Handling Risk Attitudes for Preference Learning and Intelligent Decision Support

Camilo Franco(✉), Jens Leth Hougaard, and Kurt Nielsen

IFRO, Faculty of Science, Copenhagen University, Frederiksberg, Denmark
cf@ifro.ku.dk

Abstract. Intelligent decision support should allow integrating human knowledge with efficient algorithms for making interpretable and useful recommendations on real world decision problems. Attitudes and preferences articulate and come together under a decision process that should be explicitly modeled for understanding and solving the inherent conflict of decision making. Here, risk attitudes are represented by means of fuzzy-linguistic structures, and an interactive methodology is proposed for learning preferences from a group of decision makers (DMs). The methodology is built on a multi-criteria framework allowing imprecise observations/measurements, where DMs reveal their attitudes in linguistic form and receive from the system their associated type, characterized by a preference order of the alternatives, together with the amount of consensus and dissention existing among the group. Following on the system's feedback, DMs can negotiate on a common attitude while searching for a satisfactory decision.

Keywords: Interval mutlicriteria · Fuzzy-linguistic structures · Human-system interaction · Consensus-dissention · Social decision making

1 Introduction

Uncertainty is naturally present in real-world decision problems. In fact, uncertainty is always present in human evaluations, measurements and judgments, which represent the available information that has to be dealt with for gaining relevant knowledge and making decisions. Under this view, support is required to give decision makers (DMs) useful and insightful feedback for arriving at satisfactory solutions. Based on multi-criteria decision modeling (see e.g. [5,13,22]), in particular the Weighted Overlap Dominance (WOD) procedure [13] which deals with imprecise (interval) data problems, we address the specific challenge of handling risk decision attitudes for *intelligent decision support* (see e.g. [6,10,16,27,28]).

The decision support system (DSS) process dynamics that will be examined throughout this paper is illustrated in Fig. 1, being composed by three main phases, namely INFO, WOD and IACT:

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1. INFO. All the available information is introduced into the system, consisting in a fixed set of alternatives, a given set of interval-valued criteria with their respective weights, and the risk attitudes of DMs.
2. WOD. For every DM, alternatives are ordered according to their weighted multi-dimensional interval scores, obtaining for every pair of alternatives either a dominance/outranking or an indifference relation.
3. IACT. The system learns the type of every DM according to an associated preference order, measuring the amount of consensus and dissention among types, so DMs can negotiate/rectify their attitudes, restarting the process at INFO while searching for a satisfactory/optimal solution. The process stops when no further consensus can be reached.

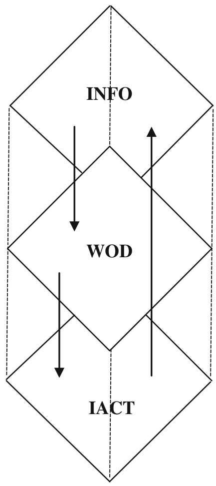

Fig. 1. The DSS process dynamics

Focusing on WOD, it is stated that one alternative outranks/dominates another one if there is *sufficient evidence* for affirming so, otherwise they are considered to be indifferent [7,13]. Under this approach, the *verification* of sufficient evidence is examined in relation to the *risk attitude* of the DM (as it will be examined in detail in Sect. 4). Hence, the inherent conflict of the multicriteria problem, associated to the *incomparability* [20] among alternatives, can be explained by learning the different attitudinal types of DMs, like opposing postures (sources of disagreement) which have to come closer together for finding a *social decision*. In this way, the objective of this paper is to establish a decision support methodology that builds useful and reliable knowledge from the linguistic interaction with DMs, aiding their negotiation process while searching for results with greater *coherence* among them, maximizing group consensus by reducing (pairwise) minimal dissention among types.

In order to do so, this paper is organized as follows. Section 2 offers an outline of the WOD inference process as it was originally presented in [13]. Section 3 introduces fuzzy-lingustic structures, presenting the preliminary concepts that are used in Sect. 4 for modeling risk attitudes. In Sect. 5 the methodology for learning the types of DMs is explained, and in Sect. 6 the DSS human-system interaction is summarized under Algorithm 3, producing decision support while searching for an agreement on the social solution. Finally there are some notes and comments concerning open problems for future research.

2 Inferring Preferences from Imprecise Data

The WOD procedure [13] allows coping with the natural imprecision of real life observations and measurements, as given by interval values. This procedure makes use of criteria weights and risk attitude parameters to make sense of the interval data, identifying the preference relations holding among the alternatives. In short, the WOD procedure consists in the following.

