Chapter 37
A Framework for Assessing Statistical Knowledge for Teaching Based on the Identification of Conceptions of Variability Held by Teachers

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37.1 Introduction

Aiming toward statistical literacy in today's information society, recent curricular reforms in many countries have brought a number of topics related to statistics and probability into the school mathematics curriculum (e.g., NCTM, 2000), it being noticeable that variability may arise in many different ways in such topics.

Variability—a property of an statistical object which accounts for its propensity to vary or change—is considered by several researchers as a fundamental concept in statistics (e.g., Pfannkuch & Ben-Zvi, 2011; Shaughnessy, 2007); and its acknowledgement and understanding are regarded as essential skills for statistical literacy, reasoning, and thinking (e.g., Sánchez, da Silva, & Coutinho, 2011; Wild & Pfannkuch, 1999). According to Gattuso and Ottaviani (2011, p. 122), "[t]o be part of a modern society in a competent and critical way requires citizens to … understand the variability, dispersion, and heterogeneity which cause uncertainty in interpreting, in making decisions, and in facing risks," and teachers are in charge to foster and develop such knowledge and skills in their students. Despite all these facts, scarce studies can be found in the literature focused on the conceptions of variability held by in-service teachers, as well as on the knowledge entailed by teaching variability-related contents, and statistics in general, to help students achieve the aims of statistics education (Shaughnessy, 2007). Hence, it is by no

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means surprising the urgent call for increasing research on these areas made for a number of concerned researchers, particularly for studies on teachers' professional knowledge and teachers' practices while teaching variability (e.g., Sánchez et al., 2011, p. 219), as well as for the developing and improvement of the models for the didactic knowledge required to teach statistics (e.g., Godino, Ortiz, Roa, & Wilhelmi, 2011, p. 281). Accordingly, the purpose of this chapter is to respond to such calls for research.

In the present article, I propose a conceptualization of SKT—the knowledge, skills, and habits of mind needed to carry out effectively the work of teaching statistics in a way that supports student learning and achievement—aiming to contribute to a better understanding of what knowledge is necessary and sufficient to teach statistics well, by addressing and helping to fill in some notable gaps in the research literature on statistics education. The proposed model focuses on investigating teachers' knowledge for teaching variability-related concepts, and its main implications are (a) preparing the ground for future empirical research on SKT at school level; (b) bringing closer together SKT and the model for MKT developed by Ball, Thames, and Phelps (2008); (c) extending such model to include teachers' beliefs about statistics, teaching and learning of the various statistical topics in the school mathematics curriculum; and (d) identifying and taking into account teachers' conceptions of variability.

37.2 Literature Review

In the next subsections, the author presents a summary of some of the research literature relevant to the development of the framework for SKT proposed in the current article.

37.2.1 The MKT Model

Influenced by the criticisms directed at the aspects of teacher knowledge identified by Shulman in his breakthrough article (Shulman, 1986), and examining ways in which Shulman's ideas could be operationalized in mathematics education, Ball et al. (2008) developed the notion of mathematical knowledge for teaching (MKT), a practice-based model of content knowledge needed for teaching mathematics effectively, focused on both what teachers do as they teach mathematics and what knowledge and skills teachers need in order to be able to teach mathematics effectively. This model describes MKT as being made up of two domains—subject matter knowledge (SMK) and pedagogical content knowledge (PCK)—each of them structured in a tripartite form, as depicted in Fig. 37.1.

