Chapter 7
How Much Interconnected Should Networks be for Cooperation to Thrive?

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Abstract While the consensus is that interconnectivity between networks does promote cooperation by means of organizational complexity and enhanced reciprocity that is out of reach on isolated networks, we here address the question just how much interconnectivity there should be. The more the better according to naive intuition, yet we show that in fact only an intermediate density of sufficiently strong interactions between networks is optimal for the evolution of cooperation. This is due to an intricate interplay between the heterogeneity that causes an asymmetric strategy flow because of the additional links between the networks, and the independent formation of cooperative patterns on each individual network. Presented results are robust to variations of the strategy updating rule, the topology of interconnected networks, and the governing social dilemma, and thus indicate a high degree of universality. We also outline future directions for research based on coevolutionary games and survey existing work.

7.1 Introduction

Network reciprocity is amongst the most well-known mechanisms that may sustain cooperation in evolutionary games that constitute a social dilemma [1]. It was discovered by Nowak and May [2], who observed that on structured populations cooperators can aggregate into compact clusters and so avoid being wiped out

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by defectors. Although the mechanism may not work equally well for all social dilemmas [3], and recent empirical evidence based on large-scale economic experiments indicate that it may be compromised or fail altogether [4, 5], there is still ample interest in understanding how and why networks influence the evolution of cooperation. Recent reviews are a testament to the continued liveliness of this field of research [6–9].

Following the explorations of evolutionary games on individual small-world [10–14], scale-free [15–29], coevolving [30–35], hierarchical [36] and bipartite [37] networks, the attention has recently been shifting towards interconnected networks [38–42]. The latter have been put into the spotlight by Buldyrev et al. [43], showing that even seemingly irrelevant changes in one network can have catastrophic and very much unexpected consequence in another network. Subsequently, interconnected networks have been tested for their robustness against attack and assortativity [44–47], properties of percolation [48–52] and diffusion [53], and they have indeed become a hot topic of general interest [54, 55], touching upon subjects as diverse as epidemic spreading [56], the appearance and promotion of creativity [57], and voting [58].

Previous research concerning evolutionary games on interconnected networks has revealed, for example, that biased utility functions suppress the feedback of individual success, which leads to a spontaneous separation of characteristic time scales on the two interconnected networks [38]. Consequently, cooperation is promoted because the aggressive invasion of defectors is more sensitive to the deceleration. Even if the utilities are not biased, cooperation can still be promoted by means of interconnected network reciprocity [39], which however requires simultaneous formation of correlated cooperative clusters on both networks. It has also been shown that the coupling of the evolutionary dynamics in each of the two networks enhances the resilience of cooperation, and that this is intrinsically related to the non-trivial organization of cooperators across the interconnected layers [40]. Perhaps most closely related to the setup of the present work is that by Wang et al. [42], who showed that probabilistic interconnections between interconnected networks can very much promote the evolution of cooperation. In our model, however, the strategy transfer between networks is prohibited. The interconnectivity is thus due solely to coupling together the payoffs of select players that reside on different networks.

Here, we wish to determine how strong the interconnectivity between the networks really ought to be for the optimal promotion of cooperation. Since existing works unequivocally declare that interconnectivity works in favor of the resolution of social dilemmas, one might intuitively assume that the stronger the interconnectivity the better. As we will show, however, this assumption is not necessarily true. To address the problem, we consider primarily the prisoner's dilemma game on two square lattices, where a certain fraction of randomly selected players is allowed to connect with the corresponding players in the other lattice (see Methods section). While strategy transfers between the two networks are not allowed, the additional connections between the corresponding players do influence their utility, and thus their ability to retain and possibly spread their strategies on the

home network. This introduces two new parameters, namely the fraction of players that is allowed to form links with the corresponding players in the other network ρ, and the strength of this links α. Together, these two parameters determine the strength of interconnectivity, and also the success of resolving social dilemmas. In the next section we provide further details with regards to the studied evolutionary games, the applied dynamical rule, and the topology of interaction networks. Independent of the strategy updating rule, the topology of interconnected networks, and the governing social dilemma, we will show that cooperation is promoted best if only an intermediate fraction of players is allowed to have external links to the other network, but also that those links should be sufficiently strong. We will also reveal mechanisms that lead to the emergence of the optimal interconnectivity.

7.2 Methods

The evolutionary game is staged on two square lattices, each of size $L \times L$, where initially each player x is designated either as a cooperator ($s_x = C$) or defector ($s_x = D$) with equal probability. Likewise randomly, a fraction ρ of players on each lattice is selected and allowed to form an external link with a corresponding player in the other lattice. Although we predominantly use the square lattice, we will also resort to using the triangle lattice, given that the difference in the clustering coefficient has been determined as a potentially key factor for the outcome of games that are governed by pairwise interactions [67, 68].

The accumulation of payoffs π_x on both networks follows the same standard procedure, depending on the type of the governing social dilemma. In both games two cooperators facing one another acquire R, two defectors get P, whereas a cooperator receives S if facing a defector who then gains T. The prisoner's dilemma game is characterized by the temptation to defect $T = b$, reward for mutual cooperation $R = 1$, and punishment P as well as the sucker's payoff S equaling 0, whereby $1 < b \leq 2$ ensures a proper payoff ranking [2]. We note that qualitatively similar results are obtained also for other values of S. The snowdrift game, on the other hand, has $T = \beta$, $R = \beta - 1/2$, $S = \beta - 1$ and $P = 0$, where the temptation to defect can be expressed in terms of the cost-to-benefit ratio $r = 1/(2\beta - 1)$ with $0 \leq r \leq 1$. Due to the interconnectivity (external links between corresponding players), however, the utilities used to determine fitness are not simply payoffs obtained from the interactions with the nearest neighbors on each individual network, but rather $U_x = \pi_x + \alpha \pi_{x'}$ for players that have an external link, and $U_x = \pi_x$ otherwise. The parameter $0 \leq \alpha \leq 1$ determines the strength of external links, i.e., the larger its value the higher the potential increase of utility of two players that are connected by the external link.

