Image Contrast Enhancement by Distances Among Points in Fuzzy Hyper-Cubes

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Abstract. A new geometrical fuzzy approach for image contrast enhancement is here presented. Synergy among ascending order statistics and entropy evaluations are exploited to get contrast enhancement by evaluation of distances among points inside fuzzy unit hyper-cube. The obtained results can be considered interesting, especially compared with consolidated techniques which encourages further studies in this direction.

1 Introduction

Contrast enhancement techniques, representing the first treatment to enhance image quality, are subdivided in two main trends. The first, direct type one, formulates a criterion of contrast measurement and enhances the image quality by improving of such measure. The second one, of indirect type (such as histogram equalization), acts on the image histogram modifying the intensity of the gray levels of pixels has a gray levels transformation in which dark pixels appear darker and light ones appear brighter. Both techniques produce a stretching of the global distribution of the intensity of the gray levels requiring the elaboration of adaptive procedures of features extraction directly and automatically from the image. Owing to the uncertainty and vagueness of the sampling techniques, the construction of an image is not free from uncertaintess and noise (loss of informative content during the transformation of objects from three-dimensional to bi-dimensional images, ambiguity of the definition of edges, regions and boundaries). So, it follows the necessity to implement an adaptive procedure of contrast enhancement which manipulates data uncertainty. For the reasons given above, scientific research has produced important results through fuzzy techniques both with direct and indirect approaches [1], [2], [3]. In addition, a lot of efforts have been done about adaptive formulation of contrast indicators enhancing the image quality by evaluations of differences of gray levels in local neighborhoods [2]. By adaptive extraction of features, meaningful contributions have been proposed where the main fuzzy entropy plays a determining role for the gray levels fuzzification [3]. Good results have been also obtained by wavelet-fuzzy techniques in which approximation/detail coefficients and transformation/saturation operators are exploited to get contrast enhancement [4], [5].

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In addition, a lot of papers deal with got contrast enhancement by fuzzy generation of histogram equalization [6], [12], [13], [14] and specific statistical applications [7], [9], [10] can be considered particularly meaningful. However, scientific literature is poor in papers in which histogram stretching is carried out by fuzzy geometrical approaches where intelligible contrast enhancement procedures have been adjusted (particularly helpful for not technical experts). So, in this work, the authors present a new approach based on a particular fuzzy formulation in which the features extraction characterizes the fuzzification of the gray levels ranges. Moreover, some statistical-entropic considerations calibrate the procedure adaptively. Finally, synergy among ascending order statistics and fuzzy geometries yields contrast enhancement steps automatically. The proposed procedure, characterized by a low computational load shows very useful for real-time applications, when applied to a lot of images with different features, being in addition the obtained results wholly qualitatively and quantitatively comparable with those obtained by consolidated techniques. The paper is organized as follows. Starting from gray levels images, the steps of the proposed procedure will be illustrated giving a reason for each operational choice. Then, the obtained results on images with different features are presented, making qualitative and quantitative comparisons with the same images treated by using already consolidated techniques. Finally, some conclusions are drawn.

2 Material and Methods

Generally, an $M \times N$ image I with L levels can be defined by means of a matrix of pixels on which, for each pixel position (i, j), $i = 0, 1, 2, ..., M-1$ and $j = 0, 1, 2, ..., N-1$, we associate its gray level x_{ij}. In a fuzzy domain, it is imperative to fix the membership value to each pixel to I by means a function $\mu_I(x_{ij}) : I \rightarrow [0, 1]$ defining the membership degree of x_{ij} to I. In particular, if $\mu_I(x_{ij}) = 1$ the corresponding pixel is totally belonging to I; if $\mu_I(x_{ij}) = 0$ x_{ij} does not belong to I totally. Intermediate values of μ show a partial membership of x_{ij} to I. If $\mu_I(x_{ij})$ is the informative content of x_{ij} in I, we can represent I by $\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[x_{ij}, \mu_I(x_{ij}) = g_{ij}], \forall x \in I, i = 0, 1, 2, ..., M-1, j = 0, 1, 2, ..., N-1$ where $\mu_I(x_{ij}) = g_{ij}$ is the gray level of x_{ij} [8], [11]. So, contrast enhancement process can be defined thinking that if x_{ij} is *dark* it has to be made darker; if x_{ij} is *light* it has to be made lighter; if x_{ij} is *gray* it has to remain gray. Stating that *dark*, *light* and *gray* are fuzzy terms, it is necessary to express them by a suitable function regulating the mapping of I in the fuzzy domain by means of the image features themself. In this way, image contrast enhancement is designed as the transformation of this function making darker the gray levels considered dark, and clearer the ones thought as clear.

