


# A Study of Flexible Energy-Saving Joint for Biped Robots Considering Sagittal Plane Motion

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**Abstract.** A flexible ankle joint for biped walking robots is proposed to investigate the influence of joint stiffness on motor's peak torque and energy consumption of the sagittal plane motion during the single support phase. Firstly, an improved model of the inverted pendulum is established, which is the theoretical foundation of the flexible ankle joint. Then the analysis of the analytic method of flexible joint is presented based on the improved model of the inverted pendulum. Finally, dynamic simulations of the flexible joint are performed to examine the correctness of analytic method. The results show that the flexible joint can reduce the joint motor's peak torque and energy consumption. Furthermore, there is an optimal joint stiffness of the flexible system, which can minimum peak torque with reduction of 45.99% and energy consumption with reduction of 51.65%.

**Keywords:** Flexible joint · Biped robot · Joint stiffness · Gait planning · Energy consumption

## 1 Introduction

Biped robot has become one of the research focuses and expected to be a promising locomotion applicable to medical, nursing, and home robot due to its advantages of high adaptability, great kinematic coordination and low energy consumption. It is composed of two legs and one platform (the waist). The function of legs is to provide mobility for the platform, while the platform is the base on which the manipulator and the control system are mounted [1]. The gait planning involves temporal and spatial coordination of different joints during walking which is the basis of stable walking, and the most important and critical issue in gait generation is the consumption of energy during walking. Studies show that the legs of biped robot consume more energy in stance phase than in swing phase [2]. Therefore the extensive improvement of the joint energetic efficiency is an important need to optimize the robot's walking.

Many researches on flexible shoe system for energy-saving have been done in recent years. WABIAN-2R [3] had one additional passive joint for bending toe motion within its foot. The passive joint was selected as the toe joint based on human gait analysis reports in order to walk with heel-contact and toe-off motions in steady

walking. Hideaki et al. [4] proposed a flexible shoe system for biped robots to optimize energy consumption of the lateral plane motion. Li et al. [5] presented a flexible foot design for their humanoid robot, including impact absorption mechanism and a foot posture estimation system. Sreeja et al. [6] presented a technique to achieve stable walking of a biped robot on an uneven terrain and they found that a biped structure with a flexible foot will be able to navigate uneven terrain more effectively than the same with a flat foot.

On the other hand, the tracking control of flexible joint robots has attracted the attention of many researchers. A neural network used by Chris et al. [7] in a direct-adaptive control scheme can achieve trajectory tracking of a (highly) flexible joint robot holding an unknown payload without need for many learning repetitions. Sung et al. [8] proposed a new method for the robust control of flexible-joint (FJ) robots with model uncertainties in both robot dynamics and actuator dynamics. The proposed control system was a combination of the adaptive dynamic surface control (DSC) technique and the self-recurrent wavelet neural network (SRWNN). Naoki et al. [9] proposed an approach of motion stabilization by using laser distance sensors for biped robots with flexible ankle joints, and proposed the vibration control method for avoiding the vibrated Zero Moment Point (ZMP) behavior due to the mechanical resonance. Abdul et al. [10] investigated the energetic effects of knee locking and addition of torsional springs to different joints of a seven-link fully actuated planar bipedal robot. The focus was on the reduction of energy consumption during walking. Nikos et al. [11] proposed the recently developed COMpliant huMANoid COMAN, which was actuated by passive compliance actuators based on the series elastic actuation principle (SEA).

However, the above researches mostly worked on the control methods of biped robot with flexible foot or joint, and only a few studies worked on the flexible joint's influences on biped robot's dynamic parameters. In addition, the sole of biped robot is where contact force generates and the contact force has a direct significant effect on ankle joint. So based on these basic studies, this paper considers the development of a flexible ankle joint to reveal the influences of ankle joint stiffness on ankle joint motor torques and energy consumption during single support phase (SSP).

This paper is organized as follows: Section 2 presents the mechanism of sagittal plane flexible ankle joint and the off-line gait planning process. The analytic method analysis and dynamic simulation analysis of the flexible joint are performed in section 3 to confirm the effectiveness of the flexible ankle joint. Finally, section 4 provides the conclusion and future work.