Importantly, while the interconnectivity affects the utility of players, it does not allow strategies to be transferred between the two networks. Thus, the evolution of the two strategies proceeds in accordance with the standard Monte Carlo simulation procedure comprising the following elementary steps for each network. First, a

randomly selected player x acquires its utility U_x by playing the game with all its nearest neighbors and taking into account also the potential addition to the utility stemming from the possible external link, as described above. Next, one randomly chosen neighbor of x within the same network, denoted by y, also acquires its utility U_y in the same way. Lastly, player x attempts to adopt the strategy s_y from player y with a probability determined by the Fermi function

$$W(s_y \rightarrow s_x) = \frac{1}{1 + \exp[(U_x - U_y)/K]} \, , \qquad (7.1)$$

where $K = 0.1$ quantifies the uncertainty related to the strategy adoption process [6, 65]. The latter is usually associated with errors in decision making and imperfect information transfer between the players. Notably, to test the robustness of our findings, we will also use the best-takes-over [2] and the proportional imitation [66] strategy updating rule. Regardless of which type of interaction network, evolutionary game, or the strategy updating rule is used, in accordance with the random asynchronous update, each player on both networks is selected once on average during a full Monte Carlo step. Moreover, sufficiently large system sizes (from $L = 200$ to $L = 800$) and relaxation times need to be used to avoid finite size effects and to ensure a stationary state has been reached. Presented results were averaged over up to 30 independent runs to further improve accuracy.

7.3   Results

To begin with, we show in Fig. 7.1 the impact of parameters ρ and α on the outcome of the prisoner's dilemma game. It can be observed that there exists an intermediate range of the fraction of players that are allowed to form an external link at which
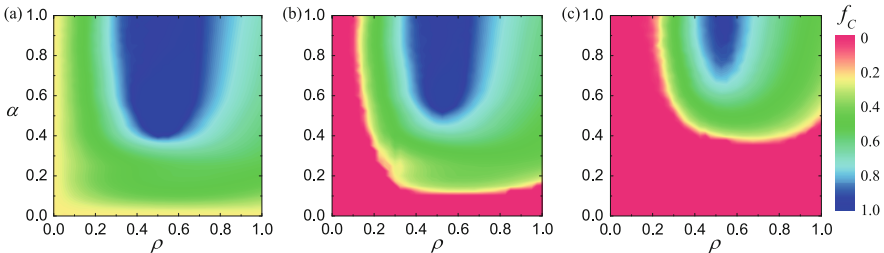

Fig. 7.1   Tuning in on the optimal interconnectivity between two square lattices for the resolution of the prisoner's dilemma. Color coded is the fraction of cooperators f_C in dependence on the fraction of players that are allowed to form an external link ρ and the strength of these links α, as obtained for $b = 1.03$ (**a**), $b = 1.05$ (**b**) and $b = 1.1$ (**c**). Irrespective of b, there exists an intermediate value of $\rho \approx 0.5$ at which cooperation is optimally promoted. But in addition to that, the value of α needs to be sufficiently large as well (After [70])

cooperators fare best. Irrespective of the temptation to defect b, values around $\rho \approx$ 0.5 yield an optimal outcome of the social dilemma. Yet the coupling strength is important too. Only if the value of α is sufficiently large are the players able to utilize the advantage of being linked to their corresponding players in the other network. Although the level of cooperation appears to fade slightly beyond $\alpha = 0.7$ if the temptation to defect is low or moderate [panels (a) and (b)], the prevailing conclusion is that the coupling strength needs to be sufficiently strong.

   To clarify the mechanism that is responsible for the promotion of cooperation, we first monitor the evolution of cooperation by measuring not just the overall average cooperation level (f_C), but separately also the average cooperation level for players with (f_{C_d}) and without (f_{C_o}) external links to the other network in dependence on the number of full Monte Carlo steps (MCS), as defined in the Methods section. For easier reference, we will refer to individuals with external links to the other network as "distinguished" and to those who have no such links as "ordinary" players (note that the subscripts in f_{C_d} and f_{C_o} are chosen accordingly). Figure 7.2 reveals that the cooperation level amongst the distinguished players who do have external links to the other network is significantly higher than the cooperation level amongst the players who are not externally linked. Expectedly, the overall cooperation level is in-between f_{C_d} and f_{C_o}. The identified difference between f_{C_d} and f_{C_o} is crucial, because it indicates that players who have the ability to collect an additional payoff from the other network are more likely to cooperate. Indeed, the natural selection
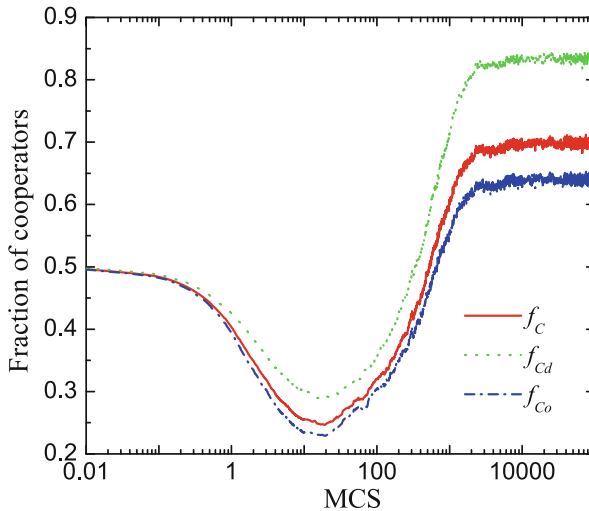

Fig. 7.2 Distinguished players who have an external link with their corresponding player in the other network are more likely to cooperate than those who are not externally linked. Depicted is the time evolution of the fraction of cooperators in the whole population (f_C), among the distinguished players (f_{C_d}), and among ordinary players, i.e., those that do not have an external link to the other network (f_{C_o}). It can be observed that $f_{C_d} > f_{C_o}$. Parameter values used were: $b = 1.05$, $\rho = 0.3$ and $\alpha = 0.8$ (After [70])

of the cooperative strategy among distinguished players is higher than in the whole population, which is a consequence of an asymmetric strategy flow that emerges between the distinguished and other players. Because of generally higher payoffs, the distinguished players are followed by the others, which results in the selection of cooperation around them. In other words, distinguished players with an external link to the other network play the role of leaders in the community, similarly as was reported many times before for hubs on heterogeneous isolated (individual) networks [6].

