

A Two Step Procedure for Integrated Inventory—Supply Chain Management Information Systems

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Abstract In this work we present a two step procedure aimed at integrating inventory and distribution functions for balancing stock levels in distribution systems. In particular, we analyse the flow of products within a multi-echelon, multi-channel distribution network, with the aim of minimizing logistic costs. The key issue of the present paper is that cost minimization is searched whilst granting a certain customers' service level, here expressed in terms of percentage of fulfilled demand. Moreover, the present paper focus on the gain that company should derive finding balanced stock levels in the whole network, that is the gain derived by using integration network management models and integrated inventory—supply chain management information systems. This integration is possible only if inventories' information related to the whole network are available. Results of computational experimentations aimed at comparing different inventory management policies are presented.

Keywords Integrated inventory management · Distribution systems · Customer satisfaction · Role of information sharing in the integration functions

1 Introduction

Few years ago, in a more and more competitive and aggressive market, companies developed the logistic function, devoted to manage the flows of information and goods in the logistic system, in order to improve the customer service level and control the logistic costs. Nowadays, these companies have to reorganise their

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logistic and supply chain management systems in order to meet changes and flexibility, and, above all, guarantee a high level of service as a key factor for being competitive. Information sharing in the whole chain is a key factor for meeting flexibility. Moreover, the introduction of the Electronic Commerce (EC) has induced changes and problems arising in distribution channels, that are completely new and impact on the increasing customer service expectations [4].

A review of supply chain management operations in a multi-channel distribution with EC channel is presented in [1], where different managerial planning tasks for the activities involved at each level of the supply chain are reported, together with the corresponding quantitative models; some strategies for the inventory management are also described. In [5] a survey on supply chain management literature that focuses on the innovative measures of Quick Response (QR) is presented.

High quality services and cost minimization are imperative goals for competitive supply chains [6]. All over the world customers pay more and more attention to the intangible value of products. Moreover, customers ask that distribution costs should not negatively impact on the price of the products.

Distribution activities connected to the customer service level and the logistic costs are, among others, the orders' management, the inventory and storage management, the material handling, and the transportation of goods. Related to the above logistic activities, companies face logistic costs involving: transportation costs, warehousing and inventory costs, and stock out costs related to impossibility of completely satisfying the demand.

Companies should redesign the optimal inventories allocation in the distribution system in such a way to avoid an uncontrolled growth of costs and the presence of overstocks in the warehouses for maintain enough inventories to satisfy customers' demand; in particular, as stressed in [5], the fundamental task is to balance the stock levels at the top and the bottom echelons. In [16] and, more recently, in [14] control rules for minimizing unbalanced stock levels are proposed. In a recent paper [17] three different inventory strategies for one manufacturer, one retailer supply chain with both a traditional channel and a e-channel are compared.

Motivated by the above considerations, in this work we devote our attention to the integration of inventory and distribution management functions in a multi-echelon, multi-channel distribution system with the main aim of balancing stock levels in the whole network. Some real multi echelon distribution systems are described in [2].

Articles dealing with similar problems generally concern simple networks (i.e. tree systems with 2 levels or n -echelon serial systems) in which demand points are usually in the last level of the network. Inventories are often included only in the facilities operating at the lower level of the network, that is at the peripheral depots. Many papers dealing with integrated inventory management have as objective function the minimization of the distribution costs and take as decision variables the order points of each facility in the network.

More precisely, we analyze the management of inventories of final goods in a distribution system where products are available in different supply channels, that is a traditional channel, in which the products are distributed through depots, and a direct channel. We describe and compare different inventory management policies in order to analyse the integration between inventory and distribution management functions in the network. We present a two phase procedure aiming at integrating in the same framework inventory and distribution functions thanks to the information sharing.

A novel contribution of this paper is that it focuses on customer value, while many supply chain management systems focus on topics like economics quantity order, and good issues. At least to the authors' knowledge, only few papers refer to the maximization of the customer service level. For instance, in [13] the authors analyze the effect of target service level in supply chains, while in [16] the authors aim at maintaining a specified customer service level, expressed as in our case as a percentage of fulfilled demand, by looking for optimal goods allocation and defining rationing policies. In a recent work [15] a simulation-optimization approach for solving a 2-echelon inventory problem with service level constraints is proposed.