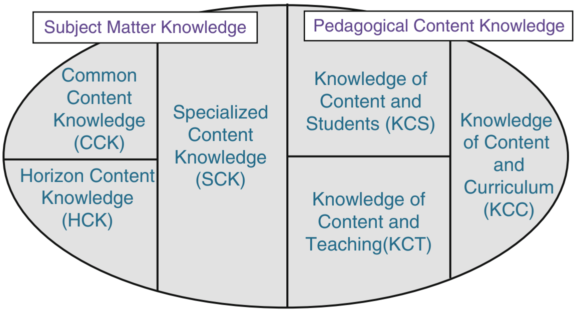


Fig. 37.1 Domains of MKT, according to Ball et al. (2008)

According to Ball et al. (2008), SMK can be divided into common content knowledge (CCK)), specialized content knowledge (SCK)), and horizon content knowledge (HCK)). The construct CCK refers to the mathematical knowledge and skills expected of any well-educated adult, which are commonly used in any setting, not necessarily the one of teaching. SCK is the mathematical content knowledge specific to the work of teaching and needed in its practice—and not in the practice of other professions. The third construct, HCK, is an awareness of where both the present learner experience and the instructional content are situated over the span of mathematics included in the school curriculum, and of what their connections are with the key mathematical practices and major disciplinary ideas and structures that lie ahead, on the curricular horizon.

Furthermore, Ball and her colleagues presented a more refined division of Shulman's PCK, comprised by knowledge of content and students (KCS), knowledge of content and teaching (KCT)), and knowledge of content and curriculum (KCC)). The construct KCS represents the teacher's amalgamated knowledge about how students come to understand mathematics and mathematics content itself. KCT refers to the knowledge about how to carry out the design of instruction in order to develop mathematical understanding in students, and about how a particular mathematical content shapes mathematics instructional practice. Finally, KCC is the knowledge that teachers have on how specific topics, procedures, and concepts are offered in school curricula at a particular grade level, along with an understanding of the grade-wise relationships among them and the variety of educational materials that can be drawn on to facilitate the development of students' mathematical understandings.

Through this model for MKT, Ball and her colleagues made significant progress in identifying the relationship between teacher knowledge and students' achievement in mathematics, as well as in developing reliable and valid measures of MKT. Nevertheless, as highlighted by some researchers (e.g., Petrou & Goulding, 2011, p. 16), Ball et al.'s (2008) model of MKT does not acknowledge the role of beliefs in teachers' taking on, and performance of, educational practices, which could be a drawback since beliefs are often regarded in the literature as important factors affecting teachers' instructional practice.

37.2.2 MKT-Based Models for SKT

It is by no means surprising that almost all the few conceptualizations of SKT proposed to date have assimilated some of the categories present in the aforementioned model for MKT developed by Ball and her colleagues, due to the considerable overlap and cooperation between mathematics and statistics, as well as between the structure of mathematics education and statistics education (see Hand, 1998). Nevertheless, due to the specificity of statistics as discipline (see Gattuso & Ottaviani, 2011; Godino et al., 2011), it is not surprising either the effort that has been made through those few conceptualizations of SKT to adapt MKT components in order to meet the particular case of statistics education. In this subsection, the author presents an overview of the MKT-based models of SKT developed by Burgess (2011), Groth (2007), and Noll (2011).

Groth (2007) developed a hypothetical framework to explain the SKT required for teaching statistics at high school level, borrowing and focusing on the constructs of CCK and SCK described by Ball et al. (2008), and merging and adapting them with the framework for statistical problem solving given in the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report (Franklin et al., 2007), in order to characterize the work of teaching statistics, make distinctions between the mathematical and nonmathematical knowledge needed for it, and differentiate such work from the one of teaching mathematics.

In his model, Groth argues that some aspects of the common and specialized knowledge entailed by the teaching of statistics require a growing research base, particularly the specialized one related to nonmathematical knowledge, which encompasses the pedagogical activities that take place in the classroom.

In order to examine, through a classroom-based approach, the knowledge that elementary school teachers need to successfully implement the teaching of statistics through projects and investigations, Burgess (2011) developed a two-dimensional framework comprised by four of the knowledge components described by Ball et al. (2008)—CCK, SCK, KCS, and KCT—and six out of eight components of Wild and Pfannkuch's (1999) model for statistical thinking in empirical enquiry. Through his model, Burgess identified the different types of knowledge that were either needed and used, or needed but not used, in the context of teaching experiences based on statistical investigations, finding, among other things, that all the aspects of knowledge included in his proposed model were necessary in the classroom.