To test this explanation further, we directly adjust the teaching activity of players, i.e., the ability to pass their strategy to a neighbor [59, 60], which can be done effectively by introducing a multiplication factor w to Eq. (7.1). We consider two options. First, we depreciate all distinguished players by using $w = 0.05$ while keeping $w = 1$ for those who do not have an external link, and second, we reverse these values. The expectation is that the first option will nullify the advantage of interconnectivity between the two networks, while the second option will further amplify the positive effects on the evolution of cooperation.

Figure 7.3 reveals nicely how the leading role of distinguished players improves the cooperation level [panels (a) and (c)], yet only if distinguished players are not depreciated by $w = 0.05$ [panel (b)]. For comparison, we show in panel (a) the results obtained with the basic model where all players have $w = 1$. There the change from $\alpha = 0$ to $\alpha = 0.5$ (note that for $\alpha = 0$ the difference between distinguished and ordinary players vanishes) introduces a noticeable increase in the critical temptation to defect where cooperators die out and an overall increase in the level of cooperation. But if $w = 0.05$ is applied to distinguished players,
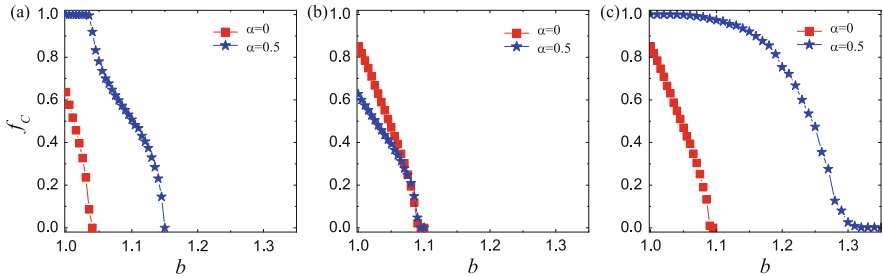

Fig. 7.3 Interconnectivity between networks favors cooperation only if the awarded additional payoffs are not counteracted by reduced teaching activity. Panel (**a**) depicts the fraction of cooperators f_C in dependence on the temptation to defect b for the basic version of the game, where all players have teaching activity $w = 1$. In panel (**b**) the teaching activity of all distinguished players who have an external link to the other network is reduced to $w = 0.05$. It can be observed that the promotion of cooperation due to the interconnectivity vanishes. In panel (**c**) the teaching activity of all ordinary players is reduced to $w = 0.05$, which further strengthens the leading role of the distinguished players and leads to the strongest promotion of cooperation. All panels feature results for $\alpha = 0$ and $\alpha = 0.5$ at $\rho = 0.5$, where $\alpha = 0$ means that effectively the two networks are isolated, i.e., there is no interconnectivity because the links between the two networks yield no additional utility to either player (After [70])

then they become unable to lead the others despite of the fact that their utilities are higher than those of their neighbors. In the absence of hierarchy that was previously warranted by the interconnectivity between networks, there will form no groups with homogeneous strategies, and hence the advantage of cooperation will not be revealed. As panel (b) shows quite interestingly, $\alpha = 0$ (absence of interconnectivity between networks) even slightly outperforms $\alpha = 0.5$, because a weak hierarchy is then restored due to the unequal teaching activity of players. Note that at $\alpha = 0.5$ the unequal teaching activity is counteracted by the additional payoff distinguished players receive from the other network. At $\alpha = 0$ this counterbalance is lost, and the ordinary players become the leaders due to their higher value of w. The positive effect on the level of cooperation is, however, rather marginal.

In the opposite case, when distinguished players are endowed with the full teaching activity $w = 1$ while ordinary players without an external link are depreciated with $w = 0.05$, the change from $\alpha = 0$ to $\alpha = 0.5$ is the strongest [panel (c)]. Here the inequality of w and the additional payoffs stemming from the other network are able to strengthen each other and fortify the leaders to yield the highest cooperation level within the framework of this model. The enforced role of distinguished players will cause an immediate reaction from the followers, and this prompt reaction will select the more successful cooperative strategy as described before. From the technical point of view, it is interesting to note that the curves for $\alpha = 0$ in Fig. 7.3b, c coincide, because in the absence of extra payoff from the other network it does not matter whether distinguished or ordinary players are endowed with $w = 1$ (or $w = 0.05$). Since the fraction of distinguished players was set to $\rho = 0.5$, both fractions are equally large, thus resulting in the same cooperation level.

Results presented in Fig. 7.3 and the pertaining interpretation can be corroborated by comparing the evolution of cooperation within different groups of players, similarly as done in Fig. 7.2 above. As Fig. 7.4b shows, when the teaching activity is reduced for distinguished players the cooperation among them is not favored by natural selection. Consequently, only a very low value of b can ensure a reasonably high cooperation level in the whole population because the latter is dragged down considerably by the low f_{C_d}. There is a slight improvement among other players, as evidenced by the higher f_{C_o}, which is due to a higher teaching activity. However, despite their higher teaching activity, ordinary players cannot lead the whole population efficiently because distinguished players are reluctant to follow them due to their higher individual utility. If either the teaching activity of all players is left intact [panel (a)] or the higher teaching activity is awarded to distinguished players [panel (c)], then $f_{C_o} < f_{C_d}$, as observed initially in Fig. 7.2. Thus, the basic mechanism of cooperation promotion is restored or even additionally fortified.