Moreover, the present paper focus on the gain that company should derive finding balanced stock levels in the whole network, that is the gain derived by using integration network management models and integrated inventory—supply chain management information systems. This integration is possible only if inventory's information related to the whole network are available. Some analysis on the private and global information in the inventory management are reported in [7, 8]. In [10] the value of real time information in inventory management is analysed. Finally, in [12] is stressed the importance of coordination and information sharing in the systems for control policies and integrated models.

In a logistic system where global information are available, the concept of inventory generally refer to the echelon stock (that is inventories at the global system), while if only local information are available commonly a different concept of inventory is used: the inventory position or installation stock (i.e. inventories at each stock point). The echelon stock concept was proposed in [9]; this work is generally considered the first attempt of introducing integration of the inventory management in distribution networks.

The organization of the remaining of the paper is the following. In Sect. 2 we describe in details the problem under investigation, focusing on the flow of goods in the multi-echelon, multi-channel distribution system we are involved with; the main characteristics of inventory policy used are also described. The proposed two phase procedure aimed at integrating the inventory and distribution functions is described in details in Sect. 3, together with the mixed integer linear programming (MILP) model used for determining a starting solution for the problem under investigation. Preliminary results referred to a multi-channel distribution network are given in Sect. 4. Finally, conclusions and outlines for future works are given in Sect. 5.

2 Problem Definition: A Multi-echelon, Multi-channel Distribution System

The supply chain network under investigation is a multi echelon, multi channel distribution system, in which there is a flow of final products from the plants (where they are produced) to the demand points, generally called customers. The network is characterized by the presence of central depots (D), peripheral depots (P) and suppliers, that in turn are split into clients, that is wholesalers (C), and big clients, that is distributors or retailers (B). In the following, plants are not included in the analysis since central depots play the role of supply points of the network.

The following channels for supplying goods are considered:

- a traditional channel, where peripheral depots (supplied by the central ones) serve customers;
- a direct channel, for serving big clients characterized by large demand, thus served directly by the central depots.

As an example of such logistic network, let us report in Fig. 1 a simple distribution system with 2 central depots (D), 3 peripheral depots (P), 4 big clients (B) and a set (C) of other customers. Note that links (arrows) in the network represent the flow of goods from depots to clients and from central depots to peripheral ones. Such links are usually predefined but, as we will see in the next section, we consider the possibility of changing the given flow assignment in order to get balanced stock levels.

We assume that balanced stock levels imply the same inventory level at each peripheral depot, for each product, in terms of number of days of stock, while a higher stock is maintained at the central depots.

Referring to Fig. 1, the central depots ($D1$ and $D2$) serve directly the peripheral depots ($P1$, $P2$ and $P3$) and the big-clients ($B1$, ... $B4$). Inventories are stocked both at the central and peripheral depots. The assignment of the peripheral depots and big clients to the central depots is known, as well as the assignment of the clients to the peripheral depots.

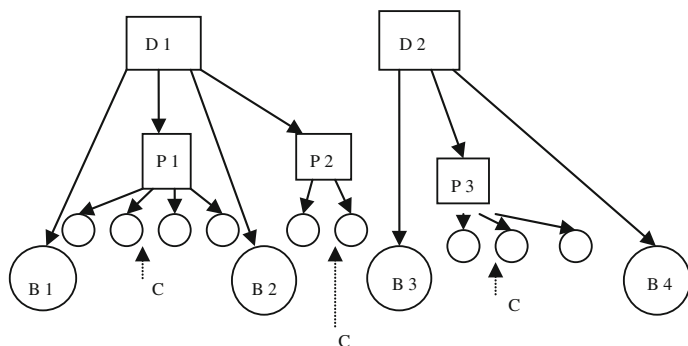


Fig. 1 The distribution network architecture under investigation

Having in mind the above distribution system, assuming a time horizon T split into t homogeneous periods ($T = \{1, 2, \dots, t\}$), and given the customers' demand for each time period t , the problem is to determine the optimal flow of goods in the network and the inventory levels to maintain at each central and peripheral depot for each time period $t \in T$; this implies to decide the emission order time and the quantity that each depot has to order. The capacity of depots and customer service level constraints have to be satisfied.