Noll (2011) investigated the SKT held by 68 American graduate teaching assistants' (TAs) using a task-based survey and a series of semistructured interviews, focusing on TAs' knowledge about distributions of data and empirical samples, as well as in their knowledge of student thinking about sampling concepts. Noll selected three of the components described by Ball et al. (2008)—CCK, SCK, and KCS—to develop her framework. Key features in her model are the interpretation of CCK and SCK as statistical literacy and statistical thinking, respectively. The findings from Noll's research indicate that TAs have a limited SKT in all the three components in study, which is particularly noticeable in their difficulty teaching certain topics —especially conceptual ideas of variability—and making sense of students' work and interpretation about variability and other sampling-related concepts.

37.3 A New Conceptualization of SKT

The purpose of this section is to make a contribution to the literature on statistics education, by proposing a conceptualization attempting to characterize some critical components related to the knowledge required to teach statistics effectively. On the basis of literature review and personal research experience, several components that were thought to be potential predictors of SKT were identified and considered for analysis, and the following arguments were raised as a result of such analysis:

(a) *The proposed model of SKT should be closely tied to a model of MKT*: On the basis that school statistics is often taught as part of mathematics curriculum by mathematics teachers, as well as due to the common grounds shared by mathematics and statistics, it is anticipated that a model of SKT should be closely tied to a model of MKT. Consequently, I argued that the six constructs necessary for having a solid MKT identified by Ball et al. (2008) in their framework would serve as a useful starting point to hypothesize what knowledge might be needed for teaching statistics effectively.

(b) *Some knowledge components in the MKT model used must be redefined to meet the requirements of teaching statistics*: Although mathematics and statistics share some common grounds, the two disciplines are different in several ways (an in-depth discussion of these differences can be found in Gattuso & Ottaviani, 2011). Therefore, in order to acknowledge such differences and meet the requirements specific to the teaching of statistics, some knowledge components in the MKT model used must be redefined. In the case of the conceptualization proposed here, CCK will be seen as statistical literacy, which development is regarded as one of the main goals of statistics education and mathematics curricula at all educational levels (e.g., Gal, 2004; Pfannkuch & Ben-Zvi, 2011), and thus its acquisition is expected from any individual after completing school education. The rest of knowledge components in this framework are defined in the same way as in the model of MKT by Ball et al. (2008), but rephrased in some cases to meet the requirements of teaching statistics.

(c) *In order to conceptualize SKT, teachers' beliefs about statistics, teaching and learning must be considered*: The relationship between beliefs—defined by Philipp (2007, p. 259) as "psychologically held understandings, premises, or prepositions about the world that are thought to be true"—and teachers' classroom practice has been well articulated in the literature by several researchers (e.g., Gal, Ginsburg, & Schau, 1997; Philipp, 2007; Pierce & Chick, 2011). Moreover, beliefs are identified by Gal (2004) as one of the dispositional elements of statistical literacy, being the latter regarded as CCK in the present conceptualization of SKT. On the basis of these facts, in this model of SKT teachers' beliefs about statistics, teaching and learning are going to be regarded as fundamental, attempting in that way to obtain a much richer and broader picture of the knowledge needed to teach statistics efficiently, as well as to overcome a common drawback in all the MKT-based frameworks of SKT reviewed previously.