Furthermore, it is important to emphasize that the optimal interconnectivity between the networks can work in favor of cooperation only if there are distinguished players in both graphs. On the other hand, if the additional utilities flow only in one direction, the mechanism will fail or yield only a marginally better outcome. Results supporting this argumentation are presented in Fig. 7.5. For reference, the
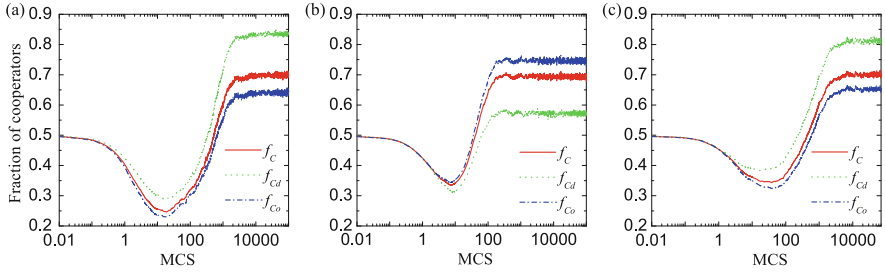
Fig. 7.4 Decreasing the teaching activity of distinguished players nullifies their higher propensity to cooperate, and it reduces the temptation to defect that still warrants a healthy cooperation level in the whole population. Depicted is the time evolution of the fraction of cooperators in the whole population (f_C), among the distinguished players (f_{C_d}), and among ordinary players (f_{C_o}). It can be observed that $f_{C_d} > f_{C_o}$ in panels (**a**) and (**c**), but not in panel (**b**). In the latter, the teaching activity of all distinguished players who have an external link to the other network is reduced to $w = 0.05$. In panel (**a**) all players have $w = 1$, while in panel (**c**) the teaching activity of all ordinary players is reduced to $w = 0.05$, which further amplifies the $f_{C_d} > f_{C_o}$ difference. We have used parameter values that yield approximately the same overall level of cooperation (f_C) in all three cases: $\rho = 0.3$, $b = 1.05$ and $\alpha = 0.8$ for panel (**a**), $\rho = 0.3$, $b = 1.0$ and $\alpha = 0.25$ for panel (**b**), and $\rho = 0.3$, $b = 1.09$ and $\alpha = 0.25$ for panel (**c**) (After [70])
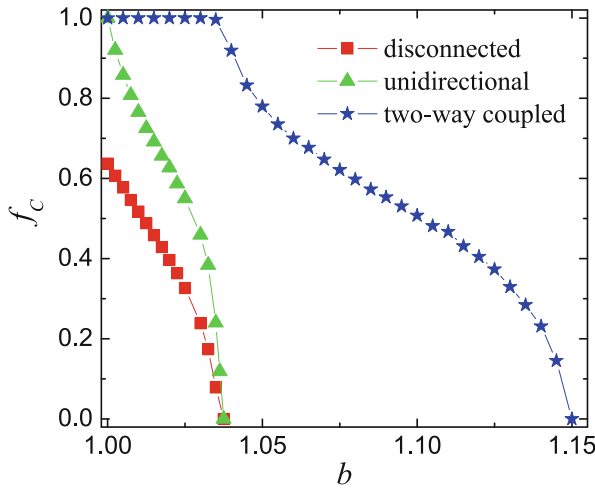

Fig. 7.5 Optimal interconnectivity between the two networks works optimally only if the links connecting them go both ways. Depicted is the fraction of cooperators f_C in dependence on the temptation to defect b as obtained on an isolated network (reference case), on two networks connected by means of unidirectional links, and on two mutually interconnected networks. Cooperation is optimally promoted only if there is independent formation of cooperative patterns on each individual network, for which the chance for heterogeneity (distinguished players) needs to be provided on both of them. Parameter values used were: $\rho = 0.5$ and $\alpha = 0.5$ (After [70])

outcome on an isolated network is depicted as well. It can be observed that if distinguished players in the first network can collect additional payoffs from the second network, but the evolution in the second network is independent from the first network because players there are unable to collect additional payoffs (the interconnectivity is unilateral), then the critical temptation to defect b at which cooperators die out does not increase at all. In fact, only the level of cooperation increases slightly in the mixed $C + D$ phase. If the interconnectivity is bilateral, i.e., players on both networks can collect an additional payoff from the other network, however, the positive impact on the evolution of cooperation is much stronger. This highlights the importance of mutual interconnectivity and the independent formation of cooperative patterns on each individual network. Players connected to their corresponding partners in the other network can support each other effectively only if homogeneous cooperative domains emerge on both networks. Notably, the importance of correlated growth and formation of cooperative domains has been emphasized already in [39], although here they need not overlap geographically. We could observe practically the same cooperation level when distinguished players collected additional payoffs from an ordinary player in the other network. Namely, there is no need to link distinguished players from different networks with one another. The crucial condition is the chance of heterogeneity on both networks [61], the positive effect of which can then be mutually amplified through the interconnectivity.

Our argument for the spreading of cooperative behavior among distinguished players can also be supported by how the border of the full D phase behaves in dependence of ρ and α at a high temptation to defect. As results presented in Fig. 7.1c suggest, a smaller density of distinguished players can be compensated by a higher value of α, but only up to a certain point. At such a high temptation value cooperators cannot survive even for large values of α if the density of distinguished players ρ is below a threshold value. Naturally, the critical ρ depends slightly on the temptation to defect [compare panels (b) and (c)], but the smallest value of the phase transition point is close to the critical $\rho_c = 0.1869(1)$ value of jamming coverage of particles during a random sequential adsorption when nearest and next nearest neighbor interactions are excluded on a square lattice [62]. Shortly, if distinguished players are too rare, then to support them via a high α will not yield the desired impact because their influence cannot percolate. The latter, however, is an essential condition to maintain cooperation, which was already pointed out in previous works [63, 64].

