Topology and Mobility Variations of a Novel Redundant Reconfigurable Parallel Mechanism

Haibo Qu and Sheng Guo

Abstract In this paper, a novel redundant reconfigurable parallel manipulator is proposed, which can be changed into three reconfigured structures. First, the mechanism structure and reconfigurable kinematic limb are described, and the constraint system applied on the moving platform is obtained. Then, the mobility of the mechanism and the connectivity between moving platform and fixed base are calculated, including the reconfigured structure. Also, a new connectivity criterion is proposed for such parallel mechanism in reconfigured structure. Finally, the possible three reconfigured structures are analyzed. Different reconfigured structure results in different topology, mobility and connectivity.

Keywords Parallel mechanism · Redundant · Mobility · Connectivity

1 Introduction

Due to potential industrial applications, the study of reconfigurable parallel mechanisms has attracted much attention in the field of mechanisms and machine theory. Two classes of reconfigurable parallel mechanisms have been proposed based on the mobility and topology variations. One is kinematotropic parallel mechanism with multiple operation modes and continuous variations in the positions of their variables [1], another is metamorphic parallel mechanism with changeable number of all effective links, mobility and topology as they move from one singular configuration into regions with different global mobility [2].

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Several reconfigurable parallel mechanisms and related analysis theories have been reported recently. For kinematotropic parallel mechanism, Kong et al. [3] and Kong [4] synthesized parallel mechanisms with multiple operation modes, which were formed by kinematotropic linkages. Zeng et al. [5] designed and analyzed a kinematotropic hybrid parallel manipulator. Gogu [6] identified four types of branching singularities in kinematotropic parallel mechanisms by using new proposed mobility formula. Ye et al. [7] proposed a family of reconfigurable parallel mechanisms with diamond kinematotropic linkages. Qin et al. [8] presented a derivative mechanism from the "queer-square" [1] and investigated the property of its multi-furcated motion.

For metamorphic parallel mechanism, Dai and Jones [9] developed formal matrix operations to describe the distinct topology of configurations found in a metamorphic mechanism and to complete transformation between them. Dai and Wang [10] proposed and analyzed a robotic hand with a metamorphic palm. Zhang and Dai [11] revealed the reconfiguration properties of metamorphic mechanism based on the Lie displacement-subgroup. Kuo and Yan [12] studied the mobility and configuration singularity of mechanisms with variable topologies. Li et al. [13] characterized a metamorphic linkage by using constraint graph of computational geometry in order to simplify the representation of its configuration changes. Zhang et al. [14] proposed a configuration synthesis methodology for metamorphic mechanisms based on constraint variation. Carbonari et al. [15] presented a class of reconfigurable modular parallel robots stemming from the 3-CPS (C: cylindrical joint, P: prismatic joint, S: spherical joint) under-actuated topology. Zhang et al. [16] analyzed the geometric constraint and mobility variation of two metamorphic parallel mechanisms with metamorphic spherical joints. Gan et al. [17, 18] presented constraint variation and mobility change of metamorphic parallel mechanisms with metamorphic universal joints.

To date, very few redundant metamorphic parallel mechanisms have been proposed. Tosi et al. [19] proposed a reconfigurable redundant parallel-serial hybrid manipulator named *Cheope*. The architecture of parallel part of *Cheope* can be configured to obtain the moving platform three degrees of freedom (DOF) with redundant actuation. Here, the word "redundant" refers to that the mobility of whole reconfigurable parallel mechanism is greater than the DOF of moving platform.

In this paper, a novel redundant reconfigurable parallel mechanism is proposed. The paper is organized as follows. In Sect. 2, the mechanism structure and reconfigurable kinematic limb are described. The constraint screw provided by the reconfigurable kinematic limb is calculated. In Sect. 3, the mobility of the mechanism and the connectivity between moving platform and fixed base are calculated. In Sect. 4, the possible reconfigured structures are analyzed. Different reconfigured structure results in different topology, mobility and connectivity. Finally, the conclusions are drawn.

