Can Values Awareness Help Teachers and Parents Transition Preschool Learners into Mathematics Learning?

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Abstract This chapter focuses on different kinds of transitioning that young children do. These are transitioning from home to preschool, from home to school and from preschool to school. With each of these transitions, children may come into contact with different perspectives on mathematics. Six mathematical activities and six mathematical values are described as a way of conceptualising the possible differences young children might experience as they transition across different contexts. A call for more research in this area is made at the end of this chapter.

Introduction

I congratulate those who research the issues of preschool mathematics learning as that is the level at which young learners either switch on to mathematics or switch off from it. To paraphrase the old Jesuit adage, 'Give me the child before they are 7 and I will give you the person'. While teachers and parents have known about the influence of the home background on learning for a long time, it is only relatively recently that we have developed the theoretical tools and research practices to help us study the influences of home/preschool/school relations and values.

My chapter is framed within the general construct of mathematics as a form of cultural knowledge. I outline some of the trends in research in this area, and in particular I present my ideas about the relatively new concept of values in mathematical education. I will also focus on ideas from research, as that is the area I know best. I am not a preschool teacher, and my parental experiences are in the distant past! However, I believe it is the theoretical content of research which is giving us the most useful practical ideas concerning the influences of preschool, school and home mathematical practices.

I am also an enthusiastic promoter of the work of researchers in the area of mathematics education. These researchers around the world, along with most workers in education, are now experiencing hard financial times, thanks to increasingly

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right-wing governments who think they know what is 'best practice' in this area. However, my emphasis on research should not be read as ignoring the excellent work done by many preschool and school teachers in real situations. Indeed, the best research in my opinion usually involves both practising teachers and practising researchers.

My talk is structured around three basic contributions from research involving cultural issues:

- The contribution to *curriculum* thinking of culturally based mathematical knowledge
- The contribution of research on culturally situated *learning*
- The contribution of mathematical values to mathematical *pedagogy*

In each case I will describe the implications for research and practice concerning mathematical transitioning of preschoolers. I will concentrate mainly on the values area since this work is relatively new and increasingly influential.

The Contribution of Ethnomathematics to a Culturally Based Mathematics Curriculum

One of the most significant areas of research development in the last three decades has been that of ethnomathematics. It has not only generated a great deal of interesting evidence, but it has fundamentally changed many of our ideas and constructs (see, e.g. Ascher and Ascher 1997; D'Ambrosio 1985; Gerdes 1997; Joseph 1991).

For me ethnomathematics is not a branch of mathematics but is the study of relationships between mathematics and culture, and the most significant influences from this research have been in relation to:

- Human interactions. Ethnomathematics concerns mathematical activities and practices in society, which mostly take place outside school, and it thereby draws attention to the roles which people other than teachers play in mathematics education, especially family members.
- Values and beliefs. Ethnomathematics makes us realise that any mathematical activity involves values, beliefs and personal choices.
- Interactions between mathematics and languages. Languages act as the principal carriers of mathematical ideas and values in different cultures.

In general, these points have provoked mathematics educators into giving more consideration to the overall structure of the mathematics curriculum and to how it responds, or sadly more usually how it does not respond, to the challenge of culturally based knowledge.

From the perspective of preschool education, my early work in this area (Bishop 1988) identified six general activities which every studied culture performs, in different ways and to different levels of sophistication. As well as researching mathematical practices outside the classroom, they also have been developed into firstly the commonly recognisable curriculum topics we know today, but also the complex academic activities we know about in universities.

The six universal activities are the following together with examples from non-mainstream mathematics cultures:

Counting This is to do with answering the question 'How many?', with inventing ways to describe numbers, recording them and calculating with them. Fingers, parts of the body, stones, sticks and string are just some of the objects which are used as 'counters'.

