An L_1-Method: Application to Digital Symmetric Type-II FIR Filter Design

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Abstract In this paper, the design of digital symmetric type-II linear-phase FIR low-pass (LP) and band-pass (BP) filter is formulated using the L_1 optimality criterion. In order to obtain better filter performance we compute the optimal filter coefficients using the L_1-norm based fitness function. The use of L_1 technique in digital filter design applications has the advantages of a flatter passband and high stopband attenuation over other gradient-based filter optimization methods. This technique is applied to optimally design type-II FIR filters. Simulations and statistical analysis have been performed for the 25th order LP and BP filters. It is observed, that the L_1-based filter results is an improved design in comparison with the filters obtained using the equiripple, least-square and window techniques.

Keywords Finite impulse response · L_1-error criterion · Stopband attenuation · Least-square · Window method

1 Introduction

Digital filtering is an important area of research from last few decades. Digital filters are applied in a variety of engineering applications such as, signal processing, communication, control systems and many more. They carry out the process of attenuating some band of frequencies and allow some frequencies to pass through them. The digital filter is implemented by the discrete convolution of input signal and filter coefficients. These are classified as: Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) [1], [2]. In this work, we intend to design the FIR filter with optimal filter coefficients using the L_1 algorithm explained below.

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FIR filter design, being an approximation problem, determines the filter solution by approximating the frequency response of designed filter to the ideal response. Such approximation techniques involves, the least-square method [3, 4], equiripple design [5, 6], windowing techniques, frequency-sampling method and maximally-flat design. The most applied techniques are based on the L_2 (least-square) [7] and L_∞ (minimax) norms. In L_2-norm based filters the stopband attenuation (A_{stop}) is high with flat passsband on the cost of high overshoot at the discontinuity of ideal function. Whereas, the L_∞-norm based filters yields equal magnitude ripples in both passband and stopband.

In 2006, Grossmann and Eldar proposed a new method for the filter design, based on the L_1-norm [8]. The linear phase FIR filters are designed exploring the problems of differentiability and uniqueness of solution associated with the L_1 approach [9, 10, 11]. Considering type-II, the designed L_1 filters possess a flat passband and stopband with high A_{stop}. On its comparison with other techniques, the L_1 filter features with a higher A_{stop} than L_2 and L_∞-norms. It also eliminates the drawback of the high overshoots at the point of discontinuity. Thus, implementing the L_1 method provides a better solution in the field of filter design [12].

In this paper, the optimal LP and BP FIR filters are designed with type-II filter response using the L_1-method so as to obtain a symmetric even length filter with high A_{stop} and a flat passband response. The purpose of designing type-II filter is to obtain an even length filter which are necessary to be implemented in some applications and are not possible to be designed using the generalized type-I filter response. The obtained results are compared to the type-II equiripple filters, least-squares design and with filters designed using the kaiser window.

The rest of the paper is organized as follows: In Section 2, the framework of type-II filter design problem in mathematical formulated. The problem specific employed L_1 algorithm is described in Section 3. The simulation results and analysis are presented in Section 4. Finally, Section 5 concludes the paper.

2 Problem Formulation

In this section, the problem of designing the FIR filters with symmetric even length (N) impulse response (Type-II) is considered. The amplitude response of such filters has zero magnitude at $\omega = \pi$. Due to this, the design of high-pass and band-stop filters is not possible with type-II frequency response. In this paper,the design of type-II LP and BS FIR filters using the L_1 method is proposed. Here, the design problem is considered as an optimization problem where the frequency response of type-II filter, $H(\omega)$ is approximated to the ideal frequency response, $H_{\mathrm{id}}(\omega)$. The ideal response for the LP and BP filters are given as

$$H_{\mathrm{id_{LP}}}(\omega) = \begin{cases} 1, & \omega \in [0, \omega_c] \\ 0, & \omega \in (\omega_c, \pi] \end{cases} \tag{1}$$

and

$$H_{\mathrm{id_{BP}}}(\omega) = \begin{cases} 1, & \omega \in [\omega_{c_1}, \omega_{c_2}] \\ 0, & \omega \in [0, \omega_{c_1}) \cup (\omega_{c_2}, \pi] \end{cases} \tag{2}$$