The objective is the minimization of ordering, inventory, stock out and transportation costs.

We assume that the customer service level, that represents one important parameter for checking the performances of the distribution system, is expressed as percentage of fulfilled demand.

Moreover, we assume to operate in a centralised control system based on global information. The centralized control allows changes in the inventory policy by modifying the flows of goods in the network in order to avoid stock out.

Our inventory policy is based on a periodic (daily) review policy in which goods are ordered when inventories are under a given level, the so-called ordering point; the quantity to order is defined for restoring inventories while minimising the logistic costs and depends on the existing stock in the whole system and, consequently, on the used inventory strategy.

Some stock controls are used for finding the best inventory strategy to use among a base stock policy, a rationing strategy as proposed in [11] and a basic stock policy modification as suggested in [3].

3 The Proposed Two Phase Procedure

The proposed two phase algorithm for solving the problem described above is now presented.

In the first phase, taking into account the logistic network under consideration and the existing assignment of the peripheral depots (P) and big clients (B) to the central depots (D), we decompose the problem into $|D|$ sub problems, thus we define the optimal flows and inventory policy by solving a Mixed Integer Linear Programming model for each central depot of the network and its sub-network. In this phase the amount of information available is considered in the definition of the echelon stock level and echelon inventory position at the central depots.

In the successive phase, denoted "integration" phase, we determine the "current stock situation" of the whole network, thus identifying the best transferring policy for managing the flow of goods and granting the higher possible customer service level. Note that in this phase an information system able to provide to all central depots of the network real time information related to the exact stock and inventory position of peripheral depots is a crucial element.

After checking if the inventory and distribution policies obtained by solving the $|D|$ MILP models are adequate with respect to the overall current stock situation,

different instruments for managing the flows of goods in the network and maintaining balanced stock levels in all depots are used. In particular, first the flows are defined by solving $|D|$ MILP models (base stock policy), otherwise the current assignment of peripheral depots (P) and big clients (B) to central depots (D) can be discussed and, finally different stock policies (i.e. the basic stock policy modification and the rationing strategy) can be used.

Let us describe in more detail the two phases of the proposed solution approach. Note that, when describing the following procedure we will refer to a representative product; anyway, the model and the other step of the procedure can be extended to include multi-products (i.e. by defining different stock levels for each product and so on).

3.1 Phase 1: Definition of Flow of Goods and Inventory Levels by Using a MILP Model

In this phase, referring to a time horizon T , we define the optimal flows in the considered network and the inventory level for each stock point (i.e. for each D and P).

Before presenting the model, let us give the required notation.

For each central and peripheral depot j , $\forall j \in D \cup P$, the following quantities are known:

| | |
|--------------|---|
| l_j | lead time for depot j ; |
| k_j | capacity of depot j ; |
| s_j | service level of depot j ; |
| o_{jt} | order point of depot j at period t , $\forall t \in T$; |
| c_j^o | fixed ordering cost of depot j ; |
| c_j^w | warehousing cost (for unit of inventory and for unit of time) of depot j ; |
| c_j^s | stock out cost (for unit of demand and for unit of time) of depot j ; |
| I_{j0} | the stock level of depot j at the beginning of the time horizon; |
| Q_{jt-l_j} | the quantity ordered by depot j in the previous $ l_j $ periods of time, with respect to the beginning of the time horizon. |

