

On the Use of Isotropic Hardening Plasticity to Model Cyclic Consolidation of Fine Grained Soils

Nina Müthing, Thomas Barciaga and Tom Schanz

Abstract Cyclic soil behavior plays an important role in geotechnical engineering, both in the installation phase as over the life span of constructions. Relevant application examples which find increasing attention nowadays are the dimensioning of on- and offshore foundation systems, the analysis of soil behavior due to mechanized tunneling processes as well as analyses of loading histories related to deep excavation walls. Thus, in the present paper fundamentals of cyclic soil behavior under partially drained, oedometric conditions are analyzed. Excess pore water pressure evolution and accumulated deformations are studied by both numerical and experimental approach. For this purpose, a new oedometer device is introduced which allows to measure complete stress state under transient loading. Additionally, by numerical experiments using FEM the influence of soil stiffness and permeability on the evolution of excess pore water pressures and accumulated deformations is studied. By comparison of numerical and laboratory experiments the ability of classical isotropic hardening plasticity to model cyclic consolidation phenomena is validated.

1 Introduction

Cyclically loaded structures are most common in different applications of geotechnical engineering. Typical structures as foundations of wind turbines or pipelines must be designed considering these nonmonotonic impacts. As structures are in contact with the surrounding soil understanding the soil-structure interaction as well as the underlying mechanisms is a major need. Thereby, an important role falls upon the constitutive behavior of cyclically loaded soils. This paper deals in particular with the cyclic behavior of fine grained soils as in fine grained soils excess pore water pressure build up and dissipation occurs and influencing the effective stresses and kinematics as well as the stability of the entire structure accordingly. Related to the

N. Müthing (✉) · T. Barciaga · T. Schanz
Chair of Foundation Engineering, Soil and Rock Mechanics,
Ruhr-Universität Bochum, 44780 Bochum, Germany
e-mail: nina.muething@rub.de

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excess pore water pressures, the accumulation of settlements is of major interest. FEM simulations are a well-established tool to analyze this type of complex soil-structure interaction boundary value problems involving consolidation process and transient states accordingly.

Existing analytical solutions, see, e.g., [6, 7], consider cyclic loading conditions. However, in most cases these solutions are based on Terzaghi's classical theory assuming geometrical as well as constitutive linearity and therefore only rather idealized boundary value problems can be analyzed. Though, in reality with changing effective stresses and void ratio under cyclic loading conditions soil stiffness and permeability will also change.

In the present paper, partially drained consolidation of fine grained soils under cyclic loading condition is analyzed. Therefore, a newly developed one-dimensional compression cell is introduced and typical, experimental results for fine grained soils are discussed. This type of experiment involves several challenges as, for example, measuring only small amount of water inflow or outflow of the sample, measuring radial total stress and sealing the cell in order to measure and/or control pore water pressures in the samples. Furthermore, numerical experiments using FEM are carried out to study the influence of soil stiffness and permeability on the evolution of excess pore water pressures and accumulated deformations. The FEM simulations comprise one-dimensional tests involving isotropic hardening plasticity which is commonly used in engineering practice [9]. In the coupled hydraulic-mechanical analysis the soil stiffness is adapted according to the change in effective stress. Additionally soil stiffness is modeled as loading direction dependent. Plastic strains are determined based on a double hardening rule. Cap hardening is calibrated to result in realistic K_0 values under one-dimensional loading paths. Experimental results are qualitatively compared to the FEM simulations.

Summarizing, the main objective of this paper is to analyze whether classical hardening plasticity is an appropriate tool to model cyclic soil-structure interaction mechanisms involving underlying partially drained consolidation processes.

2 Experimental Study

In the experimental study partially drained, oedometric tests on soft Kaolin clay were carried out to study the cyclic consolidation behavior of fine grained soils. A new oedometer device was designed and constructed at Ruhr-Universität Bochum, introduced already in [4, 5]. In the following section the oedometer device is described regarding its technical features and testing procedure. Hydraulic boundary and loading conditions, which were used to perform cyclic tests, are given. Furthermore, the tested Spergau Kaolin clay samples are characterized and the sample preparation is described.

2.1 Oedometer Device

To experimentally analyze the consolidation behavior of fine grained soils under cyclic loading conditions a new oedometer device was implemented at Ruhr-Universität Bochum and introduced in [4]. In the new oedometer device cylindric samples of 20 mm height and 70 mm diameter are tested (see Fig. 1). The oedometer ring is sealed against the top and bottom plate by rubber rings. Therefore, it allows the measurement of pore water pressure and pore water volume, which is in the focus of this study. As top and bottom of the device can be set drained or undrained independently, tests under different hydraulic boundary conditions as well as undrained compression tests can be performed.

Supplementary to the functionality of a classical oedometer the following measurements are conducted: measurement of positive and negative pore water pressures up to 1000/−100 kPa at the bottom of the sample, lateral stress measurement up to 400 kPa allowed by strain gauges attached to the oedometer ring, determination of wall friction between sample and oedometer ring by measurement of vertical stress above and below the sample as well as volume measurement of the outflowing pore water.