The transfer function of the approximating filter, $H(z)$ is specified as

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \tag{3}$$

The frequency response, derived from the transfer function is written as

$$H(\omega) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n} \tag{4}$$

Eq. (4) is written as

$$H(\omega) = \widetilde{H}(\omega) e^{-j\omega \frac{N-1}{2}} \tag{5}$$

where $\widetilde{H}(\omega)$ is the amplitude response of the filter. Considering Type-II linear phase FIR filter with even length and symmetric coefficient, $\{h(n) = h(N-1-n), 0 \le n \le N-1\}$, the the amplitude response is defined as [2]

$$\widetilde{H}(\omega) = 2 \sum_{n=1}^{M} h[M-n] \cos\left[\omega\left(n - \frac{1}{2}\right)\right] \tag{6}$$

where $M = N/2$. Assigning $b(n) = 2h[M-n]$, $1 \le n \le M$ and writing $\widetilde{H}(\omega)$ as a function of ω and filter coefficients, $\mathbf{b}$ (where $\mathbf{b} = (b(1), b(2), \ldots, b(M))$), we get

$$\widetilde{H}(\omega) = \widetilde{H}(\omega, \mathbf{b}) = \sum_{n=1}^{M} b(n) \cos\left[\omega\left(n - \frac{1}{2}\right)\right] \tag{7}$$

Various fitness function employed for the filter design as an approximation problem are

1. Weighted Least-Squares (LS)

$$\|E(\omega, \mathbf{b})\|_2 = \int_0^{\pi} W(\omega) \left|\widetilde{H}(\omega, \mathbf{b}) - H_{\mathrm{id}}(\omega)\right|^2 d\omega \tag{8}$$

2. Weighted Chebyshev

$$\|E(\omega, \mathbf{b})\|_\infty = \max_{\omega\in[0,\pi]} \left\{ \int_0^\pi W(\omega) \left| \widetilde{H}(\omega, \mathbf{b}) - H_{\text{id}}(\omega) \right| d\omega \right\} \tag{9}$$

3. Weighted L_1-norm

$$\|E(\omega, \mathbf{b})\|_1 = \int_0^\pi W(\omega) \left| \widetilde{H}(\omega, \mathbf{b}) - H_{\text{id}}(\omega) \right| d\omega \tag{10}$$

where $W(\omega)$ is the weighting function and $||.||$ denotes the norm of the function. $E(\omega, \mathbf{b})$ is the error, measured between the approximated filter response, $\widetilde{H}(\omega, \mathbf{b})$ and the ideal response, $H_{\text{id}}(\omega)$, defined as

$$E(\omega, \mathbf{b}) = \widetilde{H}(\omega, \mathbf{b}) - H_{\text{id}}(\omega) d\omega \tag{11}$$

$$= \sum_{n=1}^{M} b(n) \cos\left[\omega\left(n - \frac{1}{2}\right)\right] - H_{\text{id}}(\omega) d\omega \tag{12}$$

The error function given in eq. (10) represents the fitness function to be minimized using the L_1 method. It evaluates the fitness function and optimize the filter coefficients. The employed algorithm for the purpose of FIR filter designing is explained in next section.

3 The L_1 Algorithm

The L_1 optimization technique remained unexplored for many years in the field of filter designing due to the above mentioned reasons. With its implementation for the optimization of filter coefficients, the error function turns out to be solvable for the case of FIR filter design. The motive behind exploring and implementing L_1 optimization is due to the smaller overshoot it yields around the discontinuity as compared with the most efficient techniques, minimax and least-squares [8]. In passband, the L_1 based filter results in a flatter response than least-square which happens to be its most desirable property. The design and optimization of linear phase FIR filters using L_1 technique and its characteristic comparison with the minimax method is being demonstrated in [13].