2 Geometric Description and Reconfigurable Limb

2.1 Geometric Description

As shown in Fig. 1, the proposed redundant reconfigurable parallel mechanism consists of a moving platform, a fixed base and three reconfigurable kinematic limbs with closed loops. Each limb with closed loop can be expressed as $(\underline{P}_i R_{i1} R_{i2} R_{i3} R_{i4} \underline{P}_O) S_i$, where the underlined $\underline{P}_i$ denotes an active prismatic joint of the ith limb, $R_{ij}, i = 1, 2, 3,\ j = 1, \ldots, 4$ represents the jth revolute joint of the ith limb, the underlined $\underline{P}_O$ is an active prismatic joint connecting the central moving platform to the fixed base. The central moving platform is a common part, which connects with the three limbs through links L_{i3} and L'_{i3} simultaneously, where $L_{ij}, i = 1, 2, 3,\ j = 1, 2, 3,$ is the jth link of the ith limb, SL_i is the slider of the ith limb. Since the revolute joints R_{i3} and R'_{i3}, R_{i4} and R'_{i4} are coaxial respectively, links L_{i3} and L'_{i3} perform the same effect. Therefore, in the following text, the symbol R_{i3} will denote revolute R_{i3} and R'_{i3} simultaneously, the symbol R_{i4} represents revolute R_{i4} and R'_{i4} simultaneously, the symbol L_{i3} denotes link L_{i3} and L'_{i3} simultaneously.

2.2 Reconfigurable Kinematic Limb

The reconfigurable kinematic limb can be depicted as shown in Fig. 2. In this section, we performed the constraint analysis of the ith reconfigurable kinematic limb. We assume a coordinate system $o - xyz$ is attached to the fixed base, where the z-axis is pointed along the direction of prismatic joint $\underline{P}_O$, the x-axis is pointed along the direction of prismatic joint $\underline{P}_i$. Each basic joint is associated with a unit screw.

The reconfigurable kinematic limb is constituted by a spherical joint and a closed loop unit. The closed loop unit in each reconfigurable kinematic limb can be considered as a parallel mechanism with two sub-limbs. The $\underline{P}_i R_{i1} R_{i2}$ is the first

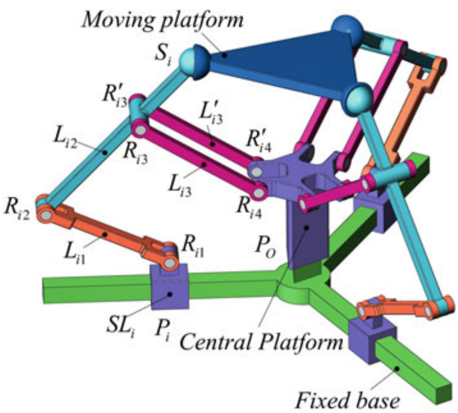
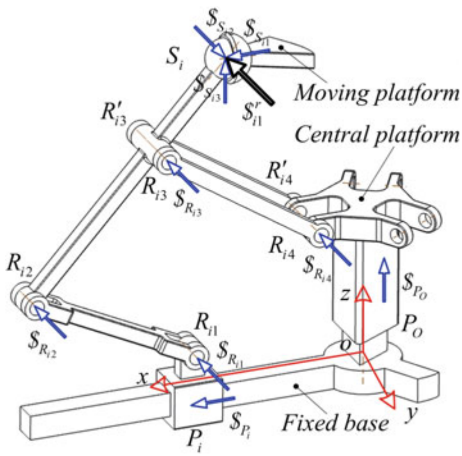


Fig. 1 The proposed redundant reconfigurable parallel mechanism

Fig. 2 The kinematic limb



sub-limb and the $R_{i3}R_{i4}P_O$ is the second sub-limb. The joint screw system of the ith kinematic limb can be expressed as,

$$\begin{cases} \$_{P_i} = [0 \quad 0 \quad 0; \quad 1 \quad 0 \quad 0]^{\mathrm{T}} \\ \$_{R_{i1}} = [0 \quad 1 \quad 0; \quad -n_{i1} \quad 0 \quad l_{i1}]^{\mathrm{T}} \\ \$_{R_{i2}} = [0 \quad 1 \quad 0; \quad -n_{i2} \quad 0 \quad l_{i2}]^{\mathrm{T}} \\ \$_{P_O} = [0 \quad 0 \quad 0; \quad 0 \quad 0 \quad 1]^{\mathrm{T}} \\ \$_{R_{i3}} = [0 \quad 1 \quad 0; \quad -n_{i3} \quad 0 \quad l_{i3}]^{\mathrm{T}} \\ \$_{R_{i4}} = [0 \quad 1 \quad 0; \quad -n_{i4} \quad 0 \quad l_{i4}]^{\mathrm{T}} \\ \$_{S_{i1}} = [1 \quad 0 \quad 0; \quad 0 \quad c_i \quad -b_i]^{\mathrm{T}} \\ \$_{S_{i2}} = [0 \quad 1 \quad 0; \quad -c_i \quad 0 \quad a_i]^{\mathrm{T}} \\ \$_{S_{i3}} = [0 \quad 0 \quad 1; \quad b_i \quad -a_i \quad 0]^{\mathrm{T}} \end{cases} \tag{1}$$