## 2 Modeling

### 2.1 Mechanism of Flexible Ankle Joint

It is suggested to introduce flexibility at the mechanism joints, which means the deformation between the motor and the joint. For biomechanical systems, it makes no sense, while with robots, it may happen that the transmission between the motor and the corresponding joint is not completely rigid but features some elasticity. Fig. 1

shows the model of the improved inverted pendulum, which is used to explain the concept of energy-saving flexible ankle joint. The joint consists of footplate, spring and mounting plate. The footplate is the base of the joint and is attached on the actual sole of an ordinary biped robot. The joint stiffness depends on the spring stiffness, and the design of spring stiffness is important to realize stable walking. If the stiffness is too large, the ankle joint motor needs too large torque for small deformation of the spring, which can cause more energy consumption. If the stiffness is too small, there will be no difference between flexible joint and rigid joint.

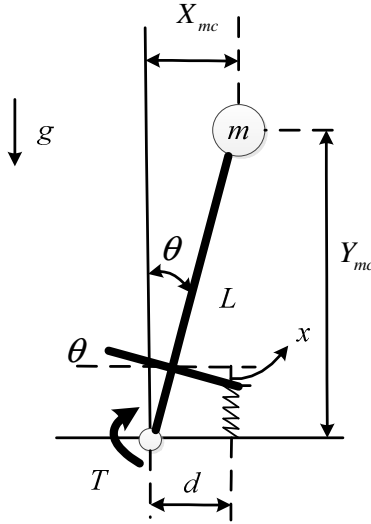


Fig. 1. Improved inverted pendulum model

The basic idea consists of the optimized use of gravity. In the case of the sagittal plane, the joint can absorb the energy that is used to swing the waist forward and backward. The ankle joint of the supporting leg can be locked if the spring stiffness is suitable and the waist motion is presented, naturally introduced by gravitational effect. Based on this improved inverted pendulum model, the Lagrange equation of the second kind can be expressed as:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}(t)} \right) - \frac{\partial L}{\partial \theta(t)} = \tau(t) \tag{1}$$

Where  $\tau(t)$  is the ankle joint motor's torque, and  $L$  can expressed as:

$$L = T - V = \frac{1}{2} m (\dot{X}_{mc}^2 + \dot{Y}_{mc}^2) - mgY_{mc} - \frac{1}{2} kx^2 \tag{2}$$

Where  $x = d \cdot \tan \theta(t)$  is deformation of the spring.

According to equation (1) and (2), the motor's energy consumption  $Q$  is as follows:

$$Q = \int_{\theta_1}^{\theta_2} \tau(t) d\theta \quad (3)$$

It is not easy to justify the introduction of such flexible joint. But the relationship between ankle joint stiffness and motor's dynamic parameters will be presented using this analytic method.

## 2.2 Off-Line Gait Planning Based on ZMP Criterion

### Process of Gait Planning

The concept of the ZMP is useful for understanding dynamic stability and also for monitoring and controlling a walking robot [12]. The ZMP is the point on the ground where the tipping moment acting on the robot, due to gravity and inertia forces, equals zero [13].

Based on the table-carrier model [14], the coordinates of ZMP can be expressed as:

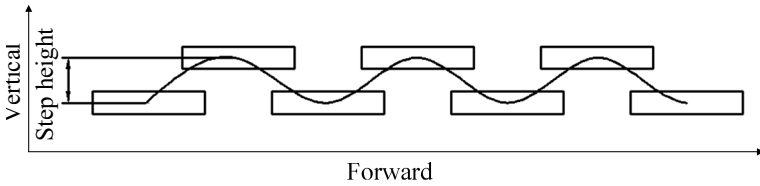
$$x_{zmp} = x - \frac{z}{g} \ddot{x} \quad (4)$$

$$y_{zmp} = y - \frac{z}{g} \ddot{y} \quad (5)$$

The procedures of gait planning based on ZMP criterion include planning the foot trajectory and ZMP trajectory, obtaining the trajectory of COM by ZMP equation, obtaining the motion trajectory of relevant parts according to forward kinematics, obtaining the angular trajectories of different joints and the spatial position of biped robots according to inverse kinematics.