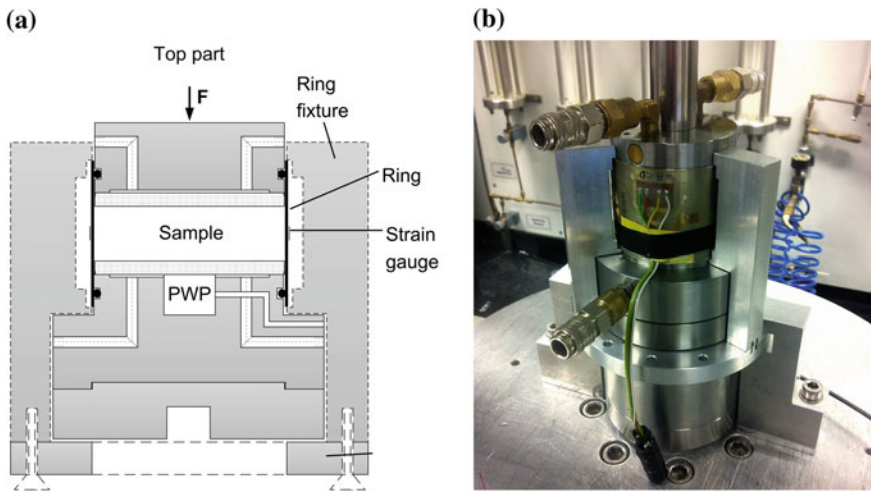


Fig. 1 a Sketch and b photograph of the new oedometer device [4]

Table 1 Plasticity of Spergau Kaolin

Liquid limit, w_L (%)	53.4
Plastic limit, w_P (%)	32.3
Shrinkage limit, w_S (%)	31.3
Plasticity index, I_P (%)	21.1

2.2 Testing Material and Sample Preparation

In the present study experiments were conducted on Spergau Kaolin clay. Spergau Kaolin is an inorganic clay of high plasticity [1]. Table 1 gives the Atterberg limit values for this clay.

The samples were prepared mixing Kaolin powder with water to a sample material with a water content slightly above liquid limit ($w = 1.1 \cdot w_L = 0.588$). The paste-like soil material with a density of $\rho = 1.63 \text{ g/cm}^3$ was placed in the oedometer ring using a spatula between two deaired filter plates. Figure 2b shows an exemplary Kaolin sample.

The clay sample was saturated by streaming it from bottom to top. This procedure guarantees better saturation and allows the determination of the sample permeability, as a permeability test with variable pressure head is performed. Thereby, the mean values of the initial permeability of the samples were determined, for details see [4].

2.3 Hydraulic Boundary Conditions

The hydraulic boundary conditions in the experiment had to be chosen such that pore water pressures could be measured during the cyclic consolidation process. In a standard oedometer configuration, where usually both top and bottom of the sample are drained (abbreviated PTPB), the measurement of pore water pressure however

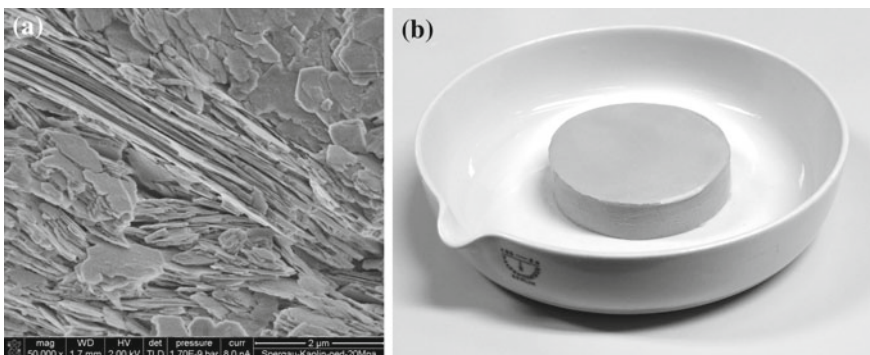


Fig. 2 a ESEM photograph of Spergau Kaolin [1] and b Spergau Kaolin sample

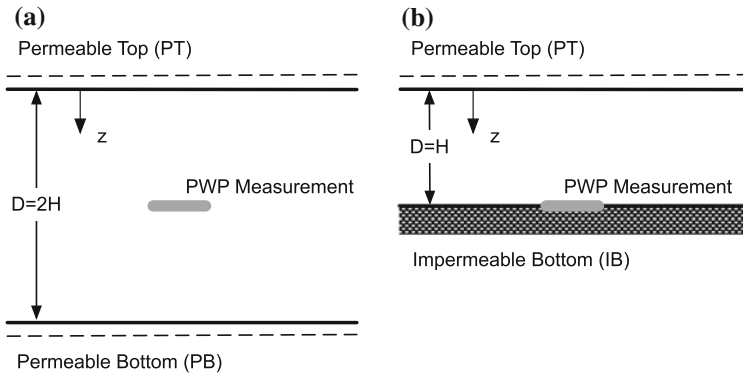


Fig. 3 Possible hydraulic boundary conditions and location of pore water pressure measurement: **a** PTPB, **b** PTIB

