)}(t_1) & (2.14) \\ \hat{x}^{(0)}(t_k) = \hat{x}^{(1)}(t_k) - \hat{x}^{(1)}(t_{k-1}) & (2.15) \end{cases},\ k = 2,3,\ldots$$

2.3 Fourier Residual Modification

In order to improve the accuracy of forecasting models, this paper proposed the Fourier series [7, 17] to modify the residuals in GM (1, 1) and NGBM (1, 1) models which reduces the values of MAPE. The overall procedure to obtain the modified model is as the followings:

Let x is the original series of n entries and v is the predicted series (obtained from GM (1, 1) or NGBM (1, 1). Based on the predicted series v, a residual series named ε is defined as:

$$\varepsilon = \{\varepsilon(k)\},\ k = 2,3,\ldots n \tag{16}$$

$$\text{Where } \varepsilon(k) = x(k) - v(k),\ k = 2,3,\ldots n \tag{17}$$

Expressed in Fourier series, $\varepsilon(k)$ is rewritten as:

$$\hat{\varepsilon}(k) = \frac{1}{2}a_{(0)} + \sum_{i=1}^{Z}\left[a_i \cos\left(\frac{2\pi i}{n-1}(k)\right) + b_i \sin\left(\frac{2\pi i}{n-1}(k)\right)\right],\ k = 1,2,3,.,\ n \tag{18}$$

Where $Z = (\frac{n-1}{2}) - 1$ called the minimum deployment frequency of Fourier series [7] and only take integer number, therefore, the residual series is rewritten as:

$$\varepsilon = PC \tag{19}$$

Where

$$P=\begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \dots & \cos\left(\frac{2\pi\times Z}{n-1}\times 2\right) & \sin\left(\frac{2\pi\times Z}{n-1}\times 2\right) \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \dots & \cos\left(\frac{2\pi\times Z}{n-1}\times 3\right) & \sin\left(\frac{2\pi\times Z}{n-1}\times 3\right) \\ \dots & \dots & & \dots & \dots & \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times n\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times n\right) & \dots & \cos\left(\frac{2\pi\times Z}{n-1}\times n\right) & \sin\left(\frac{2\pi\times Z}{n-1}\times n\right) \end{bmatrix} \quad (20)$$

And $C = [a_0, a_1, b_1, a_2, b_2, \dots, a_Z, b_Z]$ (21)

The parameter a_0, a_1, b_1, a_2, b_2… a_Z, b_Z are obtained by using the ordinary least squares method (OLS) which results in the equation of:

$$C = (P^T P)^{-1} P^T [\varepsilon]^T \quad (22)$$

Once the parameters are calculated, the modified residual series is then achieved based on the following expression:

$$\hat{\varepsilon}(k) = \frac{1}{2}a_{(0)} + \sum_{i=1}^{Z}\left[a_i \cos\left(\frac{2\pi i}{n-1}(k)\right) + b_i \sin\left(\frac{2\pi i}{n-1}(k)\right)\right] \quad (23)$$

From the predicted series v and $\hat{\varepsilon}$, the Fourier modified series $\hat{v}$ of series v is determined by:

$$\hat{v} = \{\hat{v}_1, \hat{v}_2, \hat{v}_3, \dots, \hat{v}_k, \dots, \hat{v}_n\} \quad (24)$$

Where

$$\hat{v} = \begin{cases} \hat{v}_1 = v_1 \\ \hat{v}_k = v_k + \hat{\varepsilon}_k \quad (k = 2,3,\dots, n) \end{cases} \quad (25)$$

2.4 Evaluative Precision of Forecasting Models

In order to evaluate the forecast capability of these models, Means Absolute Percentage Error (MAPE) index was used to verify the performance and reliability of forecasting technique [21]. It is expressed as follows:

$$MAPE = \frac{1}{n}\sum_{k=2}^{n}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\ \% \quad (26)$$

Where $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$ are actual and forecasting values in time period k, respectively, and n is the total number of predictions. Wang et al [22] interprets the MAPE results as a method to judge the accuracy of forecasts, where more than 10% is an inaccurate forecast, 5%-10% is a reasonable forecast, 1%-5% is a good forecast, and less than 1% is a highly accurate forecast.

