Children's Play as a Starting Point for Teaching Shapes and Patterns in the Preschool

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Abstract This chapter contributes to knowledge about the teaching and learning of mathematical content through play in preschool. The study focuses on the potential for teaching and learning mathematical content with children's play as the starting point. The question is important because according to research, children's mathematical encounters in play activities are educational experiences. The understanding of children's mathematical encounters in play and teachers' teaching is presented in terms of learnable and teachable moments in "here-and-now" situations. The data consists of video-recorded observations of young children's play in four Swedish preschools. Two 14-min-long excerpts from the recordings illustrate the potential of children's play for the teaching and learning of shapes and patterns. The results show that a teacher's questions and didactical choices in play can support children's explorations if the teacher observes and recognizes the mathematical content. The results also indicate the potential for teachable and learnable moments and dilemmas when play is the starting point.

The conclusion is that "here-and-now" situations provide teachable and learnable moments. There are also dilemmas, in that teachers have to observe and discern the mathematics in children's play and direct the child's attention towards this.

Introduction

In this chapter, children's early learning of a mathematical content in everyday activities like play is regarded as essential. Research on early mathematics highlights the importance of education and early mathematical skills (Clements and Sarama 2009; Claessens and Engel 2013; Ginsburg and Amit 2008). According to Claessens and Engel (2013), early mathematical knowledge and skills predict the learning of other content areas like language. One way of learning mathematics in the preschool is to provide children with rich and varied mathematical encounters

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and experiences (Clements and Sarama 2009). Research also shows that children learn mathematics in daily interactions with peers (e.g. Bäckman 2015), in the environment (e.g. Carruthers and Worthington 2006; Ginsburg 2006) and in culture (e.g. Starkey and Klein 2008; van Oers 2010).

Two strong discourses on education and teaching/learning in research concerning early childhood can be identified. The first discourse is based on play and children's experiences of the mathematics encountered in their activities (e.g. Carruthers and Worthington 2006; Ginsburg 2006). In this discourse, the idea is that children learn mathematics through play. This leads to informal teaching, in which the teacher follows the children's interests and strives to discern the mathematical content in everyday situations.

In the second and more formal discourse, the teacher chooses the content in advance and plans the teaching situations. Researchers maintain that in this discourse, teachers' instructions are important (e.g. Clements and Sarama 2009; Claessens and Engel 2013; Starkey and Klein 2008). Both discourses can support children's mathematical learning. With a play-based approach, as in this study, the teacher has to observe mathematics in the children's activities and identify potential teachable moments (Hyun and Marshall 2003). This means that teachers have to quickly decide which solutions are appropriate at any given moment. In the second more "teacher-directed approach", the teacher is able to plan what he or she wants the children to learn and design specific teaching situations. The planning can also include deciding which questions and instructions to use to promote the learning of certain content.

In this chapter, I explore the potential for teaching and learning mathematical content in two "here-and-now" situations, with children's play as the starting point. "Here-and-now" situations (Bäckman 2015) mean that the preschool teacher is not tied to any specific situation, but tries to target the learning situations that spontaneously arise in everyday activities, especially in children's play. In these situations, preschool teachers try to ensure that children are able to distinguish mathematical content, for example, shapes and repeat patterns in their activities. "Here-and-now" situations are closely connected with play and have a temporal aspect, which means that the teacher has to make quick decisions on the spur of the moment.

Children's encounters with mathematics in activities provide them with experiences that form a basis for education. In this chapter, the mathematical content consists of geometry, that is, shapes, geometric figures and patterns. Ginsburg (2006) argues that when children play, objects provide opportunities for mathematical thinking. Children construct their knowledge of, for example, shape by playing with shapes.

According to Clements and Sarama (2009), teachers can and should challenge children's experiences of different shapes and create environments with rich and various shapes and thereby promote learning. In Sweden, most children spend a great deal of their early childhood in preschool.[1] Play is an important part of their

[1] 83 % of Swedish children attend preschool between the ages of 1 and 5 years (National Agency for Education 2011).

daily lives, and the preschool curriculum suggests that education should be playful (National Agency for Education 2011). If preschool teachers observe and discern mathematics in children's play, their observations can form the basis for discussion and reflection together with children. In this way, education in preschool can be designed according to children's perspectives.

