Kite Sailing Platform Mathematical Model and Stabilization

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Abstract This article is devoted to the mathematical modeling of a nonconventional sailing platform, which is called a mast-free (or kite) sailing platform. The platform is based on the modern sail type called "kite". It is a semi-rigid concave wing, which is used for towing water surface objects by wind power. Systems based on the kite successfully evolved over the past 10 years as an independent high-tech water sport. Mast-free sailing platforms managed by human allow them to travel long distances in a wide range of weather conditions. It is necessary to make the platform completely autonomous. To create an automatic control system for this new sailing platform it is necessary to have a mechanical model of the platform. This model should be linear in proximity of its equilibrium states for steady state calculations. All automatic stabilization theory is applicable to use within the linearized model.

1 Introduction

In recent years, active search of mobile robotic platforms is actual. One of perspective problems is oceanic autonomous sailing platform development. Nowadays, designers from Europe and USA attempt to create autonomous sail boat mostly according to classical single-sticker scheme. The alternative approach of sailing platform design is presented in the paper. It is called a mast-free (or kite) sailing

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platform. The platform is based on the modern sail type called “kite”. It is a semi-rigid concave wing, which is used for towing water surface objects by wind power. Systems based on the kite successfully evolved over the past 10 years as an independent high-tech water sport [1]. Human-controlled mast-free sailing platforms allow long distance travelling in a wide range of weather conditions [2]. The task is to make the sailing platform completely autonomous.

To implement the mast-free autonomous sailing platform, the specific control system is required. This control system shall stabilize the platform rectilinear uniform motion under the small wind velocity and direction fluctuations conditions and under weak wave conditions. These are the control system minimal requirements. The strong waves control is the different task and is out of current paper consideration.

Kite towed object system model is complex and multiparametric [3]. Hence, complex regulators are required to control that system. This paper describes an attempt to design a simple control system. Proposed control system is based on human manual control experience.

Control system consists of few independent regulators. Each regulator should have only one input parameter, as in that case the system can be manually tuned without complicated stability calculations.

2 Mast-Free Sailing Platform Model

Mast-free sailing platform consists of two parts: a kite and a board. The kite and the board are connected with inextensible mechanical link that is referred to as line. Board is partially drowned. Kite glides above the water surface. It can easily rotate around the board connected to kite. Herewith the distance between the kite and the board is constant in any kite rotation angle condition. Kite-board system has some translator motion velocity with relation to water surface. This system is similar to the system, proposed in [4], but the towed object is not the hydrofoil; in the present paper the loaded kite-board is used. The human controlled system example is presented on Fig. 1. If human is replaced by the automatics, the forces fulcrums shifts to a single point.

2.1 General Kite Model Description

For mechanic tasks kite can be presented by model that consists of forces and torque concentrated in single point. The model feature, as in [5], is the replacement of complex kite air flow Navier Stokes equations solution with constant coefficients. In the model of [5] the system has no translatory movement, but the kite moves within the wind window.

Fig. 1 Human controlled board forces and axes

The proposed model considers system translatory movement, but it neglects the kite movement within the wind window. This assumption is acceptable for the steady states simulations, when kite moves within wind window slowly.

2.2 General Board Model Description

A board is water surface planing object towed by kite. Hence, board mechanic model should consider water planing hydrodynamic force, gravity force and kite tow force. The forces fulcrums do not match so each force causes a torque.

Water planing hydrodynamic force is directed orthogonally to board surface, as shown on Fig. 1. Its magnitude can be accurately calculated by water flow equations with considering board geometry. However, for mechanical tasks, the force can be approximately calculated by square of water surface velocity and board wetted surface area [6]. The force fulcrums is a center of board wetted surface geometry figure. On the Fig. 1 the wetted surface shown as figure shaded with red lines.

$$F_{hd} = C_{hd} \frac{\rho_w V_d^2}{2} S_d \tag{1}$$

where C_{hd} is hydrodynamic coefficient, ρ_w is water density, V_d is water surface board velocity and S_d is board wetted surface area.