The linear L_1 approximation method of continuous functions defined over an interval by a finite number of basis functions was proposed in [14]. This algorithm computes the optimal coefficients of basis functions with the use of modified Newton method. This estimate was generalized and the modified Newton method was developed for the calculation of L_1-based filter coefficients in [15]. This method is described here for the design of type-II symmetric FIR filter.

The algorithm applied to formulate the L_1 problem as a linear approximation problem [15] demands for the evaluation of first and second order derivative of the error function defined in eq. (10). The n^{th} component of gradient (first-order derivative) at $\mathbf{b}$ is given by

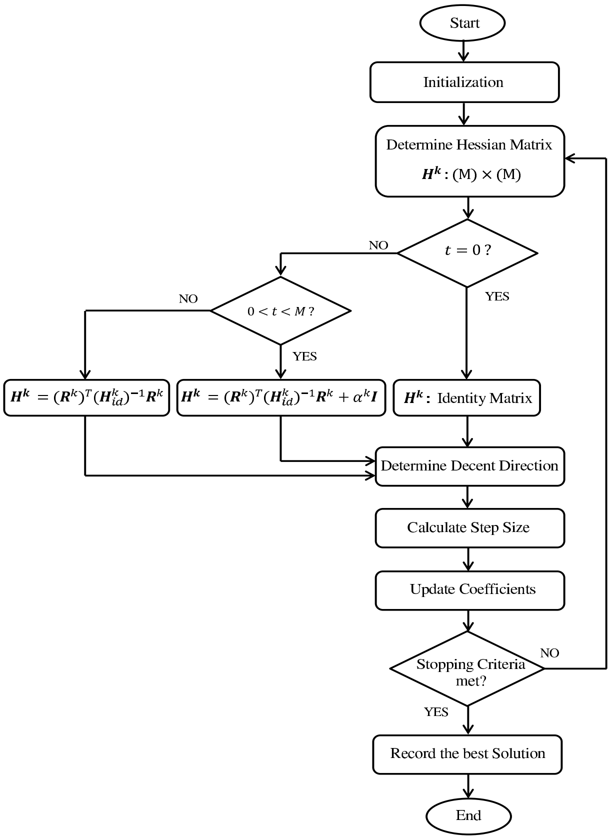

Fig. 1 Flowchart for the L_1 based FIR filter design method.

$$g_n(\mathbf{b}) = \langle \cos(n\omega), \operatorname{sgn}(E(\omega, \mathbf{b})) \rangle \tag{13}$$

where $\operatorname{sgn}(E(\omega, \mathbf{b}))$ gives the Signum result of the function $E(\omega, \mathbf{b})$.

The Hessian matrix (second-order derivative) is computed over the entire digital frequency, which takes one of three forms according to the number of zeros of $E(\omega, \mathbf{b})$ and is given by

$$\mathbf{H}(\mathbf{b}) = \mathbf{R}^T \mathbf{H}_{\text{id}}^{-1} \mathbf{R} \tag{14}$$

here $\mathbf{R}$ is a $t \times M$ matrix with $\mathbf{R}_{ij} = \cos((j - 0.5)z_i)$ and z_i denotes the zero of $E(\omega, \mathbf{b})$ at i^{th} position, equal to $\frac{(2i-1)\pi}{2M}$, $i = 1, 2, \ldots, M$. $\mathbf{H}_{\text{id}} = \operatorname{diag}\{d_1, \ldots, d_t\}$ having $d_i = \frac{2\mathbf{W}(z_i)}{E'(\omega, \mathbf{a})}$.