Moreover, for each period of time t , for each big client and for each peripheral depot i , $\forall i \in B \cup P$, $\forall t \in T$, are known:

| | |
|---------------|--|
| d_{it} | demand of big client/peripheral depot i , in period t ; |
| c_{dt}^t | transportation cost from central depot d to big client/peripheral depot i , $\forall d \in D$; |
| γ_{di} | the assignment of big client/peripheral depot i to central depot d , $\forall d \in D$ (i.e. $\gamma_{di} = 1$ if i is assigned to central depot d , 0 otherwise). |

The decisions, in each time period t , are related to: *the ordered quantity and stock out of each depot*:

$$\begin{aligned} Q_{jt} \geq 0 & \quad \text{ordered quantity of depot } j, \text{ in time period } t, \forall j \in D \cup P, \forall t \in T; \\ \beta_{dit} \geq 0 & \quad \text{stock out of central depot } d \text{ with respect to big client } i, \text{ in time period } t, \\ & \quad \forall d \in D, \forall i \in B, \forall t \in T; \\ \beta_{jt} \geq 0 & \quad \text{stock out of peripheral depot } j, \text{ in time period } t, \forall j \in P, \forall t \in T. \end{aligned}$$

When a depot orders a positive quantity $Q_{jt} > 0$, it has to pay a fixed ordering cost and the following binary decision variables are needed:

$$y_{jt} = \begin{cases} 1 & \text{if depot } j \text{ issues an order in time period } t \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in D \cup P, \forall t \in T;$$

the stock level and inventory position of each peripheral depot:

$$\begin{aligned} I_{jt} \geq 0 & \quad \text{stock level of peripheral depot } j \text{ in time period } t, \forall j \in P, \forall t \in T; \\ IP_{jt} \geq 0 & \quad \text{inventory position at peripheral depot } j \text{ in time period } t, \forall j \in P, \forall t \in T; \end{aligned}$$

the echelon stock level and echelon inventory position of each central depot:

$$\begin{aligned} I_{dt}^{ech} \geq 0 & \quad \text{echelon stock level of central depot } d \text{ in time period } t, \forall d \in D, \forall t \in T; \\ IP_{dt}^{ech} \geq 0 & \quad \text{echelon inventory position at central depot } d \text{ in time period } t, \\ & \quad \forall d \in D, \forall t \in T. \end{aligned}$$

The proposed Integrated Inventory Management (IIM) model can be now given as follows. Min

$$\begin{aligned} \sum_{t \in T} \sum_{j \in D \cup P} c_j^o * y_{jt} + \sum_{t \in T} \sum_{d \in D} c_d^w * I_{dt}^{ech} + \sum_{t \in T} \sum_{j \in P} c_j^w * I_{jt} + \sum_{t \in T} \sum_{j \in P} c_j^s * \beta_{jt} + \\ \sum_{t \in T} \sum_{d \in D} \sum_{i \in B} c_d^s * \beta_{dit} + \sum_{t \in T} \sum_{j \in P} c_{dj}^l * Q_{jt-l_j} + \sum_{t \in T} \sum_{d \in D} \sum_{i \in B} c_{di}^l * (d_{it} - \beta_{dit}) \end{aligned} \quad (1)$$

subject to

$$My_{jt} - Q_{jt} \geq 0 \quad \forall j \in D \cup P, \forall t \in T \quad (2)$$

$$I_{dt-1} + Q_{dt-l_d} - \sum_{j \in P} \gamma_{dj} Q_{jt} - \sum_{i \in B} \gamma_{di} (d_{it} - \beta_{dit}) = I_{dt} \quad \forall d \in D, \forall t \in T \quad (3)$$

$$I_{jt-1} + Q_{jt-l_j} - (d_{jt} - \beta_{jt}) = I_{jt} \quad \forall j \in P, \forall t \in T \quad (4)$$

$$I_{dt} + \sum_{j \in P} \gamma_{dj}(I_{jt} + Q_{jt}) = I_{dt}^{ech} \quad \forall j \in D, \forall t \in T \quad (5)$$

$$I_{dt}^{ech} + Q_{dt} - \sum_{j \in P} \gamma_{dj}\beta_{jt} - \sum_{i \in B} \gamma_{dj}\beta_{dit} = IP_{dt}^{ech} \quad \forall j \in D, \forall t \in T \quad (6)$$