Locating This concerns finding your way around, navigating, orienting yourself and describing where things are in relation to one another. Compass direction, stars, the sun, wind and maps are used by people all over the world to find their ways and position themselves. Many spatial geometric ideas come from this activity.

Measuring 'How much?' is a question asked and answered everywhere. Whether it is amounts of cloth, food, land or money which are valued, measuring is a skill all people develop. Parts of the body, pots, baskets, string, beads and coins have all been used as units, as have written and drawn amounts on paper or cloth.

Designing Shapes are very important in geometry, and these come originally from designing objects to serve different purposes. The objects can be small and mundane like a spoon or symbolically important like a temple. Mathematically, we are interested in the shapes and the designs which are used, together with their different properties.

Playing Everyone plays and everyone takes playing ,very seriously! Not all play is important from a mathematical viewpoint, but puzzles, logical paradoxes, rules of games, strategies for winning, guessing, chance and gambling all demonstrate how playing contributes to the development of mathematical thinking.

Explaining Understanding why things happen the way they do is a universal human quest. In mathematics, we are interested in why number patterns happen, why geometric shapes go together, why one result leads to another, why some of the natural world seems to follow mathematical laws and in the process of trying to symbolise answers to these kinds of 'why' questions. A proof is one kind of symbolic answer, but there are many others, depending on what else you believe to be true.

Certainly every family group will engage in these activities to more or less an extent, and at the micro-level of culture, we can understand the need to assist teachers with making the links between the curriculum content in-school and in-preschool and the mathematical practices of the families outside school (Civil 1998). It also is an excellent basis for building on the constructivist approach to education at this level. As I understand it, Sweden has used this six-activity structure to develop a preschool mathematics curriculum. I hope it is proving to be a useful structure to help all your homeschoolers and preschoolers make the transition into formal mathematics successfully. But for this to happen, it is necessary to embed these activities within the value-laden frameworks of the classroom or home contexts.

The Contribution of Research on Culturally Situated Learning

Compared with Australia, Sweden appears to the outside world as a monocultural society with few multicultural schools, although this situation has changed during the last few decades. This is a markedly different educational context from that in Australia where we have the second largest number of different migrant groups in the world attending our multicultural schools. We can easily see that the culture experienced by learners in their homes is rarely the same as that represented by the school through its curriculum, its pedagogy and its values.

This kind of disjunction can easily lead to what I have called for many years 'cultural conflicts' (Bishop 1994). For many children around the world, the mathematics experience in schools is in cultural conflict with their home experience. Their situation is one of cultural dissonance, and the educational process is for them one of acculturation, rather than enculturation. The social groupings in which learners exist and learn mathematics inside and outside school have their own cultures, customs, languages and values.

This argument has been the basis for the development of the research on 'situated cognition' (Lave and Wenger 1991; Kirshner and Whitson 1997). When you learn something, you learn it in a context. For example, studies of the 'failures' of bilingual learners in a monolingual classroom, or of farmers' children studying in totally urban-centred curriculum, or of handicapped learners in mainstream classrooms all help to shed light on other explanations of failure and success besides the attributes of the learners themselves.

More research needs to focus on the transitions in learning as experienced by learners in cultural conflict situations. A good start has been made with the text by Abreu et al. (2002a) which is focused around five empirical studies of learners in cultural transition, with important theoretical perspectives added. These studies illustrate the range of contexts being researched now in mathematics learning from a cultural perspective.

The first empirical study is by Gorgorio et al. (2002). Their work was with immigrant students in Catalonia, an area of Spain, in which there are many migrants, and their study deals with the classroom complexities of cultural conflict and the enormous challenges facing the teachers in those schools and preschools.

The second study, reported by Bishop (2002a), also involved immigrant students but focused on the classroom challenges facing them, as they come to terms with different expectations, norms of teacher and student behaviour and the significant values.

The third study was carried out in Cabo Verde by Santos and Matos (2002). It was carried out in the city and focused on the mathematical activities learnt and practised by the *ardinas* who are the local newspaper sellers.