where $(l_{ij}\ m_{ij}\ n_{ij}), i = 1, 2, 3,\ j = 1, \ldots, 4$, is the coordinate of the center of revolute joint R_{ij}, $(a_i\ b_i\ c_i)$ is the coordinate of the center of spherical joint S_i.

Since there exists a central platform, we should distinct the limb screw system relative to central platform or fixed base. The screw system of ith limb can be written as $\{\$_{P_i}, \$_{R_{i1}}, \$_{R_{i2}}, \$_{R_{i3}}, \$_{R_{i4}}, \$_{S_{i1}}, \$_{S_{i2}}, \$_{S_{i3}}\}$ relative to central platform, and as $\{\$_{P_i}, \$_{R_{i1}}, \$_{R_{i2}}, \$_{P_O}, \$_{R_{i3}}, \$_{R_{i4}}, \$_{S_{i1}}, \$_{S_{i2}}, \$_{S_{i3}}\}$ relative to fixed base. No matter relative to central platform or fixed base, the reciprocal screw of the ith limb can be obtained the same result through reciprocal product operation [20], as shown in Fig. 2.

$$\$^r_{i1} = [0 \quad 1 \quad 0; \quad -c_i \quad 0 \quad a_i]^{\mathrm{T}} \tag{2}$$

where the subscript r indicates screw $\$^r_{i1}$ is wrench. Since the pith of $\$^r_{i1}$ is zero, the wrench $\$^r_{i1}$ denotes a constraint force acted on the center of spherical joint S_i and with its direction parallel to the axis of revolute joint R_{i4}.

The screw system of the first sub-limb can be written as $\{\$_{P_i}, \$_{R_{i1}}, \$_{R_{i2}}\}$. Based on reciprocal product operation, the reciprocal screw system can be obtained,

$$\begin{cases} \$_{i1}^{r} = [0 \quad 1 \quad 0; \quad 0 \quad 0 \quad 0]^{\mathrm{T}} \\ \$_{i2}^{r} = [0 \quad 0 \quad 0; \quad 0 \quad 0 \quad 1]^{\mathrm{T}} \\ \$_{i3}^{r} = [0 \quad 0 \quad 0; \quad 1 \quad 0 \quad 0]^{\mathrm{T}} \end{cases} \tag{3}$$

The screw system of the second sub-limb can be written as $\{\$_{P_O}, \$_{R_{i3}}, \$_{R_{i4}}\}$. Based on reciprocal product operation, the reciprocal screw system can be obtained,

$$\begin{cases} S_{i1}^{\prime r} = [0 \quad 1 \quad 0; \quad 0 \quad 0 \quad 0]^{\mathrm{T}} \\ S_{i2}^{\prime r} = [0 \quad 0 \quad 0; \quad 0 \quad 0 \quad 1]^{\mathrm{T}} \\ S_{i3}^{\prime r} = [0 \quad 0 \quad 0; \quad 1 \quad 0 \quad 0]^{\mathrm{T}} \end{cases} \tag{4}$$

From Eqs. (3) and (4), the reciprocal screw systems $\{\$_{i1}^{r}, \$_{i2}^{r}, \$_{i3}^{r}\}$ and $\{\$_{i1}^{\prime r}, \$_{i2}^{\prime r}, \$_{i3}^{\prime r}\}$ are the same. Namely, in each kinematic limb, there are three redundant constraint screws.