Table 1 Kite model assumptions

Assumption	Grounds/comments
Kite is absolutely rigid	Real kite wing is semi-rigid. However, in fly time, relative changes in kite linear dimensions are units of percent. So kite flat area, determined by the dimensions, changes in units of percent also. Aerodynamic force is proportional to flat area, so its error is not more than units of percent. Small linear dimensions change is provided by kite producer because kite is controllable only when its geometry is maintained
Kite is joined to board by single line	Real kite has two power lines and two control lines. However, control lines do not participate in force transmission. Two real power lines can be replaced by single virtual line. This virtual line begins on real fulcrum on board and finishes in kite fulcrum
Line is joined to kite in aerodynamic force fulcrum	Kite aerodynamic force fulcrum also is a virtual point. This force is distributed on its surface. However, kite is constructed to provide match of equivalent aerodynamic force fulcrum and the virtual line in all states except the rotations around the line state
Kite is controlled by two kite angles set: kite open angle and kite yaw	In real kite there is no direct controller of yaw. The control is realized by alteration of two control lines' lengths. Control lines instantly and directly affect kite open angle and indirectly set kite yaw with a small time constant
Kite aerodynamic force is function of aerodynamic coefficient and kite flat area. Aerodynamic coefficient and flat area are functions of angle of attack	In the model air flow calculations are avoided. The coefficients are found for all incoming air flow directions
Gravity center is situated on kite fulcrum	Real kite is constructed to avoid extra torques caused by mismatch of gravity fulcrum and kite aerodynamic fulcrum. However, automation devices placed on kite can shift gravity fulcrum. In this case the additional torques should be considered. Especially, if kite with automatic device weight is much more than kite own weight

2.3 Mast-Free Platform Model Assumptions

To use simulation results correctly it is necessary to know the assumptions that are accepted in the model. Presented model considers nonzero system movement states only. Assumptions for each element of the model are presented in the Tables 1, 2 and 3 with short comments.

Table 2 Kite line model

Assumption	Grounds/comments
Line is inextensible	The relative tensile of 1.4 mm diameter "Dyneema" power line usually used in kite design is less than 5 % [7]
Line weight can be neglected	Running weight of 1.4 mm diameter "Dyneema" line is 1.2 grams per meter [7]
Line is always stretched	The model is valid for line stretched states. If line is not stretched then kite is uncontrollable. In this state, it is possible to simulate its fall only. The paper doesn't consider that state

Table 3 Board model

Assumption	Grounds/comments
Board is absolutely rigid	It is true for surfboards or wake boards. Most kiteboards are flexible
Hydrodynamic force is concentrated in single point of board wetted surface. In simulations the force is replaced by force on board gravity center and corresponding torque	In reality, the force is irregularly distributed on wetted surface. For simple geometry board it is fore of board wetted surface. For complicated geometry board, concentrated hydrodynamic force fulcrum moves with respect to angles of attack
Water planning hydrodynamic force is directed orthogonally to board plane	In reality, in the board plane the extra hydrodynamic force component exists

2.4 Forces and Torques in Kite Axes System

Kite axes system consists of three orthogonal axes. The first one matches with kite line, second one is directed along a kite central foil and third one is orthogonal to the central foil plane. Axes system origin matches with kite fulcrum. The axes system is presented on Fig. 1.

Kite simulation input parameters (Fig. 2)

- $F_T^b = \{F_{Tx}^b; \quad F_{Ty}^b; \quad F_{Tz}^b\}$—line tension force vector in board coordinate system,
- $V_w^{ws} = \{V_{wx}^{ws}; \quad V_{wy}^{ws}; \quad 0\}$—wind velocity vector in water surface coordinate system,
- $V_d^{ws} = \{V_{dx}^{ws}; \quad V_{dy}^{ws}; \quad 0\}$—board velocity vector in water surface coordinate system,
- $A^{b-k} = \begin{bmatrix} A_{xx}^{b-k} & A_{xy}^{b-k} & A_{xz}^{b-k} \\ A_{yx}^{b-k} & A_{yy}^{b-k} & A_{yz}^{b-k} \\ A_{zx}^{b-k} & A_{zy}^{b-k} & A_{zz}^{b-k} \end{bmatrix}$—board to kite coordinate systems conversion operator,