In the fourth study, Abreu et al. (2002b) describe the ways that parents participate in their children's mathematical progress at their multi-ethnic primary schools. They analyse the ways parents try to support their children's transitions from home to preschool and school, and they point to the conflicts experienced by both parents and their children as they try to come to terms with the unfamiliar, unwritten and unspoken values and 'rules' in the schools.

The fifth study, by Civil and Andrade (2002), also is concerned with the home/school relationship and features Mexican-American families and their children. As well as exploring the challenges the parents, their children and the teachers face, the study focuses on an experimental structured programme whereby teachers visit the homes of their students' families.

In all these situations and studies, the learners are clearly faced with negotiating transitions in knowledge and knowing, but they must also make transitions in values, language, customs and behaviours. These studies and perspectives enable us to see that learners are not just learning the cultural knowledge that they are being taught (as well as other knowledge that they are not taught, of course). They are in fact co-constructing that knowledge. This is the importance of the ideas behind the term 'transition' in this chapter, where it is used both as a noun and as a verb. More usually it is the noun that is emphasised, but the verb emphasises the complexity of the transition process. In particular it focuses our research attention on the contribution of the learners themselves.

This is probably also the most important contribution of constructivism—that it is not the individual who is constructing her/his own personal knowledge. Of course from a psychological point of view, that is important, but it is also rather obvious. What is much more important is the quality of the *social* situation that enables the learners to socially co-construct their new cultural knowledge. Knowledge changes with every generation, and it is mediated in that change by teachers and by learners of all cultural persuasions.

The Contribution of Research on Values to Mathematical Transitions

If we now consider research on mathematics pedagogy from the cultural perspective, one aspect that seems to be the most fundamental is, strangely, one of the most ignored, namely, that of values in mathematics teaching. It is ignored in both theory and practice. It seems that in keeping with a common idea that many people still believe, that mathematics education, like mathematics itself, is universal and therefore culture-free, it is also perceived by them to be value-free. This does not mean that they think mathematics has no value, but rather that they do not think that mathematics education conveys, or is the result of, any values over and above those values a particular society or culture is promoting through its educational, political and social institutions.

What should be of great concern to educators and parents alike is that values teaching and learning does of course occur in mathematics classrooms all the time, for example, whenever teachers or parents make decisions that affect their children's learning. Moreover, because most of this decision-making appears from our research to be done implicitly, there is only a limited understanding at present of what and how values are being developed.

So why should we study values and mathematics teaching? Surely teachers have enough to worry about teaching numbers, fractions, etc. without more abstract ideas. Here are some answers to that question:

1. Values emphasise the emotional and affective side of mathematics education, which despite its well-recognised significance is still not well researched.
2. Values are often ignored by researchers and teachers in their work context, yet they have a profound influence on the quality of mathematics learning in our preschools and schools.
3. Teaching mathematics without considering values is a nonsense—indeed I believe that the main reason many promising curriculum and teaching developments are not sustained is precisely because they do not take into account the value changes which are often implied.
4. Indeed, from a research perspective, trying to develop new curriculum and pedagogy practices without understanding the changing values involved is a futile exercise. For example, teaching the six universal mathematical activities above without considering values is to devalue their pedagogical usefulness. These activities have developed over the centuries and in diverse socio-historical mathematical contexts. The values involved are culturally hugely significant.

My research on values in mathematics education started in the 1980s as part of the concerns above. My particular interest was in the cultural dimension of mathematics education, and in the book on mathematical enculturation (Bishop 1988), I proposed six sets of values that I argued to be the main values adopted by mathematicians as they developed historically what has come to be called 'Western' mathematics. The analysis relied on various historical interpretations of the development of the activities of mathematics.