$$I_{jt} + Q_{jt} - \beta_{jt} = IP_{jt} \quad \forall j \in P, \forall t \in T \quad (7)$$

$$IP_{dt}^{ech} \geq o_{dt} \quad \forall j \in D, \forall t \in T \quad (8)$$

$$IP_{jt} \geq o_{jt} \quad \forall j \in P, \forall t \in T \quad (9)$$

$$I_{jt} \leq k_j \quad \forall j \in DUP, \forall t \in T \quad (10)$$

$$\frac{d_{jt} - \beta_{jt}}{d_{jt}} \geq s_j \quad \forall j \in P, \forall t \in T \quad (11)$$

$$\frac{(d_{it} - \beta_{dit})}{d_{it}} \geq s_d \quad \forall d \in D, \forall b \in B : \gamma_{dj} = 1, \forall t \in T \quad (12)$$

(1) is the objective function of the proposed model minimizing the four main cost components of our problem that is the ordering, warehousing, stock out and travelling costs.

Constraints (2) set binary variables y_{jt} to 1 if a positive quantity Q_{jt} is ordered in a depot of the network. Equations (3) and (4) define, for each time period t , the stock level of the central and peripheral depots, respectively. Note that the main difference between (3) and (4) is due to the fact that central depots play a dual role; in particular, as it has been already said, central depots have to serve both customers and peripheral depots.

Equations (5) and (6) define the echelon inventory position of the central depots, while (7) are related to the inventory position of the peripheral ones.

Constraints (8) and (9) concern the control of the stock level to maintain at each stock point, and force the inventory echelon position and the inventory position to be greater or equal than the established order point for central and peripheral depots, respectively.

(10) are the capacity constraints of the stock points of the network.

Finally, (11) and (12) are the customer service level constraints and impose that the percentage of satisfied demand of the depots and big clients must be greater or equal to a given predefined quantity expressed by the service level.

This model implies that information sharing is performed in the network. Otherwise, if no information are available, in the model only the stock level can be used instead of echelon stock and echelon inventory position.

3.2 Phase 2: The Integration Phase

This phase is aimed at verifying whether the solution of model (1)–(12), defining the optimal inventory allocation in the whole distribution system, is consistent with the current stock level in the whole network and thus, at defining the best inventory strategy according to the current global stock level, with the goal of avoiding unbalanced inventories at the different echelons. In fact, note that even a shortage in a part of the network may need to modify the optimal inventory allocation in the whole distribution system.

At the end of *phase1* the inventory manager of the network knows the global amount of goods that have to leave each central depot for serving peripheral depots and big clients. These quantities represent the out flow of these depots and are the result of a base stock policy obtained by solving model IIM.

The following main steps describe *phase2*, called the integration phase.

Step 1: identification of local stock out *for each central depot*.

If the existing stock level is greater than the out flow, the *base stock policy* is used.

Else: the inventory policy is re-determined by solving model IIM for the whole network, i.e. in the new model the assignment of peripheral depots and big clients to the central depots is a decision to take (we will refer to this new model as IIM-A).

At the end of Step 1, the inventory manager of the network knows the quantities to transfer from each central depot to the peripheral depots and to big clients assigned to it. These quantities represent the out flow of central depots and are the result of a base stock policy with new assignments. The new assignments guarantee a better distribution of goods in the network.

Step 2: identification of possible global stock out *in the network*.

If the global amount of inventories existing at the top level of the network is sufficient to meet the demand of the whole network (total out flow), the *base stock policy with the new assignment* is used.

Else: a modification of the basic stock policy [3] is used and a notification of the existing stock level is sent to the production function.

The *basic stock policy modification* is obtained by reducing the order point of each depot, that is by solving the model IIM-A in which a “minimum” order point (o^{min}_{jt}) (we will refer to this new model as IIM-A(o^{min}))

In this way, the demand of peripheral depots decreases while inventories are taken at the top echelon of the network.