3 Simulation and Empirical Results

3.1 Data Analysis

The historical data of total coal consumption in China for 1980-2012 are obtained from the yearly statistical data published on the website of U.S. Energy Information Administration (EIA) [23]. All historical data was described in Figure 1, and the unit of coal consumption is measured in Quadrillion Btu (Q.T).

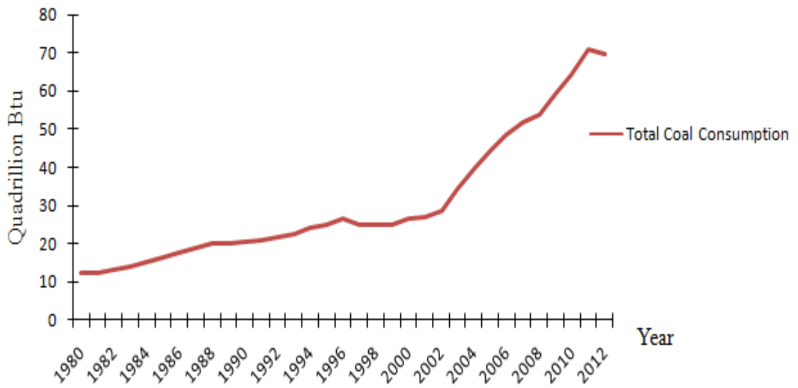

Fig. 1. Time series plots of China's coal consumption 1980-2012

Figure 1 clearly showed that the consumption demand of coal quickly increasing. The coal consumption reached to 70 Quadrillion Btu in 2012, increased 50 Quadrillion Btu compared with year of 1980.

To find out the parameters in GM (1, 1) and NGBM (1, 1) model as well as modified model of their models, computer software called Microsoft Excel is used. Beside a basic function in excel, Excel software also offers two useful functions named Mmult (array 1, array 2) to return the matrix product of two relevant arrays and Minverse (array) to return the inverse matrix. These two functions are of great help to find out the values of parameters in GM (1, 1), NGBM (1, 1) and Fourier residual modification.

3.2 GM (1, 1) Model for the Coal Consumption Demand

From historical data in the website of EIA [23] and based on the algorithm expressed in section 2.1, the coefficient parameter a and b in GM (1, 1) for the coal consumption is calculated as:

a= -0.0608, b= 8.7482 and the GM (1, 1) model for coal consumption demand are then:

$$\hat{x}^{(1)}(k) = 156.14 \times e^{-0.0608(k-1)} - 143.84$$

The evaluation indexes of GM (1, 1) model are also listed in Table 1. The residual series of GM (1, 1) is modified with Fourier series as illustrated in section 3.3

3.3 Modified GM (1, 1) Model by Fourier Series

The residual series of GM (1, 1) obtained in section 3.2 is now modified with Fourier series as per the algorithm stated in section 2.3. With this modified series, the forecasted values demand of coal consumption based on Fourier residual modified GM (1, 1) model FRMGM (1, 1) are calculated based on the equation (24). The evaluation index of FRMGM (1, 1) is summarized in Table 1.