(d) *Tasks designed to elicit teachers' conceptions of variability would be helpful to provide indicators to measure SKT as defined in this study*: In the case of teachers, conceptions—the set of internal representations and corresponding associations that a mathematical concept evokes in the individual—have been proved to influence their own approaches to teaching, and consequently their students' approaches to learning (e.g., Trigwell, Prosser, & Waterhouse, 1999). Also, the work carried out by González (2011) and Isoda and González (2012) provides empirical evidence that the use of tasks addressing variability and variability-related concepts is an effective method for eliciting, identifying, describing, and assessing not only the conceptions of variability held by teachers but also their SMK in statistics. On the basis of these facts, and because conceptions represent knowledge and beliefs working in tandem (Knuth, 2002), gaining insight into the teachers' conceptions of variability is regarded as necessary in the proposed model for SKT.

37.3.1 An Instrument to Assess SKT Based on the Identification of Conceptions of Variability Held by Teachers

Based on the four arguments outlined above, a pen-and-paper instrument, comprised by tasks addressing variability and variability-related concepts present in the school mathematics curriculum, was designed in order to assess the eight components of SKT identified and described by this study— the six knowledge components in the model for MKT developed by Ball et al. (2008); teachers' beliefs about statistics, teaching and learning; and teachers' conceptions of variability. Each item in the instrument was developed based on questions used in previous studies with similar aims reported in the literature (e.g., Ball et al., 2008; Isoda & González, 2012), which were adapted to reflect the context of the item, the case of teaching school statistics, and the specific objectives of the present conceptualization of SKT.

In order to provide a comprehensive framework for conceptualizing SKT in the context of variability, twelve indicators were identified and selected for assessing SKT from the teachers' answers to each of the designed items (see Table 37.1).

Item 1 is provided as an example of the designed items (see Fig. 37.2). The original version of the task (by Garfield, delMas, & Chance, 1999) was adapted and enriched with questions aiming to elicit all the facets of SKT identified by this framework. A mapping between the components of SKT that would be brought out by each question in Item 1 and the indicators associated to such components identified by this framework could be appreciated in Table 37.2.

Table 37.1 Set of proposed indicators to assess SKT through the answers to Item 1

A: Indicators associated to Statistical Literacy (CCK)
1. Is the teacher able to give an appropriate and correct answer to the given task?
2. Does the teacher consistently identify and acknowledge variability and correctly interpret its meaning in the context of the given task?
B: Indicators associated to SCK
1. Does the teacher show evidence of ability to determine the accuracy of common and nonstandard arguments, methods, and solutions that could be provided on a single question/task by students (especially while recognizing whether a student's answer is right or not)?
2. Does the teacher show evidence of ability to analyze right and wrong solutions that could be given by students, by providing explanations about what reasoning and/or mathematical/statistical steps likely produced such responses, and why, in a clear, accurate and appropriate way?
C: Indicators associated to HCK
1. Does the teacher show evidence of having ability to identify whether a student comment or response is mathematically/statistically interesting or significant?
2. Is the teacher able to identify the mathematically/statistically significant notions that underlie and overlie the statistical ideas involved in the given task?
D: Indicators associated to KCS
1. Is the teacher able to anticipate students' common responses, difficulties, and misconceptions on the given task?
2. Does the teacher show evidence of knowing the most likely reasons for students' responses, misconceptions, and difficulties in relation to the statistical ideas involved in the given task?
E: Indicators associated to KCT
1. In design of teaching, does the teacher show evidence of knowing what tasks, activities, and strategies could be used to set up a productive whole-class discussion aimed at developing students' deep understanding of the key statistical ideas involved in the given task, instead of focusing just on in computation methods or general calculation techniques?
2. Does the teacher show evidence of knowing how to sequence such tasks, activities, and strategies, in order to develop students' deep understanding of the key statistical ideas involved in the given task?
F: Indicators associated to KCC
1. Does the teacher show evidence of knowing at what grade levels and content areas students are typically taught about the statistical ideas involved in the given task?
2. Does the designed lesson (or series of lessons) show evidence of teacher's understanding and support of the educational goals and the intentions of the official curriculum documents in relation to the teaching of the statistical contents present in the given problem, as well as statistics in general?