Step 3: identification of emergency global stock out *in the network*.

If the global amount of inventory existing at the top level of the network is enough for satisfying the requirements of the whole network resulting by the solution of IIM-A(o^{min}), the *basic stock policy modification* is used.

Else a *rationing policy* [11] is used, that is each peripheral depot with a positive demand in the time period under investigation will receive a quantity defined in such a way that each depot has the same days of coverage (balanced distribution).

4 Results of the Application of the Two Phase Procedure

We use the solution approach described above for solving distribution and inventory problems on different networks and for evaluating new distribution strategies.

The experimental tests are based on a distribution network made up of 2 *D*, 10 *P*, 30 *B*, and 100 *C* already assigned to *P*. The time horizon is three weeks split into time periods of one day. The feasible flows of goods in the network have been described in Fig. 1. The demand of the customers of the network presents a constant trend during the considered time horizon, and the demand of big clients *B* represents the 20 % of the global demand of the network.

We simulate different scenarios by assuming different initial stock situations and customers' demands. In particular, referring to the initial stock situation we consider a standard scenarios (St.S.), in which the initial stock situation is coherent with the demand of the network, and a critical scenarios (Cr.S.), in which the initial stock situation is not enough for satisfying the demand of the network.

For the considered scenarios we compare costs and inventory levels obtained by using two above mentioned concepts of inventory in the MILP model (i.e. the inventory *I*. and the echelon stock *Ech.*).

In Fig. 2 some graphs are reported. These graphs are related to the partition of the logistic costs in the different cases analyzed. In the last row the total logistic costs are indicated. These costs are obtained by solving the MILP models.

It can be noted that when referring to the echelon stock concept the warehouses costs are lower than in the case of the inventory concept is used, while the ordering costs have an opposite trend.

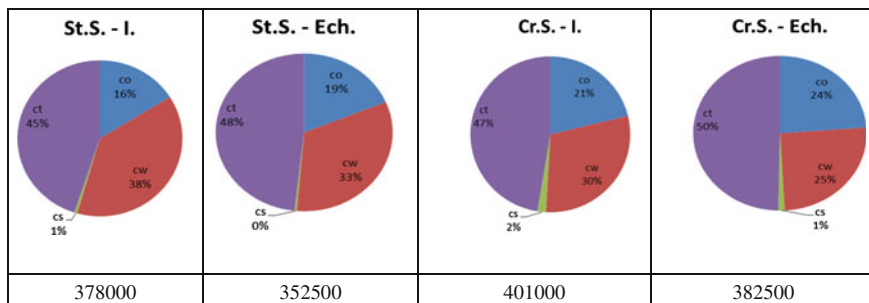
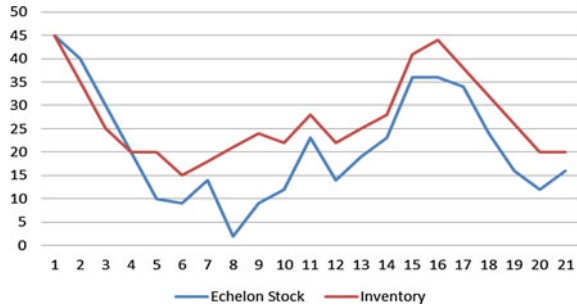


Fig. 2 Partition of logistic costs in the analysed different scenarios

Fig. 3 Comparison of inventory levels at central depots



Another difference concerns the stock levels maintained at central depots during the time horizon, as reported in Fig. 3. We noted that, when referring to echelon stocks, that is in the case of integrated inventory management obtained thanks to information sharing, costs decrease on average of 7 %.

Referring to the customers' demand we consider 2 other scenarios that differs for the % of demand of customers *C* (i.e. served by *P*) and of big clients *B* (i.e. served directly by *D*). Starting from a standard initial situation (St.S.), using the echelon stock concept (Ech.), in Fig. 4 the graphs compare the results obtained in the following cases: 100 %*C*-0 %*B*, 80 %*C*-20 %*B* and 60 %*C*-40 %*B*. The greater presence of big clients *B* in the network involves higher ordering, warehouse costs for central depots *D*, while the total ordering and warehouses costs are lower. Also the total transportation costs decrease.