When the kinematic limb performs reconfigurable characteristic, the reconfigurable configuration can be depicted as shown in Fig. 3. At this moment, the axes of revolute joints R_{i2} and R_{i4} are collinear. The moving platform is connected to the central platform by using sub-limb $R_{i4}S_i$, and then the central platform is connected to the fixed base by using central prismatic joint P_O. The prismatic joint P_i and the revolute joint R_{i1} do not contribute to the equivalent reconfigurable configuration.

Relative to fixed base, the screw system of equivalent limb configuration $P_OR_{i4}S_i$ can be written as $\{\$_{P_O}, \$_{R_{i4}}, \$_{S_{i1}}, \$_{S_{i2}}\$_{S_{i3}}\}$. Through reciprocal product operation, the reciprocal screw can be obtained as,

$$\$_{i1}^{r} = [0 \quad 1 \quad 0; \quad -c_i \quad 0 \quad a_i]^{\mathrm{T}} \tag{5}$$

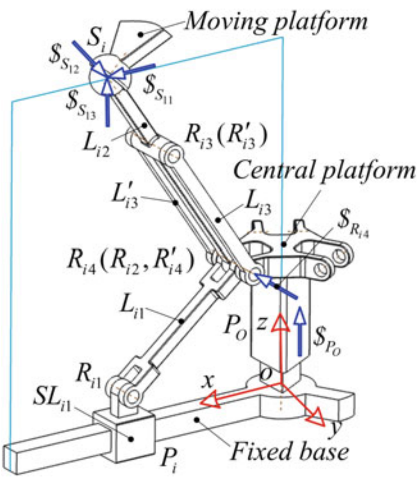


Fig. 3 The reconfigured kinematic limb

Relative to central platform, the screw system of sub-limb $R_{i4}S_i$ can be written as $\{\$_{R_{i4}}, \$_{S_{i1}}, \$_{S_{i2}}\$_{S_{i3}}\}$. The reciprocal screws of sub-limb $R_{i4}S_i$ form a 2-system. Except the reciprocal screw $\$^r_{i1}$, another reciprocal screw $\$^r_{i2}$ can be obtained, which denotes a constraint force acted on the center of spherical joint S_i and directed from spherical joint S_i to revolute joint R_{i4}.

$$\$^r_{i2} = [\, l_{i4} - a_i \quad 0 \quad n_{i4} - c_i; \quad b_i \quad c_i l_{i4} - a_i n_{i4} \quad b_i(a_i - l_{i4})\,]^{\mathrm{T}} \tag{6}$$

From Eqs. (5) and (6), we know that the reciprocal screw of the ith reconfigured kinematic limb will exhibit different results relative to central platform or fixed base.

3 Mobility and Connectivity Analysis

In this section, the mobility of the whole parallel mechanism and the number of independent motions of moving platform are analyzed. The number of independent motions of moving platform can be defined as connectivity between moving platform and fixed base. For kinematically redundant parallel mechanism, such as the proposed redundant parallel mechanism depicted in Fig. 1, the mobility of the whole parallel mechanism is larger than the connectivity between moving platform and fixed base.

3.1 Connectivity Between Moving Platform and Fixed Base

For the proposed redundant reconfigurable parallel mechanism, the centers of the three spherical joints S_1, S_2 and S_3 form an equilateral triangle, i.e. the moving platform, and are restrained to move in the vertical planes π_1, π_2 and π_3, respectively. From Eq. (2), we know that each kinematic limb provide a constraint force, $\$^r_{i1}$, acted on the center of spherical joint S_i and with its direction parallel to the axis of revolute joint R_{i4}, i.e. perpendicular to the plane π_i, as shown in Fig. 4.

The connectivity between moving platform and fixed base can be obtained through the formula as follows,

$$C = 6 - Rank(\$^r_{MP}) = 3 \tag{7}$$

where C is the connectivity between moving platform and fixed base. $\$^r_{MP}$ is the constraint system applied on the moving platform written as $\{\$^r_{11}, \$^r_{12}, \$^r_{13}\}$.