3.4 NGBM (1, 1) Model for the Coal Consumption Demand

As the way to calculate and based on the mathematical algorithm expressed in section 2.2, the coefficient parameter *a*, *b* and the power of *r* in NGBM (1, 1) for the coal demand are calculated as: *a= -0.0659*, *b=25.1403*, and *r=-0.2374*

And the NGBM (1, 1) model for coal consumption is then:

$$\hat{x}^{(1)}(k)=\left[(403.7017)e^{0.06388(1+0.2374)(k-1)}-381.3896\right]^{\frac{1}{1+0.2374}}$$

The evaluation index of NGBM (1, 1) model is also listed in Table 1. The residual series of NGBM (1, 1) is modified with Fourier series as illustrated in section 3.5

3.5 Modified NGBM (1, 1) Model by Fourier Series

The residual series of NGBM (1, 1) obtained in section 3.4 is now modified with Fourier series as per the algorithm stated in section 2.3. With this modified series, the forecasted values of coal consumption based on Fourier residual modified GM (1, 1) model FRMGM (1, 1) are calculated based on the equation (24). The evaluation index of FRMNGBM (1, 1) is summarized in Table 1.

Table 1. Summary of evaluation indexes of model accuracy

Model	MAPE (%)	Forecasting accuracy (%)	Performance
GM (1,1) with p =0.5	13.79	86.21	Good
NGBM(1, 1) with p=0.5 and r=*-0.237*	10.39	89.61	Good
FRMGM (1, 1) with p =0.5	**0.066**	**99.93**	**Excellent**
FRMNGBM (1 ,1) with p=0.5 and r=*-0.237*	**0.003**	**99.997**	**Excellent**

Table 1 shows all evaluation indexes of each model of GM (1, 1), FRMGM (1, 1), NGBM (1, 1) and FRMNGBM (1, 1) with its performance in forecasting coal consumption. The empirical results clearly show that the accuracy of both GM (1, 1) and NGBM (1, 1) forecasting models after using Fourier series revised their residual error provided more accuracy than original ones. Furthermore, this paper also indicated that the forecasting ability of FRMNGBM (1, 1) model is the better model with MAPE=0.003 (less much than FRMGM (1, 1) with MAPE =0.066) for forecasting the coal consumption demand in China. More detail was illustrated in figure 2.

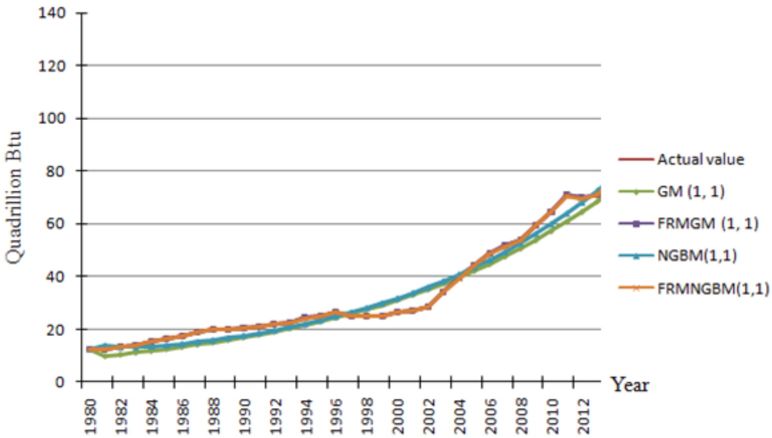

Fig. 2. Curves of actual values and simulated values using four models for coal consumption forecasting

4 Conclusion

Grey forecasting model is one of the most important parts of grey theory system. In the recent years, many scholars have been proposed new procedures or new models to improve the precision of GM (1, 1) model and NGBM (1, 1) with different ways, in this paper proposed an another way in order to improve the accuracy level of their performance by modify the residuals of GM (1, 1) and NGBM (1, 1) by Fourier series. This results displayed robust models in term of MAPE, compared with among them, FRMNGBM (1, 1) is the better model in forecast with the MAPE for forecasting the demand of cold consumption in China is 0.003%. Future researchers can be applied the proposed model on the other industries to forecast performance.