2.1 Choice of the Typology of Mapping in the Fuzzy Domain

Scientific literature suggests us different typologies of mapping functions: linear piecewise (triangular, trapezoidal, sigma-functions) and smoother ones

(S-functions, bell-shaped, broken gaussian) to guarantee a better transaction among gray levels. Here, let here choose, define and exploit an S-function as follows

$$g_{ij} = \mu_I(x_{ij}) = \begin{cases} 0 & 0 \leq x_{ij} \leq a \\ \frac{(x_{ij}-a)^2}{(b-a)\cdot(c-a)} & a \leq x_{ij} \leq b \\ 1 - \frac{(x_{ij}-c)^2}{(c-b)\cdot(c-a)} & b \leq x_{ij} \leq c \\ 1 & x_{ij} \geq c \end{cases} \tag{1}$$

in which a, b, c (b not necessarily equal to $(a+c)/2$) have to be adaptively determined.

2.2 Adaptive Setting of S-Function Parameters

S-function construction has to perform principles of noise reduction and loss information minimization. So, to determine a and c, it has been elaborated an algoritm acting directly on the image histogram to reduce the noise, starting from the approach developed in [3]. Regarding b, a consolidated approach based on maximum fuzzy entropy principle has been exploited because high value of entropy is a measure of a better fuzzy information in the image.

How Determine Parameters a and c. Let g_{max}, g_{min} and $H_{ist}(g)$ be the maximum and minimum values of gray levels in I and the relative histogram respectively. $H_{ist}(g)$ will present z maximum locals (peaks) labeled by $H_{ist}(g_1)$, $H_{ist}(g_2)$,..., $H_{ist}(g_z)$ and their mean value, $\overline{X_{ist_{max}}(g)}$, will gets the form [3] $\overline{X_{ist_{max}}(g)} = \frac{1}{z}\sum_{i=1}^{z} \overline{X_{ist_{max}}(g_i)}$. From the set of peaks we select k of them ($k \leq z$) which exceed $\overline{X_{ist_{max}}(g)}$ excluding the other ones because they can be considered less meaningful. From the selected k peaks, let consider only the first one $(\overline{X_{ist_{max}}(g_1)})$ and the last one $(\overline{X_{ist_{max}}(g_k)})$. The gray levels lower than $(\overline{X_{ist_{max}}(g_1)})$ can be considered as background, and the upper to $(\overline{X_{ist_{max}}(g_k)})$ can be considered as noise: this way we preserve the informative content of the image and, at the same time, we reduce the noise. Two particular gray levels, L_A and L_B will be determined, so that the loss of information in the ranges $[g_{min}, L_A]$ and $[L_B, g_{max}]$ is equal to a particular value f_1, with $0 < f_1 < 1$ (tipically $f_1 = 0.01$):

$$\sum_{i=g_{min}}^{L_A} H_{ist}(i) = \sum_{i=L_B}^{g_{max}} H_{ist}(i) = f_1 \tag{2}$$

However, the selection of the peaks occurs by thresholding on a mean value evaluated by Eq.(2) [3] which does not consider the mutual positions of the peaks. Here, the authors propose a weighted mean computed as

$$\overline{X_{ist_{max}}(g)} = \frac{\sum_{i=1}^{z} H_{ist_{max}}(g_i) \cdot g_i}{\sum_{i}^{z} g_i} \tag{3}$$

so that $X_{ist_{max}}(g)$ is the height of the centre of gravity of the histogram taking into account the mutual positions above mentioned. Finally, a and c parameters can be determined by $a = \frac{g_{max}-g_{min}}{2} + g_{min} \quad if \ (a > L_A) \rightarrow a = B_1$ and $c = \frac{g_{max}-g_k}{2} + g_k \quad if \ (a > L_A) \rightarrow a = B_1$.