It is lastly of interest to verify the robustness of these observations, first with regards to the strategy updating rule. As results presented in Fig. 7.6 demonstrate, our conclusions are not restricted solely to the Fermi-type strategy updating [6, 65], but remain valid also under the best-takes-over rule [2] and proportional imitation [66]. In both cases an optimal intermediate value of ρ is clearly inferable, and the positive effect on the evolution of cooperation is the stronger the larger the value of the coupling strength α. This is qualitatively identical as observed in Fig. 7.1 with the Fermi rule.
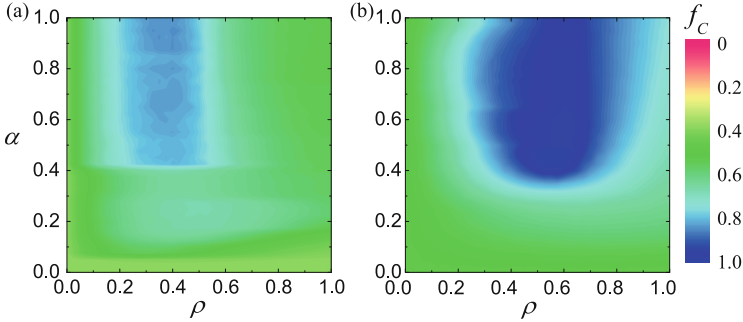
Fig. 7.6 Robustness of optimal network interconnectivity on different strategy updating rules. Color coded is the fraction of cooperators f_C in dependence on the fraction of players that are allowed to form an external link ρ and the strength of these links α, as obtained for $b = 1.4$ and best-takes-over strategy updating (**a**), and $b = 1.06$ and proportional imitation (**b**). Irrespective of the applied strategy updating rule, there exists an intermediate value of ρ at which cooperation is optimally promoted, and the dependence on α is also qualitatively the same as in Fig. 7.1, where the Fermi strategy adoption rule has been used (After [70])
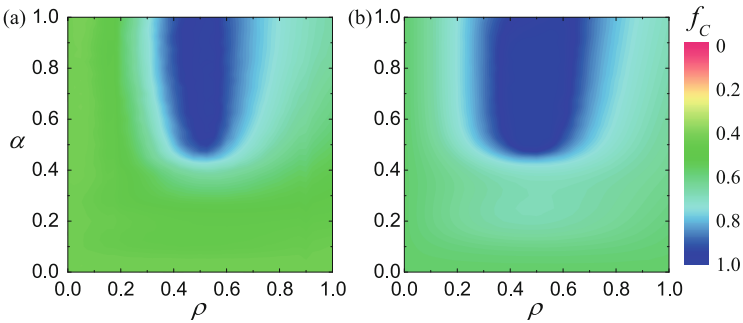

Fig. 7.7 Robustness of optimal network interconnectivity on the topology and the type of social dilemma. Color coded is the fraction of cooperators f_C in dependence on the fraction of players that are allowed to form an external link ρ and the strength of these links α, as obtained for $b = 1.1$ on the triangle lattice (**a**), and $r = 0.3$ (snowdrift game) on the square lattice (**b**). The Fermi strategy adoption rule has been used in both cases. Irrespective of the topology of each individual network and the type of the governing social dilemma, the results are qualitatively the same as in Figs. 7.1 and 7.6 (After [70])

Since previous works revealed that the clustering coefficient could be a decisive factor affecting the evolution of cooperation in games that are governed by pairwise interactions [67, 68] (note that this is not the case for games governed by group interactions [69]), it is also instructive to examine the relevance of network interconnectivity under this condition. Unlike the square lattice that has a zero clustering coefficient, the triangular lattice has a high clustering coefficient, and thus serves the purpose very well. Results presented in Fig. 7.7a attest to the fact that the existence of optimal network interconnectivity does not depend on structural properties of each individual network, as indeed the $\alpha - \rho$ dependence of f_C is the

same as observed before for the square lattice in Fig. 7.1. To conclude, we extend our exploration also to other social dilemmas, more precisely the snowdrift game, and as shown in Fig. 7.7b, the main conclusions remain intact. To reiterate, the exists and intermediate level of interconnectivity between networks that is optimal for the resolution of social dilemmas.

7.4 Discussion

Summarizing, we have studied the evolution of cooperation in the spatial prisoner's dilemma and the snowdrift game subject to interconnectedness by means of a different fraction of differently strong links between the corresponding players residing on the two interconnected networks. We have found that for cooperation to be optimally promoted, the interconnectivity should stem only from an intermediate fraction of links connecting the two networks, and that those links should affect the utility of players significantly. The existence of optimal interconnectivity has been attributed to the heterogeneity that is brought about by the enhanced utility of those players that do have external links to the other network, as opposed to those who have not. This introduces asymmetric strategy flow, which in turn leads to the emergence of influential leaders that can act as strong cooperative hubs in their respective networks. Importantly, the compact cooperative patterns that appear independently on both networks support each other mutually through the links that constitute the interconnectivity. Indeed, we have shown that the mechanism works best only if the interconnectivity is bilateral, and if the asymmetric strategy flow is not counteracted by artificial low weights assigned to the reproducibility of interconnected players. In case of unidirectional interconnectivity or if the reproducibility of players is altered, however, some marginal benefits for cooperators may still exist, but these are then far removed from the full potential of interconnected networks to aid the resolution of social dilemmas. We have tested the robustness of these conclusions by replacing the Fermi strategy adoption rule with the best-takes-over and the proportional imitation rule, as well as by replacing the square lattice having zero clustering coefficient with the triangle lattice that has a much higher clustering coefficient, as well as finally by replacing the prisoner's dilemma game with the snowdrift game. Quite remarkably, we have found that the optimal interconnectivity persist across all these different setups, thus leading to the conclusion that it ought to be to a large degree a universally valid phenomenon.