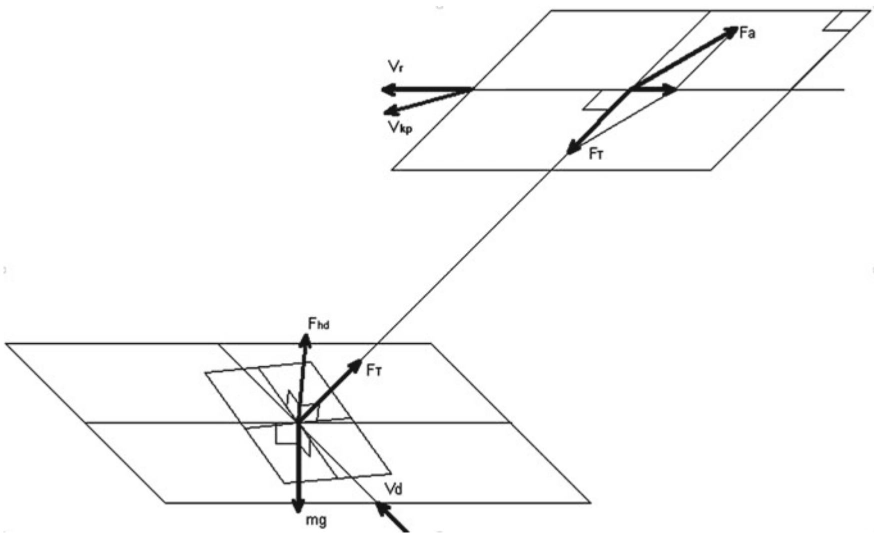

Fig. 2 Mast-free sailing platform model

- $A^{ws-k} = \begin{bmatrix} A_{xx}^{ws-k} & A_{xy}^{ws-k} & A_{xz}^{ws-k} \\ A_{yx}^{ws-k} & A_{yy}^{ws-k} & A_{yz}^{ws-k} \\ A_{zx}^{ws-k} & A_{zy}^{ws-k} & A_{zz}^{ws-k} \end{bmatrix}$—water surface to kite coordinate systems conversion operator.

Kite simulation internal parameters

- $F_T^k = \{ F_{Tx}^k; \quad F_{Ty}^k; \quad F_{Tz}^k \}$—line tension force vector in kite coordinate system,
- $F_a^k = \{ F_{ax}^k; \quad F_{ay}^k; \quad F_{az}^k \}$—aerodynamic force vector in kite coordinate system,
- $V_r^k = \{ V_{rx}^k; \quad V_{ry}^k; \quad V_{rz}^k \}$—kite rotational motion with respect to board velocity vector in kite coordinate system,
- $a^k = \left\{ a_x^k; \quad a_y^{kk}; \quad a_z^k \right\}$—kite acceleration vector.

2.5 Kite Motion Parameters Calculation Algorithm in Kite Coordinate System

To calculate a time step of kite simulation it is necessary to use kite model input parameters from board model that were calculated on previous time step and use it in kite equation in correct order, presented below and illustrated in Fig. 2.

1. Line tension force vector is converted to kite coordinate system (initially line tension force was board axis vector)

$$F_T^k = A^{b-k} \cdot F_T^b \tag{2}$$

2. The mast-free platform velocity vector relative air flow V_{kp}^{ws} is obtained by sum of board velocity vector V_d^{ws} and wind velocity vector V_w^{ws}. This operation is performed in water surface coordinate system

$$V_{kp}^{ws} = V_w^{ws} + V_d^{ws} \tag{3}$$

3. The mast-free platform velocity vector V_{kp}^{ws} is converted to kite axes

$$V_{kp}^k = A^{ws-k} \cdot V_{kp}^{ws} \tag{4}$$

4. Incoming air flow vector in kite coordinate system V^k is obtained by sum of the mast-free platform velocity vector V_{kp}^k and board relative kite rotational motion velocity vector V_r^k (obtained from previous simulation step)

$$V^k = V_{kp}^k + V_r^k \tag{5}$$

5. Incoming air flow vector direction angle is calculated by current kite open angle α and incoming air flow vector direction

$$\gamma = \alpha + arctg\left(\frac{V_y^k}{V_x^k}\right) \tag{6}$$

6. Aerodynamic force coefficients components and flat area are obtained by incoming air flow vector direction angle (kite angle of attack) $C^{xy} = f(\gamma)$. The coefficient and flat area values is the table function. The values are obtained by the measurements. The table structure is presented in Table 4.