is rather difficult as pore water pressures have to be measured in the middle of the sample (see Fig. 3). Due to the minor height of the sample the implementation of a pore pressure transducer in this place would cause a strong disturbance of the sample and flow paths. Therefore, experiments were conducted with the following testing configuration: permeable top and impermeable bottom (abbreviated PTIB). This configuration is equivalent to a standard PTPB oedometer configuration considering that the height of the sample is halved. The hydraulic boundary conditions can be described by

$$u(z = H, t) = 0 \tag{1}$$

where z is the vertical coordinate starting from the bottom of the sample and H is the sample height.

2.4 Applied Loading

For the present study a cyclic loading function of sinusoidal type was selected according to [4, 7]:

$$L(t) = q \sin^2 \frac{\pi t}{d} \tag{2}$$

where $L(t)$ = applied loading as a function of time, q = load amplitude, t = time, and d = load period.

This haversine loading function was chosen considering that in geotechnical applications mostly compression loading is considered, while tensile loading is neglected.

In the present study the load amplitude q was set to 400 kPa accounting for the soft soil behavior and the short time range of load application. The load period d was

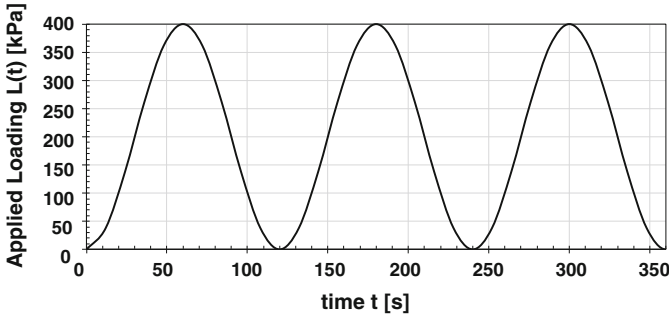


Fig. 4 Applied haversine loading function

chosen to be 15% of the reference time $t^{\text{ref}} (T_0 = 1)$, an equivalent sample (identical initial state) under static loading would need to completely consolidate:

$$d = \frac{T_0 \times H^2}{c_v} = \frac{0.15 \times (0.02 \text{ m})^2}{5 \times 10^{-7} \text{ m}^2/\text{s}} = 120 \text{ s} \quad (3)$$

where T_0 is the chosen dimensionless period, c_v is the material-dependent coefficient of consolidation assumed to be a constant value determined from static consolidation tests on the same sample material. For details see [4].

Figure 4 shows the haversine loading function over time as applied in the experimental testing.

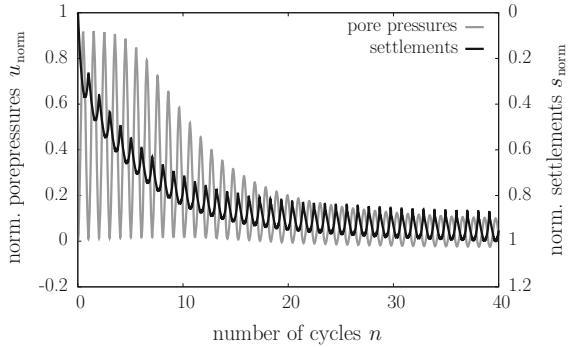
2.5 Experimental Results

In the following section selected results from the experimental study on Kaolin clay under haversine cyclic loading are presented. Thereby, the focus is set on the qualitative pore water dissipation behavior and evolution of settlements during the cyclic consolidation process.

2.5.1 Pore Water Dissipation Behavior

Figure 5 shows the evolution of excess pore water pressure u_{norm} , normalized with the applied load amplitude $q = 400 \text{ kPa}$, during the consolidation process. It can be observed that in the first cycles of the test the excess pore water pressure reaches values equal to 0.9. Considering the friction between soil sample and oedometer ring, which is among 5–10% of the applied load, at the beginning almost the entire load is carried by the pore water pressure. With ongoing consolidation time the excess pore water pressure dissipates, reaching a quasi-stationary state after approximately

Fig. 5 Evolution of pore pressures and settlements from the experimental study



40 cycles. It is important to notice that the amplitude of excess pore water pressures u is strongly damped during the consolidation process. Moreover, the pore water pressures in the quasi-stationary state reach slight negative values at the beginning and end of a loading cycle, while the average value stays clearly above zero.

2.5.2 Evolution of Settlements

As displayed in Fig. 5, the settlements s_{norm} , normalized with the maximum consolidation settlement, during the cyclic consolidation process accumulate and just as the excess pore water pressures reach a quasi-stationary state after approximately 40 cycles. The increment of settlement accumulation is significantly decreasing from larger values in the first cycles to smaller ones in quasi-stationary state.

