Distribution Supply Chain Inventory Planning under Uncertainty

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Abstract. This paper considers a multi-echelon inventory/distribution system formed by N-warehouses and M-retailers that manages a set of diverse products in a dynamic environment. Transshipment at both regional warehouses and retailers levels is allowed. A mixed integer linear programming model is developed, where product demand at the retailers is assumed to be not known. The problem consists of determining the optimal reorder policy by defining a new concept of robust retailer order in a two level programming approach, which minimizes the overall system cost, including ordering, holding in stock and in transit, transportation, transshipping and lost sales costs. A case study based on a real retailer distribution chain is solved.

Keywords: Supply chain management, inventory planning, mixed integer linear programming, uncertainty, scenario planning approach.

1 Introduction

Multi-warehouse/multi-retailer/multi-product and multi-period distribution supply chains are complex systems through which flows of products have to be correctly managed along with the inventory policies practice within each entity. In order to overcome this challenging problem, the present paper addresses the inventory planning problem in such systems by using exact optimization methods.

Research on uncertainty can be classified by the type of approach used to represent it. Two types of approaches are identified: probabilistic approach and the scenario planning approach. The choice of the best method is context-dependent, with no single theory being sufficient to model all kinds of uncertainty [1]. In this paper, our focus is the scenario planning approach applied to a distribution supply chain (SC), where future demand is described through discrete scenarios.

Scenario planning approach attempts to capture uncertainty by representing it in terms of a moderate number of discrete realizations of the stochastic quantities, constituting distinct scenarios. The objective is to find robust solutions that perform well under all scenarios. Mohamed [2] used a scenario planning approach to decide on the design of a production and distribution network that operates under varying exchange rates. Tsiakis *et al.* [3] used a scenario planning approach to design a multi-echelon distribution supply chain network under demand uncertainty using a mixed-integer

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linear programming model. Combining scenario planning with supply-chain planning achieves the best of both worlds, which leads to long-term competitive advantage, as said in [4]. More recently, [5] and [6] adopted a scenario planning approach for handling the uncertainty in product demands. They proposed an optimal design of supply chain networks using a mixed-integer linear programming (MILP).

The scenario planning approach is a two level approach that has been extensively applied in design and planning problems, whereas we are focusing in an operational problem. Thus, a new concept is derived which is the robust ordering policy that should be adopted by each retailer. In this sense, the order quantity and timing has to be uncertainty free, but considering the different realizations of the uncertain parameters. The above identified opportunity is explored along the present paper where a generic model is proposed that determines both the inventory and distribution plans of the retailers and warehouses that minimize the distribution supply chain total costs under an uncertain demand environment.

2 Inventory Planning Mathematical Model under Uncertainty

The distribution supply chain inventory planning problem presented is formulated as a MILP model. The scenario approach used follows the work of [3], where demand can be described by a set of demand scenarios including both optimistic and pessimistic demands in a given time period.

We consider that retailers have own management autonomy and, therefore, have to plan their orders deterministically - robust retailer order. One order quantity placed by one retailer may be replenished through multiple flows.

The indices, constants, sets, parameters and variables used in the model formulation are defined using the following notation:

Indices

i - product; j, k, l, m - entity node; s - product demand scenario; t - time period

Constants

NP number of products; NW number of regional warehouses; NR number of retailers; NT number of time periods; NS number of product demand scenarios

Sets

$i \in P = \{1,2,...,NP\}$ products

$j,k,l,m \in I = \{0,1,2,...,NW, NW+1, NW+2,..., NW+NR\}$ SC nodes

$t \in T = \{1,2,...,NT\}$ time periods

$s \in S = \{1,2,...,NS\}$ scenarios for uncertain product demands

$W = \{1,2,...,NW\}, W \subset I$ regional warehouses

$R = \{1,2,...,NR\}, R \subset I$ retailers

$W_o = \{0\}, W_o \subset I$ central warehouse

$DN = \{1,2,...,NW, NW+1, NW+2,..., NW+NR\}, DN \subset I$ demand nodes

$SN = \{0,1,2,...,NW\}, SN \subset I$ supply nodes

Parameters

BGM a large positive number; CD_{ikst} customer demand of the product *i* at *k* in scenario *s* in time period *t;* HOC_{ij} unitary holding cost of the product *i* at *j* per time period; HTC_{ijk} unitary holding in transit cost of the product *i* from *j* to *k*; Ito_{ij} initial inventory level of the product *i* at *j;* LSC_{ijt} unitary lost sale cost of the product *i* at *j* in time period *t;* LTT_{jk} transportation lead time from *j* to *k;* OC_{ij} ordering cost of the product *i* at *j*; SS_{ij} safety stock level of the product *i* at *j*; STC_{jt} storage capacity at entity *j* in time period *t;* $TRACMAX_{jk}$ maximum transportation capacity from *j* to *k;* $TRACMIN_{jk}$ minimum transportation capacity from *j* to *k;* TRC_{ijk} unitary transportation cost of the product *i* from *j* to *k* ; φ_s probability of customer demand scenario *s.*