7. Aerodynamic force is calculated by incoming air flow magnitude $\left|V_{xy}^k\right| = \sqrt{\left(V_x^k\right)^2 + \left(V_y^k\right)^2}$ and ρ_{air}—density of air

$$\overline{F_a} = \left\{F_a^x, F_a^y, F_a^z\right\} = \{C^x(\gamma), C^y(\gamma), 0\} \cdot \left|\overline{V_{xy}^k}\right|^2 S_{fa}(\gamma) \cdot \rho_{air} \tag{7}$$

Table 4 Aerodynamic force coefficients components and flat area table

γ(degrees)	Flat area	C^{xy}
− 30	S_{fa}	$\{C^x, C^y\}$
…	…	…
30	S_{fa}	$\{C^x, C^y\}$

2.6 Forces and Torques in Board Coordinate System

Board simulation input parameters

- $F_T^b = \{F_{Tx}^b;\quad F_{Ty}^b;\quad F_{Tz}^b\}$—kite tow force vector in board coordinate system is obtained from kite simulation.
- $M_T^b = \{M_{Tx}^b;\quad M_{Ty}^b;\quad M_{Tz}^b\}$—kite tow torque vector in board coordinate system.

Board simulation internal parameters

- $F_{hd}^b = \{F_{hdx}^b;\quad F_{hdy}^b;\quad F_{hdz}^b\}$—hydrodynamic force vector in board coordinate system.
- $m^b g^b = \{m^b g_x^b;\quad m^b g_y^b;\quad m^b g_z^b\}$—gravity force vector in board coordinate system.
- $a^b = \{a_x^b;\quad a_y^b;\quad a_z^b\}$—board acceleration vector in board coordinate system.
- $M_{hd}^b = \{M_{hdx}^b;\quad M_{hdy}^b;\quad M_{hdz}^b\}$—hydrodynamic torque vector in board coordinate system.
- $M_{mg}^b = \{M_{mgx}^b;\quad M_{mgy}^b;\quad M_{mgz}^b\}$—gravity torque vector in board coordinate system.
- $\varepsilon^b = \{\varepsilon_x^b;\quad \varepsilon_y^b;\quad \varepsilon_z^b\}$—board angular acceleration vector in board coordinate system.
- $r^b = \{r_x^b;\quad r_y^b;\quad r_z^b\}$—force radius vector in board coordinate system.
- $J^b = \{J_x^b;\quad J_y^b;\quad J_z^b\}$—rotational inertia vector in board coordinate system.

The board translatory motion equation

$$F_T^b + F_{hd}^b + m^b g^b = m^b a^b \tag{8}$$

The board rotational motion equation

$$M_T^b + M_{hd}^b + M_{mg}^b = J^b \varepsilon^b \tag{9}$$

The systems can be simplified if hydrodynamic force vector is orthogonal to board plane (as shown in Fig. 3).

- $F_{hd}^b = \{0;\quad 0;\quad F_{hdz}^b\}$.
- $M_{hd}^b = \{M_{hdx}^b;\quad M_{hdy}^b;\quad 0\}$.

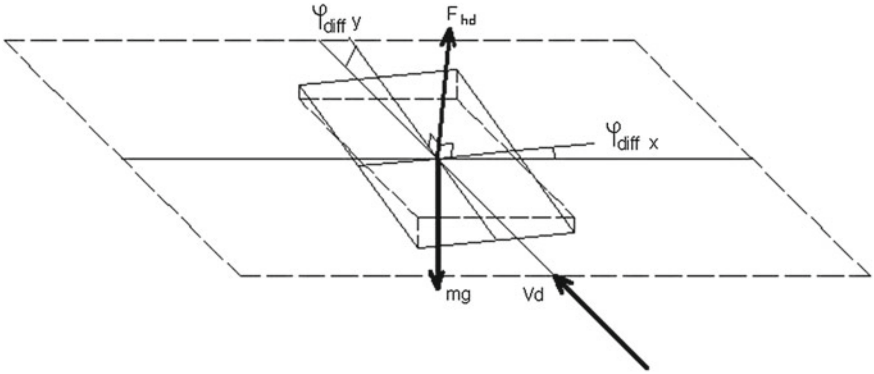

Fig. 3 Board forces model

So, the board torque can be represented as (as shown in Fig. 4):

$$\begin{cases} r^b_{Tx}F^b_{Tz} + r^b_{hdx}F^b_{hdz} + r^b_{mgx}mg^b_z = J^b_x\varepsilon^b_x \\ r^b_{Ty}F^b_{Tz} + r^b_{hdy}F^b_{hdz} + r^b_{mgy}mg^b_z = J^b_y\varepsilon^b_y \end{cases} \quad (10)$$