From Eq. (7), the connectivity can be calculated and equal to 3. The principal motion screws of moving platform can also be obtained through reciprocal product operation towards to constraint system $\$^r_{MP}$. According to the physical meaning, the three independent motions of the moving platform can be described as, one

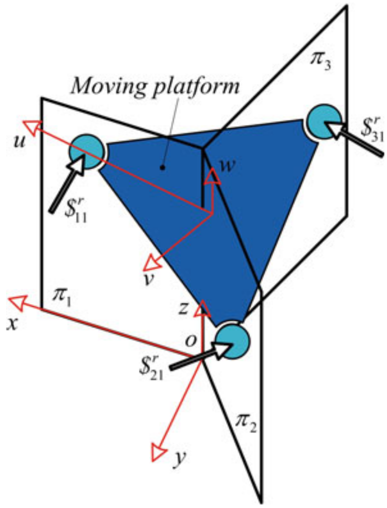


Fig. 4 Constraint system of moving platform

translation along z-axis, and two rotations with the rotational axes space spanned that each rotational axis intersects with the three constraint forces $\$^r_{11}, \$^r_{12}, \$^r_{13}$, simultaneously.

3.2 Mobility of the Whole Parallel Mechanism

The mobility of the whole parallel mechanism is defined as "Number of independent coordinates needed to define the configuration of a kinematic chain or mechanism", and can be calculated by the modified Kutzbach-Grbüler criterion [21].

$$M = d(n - g - 1)\sum_{k=1}^{g} f_k + v - \xi \tag{8}$$

where M is the mobility of the proposed parallel mechanisms, n is the number of links including frame, g is the number of kinematic joint, f_k is the freedom of the kth kinematic joint, v is the total number of the redundant constraints of the mechanism, and ξ is the number of passive degree of freedom.

From Eqs. (3) and (4), we know that each kinematic limb provides three redundant constraints. Therefore, there are nine redundant constraints for the proposed parallel mechanism. There is no passive degree of freedom in the mechanism. The mobility criterion can be rewritten as,

$$M = 6(15 - 19 - 1) + 25 + 9 - 0 = 4 \tag{9}$$

The result 4 in Eq. (9) indicates that there need four actuators to determine the unique configuration of the proposed redundant parallel mechanism.

4 Topology Variations and New Connectivity Criterion

Based on the reconfigurable kinematic limb, the proposed redundant reconfigurable parallel mechanism can be reconfigured into three different variations, as shown in Figs. 5, 6 and 7. Each reconfigured structure makes different topology, mobility and connectivity between moving platform and fixed base.

In particular, the calculation of connectivity for the reconfigured structure should take the central platform into consideration, because the reciprocal screw of the *i*th reconfigured kinematic limb will exhibit different results relative to central platform or fixed base. Therefore, a new calculation criterion for connectivity between moving platform and fixed base during the reconfigured structure should be proposed as,

$$\{\$_{MP}\} = \{\$_{P_O}\} \cup \{N[{}^{CP}\$^r_{ij}]\} \tag{10}$$

where $\$_{MP}$ is the independent motion screws of moving platform relative to the fixed base, $\$_{P_O}$ is screw of central prismatic joint, and $N[{}^{CP}\$^r_{ij}]$ denotes the null space operation on the constraint system, ${}^{CP}\$^r_{ij}$, applied on the moving platform relative to central platform. The symbol $\cup$ represents the union operation.

For these three reconfigured structures, as shown in Figs. 5, 6 and 7, the connectivity between moving platform and fixed base can be obtained by using Eq. (10),

$$\begin{cases} \{\$_{MP}\}_1 = \{\$_{P_O}\} \cup \{N[\$^r_{11}, \$^r_{12}, \$^r_{21}, \$^r_{31}]\} \\ \{\$_{MP}\}_2 = \{\$_{P_O}\} \cup \{N[\$^r_{11}, \$^r_{12}, \$^r_{21}, \$^r_{31}, \$^r_{32}]\} \\ \{\$_{MP}\}_3 = \{\$_{P_O}\} \cup \{N[\$^r_{11}, \$^r_{12}, \$^r_{21}, \$^r_{22}, \$^r_{31}, \$^r_{32}]\} \end{cases} \tag{11}$$