The modified Newton's method consist of the iterations that generates a sequence of coefficients $\mathbf{b}^k$

Table 1 Algorithm for Filter Design using L_1 norm

Step 1	Design the ideal frequency response defined in (1). Set $M = (N+1)/2$.
Step 2	Calculate initial vector $\mathbf{b}^1 \in \Re^M$, set stopping condition factor, $\epsilon > 0$, step-size selection parameters as $0 < \sigma < 1/2,\ 0 < \beta < 1$ and for the control of Hessian matrix, set $\delta_1 > 0,\ \delta_2 > 0$ and $\mu > 0$. Set $k = 1$ to determine $\mathbf{b}^1$.
Step 3	Determine the Hessian matrix $\mathbf{H}^k$ of size $(M) \times (M)$, based on the value of t. **i.** If $t = 0$ or $\mathbf{H}_{\text{id}}$ is singular, then set Hessian matrix as identity matrix. **ii.** If $t \geq M$, $\mathbf{H}_{\text{id}}$ is non-singular and rank $(\mathbf{R}^k = M)$, then set $\mathbf{H}^k = (\mathbf{R}^k)^T (\mathbf{H}^k_{\text{id}})^{-1}\mathbf{R}^k$. **iii.** If $0 < t < M$, $\mathbf{H}_{\text{id}}$ is non-singular and rank $(\mathbf{R}^k < M)$, then set $\mathbf{H}^k = (\mathbf{R}^k)^T (\mathbf{H}^k_{\text{id}})^{-1}\mathbf{R}^k + \alpha^k \mathbf{I}$, where $\alpha^k > 0$.
Step 4	Determine the descent direction $\mathbf{d}^k$ defined in eq. (16), that obtains the unique solution.
Step 5	Stop if $\lvert(\mathbf{d}^k)^T \mathbf{g}^{\mathbf{k}}\rvert$ is less than given threshold, ϵ.
Step 6	Calculate step-size, α^k detemined using Armijo rule.
Step 7	Set $\mathbf{b}^{k+1} = \mathbf{b}^k + \alpha^k \mathbf{d}^k$ and $k = k+1$. Goto Step 2.
Step 8	The M coefficients are stored and the frequency response of designed Nth order FIR LP and BP filter is calculated.

$$\mathbf{b}^{k+1} = \mathbf{b}^k - \alpha^k [\mathbf{H}^k]^{-1} \mathbf{g}^k \tag{15}$$

assuming that the Newton direction

$$\mathbf{d}^k = -[\mathbf{H}^k]^{-1} \mathbf{g}^k \tag{16}$$

is a descent direction, where $\mathbf{g}^{\mathbf{k}}$ is the gradient of function at $\mathbf{b}^k$, α^k is the step size, determined according to the Armijo rule [16] and $\mathbf{H}^{\mathbf{k}}$ is the Hessian matrix of $\|E(\omega, \mathbf{b})\|_1$. Solving $\mathbf{d}^k$, the descent direction (also called the gradient method), involves the solution of the linear equations with M unknowns (the length of $\mathbf{d}^k$). To reduce these computations, the special structure of the matrix $\mathbf{H}^k$ in eq. (14) is exploited based on the number of zeros of $E(\omega, \mathbf{b})$. This is explained in the steps for the design of FIR LP and BP filter based on L_1 criterion, summarized in Table 1. The process flow chart is pictured in Fig. 1.

4 Simulation Results and Analysis

This section presents extensive simulations performed using the MATLAB v.7.13 platform on intel core(TM) i5 CPU, 3.20GHz with 4 GB RAM for the design of type-II 25th order FIR LP and BP filters.In order to demonstrate the superiority of the proposed design based on L_1-criterion, comparative analysis is carried out with the equiripple, least-square and windowed methods. The design parameters values used in the L_1 algorithms are as follows, $\epsilon = 10^{-6}, \sigma = 10^{-3}, \beta = 0.5, \delta_1 = 10^{-15}$, $\delta_2 = 10^{15}$ and $\mu = 10^{-10}$. The design examples are analyzed below.

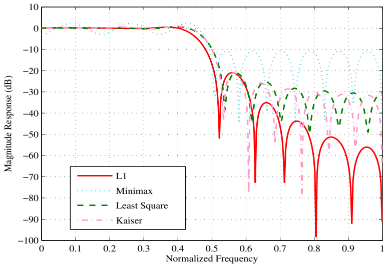

Fig. 2 Magnitude Response (dB) for 25th order FIR LPF.