How Determine Parameter b. Being fuzzy entropy a reliable measure of uncertainty of a system and high entropy values keeping anhigh informative content, the evaluation of b occurs by fuzzy entropy maximisation: $b \in [a+1, c-1]$, so its optimal evaluation, b_{opt}, can be determined by $H_{max}(I, a, b_{opt}, c) = \{H(I, a, b, c) \in' g_{min} \leq a < b < c \leq g_{max}\}$. If H is Shannon's entropy (or other entropic formulations depending on the application under study), the ambiguity of an image I, $H(I)$, can be expressed as

$$\frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} S(g_{mn}) = -g_{mn}\cdot log_2 g_{mn} - (1-g_{mn})\cdot log_2(1-g_{mn}), \quad 0 < H(I) < 1 \tag{4}$$

with Shannon's function S increasing monotonically on $[0, 0.5]$ and decreasing monotonically on $[0.5, 1]$ with a maximum falling on $g_{mn} = 0.5$.

2.3 S-Function Transformation

S-Function Partition in Partially Superimposed Portions. We subdivide the range $[g_{min}, g_{max}]$ in three sub-intervals partially superimposed, generating fuzzy rectangular patches labelled by A, B and C respectively. Specifically, A is set around 0.5 value of fuzzy membership (gray area of image); B and C are sets around dark and bright areas of the image respectively (Fig. 1). Moreover, each patch is supported on its sub-interval: $Support_A = \frac{b+c}{2} - \frac{a+b}{2}$, $Support_B = b - a$, $Support_C = c - b$ in which $Support_j$ (j=A, B, C), represents the basis of each fuzzy rectangular patch characterized by particular values of ascending order statistics which represent a set of features dirtectly extracted from the image. Contrast enhancement will be done by transformation of the S-function starting from the statistics above obtained (Table I and Fig. 1). Next section highlights the details of such idea.

Extraction and Fuzzification of Statistical Features. From $Support_j$, we evaluate mean, variance, skewness and kurtosis, labelled by $MN(\cdot)$, $VAR(\cdot)$, $SK(\cdot)$, $KU(\cdot)$, constituting the following patterns $[MN(Support_j)$, $VAR(Support_j)$, $SK(Support_j)$, $KU(Support_j)], j = A, B, C$ which, inside $\mathbb{R}^4$, represents three points

$$P_j = [MN(Support_j), VAR(Support_j), SK(Support_j) KU(Support_j)] \in \mathbb{R}^4. \tag{5}$$

To underline the fuzzy nature of the approach, it is advisable to fuzzify P_j by a sigmoidal function (anyway other types of function can be taken into account)

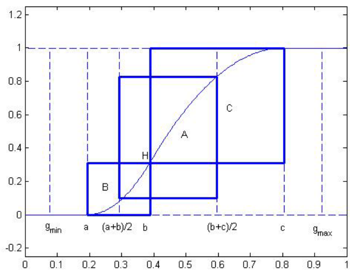

Fig. 1. Fuzzy subdivision in partially overlapped ranges A, B, C (gray, dark and bright areas respectively).

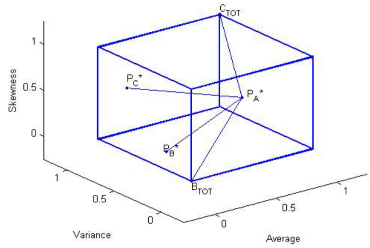

Fig. 2. Gray, dark and bright areas as points inside a Fuzzy Unit Hyper-Cube ($FUHC^4$).

as $P_j^* = \frac{1}{1+e^{-m(P_j-n)}}$ (tipically $m = 11$ and $n = 0.5$): in this way P_j^* fall inside a four-dimensional Fuzzy Unit Hyper-Cube, $FUHC^4$ (Fig. 2) [15]:

$$P_j^* = [\frac{1}{1+e^{-m(MN(Support_j)-n)}}, \frac{1}{1+e^{-m(VAR(Support_j)-n)}}, \frac{1}{1+e^{-m(SK(Support_j)-n)}}, \frac{1}{1+e^{-m(KU(Support_j)-n)}}] \in FUHC^4 \tag{6}$$

So, S-function transformation occurs by evaluation of mutual distances of the points P_j^* and two crucial fuzzy points (B_{TOT} and C_{TOT}) inside $FUHC^4$ as detailed in the following subsection.