The aim of this chapter is to explore the potential of children's play as a starting point for teachers' teaching and children's learning of shapes, geometric figures and patterns. Here, *teachable* and *learnable moments* and *dilemmas* in play activities are problematized. According to Hyun and Marshall (2003), teachable moments occur when teachers observe, discern and interpret children's spontaneous interest in play. Teachers can create learnable moments and take advantage of the teachable moments that arise (Cheeseman 2015). A moment is learnable when a child discerns the mathematical content and when the situation promotes learning. A teachable moment can also be a learnable moment, especially if a child has the same focus as the teacher and is receptive to the teacher's questions. If at the same time the teacher is able to discern children's earlier experiences and reflect with the children and challenge their existing thoughts, this could be both a learnable and a teachable moment. One dilemma is that children's own intentions in play can make it difficult for the teacher to visualize the mathematical content that is appropriate for them. It can be problematic for the teacher to direct the child's attention to a mathematical content when the child's attention is directed towards a play content. Another dilemma is whether the teacher is able to recognize the mathematical content in children's play. Teachers need mathematical knowledge and knowledge of relevant issues that can challenge the child's thinking in the moment.

Teaching and Learning Through Play

Children's learning occurs in meaningful and social contexts when they learn something new that builds on their earlier experiences (Vygotsky 1978). In the preschool context, social interactions with peers, adults and the environment all offer children mathematical experiences. Vygotsky (1978) also emphasizes play as the most important part of a child's learning. For researchers to be able to say that learning occurs, knowledge is often required about children's understanding before and after a learning situation. In this study, the interpretations of children's learning are based on whether the moment is learnable or not. According to Bruner (2002), teachers can create interesting environments by making use of such interactions and by using play materials like building blocks and geometrical figures to "scaffold" children's learning.

Play is an important aspect of teachers teaching and children's learning in preschool (Ginsburg 2006; Ginsburg and Amit 2008; Munn and Kleinberg 2003; Pramling Samuelsson and Fleer 2008; Wager and Parks 2014; Wood and Attfield 2005). Ginsburg (2006) describes play and learning as two sides of the same coin, i.e. children play and learn at the same time. The author argues that play motivates and enhances children's cognitive and socio-emotional development.

According to Wood and Attfield (2005), the potential of play in teaching has great importance because it is integrated into the learning process. In play, children can develop skills such as language, mathematics, communication and social skills. Children have opportunities to think hypothetically and follow rules. In play, children are able to guess, estimate or predict what might happen. They can also explore shapes, geometric figures and patterns, dimensions and positions and develop their reasoning about different aspects. Wager and Parks (2014) discuss children's opportunities to learn mathematics in play and how teachers can support that learning in both informal and formal settings. They argue that it is important to observe children's play at school, at home and in the community in order to understand how play facilitates mathematical learning and what the children learn.

Different play contexts provide meaningful opportunities for children to use and develop mathematical skills, such as problem-solving situations in which children can think, experiment, draw and say what they are thinking (Ahlberg 1998). Furthermore, Wood and Attfield (2005) argue that education should give children the opportunity to use flexible and creative ways of thinking and acting and that various play contexts can offer rich opportunities for this.

Preschool children's learning and spatial thinking in geometry imply an understanding of space and, for example, shapes and pattern. Van den Heuvel-Panhuizen and Buys (2005) highlight the importance of geometry and spatial thinking. Children's understanding and meaning making in geometry influence their thinking and spatial development. Van Hiele (1959) and later Tirosh et al. (2011) describe the different levels of children's geometric thinking describing the first level as the visual level. This means that children have an early experience of an object when they see its structure or form. At this level of their spatial thinking, children assess figures belonging to the same category. For example, a rectangle could be a door or a table. The second level is the descriptive level. At this level, children examine the properties of shapes, rather than their appearance. Children can verbally describe that triangles have three corners and three sides and that a circle is round. This means that at this descriptive level, language is important. The third level is the deductive level and means that children are able to formulate definitions for shapes like triangles and rectangles. When children explore various items and look at them and touch them, they have a visual and tactile experience which can support them to discern similarities and differences that will form the basis for future experiences of shapes.

