Motion Decoupling Analysis of a Kind of 2R Parallel Mechanism with Two Continuous Rotational Axes

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Abstract The two-rotational-degrees-of-freedom (2R) parallel mechanism (PM) with two continuous rotational axes (CRAs) has a simple kinematic model. Therefore, it is easy to implement trajectory planning, parameter calibration, and motion control, which allows for a very variety of application prospects. Aiming at three different structural forms of a kind of 2R-PM with two CRAs (2UPS-RR mechanism), comparative analyses on motion decoupling were carried out in this paper, the results showed that two rotational degrees of freedom are completely decoupled for a structural form of the studied mechanism, and the regular features that make it be completely decoupled were also summarized.

Keywords Parallel mechanism · 2UPS-RR · Completely decoupled · Partially decoupled

1 Introduction

In recent years, lower-mobility parallel mechanisms (PMs) have become a hotspot of research in the field of robotic mechanisms due to their desirable characteristics, such as a simple structure, low cost, and easy control [1, 2]. One of the most important of the various types of lower-mobility PMs is the two-degrees-of-freedom rotational (2R) PM, which has been applied widely in many fields. The five-bar spherical PM [3] and 2UPS-U PM are two typical 2R-PMs that have broad applications, such as haptic interface devices [4], self-stabilizing platforms on ships and planes [5], hybrid

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robots [6], and devices for ankle rehabilitation [7]. Note that R, P, S, and U stand for revolute, prismatic, spherical, and universal joints, respectively.

Because all three translations of the moving platform (MP) of the 2R-PM are constrained, the two rotations of the MP are also limited; that is to say, the two rotational axes of the MP cannot be chosen arbitrarily (incomplete rotation) [8]. Therefore, the properties and distribution of the two rotational axes are important for trajectory planning and kinematic parameter calibration. The rotational axes of PMs can be divided into two types: continuous rotational axes (CRA) and instantaneous rotational axes (IRA). In an IRA, the MP can only rotate around the rotational axes in a specific pose. In a CRA, the MP can rotate around the rotation axes continually [9]. The trajectory of the MP of the 2R-PM with two CRAs can be determined easily, thus the complexity of kinematic equations is also simplified, so allowing easier implementation of trajectory planning, kinematic parameter calibration, and motion control. The 2R-PM with two CRAs has more extensive application prospects than the ordinary 2R-PM.

However, the present analyses on motion decoupling about the 2R-PM with two CRAs are still relatively immature. Motion decoupling describes the coupling relationship between the input and output parameters. If the velocity Jacobian matrix between the inputs and the outputs of the mechanism is a triangular matrix or a diagonal matrix, the mechanism is decoupled partially or completely respectively [10]. Therefore, we will analyze the different structural forms of a kind of 2R-PM with two CRAs, i.e., 2UPS-RR PM, discuss the effects of different structural forms on decoupling performance, so as to obtain regular features of the structural forms that make the two rotational degrees of freedom (DOFs) being decoupled.

2 Motion Decoupling Analysis of a Kind of 2R-PM (2UPS-RR PM)

2.1 Three Different Structural Forms of the 2UPS-RR PM

The 2UPS-RR PM consists of two UPS branches with no constraints and a constrained branch RR. It is well known that the DOFs of MP are the intersection of the DOFs of each branch for the PM. Since the two UPS branches have full DOFs, this mechanism has only two rotational DOFs around the axes of the two R joints within RR chain. Moreover, the mechanism can always rotate around the axes of the two R joints, so they are CRAs, denoted as $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ respectively, that is, the 2UPS-RR mechanism has two CRAs. Three different structural forms of the 2UPS-RR mechanism are shown in Fig. 1a, b and c respectively.