Consider a set of decision makers D, a set of alternatives N and a set of criteria C, such that for every alternative $a \in N$ and criterion $i \in C$ there is a lower and upper bounded valuation, respectively given by $x_{ai}^L, x_{ai}^U \in [0,1]$, such that $x_{ai}^L \leq x_{ai}^U$, scoring alternatives according to the characteristic property of the criterion. Every criterion has an associated weight expressing its relative importance, given by $w_i \in \mathbb{R}^+$, and every decision maker $e \in D$ has a subjective decision attitude represented by parameters $\beta_e \in [0,1]$ and $\gamma_e \in \mathbb{R}^+$.

Therefore, for every alternative $a \in N$, the suitability of a regarding the set of criteria C, $|C| = m$, is given by the multi-dimensional (hyper) cube,

$$c_a = \left[x_{a1}^L, x_{a1}^U\right] \times \cdots \times \left[x_{am}^L, x_{am}^U\right]. \tag{1}$$

Based on this information, a pairwise comparison process is developed among alternatives $a, b \in N$, such that $\sum_{i=1}^m w_i x_{ai}^U \geq \sum_{i=1}^m w_i x_{bi}^U$. According to the amount of overlap between c_a and c_b, the WOD procedure infers the preference relation holding among a and b. There are three kinds of overlap, namely *no overlap*, *partial overlap* and *complete overlap*. In the case of *no overlap*, such that

$$\sum_{i=1}^m w_i x_{ai}^L > \sum_{i=1}^m w_i x_{bi}^U, \tag{2}$$

it certainly holds that a dominates b, which is represented by the *outranking relation* $\succ$, such that

$$a \succ b. \tag{3}$$

On the other hand, if there is *partial overlap*, such that

$$\sum_{i=1}^m w_i x_{ai}^L > \sum_{i=1}^m w_i x_{bi}^L \tag{4}$$

and

$$\sum_{i=1}^m w_i x_{ai}^U > \sum_{i=1}^m w_i x_{bi}^U, \tag{5}$$

then it holds that,

$$a \succ b \ \Leftrightarrow \ P(a,b) > \beta. \tag{6}$$

Here $P(a,b)$ expresses a proxy for the likelihood that alternative a in fact dominates alternative b, due to the possibility that some point in (or randomly taken from) c_a can be greater than another point from c_b (see [13] for a specific example on how to estimate such proxy). This likelihood has to be higher than β in order for a to outrank b. Otherwise, if

$$P \leq \beta, \tag{7}$$

then both alternatives are said to be *indifferent*, such that

$$a \sim b. \tag{8}$$

Lastly, if there is *complete overlap*, such that

$$\sum_{i=1}^{m} w_i x_{ai}^{L} < \sum_{i=1}^{m} w_i x_{bi}^{L} \tag{9}$$

and

$$\sum_{i=1}^{m} w_i x_{ai}^{U} > \sum_{i=1}^{m} w_i x_{bi}^{U}, \tag{10}$$

then it holds that,

$$a \succ b \ \Leftrightarrow \ G(a,b) > \gamma, \tag{11}$$

where $G(a,b)$ expresses the likelihood that any point belonging to c_a is greater than any other point in c_b (see again [13] for more details). Hence, if $G(a,b)$ is greater than γ, it holds that $a \succ b$. Otherwise, it either holds that $b \succ a$ or $a \sim b$ if it is respectively verified that $G(a,b)$ is less than or equal to γ.

Notice that the indifference relation of the WOD procedure, due to the interval nature of data, does not hold as a transitive or equivalence relation. Therefore, the outranking order assigned on N is *semi-transitive*, such that for every $a,b,c \in N$ it holds that $a \succ b, b \succ c \nRightarrow c \succ a$ (see again [13] but also [7]).

Under this framework, the parameters β and γ denote risk thresholds for establishing an outranking relation, such that their meaning is being modeled in direct relation to a crisp number. On the other hand, acknowledging the general character of words, concepts and perceptions, it is necessary to take a closer look at the correspondence between DMs' risk attitudes and their numerical translation/estimation. Thus, a given attitude should at least refer to a set of values, which under an explicit semantic structure, allows incorporating the gradualness and generality of its numerical estimation.