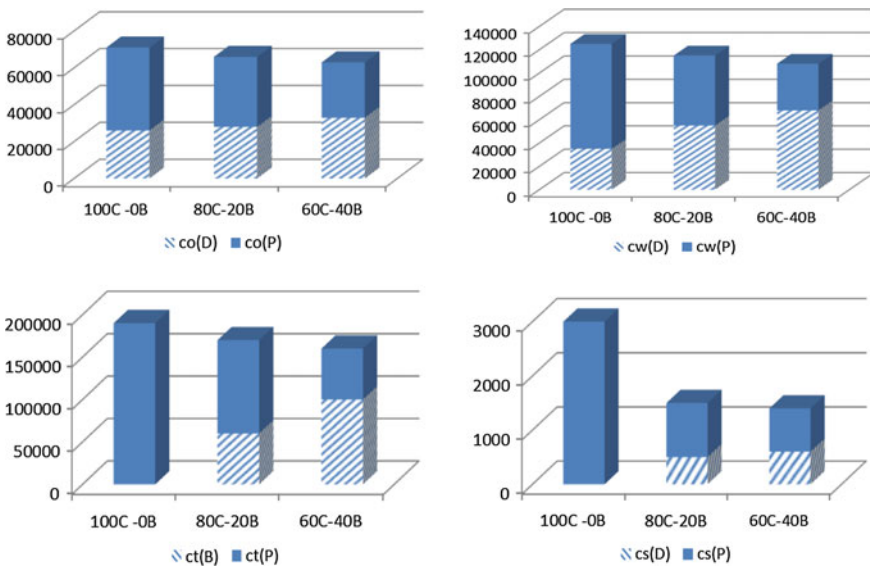


Fig. 4 Comparison of ordering, inventory, transportation, stock out, costs in case of different partitions of customers between *B* and *C*

It is also interesting to note the difference in the distribution of inventories among layers (i.e. at the central and peripheral depots) due to the presence of different percentage of big clients with respect to the total demand of the clients of the network.

Having in mind the dual role played by central depots, we can note that big clients has a positive effect on inventories taken at the central depots for supplying the depots at the lower level of the network; consequently, also inventories in the whole network results lower when an higher number of big clients is present in the network.

Finally, we evaluate the effect of using different strategies thanks to the information sharing on both total logistic cost and stock out costs. The proposed procedure seems very promising when there is a critical initial situation. In fact, for example when the initial stock situation is critical, fixing equal to 100 % the total cost obtained by solving model MII (*phase1*) i.e. by applying a base stock policy, the 5 % of this cost is related to stock out. At the end of *phase2* the total cost is reduced to 98.3 % with a 0.8 % of stock out costs. In this way we are able to increase the customer service level and generally the manager can act in such a way to return to a standard situation in a lower amount of time: in the analyzed cases, on average, 18–20 days are necessary to come back to a standard situation when a base stock policy is used, while only 1 week is necessary when a rationing policy is used. We have noted that costs increase when the rationing policy is used, since this policy is in favor of the inventory balance for increasing the customers' fulfilled demand percentage, whilst transportation is not optimize.

5 Conclusions and Future Research

In this work we have proposed a procedure aimed at integrating inventory and distribution functions for balancing stock levels in distribution networks; it seems very promising especially for reducing stock out and faster coming back to a normal stock situations. The main limit of this work is the assumption of a known demand. In fact, as generally a certain level of uncertainty characterized the demand (and other data, i.e. lead time), in the next future we will deeply analyze the capability of the proposed method of granting high service level in different market conditions, particularly when there is a greater variability in the demand. The usage of robust optimization for tackling the uncertainty of the demand will be investigated too.

Finally, different strategies for avoiding inventory unbalance could be addressed, such as lateral transshipments among facilities operating in the same level of the network, considering the order point in the proposed model as a decision variable and including production plans in the decision process.

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