Construction of the Transformed S-Function. Having obtained P_A^*, P_B^* and P_C^* points as above described, their mutual distances are evaluated as

$$d(P_i^*, P_j^*) = ||P_i^* - P_j^*||_2, \quad i, j = A, B, C. \tag{7}$$

So, the following cases can occur:
a) $d(P_A^*, P_B^*) = ||P_A^* - P_B^*||_2 > d(P_A^*, P_C^*) = ||P_A^* - P_C^*||_2$ where patch A represents brighter areas with respect to the other ones;
b) $d(P_A^*, P_B^*) = ||P_A^* - P_B^*||_2 < d(P_A^*, P_C^*) = ||P_A^* - P_C^*||_2$ in which patch A has to be considered as darker with respect to the other ones. In both cases, S-function transformation is made by an anticlockwise rotation of the tangent line to S-function (t-line in short) in a crucial point H (where brightness and darkness are concomitant) (Fig. 1) representing the superimposition among S-function and patches A, B and C: $H = (S-function) \cap Patch(A) \cap Patch(B) \cap Patch(C)$. If A is brighter than dark, in $FUHC^4$, P_A^* is closer to the point of maximum brightness, i.e. C_{TOT} (membership value equals to unity), and farther from to point

of maximum darkness darkness, i.e. B_{TOT} (membership values equals to zero), so the t-line slope will be increased by the factor $\frac{d(P_A^*,C_{TOT})}{d(P_A^*,B_{TOT})} = \frac{||P_A^*-C_{TOT}||_2}{||P_A^*-B_{TOT}||_2}$. Dually, if patch A is darker instead of brighter, t-line slope will be decreased by the factor $\frac{d(P_A^*,B_{TOT})}{d(P_A^*,C_{TOT})} = \frac{||P_A^*-B_{TOT}||_2}{||P_A^*-C_{TOT}||_2}$.

S-function transformation for patch A (brighter than dark)
Being $H = (b, (b-a)/(c-a))$, t-line can be written as:

$$\mu(g_{mn}) = \frac{b-a}{c-a} + \frac{2}{c-a}(g_{mn} - b) \tag{8}$$

Considering that $d(P_A^*, P_B^*) > d(P_A^*, P_C^*)$, the new slope $(\mu_t')_{new}$ becomes:

$$(\mu_t')_{new} = (\mu_t')_{old} + (\mu_t')_{old}\frac{d(P_A^*, C_{TOT})}{d(P_A^*, B_{TOT})} = \tag{9}$$
$$\frac{2}{c-a}(1 + d(P_A^*, C_{TOT})/d(P_A^*, B_{TOT}))$$

So, the new tangent line (r-line), written as

$$\mu(g_{mn}) = \frac{b-a}{c-a} + \frac{2}{c-a}(1 + d(P_A^*, C_{TOT})/d(P_A^*, B_{TOT}))(g_{mn} - b) \tag{10}$$

intersects $\mu(g_{mn}) = 0$ and $\mu(g_{mn}) = 1$ in

$$K = \left(b + \frac{a-b}{2 \cdot \left(1 + \frac{d(P_A^*,C_{TOT})}{d(P_A^*,B_{TOT})}\right)}, 0\right)$$

and

$$Z = \left(\left(1 - \frac{b-a}{c-a}\right) \cdot \frac{a-b}{\frac{2}{c-a} \cdot \left(1 + \frac{d(P_A^*,C_{TOT})}{d(P_A^*,B_{TOT})}\right)}, 1\right)$$

respectively where

$$g_K = b + \frac{a-b}{2 \cdot \left(1 + \frac{d(P_A^*,C_{TOT})}{d(P_A^*,B_{TOT})}\right)}$$

and

$$g_Z = \left(1 - \frac{b-a}{c-a}\right) \cdot \frac{a-b}{\frac{2}{c-a} \cdot \left(1 + \frac{d(P_A^*,C_{TOT})}{d(P_A^*,B_{TOT})}\right)}$$

are the gray levels of K and Z. So, the new S-function $\mu(g_{mn})_{new}$ becomes (Fig. 3):

$$\begin{cases} 0 & g_{mn} \leq g_K \\ \frac{b-a}{c-a} + & \\ + \frac{2}{c-a} \cdot \left(1 + \frac{d(P_A^*,C_{TOT})}{d(P_A^*,B_{TOT})}\right) \cdot (g_{mn} - g) & g_K \leq g_{mn} \leq g_Z \\ 1 & g_{mn} \leq g_Z \end{cases} \tag{11}$$

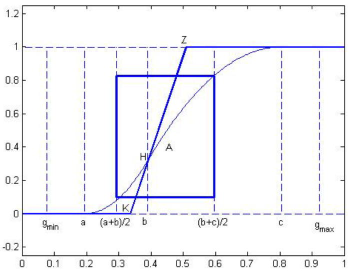

Fig. 3. Modification of the S-function.