3 Numerical Modeling

The cyclic consolidation of normally consolidated Kaolin clay is numerically modeled using the finite element method (FEM). The FEM is an appropriate tool to solve geotechnical boundary value problems under defined complex initial and boundary conditions. Simulating the cyclic consolidation under partially drained conditions in a numerical model allows the investigation of the influence of constitutive soil parameter variation on model responses (e.g., pore water pressures and settlements).

3.1 Initial and Boundary Conditions

The geometry of the simulation model is chosen in accordance with the experimental setup using same diameter and height as the samples in the oedometer test device described above. For reasons of computational simplicity the cylindrical sample is

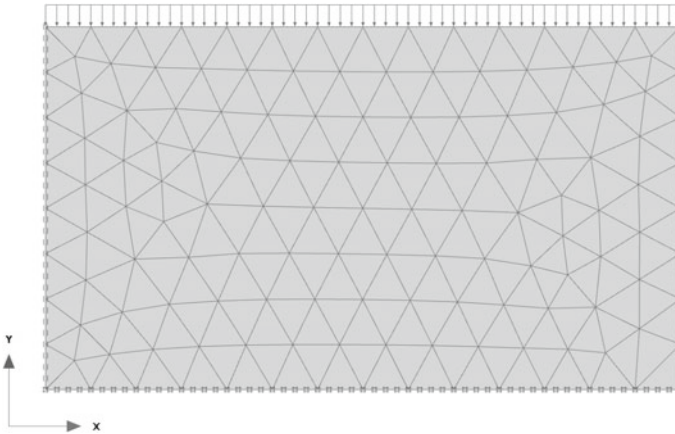


Fig. 6 Discretization for axis-symmetric FE simulation of oedometer tests ($H = 2$ cm, $D/2 = 3.5$ cm)

modeled using principles of axis symmetry with respect to the vertical centerline of the sample.

In the conducted research 15-node triangular finite elements are used for the discretization of the simulation model (see Fig. 6). Within coupled hydromechanical analyses of consolidation problems both settlements and pore water pressures are degrees-of-freedom for the nodes of a finite element.

The mechanical boundary conditions are applied with respect to the oedometer test conditions. Therefore, the prescribed boundary conditions allow for settlements of the soil sample in general, while the bottom is fixed in its vertical position. The lateral displacements are constraint at the side wall and in the symmetry axis. The numerical simulations are performed under stress controlled loading conditions. Whereat the cyclic load is applied at the top of the soil sample as a mechanical stress boundary condition referred to the haversine loading function from the laboratory experiments (same amplitude, load period, number of cycles, and equivalent consolidation coefficient).

In all conducted numerical tests, homogeneous material properties and isotropic permeabilities of the soil sample are assumed. The soil sample is fully water saturated. The chosen hydraulic boundary conditions are chosen according to the PTIB case in the experiment and therefore allow for drainage at the top of the soil sample and restrict the side wall, the bottom and the symmetry axis of the soil sample to be impermeable.

In general the numerical simulations start with a K_0 phase to initialize the total and effective stress conditions of the soil sample considering at-rest earth pressure and hydrostatic water pressure distributions. The initial density of the soil sample is initialized by introducing the initial void ratio e_0 . In the subsequent coupled hydro-mechanical calculation phase, the cyclic load is applied in terms of the haversine

loading function described above while the consolidation process is numerically analyzed according to Biot’s theory simultaneously, see [2, 3].

3.2 Constitutive Soil Model

For adequate modeling of the mechanical behavior of frictional soil, a sophisticated constitutive model, namely the hardening soil model is used. The hardening soil model has been developed in the framework of classical elastoplasticity, see [8, 9].

The highly nonlinear, hyperbolic stress–strain behavior of the soil is accurately modeled by introducing three loading path dependent stiffnesses. The tangent stiffness for primary oedometer loading $E_{\text{od}}^{\text{ref}}$, the secant stiffness for primary triaxial loading E_{50}^{ref} and the un-/reloading stiffness $E_{\text{ur}}^{\text{ref}}$ correspond to a stress-dependent reference stiffness p^{ref} . Whereat the exponential correlation is controlled by a material parameter m .

The shear failure surface in the hardening soil model obeys Mohr–Coulomb failure criterion including constitutive strength parameters, namely friction angle φ and cohesion c .

The plasticity of the soil is modeled by introducing a double hardening yield surface shown in Fig. 7 which encloses the elastic region. The yield surface is not fixed in principle stress space and is allowed to isotropically expand due to deviatoric and volumetric plastic straining.