My educational argument is that these values are then 'carried' by the mathematics that is currently taught in schools and universities all over the world. They are mathematical values, as distinct from mathematics **educational** values which are imbued with values associated with each educational situation. Furthermore, these mathematical values will always be mediated by teachers and by every education system and will receive relative emphases in their teaching. Nevertheless, I argue that wherever Western mathematics is being taught, it is reasonable to assume that these values will always be portrayed by teachers and parents to some degree.

Some values will appear recognisable, while others may be rather more obtuse. They have been further discussed in the literature, explored in research studies and recognised as being important for educational purposes. In relation to this conference, I want to see how they could contribute to the transitioning of preschool learners into school. So far, the research has been mainly with primary and secondary teachers and their students, but it will be interesting to see what the ideas could offer to learners who are in the home/school transition process of preschool learning.

My particular research on values has its conceptual basis in a seminal work by White (1959). Briefly White, a cultural anthropologist who was neither mathematician nor educator, proposed four drivers of any culture, namely, technology, ideology, sentiment and sociology, the last three of which I have argued are the value drivers of the culture of mathematics, viewed itself as a symbolic technology (Bishop 1988, 1991). Their structure is connected with the six value clusters as follows, in three sets of complementary pairs:

Ideology: rationalism and objectism
Sentiment: control and progress
Sociology: openness and mystery

In providing details of these value clusters below, note that I am talking in general educational terms, not addressing preschool mathematics education specifically. I have also included various pedagogical activities, which enable the values to be seen in the learning context. Perhaps the best way to contextualise them in this chapter is to consider them in relation to your own knowledge and values.

Ideology: Rationalism

Valuing rationalism means appreciating argument, reasoning, logical analysis and explanation. It concerns theory and hypothetical and abstract situations. It includes appreciation of the aesthetics and beauty of mathematical proofs and is the main value cluster that people think about with mathematics. Pedagogical questions might be: Do you encourage your students to argue in your classes? Do you have debates? Do you emphasise mathematical proving? Could you show the students examples of proofs from history (e.g. different proofs of Pythagoras' theorem) and discuss their beauty and elegance?

Ideology: Objectism

Valuing objectism means appreciating and creating mathematical objects, the objectifying process and application ideas in mathematics. This cluster favours analogical thinking, symbolising and the presentation and use of data. Mathematicians throughout history have created symbols and other forms of representation for their ideas and have then treated those symbols as the object source for the next level of abstraction and theorising. Encouraging students to search for different ways to symbolise and represent ideas, and then to compare their symbols for conciseness and efficiency, is a good way to encourage appreciation of this value. Do you use geometric diagrams to illustrate algebraic relationships? Could you show the students different numerals used by different cultural groups in history? Could you discuss the need for simplicity and conciseness in choosing symbols? And why that helps with further abstractions?

Sentiment: Control

Valuing control means appreciating the power of mathematical knowledge through mastery of rules, facts, procedures and established criteria. It also promotes security in knowledge and the ability to predict. The value of ‘control’ is another one of which most people are very conscious. It involves aspects such as having rules and being able to predict, and it is one of the main reasons that people like mathematics. It has right answers that can always be checked. Do you emphasise not just ‘right’ answers but also the checking of answers and the reasons for other answers not being ‘right’? Do you encourage the analysis and understanding of why routine calculations and algorithms ‘work’? Could you emphasise more the bases of these algorithms? Do you always show examples of how the mathematical ideas you are teaching are used in society?

Sentiment: Progress

Valuing progress means appreciating the ways that mathematical ideas grow and develop, through alternative theories, development of new methods and the questioning of existing ideas. This cluster is also about the values of individual liberty and creativity. Because mathematics can feel like such secure knowledge, mathematicians feel able to explore and progress their ideas. This value cluster is involved in ideas such as abstracting and generalising, which is how mathematics grows. Do you emphasise alternative, and non-routine, solution strategies together with their reasons? Do you encourage students to extend and generalise ideas from particular examples? Could you stimulate them with stories of mathematical developments in history?