Spatial thinking and spatial orientation are important for children's exploration of the world, because they indicate where things are located and placed and the distance between them. Van den Heuvel-Panhuizen and Buys (2005) describe the spatial ability and orientation that is important for children's spatial development and their discernment of shapes and patterns. This is in line with Ginsburg and Ertle's (2008) suggestions of describing spatial relations and different kinds of patterns, e.g. alternating patterns with or without repetition and growing patterns. A pattern refers to an underlying rule, such as the repetition of circular shapes. Clements and Sarama (2009) argue that children develop their geometric thinking through play that this can be carefully planned by using materials like mosaics and puzzles, but can also occur in spontaneous play.

In order to experience and learn about geometric content such as shapes, figures and patterns, environments are required that offer a variety of geometric shapes and figures. According to Clements and Sarama (2009), this environment should give children an opportunity to explore and discern the similarities and differences in the artefacts used and give rise to discussions about different kinds of shapes and their properties and the kind of geometric tasks and challenges that arise. The authors highlight four guiding features in the environment that can provide education about shapes. The first feature is that preschools should give children opportunities to experience a lot of different kinds of shapes. This includes varied examples of the characteristics of shapes and opportunities to discern the similarities and differences among them.

The second feature is that preschool teachers should encourage and challenge children's descriptions in order to enrich their language. For example, children should have opportunities to explain why a shape belongs or does not belong to a certain category.

The third feature includes the environment. Preschools should offer different classes of shape such as various sizes and orientation of circles, triangles, squares and rectangles, as well as different colours and materials. Clements and Sarama (2009) argue that this includes showing children that squares are examples of rectangles.

The fourth feature is to stimulate children by providing a wide range of interesting activities and tasks, including reflection and discussion, so that children can compare, identify and explore the different shapes and figures that are important in geometry.

Claessens and Engel (2013) suggest that when children are able to focus on pattern recognition, measurement and advanced numbers in the early years of schooling, it will benefit their learning later in school. The authors highlight that teachers' instructions are necessary for children's outcomes in mathematics. Bruner (2002) highlights scaffolding which consists of teachers' feedback and the use of different strategies, such as active listening, questions, affirmation and mathematics-related talk (Bruner 2002).

Play gives teachers the opportunity to observe children's expressions and their mathematization (actions and reflections on mathematical content, articulation of concepts and features) of shapes (Carruthers and Worthington 2006; Van Oers 2010). Play also helps children to use their imagination and creativity by, for example, considering how an object works, how a ball rolls and what they can do with a ball. Teachers and children can discuss and reflect on the characteristics of objects, e.g. whether shapes are round or curved, which forms have corners and how many corners there are (Clements and Sarama 2009). When children play with blocks and build constructions, preschool teachers have an opportunity to reason with them about different classifications and attributes and to offer opportunities that will develop their spatial abilities, such as body and spatial awareness, and knowledge about measurement. Carruthers and Worthington (2006) and Van Oers (2010) argue that there is a mathematical content in children's play and that it is up to the teacher to discern when mathematics occurs in different play contexts.

Methodology

The aim of this study is to explore the potential of children's play as a starting point for teachers' teaching of shapes and patterns. Children's actions and intentions are in focus, specifically what they direct their attention towards in their mathematical encounters during play, as these have the potential to become learnable moments. The research also focuses on teachers' approaches to and teaching in teachable moments.

This study is a part of a larger study (Bäckman 2015). Thirty-five 4-year-old children from four Swedish preschools took part in the study relating to children's experiences of mathematics in everyday situations. Video observations were used to focus on children's activities with a mathematical content. Children's and preschool teachers' formations of mathematics in the preschool constitute the study's research object and include an analysis and interpretation of children's actions and mathematical encounters. It also includes interactions with teachers.

For this chapter, two 14-min excerpts from the study (18 h of video observations) are used. These excerpts—one observation with shapes and another with geometric figures and patterns—have been chosen because they reflect common situations found in the preschool and highlight two dimensions of the teaching and learning potential of play.

The video observations make the mathematical content in children's play and children's actions visible. The observations show verbal and non-verbal language, glances, gestures, nods, smiles, the artefacts the children are using and how they use them and whether they use them on their own or together with peers/adults. The observations also highlight teachers' actions and their interactions with one child.

The research is directed towards teachers' teaching and preschool children's learning, which entails a particular responsibility to comply with applicable ethical considerations. As video observations are used to observe different situations in the preschool, it is necessary to protect the participants' identities and integrity. Both parents and teachers gave their written consent for the children's participation. In the video-recorded situations, the participating children's oral consent was obtained. The teachers also gave their consent to participate in the study.