In order to undertake computations with attitudes under the DSS (see again Fig. 1), the estimation of linguistic values for β and γ can be examined through the computing with words and perceptions paradigm (see [30–32], but also [18,23]). Thus, the following analysis is based on the intuition that *language* is the means to represent the subjective thinking process and the relation between perception and reality, enhancing the interaction with technology and the affective (decision-wise) states of DMs.

The complete procedure for the articulation of binary preference relations is specified under the WOD Algorithm 1. In the following section fuzzy-linguistic structures are introduced, which will be later used for undertaking a linguistic modelization of the attitudes explaining the β and γ parameters.

Algorithm 1. WOD algorithm

Input: For every $a \in N$ and $i \in C$, the hyper cubes c_a, the criteria weights w_i and for every $e \in D$, the risk attitude parameters β_e and γ_e.
Output: For every $e \in D$, a preference order on N.
$(WOD-1)$ For every $a, b \in N$, establish an outranking or indifference relation according to (1)-(11).

3 Fuzzy-Linguistic Structures

Fuzzy logic [30,31] allows representing the meaning of words and concepts, examining human reasoning through natural/ordinary language. Under this approach, commonly known as the Computing with Words paradigm [9,11,14,32], words are taken as linguistic terms that are susceptible of being represented by fuzzy sets. Thus, through their associated membership functions, the meaning of fuzzy sets is supported by a particular structure maintaining a specific order among them (see e.g. [6,17,18]). Such structure is here referred to as a *fuzzy-linguistic structure*.

Addressing the general character of words, and in particular of gradable predicates that are succeptible of verification *up to a certain degree*, fuzzy sets are an appropriate tool for designing the means of such verification process. In this way, a fuzzy set representing a linguistic term can be regarded as containing a *core* and a *support*, such that its core is the subset of the universe U where the term is known to hold true, while its co-support consists in the subset of U where it is known that it does not hold true. Hence, there is a space in between the core and the co-support that can be gradually filled in by a continuous and monotone transition (in fact, the specific form of this transition is a matter of design [24,25]), representing the intensity in which the elements of U verify the meaning of the fuzzy set. Thus, the elements belonging to the core are considered to have absolute intensity, while the ones belonging to the co-support have null intensity.

For a general valuation scale L, the membership function $\mu : U \to L$ can be expressed as an ordered quadruple of the ordinates $(\mu^1, \mu^2, \mu^3, \mu^4)$, such that the interval $[\mu^1, \mu^4]$ stands as the support and the interval $[\mu^2, \mu^3]$ stands as the core of the fuzzy set. So, for any pair of consecutive linguistic terms $l_t, l_{t+1} \in L$, respectively represented by μ_{l_t} and $\mu_{l_{t+1}}$, the order relation $\trianglelefteq$ is defined such that $\mu_{l_t} \trianglelefteq \mu_{l_{t+1}}$ holds only if $\mu^3_{l_t} \leq \mu^1_{l_{t+1}}$ and $\mu^4_{l_t} \leq \mu^2_{l_{t+1}}$. Now, *fuzzy-linguistic structures* can be defined as follows.

Definition 1. *Given a set of different and consecutive linguistic labels* $L = \{l_1, l_2, ..., l_T\}$*, where each label* $l_t \in L$*,* $t = 1, 2, ..., T$*, is represented by means of a fuzzy set with a membership function given by* $\mu_{l_t} = (\mu^1_{l_t}, \mu^2_{l_t}, \mu^3_{l_t}, \mu^4_{l_t})$*, a fuzzy-linguistic structure is such that for any pair of consecutive labels* $l_t, l_{t+1} \in L$*, it holds that* $\mu_{l_t} \trianglelefteq \mu_{l_{t+1}}$*.*

In this way, a fuzzy-linguistic structure contains the reference ordered set of linguistic terms, such that l_1 and l_T are respectively the minimum and maximum objects of the structure. This approach can be further developed to handle words in a manner that is more approximate to natural language and its use of gradable predicates, taking into consideration *linguistic modifiers* and *linguistic aggregation operators*, following the initial proposal of [18] (see also [6]).

Linguistic modifiers can be defined as unary functions $M : L \to L$, such that their effect on the meaning of the terms can be either *compressing* or *expanding* [18]. A compressing M is such that for any $l_t \in L$, it holds that $M(l_t) \subset l_t$, while an expanding M is such that $l_t \subset M(l_t)$. Some examples for compressing M can be "very"-l_t, "strictly"-l_t or "strongly"-l_t, while for an expanding M, they can be "around"-l_t, "almost"-l_t or "roughly"-l_t.