For structural form (a), the axes of two R joints are orthogonal and different, and they do not intersect with the two UPS branches. The base $U_1R_1U_2$ is an isosceles

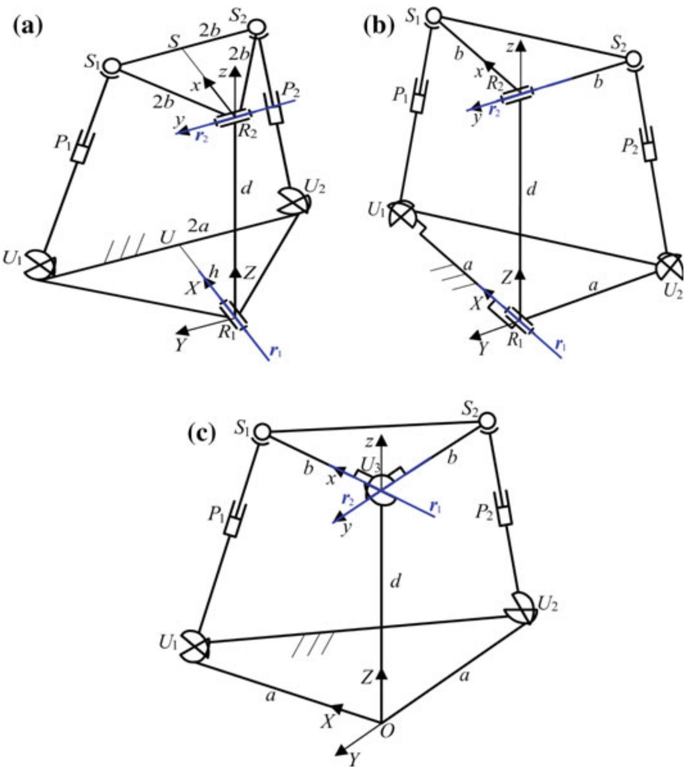

Fig. 1 Three different structural forms of the 2UPS-RR mechanism. **a** Structural form (a) . **b** Structural form (b). **c** Structural form (c)

triangle, the axis of the R_1 joint coincides with the high line R_1U, the length of the bottom side U_1U_2 is equal to $2a$, and the length of R_1U is equal to h, the MP $S_1R_2S_2$ is an equilateral triangle, and the axis of R_2 joint r_2 is parallel to the bottom side S_1S_2 in a length of $2b$. For structural form (b), the axes of two R joints r_1 and r_2 are also orthogonal and different, the axis of the R joint connecting the base passes through the center of the U joint U_1 within the $U_1P_1S_1$ branch, and the axis of R joint connecting the MP passes through the center of S pair S_2 within $U_2P_2S_2$ chain, both the base $U_1R_1U_2$ and the MP $S_1R_2S_2$ are isosceles right triangles, the length of their right-angle sides are equal to a and b respectively. For structural form (c), the axes of two R joints r_1 and r_2 are orthogonal and intersecting, the axis of the R joint connecting the base in the initial configuration passes through the center of the S joint S_1 within the $U_1P_1S_1$ branch, and the axis of the R joint connecting the MP passes through the center of the S joint S_2 within the $U_2P_2S_2$ branch, both base U_1OU_2 and the MP $S_1U_3S_2$ are isosceles right triangles, the length of their right-angle sides are equal to a and b respectively.

2.2 Kinematic Analysis of Three Different Structural Forms

2.2.1 Kinematic Analysis of Structural Form (a)

A fixed coordinate A: R_1-XYZ is attached at point R_1 on the base with X- and Z-axis coinciding with $\boldsymbol{r}_1$, i.e., the axis of R_1 joint, and vector $\boldsymbol{R_1R_2}$ respectively. And a moving coordinate frame B: R_2-xyz is attached at R_2 on the MP with y- and x-axis coinciding with $\boldsymbol{r}_2$, i.e., the axis of R_2 joint, and the vector $\boldsymbol{R_2S}$ respectively.

Assumed that the length of R_1R_2 is equal to d, then homogeneous transformation matrix from the moving frame B to the fixed frame A can be derived as

$$\begin{aligned} {}^A_B\boldsymbol{T} &= Rot(\boldsymbol{x},\alpha)Trans([0 \quad 0 \quad d]^T)Rot(\boldsymbol{y},\beta) \\ &= \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ \sin\alpha\sin\beta & \cos\alpha & -\sin\alpha\cos\beta & -d\sin\alpha \\ -\cos\alpha\sin\beta & \sin\alpha & \cos\alpha\cos\beta & d\cos\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{1}$$

In which α and β represent the rotational angle around $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ respectively.