While interconnectivity can promote the evolution of cooperation by means of the mechanisms presented in this chapter (for further details see [38–41]), and while it has been established that there in fact exists an optimal intermediate level of interconnectivity that works best in deterring defection [70, 71], it is not clear how it could have emerged from initially isolated subsystems. Many works have simply assumed some level of interconnectivity be there without considering its origin or mechanisms that might have led to its emergence. To amend this, we have recently adopted the established concept of coevolution in the realm of the prisoner's

dilemma game [8], with the aim of introducing an elementary coevolutionary rule that leads towards the spontaneous emergence of optimal interconnectivity between two initially completely independent networks [72]. The rule is simple indeed, assuming only that players who are often enough successful at passing their strategy to one of their neighbors, regardless of which strategy it is, are allowed to form an external link to the corresponding player in the other network in order to potentially increase their utility. As we have shown [72], this suffices for the system to self-organize into a two-class society in which the resolution of the prisoner's dilemma is most likely. Alternatively, it is possible to introduce rewards for high-enough evolutionary fitness of individual players in the form of additional links that bridge the gap between two initially disconnected populations [73]. These rewards effectively introduce interconnectivity between two populations, and they allow the rewarded players to increase their utility with a fraction of the utility of the player in the other population. Regardless of game-specific details, the self-organization of fitness and reward promotes the evolution of cooperation well past the boundaries imposed by traditional network reciprocity [2], as well as past the boundaries imposed by interconnected network reciprocity [39], if only the utility threshold is sufficiently large [73]. On the other hand, the threshold must not exceed a critical value, which could be well below the maximal possible utility a cooperator is able to reach if it would be fully surrounded by other cooperators. These latest results concerning coevolution on interconnected networks indicate that even seemingly irrelevant and minute additions to the basic evolutionary process might have led to the intricate and widespread interconnectivity between networks that we witness today in many social and technological systems.

While the games on interconnected networks studied here are not meant to model a particular real-life situation, they nevertheless do capture the essence of some situations that are viable in reality. For example, it is generally accepted that not all individuals are equally fond of making connections outside of their natural environment. Similarly, some would very much wish to do so, but may not have a chance. These and similar considerations may all affect the level of interconnectivity between two or more networks, and it is within the realm of these possibilities that our study predicts the existence of an optimal level of interconnectedness. Future studies could address the coupling of more complex (small world or scale-free, for example) interaction topologies, the outcome of other games on interconnected networks, such as for example the traveler's dilemma game that has recently been studied in a spatial setting [74], as well as the impact of coevolution and growth, both of which have recently been the subject of much interest [75–79]. Overall, it seems safe to conclude that the interconnectivity of interaction networks offers several exciting possibilities for further research related to evolutionary games, and it ought to bring the models a step closer to actual conditions, given that networks indeed rarely exist in an isolated state.

References

1. Nowak, M.A.: Five rules for the evolution of cooperation. Science **314**, 1560–1563 (2006)
2. Nowak, M.A., May, R.M.: Evolutionary games and spatial chaos. Nature **359**, 826–829 (1992)
3. Hauert, C., Doebeli, M.: Spatial structure often inhibits the evolution of cooperation in the snowdrift game. Nature **428**, 643–646 (2004)
4. Gracia-Lázaro, C., Cuesta, J., Sánchez, A., Moreno, Y.: Human behavior in prisoner's dilemma experiments suppresses network reciprocity. Sci. Rep. **2**, 325 (2012)
5. Gracia-Lázaro, C., Ferrer, A., Ruiz, G., Tarancón, A., Cuesta, J., Sánchez, A., Moreno, Y.: Heterogeneous networks do not promote cooperation when humans play a prisoner's dilemma. Proc. Natl. Acad. Sci. U. S. A. **109**, 12922–12926 (2012)
6. Szabó, G., Fáth, G.: Evolutionary games on graphs. Phys. Rep. **446**, 97–216 (2007)
7. Roca, C.P., Cuesta, J.A., Sánchez, A.: Evolutionary game theory: temporal and spatial effects beyond replicator dynamics. Phys. Life Rev. **6**, 208–249 (2009)
8. Perc, M., Szolnoki, A.: Coevolutionary games – a mini review. BioSystems **99**, 109–125 (2010)
9. Perc, M., Gómez-Gardeñes, J., Szolnoki, A., Floría, L.M., Moreno, Y.: Evolutionary dynamics of group interactions on structured populations: a review. J. R. Soc. Interface **10**, 20120997 (2013)
10. Abramson, G., Kuperman, M.: Social games in a social network. Phys. Rev. E **63**, 030901(R) (2001)
11. Kim, B.J., Trusina, A., Holme, P., Minnhagen, P., Chung, J.S., Choi, M.Y.: Dynamic instabilities induced by asymmetric influence: prisoner's dilemma game in small-world networks. Phys. Rev. E **66**, 021907 (2002)
12. Masuda, N., Aihara, K.: Spatial prisoner's dilemma optimally played in small-world networks. Phys. Lett. A **313**, 55–61 (2003)
13. Tomassini, M., Luthi, L., Giacobini, M.: Hawks and doves games on small-world networks. Phys. Rev. E **73**, 016132 (2006)
14. Fu, F., Liu, L.-H., Wang, L.: Evolutionary prisoner's dilemma on heterogeneous Newman-Watts small-world network. Eur. Phys. J. B **56**, 367–372 (2007)
15. Santos, F.C., Pacheco, J.M.: Scale-free networks provide a unifying framework for the emergence of cooperation. Phys. Rev. Lett. **95**, 098104 (2005)
16. Santos, F.C., Pacheco, J.M., Lenaerts, T.: Evolutionary dynamics of social dilemmas in structured heterogeneous populations. Proc. Natl. Acad. Sci. USA **103**, 3490–3494 (2006)
17. Gómez-Gardeñes, J., Campillo, M., Floría, L.M., Moreno, Y.: Dynamical organization of cooperation in complex networks. Phys. Rev. Lett. **98**, 108103 (2007)
18. Rong, Z., Li, X., Wang, X.: Roles of mixing patterns in cooperation on a scale-free networked game. Phys. Rev. E **76**, 027101 (2007)
19. Masuda, N.: Participation costs dismiss the advantage of heterogeneous networks in evolution of cooperation. Proc. R. Soc. B **274**, 1815–1821 (2007)
20. Tomassini, M., Luthi, L., Pestelacci, E.: Social dilemmas and cooperation in complex networks. Int. J. Mod. Phys. C **18**, 1173–1185 (2007)
21. Szolnoki, A., Perc, M., Danku, Z.: Towards effective payoffs in the prisoner's dilemma game on scale-free networks. Physica A **387**, 2075–2082 (2008)
22. Assenza, S., Gómez-Gardeñes, J., Latora, V.: Enhancement of cooperation in highly clustered scale-free networks. Phys. Rev. E **78**, 017101 (2008)
23. Santos, F.C., Santos, M.D., Pacheco, J.M.: Social diversity promotes the emergence of cooperation in public goods games. Nature **454**, 213–216 (2008)
24. Peña, J., Volken, H., Pestelacci, E., Tomassini, M.: Conformity hinders the evolution of cooperation on scale-free networks. Phys. Rev. E **80**, 016110 (2009)
25. Poncela, J., Gómez-Gardeñes, J., Moreno, Y.: Cooperation in scale-free networks with limited associative capacities. Phys. Rev. E **83**, 057101 (2011)
26. Brede, M.: Playing against the fittest: a simple strategy that promotes the emergence of cooperation. EPL **94**, 30003 (2011)