For example, given a linguistic term $l_t \in L$ represented by means of the membership function μ_{l_t}, and given an averaging operator k, a *compressing* M, denoted by CM, is such that

$$CM(\mu_{l_t}) = (k(\mu^1_{l_t}, \mu^2_{l_t}), \mu^2_{l_t}, \mu^3_{l_t}, k(\mu^3_{l_t}, \mu^4_{l_t})), \tag{12}$$

and an *expanding* M, denoted by EM, is such that

$$EM(\mu_{l_t}) = (\mu^1_{l_t}, k(\mu^1_{l_t}, \mu^2_{l_t}), k(\mu^2_{l_t}, \mu^3_{l_t}), \mu^4_{l_t}). \tag{13}$$

On the other hand, linguistic aggregation operators allow using the existing linguistic labels to generate new labels, such that new terms can appear *in between* any pair of consecutive terms, while maintaining the order among the linguistic components of the structure [6,18]. In this way, a new term can arise *in between* any pair $l_t, l_{t+1} \in L$, by means of an operator specifically designed for the inclusion of new linguistic labels.

Definition 2. *Given a fuzzy-linguistic structure, the in between linguistic aggregation operator is a mapping* $LA : L^2 \to L$ *such that for any pair of consecutive terms* $l_t, l_{t+1} \in L$ *and their associated fuzzy sets, it holds that* $CM(\mu_{l_t}) \trianglelefteq LA(\mu_{l_t}, \mu_{l_{t+1}}) \trianglelefteq CM(\mu_{l_{t+1}})$*.*

In this way, given two averaging operators k_1, k_2, such that for any pair of elements $u_1, u_2 \in U$ it holds that $k_1(u_1, u_2) \leq k_2(u_1, u_2)$, LA can be taken as in the following example, previously undertaking the compression of the consecutive terms, as in $\nu_{l_t} = CM(\mu_{l_t})$ and $\nu_{l_{t+1}} = CM(\mu_{l_{t+1}})$,

$$LA(l_t, l_{t+1}) = (\nu^3_{l_t}, k_1(\nu^3_{l_t}, \nu^2_{l_{t+1}}), k_2(\nu^3_{l_t}, \nu^2_{l_{t+1}}), \nu^2_{l_{t+1}}). \tag{14}$$

Under the general framework of fuzzy-linguistic structures, the design of different examples for M and LA can be further developed, including more linguistic terms and modifiers that preserve the order relation among every pair

$l_t, l_{t+1} \in L$,while enhancing the granularity of L as much as required (see [6,18]). Its application for the representation and measurement of risk attitudes will be explored next.

4 Measuring Risk Attitudes with Fuzzy Linguistic Structures

Based on fuzzy-linguistic structures, the β and γ risk attitudes can be modelized and incorporated in the articulation of preferences under the WOD interactive decision process. The incorporation of attitudes is particularly relevant for decision support under imprecision, where attitudes play a central role (see e.g. [28,29], but also [7]). In this sense, examining the meaning of risk as a concept which is used by DMs, the attitude towards risk can be measured on a linguistic scale built from the two opposite categories of *aversion* and *proneness* (see [1,15,19] for a general view on the evaluation of attitudes under different bipolar evaluation spaces).

As it has been examined in Sect. 2 and the DSS process dynamics of Fig. 1, attitudes guide the articulation of preferences through the interaction between the system and the group of DMs. In particular, attitudes towards risk refer to the amount of evidence needed to affirm an outranking relation for every pair $a, b \in N$, such that $a \succ b$ ($\succ$), instead of having that $a \sim b$ ($\sim$) or even that $b \succ a$ ($\succ^{-1}$), the latter only for the case of complete overlap and the parameter γ.

Therefore, high values of β correspond with a low risk attitude, because an outranking relation will only hold if there is a high amount of evidence existing in favor of $\succ$. In this way, β is defined over a scale with minimum element 0, denoting *high* risk, and a maximum element $K = 1$, denoting *low* risk, with an indeterminate space of *medium* risk consisting of being *in between* high and low risk attitudes (see Fig. 2). So, if β is close to 0, the attitude towards risk is considered to be of *risk proneness*, and if β is close to 1, then the attitude is considered to be of *risk aversion*, being the middle attitude regarded as *risk neutrality*. Notice that here neutrality refers to a middle attitude (as in [8,19]), although a linearity between extreme and neutral attitudes may not necessarily hold (see e.g. [15,17]).