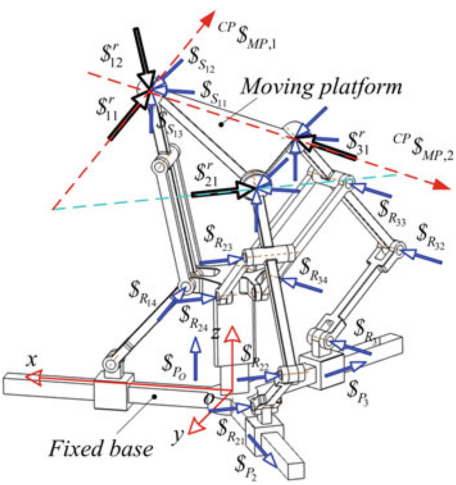


Fig. 5 The 1st reconfigured structure

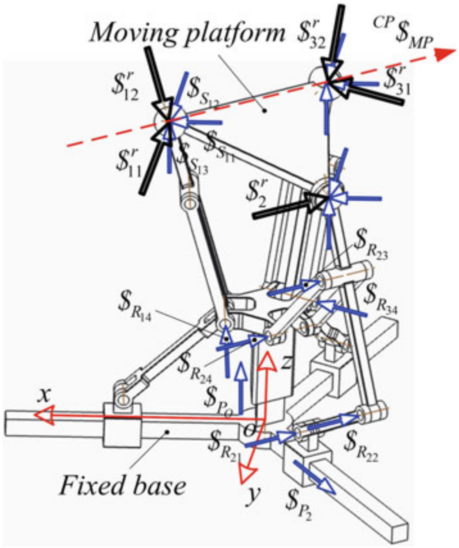


Fig. 6 The 2nd reconfigured structure

Here, the results of Eq. (11) will be given directly. The detailed operation can be performed by using union and reciprocal product operation. For the first reconfigured structure, as shown in Fig. 5, the screw motions, $\{\$_{MP}\}_1$, of the moving platform can be obtained as one translation along z-axis and two rotations with the rotational axes space spanned that each rotational axis intersects with the four constraint forces $\$^r_{11}$, $\$^r_{12}$, $\$^r_{12}$ and $\$^r_{13}$, simultaneously. For the second reconfigured structure, as shown in Fig. 6, the moving platform possesses one translation along

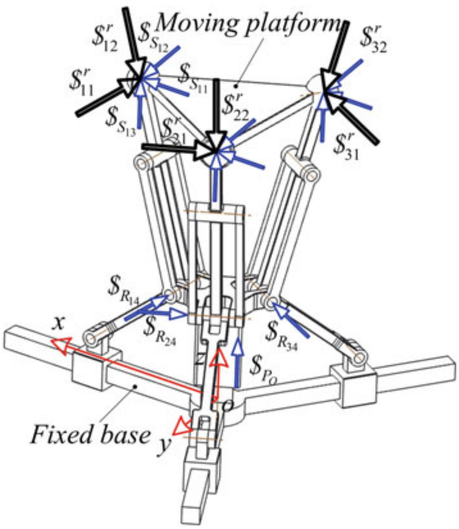


Fig. 7 The 3rd reconfigured structure

z-axis and one rotation around the axis passing through the centers of spherical joints S_1 and S_3. For the third reconfigured structure, as shown in Fig. 7, the moving platform only possesses one translation along z-axis, which is controlled by prismatic joint P_O.

The mobility of these three reconfigured structures can be calculated by using Eq. (8), and rewritten as,

$$\begin{cases} M_1 = 6(12 - 15 - 1) + 21 + 6 + 0 = 3 \\ M_2 = 6(9 - 11 - 1) + 17 + 3 + 0 = 2 \\ M_3 = 6(6 - 7 - 1) + 13 + 0 + 0 = 1 \end{cases} \tag{12}$$

From Eq. (12), we know that there need three, two and one actuators to uniquely determine the configuration of the first, second, and third reconfigured structures, as shown in Figs. 5, 6 and 7, respectively. For these three reconfigured structures, the mobility is equal to the number of independent motions of the moving platform. The actuators of each reconfigured parallel mechanism can be selected as the central vertical prismatic joint and the prismatic joints in the other two, one or zero un-reconfigured kinematic limbs. Therefore, the proposed reconfigurable parallel mechanism will be non-redundant after reconfiguration.

5 Conclusions

In this paper, a novel redundant reconfigurable parallel mechanism is proposed, which can be changed into three reconfigured structures. Since the reciprocal screw of the ith reconfigured kinematic limb will exhibit different results relative to central platform or fixed base, a new connectivity criterion is proposed to determine the independent motions of moving platform relative to fixed base by taking the central platform into consideration. The different reconfigured structure will result in different topology, mobility and connectivity.

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