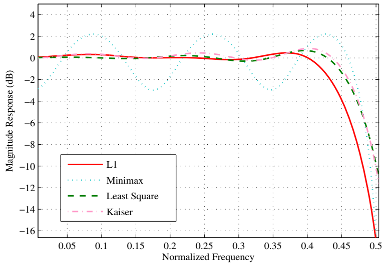

Fig. 3 Enlarged Passband Response (dB) for 25th order FIR LPF.

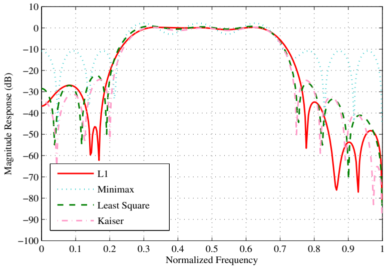

Fig. 4 Magnitude Response (dB) for 25th order FIR BPF.

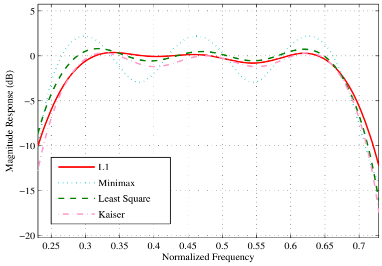

Fig. 5 Enlarged Passband Response (dB) for 25th order FIR BPF.

Filter specification specified are: For LPF, passband frequency, $\omega_p = 0.474\pi$, stopband frequency, $\omega_s = 0.493\pi$ and cut-off frequency, $\omega_c = 0.4835\pi$. For BSF, passband frequency, $\omega_{p_1} = 0.28\pi$, $\omega_{p_2} = 0.68\pi$, stopband frequency, $\omega_{s_1} = 0.23\pi$, $\omega_{s_2} = 0.28\pi$ and cut-off frequency, $\omega_{c_1} = 0.25\pi$, $\omega_{c_2} = 0.7\pi$. The magnitude response in dB of the proposed 25th order LPF and BSF is shown in Fig. 2 and 4, respectively. The response is plotted with the response obtained using already existing design methods, minimax, least-square and windowed method. Table 2 and 3 gives the optimized filter coefficients for all applied methods for the LFP and BSF design, respectively. To carry out the comparison, statistical results are analyzed and reported in Table 4. It is observed from Table 4 that the minimum stopband attenuation incurred with the L_1-method is -21.98 and -34.85, for LPF and BPF, respectively. From the stopband profile of the L_1 based filters, it can be concluded that the highest stopband attenuation is obtained with the proposed design. Furthermore, the passband ripples are 0.4672 dB and 0.3599 dB, for LPF and BPF designs, respectively. These obtained values are least among all the designs which results in the flattest passband with the L_1-based filters. The flatness in passband can be depicted from the enlarged view in Fig. 3 and 5.

Table 2 Optimized coefficients of 25th order FIR LPF filter.

Optimized Coefficients	L_1 Criterion	Minimax	Least Square	Kaiser Window
$h(0) = h(25)$	−0.004304821875159	0.015009154932070	0.003346236484176	0.003353949812098
$h(1) = h(24)$	−0.009945569482198	−0.159733218946615	−0.025832583651239	−0.026384058910212
$h(2) = h(23)$	0.002892064991698	−0.019972687908533	−0.007000222174646	−0.007087353251721
$h(3) = h(22)$	0.021425807571245	0.033344364352309	0.030978162778327	0.031656658601014
$h(4) = h(21)$	−0.004771401539193	0.013239722254575	0.012372854572430	0.012579827879823
$h(5) = h(20)$	−0.039463807888077	−0.040699493740147	−0.038297116225617	−0.039151796729370
$h(6) = h(19)$	−0.003076249234356	−0.021837804969918	−0.020951741171072	−0.021352353618087
$h(7) = h(18)$	0.057415462699534	0.051858820599784	0.050174640982908	0.051307395738706
$h(8) = h(17)$	0.019794351740752	0.037642868368773	0.036871568337463	0.037631954977186
$h(9) = h(16)$	−0.083462683176473	−0.075416760140064	−0.074476032718155	−0.076163795722620
$h(10) = h(15)$	−0.056173676201219	−0.077933990820229	−0.077508700899409	−0.079182787368566
$h(11) = h(14)$	0.179179360425898	0.161468696856027	0.161077894993653	0.164710418330001
$h(12) = h(13)$	0.424754251606035	0.438344605836526	0.440344846920456	0.448081940261748