Non-Negative Continuous Variables

FI_{ijst} inventory of product *i* at *j* in scenario *s* at time period *t;* LS_{ijst} lost sales of product *i* at *j* in scenario *s* at time period *t*; RO_{ikt} robust retailer order of product *i* at *k* at time period *t*; SQ_{ijkst} shipping quantity of product *i* from *j* to *k* in scenario *s* in time period *t.*

Binary Variables

$BV1_{ijst}$ = 1 if a regional warehouse order of product *i* is placed by *j* in scenario *s* in time period *t*, 0 otherwise; $BV2_{ikt}$ = 1 if a robust retailer order of product *i* is placed by *k* in time period *t*, 0 otherwise.

The aim is to minimize the expected value of the total cost considering all scenarios and their probability of occurrence. This leads to the objective function (1).

$$\begin{aligned} \textit{Minimize total expected cost} = & \sum_{i\in P}\sum_{k\in R}\sum_{t\in T} OC_{ik} \times BV2_{ikt} \\ & + \sum_{s\in S}\varphi_s \times \Bigg(\sum_{i\in P}\sum_{j\in W}\sum_{t\in T} OC_{ij} \times BV1_{ijst} + \sum_{i\in P}\sum_{j\in I}\sum_{t\in T}\left(HOC_{ij} \times FI_{ijst} + LSC_{ijt} \times LS_{ijst}\right) \\ & + \sum_{i\in P}\sum_{j\in I}\sum_{k\in I}\sum_{t\in T}\left(\left(HTC_{ijk} \times LTT_{jk} + TRC_{ijk}\right) \times SQ_{ijkst}\right)\Bigg) \end{aligned} \tag{1}$$

The minimization of system costs is subject to the following constraints:

$$\begin{aligned} FI_{ijs1} = & Ito_{ij} + SQ_{i,0,j,s,t-LTT_{0j} | LTT_{0j}=0} - \sum_{k\in R} SQ_{ijks1} - \sum_{l\in W \wedge l\neq j} SQ_{ijls1} \\ & + \sum_{l\in W \wedge l\neq j} SQ_{i,l,j,s,t-LTT_{lj} | LTT_{lj}=0}, i\in P, j\in W, s\in S, t=1 \end{aligned} \tag{2}$$

$$FI_{ijst} = FI_{i,j,s,t-1} + SQ_{i,0,j,s,t-LTT_{0j}|LTT_{0j}<t} - \sum_{k\in R} SQ_{ijkst} - \sum_{l\in W \wedge l\neq j} SQ_{ijlst}$$
$$+ \sum_{l\in W \wedge l\neq j} SQ_{i,l,j,s,t-LTT_{lj}|LTT_{lj}<t}, i\in P, j\in W, s\in S, t\in T\setminus\{1\} \tag{3}$$

$$RO_{ikt} = \sum_{j\in W} SQ_{i,j,k,s,t-LTT_{jk}|LTT_{jk}<t}$$
$$+ \sum_{m\in R\wedge m\neq k} SQ_{i,m,k,s,t-LTT_{mk}|LTT_{mk}<t}, i\in P, k\in R, s\in S, t\in T \tag{4}$$

$$FI_{iks1} = Ito_{ik} + RO_{ik1} - (CD_{iks1} - LS_{iks1}) - \sum_{m\in R\wedge m\neq k} SQ_{ikms1}, i\in P, k\in R, s\in S, t=1 \tag{5}$$

$$FI_{ikst} = FI_{i,k,s,t-1} + RO_{ikt} - (CD_{ikst} - LS_{ikst})$$
$$- \sum_{m\in R\wedge m\neq k} SQ_{ikmst}, i\in P, k\in R, s\in S, t\in T\setminus\{1\} \tag{6}$$

$$SQ_{i0jst} + \sum_{l\in W\wedge l\neq j} SQ_{iljst} \le BGM \times BV1_{ijst}, i\in P, j\in W, s\in S, t\in T \tag{7}$$

$$\sum_{s\in S} \varphi_s \times \left(\sum_{j\in W} SQ_{ijkst} + \sum_{m\in R\wedge m\neq k} SQ_{imkst} \right) \le BGM \times BV2_{ikt}, i\in P, k\in R, t\in T \tag{8}$$