The plastic shear behavior of the soil is modeled by a deviatoric yield surface which is a function of the current stress state, the triaxial loading stiffness E_{50}^{ref} , the un-/reloading stiffness $E_{\text{ur}}^{\text{ref}}$, and the deviatoric plastic strains γ^p which are used as internal hardening parameters. The hardening expansion of the deviatoric yield surface is controlled by the evolution of γ^p as shown in Fig. 8. The deviatoric yield surface is allowed to expand up to the Mohr–Coulomb failure surface. For the cal-

Fig. 7 Yield surface of the Hardening Soil Model in principle stress space [9]

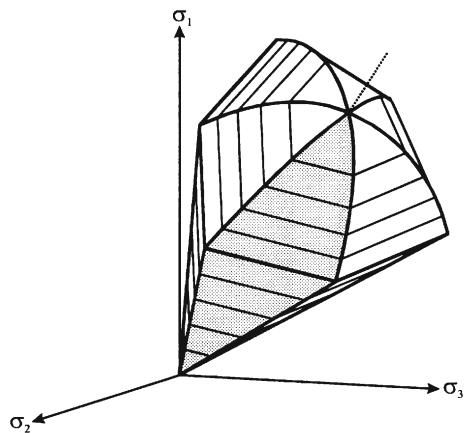
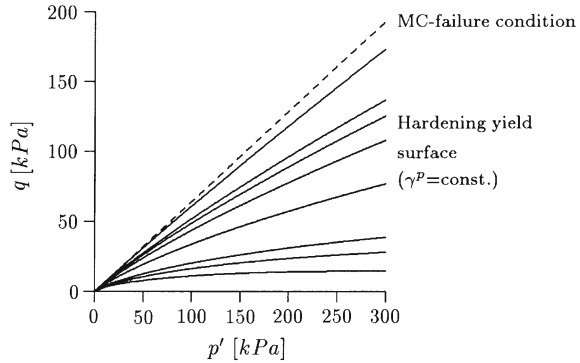


Fig. 8 Deviatoric hardening of the yield surface of the hardening soil model and Mohr–Coulomb failure surface [9]



ulation of γ^p a nonassociated flow rule is assumed where the plastic potential is a function of the dilatancy angle ψ .

The volumetric soil behavior is controlled by a cap–yield surface which separates the elastic and plastic region in the direction of the effective mean stress p' . Whereat the size of the cap is defined by the effective preconsolidation pressure p'_c which accounts for the stress history of the soil. The p'_c is used as an internal state variable and is updated with respect to the evolution of volumetric plastic strains which are calculated using an associated flow rule. The initial elliptical shape of the cap is determined internally based on initial K_0 -conditions where K_0 is the coefficient of earth pressure at rest.

3.3 Constitutive Material Parameters

The constitutive parameters for the hardening soil model are derived based on the mechanical and hydraulic parameters determined in standard laboratory tests (triaxial tests and static oedometer tests) on Kaolin clay with initial conditions equivalent to those in the experimental setup ($w = 1.1 \cdot w_L = 0.588$). An overview of the main constitutive parameters of the hardening soil model used in the numerical simulation of cyclic oedometer tests is given in Table 2.

3.4 Numerical Testing Concept and Program

Within the present research the focus is set on the analysis of the influence of consolidation coefficient c_v , permeability k , and oedometric stiffness E_{oed} on the model responses. For comparison of the numerical model responses with the experimental data of Kaolin clay the development of pore water pressures u at the bottom and settlements s at the top of the soil sample are evaluated. The numerical testing

Table 2 Main constitutive parameters for the hardening soil model

Parameter	Value	Unit
φ	22	(°)
ψ	0	(°)
c	5	(kN/m ²)
E_{50}^{ref}	1,750	(kN/m ²)
E_{oed}^{ref}	1,750	(kN/m ²)
E_{ur}^{ref}	5,250	(kN/m ²)
p^{ref}	100	(kN/m ²)
m	0.95	(-)
K_0	0.625	(-)
e_0	1.543	(-)
k	4×10^{-8}	(m/s)

program for analyzing the influence of varying the constitutive parameters on the model responses u and s is given in Table 3.

Case 1 is chosen as the reference case. The variation of the consolidation coefficient c_v is linearly controlled by the variation of the permeability k and the oedometric stiffness E_{oed} according to Eq. (4) where γ_w is the weight of water.

$$c_v = \frac{k E_{oed}}{\gamma_w} \tag{4}$$

The permeability k and the oedometric stiffness at reference stress E_{oed}^{ref} are varied by multiplication of the values from the reference test (case 1) with the factors $i = 2$ and $i = 1/2$. The cases 2 and 3 are conducted to investigate the influence of constant

Table 3 Numerical testing program—variation of constitutive parameters

Case	c_v (m ² /s)	k (m/s)	E_{oed}^{ref} (kPa)
Reference test			
1	7×10^{-7}	4×10^{-8}	1,750
Constant c_v by variation of k and E_{oed}^{ref}			
2	7×10^{-7}	2×10^{-8}	3,500
3	7×10^{-7}	8×10^{-8}	875
Variation of c_v by variation of k and constant E_{oed}^{ref}			
4	3.5×10^{-7}	2×10^{-8}	1,750
5	1.4×10^{-6}	8×10^{-8}	1,750
Variation of c_v by variation of E_{oed}^{ref} and constant k			
6	3.5×10^{-7}	4×10^{-8}	875
7	1.4×10^{-6}	4×10^{-8}	3,500

c_v on the model responses u and s while k and $E_{\text{oed}}^{\text{ref}}$ are varying contrarily. The cases 4 and 5 are used to analyze the influence of c_v controlled by variation of k and constant $E_{\text{oed}}^{\text{ref}}$. In the cases 6 and 7 the c_v is influenced by variation of $E_{\text{oed}}^{\text{ref}}$ and constant k .