References

1. Deng, J.L.: Control problems of grey systems. Systems and Control Letters **5**, 288–294 (1982)
2. Emil, S., Camelia, D.: Complete analysis of bankruptcy syndrome using grey systems theory. Grey Systems: Theory and Application **1**, 19–32 (2011)
3. Hong, W., Fuzhong, C.: The application of grey system theory to exchange rate prediction in the post-crisis era. International Journal of Innovative Management **2**(2), 83–89 (2011)
4. Chen, H.J.: Application of grey system theory in telecare. Computers in Biology and Medicine **41**(5), 302–306 (2011)

5. Yi, L., Sifeng, L.: A historical introduction to grey system theory. In: IEEE International Conference on System, Man And Cybernetics, pp. 2403–2408 (2004)
6. Sifeng, L., Forrest, J., Yingjie, Y.: A brief introduction to grey system theory. In: 2011 IEEE International Conference on IEEE Grey Systems and Intelligent Services (GSIS) (2011)
7. Huang, Y.L., Lee, Y.H.: Accurately forecasting model for the stochastic volatility data in tourism demand. Modern economy **2**(5), 823–829 (2011)
8. Chu, F.L.: Forecasting Tourism Demand in Asian-Pacific Countries. Annual of Tourism Research **25**(3), 597–615 (1998)
9. Jiang, F., Lei, K.: Grey Prediction of Port Cargo Throughput Based on GM(1,1, a) Model. Logistics Technology **9**, 68–70 (2009)
10. Guo, Z.J., Song, X.Q., Ye, J.: A Verhulst model on time series error corrected for port Cargo Throughput forecasting. Journal of the Eastern Asia Society for Transportation Studies **6**, 881–891 (2005)
11. Kayacan, E., Ulutas, B., Kaynak, O.: Grey system theory-based models in time series prediction. Expert Systems with Applications **37**, 1784–1789 (2010)
12. Askari, M., Askari, H.: Time Series Grey System Prediction-based Models: Gold Price Forecasting. Trends in Applied Sciences Research **6**, 1287–1292 (2011)
13. Tsai, L.C., Yu, Y.S.: Forecast of the output value of Taiwan's IC industry using the Grey forecasting model. International Journal of Computer Applications in Technology **19**(1), 23–27 (2004)
14. Hsu, L.C.: Applying the grey prediction model to the global integrated circuit industry. Technological forecasting and Social change **70**, 563–574 (2003)
15. Hsu, C.C., Chen, C.Y.: Application of improved grey prediction model for power demand forecasting. Energy Conversion and management **44**, 2241–2249 (2003)
16. Kang, J., Zhao, H.: Application of Improved Grey Model in Long-term Load Forecasting of Power Engineering. Systems Engineering Procedia **3**, 85–91 (2012)
17. Wang, C.N., Phan, V.T.: An enhancing the accurate of grey prediction for GDP growth rate in Vietnam. In: 2014 International Symposium on Computer, Consumer and Control (IS3C), pp. 1137–1139 (2014). doi:10.1109/IS3C.2014.295
18. Wang, Z.X., Hipel, K.W., Wang, Q., He, S.W.: An optimized NGBM (1, 1) model for forecasting the qualified discharge rate of industrial wastewater in China. Applied Mathematical Modelling **35**, 5524–5532 (2011)
19. Ou, S.L.: Forecasting agriculture output with an improved grey forecasting model based on the genetic algorithm. Computers and Electronics in Agriculture **85**, 33–39 (2012)
20. Truong, D.Q., Ahn, K.K.: An accurate signal estimator using a novel smart adaptive grey model SAGM (1, 1). Expert Systems with Applications **39**(9), 7611–7620 (2012)
21. Makridakis, S.: Accuracy measures: Theoretical and practical concerns. International Journal of Forecasting **9**, 527–529 (1993)
22. Wang, C.N., Phan, V.T.: An improvement the accuracy of grey forecasting model for Cargo Throughput in international commercial Ports of Kaohsiung. International Journal of Business and Economics Research **3**(1), 1–5 (2014). doi:10.11648/j.ijber.20140301.11
23. Website of the U.S. Energy Information Administration [Online]. http://www.eia.gov/