$$\sum_{i\in P} FI_{ijst} \le STC_{jt}, j\in DN, s\in S, t\in T \tag{9}$$

$$\sum_{i\in P} SQ_{ijkst} \le TRACMAX_{jk}, j\in DN, k\in DN, j\neq k, s\in S, t\in T \tag{10}$$

$$TRACMIN_{jk} \le \sum_{i\in P} SQ_{ijkst}, j\in DN, k\in DN, j\neq k, s\in S, t\in T \tag{11}$$

$$SS_{ij} \le FI_{ijst}, i\in P, j\in DN, s\in S, t\in T \tag{12}$$

$$FI_{ijst}, LS_{ijst}, RO_{ikt}, SQ_{ijkst} \ge 0, i\in P, j\in I, k\in I, s\in S, t\in T \tag{13}$$

$$BV1_{ijst}, BV2_{ikt} \in \{0,1\}, i\in P, j\in W, k\in R, s\in S, t\in T \tag{14}$$

Objective function (1) involves: robust retailer ordering cost (first term) and regional warehouse ordering cost (second term); holding cost at both stages of the supply chain, regional warehouses and retailers and lost sales cost (third term); and, finally, the holding in transit cost and transportation cost (fourth term). The second, third and fourth terms are affected by scenarios *s*. For $t = 1$ the inventory of product *i* at regional warehouses *j* by product demand *s* is given by constraint (2). For the remaining time periods one should use constraint (3). A robust retailer order quantity is introduced though the definition of a new variable RO_{ikt} and another binary variable is needed for robust retailer order $BV2_{ikt}$. At retailers, the robust retailer order (shipment and transshipment) quantity is given by equation (4). For $t = 1$ the inventory of product *i* at the retailers *k* is given by constraint (5). Constraint (6) is applicable for the remaining time periods. If the transportation amount between two nodes is not zero, the binary variable $BV1_{ijst}$ equals 1, constraint (7). In constraint (8), the left hand side represents the robust retailer order quantity. The total inventory stored at any node must respect the storage capacity - constraint (9). At any time period t, the sum of the shipping quantity of each product i must respect the transportation minimum and maximum limits - constraints (10) and (11). Constraint (12) ensures that the inventory of each product i must be higher or equal than the required safety stock. The model uses non-negative continuous variables (13) and binary variables (14).

3 Inventory Planning Case Study

A Retail Company case study is here analyzed using the proposed inventory planning policy. The GAMS 23.5 modeling language combined with the CPLEX 12.2 were used to solve the problem in hand. An Intel CORE i5 CPU 2.27GHz and 4GB RAM was utilized. The stopping criteria were either a computational time limit of 7200 s or the determination of the optimal solution.

The distribution supply chain involves one central warehouse, two regional warehouses and four retailers. Three family types of products are considered. The maximum storage capacity of regional warehouses and retailers is of 500 units. The storage capacity of central warehouse is unlimited. Two transportation options are considered: transportation quantities between 0 and 500 units (SQ 0-500) and transportation quantities between 0 and 30 units (SQ 0-30). A seven period planning horizon is used. Lead time among retailers (for the transshipment operation) is assumed equal to one time period. The cost parameters are (in Euro) i) the ordering cost is 20, ii) the holding costs are 0.2 (warehouses) and 0.6 (retailers), iii) the holding in transit costs are 0.3 (from central warehouse to regional warehouses and transshipment between warehouses) and 0.9 (between warehouses and retailers or transshipment between retailers) and iv) the lost sales cost is 25. Tables from 1 to 4 present the remaining implemented parameters' values.

Table 1 Unitary product transportation costs (TRC) between entities (euro)

	W1	W2	R1	R2	R3	R4
W0	0.55	0.22	-	-	-	-
W1	-	0.7	0.22	0.2	0.32	0.38
W2	0.7	-	0.68	0.52	0.34	0.1
R1	-	-	-	0.2	0.8	1.3
R2	-	-	0.2	-	0.3	1.0
R3	-	-	0.8	0.3	-	0.36
R4	-	-	1.3	1.0	0.36	-

Three probabilities of product demand scenarios are considered, i. e., $\varphi_1 = 0.5$, $\varphi_2 = 0.3$, $\varphi_3 = 0.2$. The first product demand scenario is considered in table 3 – base scenario. The product demand for the second scenario, is a pessimist one, and for the third scenario, is an optimistic one, which present values lower and higher, respectively, than the base scenario. Due to space limitations, their values are not presented.

Table 2 Initial inventory level (Ito)/Safety Stock (SS) on warehouses and retailers (unit).