If the board gravity center matches with board fulcrum one can avoid the correspondent torque use:

$$\begin{cases} r^b_{Tx}F^b_{Tz} + r^b_{hdx}F^b_{hdz} = J^b_x\varepsilon^b_x \\ r^b_{Ty}F^b_{Tz} + r^b_{hdy}F^b_{hdz} = J^b_y\varepsilon^b_y \end{cases} \quad (11)$$

If board type is surf board, its horizontal rotation axis matches with the board aft fin. In the case of horizontal rotation, equation has different radius vectors than radius vectors in first equations

$$r^b_{Tz}F^b_{Ty} + r^b_{hdz}F^b_{hdy} + r^b_{mgz}mg^b_y = J^b_z\varepsilon^b_z \quad (12)$$

If hydrodynamic force vector is orthogonal to board plane, its horizontal projection is zero $F^b_{hdy} = 0$ and equation becomes simpler:

$$r^b_{Tz}F^b_{Ty} + r^b_{mgz}mg^b_y = J^b_z\varepsilon^b_z \quad (13)$$

For stabilization simulation, board rotational inertia can be approximately calculated as

$$J^b_x \cong m_b\frac{W^2_b}{12} \quad (14)$$

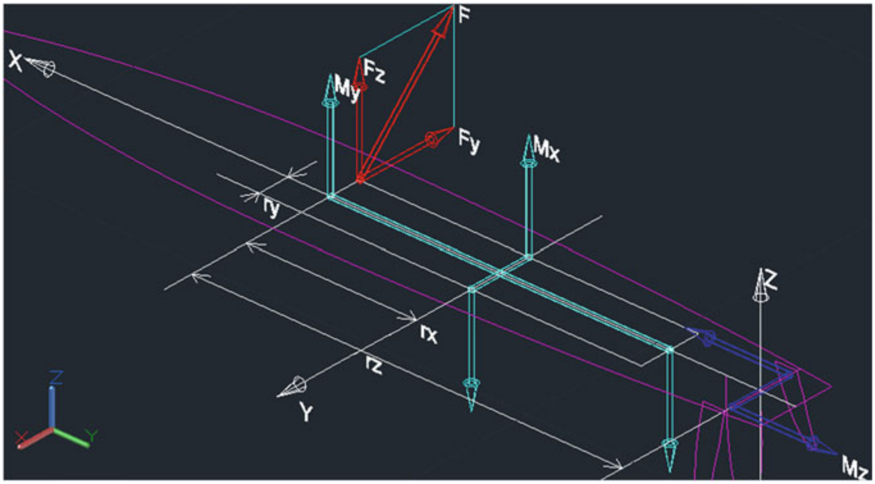

Fig. 4 Radius vectors and torques of board force

$$J_y^b \cong m_b \frac{L_b^2}{12} \tag{15}$$

$$J_z^b \cong m_b \left(\frac{L_b^2}{12} + \frac{W_b^2}{3} \right) \tag{16}$$

where L_b is board length, W_b is board width.

3 Mast-Free Sailing Platform Practical Stabilization Method Based on Kite Manual Control Experience

3.1 General Idea of Independent Parameters Control

Presented above model is a complicated dynamic system. Design stabilization control for the system is not trivial task. However, on practice the kite-board system is easily stabilized by a human. Nevertheless, human uses another simplified model.

According to authors' experience, manual stabilization is based on independent stabilizations of each controlled parameter. Human controls kite tow power and kite position separately. It is possible because kite power regulation and kite position regulation have different time constants. Kite tow force changes almost instantly after control action. It is provided by kite design. Hence, kite moves slowly enough with respect to board with the time constants conditioned by line length (from 25 to 30 meters). [8].

Therefore, human regulates all kite-board system parameters with different priorities, fast alternating parameters first. Human controls kite tow power constantly. If power is acceptable, human controls kite position. If power is different from the acceptable, then human corrects kite power. In this method, kite power regulation process doesn't overlap kite position regulation.