On the other hand, on the contrary to the partial overlap case of β, γ refers to the three possibilities of obtaining $\succ$, $\sim$ or the inverse relation $\succ^{-1}$, where every time that $\succ$ does not hold, it reciprocally holds that $\succeq^{-1}$, such that $\succeq = \langle \succ, \sim \rangle$. Hence, γ is measured over a scale with a minimum element 0, denoting *high* risk for affirming $\succ$ (or inversely, low risk for affirming $\succ^{-1}$), and a maximum element $K \in \mathbb{R}^+$, denoting *low* risk for affirming $\succ$ (or inversely, high risk for affirming $\succ^{-1}$). Thus, there is some space for a *medium* state of risk consisting of being *in between* high and low risk attitudes (see again Fig. 2), where low values of γ denote a *risk prone* attitude, high values denote a *risk averse* attitude, and intemediate values denote a *risk neutral* attitude.

Overall, the risk attitude parameters β and γ refer to the measurement of attitudes with respect to three basic components, namely *proneness*, *neutrality*

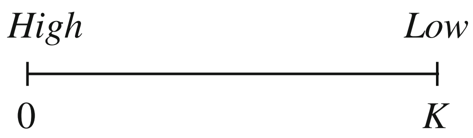

Fig. 2. Measuring risk attitudes for affirming $\succ$ on a commonl linear scale for β and γ

and *aversion*, ordered according to a specific structure that holds among them. Acknowledging the general character of words, those terms naturally refer to a region or interval of the numerical scale, suggesting their correspondence with a set of numbers instead of a correspondence with a unique number. Even more, adjacent terms suggest a gradual intersection between them, where e.g. diminishing intensities of risk proneness may coincide with increasing intensities of risk neutrality.

In consequence, a risk attitude R can be measured with respect to a basic fuzzy-linguistic structure L^R, composed of at least the two opposite and most extreme linguistic labels (l_1, l_T) of proneness (l_1) and aversion (l_T), such that,

$$L^R = \{l_1 = prone, l_T = averse\}. \tag{15}$$

Based on this basic structure, the meaning of the terms can be modified, where it is possible for the decision maker to express linguistic grades of risk by attaching different words to the terms, such as "very"or "strictly" in the case of the compressing modifiers CM, or of "roughly"or "around" in the case of expanding EM. Besides, with the use of aggregation operators, such as the *in between* operator LA, new terms can emerge from any pair of consecutive terms, enabling the decision maker to create and use a new term for valuing attitudes. For example, the first new term consists in being neither "prone"nor "averse", but "in between"them, denoting the state of l_2 =*risk neutrality* (see Fig. 3, where the opposite terms l_1 and l_T compress, making room for l_2). Following the same line of reasoning, the decision maker can be as specific as required, e.g. being "in between neutral and prone" or "roughly strongly-averse".

As a result, the attitude towards risk for DMs can be expressed by some (M or LA) modifed term in L^R, assigning linguistic values for computing with β and γ parameters. In this way, for every ordinate of the fuzzy set representing a given attitude, the WOD phase infers an order, so in the next phase the different types of DMs can be identified, as it will be examined in the next section.

5 Learning Types for Decision Support

Following the decision process, the system computes a preference order for every DM according to their attitudes. As it has been pointed out in Sect. 2, the outranking order resulting from Algorithm 1 is a semi-transitive one, such that a definite procedure can be used to further refine it and learn a weak order or ranking.

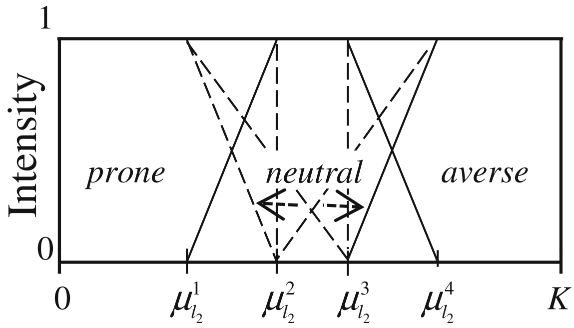

Fig. 3. Emergence of the middle term denoting a "neutral" attitude

Alternatives are ranked according to their *relevance* [7], taking into account the amount and the importance of the alternatives that they outrank. In this way, for every alternative $a \in N$, the relevance of a is given by $\sigma(a)$, such that,

$$\sigma(a) = s_a + \sum_{\forall b \in S_a} s_b, \tag{16}$$

where S_a is the set of alternatives that are dominated/outranked by a, and $s_a = |S_a|$.