	W1	W2	R1	R2	R3	R4
Product1	45/14	30/14	24/7	22/3	20/3	18/3
Product2	15/11	11/11	16/2	14/2	12/2	10/2
Product3	11/8	9/8	8/2	4/2	6/1	9/1

Table 3 Customer Demand at the retailers of product1/product2/product3 in each period (unit).

	R1	R2	R3	R4	Total
Period1	12/4/3	11/3/7	14/6/9	3/1/2	40/14/21
Period2	10/12/2	9/11/9	6/1/0	15/9/5	40/33/16
Period3	14/2/4	13/1/1	9/11/1	9/5/1	45/19/7
Period4	8/14/1	7/13/3	11/2/6	8/4/6	34/33/16
Period5	16/8/7	15/9/4	20/10/2	10/6/8	61/33/21
Period6	6/10/5	5/5/5	0/5/3	7/2/9	18/22/22
Period7	18/6/6	17/7/2	10/7/6	11/8/3	56/28/17
Total	84/56/28	77/49/31	70/42/27	63/35/34	294/182/120

Table 4 Lead Time Transportation (LTT) of product (time period).

	W1	W2	R1	R2	R3	R4
W0	2	1	-	-	-	-
W1	-	1	1	1	1	1
W2	1	-	2	2	1	0

The aim of such study is to propose an inventory planning that can handle with all three scenarios while minimizing the total expected cost. The most representative results are shown in table 5, where the total expected cost and the global service level are illustrated. It can be seen that the total expected cost grows from SQ 0-500 to SQ 0-30. For the global service level, generally SQ 0-500 returns a higher service level than SQ 0-30.

Total lost sales per product demand scenario and per product, for both settings for case study under uncertain product demands are presented in table 6. The optimistic product demand scenario (s3) leads to the worst results, as it has the higher number of lost sales. The pessimistic product demand scenario (s2) results lead to be best lost sale solution.

Table 5 Total expected cost and global service level per retailer and per product1/ product2/ product3 (aggregated on time horizon) under uncertain product demands.

	SQ 0-500	SQ 0-30
Total expected cost	2398.06	2598.90
Retailer 1	1/0.99/0.98	1/0.98/0.97
Retailer 2	1/0.99/0.82	0.98/0.99/0.82
Retailer 3	1/0.99/0.85	1/0.99/0.82
Retailer 4	1/1/1	1/1/1

Table 7 shows the total robust retailer order (shipment and transshipment) quantity per product demand scenario and per product, which is aggregated on time horizon and on retailers' echelon, for both transportation options under uncertain product demands. The total robust retailer order quantity is scenario independent. However, the shipment quantity (from the regional warehouses) and the transshipment quantity (from the others retailers) are scenario dependent. Note that both transportation quantities (shipment and transshipment) are capacity limited. In general, the pessimistic scenario (s2) makes more use of transshipment than the others. Regarding the total number of robust retailer orders per product1/product2/product3 these are 11/7/7 and 14/11/9 respectively for SQ 0-500 and SQ 0-30. It is an expected result, since with more constrained distribution flows, orders sizes are more restricted leading to the need of being replenished through a higher number of orders.

Table 6 Total lost sales per product demand scenario and per product1/ product2/ product3 (aggregated on time horizon and on retailers' echelon) (unit).

	SQ 0-500	SQ 0-30
s1	0/0/9	1/0/10
s2	0/0/5	0/0/5
s3	0/7/24	5/11/28

Table 7 Total robust retailer order (shipment and transshipment) quantity per product demand scenario and per product1/ product2/ product3 (aggregated on time horizon and on retailers' echelon) under uncertain product demands (unit).

	SQ 0-500			SQ 0-30		
	shipment	transshipment	total	shipment	transshipment	total
s1	226/138/90	78/74/58	304/212/148	225/138/91	56/50/47	281/188/138
s2	172/85/54	132/127/94	304/212/148	172/93/58	109/95/80	281/188/138
s3	282/187/131	22/25/17	304/212/148	277/183/127	4/5/11	281/188/138

4 Conclusions

This paper proposes a generic inventory planning model for a multi-period/multi-warehouse/multi-retailer/multi-product distribution supply chain. An inventory and distribution plan is obtained, which minimizes the total costs under demand uncertainty through the determination of a robust retailer order. The definition of the robust retailer order concept can be considered the main contribution of this paper.

Future research should include further detailed validation of the proposed model through the study of more complex distribution supply chain structures over larger periods of time with more products. The definition of the safety stock levels and the comparisons between centralized and decentralized systems under uncertainty accounting for risk pooling advantages should also be the focus of further research.

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