27. Tanimoto, J., Brede, M., Yamauchi, A.: Network reciprocity by coexisting learning and teaching strategies. Phys. Rev. E **85**, 032101 (2012)
28. Pinheiro, F., Pacheco, J., Santos, F.: From local to global dilemmas in social networks. PLoS One **7**, e32114 (2012)
29. Simko, G.I., Csermely, P.: Nodes having a major influence to break cooperation define a novel centrality measure: game centrality. PLoS One **8**, e67159 (2013)
30. Ebel, H., Bornholdt, S.: Coevolutionary games on networks. Phys. Rev. E **66**, 056118 (2002)
31. Zimmermann, M.G., Eguíluz, V., Miguel, M.S.: Coevolution of dynamical states and interactions in dynamic networks. Phys. Rev. E **69**, 065102(R) (2004)
32. Pacheco, J.M., Traulsen, A., Nowak, M.A.: Coevolution of strategy and structure in complex networks with dynamical linking. Phys. Rev. Lett. **97**, 258103 (2006)
33. Santos, F.C., Pacheco, J.M., Lenaerts, T.: Cooperation prevails when individuals adjust their social ties. PLoS Comput. Biol. **2**, 1284–1290 (2006)
34. Fu, F., Chen, X., Liu, L., Wang, L.: Promotion of cooperation induced by the interplay between structure and game dynamics. Physica A **383**, 651–659 (2007)
35. Tanimoto, J.: Dilemma solving by coevolution of networks and strategy in a 2×2 game. Phys. Rev. E **76**, 021126 (2007)
36. Lee, S., Holme, P., Wu, Z.-X.: Emergent hierarchical structures in multiadaptive games. Phys. Rev. Lett. **106**, 028702 (2011)
37. Gómez-Gardeñes, J., Romance, M., Criado, R., Vilone, D., Sánchez, A.: Evolutionary games defined at the network mesoscale: the public goods game. Chaos **21**, 016113 (2011)
38. Wang, Z., Szolnoki, A., Perc, M.: Evolution of public cooperation on interdependent networks: the impact of biased utility functions. EPL **97**, 48001 (2012)
39. Wang, Z., Szolnoki, A., Perc, M.: Interdependent network reciprocity in evolutionary games. Sci. Rep. **3**, 1183 (2013)
40. Gómez-Gardeñes, J., Reinares, I., Arenas, A., Floría, L.M.: Evolution of cooperation in multiplex networks. Sci. Rep. **2**, 620 (2012)
41. Gómez-Gardeñes, J., Gracia-Lázaro, C., Floría, L.M., Moreno, Y.: Evolutionary dynamics on interdependent populations. Phys. Rev. E **86**, 056113 (2012)
42. Wang, B., Chen, X., Wang, L.: Probabilistic interconnection between interdependent networks promotes cooperation in the public goods game. J. Stat. Mech. **2012**, P11017 (2012)
43. Buldyrev, S.V., Parshani, R., Paul, G., Stanley, H.E., Havlin, S.: Catastrophic cascade of failures in interdependent networks. Nature **464**, 1025–1028 (2010)
44. Huang, X., Gao, J., Buldyrev, S.V., Havlin, S., Stanley, H.E.: Robustness of interdependent networks under targeted attack. Phys. Rev. E **83**, 065101(R) (2011)
45. Zhou, D., Stanley, H.E., D'Agostino, G., Scala, A.: Assortativity decreases the robustness of interdependent networks. Phys. Rev. E **86**, 066103 (2012)
46. Baxter, G.J., Dorogovtsev, S.N., Goltsev, A.V., Mendes, J.F.F.: Avalanche collapse of interdependent networks. Phys. Rev. Lett. **109**, 248701 (2012)
47. Peixoto, T.P., Bornholdt, S.: Evolution of robust network topologies: emergence of central backbones. Phys. Rev. Lett. **109**, 118703 (2012)
48. Parshani, R., Buldyrev, S.V., Havlin, S.: Interdependent networks: reducing the coupling strength leads to a change from a first to second order percolation transition. Phys. Rev. Lett. **105**, 048701 (2010)
49. Son, S.-W., Bizhani, G., Christensen, C., Grassberger, P., Paczuski, M.: Percolation theory on interdependent networks based on epidemic spreading. EPL **97**, 16006 (2012)
50. Lau, H.W., Paczuski, M., Grassberger, P.: Agglomerative percolation on bipartite networks: nonuniversal behavior due to spontaneous symmetry breaking at the percolation threshold. Phys. Rev. E **86**, 011118 (2012)
51. Schneider, C.M., Araújo, N.A.M., Herrmann, H.J.: Algorithm to determine the percolation largest component in interconnected networks. Phys. Rev. E **87**, 043302 (2013)
52. Zhao, K., Bianconi, G.: Percolation on interacting, antagonistic networks. J. Stat. Mech. **2013**, P05005 (2013)