The context of the task posed in Item 1—comparing distributions—has been acknowledged as "a fruitful arena for expanding teachers' understanding of distribution and conceptions of variability" (Makar & Confrey, 2004, p. 371). Moreover, giving an appropriate answer to Item 1 requires from teachers, among others things, knowledge and understanding of several fundamental concepts and ideas in school statistics—as in Questions (a) and (b)—; ability to connect and represent such concepts and ideas—as in Questions (a), (b) and (e)—; ability to make sense of students' answers and to sort out the reasonable ones from those that are incorrect—as in

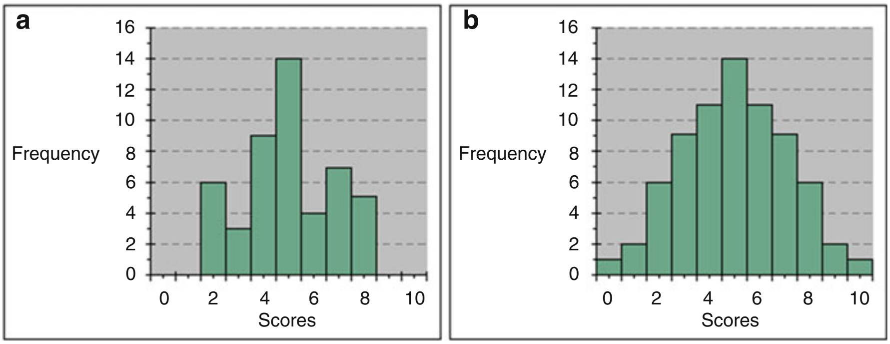
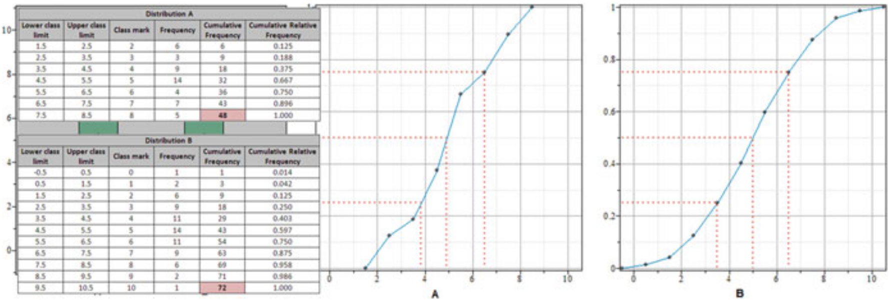


Fig. 37.2 Item 1—“Choosing the distribution with more variability” task

Table 37.2 Knowledge components of SKT elicited by each of the questions posed in Item 1

Elicited knowledge component of SKT	Associated indicator of SKT	Question
Statistical literacy (as CCK)	A1	(b)
	A2	(b)
Specialized content knowledge (SCK)	B1	(c)
	B2	(c)
Horizon content knowledge (HCK)	C1	(e)
	C2	(a)
Knowledge of content and students (KCS)	D1	(d)
	D2	(d)
Knowledge of content and teaching (KCT)	E1	(g)
	E2	(g)
Knowledge of content and curriculum (KCC)	F1	(f)
	F2	(g)

Question (c)—; understanding of how students reason in the context of the given task—as in Question (d)—; knowledge about how the concepts and ideas involved in the posed task are developed curriculum-wise as one move up the education ladder—as in Question (f)—; knowledge about how to interpret and teach different, but interconnected and interdependent, variability-related concepts used in statistics, as well as how to teach and put into practice the statistical habits of mind related to them—as in Question (g). Also, since teachers are expected to know how to map the characteristics of the given histograms to alternate representations in order to provide an evidence-based statistical argument to justify and defend their answers (see Fig. 37.3), it is anticipated that Question (g) will also elicit how teachers promote the development of statistical discourse and argumentation into the classroom, which is of crucial importance to develop statistical literacy and avoid students' misperceptions of statistics (Gal, 2004; Pfannkuch & Ben-Zvi, 2011), as well as to make visible teachers' ability to recognize what concepts can be addressed through a particular data set, and to plan and implement effective learning in the