Sociology: Openness

Valuing openness means appreciating the democratisation of knowledge, through demonstrations, proofs and individual explanations. Verification of hypotheses, clear articulation and critical thinking are also significant in this cluster, as is the transparency of procedures and assumptions. Mathematicians believe in the public verification of their ideas by proofs and demonstrations. Asking students to explain their ideas to the whole class is good practice for developing the openness value. Do you encourage your students to defend and justify their answers publicly to the class? Do you encourage the creation of posters, for example, so that the students can display their ideas?

Sociology: Mystery

Valuing mystery means appreciating the wonder, fascination and mystique of mathematical ideas. It promotes thinking about the origins and nature of knowledge and of the creative process, as well as the abstractness and dehumanised nature of mathematical knowledge. Do you tell the students any stories about mathematical puzzles in the past, about, for example, the search for negative numbers or for zero? Do you stimulate their mathematical imagination with pictures, artworks, images of infinity, etc.?

These then are what I consider to be the fundamental values underlying so-called Western mathematics. But do they accord with others' ideas? I have preferred an approach (White's categories) which gives a good theoretical basis for the clusters and categories. However, the real issue is: Can those descriptions give a complete value curriculum picture, and framework, such as already exists in the traditional conceptual and technique curriculum? Is this values listing both necessary and sufficient? And how can that question best be answered?

A way of testing the ideas is to consider them in relation to various past curriculum projects. Changing curricula means changing values, or rather changing the balance between the six values. In Table 1, I have used a simple reference device, and I have assumed that each curriculum project emphasises the three clusters but with differences within each cluster. For example, if we assume that current/traditional curricula emphasise Ob, C and M , then other projects have emphasised other balances (note: Rat = Rationalism, Obj = Objectism, etc.).

So in Table 1 we can see that:

'New Math' emphasised R, C and M.
'Realistic Math' emphasised Ob, C and Op.
'Critical Math' emphasised R, C and Op.
'Investigations approach' emphasised R, P and Op (the complete opposite to the current curriculum emphasis.

In relation to the preschool (home?) phase, I would suggest that there is a need to emphasise Ob, C and M, while the in-school phase should transition the learners into R, P and Op. The research work goes on!

In another research context, concerning teachers and their values, several research projects based at Monash University from 1999 have been specifically concerned with mathematics teachers' values in primary and secondary schools. These were under the umbrella of the Values and Mathematics Project (VAMP) and included Bishop et al. (1999), Clarkson et al. (2000) and FitzSimons et al. (2000). In these studies we rarely found any explicit values teaching going on in mathematics classrooms.

Table 1 Mathematics curricula projects and the six values

	Rat	Obj	Con	Pro	Ope	Mys
New math	*		*			*
Realistic math		*	*		*	
Critical math	*		*		*	
Investigate	*			*	*	
Preschool		*	*			*
School	*			*	*	

Additionally, few mathematics teachers admitted to any explicit values teaching. Here are some other general conclusions that have come from these studies:

1. Teachers find it difficult to discuss values and mathematics, because they are not used to doing so. Often they do not have the words. But primary teachers and teachers in schools which incorporate values programmes are, generally, more able to do so than most secondary mathematics teachers.
2. Mathematics teachers do hold values about mathematics and about mathematics education; some of these are explicitly recognised and able to be articulated; others are tacitly or implicitly held, or perhaps not even recognised.
3. Teachers have many goals in planning for lessons, all of which involve value judgements, yet the values behind the goals and plans are rarely articulated.
4. Teachers may choose to make explicit certain mathematics or mathematics education values, or they may deal with them implicitly, although it is unclear just how consciously they make this choice.
5. In the actual classroom situation, teachers face a constantly evolving and unpredictable situation as each lesson unfolds. They are often faced with value conflicts which have to be resolved immediately and pragmatically, revealing the obvious need for some coherent structuring of teachers' values frameworks.
6. It is easier for teachers to think about and recognise the values they are teaching than to implement new values. This conclusion is, of course, not really surprising, but it carries huge implications for any curriculum development process. No teacher is autonomous and any developmental process must bear in mind the influences that affect the values that teachers can impart and develop in their classrooms.