### Foot Trajectory

A walking cycle of human beings is composed of two phases: 1) a single support phase (SSP) and 2) a double support phase (DSP). Biped robots realize movement by alternately treading on the ground. At the moment when the foot contacts the ground, there is an impact force produced on the sole. The higher the vertical impact velocity is, the bigger the impact force is, which affects the walking stability of the robots. To reduce the impact force, the motion trajectory of the foot in vertical plane shall be a sine curve. In this way, the transient velocity of foot is the maximum at the highest position and the minimum when the foot leaves or hits the ground, which can effectively reduce impact force. The motion trajectory of the biped robot's feet in vertical plane is shown in Fig. 2.



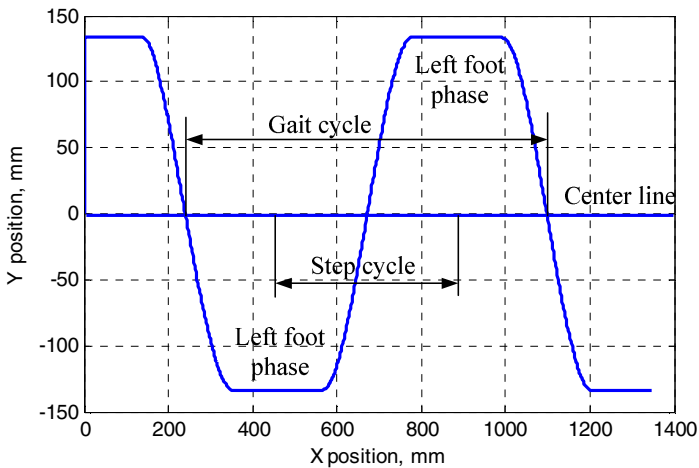
**Fig. 2.** Vertical trajectory of biped robot's feet

### ZMP trajectory

Here forward, lateral and vertical directions were defined as axis X, Y and Z. To facilitate gait planning and analysis, the following assumptions are applied in the gait planning:

- 1) The forward movement of the biped robot is in the positive direction of axis X.
- 2) During walking, the two feet are symmetrical to each other on axis X.
- 3) During walking, the height of the biped robot's COM is constant on axis Z.
- 4) During walking, the sole's plane is parallel to the path.

This paper applied the reference ZMP trajectory as shown in Fig. 3.



**Fig. 3.** Trajectory of reference ZMP

The SSP and DSP alternate incessantly when walking. During the SSP, only one foot is stationary on the ground, and the locomotion system can be regarded as an open treelike kinematic chain. During the DSP, both feet are fixed on the ground, and the locomotion system can be regarded as an over actuated closed kinematic chain. The ZMP response must be within the convex hull of the support polygon which is a prerequisite for stable walking of the robot.

**COM Trajectory**

ZMP equation is discretized applying sampling time  $\Delta t$  [15]. Where

$$\ddot{x}_i = \frac{x_{i-1} - 2x_i + x_{i+1}}{\Delta t^2} \tag{6}$$

$$\ddot{y}_i = \frac{y_{i-1} - 2y_i + y_{i+1}}{\Delta t^2} \tag{7}$$

After discretization, ZMP equation is:

$$x_{zmp} = a_i x_{i-1} + b_i x_i + a_i x_{i+1} \tag{8}$$

$$y_{zmp} = a_i y_{i-1} + b_i y_i + a_i y_{i+1} \tag{9}$$

Where

$$a_i = -z_c / (g \Delta t^2)$$

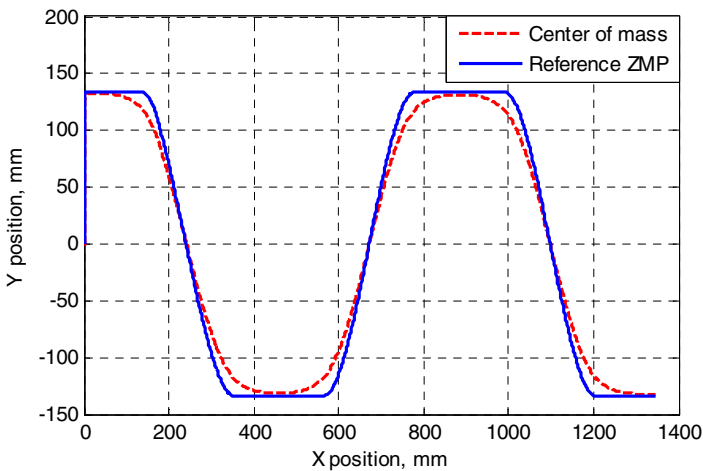
$$b_i = 2z_c / (g \Delta t^2) + 1$$

Combining equation (8) and (9) of time segments 1, 2, ..., N, a matrix form is built and the coordinates of the COM is as follows:

$$x = (A^*)^{-1} x_{zmp} \tag{10}$$

$$y = (A^*)^{-1} y_{zmp} \tag{11}$$

Equation (6)-(11) indicates that if the ZMP trajectories of the biped robot on axis X and Y are given, the trajectory of COM shown in Fig. 4 can be calculated by equation (10) and (11).