Results

The results demonstrate children's experiences with building blocks and geometric figures and their creation of patterns. One result is the identification of the potential of learnable and teachable moments in play. Children often play on their own, with material that the teacher provides. The boy in the first excerpt focuses on the different attributes of the shapes from a stable and durable perspective. He seems to have set goals in his building and tries to put different shapes on top of each other.

The observation started early one morning in a Swedish preschool when Erik, aged 4, was playing with building blocks in the hall. One of the preschool teachers was standing beside him talking to a parent.

Erik does not seem to be paying any attention to the adults' conversation. He is using different shaped blocks to build a high stack of eight blocks. The blocks consist of seven cubes in three different sizes, as well as a pyramid. Every second block is a small cube, and the alternative block is a larger cube. A pyramid is placed on the top of each stack. Erik then started to make a shorter construction consisting of six half cylinders. He placed half a cylinder with the short edge towards the floor on one side of the stack and another half cylinder opposite the first one with the long surfaces against each other. He looked at a third half cylinder, twisted and turned the block, looked at the construction and then put the half cylinder between the first two.

Erik is totally focused on the construction. He seems to want to build both horizontally and vertically. He twists and turns the blocks in an attempt to find a stable and durable way of placing them on top of each other. At the same time, there also seems to be a desire to make his building work symmetrical.

He picks up a fourth half cylinder and places it on the other side of the stack. He then creates a similar construction as the one on the opposite side of the stack with identical half cylinders on each side of the half cylinder in the middle. When the blocks are in place and everything is stable, he places a pyramid on the top of both constructions.

Erik explores the various geometric shapes by placing them on top of and next to each other in different ways. He distinguishes the various qualities of the blocks as he twists and turns and builds with them. The observation shows the potential of learning about the critical aspects of geometric shapes in the play context. Erik seems to have an idea or intention for the construction and experiences how these differently shaped blocks can or cannot be stacked. Erik is focused on his construction, although he does look up from time to time. He seems to have specific goals in mind with his building and is not disturbed by the adults and children talking to each other next to him. Erik continues to create the construction:

Erik points to two cubes in the high stack in the middle and says,

E: "It's over and it's over".

Then he points to the two top blocks and says,

E: "Those should be removed".

He takes the top two blocks from the highest tower and places them on the floor, a cube with a pyramid top. He then takes a pyramid lying on the floor and places it on top of the tall tower. He does this while holding a little blue car in his hand. He looks at the high tower and says,

E: "There you go".

Erik's building seems to give him experiences of the similarities and differences between the various blocks. He does not talk about the shapes or the features, but is totally engaged in the construction. The teacher talks to one parent in the hall and later on walks through the hall passing Erik and his construction. She stops and says:

Teacher: "What a nice building. Very tall!"

Erik looks up when the teacher is talking to him but doesn't say anything. The teacher then leaves the room.

Erik is engaged in his construction work and the teacher observes, and at a later moment, the teacher did make use of these experiences in discussions with Erik about different shapes.

Another excerpt from the data shows a 4-year-old girl, Meg, playing with geometric figures. The figures in the stack on the table in front of her have different shapes and colours, such as circles, triangles, squares, pentagons and polygons.

Meg has started to twist and turn the geometric figures into different shapes. She then selected only the red figures of different shapes (circles, triangles, squares, pentagons and polygons) from the stack in front of her. She created a red pattern with the figures and after that she selected a blue circle from the stack with the geometric figures. She put the blue circle on the table in front of her, then a yellow circle, an empty space and then a yellow circle.

The teacher, who is sitting at another table observing Meg's designs, asks Meg,

Teacher: "What should be put in the empty space in your pattern now?"

Meg looks at the geometric figures and picks up a blue circle, which she places in the empty space.

Teacher: "What is next in your pattern?"

Meg looks at the teacher and smiles as she picks up the last yellow circle.

Teacher: "What colour is the circle?"

Meg: "Yellow", she says as she puts down the shape. Meg then chooses a blue circle.

Teacher: "Blue".

Meg laughs and adds the blue circle to the yellow one in the pattern.