We currently are lacking comparable studies which explore family's values relating to mathematics, although Abreu et al. (2002a, b) could be considered an example. Also Civil and Andrade's (2002) research indicates that teachers can be encouraged to visit the homes of their students and to find ways to bridge the gaps between the values taught in school and in the home. In some other research, reference is made to the 'elders' of the family and the community, and perhaps some of the research on community values may be helpful here, rather than focusing solely on the family unit. The 'community of practice' research may also inform this part of our research (see, e.g. César and Santos 2006).

The Adequacy or Otherwise of Our Research Methods

The essential goal of research in mathematics education is to help us understand phenomena in richer ways so that we can improve the mathematics teaching and learning situation for as many students as possible. But as we embrace fully the implications of a cultural perspective on mathematics curriculum, teaching and learning, are our research methods and procedures themselves adequate for the task?

There are several researchers who argue 'no' and that we need to change how research is carried out and conceptualised if we are to address these cultural aspects in the thorough way that they need to be addressed. As an example of this, at the PME conference in 1998, Valero and Vithal (1998) criticised the mathematics education research community for its imposition of research methods from the relatively developed 'north' onto researchers and students from the relatively underdeveloped 'south' part of the world. They argued that methods developed in one cultural context are not necessarily appropriate or helpful in another cultural context, in terms of what is considered 'normal'.

It is possible to use, for example, questionnaire approaches (see Appendix for an example of two questions which are based on the six value clusters). However, to develop more sensitive research on preschool mathematical transition, we clearly need to take on board the procedures and practices of anthropological and social psychological research. In general, our research approaches must move to a more collaborative teamwork style, involving not only practitioners and researchers, but also to include the learners and their families as partners in the research process (see Flecha and Gómez 2004; Goos et al. 2005; Stathopoulou 2007).

Just as we have found it necessary and beneficial to do research 'with' rather than 'on' teachers, so there is a need to develop ways of researching 'with' rather than 'on' learners and their families. Already qualitative methodologies have moved us closer to that goal, and if we are really serious about trying to improve our understanding of family mathematics education, then we have little choice but to engage them fully in the inquiry process.

This implies, as well as taking into account the cultural contexts of the learners and their teachers, we must also take into account our contexts as researchers. Just as we recognise the influences that their cultural contexts have on their situations, so we need to recognise the influences that our cultural contexts and values have on us and on our research. This is particularly the case with values which are often hidden: values are often the 'hidden persuaders' of mathematics education (Bishop et al. 2003). Fortunately thanks to the kinds of research illustrated here, values are hidden no longer. What is needed now is for these ideas to be accepted and to be explored throughout the mathematics education community. Values awareness can certainly help parents and teachers transition learners into formal mathematics education.

Appendix: Two Items from the VAMP Teacher Questionnaire

3. "For me, Mathematics is valued in the school curriculum because…"	
……………………………………*Ranking*	
It develops creativity, basing alternative and new ideas on established ones	
It develops rational thinking and logical argument	

It develops articulation, explanation and criticism of ideas	
It provides an understanding of the world around us	
It is a secure subject, dealing with routine procedures and established rules	
It emphasises the wonder, fascination and mystique of surprising ideas	

4. "For me, Mathematics is valuable knowledge because…"	
…………………………………… *Ranking*	
It emphasises argument, reasoning and logical analysis	
It deals with situations and ideas that come from the real world	
It emphasises the control of situations through its applications	
New knowledge is created from already established structures	
Its ideas and methods are testable and verifiable	
It is full of fascinating ideas which seem to exist independently of human actions	

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