The procedure for learning types of DMs is summarized under Algorithm 2. First, for every DM $e \in D$, there is a linguistic term denoting e's attitude, given by $l^e_t \in L^R$. The system then computes an (outranking semi-transitive) order on N, resulting from the WOD evaluation of every ordinate of the membership function representing l^e_t. All four ordinates are then aggregated into an overall ranking by means of (16). Having identified all the rankings that follow from the information given by DMs, the system returns the set of types Θ explaining the different attitudes.

Thus, different attitudes can obtain the same characteristic order, implying that the *type* of a DM can be completely described by a unique order and all the attitudes associated to it.

Algorithm 2. Learning Types (LT) algorithm

Input: For every $e \in DM$, the linguistic value l^e_t denoting their risk attitude.
Output: The set of types Θ.
$(LT-1)$ Compute the WOD algorithm (1) for every ordinate of μ_{l_t}.
$(LT-2)$ Aggregate the outranking orders associated to l^e_t by means of the relevance ranking operator (16).
$(LT-3)$ Assign to every non-equivalent ranking a distinct type $q \in \Theta$.

Therefore, types $q \in \Theta$ are completely described by a ranking ρ^q of N and their associated risk attitudes $\{l_t\}_{\rho^q}$. Once the types of DMs are known, the system can offer support for resolving conflict among them, aiming at reducing the number of incomparable alternatives as it will be explained in the next section. The goal of the system focuses on using the linguistic interaction with and within DMs for maximizing consensus and arriving at a social satisfactory solution.

6 Intelligent Decision Support

The interactive human-system dynamics of Fig. 1 can now be addressed under the setting described by Algorithm 2, such that the decision process is guided towards reducing discrepancies among DMs. The system aids in identifying the predominant types and suggesting negotiation paths to arrive at an agreement or socially acceptable solution maximizing consensus.

Given the set of types θ, the group consensus is measured by the *general consensus index* $CI = 1/|\Theta|$. Complementing the information on consensus, *dissention degrees* are introduced here to measure the distances among pairs of types.

In this way, for every $a \in N$ and $q, q' \in \Theta$, the system computes the position of a in rankings ρ^q and $\rho^{q'}$, denoted respectively by ρ^q_a and $\rho^{q'}_a$, and obtains the overall dissention degree $ds(q, q')$, such that,

$$ds(q, q') = \sum_{\forall a \in N} dist(\rho^q_a, \rho^{q'}_a), \tag{17}$$

where *dist* represents a given distance measure (see e.g. [2]), like e.g. the 1-norm distance,

$$dist(\rho^q_a, \rho^{q'}_a) = |\rho^q_a - \rho^{q'}_a|. \tag{18}$$

The decision process aims at maximizing consensus (see Algorithm 3), based on the previous calculation of dissention degrees among all the different pairs of types. Thus, the system identifies all pairs $q, q' \in \Theta$ with minimal dissention, so DMs can look for an agreement among the nearest types, negotiating a common attitude that increases the general consensus index CI. In consequence, under the complete DSS process dynamics, attitudes not only guide the articulation of preferences through the interaction between the system and the group of DMs, but also (and under the same linguistic form) guide the negotiation among the different DMs.

Algorithm 3. Minimal dissention (MD) algorithm

Input: For every $e \in D$, the risk attitudes associated to e.
Output: All pairs $q, q' \in \Theta$ with minimal dissention.
$(MD-1)$ For every $e \in D$, learn the type for e according to the LT-algorithm, identify the pairs $q, q' \in \Theta$ with minimal dissention and repeat for every new input until $CI = 1$ or no further negotiation is possible (CI remains constant for a fixed number of iterations).

7 Conclusions

A DSS methodology has been provided with the purpose of aiding the consensus/negotiation process between different DMs. The system infers individuals' preferences from their attitudes towards risk, learns the predominant types of DMs and measures the dissention and consensus among them. DMs can use the knowledge generated by the system to search for a satisfactory decision or identify the source of the conflict making it impossible to arrive at a unique (optimal) social solution.

It remains for further research to test and implement the DSS dynamic process in a real-case scenario, exploring the difficulties that may emerge in a real negotiation process. From a theoretical standpoint, the estimation for the likelihood of dominance between intervals/hypercubes remains to be explored in more detail, as well as the ranking and consensus procedures, which should be compared with other techniqes found in literature (see e.g. [3,4,12,21,26]).

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