Besides the variation of c_v , k , and $E_{\text{oed}}^{\text{ref}}$, the other parameters are kept constant in compliance with Table 2. It is assumed that during the variation of oedometric stiffness $E_{\text{oed}}^{\text{ref}}$ the correlations $E_{50}^{\text{ref}} = E_{\text{oed}}^{\text{ref}}$ and $E_{\text{ur}}^{\text{ref}} = 3 E_{\text{oed}}^{\text{ref}}$ are always valid.

To generalize the comparison of the different cases, all pore pressures are normalized to u_{norm} with the maximum total stress applied during the cyclic oedometer test. Furthermore, all settlements are normalized to s_{norm} with the maximum settlements from the reference case (case 1).

4 Numerical Test Results

In the following section the results of the numerical experiments simulating cyclically loaded oedometer tests with partially drained boundary conditions are analyzed. Thereby, the main focus is set on the influence of soil stiffness $E_{\text{oed}}^{\text{ref}}$ and permeability k on the normalized excess pore pressure dissipation behavior $u_{\text{norm}}(t)$ at the bottom (undrained side) and the normalized accumulated settlements $s_{\text{norm}}(t)$ at the top of the samples (drained side). To do so case 1 is used as the reference configuration, using typical constitutive parameters of the soft Kaolin soil samples in the experimental study. As explained above cases 2–7 are derived from the reference case by changing one or two parameters (soil stiffness and/or permeability) while keeping all the other parameters the same for all cases as shown in Table 3.

Figure 9 gives the results for three cases with the same c_v but increasing and decreasing stiffness and permeability. First of all, it can be seen that for a constant value of c_v the excess pore water pressure dissipation is independent of soil stiffness $E_{\text{oed}}^{\text{ref}}$ and permeability k . This observation holds for both the amplitude and the average value. Moreover, it can be observed that the final average settlement clearly depends on the reference oedometer stiffness $E_{\text{oed}}^{\text{ref}}$ and that the stiffer the soil is the smaller is the deformation amplitude.

In Fig. 10 results showing the influence of the soil permeability k are displayed. As soil stiffness $E_{\text{oed}}^{\text{ref}}$ remains constant final average settlements are the same for all three cases. However, the less permeable the soil is the smaller is also the amplitude of vertical deformation. Amplitude and average value of the excess pore water pressure are independent of the permeability. However, the more permeable the soil is the earlier the quasi-stationary state is reached.

In Fig. 11 results showing the influence of soil stiffness $E_{\text{oed}}^{\text{ref}}$ are displayed. It can be seen that the final settlements s_{norm} linearly depend on the stiffness, while the amplitude of cyclic deformation increases with decreasing stiffness $E_{\text{oed}}^{\text{ref}}$. The dissipation of excess pore water pressure accelerates with increasing soil stiffness and increasing c_v , respectively.