Fig. 4. Low contrast image of a religious building with a lot of architectural details.

Defuzzification procedure to obtain the enhanced image
Finally, we need to apply an inverse transformation μ^{-1} to extract the new values of fuzzy membership $(\mu'_t)_{new}$ to come back in the space domain with the enhanced gray levels (g'_{mn}). Specifically, the procedure can be expressed as $\mu^{-1}((\mu'_t)_{new})$.

Defuzzification for Patch A Darker Instead of Brighter. Dually, if $d(P_A^*, P_B^*) < d(P_A^*, P_C^*)$, t-line slope is reduced to the value:

$$(\mu'_t)_{new} = (\mu'_t)_{old} - (\mu'_t)_{old} \cdot \frac{d(P_A^*, B_{TOT})}{d(P_A^*, C_{TOT})} = \qquad (12)$$
$$= \frac{2}{c-a} \cdot \left(1 + \frac{d(P_A^*, B_{TOT})}{d(P_A^*, C_{TOT})}\right)$$

So, S-function transformation and defuzzification are analogous to the above detailed ones as for $d(P_A^*, P_B^*) > d(P_A^*, P_C^*)$.

Table 1. Features of localization of each patch covering S-function

Fuzzy Patches	*Localizzation*	*Support Range*
A	around membership value=0.5	$\frac{b+c}{2} - \frac{a+b}{2}$
B	around dark areas	$b - a$
C	around bright areas	$c - b$

3 Results and Discussion

The proposed algorithm have been applied to a wide set of images with different features. In particular, qualitative/quantitative comparisons with histogram equalization and the approach elaborated in [3] have been carried out to evaluate the goodness of the procedure. From Fig. 4 to Fig.19 the most meaningful examples of the elaborations are shown. In particular, Fig.4 refers to a very low contrast image of a religious building with many architectural details; Fig. 8 visualizes a monument showing extended shadowy zones; Fig. 12 concerns a human face poorely lighted and Fig. 16 displays a low contrast seascape. In Table 2, for each image under study, sizes and adaptive setting of S-function parameters are reported. Figs. 5-9-13 and 17 refer to the image obtained by the proposed algorithm, while Figs. 6-10-14 and 18 refer to the treatment by [3]. Finally, Figs. 7-11-15 and 19 show the contrast enhancement by histogram equalization. After treatments, owing to the excessive brightness, the religious building preserves its architectural details but with a loss of sharpness (Fig. 6) and, owing to the low increase of contrast, loss of architectural shadings (Fig. 7). Fig. 8, after the treatment, was subjected to an increase of the contrast with a slight darkening of the shadowy areas and a meaningful increase of the shading details (Fig. 9), while the following elaborations enhance a bit the quality of the image (Fig. 10) and in a remarkable way the contrast (Fig. 11) respectively. Fig. 12 is a quite dark image with a low contrast; after the treatment with the algorithm proposed (Fig. 13), the contrast is sensibly enhanced, even if original obscurity remains. The outlines are well defined with a good presence of luminosity in the top of the hat. Elaborations reported in Figs. 14 and 15 enhance sensibly luminosity and contrast, but there is no trace of such peculiarities in Fig. 13. Finally, the proposed algorithm on the seascape image (Fig. 16) gets good results in terms of contrast enhancement and levels of details in the sea areas, but the perception of fog (Fig. 17) remains ans persists even applying other elaborations (Figs. 18 and 19). Such qualitative analysis is confirmed by qualitative

Fig. 5. Contrast enhancement carried out by the proposed algorithm: good highlighting of dark areas, architectural and chiaroscuro details.

Fig. 6. Contrast enhancement by [3] where an excessive increase of bright with loss of details and chiaroscuro effects are shown.