**Fig. 4.** Trajectory of COM based on reference ZMP

### 3 Flexible Ankle Joint Analysis

#### 3.1 Analysis of Analytic Method

Considering that the research on left ankle joint during left leg support phase has the similar effect with the right one, this paper takes the left ankle joint as an example. Fig. 5 illustrates the stiffness analysis concerning ankle joint motor's torque according to equation (1) when the angle of joint changes from starting point to maximum point. In this process, every kind of joint stiffness corresponds to a peak value of motor torque, as shown in Fig. 6. If the joint motor's peak torque is too large, impulsive effect will vibrate the robot system, which affects robot's walking stability and shortens motor's life.

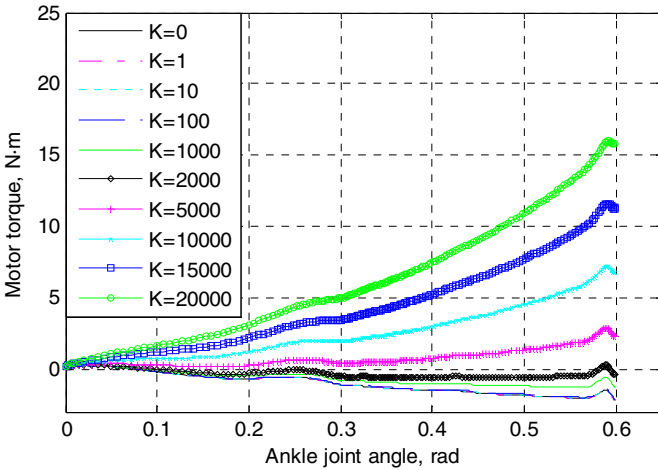


Fig. 5. Ankle joint stiffness and motor's torque

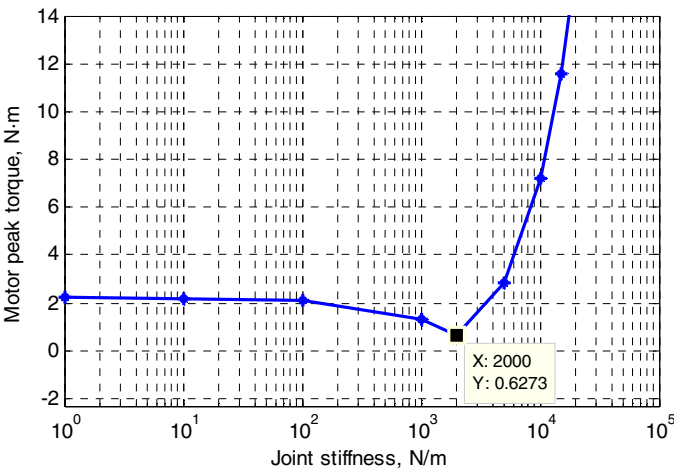


Fig. 6. Ankle joint stiffness and motor's peak torque

Fig. 7 shows the relationship between ankle joint stiffness and motor's energy consumption according to equation (3) and Fig. 5.

It is obvious that ankle joint stiffness has great significance on ankle motor's torque and energy consumption. In the case of optimal ankle joint stiffness, the joint motor's peak torque, average torque and energy consumption become the smallest. In the case of too small joint stiffness, almost 0% sharing (driving torque shared by joint motor and spring) is obtained and ankle motor's peak torque, average torque and energy consumption are not reduced. In the case of too large stiffness, over-sharing has occurred, so the ankle joint's peak torque, average torque and energy consumption increase.