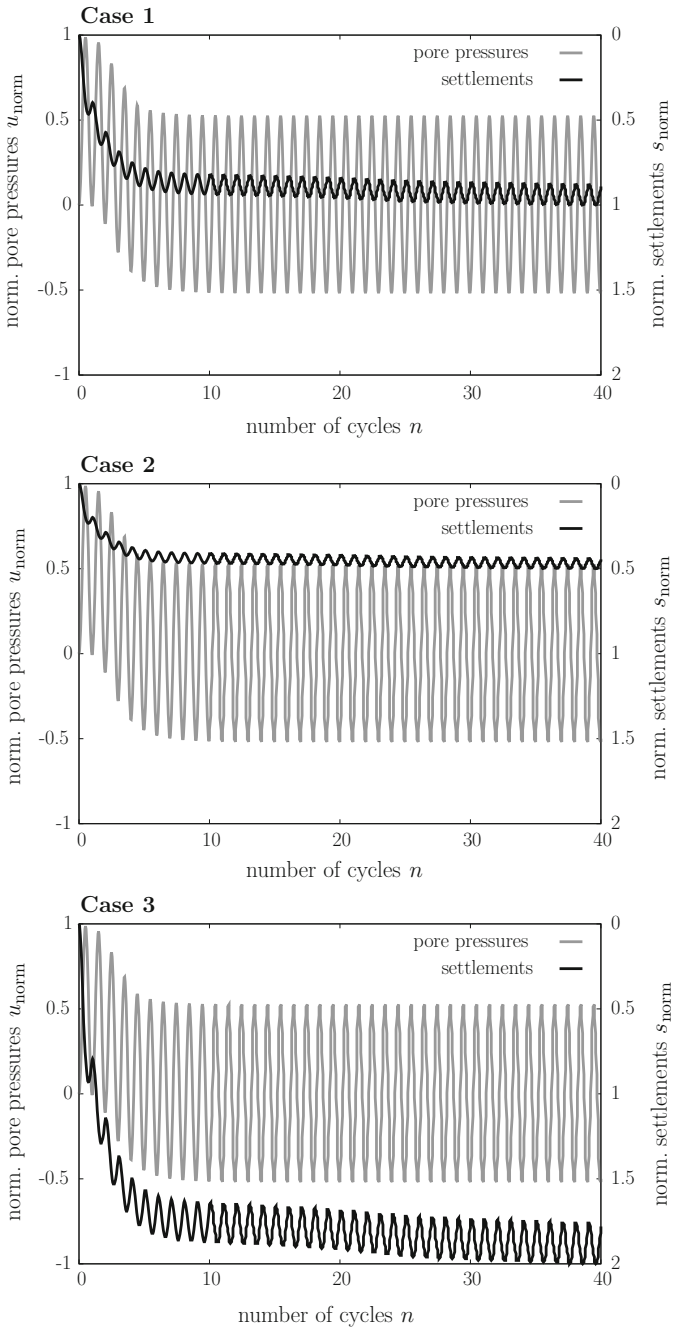


Fig. 9 Evolution of pore pressures and settlements during numerical cyclic loading test—variation of permeability and oedometric stiffness

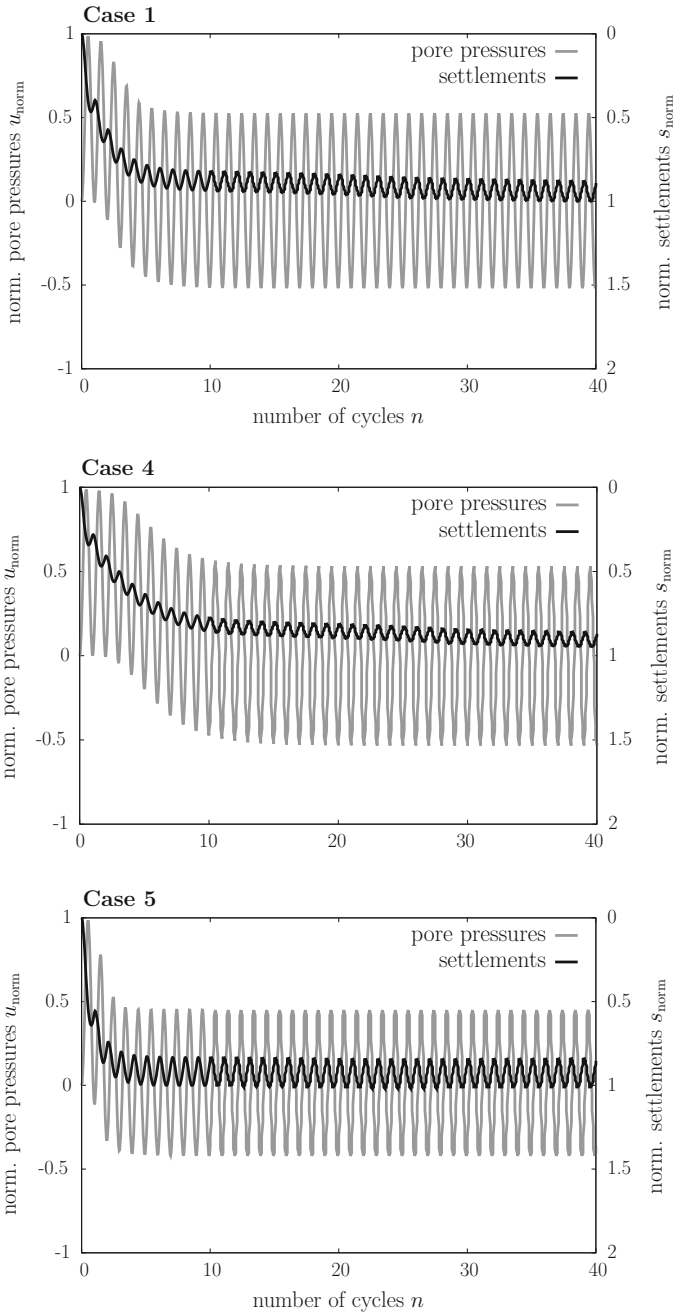


Fig. 10 Evolution of pore pressures and settlements during numerical cyclic loading test—variation of permeability