The teacher makes Meg's pattern visible to her by verbally supporting and confirming the girl's actions. The use of questions and colours are strategies employed in the feedback process to make the pattern visible. Meg is able to think abstractly and reflect on the pattern. The interaction between the teacher and the child highlights colour as a criterion for the circles in the pattern. It seems that Meg has not noticed that the coloured circles make a potential repeat pattern, but the teacher does and draws this to her attention. In this case, the teacher scaffolds and gives feedback using questions and by giving the features names. In the beginning of the observation, Meg explores different geometric figures and has an opportunity to discern the similarities and differences between them. She starts by focusing on the differently shaped red figures and then starts to create a potential repeat pattern. Maybe it is the teacher's comments that make Meg think what kind of circle is appropriate.

The observations show the kind of experiences children engage in. The observation of Erik illustrates a common situation in the preschool. Teachers talk to parents and other adults at the same time as they take care of many children. Despite this they want to give the children some kind of feedback. Erik seems to study the characteristics of the blocks before he puts them together, which may make him wonder about the various features and if the construction is stable. Erik's experiences in this play setting provide valuable opportunities to explore different shapes, which makes the situation learnable. The play also has the potential to be teachable if the teacher stays and reflects with the child. The provided material and the teacher's observation can be used again in a new teachable moment. The teacher did make use of Erik's experiences at a later occasion when they reflected on different shapes.

The example with Meg highlights a play situation in which a child explores coloured circles and puts some of them together in what appears to be a repeat pattern. Like Erik, Meg is engaged in the exploration of shapes and patterns. Here, the teacher draws Meg's attention to the possibility of repeating the colours of the circles to make a pattern and uses this as a teachable moment to guide Meg into recognizing a repeat pattern. The teacher does this by asking questions such as "what is in your pattern now?" The girl picks a blue figure and the teacher says "blue!" The

situation can also be said to be learnable in that the girl seems to observe the possibility of repeating the colours and perhaps also discerning the pattern.

These two observations take account of both the child's perspective and the teacher's perspective. Erik initiates the play himself and he seems to have set goals in sight. The teacher gives him brief feedback about the height of his construction and says that it looks nice. In Erik's case, the teacher has an opportunity to really pay attention to the mathematical aspects of his construction. She observes Erik's play but does not ask any questions and only briefly comments on his construction. However, she does have an opportunity to elaborate on Erik's construction at a later date. The teacher in Erik's case could have asked him how many sides and how many corners the different shapes had and about his choices of different shapes. Here, Erik is only exposed to the qualities of shapes when he works and plays with them.

Meg does not seem to be as goal-oriented as Erik. She looks at the different shapes and places them on the table. The teacher recognizes a possible pattern in the coloured shapes. She draws the child's attention to the possibility of repeating the colours by asking questions and stating the fact that there is a pattern, i.e. both whole and in part. The girl's actions display that she seems to be aware of the patterns she made. In Meg's case, the teacher's intention is to support the girl's discernment of patterns by asking her about the colours of the shapes. From the child's perspective, the teacher's interest, questions and statements serve as positive feedback and are perhaps enough for Meg's discernment at that particular moment.

Discussion

In this section, I reflect on and discuss the potential for teachable and learnable moments in two "here-and-now" situations with the children's play as the starting point. This includes children's experiences and the teachers' scaffolding. The designed environment with interesting play material like building blocks and geometrical figures can be part of a teacher's scaffold (Bruner 2002).

Ginsburg's (2006) suggestion that play gives children an opportunity to explore shapes, geometric figures and patterns, dimensions and positions seems to fit these situations. When the teacher observes Meg's play and the spontaneously emerging situation, she is able to exploit this and turn it into a teachable moment. Guided by the teacher's questions, children can be challenged, stimulated and acquire new experiences, which makes the situation learnable. In the examples with Erik and Meg, the teacher's attention and reaction to the child's actions are aspects of the guiding and feedback process. This agrees with Clements and Sarama's (2009) research, which points out that preschool teachers' use of questions, feedback and the provision of rich environments can lead to deeper understanding and learning. Both children in the described observations experience shapes in their different constructions. Such experiences can form the basis for children's learning. Children's cognitive processes in activities like these affect their learning, as does observing

and participating with others in play. Children are not passive recipients, but are active in the processes in which they are involved. The two observations also highlight some of the dilemmas that can arise with teachable and learnable moments in "here-and-now" situations. One such dilemma is time like the teacher in the example with Erik's construction and another is teachers' knowledge. Ginsburg and Amit (2008) argue that teaching mathematics to young children is almost the same as teaching mathematics to older children. They maintain that a preschool teacher must know what the content is and how this can be made visible to the children. They also stress that the teacher must have pedagogical content knowledge to know how to teach the content and in this study to preschool children. The teacher in the example with Erik observed a mathematical content in the boy's construction and did make use of the moment. She was able to reconnect to the boy's experiences at a later time. Wager and Parks (2014) highlight the importance of observing children's play in order to understand how play can facilitate mathematical learning.