The inputs of the active P joints within the first $U_1P_1S_1$ branch and second $U_2P_2S_2$ branch, i.e., q_1 and q_2, can then be obtained as

$$\begin{cases} q_1 = \sqrt{(a - bc\beta)^2 + (-bs\alpha s\beta + \sqrt{3}bc\alpha - ds\alpha - h)^2 + (bc\alpha s\beta + \sqrt{3}bs\alpha + dc\alpha)^2} - l_{10} \\ q_2 = \sqrt{(a - bc\beta)^2 + (bs\alpha s\beta + \sqrt{3}bc\alpha - ds\alpha - h)^2 + (-bc\alpha s\beta + \sqrt{3}bs\alpha + dc\alpha)^2} - l_{20} \end{cases} \tag{2}$$

where s and c stand for 'sin' and 'cos' respectively, and l_{10}, l_{20} represent the initial length of the $U_1P_1S_1$ branch and $U_2P_2S_2$ branch respectively.

Solving the derivative of the above equations with respect to time, we can yield

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \boldsymbol{J}\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} \tag{3}$$

where $J_{11} = \frac{h\left(bc\alpha s\beta + \sqrt{3}bs\alpha + dc\alpha\right)}{q_1 + l_{10}}$, $J_{12} = \frac{b(as\beta + dc\beta + hs\alpha c\beta)}{q_1 + l_{10}}$, $J_{21} = \frac{h\left(-bc\alpha s\beta - \sqrt{3}bs\alpha + dc\alpha\right)}{q_2 + l_{20}}$, and $J_{22} = \frac{-b(as\beta + dc\beta + hs\alpha c\beta)}{q_2 + l_{20}}$.

From the above equation, it can be seen that the two rotational DOFs are not decoupled but **fully coupled** for this structural form.

2.2.2 Kinematic Analysis of Structural Form (b)

Similarly, a fixed coordinate A: R_1-XYZ is attached at point R_1 on the base with X-and Y-axis coinciding with $\boldsymbol{r}_1$ and the vector $\boldsymbol{U}_2\boldsymbol{R}_1$ respectively, and Z-axis coincides with vector $\boldsymbol{R}_1\boldsymbol{R}_2$. And a moving coordinate frame B: R_2-xyz is attached at R_2 on the MP with y- and x-axis coinciding with $\boldsymbol{r}_2$ and the vector $\boldsymbol{R}_2\boldsymbol{S}_1$ respectively.

Assumed that the length of R_1R_2 is equal to d, the homogeneous transformation matrix from the moving frame B to the fixed frame A can be obtained, which is the same as Eq. (1). The q_1 and q_2 can then be obtained as

$$\begin{cases} q_1 = \sqrt{a^2 + b^2 + d^2 - 2ab\cos\beta - 2bd\sin\beta} - l_{10} \\ q_2 = \sqrt{a^2 + b^2 + d^2 - 2ab\cos\alpha + 2ad\sin\alpha} - l_{20} \end{cases} \tag{4}$$

Differentiating both sides of the above equation, yields

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \boldsymbol{J}\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} 0 & J_{12} \\ J_{21} & 0 \end{pmatrix}\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} \tag{5}$$

where $J_{12} = \frac{abs\beta - bdc\beta}{q_1 + l_{10}}$, and $J_{21} = \frac{abs\alpha + adc\alpha}{q_2 + l_{20}}$.

From the above equation, it can be found that the two rotational DOFs are **completely decoupled** for this structural form.

2.2.3 Kinematic Analysis of Structural Form (c)

Similarly, a fixed coordinate A: O-XYZ is attached at point O on the base with X- and Y-axis coinciding with the vector $\boldsymbol{OU}_1$ and the vector $\boldsymbol{U}_2\boldsymbol{O}$ respectively, and Z-axis coincides with the vector $\boldsymbol{R}_1\boldsymbol{R}_2$. And a moving coordinate frame B: U_2-xyz is attached at U_2 on the MP with y- and x-axis coinciding with $\boldsymbol{r}_2$ and the vector $\boldsymbol{U}_3\boldsymbol{S}_1$ respectively.