Manual board control is implemented at the similar way. Human moves kite line fulcrum along the board. It causes additional torque. The torque changes board course. Human continuously controls board hydrodynamic force also. To implement that control, human moves kite line fulcrum from the center to the side of the board. This action changes board angle of attack that directly influences hydrodynamic force magnitude. Hydrodynamic force regulation speed is higher than board course regulation speed. Board width is about 0.4 m so hydrodynamic force regulation takes a split second, by authors' experience. Course correction takes seconds because it is conditioned by board rotation radius measured in tens of meters on nominal speed. So, hydrodynamic force and board course can be regulated independently. [8].

3.2 Implementation of Independent Parameters Control in Automatic System

Automatic control system for mast-free sailing platform could be implemented by the same algorithm as human manual control described above. Multiple regulators could control the whole system independently if each regulator acts on the different frequencies [9, 10]. The slow regulator is not capable to perform significant control action affecting the fast alternating parameter. The fast regulator could approximately consider the slow alternating parameter as constant or nonalterating at all. Therefore it doesn't affect the alteration of slow alternating parameter. Therefore, the independency of control actions is performed if the regulators act on the significantly different frequencies.

Mast-free sailing platform automatic regulation system includes four independent regulators. Each regulator is simple PI regulator with only one controlled parameter and only one control action. Other possible control actions are neglected in the proposed control system.

The first regulator presented on the Fig. 5 is used for kite tow force stabilization. Tow force is measured by tension sensor mounted on board. Control action is kite open angle (angle between kite line and kite air foil axis, or α). Regulation time delay is practically zero.

The second regulator presented on Fig. 6 is used for kite position stabilization. Regulated kite position angle is calculated by three axes accelerometer measured parameters. Control action of the regulator is indirect. Regulation delay is measured in seconds. Controlled parameter is filtered by frequency filter with cut off level corresponding fluctuations period measured in seconds. Therefore, the control will be independent from kite tow force stabilization.