Table 3 Optimized coefficients of 25th order FIR BPF filter.

Optimized Coefficients	L_1 Criterion	Minimax	Least Square	Kaiser Window
$h(0) = h(25)$	0.013080208237528	0.062876101398056	0.015422936597621	0.024487776955131
$h(1) = h(24)$	−0.005575502498038	−0.116269220084882	−0.015287846989148	−0.005579104536685
$h(2) = h(23)$	−0.032670100914959	−0.085345777430859	−0.051481378623170	−0.049445991844001
$h(3) = h(22)$	0.001530829610940	−0.039914129234188	0.001827636127389	−0.000997752658279
$h(4) = h(21)$	−0.011134308330060	−0.001174900404477	−0.011267776603452	−0.018418438692279
$h(5) = h(20)$	−0.018306785726551	−0.005719565444857	−0.002754812073954	−0.012640620625559
$h(6) = h(19)$	0.075542485873412	0.081156678895356	0.093059429848611	0.086412261623438
$h(7) = h(18)$	0.032384920124126	0.042306512734840	0.022096288818837	0.025221316879272
$h(8) = h(17)$	−0.007921998215940	−0.024729352404448	−0.01246705795282	−0.004696219641479
$h(9) = h(16)$	0.048396103447932	0.041439930403364	0.044780368623445	0.051389148435767
$h(10) = h(15)$	−0.195302535995375	−0.196488532016674	−0.210850541558666	−0.194403327017152
$h(11) = h(14)$	−0.221064614224406	−0.241391281601754	−0.223813361579707	−0.214944678903213
$h(12) = h(13)$	0.328305105982610	0.339512411966533	0.331916468646392	0.303651115520570

Table 4 Statistical results for the 25th order FIR LP and BP filter.

Filter	Method	Stopband Attenuation (dB)				Passband ripple
		Minimum	Mean	Variance	Standard deviation	(dB)
Low-Pass	L_1 Criterion	-21.98	-24.8977	-35.1392	-17.5556	0.4672
	Least-Square	-21.14	-23.2482	-35.9720	-17.9926	0.6806
	Kaiser Window	-21.04	-23.7017	-35.7562	-17.8626	0.8746
	Minimax	-10.80	-14.1637	-38.1315	-19.0467	2.2081
Band-Pass	L_1 Criterion	-34.85	-20.1843	-30.4865	-15.2441	0.3599
	Least-Square	-25.88	-20.7526	-31.8352	-15.9122	0.7977
	Kaiser Window	-24.62	-21.0611	-31.6042	-15.8043	0.3232
	Minimax	-10.82	-13.2948	-36.1933	-18.0896	2.1980

5 Conclusion

In this paper, the efficient design of type-II FIR LP and BP filter using the L_1-method is presented. The requirement of even length filters for specific application can be fulfilled using the proposed type-II filter design. The performance assessment for the designed filters is expressed in terms of minimum stopband attenuation and highest passband ripple. The obtained results with the L_1-method attained highest stopband attenuation and the passband with least ripples as compared to the renowned minimax, least-squares and windowed method. This method can be applied to design 2-D filters to enhance their applicability in other fields of engineering like image processing. In addition, the method can also be used to design digital differentiator and Hilbert transformer.

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