Assumed that the length of R_1R_2 is equal to d, then homogeneous transformation matrix from the moving frame B to the fixed frame A can be derived as

$$\begin{aligned} {}^A_B\mathbf{T} &= Trans(\begin{bmatrix} 0 & 0 & d \end{bmatrix}^T)Rot(\boldsymbol{x}, \alpha)Rot(\boldsymbol{y}, \beta) \\ &= \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ \sin\alpha\sin\beta & \cos\alpha & -\sin\alpha\cos\beta & 0 \\ -\cos\alpha\sin\beta & \sin\alpha & \cos\alpha\cos\beta & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{6}$$

The q_1 and q_2 can then be obtained as

$$\begin{cases} q_1 = \sqrt{a^2 + b^2 + d^2 - 2ab\cos\beta - 2bd\cos\alpha\sin\beta} - l_{10} \\ q_2 = \sqrt{a^2 + b^2 + d^2 - 2ab\cos\alpha + 2bd\sin\alpha} - l_{20} \end{cases} \tag{7}$$

Solving the derivative of the above equations with respect to time, we can obtain

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \boldsymbol{J} \begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & 0 \end{pmatrix} \begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} \tag{8}$$

where $J_{11} = \frac{bds\alpha s\beta}{q_1 + l_{10}}$, $J_{12} = \frac{abs\beta - bdc\alpha c\beta}{q_1 + l_{10}}$, and $J_{21} = \frac{abs\alpha + adc\alpha}{q_2 + l_{20}}$.

Therefore, the two rotational DOFs are **partially decoupled** for this structural form.

2.3 The Characteristics of the Structural Forms When the Two Rotational DOFs Are Decoupled

The above analyses about three different structural forms of the 2UPS-RR PM show that the structural forms (b) and (c) are able to make two rotational DOFs partially decoupled and completely decoupled respectively, while the structural form (a) is fully coupled. Through comparative analysis, it can be found that:

Compared with the structural form (a), the two CRAs of the structural forms (b) and (c), respectively, pass through the center of the S joint or U joint which connects the MP or the base with the two driving branches, while both of the two CRAs of the structural (a) do not pass through the center of the joint which connects the MP or the base with the two driving branches.

Compared with the structural form (c), for the structural form (b), the axes of two R joints within the constrained branch RR are orthogonal and different, the axis of the R joint connecting the base passes through the center of joint connecting the base within one driving branch, and the axis of the R joint connecting the MP passes through the center of the joint connecting the MP within the other driving chain. While for the structural form (c), the axes of two R joints are orthogonal and intersecting, the one connecting the MP passes through the center of the joint, which connects the MP within one driving branch, and it holds only in the initial configuration. The other one connecting the base passes through the center of the joint, which connects the MP within the other driving branch. Once the MP rotates around $\boldsymbol{r}_2$, $\boldsymbol{r}_1$ will no longer pass through the center of the joint, which connects the MP within the driving branch.

To sum up, the conditions that make the two rotational DOFs of the 2UPS-RR PM be completely decoupled can be summarized as: the axes of two R joints within the constrained branch RR always pass through the centers of the joints connecting the MP or the base within the two driving branches respectively; and the conditions that make the two rotational DOFs of the 2UPS-RR PM be partially decoupled can be concluded as: the axis of one R joint within the constrained branch always passes through the center of the joint connecting the MP or the base within one driving branch.

3 Conclusion

Aiming at the 2UPS-RR 2R-PM with two continuous rotational axes, this paper discusses the influence of the different structural forms on its motion decoupling, and obtains the conditions that make two rotational DOFs of this 2R-PM be completely decoupled and partial decouple respectively. The analysis results can also provide an important reference value for motion decoupling study of other PMs.

The obtained 2UPS-RR PM whose two rotational DOFs are completely decoupled has simpler kinematic model, easier to control, and broader application prospects.

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