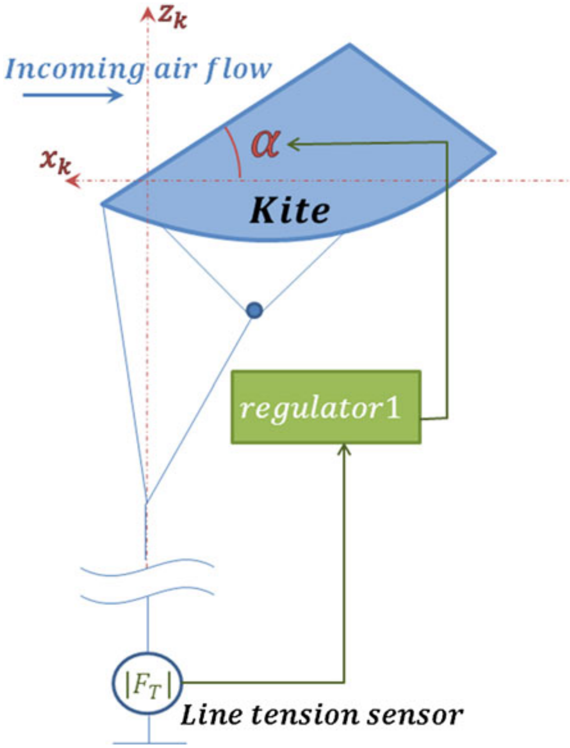

Fig. 5 Kite tow force regulator block diagram

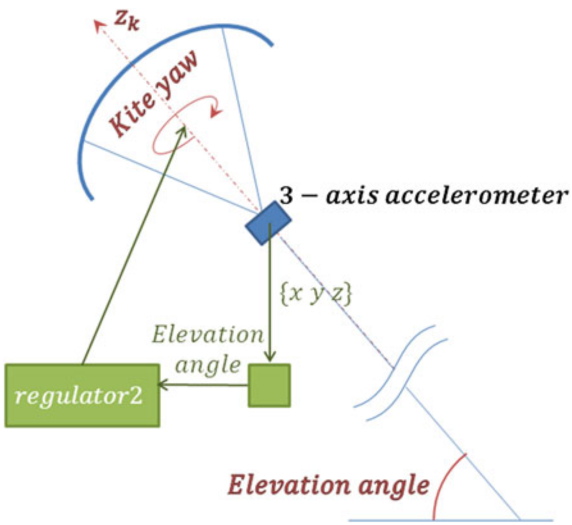

Fig. 6 Kite elevation regulator block diagram

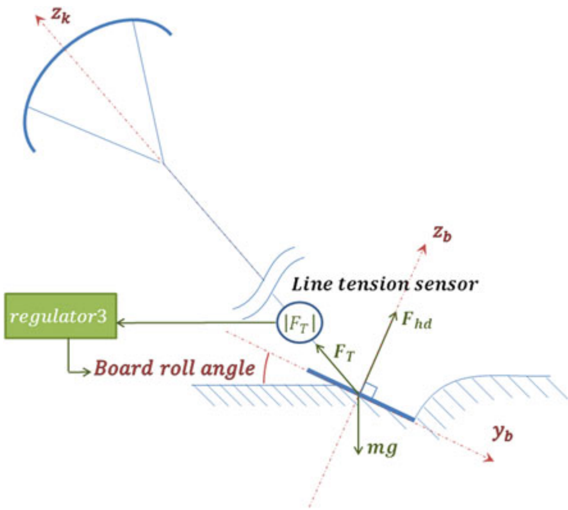

Fig. 7 Board roll regulator block diagram

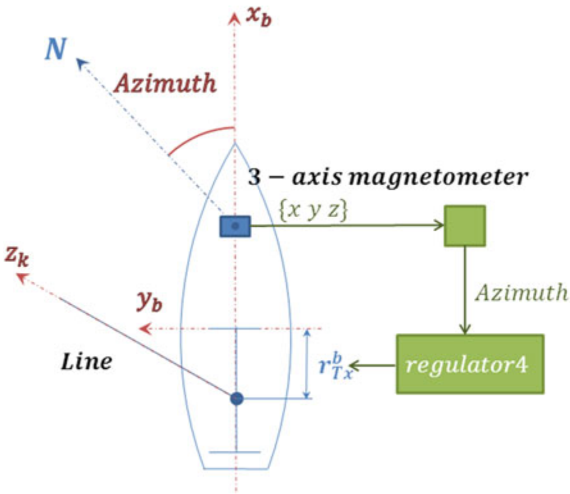

Fig. 8 Board course regulator block diagram

The third regulator presented on Fig. 7 is used for board drowning stabilization. Control parameter is kite line tow force. It should compensate board hydrodynamic force correspondent component. Control action is line fulcrum transverse shift. Time delay for the regulator should be higher than kite position regulator to implement the independency of regulations. The regulator should set correct board attack angle when kite position is set.

The fourth regulator presented on Fig. 8 is used for board course stabilization. Control parameter is course obtained from magnetic sensor. Control action is line fulcrum lengthwise shift. Time delay for the regulator is the highest, and is about tens of seconds.

To perform the acceptable regulation the regulators should be tuned for the steady state of the kite-board system. The control system requires target values' acceptable intervals of parameters such as: kite tow, kite position, board course. Acceptable target parameters values obtaining is a separate task.

Each regulator could be tuned with the help of the relatively simple empirical methods, Ziegler-Nichols method, for example. In the methods, the regulation coefficient values of the operating regulator are gradually increased to the self-oscillations occurrence moment. The acceptable regulator coefficients are calculated by the regulation coefficient value in the moment of self-oscillations occurrence.

4 Conclusion

Mast-free sailing platform simulation is a complicated mechanical task. However, it can be solved if assumptions are pointed out. Solution results can be practically used if the assumptions are suitable for simulation target.

In mast-free sailing platform practical stabilization task it is easier to use manual control experience than correct kite-board model. Regulation system can be tuned by clear heuristic methodic. Errors found in tests of regulation system tuning can be easily localized by regulator factors correction. The proposed model analysis allows obtaining board-kite system steady state parameters. The steady states, even unstable ones, could be stabilized with proposed four regulators use. The stabilized board-kite system can follow the required preset course if the preset course and wind direction satisfy the attainability domain of the model. Hence, there is possibility of the practical board-kite system control automatics realization providing the straight line motion in the preset direction in the weak waves condition.

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