53. Gómez, S., Díaz-Guilera, A., Gómez-Gardeñes, J., Pérez-Vicente, C., Moreno, Y., Arenas, A.: Diffusion dynamics on multiplex networks. Phys. Rev. Lett. **110**, 028701 (2013)
54. Gao, J., Buldyrev, S.V., Stanley, H.E., Havlin, S.: Networks formed from interdependent networks. Nat. Phys. **8**, 40–48 (2012)
55. Havlin, S., Kenett, D.Y., Ben-Jacob, E., Bunde, A., Hermann, H., Kurths, J., Kirkpatrick, S., Solomon, S., Portugali, J.: Challenges of network science: applications to infrastructures, climate, social systems and economics. Eur. J. Phys. Spec. Top. **214**, 273–293 (2012)
56. Wang, Y., Xiao, G.: Epidemics spreading in interconnected complex networks. Phys. Lett. A **376**, 2689–2696 (2012)
57. Csermely, P.: The appearance and promotion of creativity at various levels of interdependent networks. Talent Dev. Excell. **5**, 115–123 (2013)
58. Halu, A., Zhao, K., Baronchelli, A., Bianconi, G.: Connect and win: the role of social networks in political elections. EPL **102**, 16002 (2013)
59. Szolnoki, A., Szabó, G.: Cooperation enhanced by inhomogeneous activity of teaching for evolutionary prisoner's dilemma games. EPL **77**, 30004 (2007)
60. Szolnoki, A., Perc, M.: Coevolution of teaching activity promotes cooperation. New J. Phys. **10**, 043036 (2008)
61. Santos, F.C., Pinheiro, F., Lenaerts, T., Pacheco, J.M.: Role of diversity in the evolution of cooperation. J. Theor. Biol. **299**, 88–96 (2012)
62. Dickman, R., Wang, J.-S., Jensen, I.: Random sequential adsorption: series and virial expansions. J. Chem. Phys. **94**, 8252–8257 (1991)
63. Wang, Z., Szolnoki, A., Perc, M.: Percolation threshold determines the optimal population density for public cooperation. Phys. Rev. E **85**, 037101 (2012)
64. Wang, Z., Szolnoki, A., Perc, M.: If players are sparse social dilemmas are too: importance of percolation for evolution of cooperation. Sci. Rep. **2**, 369 (2012)
65. Blume, L.E.: The statistical mechanics of strategic interactions. Games Econ. Behav. **5**, 387–424 (1993)
66. Schlag, K.H.: Why imitate, and if so, how? A bounded rational approach to multi-armed bandits. J. Econ. Theory **78**, 130–156 (1998)
67. Szabó, G., Vukov, J., Szolnoki, A.: Phase diagrams for an evolutionary prisoner's dilemma game on two-dimensional lattices. Phys. Rev. E **72**, 047107 (2005)
68. Kuperman, M.N., Risau-Gusman, S.: Relationship between clustering coefficient and the success of cooperation in networks. Phys. Rev. E **86**, 016104 (2012)
69. Szolnoki, A., Perc, M., Szabó, G.: Topology-independent impact of noise on cooperation in spatial public goods games. Phys. Rev. E **80**, 056109 (2009)
70. Wang, Z., Szolnoki, A., Perc, M.: Optimal interdependence between networks for the evolution of cooperation. Sci. Rep. **3**, 2470 (2013)
71. Jiang, L.-L., Perc, M.: Spreading of cooperative behaviour across interdependent groups. Sci. Rep. **3**, 2483 (2013)
72. Wang, Z., Szolnoki, A., Perc, M.: Self-organization towards optimally interdependent networks by means of coevolution. New J. Phys. **16**, 033041 (2014)
73. Wang, Z., Szolnoki, A., Perc, M.: Rewarding evolutionary fitness with links between populations promotes cooperation. J. Theor. Biol. **349**, 50–56 (2014)
74. Li, R.-H., Yu, J.X., Lin, J.: Evolution of cooperation in spatial traveler's dilemma game. PLoS One **8**, e58597 (2013)
75. Poncela, J., Gómez-Gardeñes, J., Floría, L.M., Sánchez, A., Moreno, Y.: Complex cooperative networks from evolutionary preferential attachment. PLoS One **3**, e2449 (2008)
76. Poncela, J., Gómez-Gardeñes, J., Traulsen, A., Moreno, Y.: Evolutionary game dynamics in a growing structured population. New J. Phys. **11**, 083031 (2009)
77. Portillo, I.G.: Building cooperative networks. Phys. Rev. E **86**, 051108 (2012)
78. Li, G., Jin, X.-G., Song, Z.-H.: Evolutionary game on a stochastic growth network. Physica A **391**, 6664–6673 (2012)
79. Brede, M.: Short versus long term benefits and the evolution of cooperation in the prisoner's dilemma game. PLoS One **8**, e56016 (2013)