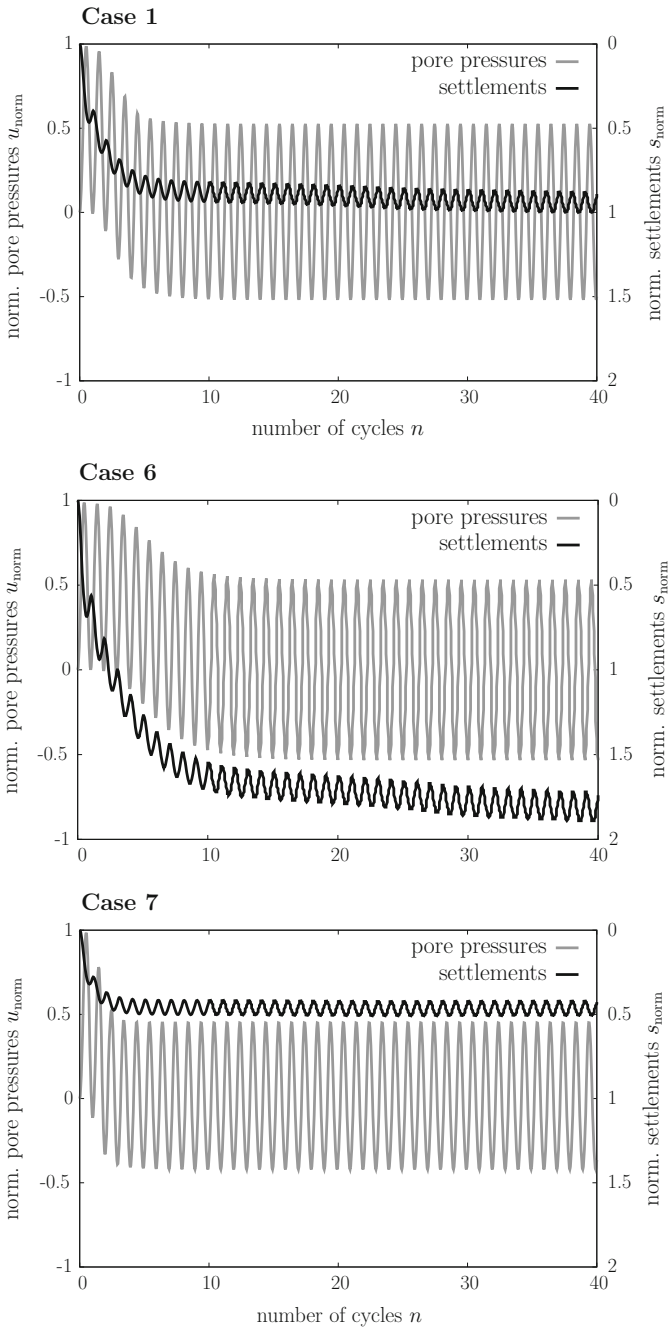


Fig. 11 Evolution of pore pressures and settlements during numerical cyclic loading test—variation of oedometric stiffness

In general it can be concluded that both—normalized excess pore water dissipation and normalized accumulation of settlements—depend on soil permeability and stiffness. The soil deformation is more significantly influenced by a change of the stiffness than the permeability, as a change in permeability with constant stiffness only effects the amplitude of the deformations but not the average value. The excess pore water dissipation is most significantly influenced by a change of c_v as for higher c_v the quasi-stationary state is reached earlier.

5 Validation of Numerical Results by Experiments

In the following section a validation of the constitutive model used is performed by a qualitative comparison of excess pore water dissipation and accumulation of settlements from the laboratory experiments. As typical numerical results for soft soils we use the reference case one, as displayed in Fig. 9a. From such a comparison it becomes obvious that accumulation of vertical settlements is captured rather well by the constitutive approach employed. For about 20 cycles amplitude of deformation decreases and quasi-stationary state seems to be reached for number of cycles larger than 40. On the other hand, a clear discrepancy must be stated when comparing results from excess pore water pressure dissipation. First of all, a significant damping of the amplitude can be observed for the experimental results. This results from a strong decrease of the maximum pore pressures reached during one cycle. Moreover, the excess pore water pressure minima only slightly reach the negative range with an average value for quasi-stationary state clearly different from zero. On the opposite, the numerical results show a remaining constant amplitude and an average of about zero for quasi-stationary state. Consequently, it can be concluded that constitutive models based on isotropic hardening plasticity are not adequate to simulate excess pore water pressure dissipation behavior during cyclic consolidation processes in a qualitatively realistic manner.

6 Conclusion and Outlook

In the present research the capability of isotropic hardening plasticity to model correctly the cyclic consolidation behavior under partially drained hydraulic boundary conditions was studied. In order to perform the experimental study a new oedometer cell was used allowing measurement of the complete stress state. Numerical experiments were conducted using FEM to model the coupled hydromechanical consolidation process. A well-established isotropic double hardening model was used to describe constitutive behavior of fine grained soil.

From the comparison of experimental and numerical results it can be concluded that the applied models based on isotropic hardening plasticity are not completely able to qualitatively simulate the excess pore water pressure dissipation behavior

during cyclic consolidation processes. This holds for the amplitude as well as for the average value in the quasi-stationary state. Final settlements in the quasi-stationary state are too large, as the accumulation of deformation is not modeled adequately. A decrease of deformation increment as observed from the experiments is not observed from the numerical simulations.

An alternative concept to simulate this type of cyclic consolidation processes is the use of constitutive models based on bounding surface plasticity. In further research, it will be studied whether this model class is able to represent accumulation of deformations and degradation of excess pore water pressure amplitude in a more realistic way.

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