Fig. 7. Contrast enhancement by histogram equalization where a good preservation of details takes place. Nevertheless, the contrast increase is reduced.

Fig. 8. Particular of a sculpture with a good presence of details.

Fig. 9. The proposed algorithm shows a good differentiation of details both in bright and shadowy areas with shaded effects (poorly highlighted in the original image).

Fig. 10. Contrast enhancemen by [3], which increases the brightness of a lot of details.

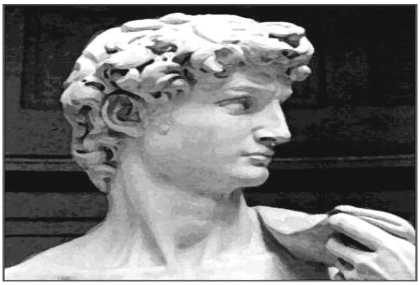

Fig. 11. Contrast enhanement by histogram equalization. It comes out a further increase in brightness together with a better contrast in shadowy zones.

Fig. 12. Low contrast human face with high presence of shadowy areas.

Fig. 13. The proposed algorithm brings about a good increase of the contrast keeping the shadiness of the image.

Fig. 14. Contrast enhancement by [3] where details, hardly recognizable in the starting-image, are here in the limelight.

Fig. 15. Contrast enhancement by histogram equalization: light increase of the performance with respect to the proposed algorithm.

Fig. 16. Low contrast seascape with presence of details and background line extremely darkened.

Fig. 17. Contrast enhancement by the proposed procedure. Good global image enhancement, with clearness of the details of the rocks and a remarkable reduction of the fog.

Fig. 18. Contrast enhancement by [3]. Image enhancement acceptable even if the foggy effect remains.

Fig. 19. Good contrast quality obtained by histogram equalization.

evaluations of $MSE/PSNR$ as reported in Table 2. If $N \times M$ is the number of pixel contaned in the image, and referring to the i-th pixel in the original image (g_{mn}) and in the image to evaluate (g'_{mn}), MSE can be computed as $MSE = \frac{1}{N \times M} \sum_{m=1}^{N} \sum_{n=1}^{M} (g_{mn} - g'_{mn})^2$. Moreover, if L is the dinamic range of the pixel values (tipically equal to $2^n - 1, n = bits/pixel$), PSNR gets the form $PSNR = 10 \cdot log_{10} \frac{L^2}{MSE}$.

Table 2. Adaptive setting of S-function parameters for each approach and quantitative evaluations of the contrat enhancement quality

Image	Size	a	b	c	MSE/PSNR proposed algorithm	MSE/PSNR approach [3]	MSE/PSNR histogram equalization
Church	480 × 320	39	85	136	1229/17.26	1421/16.638	343/22.81
Statue	480 × 320	19	55	145	924/18.508	739/19.478	895/18.645
Male Face	480 × 320	37	128	177	1153/17.546	922/18.517	883/18.705
Seascape	480 × 320	4	17	122	1224/17.657	1261/17.157	1039/17.998

4 Conclusions

The proposed approach may find wide applications in image processing, pattern recognition and computer vision. In particular, contrast enhancement of the image quality represents the first step in image processing. To face this topic, Scientific Community has consolidated two main categories of approach: direct and indirect ones, both producing noteworthy results in several application fields. Starting form the assumption that gray levels structure of an image can be formalized by fuzzy algoritms for contrast enhancement, the presence of fuzziness in the informative content of an image needs *ad-hoc* procedures for elaborating suitable protocols for getting satisfactory performance. In addition, the increasing requirement from non technical experts to hand simple contrast enhancement protocols (for example, medical and paramedical staff in biomedical ambit) it is imperative to tune intelligible algorithms characterized by a low computational complexity (useful for real-time applications). For the reasons mentioned above, the authors have proposed a new geometrical adaptive approach to get the purpose. In particular, has been here presented an indirect fuzzy approach to enhance the contrast, based on statistical-geometrical considerations and entropic formulations, to modify the histogram distribution of the original gray levels of an image: an adaptive setting of the parameters of a properly defined S-function has been done, which reduces the possibility over and/or under enhancement. The experimental results can be considered, both qualitatively and qualitatively, particularly encouraging in sight of further studies in this direction.

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