The teachers in the excerpts could have given more feedback by asking the children explain their thinking and actions. In the example with Erik, the teacher could have asked him about his thoughts and suggestions and could also have provided specific information about the different shapes he was playing with. They could have talked together about the different features of the shapes, but in this situation, they did not. In Meg's case, the teacher pointed to the possible repeat pattern as a way of scaffolding. Teachable moments can provide learning experiences for the child, although as Hyun and Marshall (2003) argue, it can be difficult for the teacher to respond to the teachable moments that arise, especially if there are a lot of children in the group or parents like in the example with Erik. This is a dilemma for the teacher, and in the example with Erik, the teacher talked to a parent which means that she could not respond to him directly.

Bäckman (2015) has highlighted and provided insights into the importance of continuing to raise awareness among preschool teachers regarding preschool children's mathematical experiences in everyday life. The most important thing to note is that mathematical content is present in a variety of situations in the preschool. It may not always be the mathematical content that is focused on by the children, but the activity itself. It is the teacher who can direct the child's intention to the mathematics and make the situation teachable and learnable. This can also be a dilemma, because children like Erik and Meg have their own intentions in play, and it can be difficult for the child to have the same focus as the teacher.

"Here-and-now" situations like the situations with Erik and Meg can be both teachable and learnable moments depending on what children express in their actions and what opportunities the preschool teacher have to exchange thoughts and reasoning around the object's various features. When children are at this visual level of their spatial thinking, shapes that look similar belong to the same category (Clements and Sarama 2009). Both Erik and Meg show by their actions that they reflect on differences and similarities. According to Carruthers and Worthington (2006) and Van Oers (2010), it is up to the teachers to observe the mathematical aspects of children's play and support them by providing the relevant material and

giving appropriate feedback. Sometimes, like in Erik's case, the material gives feedback in the moment, and the teacher observes and gives feedback later on.

The language that teachers use when talking to children about shapes is important, because it helps them to make the necessary connections. Teachers can also ask children to describe and reflect on the various features of the shapes they are playing with in the moment or afterwards like in Erik's case. In this context, the teacher's questions and feedback may be more important than instructions. The use of questions like in Meg's case can direct the child's attention to the similarities and differences among the coloured circle shapes, but the teacher lost the opportunity to ask about the specific characteristics and attributes of shapes and figures.

Clements and Sarama (2009) suggest that a carefully designed learning environment in preschool supports children's spatial development and provides opportunities for children like Erik and Meg to mathematize in play. Various play materials in the learning environment that offer reflection about similarities and differences among shapes are important for children's mathematical thinking (Ginsburg 2006). Teachers can help them by using different didactic strategies and making didactic choices even if they did not in these two examples. The presence of teachers offers the opportunity for teachable and learnable moments in "here-and-now" situations such as play like in the example with Meg. In the other example, the teacher observed Erik's play with blocks, and she had the opportunity later on to reflect with him about his choices. Maybe it could have been more mathematically useful if the teacher talked to him in the moment about how different shapes can be used to ensure that the construction is tall and stable. The didactic choices include questions directed at both the mathematical content and children's perceptions of the specific content. Here, flexibility around the mathematical content in children's activities "in the moment" and responsiveness to what the children direct their attention to are important aspects of teaching.

Conclusion

The study shows that in Erik's and Meg's exploration of shapes, possible patterns and so on, children create teachable and learnable moments in "here-and-now" situations, with play as the starting point. Teachers can observe and reflect on the experiences that children have in play and thereby provide teachable and learnable moments. Teacher's use of feedback strategies like attention, questioning and statement is also of importance. It could be teachers' questions together with various play materials that support children's mathematizing and learning in preschool rather than instructions. Play is a valuable part of children's everyday lives and can give teachers opportunities to encounter and reflect on mathematics from a child's perspective. Children have their own intentions in play, and teachers need to be attentive to these intentions and the child's experiences in the teaching situation.

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