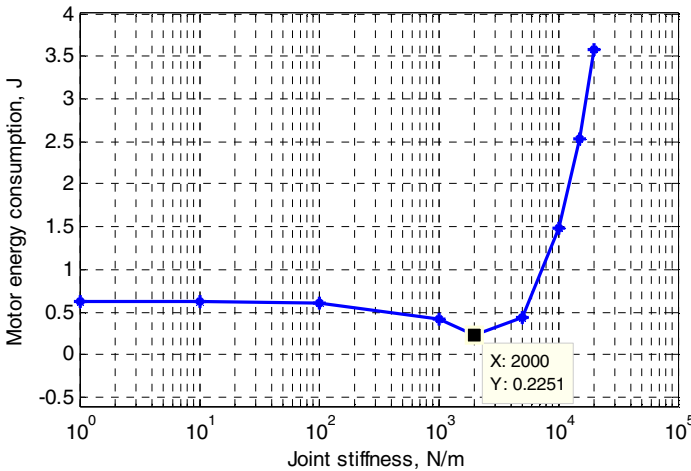
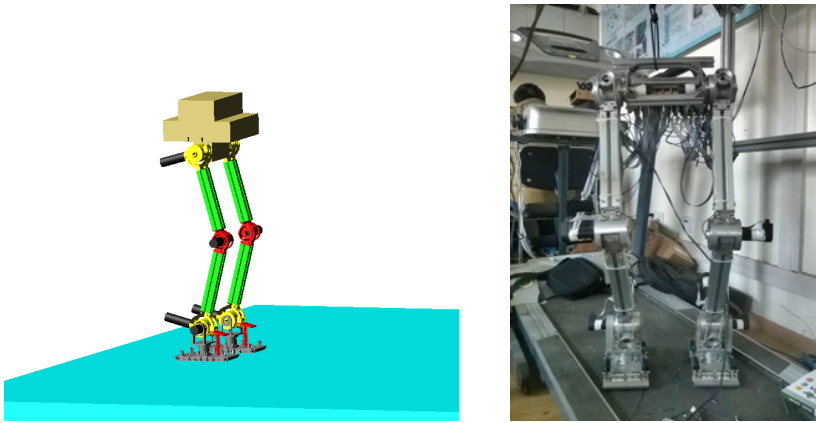


Fig. 7. Ankle joint stiffness and motor's energy consumption

### 3.2 Analysis of Dynamic Simulation

The analytic method has been adopted in section 2.1 to reveal the influences of ankle joint stiffness on motor's torque and energy consumption. In order to evaluate the analytic method, computer dynamic simulations are performed in ADAMS, and the simulation model is shown in Fig. 8 (a). The simulations are based on spatial coordination of different joints. In MATLAB, the discrete values of the joint motions are calculated with the off-line planned gait. In ADAMS, the discrete points are fitted into a curve with cubic spline interpolation function, and a continuous angle-time function is created, which serves as drives of the joints. At the same time, in order to increase the flexible ankle joint adaptability on various walking ground with different stiffness, the ground includes the materials of cement concrete, asphalt and rubber in dynamic simulations.





(a) Visual Prototype model with flexible ankle joint

(b) Prototype of biped robot

**Fig. 8.** Photos of biped robot model

This analytic method (as shown in Fig. 1) assumes that the robot has legs with no weight to clarify the effectiveness of the flexible ankle joint. Besides, the contact between sole and floor is perfect and there are no slips during walking. The gait is expressed as straightforward walking on a level flat floor. Table 1 shows the parameters of the simulation model, and these parameters are decided based on the actual robot prototype displayed in Fig. 8 (b).

**Table 1.** Simulation parameters

Parameter	Value	Parameter	Value
$m$	19.2kg	$g$	$9.8\text{m/s}^2$
$L$	0.6m	$d$	0.06m
Gait cycle	3s	Step cycle	1.5s
Step length	0.2m	Step height	0.05m
Upper leg length	0.35m	Lower leg length	0.35m

In this paper, change the ankle joint stiffness by changing the spring stiffness and create ten types of flexible ankle joint for simulations. For every kind of flexible ankle joint, dynamic simulations are performed on cement concrete ground, asphalt ground and rubber ground. The results just show the research on left flexible ankle joint during left leg support phase. Fig. 9 and Fig. 10 compare the effects of the flexible ankle joints with different stiffness on the motor's peak torque and energy consumption. With the joint stiffness increasing, the variation trend of motor's peak torque and energy consumption in dynamic simulations is similar to the analytic method.