Distribution A					
Lower class limit	Upper class limit	Class mark	Frequency	Cumulative Frequency	Cumulative Relative Frequency
1.5	2.5	2	6	6	0.125
2.5	3.5	3	3	9	0.188
3.5	4.5	4	9	18	0.375
4.5	5.5	5	14	32	0.667
5.5	6.5	6	4	36	0.750
6.5	7.5	7	7	43	0.896
7.5	8.5	8	5	48	1.000

Distribution B					
Lower class limit	Upper class limit	Class mark	Frequency	Cumulative Frequency	Cumulative Relative Frequency
-0.5	0.5	0	1	1	0.014
0.5	1.5	1	2	3	0.042
1.5	2.5	2	6	9	0.125
2.5	3.5	3	9	18	0.250
3.5	4.5	4	11	29	0.403
4.5	5.5	5	14	43	0.597
5.5	6.5	6	11	54	0.750
6.5	7.5	7	9	63	0.875
7.5	8.5	8	6	69	0.958
8.5	9.5	9	2	71	0.986
9.5	10.5	10	1	72	1.000

Fig. 37.3 Frequency distribution tables, boxplots, and ogives are some of the alternate representations and connections expected from teachers when dealing with Item 1

classroom with data, abilities required from teachers in order to be competent in developing statistical literacy in their students (Batanero & Díaz, 2010). Based on these arguments, all of them strongly related to specific components identified in the model of SKT proposed by this chapter, the selection of the task and questions posed in Item 1 is amply justified.

Regarding teachers' beliefs, a useful way to identify them is from how teachers answer to students' thinking in the classroom, interpret official curriculum documents, and design learning activities. For example, by answering to Question (g) in Item 1, it is anticipated that teachers' personal approaches to teach specific statistical contents will give evidence of their beliefs about whether, for example, teaching and learning of statistics is better accomplished through emphasizing the memorization of formulas and procedures, rather than through the developing of students' conceptual understanding of statistics and their ability to apply and interpret statistics in meaningful ways (Pierce & Chick, 2011).

Finally, regarding teachers' conceptions of variability, it is anticipated that several characteristics about how teachers acknowledge and describe variability in the context of the given task will emerge through their answers to Question (b) in Item 1. The types of conceptions of variability identified by Shaughnessy (2007, pp. 984–985) will be used in this study to classify those distinguished in teachers' answers through the proposed framework.

37.4 Conclusions

To teach statistics at any educational level, teachers must grapple with the concept of variability, one with which students often struggle. In this article, it is argued that teachers' SMK and PCK (as in Ball et al., 2008), beliefs, and conceptions of variability (as in Shaughnessy, 2007) play altogether an important role in the shaping and effectiveness of the teaching practice in statistics. Therefore, after a literature review and theoretical considerations, a model for SKT combining the aforementioned facets was developed, and an approach for the upcoming empirical research was presented.

The conceptualization of SKT being arisen in this article not only attempts to respond to the calls that have been made for more research on particular issues in statistics education, but also proposes posing tasks that involve dealing with variability as a way to assess specific components of the knowledge needed by teachers to teach school statistic effectively.

Since new school mathematics curricula worldwide require from teachers competence to build and scaffold students' statistical knowledge and conceptions, and to help their students to develop both their ability to think and reason statistically and their statistical argumentation (Pfannkuch & Ben-Zvi, 2011), answers to Item 1 are anticipated to provide enough information on how developed these knowledge and skills are in our school mathematics teachers.

Finally, the proposed conceptual framework attempts to serve as a useful tool for discussing about in-service teachers' knowledge and skills in contexts in which variability may arise, and upcoming empirical research using this framework, as well as continued work in this area, may bring about further refinements to the conceptualization proposed.

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