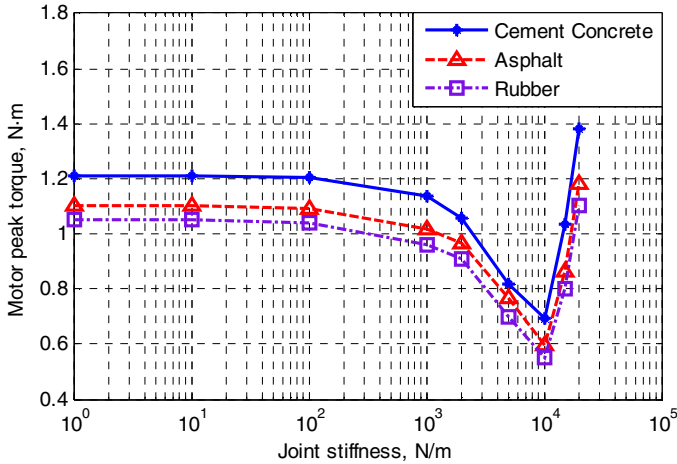


Fig. 9. Ankle joint stiffness and motor’s peak torque in simulations

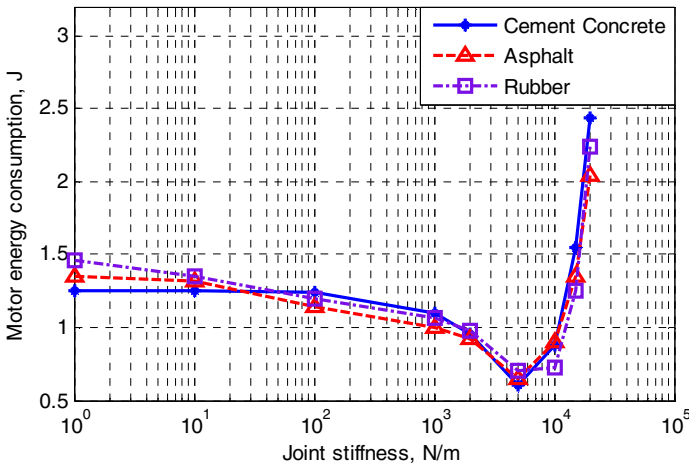


Fig. 10. Ankle joint stiffness and motor’s energy consumption in simulations

In this study, the influences of a flexible ankle joint on motor’s peak torque and energy consumption are investigated both in analytic method and dynamic simulations. As can be seen in the results, the similar variation trend of motor’s peak torque and energy consumption with joint stiffness changing occurs in analytic method and dynamic simulations.

As the joint stiffness increasing, the motor’s peak torque has a minimum value in case of optimal ankle joint stiffness. In analytic method, the optimal stiffness is around 2000 N/m with motor’s peak torque reduction of 71.57% compared with the joint without flexibility. But in dynamic simulations, the optimal stiffness is around 10000 N/m, and the motor’s peak torque reductions on different grounds are shown in Table 2.

**Table 2.** Motor's peak torque reduction on grounds

Material of ground	Peak torque reduction
Cement concrete	42.65%
Asphalt	45.99%
Rubber	47.60%

As the joint stiffness increasing, the motor's energy consumption has a minimum value in case of optimal ankle joint stiffness. In analytic method, the optimal stiffness is around 2000 N/m with motor's energy consumption reduction of 64.02% compared with the joint without flexibility. But in dynamic simulations, the optimal stiffness is around 5000 N/m, and the motor's energy consumption reductions on different grounds are shown in Table 3.

**Table 3.** Motor's energy consumption reduction on grounds

Material of ground	Energy consumption reduction
Cement concrete	52.02%
Asphalt	51.65%
Rubber	51.88%

## 4 Conclusion

This paper proposed a flexible ankle joint considering sagittal plane for biped walking robots. This system is designed to reduce the ankle joint motor's peak torque and energy consumption during the SSP. The effectiveness of this sagittal plane flexible joint is confirmed by an analytic method and dynamic simulations in ADAMS. The results show that the flexible joint can reduce the joint motor's peak torque and energy consumption. Furthermore, there is an optimal joint stiffness of the flexible system, which leads to the minimum values of motor's peak torque and energy consumption.

For future work, we will design a flexible ankle joint system considering lateral plane and discuss the interference between sagittal and lateral plane motion. And we will optimize the walking control strategies based on the flexible ankle joint system.

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